~ NEWTON'S METHOD

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$E_1 = |x - x_1|$ $\qquad$ $x^2 = 5$: $E_2 \sim E_1^2 \dots$

$E_2 = |x - x_2|$

$\vdots$

$E_n = |x - x_n|$

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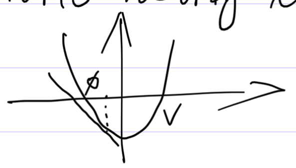
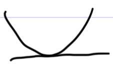
works (very) well if ① $|f'|$ not small and ② $|f''|$ not too big and ③ x_0 starts nearby x.

~~③~~ :

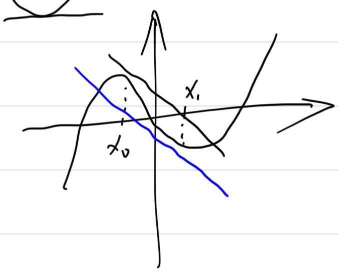


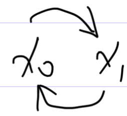
run program and trial.

~~①~~ : $f'(x_0) = 0$



~~②~~ :





Mean Value Theorem

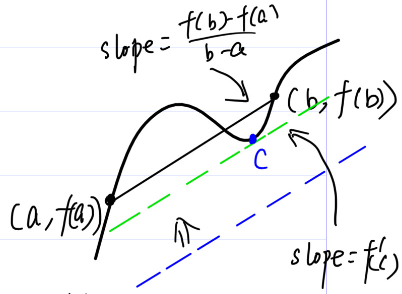
$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

for some c, $a<c<b$.

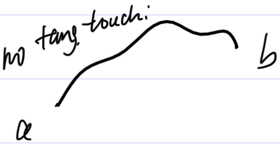
$\Uparrow$

provided f is differenciable in (a,b)
and f is continuous in $[a,b]$.

Proof: ①



② . if no tang touch:



then move from above.

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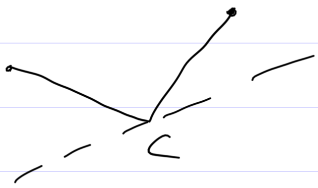
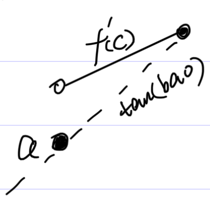
hypothesis ①:

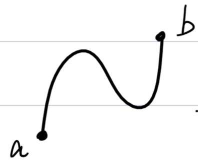
no $f'(c)$ exists



hy ② : f' exists in $(a,b) \Rightarrow f$ conti in (a,b)
need to conti at a & b or else:



but allow f' not exist at a & b:



still works

MVT App to graphing

1. If $f' > 0$, then f is increasing.
2. If $f' < 0$, then f is decreasing.
3. If $f' = 0$, then f is constant.

Proof: for any a, b. (let's assume $a < b$)

$$\frac{f(b) - f(a)}{b-a} = f'(c)$$

$$f(b) - f(a) = f'(c)(b-a)$$

$$f(b) = f(a) + f'(c)(b-a)$$

$\oplus$

$f'(c) > 0 \Rightarrow f(b) > f(a)$
$f'(c) < 0 \Rightarrow f(b) < f(a)$
$f'(c) = 0 \Rightarrow f(b) = f(a)$

for any p. a, b.

comp with linear approx.

$\frac{\Delta f}{\Delta x} \approx f'(a)$, b near a $b-a = \Delta x$

$\frac{\Delta f}{\Delta x} = f'(c)$, some c between a, b

L: avg speed $\approx$ initial speed.

MVT: min $\leq$ avg speed $\leq$ max

$$\min \leq \frac{f(b) - f(a)}{b-a} \left(= f'(c)\right) \leq \max$$

Inequalities

1. $e^x > 1+x \quad (x>0)$

Proof: let $f = e^x - x - 1$

$f' = e^x - 1 \quad > 0$ when $x>0$.

$\left.\begin{array}{l} \text{MVT} \Rightarrow f \nearrow \\ f(0) = e^0 - 0 - 1 = 0 \end{array}\right\} \Rightarrow e^x - x - 1 > 0 \Rightarrow e^x > 1+x$

$f(x) > f(0)$

2. $e^x > 1 + x + \frac{x^2}{2} \quad (x>0)$ $\quad f > f(0) = 0$

$f' = e^x - 1 - x = g \qquad f' > f'(0) = 0$

$g' = e^x - 1 \quad > 0$

3. $e^x > 1 + x + \frac{x^2}{2} + \frac{x^3}{3\cdot 2\cdot 1} + \cdots$