The Weird and Wonderful World of the Qubit

In this section we will introduce you to the qubit. A qubit (pronounced “cue-bit” and short for quantum bit) is the physical carrier of quantum information; in other words, It is the quantum version of a bit, and is how quantum computing is done.

By the end of this section you will have a basic understanding of

- Qubits and their measurements
- Excited states
- Superpositions of qubit states
- The Bloch sphere representation of a qubit
- Decoherence

The Quantum Bit (Qubit)

In this section, get ready to meet the qubit. We will start to use a bit of mathematical notation, including some concepts from linear algebra.

A qubit is a quantum system consisting of two levels, labeled $|0\rangle$ and $|1\rangle$ (here we are using Dirac's bra-ket notation) and is represented by a two-dimensional vector space over the complex numbers $\mathbb{C}^2$. This means that a qubit takes two complex numbers to fully describe it. The computational (or standard) basis corresponds to the two levels $|0\rangle$ and $|1\rangle$, and corresponds to the following vectors

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The qubit does not always have to be in just $|0\rangle$ or $|1\rangle$ but can be in any quantum state, denoted $|\psi\rangle$, which can be any superposition $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, of the basis vectors. The superposition quantities α and β are complex numbers; together they obey $|\alpha|^2 + |\beta|^2 = 1$.

Interesting things happen when quantum systems are measured, or observed. Quantum measurement is described by the Born rule. In particular, if a qubit that is in some state $|\psi\rangle$ is measured in the standard basis, the result 0 is obtained with probability $|\alpha|^2$ and the result 1 is obtained with the complementary probability $|\beta|^2$. A point of fascination is that a quantum measurement takes any superposition state of the qubit, and projects it to either the state $|0\rangle$ or the state $|1\rangle$ with a probability determined from the parameters of the superposition.

What we have described here is the abstract notation of a qubit. The prototype quantum computer you can use here in the IBM Quantum Experience uses a type of qubit called a superconducting transmon qubit, which is made from superconducting materials such as niobium and aluminum, patterned on a silicon substrate.

Physically, for this superconducting qubit to behave as the abstract notion of the qubit, we actually need to cool down the device considerably. In fact, in the IBM Quantum Lab, we are able to keep the temperature cold enough (15 milliKelvin in a dilution refrigerator) that there is no ambient noise or heat to excite the superconducting qubit; after our system has gotten cold enough (for a few days), the superconducting qubit reaches equilibrium down to the ground state $|0\rangle$.

To get a sense for what this ground state of a qubit means, try running the first score file below in a simulation mode (or look at some real cached runs). Here, the qubit is initially prepared in the ground state $|0\rangle$, then is followed by the standard measure. From your execution results, you should find in the ideal case, and with very high probability for the cached runs, that the qubit is still in the ground state. In the actual experiment runs, you can observe that there is some error, with some shots giving a $|1\rangle$ instead, which is due to imperfect measurements and some residual heating of the qubit.

The output for every score you run will be in the My Scores tab. Click the little bar graph icon next to the time stamp for your quantum score to see the results (if the results are not yet ready, or if there has been an error, you will see a yellow or red symbol on the bar graph icon). This will take you to the Results screen, where you can see the latest results.

Qubit Measurement

Excited State and Pauli Operators

As you may have guessed, a qubit does more than sit around just in the $|0\rangle$ ground state. To put it into the $|1\rangle$ *excited* state, we need a *quantum gate* -- so in this section we will introduce gates, and how to use them in the Composer.

Quantum gates, or operations, are typically represented as matrices. A gate that acts on one qubit is represented by a 2×2 unitary matrix (https://en.wikipedia.org/wiki/Unitary_matrix). Since quantum operations need to be reversible and preserve probability amplitudes, the matrices must be unitary. The result of the quantum gate is found by multiplying the matrix representing the gate with the vector representing the quantum state.

$|\psi'\rangle = U|\psi\rangle$ where $U^\dagger U = 1$ ($A^\dagger$ represents the complex conjugation and transpose of any matrix A).

