STATE-EXAM

[A] Theoretical Fundamentals.

a) TFAI:

1) First Order Logic (FOL):

Propositional Logic: Assumes that the world contains facts

☆ First-order Logic: Assumes the world contains
→ OBJECTS: peoples, houses, numbers, etc
→ RELATIONS: red, round, brother of, bigger than, etc.
→ FUNCTIONS: +, middle of, father of

★ ~~Symbo~~ Syntax of FOL

Symbols:

Our convention all lowercase

- Constants ⟶ KingJohn, 2, Koblenz, ...
- Predicates ⟶ Brother, >, =, ...
- Functions ⟶ Sqrt, LeftLegOf, ...
- Variables ⟶ x, y, a, b, ...
- Connectives ⟶ $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$
- Quantifiers ⟶ $\forall, \exists$

Atomic Sentence:

$$\text{predicate}(\text{term}_1, \text{term}_2, \ldots \text{term}_n)$$

or

$$\text{term}_1 = \text{term}_2$$

Term:

function ($term_1, \ldots term_n$)

\+ EXAMPLE:

$$\underbrace{\underbrace{\text{Brother}}_{\text{predicate}}(\underbrace{\text{kingjohn}}_{\text{constant/term}}, \underbrace{\text{richard}}_{\text{constant/term}})}_{\text{atomic sentence}}$$

$$\underbrace{\underbrace{>}_{\text{predicate}}(\underbrace{\underbrace{\text{Length}}_{\text{function}}(\underbrace{\text{LeftLegOf}}_{\text{function}}(\underbrace{\text{richard}}_{\text{constant}}))}_{\text{term}}, \underbrace{\underbrace{\text{Length}}_{\text{function}}(\underbrace{\text{LeftLegOf}}_{\text{function}}(\underbrace{\text{kingjohn}}_{\text{constant}}))}_{\text{term}})}_{\text{atomic sentence}}$$

Complex sentences:

Sibling (kingjohn, richard) $\longrightarrow$ sibling (richard, kingjohn)

\+ Universal Quantifier ($\forall$)

$\forall x$ (Studiesat (x, Koblenz) $\longrightarrow$ Smart (x))

Everyone studying in Koblenz is smart

Dont use $\wedge$ with $\forall$

~~$5x+10 = 2xy+4y$~~
~~$5x+10 = y(2x+4)$~~

$$\frac{5x+10}{2x+4} = y \qquad \frac{\cancel{37}}{100} \times \cancel{86}$$

$$\frac{5(x+2)}{\cancel{2(x+2)}} = y$$

40

8

3850
~~2300~~
1550

~~4,550,000~~

+ Existential quantifier

Someone studying in London is smart

$\exists x$ (Studies at (x, London) $\wedge$ Smart (x))

Use $\wedge$ with $\exists$

Eg. Prove A from $(B \wedge A) \vee (A \wedge C)$

1	$(B \wedge A) \vee (A \wedge C)$	
2	$B \wedge A$	Assumption
3	A	$\wedge E: 2$
4	$A \wedge C$	
5	A	$\wedge E: 4$
6	A	$\vee E$ 1, 2-3, 4-5

+ Universal Quantifier.

Elimination Rule

1	$\forall x\ P(x)$	
2	$P(c)$	$\forall E: 1$ for some c

Introduction Rule

1	c	
	$\vdots$	
2	$P(c)$	
3	$\forall x\ P(x)$	$\forall I: 1-2$

+ Existential Quantifier ($\exists$)

Elimination Rule

1	$\exists x\, P(x)$
2	c, $P(c)$
	⋮
3	Q
	Q $\quad\exists E$: 1, 2-3

Introduction Rule

1.	$P(c)$
	⋮
2	$\exists x\, P(x)$ $\quad\exists I$: 1

+ AND ($\land$) Connective (Conjunction)

$\land$ Elimination

1	$P \land Q$	
2	P	$\land E$: 1
3	Q	$\land E$: 1

$\land$ Introduction

1.	P	
2.	Q	
3	$P \land Q$	$\land I$: 1, 2

+ OR (∨) Connective (Disjunction)

