Introduction to SUNSET/FFAPL

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What is SUNSET/FFAPL?

Motivation of the (former) project/master thesis:

- Development of a programming language that supports operations on algebraic structures like finite fields, polynomial rings, residue classes, etc. Operations should work without explicit library calls.
- Idea: Compiler that reads protocol source code and outputs executable Java code.
- Realized: full integrated development environment with language interpreter
- Advantages:
  - Very simple handling of algebraic structures via native data types.
  - No explicit library or system calls necessary
  - Programming “close” to notation on the paper

Results

- Programming language FFAPL (Finite Field Application Language) .
- Parser and Interpreter reads, analyzes, and executes FFAPL programs.
- Integrated development environment for FFAPL code, called SUNSET.
- NSIS (Nullsoft Scriptable Install System) Windows installer for Sunset.

Why use Sunset/FFapl?

- …if there is so many alternatives out there (SageMath, Python, …)
- Well, it depends on how you answer two questions:
  - How good are you in programming?
  - How good are you in (basic) number theory?
- Locate yourself on the following grid to decide about using Sunset/FFapl:

how good are you in programming?		*basic to little knowledge*	*good in-depth knowledge*
	skilled/experienced	it may take quite some effort to run the libraries properly, to give the results you desire. Unless you know very well what you are doing, be careful to "buy" what the libraries give you. Alternatively: let Sunset/FFapl take care that your arithmetics works correct in the proper algebraic structure, lean back and just enjoy since you just learn an simple new language	congrats! You're good to go with any programming language and can probably do any algorithm in the language of your choice
	rather unskilled/only occasional programming	Sunset/FFapl is the system for you, since it requires to learn (i) only a very basic and simple language, and (ii) handles all the complicated math silently in the background	even if you know what you want, it can be challenging digging yourself through lengthy library API documentation (if there is any). Instead, you may consider Sunset/FFapl, since it does not require to learn complex APIs and connect libraries, and natively comes with the nice algebraic structures that you need, in a notation that you may recognize as quite familiar

how good are you in number theory?

FFAPL – Finite Field Application Language

- Long-integer and modulo-arithmetic in (finite) algebraic structures like residue classes, polynomial rings, finite fields and elliptic curves
- Creation of (Pseudo-)random number generators
- Boolean operations: Conjunction, Disjunction, NOT.
- Comparison operators: ==, <=, >=, !=, <, >
- Control structures: While- and For-loops.
- Conditional branching: if-else-constructs
- Declaration of Functions and Procedures
- Handling sets of equivalent data types in Array.
- Handling sets of different data types in Records.
- Global constants and local variables.
- Predefined functions and procedures.
- Support of comments

Native Data Types

• `String`	... Alphanumeric text
• `Boolean`	... Boolean value
• `Integer`	... Long integer (no numeric upper limit) hexadecimal notation `0x`... is supported
• `Prime`	... Prime
• `Polynomial`	... Polynomial
• `Z(n)`	... Residue class modulo n
• `Z(n)[x]`	... Polynomial ring modulo n
• `GF(p,ply)`	... Galois Field of characteristic p and with irreducible polynomial ply
• `EC(GF(…), …)`	… elliptic curve over the finite field GF and with the Weierstraß-equation determined by the coefficients a_0, a_1, a_2, a_3, a_4 und a_6 (affine coordinates required)

Special Data Types – Random Number Generators

- No notational difference to native data types
- Read-only access (like constants)
- Every access returns another random value
- Types:
  - **`PseudoRandomGenerator`**`(seed, max)`
    pseudorandom sequence, initialized with seed. Returns pseudorandom numbers between 0 (incl.) and max (incl).
  - **`RandomGenerator`**`(max)`
    Returns random numbers between 0 (incl.) and max (incl). Interface to hardware random generators prepared.
  - **`RandomGenerator`**`(min : max)`
    Returns random numbers between min (incl.) and max (incl). Interface to hardware random generators prepared.

