20210926 4:00 pm.

$\sec = \frac{1}{\cos}$ $\tan = \frac{\sin}{\cos}$

$\csc = \frac{1}{\sin}$ $\text{ctn} = \frac{\cos}{\sin}$

$\sec^2 = 1 + \tan^2$

$\tan' = \sec^2$ *1

$\sec' = \sec\tan$ *2

$\int \tan x \, dx = -\ln(\cos x) + C$

$\int \sec x \, dx = \ln(\sec x + \tan x) + C$

$\left\{ \begin{array}{l} *1 \\ *2 \end{array} \right. \Rightarrow$ $(\sec x + \tan x)' = \sec\cdot(\sec + \tan)$

$u' = \sec \cdot u$

$\frac{u'}{u} = \sec$

logrithmic derivative. $(\ln u)' = \sec$

$\int \sec = \ln u + C$

$= \ln(\sec + \tan) + C$

---

now we can $\int \text{any}(\sin, \cos, \tan, \sec)^n \ldots$

Ex: $\int \sec^4 x \, dx$

$= \int (1 + \tan^2 x) \sec^2 x \, dx$

$= \int (1 + \tan^2 x) \, d\tan x$

let $u = \tan x$

$= \int (1 + u^2) \, du$

$= u + u^3/3$

$= \tan x + \tan^3 x / 3$

More trig. sub. integrals.

Ex: $\int \frac{dx}{x^2\underbrace{\sqrt{1+x^2}}_{\text{ugly, better } \sqrt{\square^2}\text{, e.g. } \sqrt{\sec^2}}}$ → looked like $1+\tan^2$ ($=\sec^2$)

let $x=\tan u$. $dx=\sec^2 u\,du$

$$\int \frac{\sec^2 u\,du}{\tan^2 u\sqrt{1+\tan^2 u}}$$

$$= \int \frac{\sec u}{\tan^2 u}\,du$$

$$= \int \frac{\cos u}{\sin^2 u}\,du$$

← something's differencial. (total 2 ways that du can combine with.) try both way.

← complex, get rid of

let $v=\sin u$ $dv=\cos u\,du$

$$= \int \frac{dv}{v^2}$$

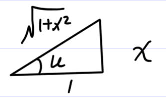
$$= v^{-1}/-1 + C$$

$$= -\frac{1}{v} + C$$

$$= -\frac{1}{\sin u} + C$$

$$= -\frac{\sqrt{1+x^2}}{x} + C$$

$x = \tan u$

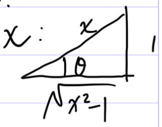


$\sin u = \frac{x}{\sqrt{1+x^2}}$

$\underbrace{\sqcup\,(\text{arc}\,\underbrace{\sqcup\ x)}_{\text{angle}}}_{\text{solveable.}}$

e.g. $\tan \text{arc}\csc x$:

$\theta \Rightarrow \csc\theta = x$:



$\tan\theta = \frac{1}{\sqrt{x-1}}$

Summary of Trig Subs.

If integrand contains	make substitution	to target
$\sqrt{a^2-x^2}$	$x=a\sin\theta$ or $x=a\cos\theta$	$a\sin\theta$ $a\cos\theta$
$\sqrt{a^2+x^2}$	$x=a\tan\theta$	$a\sec\theta$
$\sqrt{x^2-a^2}$	$x=a\sec\theta$	$a\tan\theta$

$(\tan')=\sec^2=1+\tan^2$ $(1+x^2)$

$\sec^2-1=\tan^2$ (x^2-1)

$\tan^2=1-\sec^2$ $(1-x^2)$ (also $1-\sin^2=\cos^2$)

Completing the square

Ex $\displaystyle\int \frac{dx}{\sqrt{x^2+4x}}$

Game: rewrite quadratic as $(x+a)^2+c$

$x^2+4x=(x+2)^2-4 \Rightarrow \square^2-a$

A: let $x+2=2\sec\theta$

$dx=2\cdot\sec\theta\cdot\tan\theta\,d\theta$

$$\int\frac{dx}{\sqrt{(x+2)^2-4}}$$

$$=\int\frac{2\sec\theta\tan\theta\,d\theta}{\sqrt{2^2\cdot\sec^2\theta-4}}$$

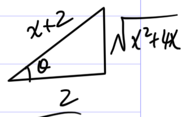
$$=\int\frac{2\sec\theta\tan\theta\,d\theta}{\sqrt{4\cdot\tan^2\theta}}$$

$$=\int\sec\theta\,d\theta$$

$$=\ln(\sec\theta+\tan\theta)+C$$

$$=\ln\left(\frac{x+2+\sqrt{x^2+4x}}{2}\right)+C$$

$\frac{x+2}{2}=\sec\theta$



$\frac{x+2}{2}+\frac{\sqrt{x^2+4x}}{2}$

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