A common group of gates, known as the Pauli Operators (https://en.wikipedia.org/wiki/Pauli_matrices), are represented by the matrices

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The Pauli X gate is known as an X_π-rotation and it takes $|0\rangle \rightarrow X|0\rangle = |1\rangle$; in other words, it flips the zero to a one, or vice versa (this is why it is also commonly referred to as a bit-flip). You can enter it into the Composer with the score already made for you below! Did you find that (unlike in the previous tutorial's example) now the qubit was in the *excited state* $|1\rangle$ with high probability? Any deviation from the excited state is likely due to decoherence and imperfect measurements.

In the other examples below, explore what the Pauli Operators do. What do you get when you try a Y or Z gate? Did you find that Y gave you an excited state and Z did not do anything?

Pauli X

Pauli Y

Pauli Z

Superposition

Now that we've got the $|0\rangle$ and $|1\rangle$ states under our belt, let's explore superposition, which is the concept that adding quantum states together (similar to overlaying two waves) results in a new quantum state. To make superpositions we need to expand our set of gates to include $\{H, S, S^\dagger\}$. In the Quantum Composer these are the blue set of gates and are represented by the matrices

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \qquad S^{+} = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$$

In the first score below, we apply H, known as the Hadamard gate, on one of the qubits that has been prepared in the $|0\rangle$ state and then followed by the standard measurement. Run the circuit and observe the result. The qubit should spend half its time in the $|0\rangle$ state and the other half in the $|1\rangle$ state. Before the measurement forced it to choose, the qubit was in both states at once. This is part of the reason for the often-misused analogy that a quantum computer does everything at once.

What is happening here?

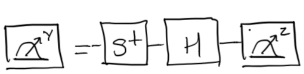
Applying the H gate to $|0\rangle$ does a size-2 discrete Fourier transform, making the state $|+\rangle = \dfrac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. This is the standard representation of a superposition state. We can define the state $|-\rangle = \dfrac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, which with $|+\rangle$ forms a new basis called the diagonal (or conjugate) basis. It is made using the second circuit below. The H makes the above superposition and then the Z flips the phase ($|1\rangle$ to $-|1\rangle$). If you run this circuit you will find that, like before, the outcomes are equal. Different states give the same outcomes!

To tell the difference between these states we need to measure in the diagonal basis. In our experiments we cannot physically change the measurement; however, we can effectively change the measurement using gates before measurement. To measure in the diagonal basis, the standard basis (Z) is rotated to the diagonal basis (X) with a Hadamard gate before the measurement.

Try Superposition (+) X Measurement and Superposition (-) X Measurement below. You should find that nearly 100% of the time the outcome is 0 and 1 respectively. That is, if we make a measurement in the standard basis, the outcome is completely uniform -- but in the diagonal basis, it has deterministic outcome. No measurement can distinguish all four kinds of states $|0\rangle$, $|1\rangle$, $|+\rangle$, $|-\rangle$. This is not a limitation of the measurement, but a fundamental consequence of the uncertainty principle. (This limitation gives rise to the possibility of quantum money and quantum cryptography.)

A third commonly-used basis is the circular (or Y) basis: $|\circlearrowright\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$, $|\circlearrowleft\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$. To make the $|\circlearrowright\rangle$ state we need to use the additional gate S, the phase gate. This gate applies a complex phase to $|1\rangle$, via the application of an H followed by an S gate. Try to figure out how to get the $|\circlearrowleft\rangle$ on your own.

Like the above example, measurement in the standard basis will not give you different statistics. Even measurement in the diagonal basis will be random. To measure in this basis we must rotate the standard basis (Z) to the circular basis (Y). To do this, use an $S^\dagger$ followed by H before your measurement.



Try out the last example. It should give you close to 1 as it is a measurement $|\circlearrowright\rangle$ in the circular basis.