Declaration of Constants and Variables

- Global constants
  - Read only access
  - Examples:

```
const p : Prime := 2;
const ply : Polynomial := [1 + x + x^2];
const gf : GF(p, ply) := [1 + x];
```

- Local variables:
  - Read- and Write-Access
  - Variable shadowing: local variables override global constants.
  - Examples:

```
a,b : Z(3)[x]; //polynomial ring modulo 3
primG : PseudoRandomGenerator(5, 100);
ply : Polynomial; // overrides global constant ply
f : Z(3)[][]; // two-dimensional array
r : Record a: Integer; b: Polynomial; EndRecord;
u : Z(3)[x];
```

Functions and Procedures

- Functions have a return-value (and type), procedures don‘t.
- Recursion and overloading of functions/procedures is legal
- Calls by reference and calls by value

Examples:

```
//function
function func(val1 : Integer) : Integer {
    ...
    return ... ;
}
//overloaded function
function func(val : Polynomial) : Z()[x] {
    ...
    return ... ;
}
//procedure
procedure proc(val : Integer; val2 : Polynomial) { ... }
```

Error messaging 1

- Multiple languages supported (Deutsch and English).
- Errors can be localized by row and column.

Example:

```
program calculate{
    r : Z(6);
    r := 4^-1;
}
```

causes the following German error:

```
FFapl Kompilierung: [calculate] Algebraic Error 106 (Zeile
3, Spalte 15)
Es existiert kein multiplikatives Inverses für 4 in Z(6)
```

causes the following English error:

```
FFapl compilation: [calculate] Algebraic Error 106 (line
3, column 15)
there exists no multiplicative inverse for 4 in Z(6)
```

Error messaging 2

Parser tells the expected syntax.

Example:

```
program calculate{
    r : Z(3)
}
```

causes the error

```
FFapl compilation : [calculate] ParseException 102 (Row 3,
Column 1)
"}" found in row 3, column 1. Expected one of:
    "[" ...
    ";" ...
    "[" ...
```

Console-I/O

- Console output via ***print*** or ***println***:
  ```
  x: Z(11); x := 7; println(x);
  ```
  creates the output:
  ```
  Z(11): 7
  ```
- Algebraic structure that stores the value is printed by default. To suppress this, just convert the value into a string via the pre-defined function ***str*(…)**:
  ```
  x: Z(11); x := 7; println(str(x));
  ```
  creates the output
  ```
  7
  ```
- Reading values from the console (user-input) can be done by the following routines: ***readInt*(…)**, ***readPoly*(…)**, ***readEC*(…)** and ***readStr*(…)**, each of which takes a string to prompt the user, and returns a value of the type specified by the suffix of the function's name (integer, polynomial, elliptic curve or string)

SUNSET

- Integrated development environment for FFAPL.
- Functionality covers:
  - File management and printing
  - Undo/Redo
  - Multi-Language support
  - Syntax- and Error-highlighting
  - Execution of FFAPL-code in separate threads
  - Interruption (abortion) of running executions
  - Individual console windows for each open FFAPL-program
  - Management of Code-Snippets
  - Integrated FFAPL-API for data types, predefined functions and snippets
  - Procedure templates and example code
  - Drag- und Drop (file opening and FFAPL-API)
  - Shortcut-Keys

Polynomials

- Polynomials are treated as literals, just like numbers:

```
p : Z(2)[x];
p := [1+x];
```

- The symbol „x“ marks the polynomial‘s variable, but using „x“ as a local program variable is allowed, even inside a polynomial literal. In that case, just enclose x into brackets:

Examples:

```
a,x: Integer;
x := 3;
a := 4;
```

syntax	evaluates to…
`[1 + 3x + x^2]`	polynomial $1+3x+x^2$
`[1 + (x)]`	1 + 3 = 4 (constant polynomial)
`[1+(a+1)x + x^2]`	$1+5x + x^2$
`[1+(x)x + x^2]`	$1 + 3x + x^2$
`[x^x]`	x^3 (exponents always evaluate)
`[(x)^x]`	27 (basis and exponent evaluated)

