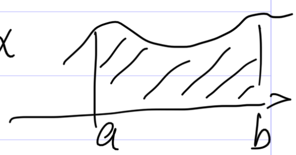
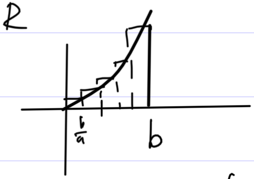
Intro to Integration.

Today: Definite Integrals.

$\int_a^b f(x)\,dx$



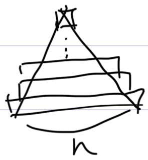
Ex 1: $f(x)=x^2$, $a=0$, $b \in R$



$$S = (b/n)(b/n)^2 + \cdots + (\tfrac{b}{n})\cdot(\tfrac{nb}{n})^2$$
$$= (\tfrac{b}{n})^3(1+2^2+3^2+\cdots+n^2)$$

x	$f(x)$
b/n	$(b/n)^2$
$2b/n$	$(2b/n)^2$
$3b/n$	$(3b/n)^2$
$\vdots$	

$$\because \frac{\frac{1}{3}n^3}{n^3} < \frac{1+2^2+3^2+\cdots+n^2}{n^3} < \frac{\frac{1}{3}(n+1)^3}{n^3}$$



$$\frac{1}{3} < \boxed{} < \frac{1}{3}\left(1+\frac{1}{n}\right)^3$$

$\downarrow$ $n \to \infty$

$\frac{1}{3}$

$$S = \frac{b^3}{n^3}\boxed{} = b^3 \cdot \frac{1}{3} = \frac{1}{3}b^3$$

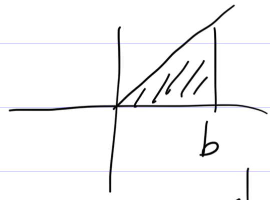
$$\int_0^b x^2\,dx = \frac{1}{3}b^3$$

Short-hand notation : $\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$
compact

$$\boxed{} = \frac{1}{n^3}\sum_{i=1}^{n} a_i$$

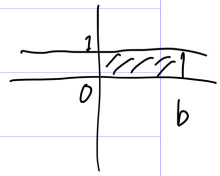
Ex 2 : $f(x) = x$

$A = \frac{1}{2} b \cdot b = \frac{1}{2} b^2$



Ex3 : $f(x) = 1$

$A = b \cdot 1 = b$

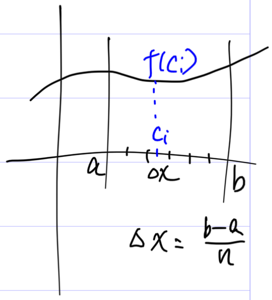


Pattern

$f(x)$	$\int_0^b f(x)dx$
x^2	$b^3/3$
x	$b^2/2$
1	b

Notation (Riemann Sums)

$$\sum_{i=1}^{n} \underbrace{f(c_i)}_{h} \underbrace{\Delta x}_{base} \xrightarrow[\Delta x \to 0]{} \int_a^b f(x)dx$$



Integrals as cumulated sums.