

Hamming code for Error Detection & Correction

{7, 4}

7 bit Hamming code

4 bit data

3 bit parity

7 bit

7	6	5	4	3	2	1	Position
d_3	d_2	d_1	p_2	d_0	p_1	p_0	Bit

$2^n \Rightarrow$ formula for parity
 $n = 0, 1, 2$

11 bit

11	10	9	8	7	6	5	4	3	2	1
d_7	d_6	d_5	p_3	d_4	d_3	d_2	p_2	d_1	p_1	p_0

2^n
 $n = 0, 1, 2, 3$

XOR

example

7	6	5	4	3	2	1
d_7	d_6	d_5	p_4	d_3	p_2	p_1

even parity

$$p_1 = \text{XOR}(1, 3, 5, 7)$$

$$p_2 = \text{XOR}(2, 3, 6, 7)$$

A	B	O
0	0	
0	1	1
1	0	1
1	1	0

$$P_3 = \text{XOR}(4, 5, 6, 7)$$

$$\begin{aligned} P_1 &= D_3 \oplus D_5 \oplus D_7 \\ P_2 &= D_3 \oplus D_6 \oplus D_7 \\ P_3 &= D_5 \oplus D_6 \oplus D_7 \end{aligned}$$

example: 1011

7	6	5	4	3	2	1
1	0	1	?	1	?	?
d_7	d_6	d_5	d_4	d_3	P_2	P_1

$$\begin{aligned} P_1 &= 1 \oplus 1 \oplus 1 \\ P_2 &= 1 \oplus 0 \oplus 1 \\ P_3 &= 1 \oplus 0 \oplus 1 \end{aligned}$$

$$\begin{aligned} &= 0 \oplus 1 = 1 \\ &\Rightarrow 1 \oplus 1 = 0 \\ &= 0 \oplus 1 = 1 \end{aligned}$$

R → 1010101 → Transmitter

Position	7	6	5	4	3	2	1
Bit	d_6	d_5	d_4	d_3	P_2	P_1	

$$2^n = (0, 1, 2)$$

$$\begin{aligned} P_1 &= 0001 = \text{Covers } (1, 3, 5, 7) \\ P_2 &= 0010 = \text{Covers } (2, 3, 6, 7) \\ P_3 &= 0100 = \text{Covers } (4, 5, 6, 7) \end{aligned}$$

$$\begin{aligned} P_1 &= \text{XOR}(1, 3, 5, 7) = \\ P_2 &= \text{XOR}(2, 3, 6, 7) = \\ P_3 &= \text{XOR}(4, 5, 6, 7) = \end{aligned}$$

example: $d_3 = 1, d_5 = 1, d_6 = 0, d_7 = 1$

$$\begin{aligned} P_1 &= \text{XOR}(3, 5, 7) = \text{XOR}(1, 1, 1) = 1 \\ P_2 &= \text{XOR}(3, 6, 7) = \text{XOR}(1, 0, 1) = 0 \\ P_3 &= \text{XOR}(5, 6, 7) = \text{XOR}(1, 0, 1) = 0 \end{aligned}$$

eg: Received: 1110101
 $d_7 d_6 d_5 P_2 d_3 P_1 P_3$
 Receiver check parity
 $P_1 = 1 \oplus 1 \oplus 1 = 1$
 $P_2 = 1 \oplus 1 \oplus 1 = 1$
 $P_3 = 1 \oplus 1 \oplus 1 = 1$
 error since it is transmitted as 0

Final encoded code =

$P_1 P_2 D_3 P_4 P_5 D_6 D_7$
1 0 1 0 1 0 1

Hamming distance: distance b/w 2 binary strings
of equal length

$B S_1 = 101011$

$B S_2 = 101101$

$HD = 2$