Exact Formula for RX-Differential Probability Through Modular Addition for All Rotations

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Outline

Rotational-XOR Cryptanalysis

Exact Probability Formula for all Rotations k

Modeling and Applications

- New best RX-trails for Alzette
- RX-backdoor from malicious constants - Malzette

Conclusions

Plan

Rotational-XOR Cryptanalysis

Exact Probability Formula for all Rotations k

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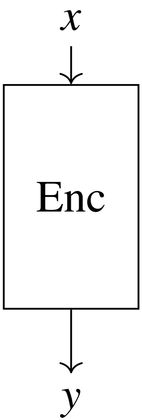
Rotational Cryptanalysis [Khovratovich and Nikolic 2010 FSE]



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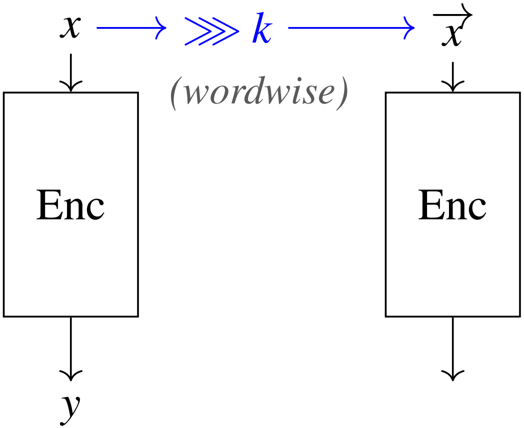

Rotational Cryptanalysis [Khovratovich and Nikolic 2010 FSE]

$$
\begin{tikzcd}
x \arrow[r, "\ggg k"] \arrow[d] & \overrightarrow{x} \arrow[d] \\
\mathrm{Enc} \arrow[d] & \mathrm{Enc} \arrow[d] \\
y \arrow[r, "\ggg k"] & \overrightarrow{y}\,?
\end{tikzcd}
$$

Rotational Cryptanalysis [Khovratovich and Nikolic 2010 FSE]

Through XOR

$$
\begin{tikzcd}
\overrightarrow{x} \arrow[d] & \\
\oplus \arrow[d] & \overrightarrow{y} \arrow[l] \\
\overrightarrow{x \oplus y} &
\end{tikzcd}
$$

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$$\begin{tikzcd} \overrightarrow{x} \arrow[d] & \\ \oplus \arrow[d] & \textcolor{red}{\overrightarrow{k}} \arrow[l] \\ \overrightarrow{x \oplus \textcolor{red}{k}} & \end{tikzcd}$$

Related-Key

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Related-Key

Through ADD

$$\begin{tikzcd}
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$$\underset{(k=\frac{n}{2})}{\frac{2}{8}} \leq p \leq \underset{(k=1)}{\frac{3}{8}} + \epsilon$$

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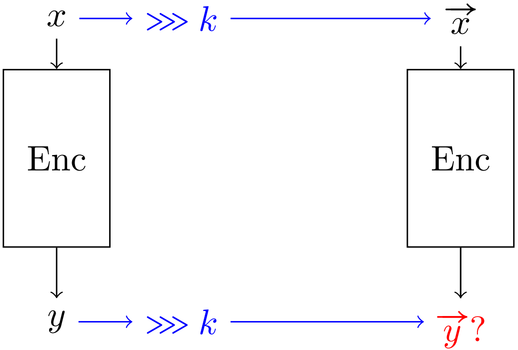
\end{tikzcd}$$

Rotational-XOR (RX) Cryptanalysis [Ashur and Liu 2016 FSE]



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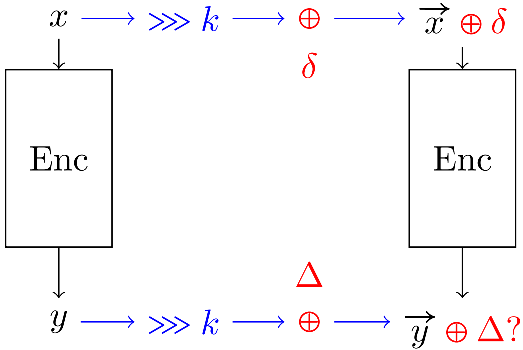

