Exp & Log

{ (exp)'

$$\frac{d}{dx} a^x = \lim_{\Delta x \to 0} \frac{a^{x+\Delta x} - a^x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} a^x \frac{a^{\Delta x} - 1}{\Delta x}$$

$$= a^x \lim_{\Delta x \to 0} \frac{a^{\Delta x} - 1}{\Delta x}$$

$(a^x)' = M(a)\, a^x$
$x=0$, $(a^x)' = M(a) \cdot 1$
is slope at $x=0$.

Define e: s.t. when $a = e$, $M(a) = \lim_{\Delta x \to 0} \frac{e^{\Delta x} - 1}{\Delta x} = 1$

① we defined that $(e^x)' = e^x$.

② when $a \neq e$, $a^x = (e \cdot \frac{a}{e})^x = e^x \cdot (\frac{a}{e})^x$ dead end

right way: $= (e^{\log_e a})^x$, * $a > 0$

$= e^{cx}$

so, $(a^x)' = (e^{cx})' = e^{cx} \cdot c = \log_e a \cdot e^{\log_e a \cdot x}$

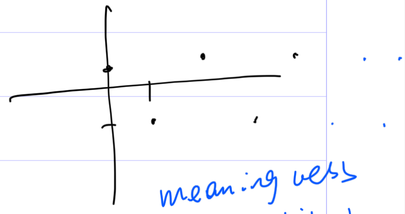
$= \log_e a \cdot a^x$

* $a = 0$: $0^x = 0$. simple

* $a < 0$: $[(-a)^x]' = \log_e(-a) \cdot (-a)^x$

question for future.

e.g. $(-1)^x$



meaning less

but may it ?

}ANS →

$$(a^x)' = \ln a \cdot a^x$$

$$\frac{d}{dx} \ln x$$

$y = \ln x$

use implicit (chain) differentiate method.

$x = e^y$

$1 = e^y \, y'_{(x)}$

$(\ln x)'_{(x)} = y'_{(x)} = \frac{1}{e^y} = \frac{1}{x}$. solved.

$\frac{d}{dx} \log_a x$ (general case)

$= \frac{d}{dx} (\log_a e \cdot \log_e x)$

$= \log_a e \cdot \frac{d}{dx} \ln x$

$= \log_a e \cdot \frac{1}{x}$

My trial on log'

$(\exp)'$ $\quad \frac{d}{dx} e^x$:

改写

① use base e: $[a^x]' = [(e^{\ln e})^x]' = \cdots = \ln a \cdot a^x$

② use logarithmic diff. 反函数 隐式求导

$\frac{d}{d}$ u(complex), sometimes it's easier to:

$\frac{d}{dx} \ln u$.

$$\frac{d}{dx} \ln u = A = \frac{d}{du} \ln u \cdot \frac{d}{dx} u = \frac{1}{u} u'$$

$$(\ln u)' = u'/u$$

$$u' = (\ln u)' \cdot u$$

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e.g.: $\frac{d}{dx} a^x = \frac{d}{dx}(\ln a^x) \cdot a^x$

$$= \frac{d}{dx} \ln a \cdot x \cdot a^x$$

$$= \ln a \cdot a^x$$

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e.g. 2: $v = x^x$

$(x^x)'$ ① right side

$$= (\ln x^x)' \cdot x^x$$

$$= (x \ln x)' \cdot x^x$$

$$= (1 \cdot \ln x + x \cdot \tfrac{1}{x}) \cdot x^x$$

$$= x^x \ln x + x^x$$

He solves Exp'. And the use of $(\ln f)'$:

$$u' \cdot \frac{1}{u} = (\ln u)'$$

apply ln to both side, then do diff.

② or $\ln v = \ln x^x$

$\ln v = x \ln x$ $\qquad v'_{(x)} = ?$

$(\ln v)' = (x \ln x)'$

$\frac{1}{v} v' = \ln x + 1$

$v' = (\ln x + 1) v$

$= (\ln x + 1) \cdot x^x$

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e.g. 3: $\lim_{n \to \infty} (1 + \frac{1}{n})^n$

if we find out $\lim_{n\to\infty} \ln \square = a$, then we know $\lim_{n\to\infty} \square$.

$\lim_{n \to \infty} \ln[(1+\frac{1}{n})^n]$

$= \lim_{n\to\infty} n \ln(1 + \frac{1}{n})$

Let $\Delta x = \frac{1}{n}$ $\to$ $= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \cdot \ln(1 + \Delta x)$

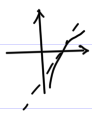
$= \frac{\ln(1+\Delta x)}{\Delta x}$

x at 1 $\qquad$ x at 1

$= \lim_{\Delta x \to 0} \frac{\ln(1+\Delta x) - \ln(1)}{\Delta x}$

generally like: $\frac{\ln(x + \Delta x) - \ln x}{\Delta x}$ $\qquad$ it is $f = \ln(x)$.

$f' = \frac{d}{dx} \ln x \big|_{x=1} = \frac{1}{x}\big|_{x=1} = 1$ (f' at $x = 1$).



Finally, $\ln \square = f' = 1$

$\square = e^1 = e$

Now we defined e.
numerical approx:
$e \approx (1 + \frac{1}{100})^{100}$

more usage on $\frac{u'}{u} = (\ln u)'$.

and what is e.