2021 09 15

Derivative:

Rate of change.

Ex. $h = 80 - 5t^2$

1. $\frac{\Delta h}{\Delta t} = \frac{-80-0}{4-0} = -20 m/s$ (average speed)
2. $\frac{d}{dt} h = 0 - 10t$

$t = 4 \Rightarrow h' = -40 m/s$ (instant speed)

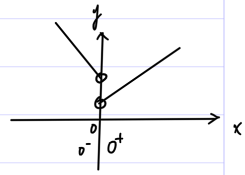
Ex. T. $\frac{dT}{dx}$: temperature gradient.

Limits v.s. Continuity

Limits:

$\lim_{x \to x_0^+} f(x)$: right-hand limit

$\lim_{x \to x_0^-} f(x)$: left-hand limit



Continuity:

DEF: f is continues at x_0 if:

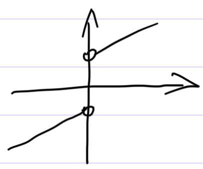
$\lim_{x \to x_0} f(x) = f(x_0)$.

which means:

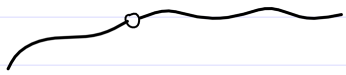
1. left-hand limit exists (can be written down).
2. right-hand limit exists (can be written down).
3. (moving value) tends to the fixed value: $f(x_0)$.

Types of dis-continuity:

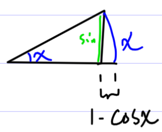
① Jump Dis-C



② Removable Dis-C

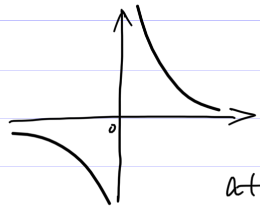


e.g. $\lim_{x \to 0} \frac{\sin x}{x} = 1$



$\lim_{x \to 0} \frac{1-\cos x}{x} = 0$

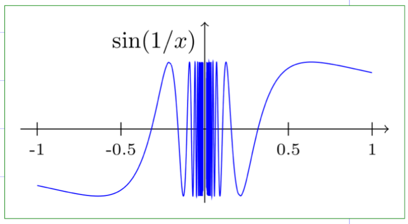
③ Infinite Dis-C



at $x=0$.

④ Other (Ugly) Dis-C

eg. $y = \sin\frac{1}{x}$ $(x \to 0)$



Diff $\Rightarrow$ Conti

: derivative exists

$\downarrow$

can be written down as $f'(x_0)$

$\downarrow$

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

: $\lim_{x \to x_0} f(x) = f(x_0)$

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proof:

$$\because \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x - x_0} \cdot 0 = 0$$

$$\lim_{x \to x_0} \frac{f(x)-f(x_0)}{x - x_0} \cdot \lim_{x \to x_0} (x - x_0) = 0$$

$$\lim_{x \to x_0} \left[\frac{f(x)-f(x_0)}{x - x_0} \cdot (x - x_0) \right] = 0$$

$$\lim_{x \to x_0} [f(x) - f(x_0)] = 0$$

$$\lim_{x \to x_0} f(x) - \lim_{x \to x_0} f(x_0) = 0$$

$$\lim_{x \to x_0} f(x) - f(x_0) = 0$$

$$\therefore \lim_{x \to x_0} f(x) = f(x_0)$$

E.O.P.

Or proof it reversely:

要使 $\lim_{x \to x_0} f(x) = f(x_0)$

$$\Leftarrow \lim_{x \to x_0} [f(x) - f(x_0)] = 0$$

$$\Leftarrow \lim_{x \to x_0} \left[\frac{f(x) - f(x_0)}{(x - x_0)} \cdot (x - x_0) \right] = 0$$

$$\Leftarrow \lim_{x \to x_0} \left[\frac{f(x) - f(x_0)}{x - x_0} \right] \cdot \lim_{x \to x_0} (x - x_0) = 0$$

$$\Leftarrow \sim \cdot \underbrace{0 = 0}$$ ✓ checked.

Forever true, even if "~" does not exist or is non-sense.

$$(\Leftarrow f'(x_0) \cdot 0 = 0)$$

End of proof.

all combinations :

	Differentiable	Continuous	
①	✓	✓	$\Rightarrow$ ✓ $\Leftarrow$ ✗ . See above proof.
	✓	✗	$\emptyset$, impossible.
	~~✗~~	~~✓~~	pre-Condiction says ✓ & focus on ✓ only
	~~✗~~	~~✗~~	pre-Condiction says ✓ & focus on ✓ only