* ∨ Elimination

1. | $P \lor Q$
2. | | P
   | | ⋮
3. | | R

4. | | Q
   | | ⋮
5. | | R

| R $\lor E$ 1, 2-3, 4-5

∨ Introduction

1 | P
  | ⋮
2 | $P \lor Q$ $\lor I$: 1

1 | Q
  | ⋮
2 | $P \lor Q$ $\lor I$: 1

+ Negation & [Contradiction]

¬ Introduction

1 | | P
  | | ⋯
2 | | $\bot$
  | $\neg P$ $\neg I$: 1-2

¬ elimination

1. | $\neg\neg P$
   | ⋯
2 | P $\neg E$: 1

+ Conditional connective ($\rightarrow$)

P	Q	$P \rightarrow Q$
T	T	T
$\boxed{\text{T}}$	$\boxed{\text{F}}$	$\boxed{\text{F}}$
F	T	T
F	F	T

$\rightarrow$ Introduction

$$\begin{array}{r|l|l}
 & \dots & \\
1 & \quad P & \\
 & \quad \dots & \\
2 & \quad Q & \\
3 & P \rightarrow Q & \rightarrow I\ 1-2
\end{array}$$

$\rightarrow$ Elimination

$$\begin{array}{r|l|l}
1 & P \rightarrow Q & \\
 & \vdots & \\
2 & P & \\
 & \vdots & \\
3 & Q & \rightarrow E : 1, 2
\end{array}$$

+ Biconditional connective ($\leftrightarrow$)

P	Q	$P \leftrightarrow Q$
$\boxed{\text{T}}$	$\boxed{\text{T}}$	T
T	F	F
F	T	F
$\boxed{\text{F}}$	$\boxed{\text{F}}$	T

$\leftrightarrow$ Introduction

$$\begin{array}{r|l|l}
1 & \quad P & \\
2 & \quad Q & \\
 & & \\
3 & \quad Q & \\
4 & \quad P & \\
 & P \leftrightarrow Q & \leftrightarrow I : 1-2, 3-4
\end{array}$$

$\leftrightarrow$ Elimination

$$\begin{array}{r|l|l}
1 & P \leftrightarrow Q & \\
2 & P & \\
3 & Q & \leftrightarrow E\ 1, 2
\end{array}$$

Examples:

1) $\neg(P \vee Q) \dashv \neg P \wedge \neg Q$

1	~~AP(~~ $\neg(P \vee Q)$	
2	$\quad P$	
3	$\quad P \vee Q$	$\vee I: 2$
4	$\quad \bot$	$\bot I: 1, 3$
5	$\neg P$	
6	$\quad Q$	
7	$\quad P \vee Q$	$\vee I: 6$
8	$\quad \bot$	$\bot I: 1, 7$
9	$\neg Q$	
10	$\neg P \wedge \neg Q$	$\wedge I\ 5, 9$

SEMANTICS:

Vocabulary is a list of words or terms

words in vocab could be proper nouns, nouns, adjectives, verbs and adverbs these are grammar.

A reference function maps each word in the vocabulary onto the object to which it refers

eg. Samuel Clemens refers to Mark Twain.

Model:

A model for the language is an image of its vocabulary under its reference function

$[f(\omega_0), f(\omega_1), f(\omega_2) \ldots f(\omega_n)]$ is the model

Syntatic form means types of words in a certain order.

for example → <NAME is ADJ>

→ <Socrates is male>

Each syntatic form is associated with a truth condition that spells the circumstances under which the sentence is true.

eg - $f(NAME) \in f(ADJ)$

In semantics, modality refers to linguistic devices that indicate degree to which an observation is possible, probable, likely or prohibited

Possible world semantics is a framework used in philosophical logic and semantics to analyze and interpret the meaning of statements about necessity and possibility as well as other modalities

Each possible world is a comprehensive scenario that specifies the truth value of every proposition

★ **Probability:**

Sample Space of an experiment is a set of possible outcomes

An event is a collection of possibilities, i.e. a subset of sample space of an experiment
The entire sample space or an empty set is also an event.

