~ Probability

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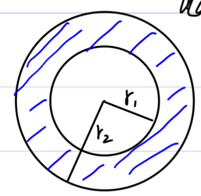
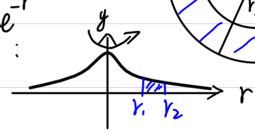
dart



Ex : distri: # hits (frequency is related to r) $= ce^{-r^2}$

side view of e^{-r^2}:



by ⊛ method of shells:

$$P(r_1<r<r_2) = \frac{\int_{r_1}^{r_2}\left[(ce^{-r^2}-0)\,dr\cdot 2\pi r\right]}{\int_0^{+\infty}\left[(ce^{-r^2}-0)\,dr\cdot 2\pi r\right]}$$

$$= \frac{\int_{r_1}^{r_2} re^{-r^2}\,dr}{\int_0^{+\infty} re^{-r^2}\,dr}$$ ⟸ there's an r, hence model: ce^{-r^2} has square.

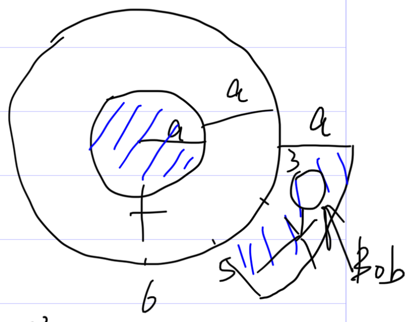
$$= \frac{\left.\frac{e^{-r^2}}{-2}\right|_{r_1}^{r_2}}{\left.\frac{e^{-r^2}}{-2}\right|_0^{+\infty}}$$

$$= \frac{e^{-r_2^2}-e^{-r_1^2}}{0-e^{-0}}$$ $\frac{\text{PART} = c\pi(e^{-r_1^2}-e^{-r_2^2})}{\text{TOTAL} = c\pi(1-0)}$

$$= e^{-r_1^2}-e^{-r_2^2}$$

$P(0 \le r < \infty) = 1$

Ex 2 :



$P(0 \le x < a) = \frac{1}{2}$

$P(\text{hit Bob}) = ?$

A: $P(0 \le x < a) = e^{-0^2} - e^{-a^2} = \frac{1}{2}$

$\frac{1}{2} \Leftarrow 1 - \frac{1}{2} = e^{-a^2}$

$P(h\,B) = \frac{2}{12} \cdot P(2a < x < 3a)$

$= \frac{1}{6} \cdot \left[e^{-(2a)^2} - e^{-(3a)^2}\right]$

$= \frac{1}{6} \cdot \left[e^{-4a^2} - e^{-9a^2}\right]$

$= \frac{1}{6} \cdot \left[(e^{-a^2})^4 - (e^{-a^2})^9\right]$

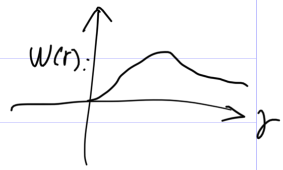
$= \frac{1}{6}\left[(\frac{1}{2})^4 - \underbrace{(\frac{1}{2})^9}_{\approx 0}\right]$

$\approx \frac{1}{6} \cdot \frac{1}{16}$

$= 1/100$

weight for $Z\alpha 1$:

$$w(r) = 2\pi r \cdot c e^{-r^2}$$

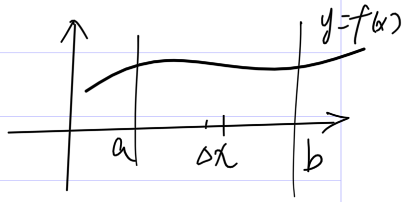


Numerical Integration

1. Riemann sums. $\sum_{i=} \cdots \Delta x$
2. trapezoidal rule.
3. Simpson's rule.

① $a = x_0 < x_1 < \cdots < x_n = b$

$y_0 = f(x_0)$, $y_1 = f(x_1) \cdots$



$\Delta x = x_i - x_{i-1}$

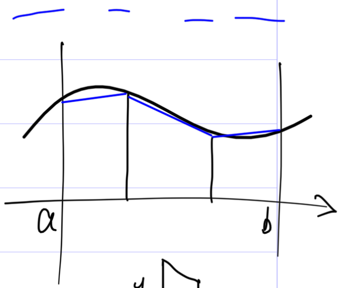
Goal: add up y to get approx integral.

Riemann sum:

$$(y_0 + y_1 + \cdots + y_{n-1})\Delta x$$

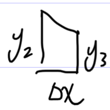
$$= (y_1 + y_2 + \cdots + y_n)\Delta x$$

②. Trape~ rule: $\frac{(y_2 + y_3)\cdot \Delta x}{2}$, ...



$$\Delta x \cdot \left(\frac{y_0 + y_1}{2} + \frac{y_1 + y_2}{2} + \cdots + \frac{y_{n-1} + y_n}{2} \right)$$

$$= \boxed{\Delta x \left(\frac{y_0}{2} + y_1 + y_2 + \cdots + y_{n-1} + \frac{y_n}{2} \right)}$$



$$= \frac{\text{Left Reim} + \text{Right Reim}}{2}$$

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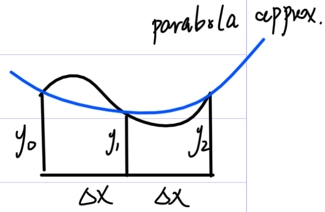
③. Simpson's rule:



area under parabola:

$$\underbrace{2\Delta x}_{\text{base}} \cdot \underbrace{\left(\frac{y_0+4y_1+y_2}{6}\right)}_{\text{average height}}$$

$$\therefore \frac{2\Delta x}{6}\left[(y_0+4y_1+y_2)+(y_2+4y_3+y_4)+\cdots+(y_{n-2}+4y_{n-1}+y_n)\right]$$

1 4 1

1 4 1

1 4 1

1 4 2 4 2 4 2 ⋯ 1

$$\therefore \frac{\Delta x}{3}(y_0+4y_1+2y_2+4y_3+\cdots+y_n)$$

way better.