$\sim$ Approx.

for simplification on Engineering.

$\sin x \approx x$ (near 0)

$\cos x \approx 1 - \frac{1}{2}x^2$

$e^x \approx 1 + x + \frac{1}{2}x^2$

$\ln(1+x) \approx x - \frac{1}{2}x^2$

$(1+x)^r \approx 1 + rx + \frac{r(r-1)}{2}x^2$

Ex. $a_k = \left(1 + \frac{1}{k}\right)^k \to e$

$\ln a_k = k \ln\left(1 + \frac{1}{k}\right)$

$\approx k\left(\frac{1}{k}\right)$ ← linear approx.

$= 1$

$a_k = e^1 = e$

Judge on your own. always prefer a linear approx first even quad.

Ex 2: Find quadratic approx. for x near 0 to $e^{-3x}(1+x)^{-1/2}$

$\approx \left[1 + (-3x) + \frac{1}{2}(-3x)^2\right] \cdot \left[1 + \left(-\frac{1}{2}\right)x + \frac{-\frac{1}{2}\cdot-\frac{3}{2}}{2}x^2\right]$

$= \left[1 - 3x + \frac{9}{2}x^2\right]\left[1 - \frac{1}{2}x + \frac{3}{8}x^2\right]$

$\approx 1 - \frac{1}{2}x + \frac{3}{8}x^2 - 3x + \frac{3}{2}x^2 + \frac{9}{2}x^2$

$= 1 - \frac{7}{2}x + \frac{51}{8}x^2$

Curve Sketching

$f' > 0$: f ↗

$f' < 0$: f ↘

$f'' > 0$: f' ↗, f ∪ (concave up)

$f'' < 0$: f' ↘, f ∩ (concave down)

Ex 1: $f(x) = 3x - x^3$

$f'(x) = 3 - 3x^2$

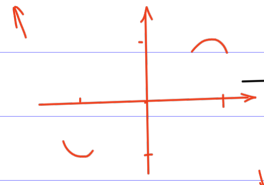
$= 3(1+x)(1-x)$

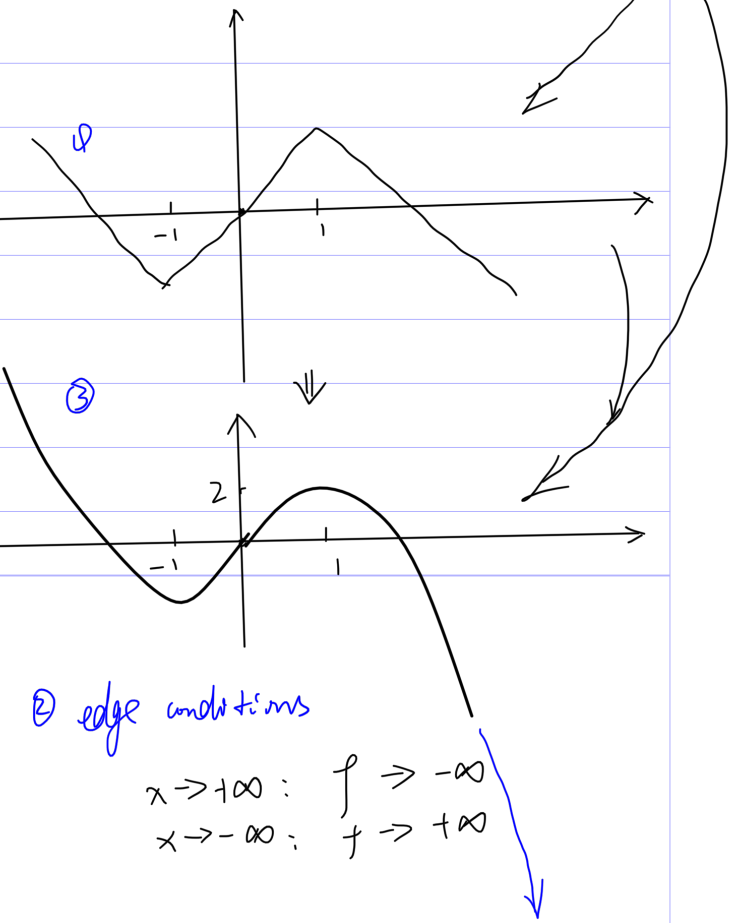
$f''(x) = -6x$

x	$-\infty$		-1		0		1		$+\infty$
f	$+\infty$								$-\infty$
f'		$-$		$+$		$+$		$-$	
f''		$+$		$+$		$-$		$-$	

Don't forget pre-calc & common sense.

e.g. (0,0), odd.





② edge conditions

$x \to +\infty$: $f \to -\infty$

$x \to -\infty$: $f \to +\infty$

if $f'(x_0) = 0$: x_0 a critical point

$y_0 = f(x_0)$ critical value