Simple Probability $(P(E) = |E| / |S|)$

For eg: $S = \{1, 2, 3, 4, 5\}$
$E = \{1, 2\}$

$$P(E) = \frac{|E|}{|S|} = \frac{2}{5}$$

Complement of E, denoted by E^c (the probabality of not E)

$$P(E^c) = 1 - P(E)$$

When two events have an empty intersection, they are said to be mutually exclusive.

A probality distribution is a function

$$P : S \longrightarrow [0, 1]$$

such that $\sum_{x \in S} P(x) = 1$

A probality distribution function (pdf) is a mathematical function that gives the probabilities of occurence of different possible outcomes for an experiment.

for eg → Coin toss $S = \{\text{"heads"}, \text{"tails"}\}$

$pdf(X) = 0.5$ for X = heads or tails

Conditional probality

Probality of an event H given event E $[p(H|E)]$
its not defined if $p(E) = 0$

for eg

	Opaque	Translucent	Total
Red	180	120	300
Blue	320	80	400
Total	500	200	700

$$P(\text{Red} \mid \text{Opaque}) = \frac{\text{Total Red opaque}}{\text{Total Opaque}} = \frac{180}{500}$$

Independent events:

$$P(E|F) = \frac{P(E\cap F)}{P(F)}$$

So for independent event,

$$P(E|F) = P(E)$$

$$\frac{P(E\cap F)}{P(F)} = P(E)$$

So, E & F are independent if

$$\boxed{P(E\cap F) = P(E) \cdot P(F)}$$

Mutually exclusive events are only independent if one of them is impossible.

Bayes Theorem

First form of Bayes Theorem

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$

Second form of Bayes Th^{m}:

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E|H) \cdot P(H) + P(E|H^c) \cdot P(H^c)}$$

degree of belief in probability is a concept from bayesian prob. which interprets probabilty as a measure of confidence in the occurence of an event.

Bayesian confirmation theory applies Bayesian prob. to evaluation in empirical evidence in order to update the degree of belief. Degree of belief increases when new evidence confirms and decreases when new evidences disconfirms.

Information Theory:

Efficient communication

where expected length of encoded messages is minimized

Huffman coding.

1. Probality of each occurence
2. Order it in descending order
3. Fork (Add two occurences forming two leaf nodes)
4. Form a tree from bottom-up.

Shannon entropy:

It is the average (or expected) amount of information contained in a set. provides a lower bound on the average length of lossless encoding.

$$H(A) = -\sum_{x=0}^{i} P(x_i) \cdot \log_2 P(x_i)$$

Entropy is the measure of randomness/uncertainty in a set

Entropy is maximum when distribution is uniform

Joint Probabity:

Taking marble out of an urn and tossing a coin
→ it shows the probality of different outcomes (two or more) random variable occuring simultaneously. Table for discrete variables and formula for continuous.

Joint entropy

measures the set of uncertanty associated with a set of variables

Interdependence:

Occurence of one variable affects the probability of occurence of other variable

If independent, $H(X,Y) = H(X) + H(Y)$

If interdependent, $H(X,Y) < H(X) + H(Y)$

Mutual information

it is a measure of amount of information gained about one random variable by observing another.
measures the reduction in uncertainty about one variable given we know the value of another.

$$I(X,Y) = H(X) + H(Y) - H(X,Y)$$

if X & Y are independent then $I(X;Y) = 0$

★ Decisions & Games:

Utility:
it represents the measure of satisfaction, benefit, or preference an individual associates with a particular outcome.

provides a framework for making rational decisions.

Expected utility:
it provides a way to make rational decisions when outcome is not certain but prob. are known.

$$E(U_a) = \sum_{x \in c(a)} U(x) \cdot (P_{a_x})$$

a rational agent will take a decision that maximizes EU

Decision Theory is what a rational agent makes decisions in the face of uncertainty.