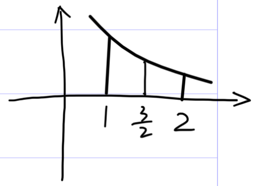
20210926 10:43

~ Numerical Methods

Ex $\int_1^2 \frac{dx}{x} = \ln x \Big|_1^2 = \ln 2 - \ln 1 = \ln 2 \approx 0.693147$

$\frac{1}{x}$

① 2 intervals.



B. trapezoidal rule:

$\Delta x \left(\frac{1}{2}y_0 + y_1 + \frac{1}{2}y_2\right)$

$= \frac{1}{2}\left(\frac{1}{2}\cdot\frac{1}{1} + 1\cdot\frac{1}{\frac{3}{2}} + \frac{1}{2}\cdot\frac{1}{2}\right)$

$= \frac{17}{24}$

≈ 0.708

C. Simpson's rule:

$\frac{\Delta x}{3}\left(y_0 + 4y_1 + y_2\right)$

$= \frac{1/2}{3}\left(1 + 4\cdot\frac{2}{3} + \frac{1}{2}\right)$

$= \frac{35}{12}$

$\approx 0.69444\ldots$

$|\text{Simpson's} - \text{ExactAns}| \approx (\Delta x)^4$

Simpson's rule use all exact ans of all parabolas.

And is also exact for cubics.

But works bad for $\frac{1}{x}$ (x near 0) (singlar, large f')

better f is nice and smooth.

Memonic device:

Check for $f(x)=1$

$\frac{\Delta x}{3}\left(y_1 + 4y_2 + 2y_3 + \cdots + y_n\right)$

$= \frac{\Delta x}{3}\left(1+4+2+\cdots+1\right)$

$= \frac{\Delta x}{3}\left(2 + \frac{n-2}{2}\cdot 6\right)$

$= \Delta x\left(n - \frac{4}{3}\right)$

$= \frac{b-a}{n}\left(n - \frac{4}{3}\right)$

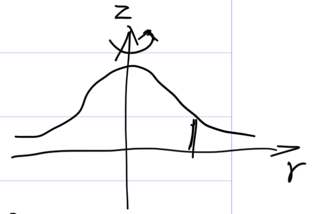
$\approx b-a$

$n-2$

$3n-6+2$

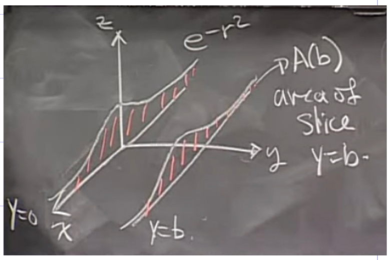
$3n-4$

$$V = \int_0^\infty 2\pi r \cdot e^{-r^2} dr$$
$$= \pi e^{-r^2} \Big|_0^\infty = \pi$$



$$Q = \int_{-\infty}^{\infty} e^{-t^2} dt$$ (without rotate)

$$V = \int_{-\infty}^{\infty} A_{(,y)} dy$$



Fix $y=b$, and find $A_{(b)}$:

$$\begin{cases} V = \int_0^\infty 2\pi r \cdot e^{-r^2} \cdot dr \\ y = b \end{cases}$$

$$h_{(x,y=b)} = e^{-r^2} = e^{-(x^2+y^2)} = e^{-(x^2+b^2)} = e^{-b^2} \cdot e^{-x^2} = Ce^{-x^2}.$$

(r is randomly moving, x, y free in 2D. Fixing y, then x is left free.)

$$A_{(b)} = A_{(x,y=b)} = \int_{-\infty}^{\infty} h_{(x,y=b)} dx$$
$$= \int_{-\infty}^{\infty} e^{-b^2} \cdot e^{-x^2} dx$$
$$= e^{-b^2} \int_{-\infty}^{\infty} e^{-x^2} dx$$
$$= e^{-b^2} Q$$

(this Q is only depends on x, or is constant ✓)

$$V = \int_{-\infty}^{\infty} A_{(y)} dy$$
$$= \int_{-\infty}^{\infty} e^{-y^2} Q \, dy$$
$$= Q \cdot \int_{-\infty}^{\infty} e^{-y^2} dy$$
$$= Q \cdot Q$$
$$V = Q^2$$
$$Q = \sqrt{V} = \sqrt{\pi}$$

[x, y independent on probability. so Q_x, Q_y are also independent, don't change with each other. $Q_x \cdot Q_y = C \Rightarrow Q_x = C_1,\ Q_y = C_2$.]

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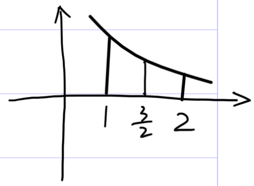

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$\approx b - a$

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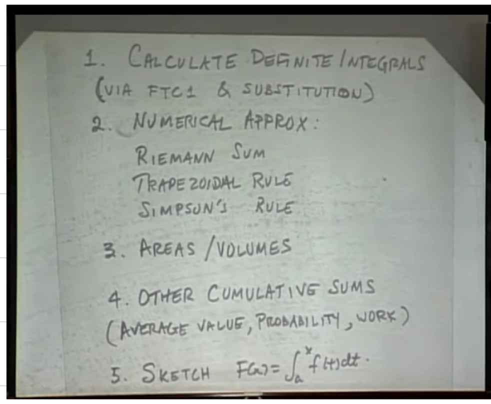
$3n - 6 + 2$

$3n - 4$

Review Sheet (Exam)


1. CALCULATE DEFINITE INTEGRALS (VIA FTC1 & SUBSTITUTION)
2. NUMERICAL APPROX:
   RIEMANN SUM
   TRAPEZOIDAL RULE
   SIMPSON'S RULE
3. AREAS / VOLUMES
4. OTHER CUMULATIVE SUMS (AVERAGE VALUE, PROBABILITY, WORK)
5. SKETCH $F(x) = \int_a^x f(t)dt$.

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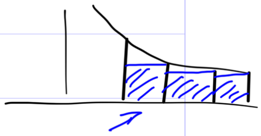
Reimmen sum:

left, right, upper, lower:



lower sum, also right-hand sum.