Construction of Finite Algebraic Structures

- Residue class groups $\mathbb{Z}_n$: **`Z`**`(n)`, arithmetic via +, -, `*`, `/` and ^
- Residue class rings $\mathbb{Z}_n[X]$: **`Z`**`(n)[x]`, arithmetic via +, -, `*`, and ^
- Finite fields: GF(p^n): **`GF`**`(p, ply)`, where `ply` is an irreducible (or primitive) polynomial of degree n over $\mathbb{Z}_p$, which is constructible via `ply := `***`irreduciblePolynomial`***`(n,p)` (predefined function). Arithmetic via +, -, `*`, `/` and ^.
- Elliptic curves: E(F) over a finite field F and Weierstraß-equation $y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6$:
  `EC``(`**`GF`**`(…), `**`a1`**` := …, `**`a3`**` := …, …).`
  Points on the elliptic curve are (in affine coordinates):
  `P := <<x, y>>`, where x, y $\in$ GF(…) and the Weierstraß-equation must be satisfied (type-checking done at compile- and runtime)
  Arithmetic via + und `*`, point at infinity symbol: <<**`PAI`**>>

Working with Variables

- In an assignment **`LHS := RHS`**, the expression **`RHS`** is always evaluated in the home algebraic structure (type) of **`LHS`**.
- Examples:
  - `m: `**`Z`**`(n)` causes all subsequent operations in instructions of the form `m := …` to be executed in $\mathbb{Z}_n$
  - inline expressions (e.g., in `print[ln]`-statements) can be used only if the data type can be inferred from left to right. For example:
    - ***`println`***`(P + <<2,3>>)` works, since the elliptic curve on which P lives determines how to compute the + within the ***`println`***.
    - ***`println`***`(<<2,3>> + P)`, or ***`println`***`(<<2,3>> + <<4,5>>)` do not work, since the host structure of `<<2,3>>` cannot be determined uniquely (the statement must be converted to the previous form)
  - Elliptic curve points admit a special syntax that allows for easy extraction of the curve coordinates; for a point P = (x, y), we can write

```
x, y: BaseGF(P); // copy the EC's base field
                 // use BaseZ for curves over Z(p)
<<x,y>> := P;  // copy the coordinates of P into x and y
```

Special Data Types 1

- Strings: explicit conversion to string via ***str***(…). Manipulation only by concatenation via +
  Example: `println("Ciphertext = " + str(c))`
- Random number generators:
  - Integer values: only declaration required, every access yields a new value:

```
X : RandomGenerator(0: (2^128-1));
for i = 1 to 10 {     // get 10 random numbers
 println("yet another AES-Key is " + X)
}
```

  - On elliptic curves over the finite field P $\in$ GF(p^n) with n $\geq$ 1, a special syntax is available:
    - `P := <<RandomPoint>>` delivers a random point on the curve
    - `P := <<RandomPointSubfield>>` returns a random point in the elliptic curve defined by the same equation as with (the declaration of) P, but over the subfield $\mathbb{Z}_n \subseteq$ GF(p^n)

Special Data Types 2

- Arrays: 0-based; size can be set at runtime, initialization via the **new** operator:

```
arr: Z(3)[];  // array of elements from Z(3)
arr := new Z()[10];  // allocate space for 10 elements
```

  Direct declaration with values is possible:

```
a: Prime[][];     // Matrix of primes
a:= {{2,5,7},{3,11,13}};
```

- Records: unify variables of different data types

```
Certificate: Record
                 e, n: Integer; // RSA public key
                 ID: String;   // Identity
                 s: Integer;   // Signature of the CA
             EndRecord
```

Subroutine Parameter

- Finite fields are determined by several parameters (characteristic, dimension, …).
- Passing such elements to functions works by generic data types having no explicit parameters:

```
function f(m: GF; a: Z(); p: Z()[x]; C: EC) : Integer {
  // local re-construction of the finite field
  M: GF(getCharacteristic(m),getIrreduciblePolynomial(m));
  E: SameAs(C) //Type-Cloning: declare E to be the same
               //elliptic curve as C (SameAs works for all
  …            //primitive data types (not arrays or records)
}
```

- Arrays can be passes to a subroutine as well:

```
procedure p(x: Integer[]; matrix: Integer[][]) {
  n := #x; // number of elements in "x"
  n2 := #x[0];   // number of columns in "matrix"
}
```

- Records cannot be passed to subroutines as parameters!

Language Conventions

- Strict separation of declarative and procedural part
- No „early-exit“ from functions; **`return`**-Statement must always be the last instruction
- No implicit typecasting, except in these cases (only):
  - Conversion from residue class type **`Z`**`(n)` to **`Integer`** during exponentiation.
  - Conversion to string for console output.

