Fundamental Theorem of Calculus (FTC1)

If $F_x' = f(x)$, then $\int_a^b f(x)dx = F(b) - F(a)$

Notation: $F(b) - F(a) = F(x)\Big|_a^b = F(x)\Big|_{x=a}^{x=b}$

Ex: $F(x) = \frac{x^3}{3}$

$F'(x) = x^2$

$\Rightarrow$ (FTC) $\int_a^b x^2 dx = F(x)\Big|_a^b = \frac{b^3}{3} - \frac{a^3}{3}$

Ex2: Area under 1 hump of $\sin x$.

$\int_0^\pi \sin x\, dx = -\cos x\Big|_0^\pi = 2$

Ex3: $\int_0^1 x^{100} dx$

$= \frac{1}{101} x^{101}\Big|_0^1 = \frac{1}{101}$

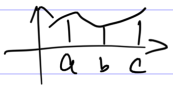
Intuitive interpretation of FTC 1:

$\int_a^b v(t)dt = L(b) - L(a)$

Extend integration to the case $f < 0$

$\int_0^{2\pi} \sin x\, dx = 0$

Properties of intergals.

1. $\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
2. $\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$
3. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$ 

4. $\int_a^a f(x)\,dx = 0$
5. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
6. (estimation) If $f(x) \le g(x)$ then $\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$ $(a<b)$

Ex: $e^x \ge 1 \quad (x \ge 0)$

$\Rightarrow \int_0^b e^x\,dx \ge \int_0^b 1\,dx$?

$\Rightarrow e^x\big|_0^b \ge x\big|_0^b$

$\Rightarrow e^b - 1 \ge b$

$\Rightarrow e^b \ge 1+b \quad (b \ge 0)$

Ex2: repeat Ex1, when $e^b \ge 1+b$ then $e^b \ge 1+b+\frac{b^2}{2}$

⋮

T. $\int_{u_1}^{u_2} g(u)\,du = \int_{x_1}^{x_2} g(u(x))\cdot u'(x)\,dx \quad (u_1 = u(x_1),\ u_2 = u(x_2))$

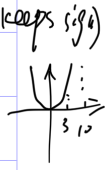
↑ only works if $u'(x)$ doesn't change sign.

單調可換元

Ex: $\int_1^2 (x^3+2)^5 x^2\,dx$

$u = x^3+2 \qquad \frac{du}{3} = x^2\,dx$

$(u' = 3x^2\big|_{x=3}^{x=10}$ keeps sign)



$\int_{u=3}^{u=10} u^5 \cdot \frac{1}{3}\,du$

$= \frac{1}{18} u^6\big|_3^{10}$

$= 10^6/18 - 3^6/18$

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