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Theorem ([AL16], $k = 1$)

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p = {} & \mathbb{1}_{(I \oplus \mathrm{SHL})(\chi_L) \oplus 1 \preccurlyeq \mathrm{SHL}(\nu_L)} \cdot 2^{-\mathrm{wt}(\mathrm{SHL}(\nu_L))} \cdot 2^{-3} \\
& + \mathbb{1}_{(I \oplus \mathrm{SHL})(\chi_L) \preccurlyeq \mathrm{SHL}(\nu_L)} \cdot 2^{-\mathrm{wt}(\mathrm{SHL}(\nu_L))} \cdot 2^{-1.415}
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where

$(\chi_L || \chi_0) = \alpha \oplus \beta \oplus \Delta$

$(\nu_L || \nu_0) = (\alpha \oplus \beta) \vee (\alpha \oplus \Delta)$ *(not-all-equal)*

SHL : shift left by 1 position (drop MSB)

$2^{-\mathrm{wt}(\mathsf{SHL}(\nu_L))}$ is a normal ARX differential prob. (excl. LSB)

Our contribution

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Ours: probability, any k

$$p = T_{n-k}(\chi_L, \nu_L, \chi_0) \times T_k(\chi_R, \nu_R, \chi_k)$$

$$T_m(\boldsymbol{\chi}, \boldsymbol{\nu}, \hat{\chi}_i) = 2^{-\mathrm{wt}(\mathsf{SHL}(\boldsymbol{\nu}))-1} + \mathbb{1}_{\boldsymbol{\chi} \in \{0\ldots0,1\ldots1\}} \times (-1)^{\hat{\chi}_i} \times 2^{-m-1}$$

Our contribution

[AL16], $k = 1$

Not fully correct:

$\exists$ class of transitions with probability

2x lower or **1.5x** higher

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large discrepancies with experiments,

imprecise validity condition

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Ours: validity, any k

$p > 0$ if and only if $u_i \leq v_i \quad \forall i \neq 0, k$

$$u = (I \oplus \mathsf{SHL})(\alpha \oplus \beta \oplus \Delta)$$
$$v = \mathsf{SHL}((\alpha \oplus \Delta) \vee (\beta \oplus \Delta))$$

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Extensively verified by experiments!

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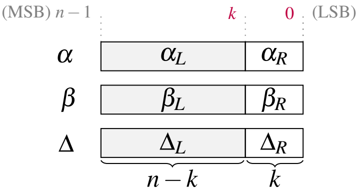
Our result (probability, any k)



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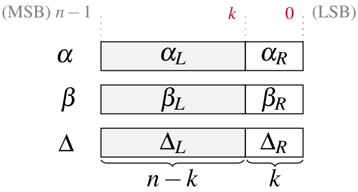
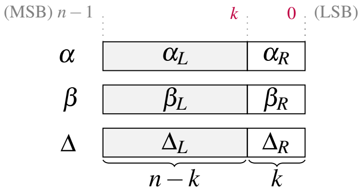


Theorem (Main, if $p > 0$)

$$p = T_{n-k}(\alpha_L, \beta_L, \Delta_L, \alpha_0 \oplus \beta_0 \oplus \Delta_0) \ \times \ T_k(\alpha_R, \beta_R, \Delta_R, \alpha_k \oplus \beta_k \oplus \Delta_k)$$



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where

$$T_m(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Delta}, \mathsf{w}) = 2^{-d-1} + \mathbb{1}_{\boldsymbol{\alpha}\oplus\boldsymbol{\beta}\oplus\boldsymbol{\Delta}\in\{0\ldots0,1\ldots1\}} \times (-1)^{\mathsf{w}} \times 2^{-m-1}$$

$$d = \mathrm{wt}(\mathsf{SHL}(\boldsymbol{\nu})) = \mathrm{wt}(\mathsf{SHL}((\boldsymbol{\alpha} \oplus \boldsymbol{\beta}) \vee (\boldsymbol{\alpha} \oplus \boldsymbol{\Delta})))$$

