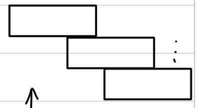
20211003 2:45

C_N : center of mass of $\underrightarrow{x}$.



does this fall down ?

$C_{N+1} = N \cdot C_N + 1(C_N+1)$ weighted avg.

$= C_N + \frac{1}{N+1}$

$C_1 = 1$

$C_2 = 1 + \frac{1}{2}$

$C_3 = 1 + \frac{1}{2} + \frac{1}{3}$

$C_N = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{N}$. $\iff$ $\int_1^N \frac{dx}{x} = \infty$

Series Example.

Power Series.

$$1+x+x^2+\cdots+ \;=\; \frac{1}{1-x} \qquad (|x|<1)$$

$1.414\cdots \leftarrow \sqrt{2}$

General Power Series.

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$
$$= \sum_{n=0}^{\infty} a_n x^n$$

① $|x|<R$ (radius of convergence)

where series convergence.

② $|x|>R$

diverges.

(③. $|x|=R$ very delicate border line, not used by us)

Rules for convergent power series

are just like polynomials.

$f(x)+g(x)$, $f(x)\cdot g(x)$, $f(g(x))$, $f(x)/g(x)$,

$\frac{d}{dx}f(x)$, $\int f(x)dx$.

- $\frac{d}{dx}(a_0+a_1x+a_2x^2+\cdots) = a_1+2a_2x+3a_3x^2\cdots$
- $\int(a_0+a_1x+a_2x^2+\cdots) = C + a_0x + a_1x^2/2 + a_2x^3/3.$

Power Series & R of convergence.

Taylor's formula.

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

$\left(\begin{array}{l} \sim \sin x \ + - \\ \sim \cos x \ + - \end{array} \right)$

$$\left(e^1 = 1 + \frac{1}{1} + \frac{1}{2!} + \frac{1}{3!} + \cdots \right)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots$$

20211003 3:45

Taylor Expansion