Example: RSA-Cipher ($n = pq$, $\varphi = (p-1)(q-1)$, $m \in \mathbb{Z}_{n,}$ $e,d \in \mathbb{Z}_{\varphi(n)}$)

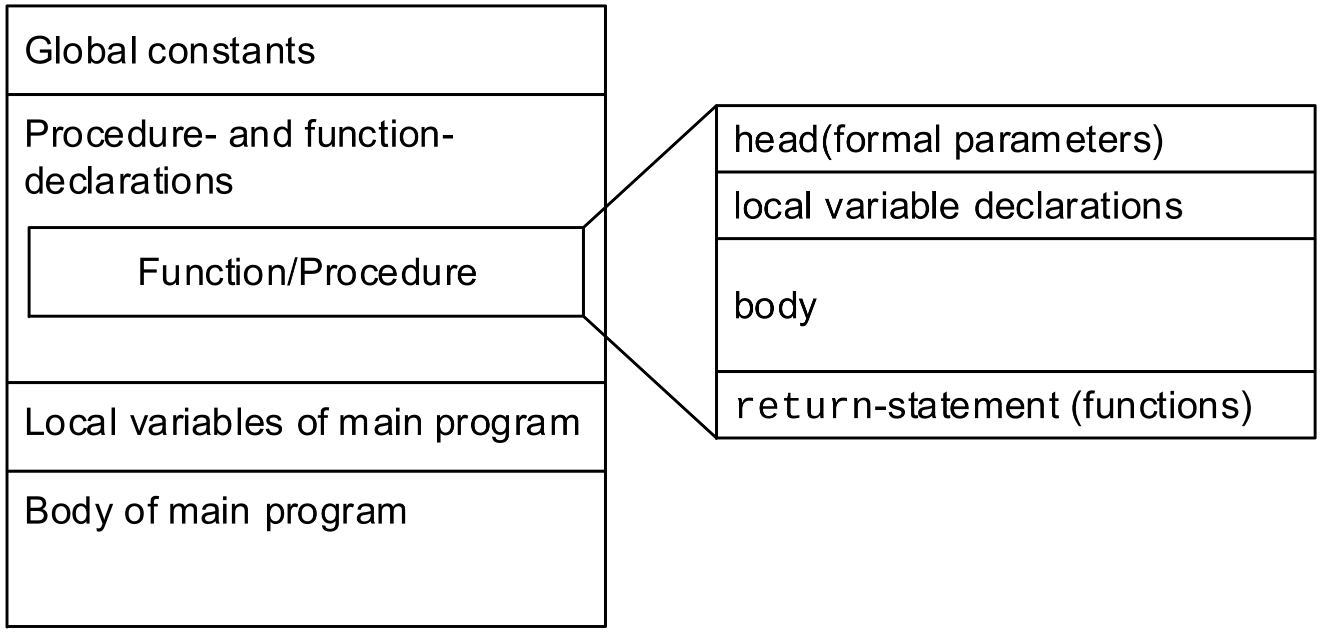
`m, c: Z(n); e: Z(phi);`

Upon evaluation of `c := m^e`, e is cast from **`Z`**`(phi)` to **`Integer`** (a compiler warning is issued, though).

- Explicit type-casting possible via predefined functions ***`int`***`(…)` ***`ply`***`(…)` and ***`str`***`(…)`.

Structure of FFAPL-Programs

Any FFAPL code must obey the following schema:


Global constants
Procedure- and function-declarations
Function/Procedure
Local variables of main program
Body of main program
head(formal parameters)
local variable declarations
body
return-statement (functions)

Elliptic Curve Pairings

Pairings are naturally supported by SUNSET/FFAPL via the function ***`TLPairing(A, B [, n])`***, but subject to the following constraints:

- **`A`** must be declared of type **`EC(Z(p), …)`**
- **`B`** must be declared of type **`EC(GF(p, …), …)`** using the same coefficients for the Weierstraß-polynomial
- In example programs, the following random choices are admissible:
  - **`A := <<RandomPointSubfield>>`**
  - **`B := <<RandomPoint>>`**
- The order of the point **`A`** in its home EC-group is determined brute-force, so it is advisable to pass this value as a third parameter **`n`** to ***`TLPairing`***. The curve should thus be constructed so that the order of ist subgroups is known a-priori.