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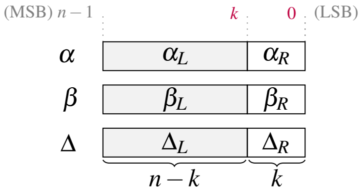

$$\chi = \alpha \oplus \beta \oplus \Delta$$

$$\nu = (\alpha \oplus \beta) \vee (\alpha \oplus \Delta)$$

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Our result (validity criterion, any k)

Theorem (RX-differential, $0 < k < n$)

$$p = \Pr\left[(\overleftarrow{x} \oplus \alpha) \boxplus (\overleftarrow{y} \oplus \beta) \;\oplus\; \overleftarrow{x \boxplus y} = \Delta\right] > 0$$

if and only if $u_i \leq v_i$ *for all* $i \neq 0, k,$ *where*

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Theorem (Normal differential ($k = 0$), Lipmaa and Moriai 2002)

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Impact of correction for $k = 1$

Theorem ([AL16], $k = 1$)

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p = {} & \mathbb{1}_{(I \oplus \mathrm{SHL})(\chi_L) \oplus \mathbf{1} \preccurlyeq \mathrm{SHL}(\nu_L)} && \cdot 2^{-\mathrm{wt}(\mathrm{SHL}(\nu_L))} \cdot 2^{-3} \\
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Thm [AL16] holds exactly when $\chi_L \notin \{0\dots0, 1\dots1\}$, *where* $(\chi_L || \chi_0) = \alpha \oplus \beta \oplus \Delta$.

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Conclusion: concrete trails are probably not affected, optimality claims do

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Exact Probability Formula for all Rotations k

Modeling and Applications

- New best RX-trails for Alzette
- RX-backdoor from malicious constants - Malzette

Conclusions

MILP Model

Model 1 - Heuristic (NEQ)

- Ignore the approximation factor: $p \approx 2^{-\operatorname{wt}\operatorname{SHL}\boldsymbol{\nu_L}-\operatorname{wt}\operatorname{SHL}\boldsymbol{\nu_R}-2}$
- A special case of the standard ARX model
- Bonus: model $[y = 1 \text{ if and only if } x_1 = \ldots = x_m]$ with 4 inequalities for any m

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Model 2 - Precise

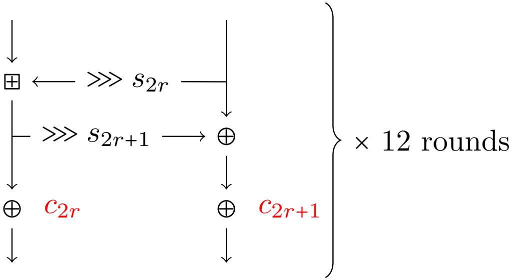
- Model the weight of the correction factor using logarithm tables (PieceWise-Linear constraints - PWL)
- "Flag" variables to determine if the correction is needed

Alzette (64-bit ARX-box, 4 32-bit modular additions)

	CASCADA,[LWRA17] ($k = 1$)	This work ($k = 1$)	This work ($k > 1$)	[HXW22] ($k = 1$)	[HXW22] ($k > 1$)
c_i	wt	wt	wt	wt	wt
c_0	**33.66**	**33.66**	33.93	37.66	43.00
c_1	**31.66**	**31.66**	33.01	38.66	-
c_2	37.66	37.66	**34.00**	52.66	-
c_3	38.66	38.66	**32.75**	45.66	-
c_4	35.66	35.66	**33.00**	45.66	-
c_5	32.66	33.66	**30.89**	44.66	-
c_6	**30.66**	**30.66**	32.97	40.66	-
c_7	37.66	37.66	**32.45**	49.66	-