Superposition (+)

Superposition (-)

Superposition (+) X Measurement

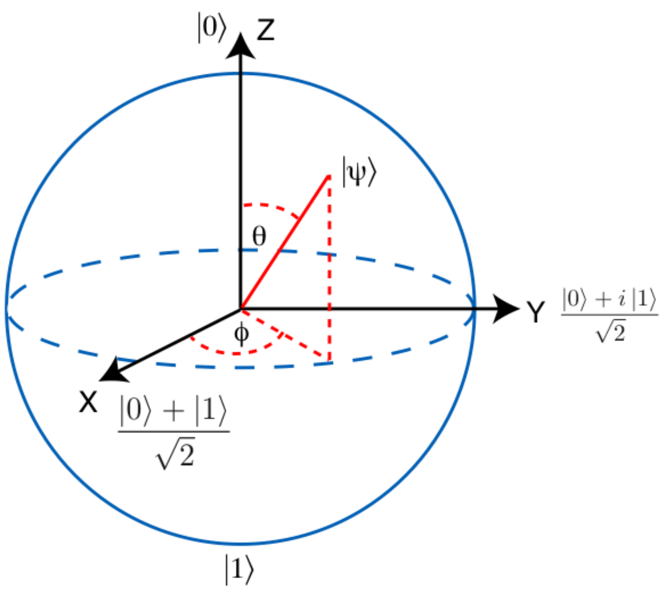
Superposition (-) X Measurement

Superposition (+i) Y Measurement

Superposition (-i) Y Measurement

The Bloch Sphere

As we observed in the previous section, probabilities in the standard basis are not enough to specify a quantum state because it cannot capture the phase of the superposition. A convenient representation for a qubit is the *Bloch sphere*.



If we define a qubit state by $|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle$ where θ and ϕ are defined in the picture, we see that there is a one-to-one correspondence between pure qubit states ($\mathbb{C}^2$) and the points on the surface of a unit sphere ($\mathbb{R}^3$). This is significant because now we can simply visualize qubit states and gates. We can reconstruct an arbitrary unknown qubit state $|\psi\rangle$ by measuring the *Bloch vector*, whose vector components are the expectation values (https://en.wikipedia.org/wiki/Expectation_value_%28quantum_mechanics%29) of the three Pauli operators, given by $\langle X\rangle = \mathrm{tr}(|\psi\rangle\langle\psi|X)$, $\langle Y\rangle = \mathrm{tr}(|\psi\rangle\langle\psi|Y)$, and $\langle Z\rangle = \mathrm{tr}(|\psi\rangle\langle\psi|Z)$. The state is given by $|\psi\rangle\langle\psi| = (I + \langle X\rangle X + \langle Y\rangle Y + \langle Z\rangle Z)/2$.

Each expectation value $\langle Q\rangle$ can be obtained experimentally by first preparing the state, rotating the standard basis frame to lie along the corresponding axis, and making a measurement in the standard basis. The probabilities of obtaining the two possible outcomes 0 and 1 are used to evaluate the desired expectation value via $\langle Q\rangle = P(0) - P(1)$. As an example, let's look at measuring the expectation value of X, $\langle X\rangle = \mathrm{tr}(|\psi\rangle\langle\psi|X)$, depicted in the circuit below. Once $|\psi\rangle$ is prepared we implement the gate H that exchanges Z to X, then we make a measurement in the standard basis. The desired expectation value is given by $\langle X\rangle = P(0) - P(1)$. Similarly, we can use the $S^\dagger - H$ gate to measure $\langle Y\rangle$.

To simplify performing these kinds of experiments, we provide a *Bloch measurement* circuit element that implements your quantum circuit three times with each of the above measurements, then plots the results on the Bloch sphere. The Bloch measurement element also takes some error into account, and corrects for it by scaling the Bloch vector using calibration experiments. A demonstration of the Bloch measurement element is given below.

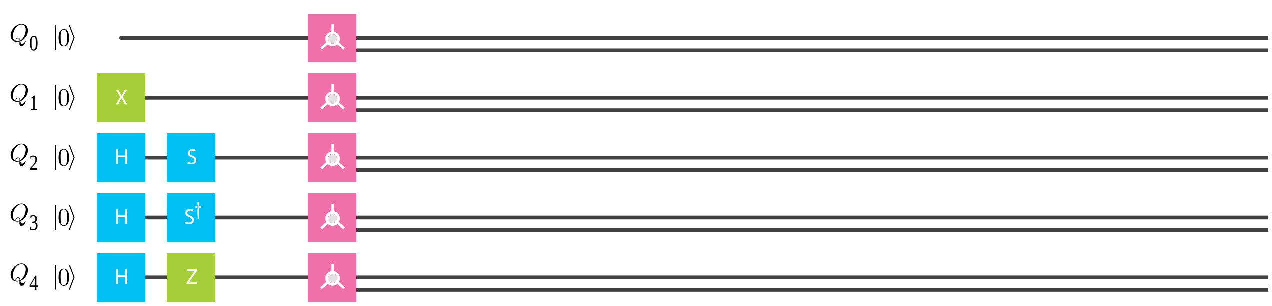
Superposition (+)