Operator Precedence

Arithmetic operations are executed as usual in mathematics, i.e., in the following sequence:

1. unitary operations (Boolean negation „!“, or sign change)
2. powers and exponentiations
3. *, / and **`MOD`** (note that modulo arithmetic is done implicitly by declaring the variables/expressions in the proper structure)
4. +, -
5. **`AND`**
6. **`OR`**
7. **`XOR`**
8. conditional statements (==, <=, >=, !=)

Other precedences must be enforced by embracing expressions in brackets.

Practical Part

Programming Exercises

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Chinese Remainder Theorem[1]

Theorem 2.1:

Let $m_1, m_2, \ldots, m_k \in \mathbb{N}+1$ with $(m_i,m_j) = 1$ for $i \neq j$ and $a_1, a_2, \ldots, a_k \in \mathbb{Z}$, $k \in \mathbb{N}+1$.
Then there is exactly one $x \in [0:m-1]$ satisfying
$(*)$ $x = a_i \pmod{m_i}$, $i \in [1:k]$, $m = m_1 \cdot m_2 \cdot \ldots \cdot m_k$.

Proof:

Existence: For $n_i := m/m_i$ we have $(n_i,m_i) = 1$, so there is some x_i, for which $x_i \cdot n_i = 1 \pmod{m_i}$. With $r_i := x_i \cdot n_i$ we get for all $i \in [1:k]$ that $r_i = 0 \pmod{m_j}$ $(i \neq j)$ and $r_i = 1 \pmod{m_i}$.

$$x := \left(\sum_{i=1}^{k} a_i \cdot r_i\right) \text{MOD}\, m \quad \Rightarrow \quad x = a_j (\text{mod}\, m_j) \text{ für alle } j \in [1:k];$$

so x is a solution to $(*)$.

[1] from VO „Basismechanismen der Kryptologie“, WS 2011

Chinese Remainder Theorem[1]

- Example:

  Let $m_1 = 17$, $m_2 = 21$ and $m_3 = 97$, giving the module $m = m_1 \cdot m_2 \cdot m_3 = 34.629$.

  With $n_i = m/m_i$ we get $n_1 = 2.037$, $n_2 = 1.649$ and $n_3 = 357$.

  By the extended Euclidian algorithm,
  $x_1 = -6$, $x_2 = 2$, $x_3 = 25$
  $r_1 = -12.222$, $r_2 = 3.298$, $r_3 = 8.925$

  If a_1, a_2 and a_3 are given, the solution is
  $x = (-12.222 \cdot a_1 + 3.298 \cdot a_2 + 8.925 \cdot a_3)$ MOD 34.629

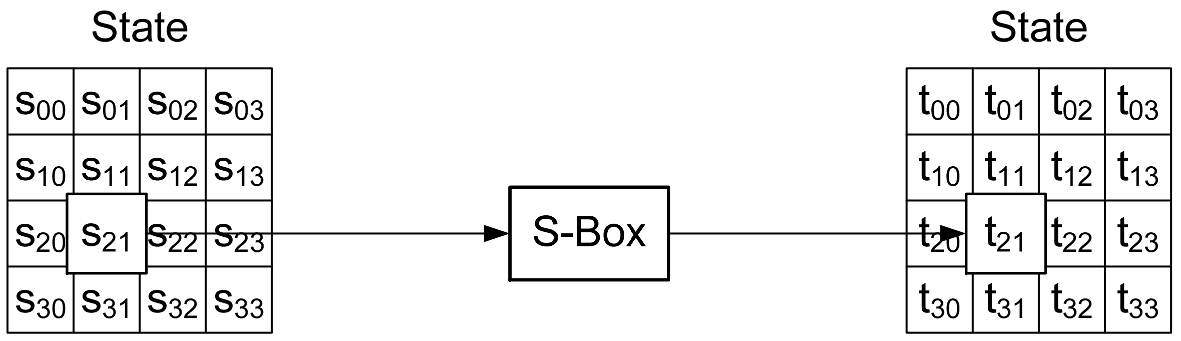
  Using $a_1 = 7$, $a_2 = 6$ and $a_3 = 25$, we get $x = 18.843$.
  From $a_1 = 2$, $a_2 = 3$ and $a_3 = 5$, we get $x = 30.075$.

