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# Average Value

$$\frac{y_1+y_2+\cdots+y_n}{n} \xrightarrow[n\to\infty]{} \frac{1}{b-a}\int_a^b f(x)\,dx$$

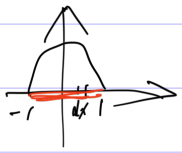
$$\frac{(f(x_1)+f(x_2)+\cdots+f(x_n))\Delta x}{b-a} \xrightarrow[\Delta x\to 0]{} \frac{\int_a^b f(x)\,dx}{b-a}$$

$$\left(\Delta x = \frac{b-a}{n}\right)$$

Ex 1: $f(x) = C$

$$\text{Avg} = \frac{1}{b-a}\int_a^b c\,dx = C$$

Ex 2: Avg h of unit circle:




$$\text{Avg} = \frac{1}{1-(-1)}\underbrace{\int_{-1}^{1}\sqrt{1-x^2}\,dx}_{\text{interpre. half circle}}$$

定积分=原函数相减
但这积分原本就是面积的定义/积累.

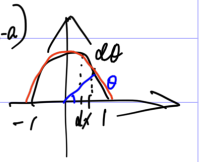
$$= \frac{1}{2}\cdot(\pi\cdot(1)^2/2)$$

$$= \frac{1}{4}\pi \quad \text{respect to } x$$

Ex 3: Ex 2 Avg h with respected to arclength $\theta$.

$y = \sin\theta$

$$\text{Avg}(y) = \frac{1}{\pi}\int_0^{\pi}\sin\theta\,d\theta \quad (\text{对应曲线下面积}/(b-a))$$




probability interp.

$$= \frac{1}{\pi}(-\cos\theta\,|_0^{\pi})$$

$$= \frac{1}{\pi}[(-\cos\pi)-(-\cos 0)]$$

$$= \frac{2}{\pi} \quad \text{respect to } \theta$$

积分值的意义，取决于单位视角，和自变量的视角.

# Weighted Average

$$\frac{\int_a^b f(x)w(x)\,dx}{\int_a^b w(x)\,dx}$$

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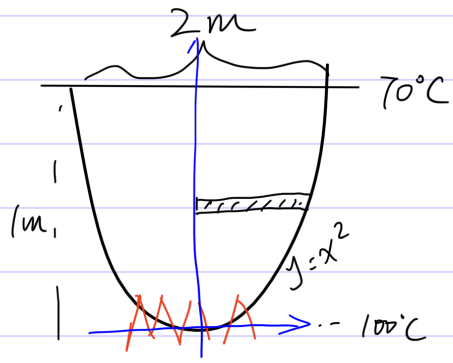


initial: 0°C

Final: $T=100-30y$

How much energy needed?

$$Q=\int_0^1 \pi x^2 \cdot T_y\,dy$$

$$=\int_0^1 \pi\cdot y\cdot(100-30y)\,dy$$

$$=\pi\int_0^1 (100y-30y^2)\,dy$$

$$=\pi\left[50y^2-10y^3\right]\Big|_0^1$$

$$=40\pi \quad (^\circ C\cdot m^3)$$

$$=(40\pi)\,^\circ C\cdot m^3\cdot\left(\frac{1\,cal}{^\circ C\cdot cm^3}\right)\cdot\left(\frac{1000\,cm}{1\,m}\right)^3$$

$$=40\pi\cdot 10^6\,cal$$

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$$Avg(T)=\frac{\int_0^1 T_y\cdot\pi x^2\,dy}{\int_0^1 \pi x^2\,dy}$$

weighted to the dV

$\pi\int_0^1 y\,dy$

$\frac{y^2}{2}\Big|_0^1$ $\frac{1}{2}\pi$

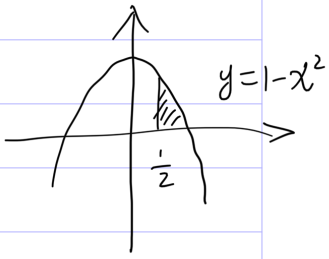
$$=\frac{40\pi}{\frac{\pi}{2}}$$

$$=80\,(^\circ C)$$

# Probability

Ex: drop point.

$P(x > \frac{1}{2})$ ?




$$= \frac{\int_{0.5}^{1} (1-x^2)\,dx}{\int_{-1}^{1} (1-x^2)\,dx}$$

$F: x - \frac{x^3}{3}$

$$= \frac{x - \frac{x^3}{3}\Big|_{0.5}^{1}}{x - \frac{x^3}{3}\Big|_{-1}^{1}}$$

$$= \frac{1 - \frac{1}{3} - (0.5 - \frac{0.5^3}{3})}{1 - \frac{1}{3} - (-1 + \frac{1}{3})}$$

$\frac{2}{3} - \frac{1}{2} + \frac{1}{24}$

$\frac{2}{3}$

$$= \frac{\frac{5}{24}}{\frac{4}{3}}$$

$$= \frac{5}{32}$$

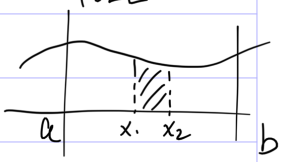
$$\frac{\int_{0}^{0.5} 0 \cdot 1 \cdot dx + \int_{0.5}^{1} (1-x^2) \cdot 1 \cdot dx}{\int_{-1}^{1} (1-x^2)\,dx}$$

$x$ $(dx)$ weighted by $w(x) = 1-x^2$

# General formula for probability

$$a \le x_1 \le x_2 \le b$$

$$P(x_1 < x < x_2) = \frac{\int_{x_1}^{x_2} w(x)\,dx}{\int_a^b w(x)\,dx} = \frac{\text{PART}}{\text{WHOLE}}$$

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