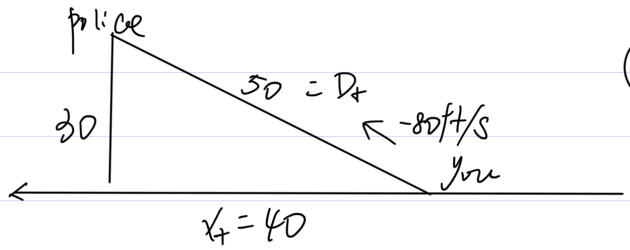
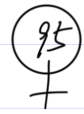
~ Related Rates





$$\begin{cases} 30^2 + x^2 = D^2 \\ \frac{d}{dt} D = -80 \end{cases}$$

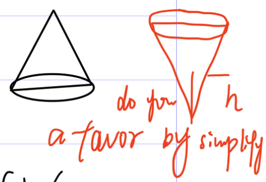
$$\frac{d}{dt}\left(30^2 + x^2 = D^2\right)$$

$$2x\,x'_t = 2D\,D'_t$$

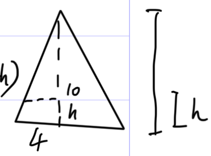
$$2\cdot 40\, x'_t = 2\cdot 50\cdot(-80)$$

$$x'_t = -100 \text{ ft/s} \quad > 95 \text{ ft/s}$$

Ex 2. A conical tank of $r = 4$ ft, $d = 10$ ft.
Filling with water at 2 cuft/min.
How fast is the water rising at $h = 5$ ft?



Ans:
$$\begin{cases} h = 5 \\ V = \frac{1}{3}\cdot \pi 4^2 \cdot 10 - \frac{1}{3}\pi\left[\frac{(10-h)4}{10}\right]^2\cdot(10-h) \\ V'_t = 2, \quad h'_t = ? \end{cases}$$



$$V'_t = \left[-\frac{\pi}{3}\cdot\frac{16(10-h)^3}{100}\right]'$$

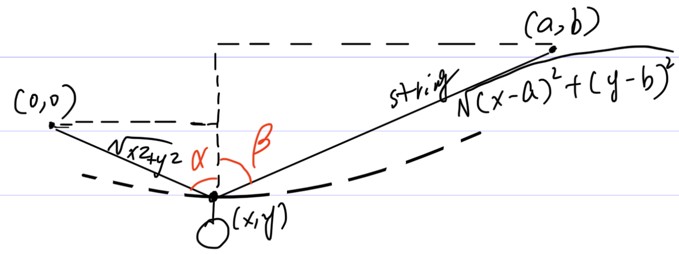
$$2 = \left[-\frac{4\pi}{75}\cdot(10-h)^3\right]'$$

$$2 = -\frac{4\pi}{75}\cdot 3\cdot(10-h)^2\cdot(-1)\,h'_t$$

$$2 = \frac{4\pi}{25}\cdot 5^2 \quad h'_t$$

$$\frac{1}{2\pi} = h'_t \quad (\text{ft/min})$$

Ex 3:



$$\sqrt{x^2+y^2} + \sqrt{(x-a)^2+(y-b)^2} = L \quad (\text{const})$$

Aim: Find min of y.

y has relation to x. a, b, L are const.

stays at $y'_x = 0$

diff on both side regarding x.

solve x, then y, then (x,y)

(A) $2(x-a)\cdot x'_x$

$$\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\cdot(2x+2yy'_x) + \frac{1}{2}\left[(x-a)^2+(y-b)^2\right]^{-\frac{1}{2}}\cdot\left[\cancel{2x}+2(y-b)y'_x\right] = 0$$

$$(x^2+y^2)^{-\frac{1}{2}}\cdot(2x+0) + \left[(x-a)^2+(y-b)^2\right]^{-\frac{1}{2}}\cdot\left[2x+0\right] = 0$$

$$(x^2+y^2)^{-\frac{1}{2}} + \left[(x-a)^2+(y-b)^2\right]^{-\frac{1}{2}} = 0$$

(B) ~~and dead end~~

$$(x^2+y^2)^{\frac{1}{2}} = -\left[(x-a)^2+(y-b)^2\right]^{\frac{1}{2}}$$

$$x^2+y^2 = (x-a)^2+(y-b)^2$$

$$= x^2-2ax+a^2+y^2-2by+b^2$$

$$0 = a^2-2ax+b^2-2by$$

$$\frac{x}{\sqrt{x^2+y^2}} = \frac{a-x}{\sqrt{(x-a)^2+(y-b)^2}}$$

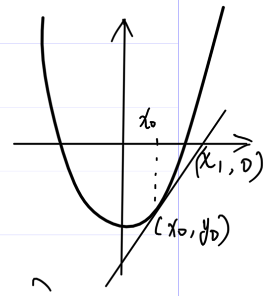
$$\Downarrow$$

$$\sin\alpha = \sin\beta$$

$$\Downarrow$$

$$\alpha = \beta$$

Newton's Method



Ex. solve $x^2=5$

let $f(x)=x^2-5$

find $f(x)=0$.

A: ① let $x_0=2$ (near guess)

$$\begin{cases} y-y_0=m(x-x_0) & \text{截距式} \\ (x,y)=(x_1,0) & \text{过x轴} \end{cases}$$

$$0-y_0=f'(x_0)(x_1-x_0)$$

$$-\frac{y_0}{f'(x_0)}=x_1-x_0$$

$$x_0-\frac{y_0}{f'(x_0)}=x_1$$

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)} \qquad \frac{y_1-y_0}{x_1-x_0}=m$$

② let $x_1=\nearrow$ (a better guess)

⋮

③ $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$ f_n dependent, (for any f_n)

$f(x)=x^2-5$ $f'(x)=2x$

$x_0=2$

$x_1=2-\frac{-1}{4}=\frac{9}{4}$

$x_2=\frac{9}{4}-\frac{(\frac{9}{4})^2-5}{2\cdot\frac{9}{4}}=\frac{161}{72}$ ($\Delta\approx 4\times10^{-5}$)

Newton's method

$$\frac{y_1-y_0}{x_1-x_0}=m.$$