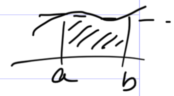
Comp: $F(b)-F(a) = \Delta F = \int_a^b f(x)\,dx$

$$\frac{\Delta F}{b-a} = \frac{1}{b-a}\cdot\int_a^b f(x)\,dx$$



$$\frac{\Delta F}{\Delta x} = \mathrm{Avg}(F')$$

$$\Delta F = \mathrm{Avg}(F')\,\Delta x \quad (FTC1)$$

$$(\min F')\Delta x \le \Delta F = F'(c)\,\Delta x \le (\max F')\Delta x \quad (MVT)$$

Exam2: $F' = \frac{1}{1+x}$. $F(0) = 1$

(1) By MVT, $A < F(4) < B$, find A, B.

A: some $0<c<4$, s.t. $F'(c) = \frac{F(4)-F(0)}{4-0}$

provided

$$\frac{1}{1+c} = \frac{F(4)-1}{4}$$

to solve at $x=4$

$$\begin{cases} \frac{4}{1+c}+1 = F(4) \\ 0<c<4 \end{cases}$$

at most we can expand $c \in (0,4)$.

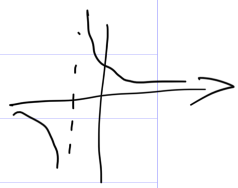
if $(0,5)$ $\frac{F(5)-F(0)}{5-0}$

we don't know $F(5)$

& we don't need to solve $F(5)$.

$$\frac{4}{1+4}+1 < F(4) < \frac{4}{1+0}+1$$

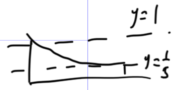
$$\underset{A}{\frac{9}{5}} < F(4) < \underset{B}{5}$$



(2) By FTC1, find A, B again.

A: for $(0,4)$. $\int_0^4 \frac{1}{1+x}\,dx = F(4)-F(0)$

$$\int_0^4 \frac{1}{1+4}\,dx < \quad < \int_0^4 \frac{1}{1+0}\,dx$$



$$\frac{x}{5}\Big|_0^4 < \quad < x\Big|_0^4$$

$$\frac{4}{5} < F(4)-1 < 4$$

$$\frac{9}{5} < F(4) < 5$$

FTC 2.

FTC 1：定积分 = 原函数的差

FTC 2：如果 $f=\int g$，那么他们是原函数和导数的关系 $f'=g$

(或：若 G 是 f 下逐渐积累起来的面积，那么 G' 就是 f.)

1. 如果有原函数，定积分可以用原函数来算.
2. 如果 f 连续，积分再求导互逆

If f is continuous,

and $G(x) = \int_a^x f(t)\,dt \quad (a \le t \le x)$

then $G'(x) = f(x)$.

Can always solve $\begin{cases} y' = f \\ y(a) = 0 \end{cases}$ by FTC2. ($\because G(a)=0$)

Ex : $\dfrac{d}{dx}\int_1^x \dfrac{dt}{t^2}$ $\quad$ guess $= \dfrac{1}{x^2}$? ✓

① first inte. $= \dfrac{d}{dx}\left(1 - \dfrac{1}{x}\right)$

$= +\dfrac{1}{x^2}$.

② Or use FTC2 :

Let $G(x) = \int_1^x \dfrac{dt}{t^2}$

then $G'(x) = \dfrac{1}{x^2}$

Proof of FTC2.

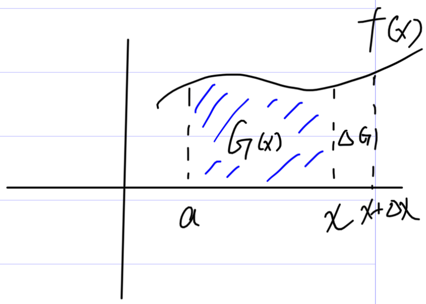
(given) $G = \int_a^x f(t)\,dt$ is Area :

$\Delta G \approx \Delta x \cdot f(x)$

$\dfrac{\Delta G}{\Delta x} \approx f(x)$

$G' = \lim\limits_{\Delta x \to 0} \dfrac{\Delta G}{\Delta x} = f(x)$ $\quad$ ($\because f$ is conti.)

Q.E.D.



Proof of FTC1 : $\quad$ (assume f is continous)

Given $F' = f(x)$.

P: cook up $G(x) = \int_a^x f(t)\,dt$ $\Big\}$ FTC2 $\Rightarrow$ $G'(x) = f(x) = F'$

$\Downarrow$ MVT

$F_x = G_x + C$

so $F(b) - F(a) = G(b) + C - (G(a) + C) = G(b) - G(a)$

$= \int_a^b f(t)\,dt - \int_a^a \cdots = \int_b^a f(t)\,dt$ $\quad$ Q.E.D.

Ex: $L'(x) = \frac{1}{x}$, $L(1) = 0$. $L = ?$

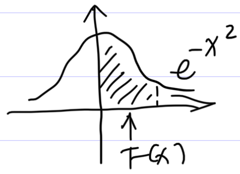
FTC 2 says the solution is $\left(\text{is it?} \ \Leftrightarrow ? \right)$

$$L(x) = \int_1^x \frac{1}{t}\,dt.$$

"new functions" ($\ln x$) by $\int$,
outside of x^n, a^x, sin, cos.

e.g. PDF: $y' = e^{-x^2}$, $y(0) = 0$

$$\Rightarrow F(x) = \int_0^x e^{-t^2}\,dt.$$



F(x) can not be expressed in terms of
log, exp, sin, cos, x^n, a^x.

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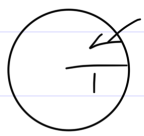
analog: "new number"



$\pi \neq \frac{m}{n}$