Optimization Algorithms :

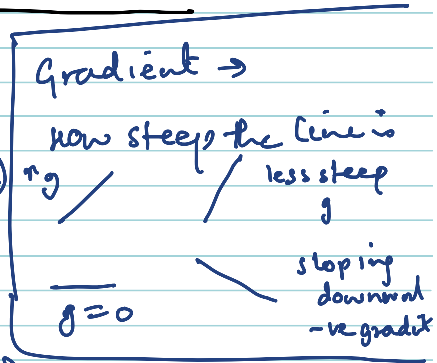
Mathematical method to find the best solution (max or min) for a given objective function within the set of constraints.

- Improve Performance
- minimizes loss
- maximizes efficiency

} by adjusting parameters within a system

• Gradient Descent Algorithms for Optimization :

1st order Optimization Algorithm



Gradient ∇f for function $f(x_1, x_2 - x_n)$ is vector of partial derivatives of f wrt to all the variables

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_3} - \frac{\partial f}{\partial x_n}\right)$$

"

Gradient Descent → 1st order
→ Iterative optimisation algorithm
→ to find a local/global minima of a differentiable function.

In linear R: we have to find a line that gives us least cost by using GA

Gradient Descent moves in direction of the steepest descent ie negative gradient $(-\nabla f(x))$

update rule

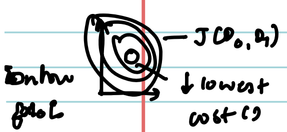
$$\boxed{x_{new} = x_{old} - \alpha \nabla f(x)_{old}}$$

(Step size) α → Too small → slow convergence

α → Too big → overshooting the minima

Local v/s Global Minima: GD may get stuck at local minima

Varients: SGD
Mini batch Gradient Descent



Constrained Optimization:

Optimizing (max/min) an objective function where subject to constaint on variables are involved.

Goal: find solution that satisfies both obj () and constraint

Types of constraints

- equality: $g_i(x) = 0$
- inequality ($\geq$ or $\leq$): $h_i(x) \leq 0$

example:

find values of x that minimize or max the objective function $f(x)$ subject to given constraints

$$\min_x f(x) \quad \text{subject to} \quad g_i(x) = 0 \quad h_i(x) \leq 0$$

Method for solving constrained optimization problem

$\boxed{\text{Lagrange Multipliers}}$

→ For equality constraints

→ Lagrange multiplier are used to solve constrained optimizatn problems

→ basically → idea is to convert constrained problem into a form that the derivative test can be applied (similar to unconstraint one)

Lagrangian Multipliers :

$$\mathcal{L}(x, \lambda) = f(x) + \lambda^T g(x)$$

Example of lagrange multiplier:

Find maximum & min values of

$f(x,y) = 81x^2 + y^2$ subject to constraint

$g(x,y) \Rightarrow 4x^2 + y^2 = 9$

when $y = 0$

$x^2 = \frac{9}{4}$ $\therefore -\frac{3}{2} < x < \frac{3}{2}$

when $x = 0$

$y^2 = 9$ $\therefore -3 < y < 3$

Solve the following equation:-

$$\nabla f(x,y) = \lambda \nabla g(x,y)$$

$$g(x,y) = c$$

$g(x,y) = 4x^2 + y^2$ & $c = 9$

Gradient of ∇f is $\parallel$ to ∇g

$$\nabla f(x,y) = [162x \quad 2y]$$

$$\nabla g(x,y) \quad [8x \quad 2y]$$

w.r.t x & y

We need to satisfy the system of equations:

$$162x = 8x\lambda \quad \text{— (1)}$$

$$2y = \lambda 2y \quad \text{— (2)}$$

$$4x^2 + y^2 = 9 \quad \text{— (3)}$$

$(2y - 2y\lambda) = 0$

$2y(1-\lambda) = 0$

either $y = 0$ or

$\lambda = 1$

subst. $y = 0$ in 3 we get

$4x^2 + 0 = 9$

$4x^2 = 9$

$x^2 = \frac{9}{4} \Rightarrow x = \pm \frac{3}{2}$

2 values extrema $P_1 = \left(-\frac{3}{2}, 0\right)$ $P_2 \left(\frac{3}{2}, 0\right)$

put $\lambda =$ in 1

$162x = 8x\lambda$

$162x = 8x$

$162x - 8x = 0$

$x = 0$

Put $x = 0$ in 3 & 2 potential extreme

$y^2 = 9$ $P_3 = (0, -3)$

$y = \pm 3$ $P_4 = (0, 3)$

Evaluate $f(x,y)$

Put $P_1\ P_2\ P_3, P_4$ in $f(x,y)$

$g(x,y) \Rightarrow x^2+y^2 = 136$

$f(x,y) = 5x-3y$

Q:

$\nabla f(x,y) = \lambda \nabla g(x,y)$

$\nabla f(x,y) = [5 \quad -3]$

$\nabla g(x,y) = [2x \quad 2y]$

$5 = 2x\lambda \quad - (1)$

$-3 = 2y\lambda \quad - (2)$

$g(x,y) \quad x^2+y^2 = 136 \quad - (3)$

$\lambda \neq 0$ as then (1) will not be true

$x = \frac{5}{2\lambda}, \quad y = \frac{-3}{2\lambda}$

Plug these value to constraint (3)

$$\frac{25}{4\lambda^2} + \frac{9}{4\lambda^2} = 136$$

$$25+9 = 136(4\lambda^2)$$

$$\lambda^2 = \frac{1}{16}$$

$$\lambda = \pm\frac{1}{4}$$

We can fit potential max & mins

$\lambda = -\frac{1}{4}$ $\quad x = -10$

$\quad y = 6$

$\lambda = \frac{1}{4}$ $\quad x = 10$

$\quad y = -6$

Put values in $f(x, y)$

$f(-10, 6) = -68$

$f(10, -6) = 68$