# Lec 5

(chain-rule) $\Rightarrow$ Implicit Diff

Ex: $\frac{d}{dx} x^a = a x^{a-1}$

$a = 0, \pm 1, \pm 2, \cdots$

today: $a = m/n$, $m, n \in N$.

$y = x^{m/n}$

$y^n = x^m$

let $t = y^n$

$t'_{(x)} = m x^{m-1} \cdot x'_{(x)}$

$t'_{(x)} = n y^{n-1} \cdot y'_{(x)}$

$n y^{n-1} \cdot y'_{(x)} = m x^{m-1}$

$$y' = \frac{m x^{m-1}}{n y^{n-1}}$$

$$= \frac{m}{n} \cdot \frac{x^{m-1}}{x^{m/n \cdot (n-1)}}$$

$$= \frac{m}{n} \cdot x^{(m-1) - \frac{m(n-1)}{n}}$$

$$= \frac{m}{n} x^{\frac{m}{n} - 1}$$

$$(x^{m/n})' = \frac{m}{n} x^{m/n - 1} \quad \Big| \; m, n \in N$$

Q: $\frac{d}{dx} y^n = \frac{d}{dx} x^m$

$$\left(\frac{d}{dy} y^n\right) \cdot \frac{dy}{dx} = m x^{m-1}$$

Non-Cal problem $\rightarrow$ $n y^{n-1} \cdot \frac{dy}{dx} = m x^{m-1}$

$$\frac{dy}{dx} = \frac{m}{n} \frac{x^{m-1}}{(x^{m/n})^{n-1}}$$

$\vdots$

Ex 2: $x^2 + y^2 = 1$

find D-to $x$: $2x \cdot 1 + 2y \cdot y'_{(x)} = 0$

$$y' = -\frac{x}{y}$$

$$= -\frac{x}{\sqrt{1-x^2}}$$

Ex 3: $y^4 + xy^2 - 2 = 0$

Diff on both sides: $4y^3 \cdot y' + 1 \cdot y^2 + x \cdot 2y \cdot y' = 0$

$$(4y^2 + 2x) y' + y = 0$$

$$y' = -\frac{y}{4y^2 + 2x} \qquad \left(y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

Solve using Implicit Diff.

# Deri of Inversed functions

Ex: $y = \tan^{-1} x$

$\frac{d}{dx}(x = \tan y)$

$1 = \left(\frac{\sin y}{\cos y}\right)'$

$1 = \frac{y' \cos^2 y + \sin^2 y \cdot y'}{\cos^2 y}$

$\cos^2 y = y'$

$y' = \cos^2(\tan^{-1} x)$

$\frac{d}{dy}(x = \tan y)$

$x'_{(y)} = \frac{1}{\cos^2 y}$

$y = \tan x$

$y' = \frac{\sin x}{\cos x}$

$y' = \frac{1}{\cos^2 x}$

$$\frac{d}{dy} \tan y = \frac{d}{dy} \frac{\sin y}{\cos y}$$

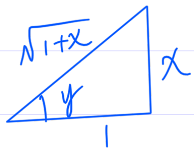
$$= \frac{\frac{d}{dy} \sin y \cdot \cos y - \frac{d}{dy} \cos y \cdot \sin y}{\cos^2 y}$$

$$= \frac{1}{\cos^2 y} = \sec^2 y$$

$$\frac{d}{dx}(\tan y = x)$$

$$\Rightarrow \frac{d}{dy} \tan y \cdot \frac{dy}{dx} = 1$$

$$\frac{1}{\cos^2 y} y'_{(x)} = 1$$




$$y'_{(x)} = \cos^2 y$$

$$y'_{(x)} = \cos^2(\tan^{-1} x)$$

$$= \frac{1}{1+x}$$