Differential Equation

Ex: $\frac{dy}{dx} = f(x) \Rightarrow y = \int f(x)\,dx$

by 1. substitution

Ex2: $(\frac{d}{dx} + x)\,y = 0$

annihilation operator

A: $\frac{dy}{dx} = -xy$ ② sep. by x, y. $\frac{dy}{=y}$ hint is to $\int$.

$\frac{1}{y}\,dy = -x\,dx$

$\int \frac{1}{y}\,dy = \int -x\,dx$

$\ln y + C_1 = -\frac{1}{2}x^2 + C_2$ * $y > 0$ $\ln|y|$

$\ln y = -\frac{1}{2}x^2 + C$

$y = e^{-\frac{1}{2}x^2 + C}$

$= A e^{-\frac{1}{2}x^2}$

Separation of Variables

$\frac{dy}{dx} = f(x)\,g(y)$

$\frac{dy}{g(y)} = f(x)\,dx$

$H(y) = \int \frac{dy}{g(y)}$; $F(x) = \int f(x)\,dx$

$H(y) = F(x) + C$ → implicit form.

$y = H^{(-1)}(F(x) + C)$

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② Ex2 could have written

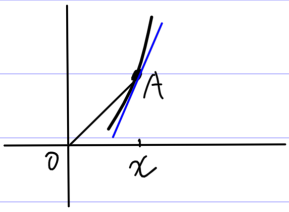
$\ln|y| = -\frac{1}{2}e^2 + C \quad (y \neq 0)$

$|y| = A e^{-\frac{1}{2}x^2}$

$y = \pm A e^{-\frac{1}{2}x^2}$

③ $y = 0$

Ex 3 : $\tan\theta = 2 \times OA$



$$\frac{dy}{dx} = 2 \cdot y/x$$

$$\frac{1}{y}dy = \frac{2}{x}dx$$

$$\int \frac{1}{y}dy = \int \frac{2}{x}dx$$

$$\ln|y| = 2\ln|x| + C$$

$$|y| = e^{\ln|x|^2 + C}$$

$$y = \pm A x^2$$

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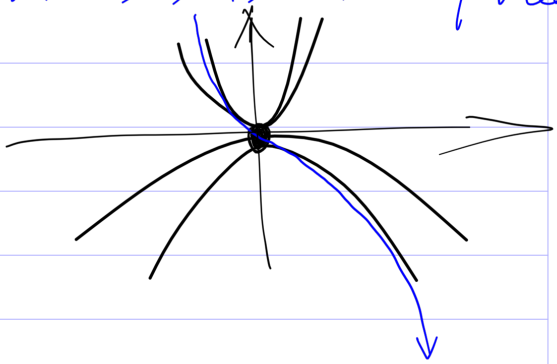
note slope at (0,0) is not defined.



Ex 4 : Find curves $\perp$ to parabolas.

$y = ax^2$

$y' = 2ax$

$y'u' = -1 \Rightarrow 2ax \cdot u'_x = -1$

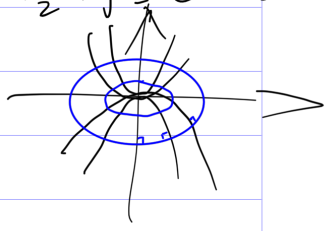
has common var x so can join.

$$\frac{dy}{dx} = \frac{-1}{2(yx)} \quad (y \neq 0)$$

$$2y\,dy = -x\,dx$$

$$y^2 = -\frac{x^2}{2} + C$$

$$\frac{x^2}{2} + y^2 = C = a^2$$



UNIT 2 EXAM

1. Linear / Quadratic Approx.
2. sketch a graph
3. MAX/MIN
4. Related Rates
5. Antiderivative + solve O.D.E. (ordinary differential equation)
6. MVT

info of f' tell's info about f.