Superposition (+) X Measurement

Superposition (+) Y Measurement

Superposition (+) Bloch Tomography

Cardinal States



Decoherence

Real quantum computers must deal with *decoherence* (https://en.wikipedia.org/wiki/Quantum_decoherence), or the loss of information due to environmental disturbances (noise). The Bloch vector formalism we introduced in the previous section is sufficient to describe the state of the system under decoherence processes. The *pure states* (https://en.wikipedia.org/wiki/Quantum_state#Pure_states)we have studied so far have $\rho = |\psi\rangle\langle\psi|$ and a Bloch vector of length 1, touching the surface of the Bloch sphere. Decoherence causes our quantum states to become *mixed states* (https://en.wikipedia.org/wiki/Quantum_state#Mixed_states), which have a *density matrix* (https://en.wikipedia.org/wiki/Density_matrix) ρ that can be written as a sum over pure states

$$\rho = \sum_k p_k |\psi_k\rangle\langle\psi_k|$$

and a Bloch vector that sits inside the Bloch sphere

$$|\langle X\rangle|^2 + |\langle Y\rangle|^2 + |\langle Z\rangle|^2 < 1.$$

Energy relaxation and T_1

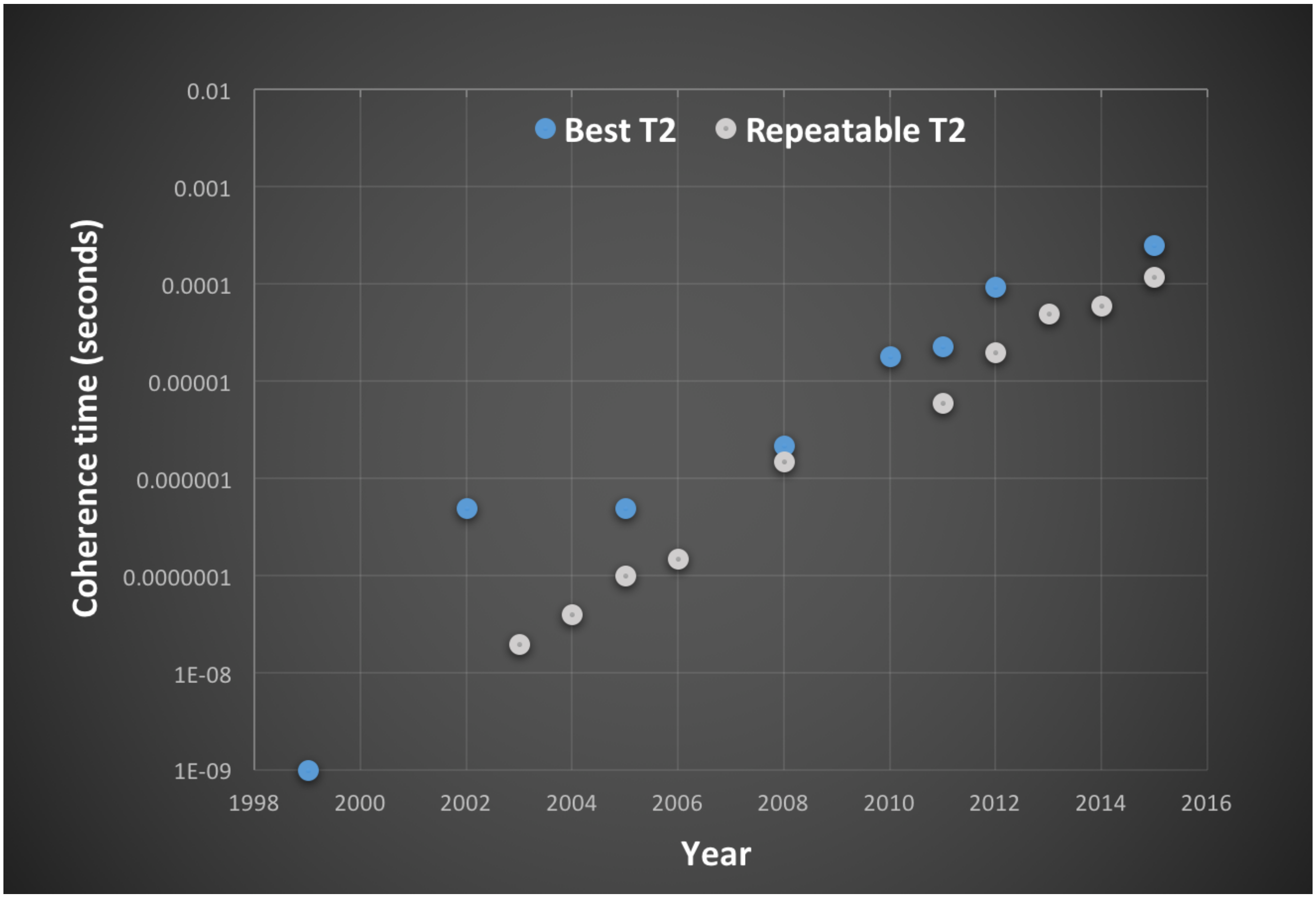
One important decoherence process is called *energy relaxation*, where the excited $|1\rangle$ state decays toward the ground state $|0\rangle$. The time constant of this process, T_1, is an extremely important figure-of-merit for any implementation of quantum computing, and one in which IBM has made great progress in recent years, ultimately leading to the prototype quantum computer you are now using. Experiment with the circuits below to see how adding many repetitions of additional do-nothing Idle gates (or Identity gates; these are gates that do nothing but wait) Id before measurement causes the state to gradually decay towards $|0\rangle$.

