~ Power Series Ops & Apps

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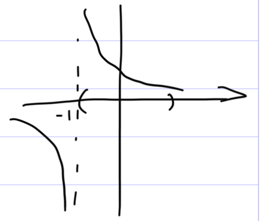
$f(x) = a_0 + a_1 x^1 + a_2 x^2 + \cdots$ ($|x| < R$, sum convergence)

(e.g. polynomial $f(x) = a_0 + a_1 x + \cdots + a_n x^n + 0 \cdot x^{n+1} + \cdots$)

Ex1. $e^x = 1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ ($R = \infty$)

Ex2. $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$ ($R = 1$)

Ex3. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots$ ($R = \infty$)



R: x fixed, $\left| \frac{x^{2n+1}}{(2n+1)!} \right|$

$= \frac{x}{1} \cdot \frac{x}{2} \cdot \frac{x}{3} \cdot \cdots \cdot \frac{x}{2n+1}$ ($n \to \infty$) $R = |-1| = 1$

$= 0$

$\to 0$ for any value of x. $\Rightarrow R = \infty$.

Common func's power series expansion.

Ops. (old series → new series)

① mul

e.g. $x\sin x = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} \cdots$ ($R=\infty$, smaller of R_1, R_2)

② diff

$\sin' x$ ($R = R_1$ when diff)

$= (x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots)'$

$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \cdots$

$= \cos x$

③ Inte

$\ln(1+x) = \int_0^x \frac{dt}{1+t}$ $(x>-1)$ $(R=1)$

$= \int_0^x (1-t+t^2-t^3\cdots)\,dt$ $|t^n| \to 0,\ t<1.\ R=1.$

$= x - x^2/2 + x^3/3 - x^4/4 \cdots$

④. Substitute

e^{-t^2}

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots$

$= 1 - t^2 + \frac{(t^2)^2}{2!} - \frac{(t^2)^3}{3!} \cdots$

$= 1 - t^2 + \frac{t^4}{2!} - \frac{t^6}{3!} \cdots$

$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt$ (so that $\lim_{x\to\infty} \mathrm{erf}(x) = 1$)

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Operations to power series.