Geometry

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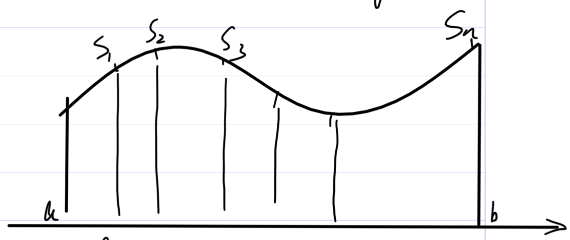
ds formula?

Arc length.

mileage.

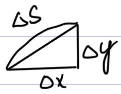


$ds^2 = dx^2 + dy^2$

$ds = \sqrt{dx^2+dy^2}$

other forms:

$ds = \sqrt{1+(\frac{dy}{dx})^2}\, dx$



$(\Delta S)^2 \approx (\Delta x)^2 + (\Delta y)^2$

$(dS)^2 = (dx)^2 + (dy)^2$

habbit ↳ $ds^2 = dx^2 + dy^2$

↓

$(ds)^2$. not $d(s^2) = 2sds$

(nor $d^2s = dds$ to be checked.)

$S_n - S_1 = \int_a^b \sqrt{1+(\frac{dy}{dx})^2}\, dx = \int_{s_0}^{s_n} ds$

$= \int_a^b \sqrt{1+[f'(x)]^2}\, dx \quad (y=f(x))$

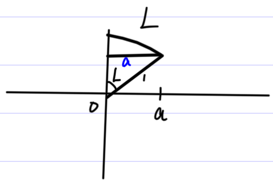
Ex 1: $y = mx$

$y' = m$

$S = \int \sqrt{1+m^2}\, dx$

$= \sqrt{1+m^2}\, x$

Ex 2: $y = \sqrt{1-x^2}$



$y' = \frac{1}{2}\cdot(1-x^2)^{-\frac{1}{2}}\cdot(-2x)$

$= \frac{-x}{\sqrt{1-x^2}}$

$L = \int \sqrt{1+y'^2}\, dx$

$= \int \sqrt{1+(\frac{-x}{\sqrt{1-x^2}})^2}\, dx$

$= \int \sqrt{\frac{1-x^2+x^2}{1-x^2}}\, dx$

$= \int \sqrt{\frac{1}{1-x^2}}\, dx$

$= \int \frac{1}{\sqrt{1-x^2}}\, dx$

let $x = \sin\theta$, $dx = \cos\theta\, d\theta$

$= \int \frac{\cos\theta\, d\theta}{\cos\theta}$

$= \theta$

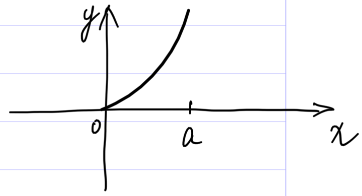
$= \arcsin x$ (def of radians)

$x = \sin L$

$ds = \sqrt{dx^2+dy^2}$

Ex 3: length of parabola. $(0 \le x \le a)$

$y = x^2, \; y' = 2x$



$ds = \sqrt{1+(2x)^2}\, dx$

$L = \int ds = \int_0^a \sqrt{1+(2x)^2}\, dx$

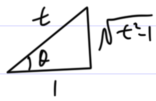
let $2x = \tan\theta$

$2dx = \sec^2\theta\, d\theta$

$L = \int_{\square}^{\square} \sec\theta \cdot \frac{1}{2}\sec^2\theta\, d\theta$

$= \frac{1}{2}\int_0^{\square} \sec^3\theta\, d\theta$

$\int \sec^3\theta\, d\theta = ?$



let $t = \sec\theta$

$dt = \sec\theta \cdot \tan\theta \cdot d\theta$

$= t \cdot \sqrt{t^2-1}\, d\theta$

$\frac{dt}{t\sqrt{t^2-1}} = d\theta$

$\frac{t^2 \cdot dt}{\sqrt{t^2-1}}$

$\frac{1}{\sqrt{1-(\frac{1}{t})^2}}\, dt$

$\frac{1}{t} = \sin\varphi$

$-\frac{1}{t^2}\, dt = \cos\varphi\, d\varphi$

$\frac{-t^2\cos\varphi\, d\varphi}{\cos\varphi}$

$-t^2\, d\varphi$

$= -\left(\frac{1}{\sin\varphi}\right)^2 d\varphi$

$= -\frac{1}{1-\cos^2\varphi}\, d\varphi$

$= \frac{1}{\cos^2\varphi - 1}\, d\varphi$

$=$

$\sec^3\theta \cdot 1\, d\theta$

$u = \sec^3\theta \quad u' = 3\sec^2\theta \sec\theta\tan\theta$

$v = x \quad v' = 1$

$\frac{1}{\cos^3\theta}\, d\theta$

$\frac{1}{\cos^3\theta}\, d\theta$

$\left(\frac{1}{\cos\theta}\right)^3 d\theta$

$\frac{1}{\cos^2\theta \quad \cos\theta}\, d\theta$

$\frac{1}{(1-\sin^2\theta)\cos\theta}\, d\theta$

$\sec\theta\, d\theta$

$\sec' = \sec\tan$

$\sec^3\theta\, d\theta \quad \sec^3\theta\, d\theta$

$t = \sec\theta$

$dt = \sec\theta\tan\theta\, d\theta$

$\frac{1}{\cos^3\theta}\, d\theta$

$t = \cos^3\theta$

$dt = -3\cos^2\theta\sin\theta\, d\theta$

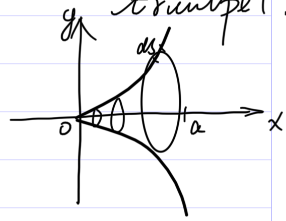
example

Surface area

Ex. surface of rotation

$y = x^2$ ↻ x.

trumpet.



method & set up

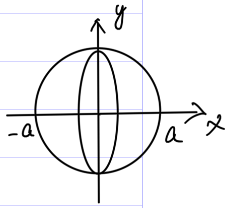
$$A = \int_{x=0}^{x=a} ds \cdot 2\pi y \quad , \; y' = 2x$$

$$= \int_{x=0}^{x=a} 2\pi x^2 \cdot \sqrt{1+(2x)^2}\, dx$$

actual computation can solve by program.

Ex 3: Surface area of sphere.

$y = \sqrt{a^2-x^2}$



$$A = \int_{-a}^{a} 2\pi y \cdot ds \quad , \; y' = -\frac{x}{\sqrt{a^2-x^2}}$$

$$= 2\pi \int_{-a}^{a} \sqrt{a^2-x^2} \cdot \sqrt{1+\left(-\frac{x}{\sqrt{a^2-x^2}}\right)^2}\, dx$$

$$= 2\pi \int_{-a}^{a} \sqrt{a^2-x^2+\frac{x^2}{1}}\, dx$$

$$= 2\pi \int_{-a}^{a} a\, dx$$

$$= 2\pi \cdot ax \Big|_{-a}^{a}$$

$$= 2\pi \cdot (a^2+a^2)$$

$$= 4\pi a^2$$

$$V = \frac{4}{3}\pi R^3$$

Surface area by rotating ds.

Parametric Curves (parameters)

$x = x(t)$ t = parameter.
$y = y(t)$

Ex 1: $x = a\cos t$
$y = a\sin t$
is a circle, counter clock wise

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parametric f exposure.