

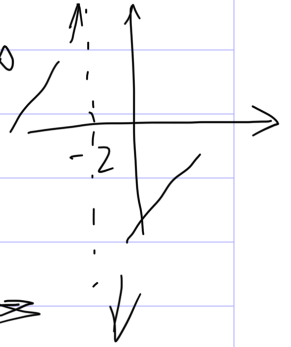
~ sketching

$$Z_x: f(x) = \frac{x+1}{x+2}, \quad f'(x) = \frac{1}{(x+2)^2} > 0$$

A: 0 no critical point, plot points not there.

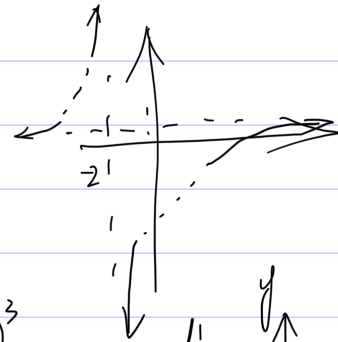
$$f(-2^+) = \frac{-2^++1}{-2^++2} = \frac{-1}{0^+} = -\infty$$

$$f(-2^-) = \frac{-2^-+1}{-2^-+2} = \frac{-1}{0^-} = +\infty$$



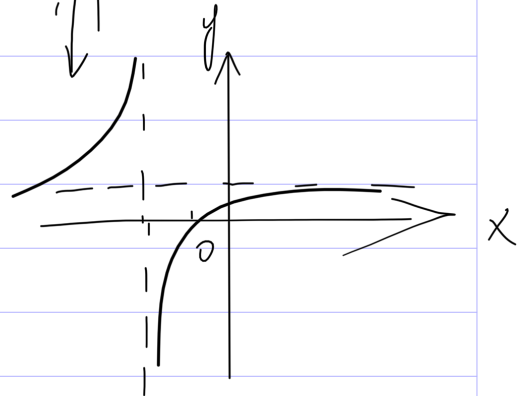
② ends  $f(+\infty) \rightarrow 1$

$$f(-\infty) \rightarrow 1$$



③  $f''(x) = -2 \frac{1}{(x+2)^3}$

$$\begin{array}{c|c} + & - \\ \hline & -2 \\ \hline - & + \end{array}$$



# General Strategy for Sketching

1. plot
  - a) discontinuities
  - b) end points ( $x \rightarrow \pm\infty$ )
  - c) easy pts (optional)
2.
  - a) solve  $f'(x) = 0$
  - b) plot critical pts & values
3. Decide  $f' \geq 0$  on intervals sep. by Cpts.
4.  $f'' \geq 0$  concave up/down.  
 $= 0$  inflection points
5. combine everything.

① 简单点.

② 间断点,  $\pm\infty$

③  $f' = 0$  各区间趋势  $\wedge \vee$

④  $f'' = 0$  各区间形状  $\cap \cup$

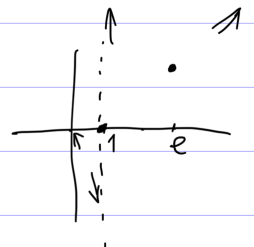
Ex.  $f(x) = \frac{x}{\ln x}$ ,  $x > 0$

A: ①  $f(1) = \frac{1}{\ln 1} = \infty \xrightarrow{x \rightarrow 1^+} \infty$

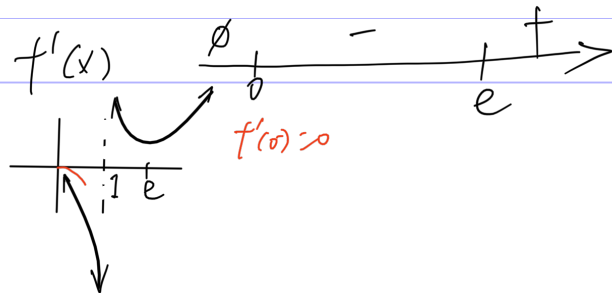
$f(0^+) = \frac{0^+}{\ln 0^+} = \frac{0^+}{-\infty} = 0^-$

$f(e) = e$

$f(+\infty) = \frac{+\infty}{\ln(+\infty)} = +\infty$



②.  $f'(x) = \frac{\ln x - x \cdot \frac{1}{x}}{\ln^2 x} = \frac{\ln x - 1}{\ln^2 x}$ , sep.  $\frac{\square}{\square} = 0 \Rightarrow x = e$   
 $\frac{\square}{\square} = 0 \Rightarrow x = 1$   
 $\Delta^2 \geq 0$



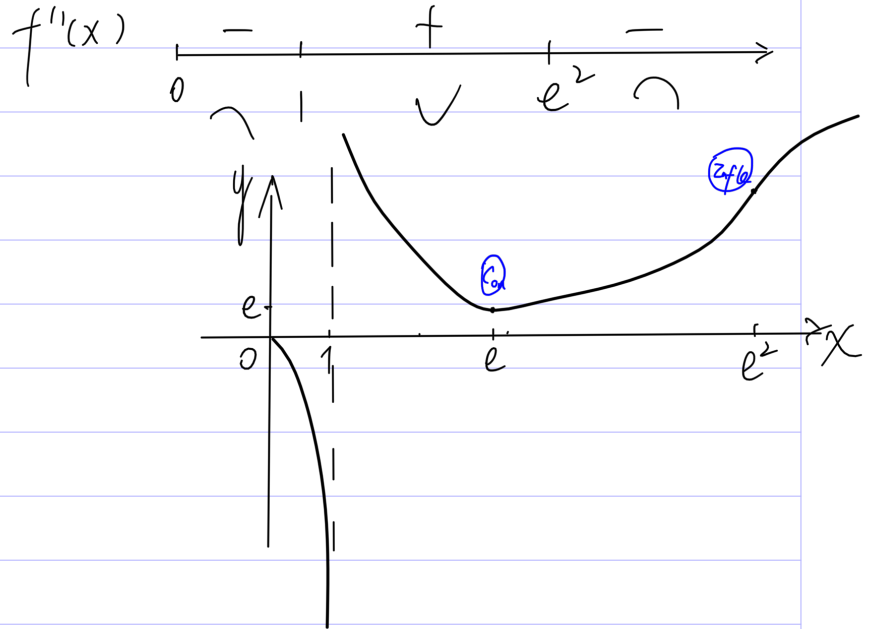
Steps to do curve sketch

$$\textcircled{12}. f''(x) = \left( \frac{1}{\ln x} - \frac{1}{\ln^2 x} \right)' = -\frac{1}{x \cdot \ln^2 x} - (-2) \cdot \frac{1}{x \ln^3 x}$$

$$= \frac{2 - \ln x}{x \ln^3 x}$$

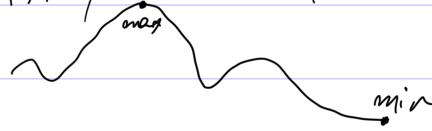
$$\frac{f''}{x} = 0 \quad 2 - \ln x = 0 \Rightarrow \ln x = 2 \quad x = e^2$$

$$\frac{f''}{x} = 0 \quad x \ln^3 x = 0 \Rightarrow x = 0 \parallel x = 1$$



example

# MAX / MIN Problems.



only need to look at Critical points  
& end points  
& discontinuity

max / min points