Derivative Formulas

20210915

- For specific funcs, $f'(x)$
- In general form: $(u+v)'$, $(cu)'$.

$\sin$ is Scrambled.
$\sin(\alpha+\beta) = S_\alpha C_\beta + C_\alpha S_\beta$
$\cos(\alpha+\beta) = C_\alpha C_\beta - S_\alpha S_\beta$
$\cos$ is Common.

Ex $\frac{d}{dx}\sin x$

$$= \frac{\sin(x+\Delta x) - \sin x}{\Delta x}$$

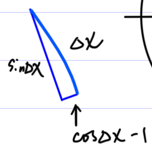
$$= \frac{\sin x\cos\Delta x + \cos x\sin\Delta x - \sin x}{\Delta x}$$

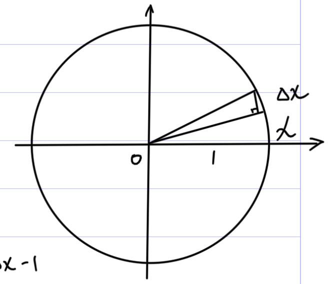
$$= \frac{1}{\Delta x}\cdot\left[\sin x(\cos\Delta x - 1) + \cos x\sin\Delta x\right]$$

$$= \sin x\cdot\underbrace{\frac{\cos\Delta x - 1}{\Delta x}}_{\to 0} + \cos x\underbrace{\frac{\sin\Delta x}{\Delta x}}_{\to 1}$$

$$\xrightarrow[\Delta x\to 0]{} \sin x\cdot 0 + \cos x\cdot 1$$

$$= \cos x \ \blacksquare$$



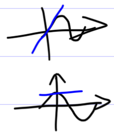


proof ①

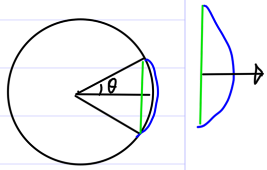
proof ②: from slope point of view:

$$\sin'0 = 1 = \lim_{\Delta x\to 0}\frac{\cos\Delta x - 1}{\Delta x}$$

$$\cos'0 = 0 = \lim_{\Delta x\to 0}\frac{\sin\Delta x}{\Delta x}$$



proof ③: $\frac{\sin\Delta x}{\Delta x} = \frac{2\sin\Delta x}{2\Delta x} = \frac{\text{bow string}}{\text{bow}} \xrightarrow[\theta\to 0]{} 1$



$$\frac{1-\cos\Delta x}{\Delta x} \sim \frac{1-\cos\Delta x}{2\cdot\Delta x} = \frac{1-\cos\Delta x}{\text{bow}}$$

$\Delta x = \theta_1 \searrow \theta_2$: (zoom till arc length for comparison)

$1-\cos\Delta x$ is more & more lesser than bow, as $\Delta x\to 0$. tends to 0 faster.

$\frac{1-\cos\Delta x}{\text{bow}} \leftarrow$ higher order of small testimal.

$\xrightarrow[\Delta x\to 0]{} 0$

So $\frac{1-\cos\Delta x}{\Delta x} \xrightarrow[\Delta x\to 0]{} 0$

Proof ④. Geometric

$y = \sin\theta$

$\Delta f = \Delta y = PR$,

$\Delta\theta = \widehat{PQ} \approx PQ$

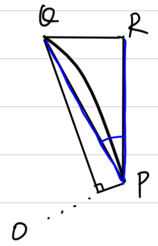
$\Delta\theta \to 0$,

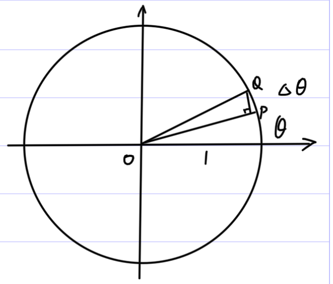
so $\angle OPQ = \angle OQP \approx 90^\circ$

$\therefore OP \perp PQ$.

$\therefore \angle QPR \approx \theta$

$$y' = \frac{\Delta f}{\Delta x} = \frac{\Delta y}{\Delta\theta} \overset{1}{\underset{2}{=}} \frac{PR}{PQ} = \cos\angle QPR \overset{3}{\approx} \cos\theta$$





Ex2. $\frac{d}{dx}\cos x$

$$= \frac{\cos(x+\Delta x) - \cos x}{\Delta x}$$

$$= \frac{\cos x\cos\Delta x - \sin x\sin\Delta x - \cos x}{\Delta x}$$

$$= \cos x \cdot \left(\underbrace{\frac{\cos\Delta x - 1}{\Delta x}}_{\downarrow\, \Delta x \to 0 \;\; 0}\right) - \sin x \cdot \underbrace{\frac{\sin\Delta x}{\Delta x}}_{\downarrow\, \Delta x \to 0 \;\; 1}$$

$$\xrightarrow[\Delta x \to 0]{} -\sin x$$

General Derivative Rules

- Product rule:

$$(uv)' = u'v + uv'$$

- Quotient rule:

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \quad (v \neq 0)$$