(all values are $-\log_2 p$*)*

Malzette



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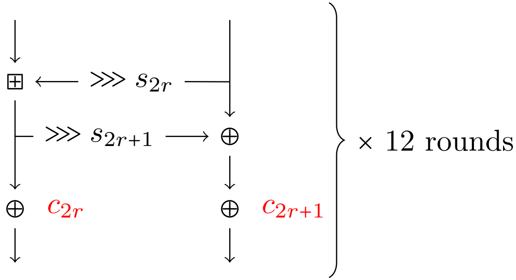
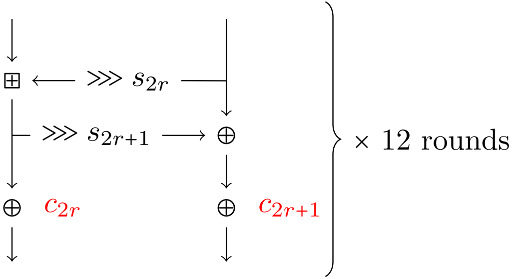


Round	Constants	$\log_2$(prob)
1	`1c71c924:249cad47`	-2.83
2	`49249c71:1249871c`	-1.83
3	`6db6c71c:5b127ffe`	-3.19
4	`38e39249:152ad249`	-1.83
5	`638e36db:649cad55`	-2.83
6	`1c71c7ff:471c9492`	-1.83
7	`36db6d55:63f1c71d`	-2.83
8	`471c7249:36a4ff1c`	-2.19
9	`4924938e:5b6c8e47`	-3.19
10	`2aab6db6:71c736db`	-1.83
11	`6db638e3:55b9c71d`	-2.83
12	`fb3d2330:b6da4b61`	-2.19
Total		-29.41

Malzette



- Diff./lin. lower bounds 2^{54} and 2^{38}
- RX-differential prob. $2^{-29.41}$ ($k = 3$)
- Verified experimentally

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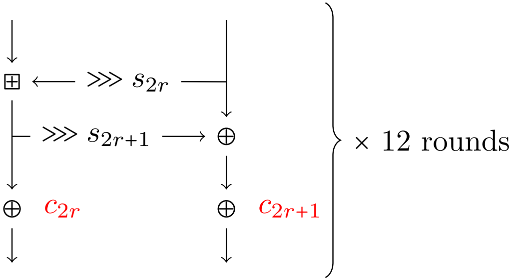
7	`36db6d55:63f1c71d`	-2.83
8	`471c7249:36a4ff1c`	-2.19
9	`4924938e:5b6c8e47`	-3.19
10	`2aab6db6:71c736db`	-1.83
11	`6db638e3:55b9c71d`	-2.83
12	`fb3d2330:b6da4b61`	-2.19
Total		-29.41

Malzette



- Diff./lin. lower bounds 2^{54} and 2^{38}
- RX-differential prob. $2^{-24.86}$ ($k = 3$)
- Verified experimentally

Round	Constants	$\log_2$(prob)
1	`00000000:4e381c1c`	-2.19
2	`2aaaaaaa:36dbe492`	-2.19
3	`7fffffff:1236db6c`	-1.83
4	`55555555:0763638e`	-1.83
5	`2aaaaaaa:1b6d4949`	-2.19
6	`55555555:638ef1c7`	-1.83
7	`00000000:47638e39`	-2.19
8	`2aaaaaaa:5236b6db`	-2.19
9	`55555555:4e381c1c`	-1.83
10	`7fffffff:638eb1c7`	-2.19
11	`7fffffff:47638e39`	-2.19
12	`3f2bb31e:b6c004cc`	-2.19
Total		-24.86

Plan

Rotational-XOR Cryptanalysis

Exact Probability Formula for all Rotations k

Modeling and Applications

Conclusions

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Theory

- Compact exact probability for all rotations k
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- RXDP with constant addition - state machine (Q: can be simplified?)

