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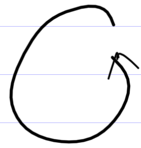
~ Parametric Curves

$x = a\cos t$

$y = a\sin t$



arclength:

$$L = \int ds$$

$$= \int \sqrt{dx^2 + dy^2}$$

$dx = -a\sin t\, dt$

$dy = a\cos t\, dt$

$$= \int \sqrt{a^2\sin^2 t\, dt^2 + a^2\cos^2 t\, dt^2}$$

$$= \int a\, dt$$

$$= at$$

$$ds^2 = dx^2 + dy^2$$

$$ds = \sqrt{\left(\frac{dx}{dt}dt\right)^2 + \left(\frac{dy}{dt}dt\right)^2}$$

$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$$

$ds/dt = a$ (speed) 1 rad / sec.

$\begin{cases} x = a\cos kt \\ y = a\sin kt \end{cases} \Rightarrow \frac{ds}{dt} = ak$. faster. $k = 1, 2, 3 \cdots$ fourier.

notation:

$$\Delta s^2 \approx \Delta x^2 + \Delta y^2$$

$$\left(\frac{\Delta s}{\Delta t}\right)^2 \approx \left(\frac{\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2$$

$$\left(\frac{ds}{dt}\right)^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$$

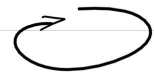
$\left(\frac{dx}{dt}\right)^2$ is not $\cancel{\frac{dx^2}{dt^2}}$

$\frac{d^2x}{dt^2}$: second derivative. $= \left(\frac{d}{dx}\right)^2 x$

example . notation convention

Ex 2: $x = 2\sin t$

$y = \cos t$



$$dL = \sqrt{{x'_t}^2 + {y'_t}^2}\, dt$$

$$= \sqrt{4\cos^2 t + \sin^2 t}\, dt$$

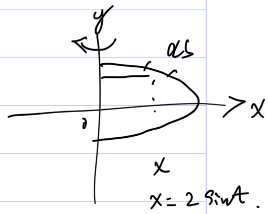
$$= \sqrt{3\cos^2 t + 1}\, dt$$

$$\text{arc length} = \int_0^{2\pi} \sqrt{4\cos^2 t + \sin^2 t}\, dt$$

(not an elementary integral, is final ans)

Surface Area.

Ex 3: rotate of Ex 2 on y-axis



$$dA = 2\pi x \cdot ds$$

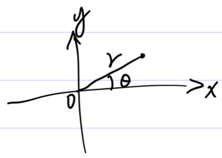
$$= 2\pi \cdot 2\sin t \cdot \sqrt{4\cos^2 t + \sin^2 t}\, dt$$

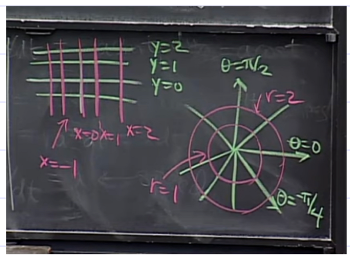
$(0 \le t \le \pi)$

can be calculated.

Polar Coordinates

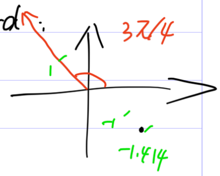
2D: $\boxed{\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}}$





Param fs formed line & area.

Ex 1. $(x,y) = (1,-1)$ in polar coord:



a) $r = \sqrt{2}$, $\theta = 7\pi/4$

b) $r = \sqrt{2}$, $\theta = -\pi/4$

c) $r = -\sqrt{2}$, $\theta = 3\pi/4$

Ex 2: $r = a$ (Circle)

Ex 3: $\theta = C$ (ray)

when $0 \le r < \infty$: ray

when $-\infty < r < \infty$: line

typical conventions:

$0 \le r < \infty$

$0 \le \theta < 2\pi$ (or $-\pi \le \theta < \pi$)

Ex 4: $y = 1$

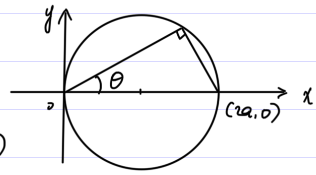
$y = r\sin\theta = 1$

$r = \frac{1}{\sin\theta}$ $(0 < \theta < \pi)$ | $r = r(\theta)$

Ex 5: off center circle.

$(x-a)^2 + y^2 = a^2$

$r = 2a\cos\theta$ $(-\pi \le \theta < \pi)$



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Polar coord get familiar.