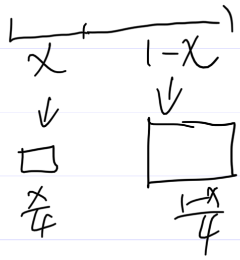
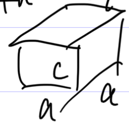
~ MAX / MIN , Related Rates

Ex 1



have max areas.

Ex 2: Fixed V, find topless min surface area, with a square bottom



$a^2 c = V$

$f = 4ac + a^2$

$f = 4a \cdot \frac{V}{a^2} + a^2$

$f = 4\frac{V}{a} + a^2$

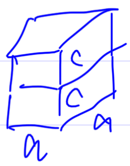
$f' = 2a - 4V/a^2$

let $2a - 4V/a^2 = 0$

$2a^3 = 4V$

$a = \sqrt[3]{2V}$ $\quad$ $C = \frac{V}{a^2} = V \cdot (2V)^{-\frac{2}{3}} = 2^{-\frac{2}{3}} \cdot V^{\frac{1}{3}}$

$C = 1/2\, a$



$a \in (0, +\infty)$

$f'' = 2 + 8V/a^3$

cp: $f''(\sqrt[3]{2V}) = 6$

e: $f(+\infty) = +\infty$

e+ø: $f(0^+) = +\infty$

min at $a = \sqrt[3]{2V}$

$f = 4\frac{V}{\sqrt[3]{2V}} + (2V)^{\frac{2}{3}}$

$= 2^{-\frac{1}{3}} \cdot 4 \cdot V^{\frac{2}{3}} + 2^{\frac{2}{3}} \cdot V^{\frac{2}{3}}$

$= (2^{\frac{5}{3}} + 2^{\frac{2}{3}}) \cdot V^{\frac{2}{3}}$

$= 3 \cdot 2^{\frac{2}{3}} \cdot V^{\frac{2}{3}}$

Ex2 by implicit diff

$V = x^2 y$, $A = x^2 + 4xy$

Goal: find min A while V is const.

$A'_{(x)} = 2x + 4y + 4x\,y'_x$

y'_x: 导数(变化率)在两变量存在的情况下才有意义.

$$\begin{cases} 0 = 2xy + x^2 y'_x \\ -2y = xy' \end{cases}$$

$A'_{(x)} = 2x + 4y + 4\cdot(-2y)$

$= 2x + 4y - 8y$

$= 2x - 4y$ $\qquad 2x = 4y,\ x = 2y$

$= 2x - 4\frac{V}{x^2}$ $\qquad \uparrow$

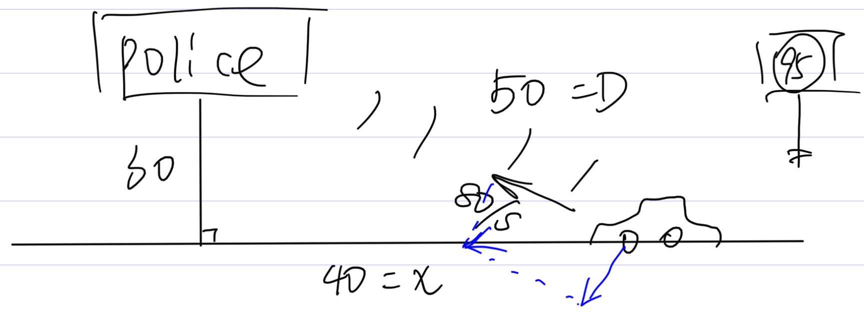
let $A' = 0$, then $x = \cdots$.

implicit method:

Good: faster, nicer.

Bad: didn't check endpoints & break pts.

Related Rates.



$$\frac{dD}{dt} = 80 \text{ ft/sec}$$

$$\frac{dx}{dt} = \frac{dx}{dD} \frac{dD}{dt}$$

$$= \frac{50}{40} \cdot 80$$

$$= 100 \text{ ft/s}$$