Methods of Integration

20210926 9:45 PM

$$\frac{P(x)}{Q(x)} = \text{rational function}$$

ratio of two polynomials $P(x)$ & $Q(x)$

Partial fractions

split P/Q into "easy" pieces.

Ex 1. $\int \left(\frac{1}{x-1} + \frac{3}{x+2} \right) dx$ ← "easy"

$$= \ln|x-1| + 3\ln|x+2| + C$$

$$\frac{1}{x-1} + \frac{3}{x+2} = \frac{4x-1}{x^2+x-2}$$ ← disguised.

Algebra problem
to detect "easy" pieces.

Cover-up method:

$$\frac{4x-1}{x^2+x-2} \overset{①}{=} \frac{4x-1}{(x-1)(x+2)} \overset{②}{=} \frac{A}{x-1} + \frac{B}{x+2}$$

③ solve for A & B.

solve for A by multiply by $(x-1)$:

$$\frac{4x-1}{x+2} = A + \frac{B}{x+2}(x-1)$$

let $x = 1$,

$$\frac{4-1}{1+2} = A$$

$$1 = A$$

Solve for B by $\times (x+2)$

$$\frac{4x-1}{x-1} = \frac{A}{x-1}(x+2) + B. \quad \text{let } x = -2$$

$$\frac{-8-1}{-2-1} = B = \frac{-9}{-3} = 3$$

Steps: ① Factor Q (denom)

② set-up $A, B, C, \cdots$.

③ cover-up.

$$1\,\square \overset{①}{=} \frac{4x-1}{\cancel{(x-1)}(x+2)} \overset{②}{=} \frac{A}{x-1} + \frac{B}{x+2}$$

③ cover $\cancel{(x-1)}$ and focus on black.

Cover-up method works if

① $Q(x)$ has distinct linear factors, and

② degree $P <$ degree Q.

Ex:
$$\frac{x^2+3x+8}{(x-1)(x-2)(x+5)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+5}$$

$$A = \frac{1+3+8}{-1 \cdot 6} = -2$$

$$B = \frac{4+6+8}{1 \cdot 7} = \frac{18}{7}$$

$$C = \frac{25-15+8}{-6 \cdot -7} = \frac{3}{7}$$

Ex 2: ($\deg P < \deg Q$)

Q has repeated linear factors.

$$\frac{x^2+2}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$$

Cover-up method works for B & C. not A.

$$C = \frac{4+2}{9} = \frac{2}{3}$$

$$B = \frac{1+2}{3} = 1$$

For A: plug in one favorite num (one solution) e.g. $x=0$.

$$\frac{0^2+2}{(-1)^2 \cdot 2} = \frac{A}{-1} + \frac{1}{(-1)^2} + \frac{2/3}{2}$$

$$A = \frac{1}{3}$$

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