18.01 CALCULUS

2021 09 08

Differenciation

Derivative $\begin{cases} \text{Geo interpretation: tangent line: limit of secant line PQ, as } Q \to P. \\ \text{DEF: } f'(x)\text{: the slope of line to a func } f \text{, at } P. \end{cases}$

$$m = f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Ex. $f(x) = \frac{1}{x}$

A my: $f'(x) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$ 定义

$= \lim_{\Delta x \to 0} \frac{\frac{1}{x_0+\Delta x} - \frac{1}{x_0}}{\Delta x}$ 代入原式

$= \lim_{\Delta x \to 0} -\frac{1}{(x_0+\Delta x)x_0}$ 化简

$= -\frac{1}{x_0^2}$ 逻辑判断 $\Delta x \to 0$

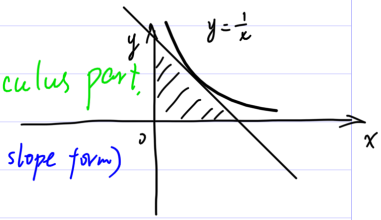
A: 差商: $\frac{\Delta f}{\Delta x} = \underbrace{\frac{\frac{1}{x_0+\Delta x} - \frac{1}{x_0}}{\Delta x} = \cdots = -\frac{1}{(x_0+\Delta x)x_0}}_{\text{preliminary algebra.}} \underbrace{\xrightarrow[\Delta x \to 0]{\text{infere}}}_{\text{only calculus part.}} -\frac{1}{x_0^2}$

abstraction layers of algebras

$\begin{cases} \text{abstraction} \\ \text{isolation} \\ \text{devide \& conquer} \\ \text{fold} \\ \text{ignore} \end{cases}$

Usually it is basic algebra that's error-prone, set you back and knock your confidence. Actually it's not big deal.
And the only calculus part is simple.
it's just a mind set, a way of thinking.

Q. find area of ⊿:



A: tangent $m = -\frac{1}{x_0^2}$ ← only calculus part.

$y - y_0 = m(x - x_0)$ ① (point-slope form)

so x sec point: $\begin{cases} ① \\ y = 0 \end{cases} \Rightarrow x = x_0^2 y_0 + x_0 = 2x_0$

y sec point: $\begin{cases} ① \\ x = 0 \end{cases} \Rightarrow y = \frac{1}{x_0} + y_0 = 2y_0$

area $= \frac{1}{2} Se_x \cdot Se_y = \frac{1}{2} \cdot 2x_0 \cdot 2y_0 = 2$.

Notation:

$$f'(x) = y' = y'_x = \underbrace{\frac{df}{dx} = \frac{dy}{dx} = \frac{d}{dx} f = \frac{d}{dx} y}_{\text{Leibniz's notation.}}$$

Ex. $f(x) = x^n$, $n = 1, 2, 3, \cdots$

$$\frac{d}{dx} x^n = ?$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^n - x^n}{\Delta x}$$

(treat x_0 (x) as fixed; Δx is moving.)

$$= \lim_{\Delta x \to 0} \frac{C_n^0 x^n + C_n^1 x^{n-1}\Delta x + \cdots + C_n^n (\Delta x)^n - x^n}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{C_n^1 x^{n-1}\Delta x + \cdots + C_n^n (\Delta x)^n}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left[n x^{n-1} + C_n^2 x^{n-2} \Delta x + \underbrace{\cdots + C_n^n (\Delta x)^{n-1}}_{O((\Delta x)^2)} \right]$$

$$\overset{\text{when}}{\underset{\Delta x \to 0}{=}} n x^{n-1}$$

only calculus part.