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github.com/cryptolu/RX-Differentials-Probability

tosc.iacr.org/index.php/ToSC/article/view/12087

References i

- Ashur, Tomer and Yunwen Liu (2016). **"Rotational Cryptanalysis in the Presence of Constants".** In: *IACR Trans. Symm. Cryptol.* 2016.1, pp. 57–70. issn: 2519-173X. doi: 10.13154/tosc.v2016.i1.57-70. url: https://tosc.iacr.org/index.php/ToSC/article/view/535.
- Daum, Magnus (2005). **"Cryptanalysis of Hash functions of the MD4-family".** PhD thesis. Ruhr University Bochum. url: http://www-brs.ub.ruhr-uni-bochum.de/netahtml/HSS/Diss/DaumMagnus/.
- Huang, Mingjiang, Zhen Xu, and Liming Wang (2022). **"On the Probability and Automatic Search of Rotational-XOR Cryptanalysis on ARX Ciphers".** In: *Comput. J.* 65.12, pp. 3062–3080. doi: 10.1093/COMJNL/BXAB126.

- Khovratovich, Dmitry and Ivica Nikolic (Feb. 2010). **"Rotational Cryptanalysis of ARX".** In: *FSE 2010*. Ed. by Seokhie Hong and Tetsu Iwata. Vol. 6147. LNCS. Springer, Berlin, Heidelberg, pp. 333–346. doi: 10.1007/978-3-642-13858-4_19.
- Lipmaa, Helger and Shiho Moriai (Apr. 2002). **"Efficient Algorithms for Computing Differential Properties of Addition".** In: *FSE 2001*. Ed. by Mitsuru Matsui. Vol. 2355. LNCS. Springer, Berlin, Heidelberg, pp. 336–350. doi: 10.1007/3-540-45473-X_28.
- Liu, Yunwen, Glenn De Witte, Adrián Ranea, and Tomer Ashur (2017). **"Rotational-XOR Cryptanalysis of Reduced-round SPECK".** In: *IACR Trans. Symm. Cryptol.* 2017.3, pp. 24–36. issn: 2519-173X. doi: 10.13154/tosc.v2017.i3.24-36.

Ranea, Adrián and Vincent Rijmen (2022). **"Characteristic automated search of cryptographic algorithms for distinguishing attacks (CASCADA)".** In: *IET Inf. Secur.* 16.6, pp. 470–481. doi: 10.1049/ise2.12077.

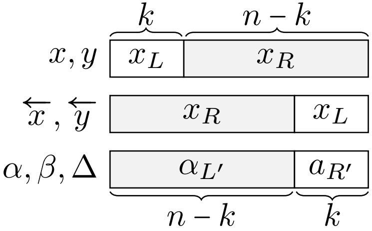
Proof ideas - Decomposition

$$x, y \quad \overbrace{\boxed{x_L}}^{k}\overbrace{\boxed{\qquad x_R \qquad}}^{n-k}$$

$$\overleftarrow{x}, \overleftarrow{y} \quad \boxed{\qquad x_R \qquad}\boxed{x_L}$$

$$\alpha, \beta, \Delta \quad \underbrace{\boxed{\qquad \alpha_{L'} \qquad}}_{n-k}\underbrace{\boxed{a_{R'}}}_{k}$$

Proof ideas - Decomposition



$$\begin{cases} (x_R \oplus \alpha_{L'}) \boxplus (y_R \oplus \beta_{L'}) \boxplus c_L \;\oplus\; x_R \boxplus y_R = \Delta_{L'} \\ (x_L \oplus \alpha_{R'}) \boxplus (y_L \oplus \beta_{R'}) \;\oplus\; x_L \boxplus y_L \boxplus c_R = \Delta_{R'} \\ c_R = \mathbb{1}_{x_R + y_R \geq 2^{n-k}} \\ c_L = \mathbb{1}_{(x_L \oplus \alpha_{R'}) + (y_L \oplus \beta_{R'}) \geq 2^k} \end{cases}$$

