Appl. of Diff.

1. linear approx.

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$x \approx x_0$

proof: $\frac{\Delta f}{\Delta x} \approx f'(x_0)$

$$\Delta f \approx f'(x_0)\,\Delta x$$

$$f(x) - f(x_0) \approx f'(x_0)(x - x_0)$$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$(x_0 = 0)$ $f(x) \approx f(0) + f'(0)x$

$x \approx 0$

	$f(0)$	$f(x)$	$f'(0)$
$\sin x$	0	$\cos x$	1
$\cos x$	1	$-\sin x$	0
e^x	1	e^x	1

$\sin x \approx 0 + x$ $\cos x = 1 - 0 \cdot x$

$e^x \approx 1 + x \cdot 1$

near 0 : $\sin x \approx x$ $\cos x \approx 1$ $e^x \approx 1 + x$

	$f(0)$	$f'(x)$	$f'(0)$	
$\ln(1+x)$	0	$\frac{1}{1+x}$	1	near 0
$(1+x)^r$	1	$r(1+x)^{r-1}$	r	

$\ln(1+x) \approx x \Leftarrow 0 + 1\cdot x$

$(1+x)^r \approx 1+rx \Leftarrow 1 + rx$

Ex 2 $\ln 1.1 = \ln(1+x)\big|_{0.1} \approx x\big|_{0.1} = 0.1$

hard — easy linear

Ex 3: $\dfrac{e^{-3x}}{\sqrt{1+x}}$ near 0.

$e^x \approx 1+x$

$(1+x)^{\frac{1}{2}} \approx 1+\frac{1}{2}x$

$\approx \dfrac{1-3x}{1+\frac{1}{2}x}$ ①

Also: $e^{-3x}\cdot(1+x)^{-\frac{1}{2}}$

$\approx (1-3x)(1-\frac{1}{2}x)$ ②

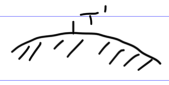
$= 1-3.5x+1.5x^2$

$\approx 1-3.5x$

① also $\approx$ ②

this is for approx, and ② is easier for computer to calc.

$\oint T$



$T' = \dfrac{T}{\sqrt{1-v^2/c^2}}$

$= T\cdot(1-v^2/c^2)^{-\frac{1}{2}}$

$\approx T\cdot\left(1+\frac{1}{2}\cdot\frac{v^2}{c^2}\right)$, $(v^2 \approx 0)$

real life: $v = 4\,km/s$

$c = 3\times10^8\,m/s$

Quadratic Approx.

$$f(x) \approx f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2\cdot 1}(x-x_0)^2$$

$$(x_0 = 0)\cdot\ f(x) \approx f(x_0) + f'(x_0)x + \frac{f''(x_0)}{2\cdot 1}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

Ex2. $\ln(1.1) = \ln(1+0.1) \approx 0.1 - \frac{(0.1)^2}{2}$