~ Polar Coordinates

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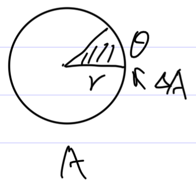
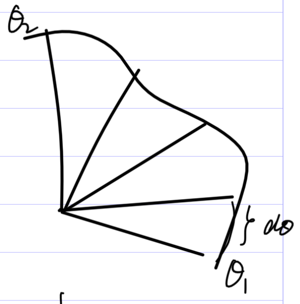
▷ ×1.5

$$\Delta A = \frac{1}{2} r^2 \theta$$

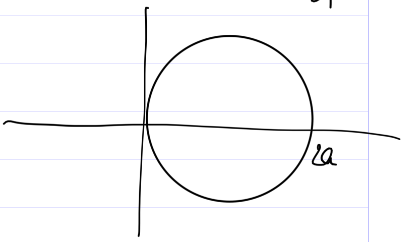


Ex: $r = r(\theta)$

$$A = \int dA$$
$$= \int \frac{1}{2} r^2 d\theta$$
$$= \int \frac{1}{2} r^2(\theta)\, d\theta$$



Ex: $r = 2a\cos\theta$



$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} r^2 d\theta$$
$$= \int_{\cdot}^{\cdot} \frac{1}{2} (2a\cos\theta)^2 d\theta$$
$$= 2a^2 \int_{\cdot}^{\cdot} \cos^2\theta\, d\theta$$
$$= 2a^2 \int_{\cdot}^{\cdot} \frac{1+\cos 2\theta}{2} d\theta$$
$$= a^2 \int_{\cdot}^{\cdot} (1+\cos 2\theta) d\theta$$
$$= a^2 \cdot \left[\theta + \frac{1}{2}\sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$
$$= a^2 \cdot \left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$$
$$= \pi a^2$$

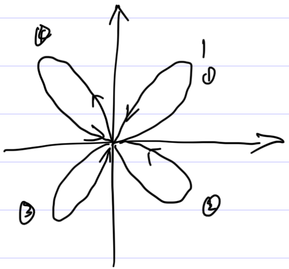
Ex 2. (drawing)