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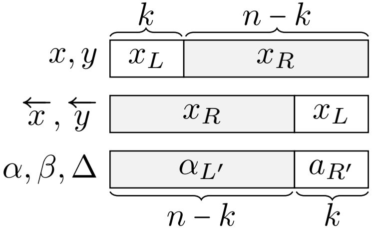

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$$\boxed{\begin{cases} (x \oplus \boldsymbol{\alpha}) \boxplus (y \oplus \boldsymbol{\beta}) \boxplus (\boldsymbol{\alpha}_0 \oplus \boldsymbol{\beta}_0 \oplus \boldsymbol{\Delta}_0) \oplus x \boxplus y = \boldsymbol{\Delta} \\ \mathbb{1}_{x+y \geq 2^m} = \mathrm{w} \end{cases}}$$

Proposition (Carry-constrained Differential through $\boxplus$)

Let

$$XDS_n = \#\{(x, y) \mid x \boxplus y \quad \oplus \quad (x \oplus \alpha) \boxplus (y \oplus \beta) = \Delta\} \text{ (Lipmaa-Moriai)}$$

$$R_n(\alpha, \beta, \Delta) = \#\{(x, y) \in XDS_n(\alpha, \beta, \Delta) \mid x + y < 2^n\}$$

Then, for $\tilde{\alpha} = (\alpha'||\alpha), \tilde{\beta} = (\beta'||\beta), \tilde{\Delta} = (\Delta'||\Delta), \chi' = \alpha' \oplus \beta' \oplus \Delta'$ *we have*

$$R_{n+1}(\tilde{\alpha}, \tilde{\beta}, \tilde{\Delta}) = \begin{cases} 2R_n(\alpha, \beta, \Delta) \text{ if not } (\alpha_{n-1} = \beta_{n-1} = \Delta_{n-1}) \text{ and } \chi' = 0 \\ \#XDS_n(\alpha, \beta, \Delta) \text{ if not } (\alpha_{n-1} = \beta_{n-1} = \Delta_{n-1}) \text{ and } \chi' = 1 \\ \#XDS_n(\alpha, \beta, \Delta) + 2R_n(\alpha, \beta, \Delta) \text{ if } \alpha_{n-1} = \beta_{n-1} = \Delta_{n-1} = 0 \text{ and } \chi' = 0 \\ 2 \times \#XDS_n(\alpha, \beta, \Delta) \text{ if } \delta_{n-1} = \alpha_{n-1} = \beta_{n-1} = \Delta_{n-1} = 1 \text{ and } \chi' = 1 \end{cases}$$

Impact of correction for $k = 1$

Theorem ([AL16], $k = 1$)

$$p = \mathbb{1}_{(I \oplus \mathsf{SHL})(\chi_L) \oplus 1 \preccurlyeq \mathsf{SHL}(\nu_L)} \cdot 2^{-\operatorname{wt}(\mathsf{SHL}(\nu_L))} \cdot 2^{-3}$$
$$+ \mathbb{1}_{(I \oplus \mathsf{SHL})(\chi_L) \preccurlyeq \mathsf{SHL}(\nu_L)} \cdot 2^{-\operatorname{wt}(\mathsf{SHL}(\nu_L))} \cdot 2^{-1.415}$$

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Thm [AL16] holds exactly when $\chi_L \notin \{0\ldots0, 1\ldots1\}$, *where* $(\chi_L || \chi_0) = \alpha \oplus \beta \oplus \Delta$.

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T_1(\chi_0, \nu_0, \chi_1) &= 2^{-\mathrm{wt}(\mathrm{SHL}(\nu_0))-1} + \mathbb{1}_{\chi_0 \in \{0\ldots0, 1\ldots1\}} \times (-1)^{\chi_1} \times 2^{-2} \\
T_{n-1}(\chi_L, \nu_L, \chi_0) &= 2^{-\mathrm{wt}(\mathrm{SHL}(\nu_L))-1} + \mathbb{1}_{\chi_L \in \{0\ldots0, 1\ldots1\}} \times (-1)^{\chi_0} \times 2^{-n}
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Conclusion: concrete trails are probably not affected, optimality claims do