Dephasing and T_2

Dephasing is another decoherence process, and unlike energy relaxation, it affects only superposition states. It can be understood solely in a quantum setting as it has no classical analog. The time constant T_2 includes the effect of dephasing as well as energy relaxation, and is another crucial figure-of-merit. Again, IBM has some of the world's best qubits by this metric. Experiment with the circuits below to see that when a state starts as a superposition (Bloch vector on the "equator" of the Bloch sphere) the qubit is subjected to more decay channels than when it starts in the computational state $|1\rangle$.

Progress in decoherence with superconducting qubits

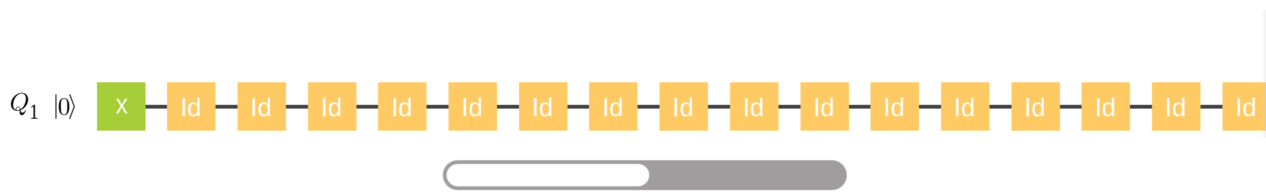
Because T_2 is such an important quantity, it is interesting to chart how far the community of superconducting qubits have come over the years. Here is a graph depicting T_2 versus time.



Excited Bloch Tomography

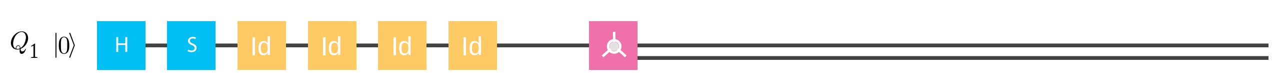
Excited (4 Idle) Bloch Tomography

Excited (16 Idle) Bloch Tomography



Superposition (+i) Bloch Tomography

Superposition (+i) (4 Idle) Bloch Tomography



Superposition (+i) (16 Idle) Bloch Tomography