$r = \sin 2\theta$

$\theta: 0 \sim \frac{\pi}{2}$

$2\theta = 0 \sim \pi$

$r: 0 \overset{1}{\frown} 0$

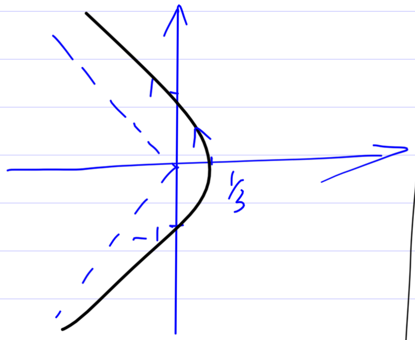


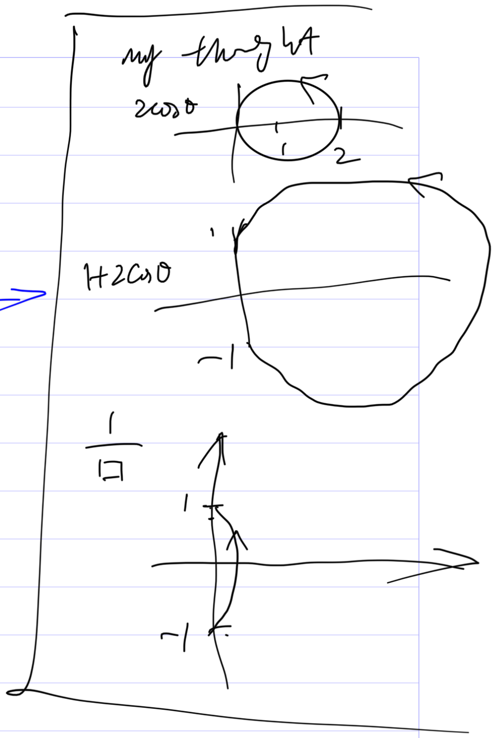
4-leaf rose

$$A = \int_0^{\frac{\pi}{2}} \frac{1}{2} r^2 d\theta$$
$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} \sin^2 2\theta\, d\theta$$
$$= \frac{1}{2} \int_{\cdot}^{\cdot} \frac{1-\cos 4\theta}{2} d\theta$$
$$= \frac{1}{4} \int_{\cdot}^{\cdot} (1-\cos 4\theta)\, d\theta$$
$$= \frac{1}{4} \cdot \left[\theta - \sin 4\theta / 4\right]_0^{\frac{\pi}{2}}$$
$$= \frac{1}{4} \cdot \frac{\pi}{2}$$
$$= \frac{\pi}{8}$$

Polar areas & drawing

Ex 3 : $r = \frac{1}{1+2\cos\theta}$





example

Ex: Find rectangular equation for $r = \frac{1}{1+2\cos\theta}$

$(r,\theta) \Rightarrow (x,y)$

A: $r + 2r\cos\theta = 1$

$r + 2x = 1$

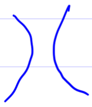
$r = 1 - 2x$

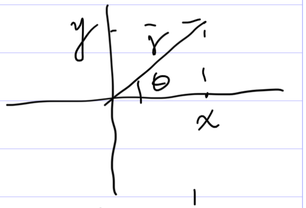
$r^2 = (1-2x)^2$

$x^2+y^2 = 1 + 4x^2 - 4x$

$-3x^2 + y^2 + 4x - 1 = 0$

(hypobala)





$\sqrt{x^2+y^2} = \frac{1}{1+2\cdot\frac{x}{\sqrt{x^2+y^2}}}$

r=0 is focus (center sun)

Kepler's law

$\frac{dA}{dt} = \text{const}$

$dA = \frac{1}{2}r^2 d\theta$

$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt}$

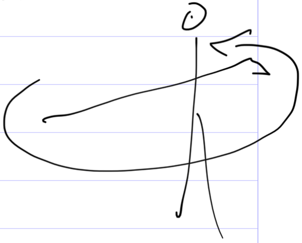
hence $r^2\frac{d\theta}{dt} = C$ (角动量守恒)

$(x,y) \Rightarrow (r,\theta)$:

$$\begin{cases} y = f(x) \\ x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$r\sin\theta = f(r\cos\theta)$

$\Rightarrow r = g(\theta)$



$(x,y) \Longleftrightarrow (r,\theta)$

Exam.

1. Techniques of integration.
   - trig subs
   - integration by parts $\int = uv - \int$
   - partial fractions $\frac{A}{} + \frac{B}{} \ldots$
2. parametric curves
   - arc length
   - area of surface of revolution
3. polar coordinates
   - including area

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ex: $\int x \tan^{-1} x \, dx$

$$\begin{cases} u = \tan^{-1} x, & u' = \frac{1}{1+x^2} \\ v = x^2/2 & v' = x \end{cases}$$

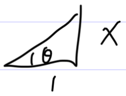
$(\tan^{-1} x)'$

$\theta = \tan^{-1} x$

$x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

$\frac{1}{1+x^2} \Leftarrow \cos^2\theta \Leftarrow \frac{1}{\sec^2\theta} = \frac{d\theta}{dx}$



$A = \tan^{-1} x \cdot x^2/2 -$

$\int \frac{1}{1+x^2} \cdot \frac{x^2}{2} \, dx$

$= \frac{x^2 \tan^{-1} x}{2} - 2\underbrace{\int \frac{x^2}{1+x^2} \, dx}$

$\Leftarrow$

$$\begin{array}{r} 1 \\ x^2+1 \overline{) \; x^2 \qquad} \\ x^2+1 \\ \hline -1 \end{array}$$

$\int \frac{x^2}{1+x^2} dx = \int (1 - \underbrace{\frac{1}{1+x^2}) \, dx}$

$\Leftarrow$

let $x = \tan\theta$, $dx = \sec^2\theta \, d\theta$

$\int \frac{1}{\sec^2\theta} \sec^2\theta \, d\theta$

$= \theta$

$= \tan^{-1} x$

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Test content