~ FTC 2.

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

(the derivative of integral, gives u the function back again)

Can solve dofferencial equations.

Ex. solue $y' = \frac{1}{x}$

consider integral $(y \sim \int y')$

let's cook up another integral $L(x) = \int_a^x \frac{1}{t}\,dt$

by FTC2, $\Rightarrow L'(x) = \frac{1}{x} = y'$

by MVT, $\Rightarrow y - L(x) = C$

$$\begin{aligned} y &= L(x) + C \\ &= \int_a^x \frac{1}{t}\,dt + C \\ &= \ln t \Big|_a^x + C \\ &= \ln x - \ln a + C \\ &= \ln x + C_2 \end{aligned}$$

$L(x) =$ definition of log. $\boxed{\int_1^x \frac{1}{t}\,dt}$ $L'(x) = \frac{1}{x}$, $L(1) = 0$

Claim $L(ab) = L(a) + L(b)$

Proof: $\int_1^{ab} \frac{1}{t}\,dt = \int_1^a \frac{1}{t}\,dt + \int_a^{ab} \frac{1}{t}\,dt$ ‖?

for $\int_a^{ab} \frac{1}{t}\,dt$:

let $u = \frac{t}{a} \Rightarrow u_2 = \frac{t_2}{a} = \frac{ab}{a} = b,\ u_1 = \frac{t_1}{a} = \frac{a}{a} = 1$, $du = \frac{1}{a}\,dt$

$$\int_a^{ab} \frac{1}{t}\,dt = \int_{u=1}^{u=b} \frac{1}{au}\cdot a\,du = \int_1^b \frac{1}{u}\,du = L(b)$$

Ex 2: $F = \int_0^x e^{-t^2}\,dt$

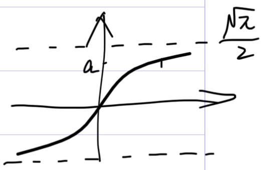
$F' = e^{-t^2}$, $F(0) = 0$

$F'' = -2x\,e^{-x^2}$ $\left(\begin{array}{l} x>0: \cap \\ x<0: \cup \end{array}\right)$

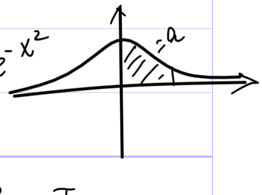
$F'(0) = 1$

$\int_0^a F' = F(a)$ $\quad \lim_{x\to+\infty} F = \frac{\sqrt{\pi}}{2}$ $\quad \lim_{x\to-\infty} = -\frac{\sqrt{\pi}}{2}$

F:



$F' = e^{-x^2}$



$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\cdot\int_0^x e^{-t^2}\,dt = \frac{2}{\sqrt{\pi}}\cdot F(x)$$

Other example of new funcs (cannot be expressed as primary terms)

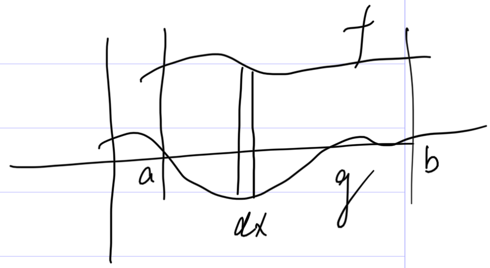
$$C(x) = \int_0^x \cos(t^2)\,dt$$
$$S(x) = \int_0^x \sin(t^2)\,dt$$
Fresnel Integrals

$$H(x) = \int_0^x \frac{\sin t}{t}\,dt$$
$$Li(x) = \int_2^x \frac{dt}{\ln t}$$
$$Li(x) \approx \#\text{primes in } (0, x)$$

Areas Between Curves

Area = $\int_a^b [f(x) - g(x)]\,dx$

limits integrand

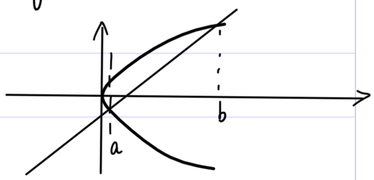


Set the $\int$ up w. limits and solve $\int$.

Ex: Find area btw. $x = y^2$ & $y = x-2$

A: $A = \int_0^a [\sqrt{x} - (-\sqrt{x})]\,dx + \int_a^b [\sqrt{x} - (x-2)]\,dx$

$\begin{cases} x = y^2 \\ y = x-2 \end{cases} \Rightarrow \begin{cases} a \\ b \end{cases}$



Or $A = \int_{y=a_y}^{y=b_y} x_{line} - x_{para}\,dy = \int_{a_y}^{b_y} [(y+2) - y^2]\,dy$

(reverse the roles of x & y)