Rules of Differenciation

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$c, +, \times, \div, f(g(x))$ composition, higer derivatives.

- $(uv)' = u'v + uv'$

Proof: $\Delta(uv)$

$= u(x+\Delta x)\cdot v(x+\Delta x) - u(x)\cdot v(x)$

$= u(x+\Delta x)\cdot v(x+\Delta x) - u(x)\cdot v(x+\Delta x) + u(x)\cdot v(x+\Delta x) - u(x)\cdot v(x)$

$= [u(x+\Delta x) - u(x)]\cdot v(x+\Delta x) + u(x)\cdot[v(x+\Delta x) - v(x)]$

$= \Delta u \cdot v(x+\Delta x) + u(x)\cdot \Delta v$

$$\therefore \frac{\Delta(uv)}{\Delta x} = \frac{\Delta u}{\Delta x}\cdot v(x+\Delta x) + u(x)\cdot\frac{\Delta v}{\Delta x}$$

$\Downarrow \Delta x \to dx$

$$\frac{d(uv)}{dx} = \frac{du}{dx}\cdot v(x+dx) + u(x)\frac{dv}{dx}$$

$\Downarrow dx \to 0$ $\quad \because v$ is derivable, $\therefore v$ is continues, $\therefore = v(x)$.

$$\frac{d(uv)}{dx} = \frac{du}{dx}\cdot v(x) + u(x)\frac{dv}{dx}$$

$$= u'v + uv'$$

- $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

proof: $\Delta\left(\frac{u}{v}\right)$

$$= \frac{u(x+\Delta x)}{v(x+\Delta x)} - \frac{u(x)}{v(x)}$$

$$= \frac{u(x+\Delta x)\cdot v(x) - u(x)\cdot v(x+\Delta x)}{v(x+\Delta x)\cdot v(x)}$$

$$= \frac{u(x+\Delta x)\cdot v(x) - u(x)\cdot v(x) + u(x)\cdot v(x) - u(x)\cdot v(x+\Delta x)}{v(x+\Delta x)\cdot v(x)}$$

$$= \frac{[u(x+\Delta x) - u(x)]\cdot v(x) - u(x)\cdot[v(x+\Delta x) - v(x)]}{v(x+\Delta x)\cdot v(x)}$$

$$= \frac{\Delta u\cdot v - u\cdot\Delta v}{v(x+\Delta x)\cdot v(x)}$$

$$\therefore \frac{\Delta\left(\frac{u}{v}\right)}{\Delta x} = \frac{\frac{\Delta u}{\Delta x}\cdot v - u\frac{\Delta v}{\Delta x}}{v(x+\Delta x)\cdot v(x)}$$

$\Downarrow \Delta x \to dx$

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v(x+dx)\cdot v(x)}$$

$\Downarrow dx \to 0$

Assume / precondiction is that v is derivable, then v is continues, then $v(x+dx) \underset{dx\to 0}{=} v(x)$.

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{u'v - uv'}{v^2}$$

Composition Rule

$y = f(x(t))$

$\frac{\Delta y}{\Delta t} = \frac{\Delta f}{\Delta t} = \frac{\Delta f}{\Delta x} \cdot \frac{\Delta x}{\Delta t}$ (Geometric)

$\Rightarrow f'_x \cdot x'_t$

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e.g. $y = (\sin t)^{10}$ ($y = x^{10}$, $x = \sin t$)
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\;\;\; = f(x) \quad\;\; = x(t)$

$y' = f'(x) \cdot x'(t)$

$= 10x^9 \cdot \cos t$

$= 10 \cdot (\sin t)^9 \cdot \cos t$

$\therefore y' = 10 \cdot (\sin t)^9 \cdot \cos t$

Higher Derivatives

$$u'' = (u')'$$

Notation:

$$u' = \frac{du}{dx} = \frac{d}{dx} u = \boxed{\frac{d}{dx}} u = Du \qquad (\text{abstraction})$$

Higher:

$$u'' = \frac{d}{dx}\left(\frac{d}{dx}u\right) = \left(\frac{d}{dx}\right)^2 u = \frac{d^2}{(dx)^2} u = \frac{d^2 u}{\boxed{dx^2}} = D^2 u$$

Is not $d(x^2)$.
Is conventional notation.

$$u''' = \frac{d^3 u}{dx^3} = D^3 u$$

$$\vdots$$

(don't worry, do it one at a time.
use mind set of abstraction, folding,
devide and conquer.
Current layer is easy to solve,
while assumming sub-layers were taking cared of.)