Differencials

微分

$y = f(x)$

$\frac{dy}{dx} = f'(x)$

derivative 的 ~~导数~~

Differencial of y (of f): $dy = f'(x)\,dx$

differenciation ~~求导~~过程

process

Ex: $(64.1)^{1/3} \approx ?$

$y = x^{\frac{1}{3}}$ at $x=64$.

$$dy = \frac{1}{3} x^{-\frac{2}{3}} \cdot dx$$

$$= \frac{1}{3 \cdot 4^{3\cdot\frac{2}{3}}} \cdot 0.1$$

$$= \frac{1}{480}$$

$$y_0 + \Delta y = y_1 \approx y_0 + dy$$

$$\approx 4 + \frac{1}{480}$$

Anti derivatives.

$$G(x) = \int g(x)\,dx$$

$\uparrow$

antiderivative of g. = indefinite integral of g.

1. $\int \sin x\,dx = -\cos x + C \longrightarrow$ so indefinite.

2. $\int x^a\,dx = \frac{1}{a+1}x^{a+1} + C$ $\quad (a \neq -1)$

3. $\int \frac{dx}{x} = \ln|x| + C$

$$\left(x<0: \ \frac{d}{dx}\ln|x| = \frac{d}{dx}\ln(-x) = \frac{1}{-x}\frac{d}{dx}(-x) = \frac{1}{x} \right)$$

e.g. $\int \sec^2 x\,dx = \tan x + C$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{(-1)} x + C$$

$$\int \frac{dx}{1+x^2} = \tan^{(-1)} x + C$$

Theorem If $F' = G'$, then

$$F(x) = G(x) + C$$

proof: if $F' = G' \Rightarrow (F-G)' = F' - G' = 0$

$$\overset{MVT}{\Rightarrow} F - G = C$$

$$\Rightarrow F = G + C$$

Ex: $\int x^3(x^4+2)^5\,dx = \frac{1}{24}(x^4+2)^6 + C$

method of substitution:

$u = x^4+2, \quad du = 4x^3\,dx$

$I = \int u^5 \cdot \frac{du}{4} = \frac{1}{4}\int u^5\,du = \frac{1}{4}\cdot\frac{1}{6}u^6 + C$

$= \frac{1}{24}(x^4+2)^6 + C$

Ex2: $\int \frac{x\,dx}{\sqrt{1+x^2}}$

$u = \sqrt{1+x^2},$

$du = \frac{1}{2}\frac{1}{\sqrt{1+x^2}}\cdot 2x\,dx$

$I = \int du = u + C$

$= \sqrt{1+x^2} + C$

Ex3: $\int e^{6x}\,dx$

$\because$ guess: $(e^{6x})' = e^{6x}\cdot 6$

$I = \frac{1}{6}e^{6x} + C$

显露/保留了

原形可供猜测

Ex4: $\int x\cdot e^{-x^2}\,dx$

guess: $(e^{-x^2})' = e^{-x^2}\cdot(-1)\cdot 2x$

$\frac{(e^{-x^2})'}{-2} = e^{-x^2}\cdot x$

$I = \frac{e^{-x^2}}{-2} + C$

Ex5: $\int \sin x\cos x\,dx$

$(\sin^2 x)' = 2\sin x\cos x$

$I = \frac{1}{2}\sin^2 x + C$

Ex 6: $\int \frac{dx}{x \ln x}$

let $u = \ln x$

$du = \frac{1}{x} dx$

$I = \int \frac{1}{u} du = \ln u + C$

$= \ln \ln x + C$