[1] from VO „Basismechanismen der Kryptologie“, WS 2011

AES – SubBytes – S-Box[1]

- SubBytes is a nonlinear Byte-substition that operates on a single byte of an AES state.
- The S-Box is defined over GF(2^8) with module $m(x) = x^8 + x^4 + x^3 + x + 1$, where m(x) is irreducible; the S-Box can be constructed as follows:
    1. Compute the multiplicative inverse a(x) of s_{ij} in GF(2^8), where {00} = 00_h is self-inverse by convention.
    2. The i-th coefficient in the result term b(x) = t_{ij} is found from a(x) and $c(x) = x^6 + x^5 + x + 1$ (byte-representation: {63}) as:

       $b_i = a_i \oplus a_{(i+4) \text{ MOD } 8} \oplus a_{(i+5) \text{ MOD } 8} \oplus a_{(i+6) \text{ MOD } 8} \oplus a_{(i+7) \text{ MOD } 8} \oplus c_i$


State
s_{00} s_{01} s_{02} s_{03}
s_{10} s_{11} s_{12} s_{13}
s_{20} s_{21} s_{22} s_{23}
s_{30} s_{31} s_{32} s_{33}
S-Box
State
t_{00} t_{01} t_{02} t_{03}
t_{10} t_{11} t_{12} t_{13}
t_{20} t_{21} t_{22} t_{23}
t_{30} t_{31} t_{32} t_{33}

[1] from VO „Basismechanismen der Kryptologie“, WS 2011

AES Sbox in FFapl

```
function Sbox(s: GF) : GF {
	a, c, result: GF(2, [x^8+x^4+x^3+x+1]);
	b: Integer[];
	b := new Integer[8];
	if (s == [0]) { a := [0]; } // 0 is self-inverse
	else { a := s^-1; }
	c := [x^6 + x^5 + x + 1];
	for i = 0 to 7 {
		b[i] := (coefficientAt(a, i)
		           + coefficientAt(a, (i+4) MOD 2)
		           + coefficientAt(a, (i+5) MOD 2)
		           + coefficientAt(a, (i+6) MOD 2)
		           + coefficientAt(a, (i+7) MOD 2)
		           + coefficientAt(c, i)) MOD 2;
	}
	result := [(b[0]) + (b[1])x + (b[2])x^2 + (b[3])x^3 +
		(b[4])x^4 + (b[5])x^5 + (b[6])x^6 + (b[7])x^7];
	return result;
}
```

Guillou-Quisquater Protocol[2] 1

- Purpose: Entity A interactively proves ist identity to another entity B, by showing knowledge of a secret s_A.
- Protocol runs in rounds, each of which has 3 phases.
- Interactive zero-knowledge proof
- In the following, we consider the literal description of the protocol as found in the crypto textbook [2], and its transcription to SUNSET/FFAPL.

[2] A. Menezes, P. van Oorschot, S. Vanstone: *Handbook of Applied Cryptology*, CRC Press, 1997, p. 412

Guillou-Quisquater Protocol[2] 2

1. *Selection of system parameters.*
   (a) An authority T, trusted by all parties with respect to binding identities to public keys, selects secret RSA-like primes p and q yielding a modulus $n = pq$. (as for RSA, it must be computationally infeasible to factor n.)
   (b) T defines a public exponent $v \geq 3$ with $\gcd(v, \phi) = 1$ where $\phi = (p-1)(q-1)$ and computes its private exponent $s = v^{-1} \bmod \phi$. […]
   (c) System parameters (v, n) are made publicly available (with guaranteed authenticity) for all users.

```
const p: Prime := getNextPrime(2^512);
const q: Prime := getNextPrime(2^100 * p);
const n: Integer := p*q;
const phi: Integer := (p-1)*(q-1);
v, s: Z(phi);
e: Integer; // auxiliary variable for constructing v
X: RandomGenerator(4:phi-1); // assure v >= 3
e := X;       // draw a random value
while(gcd(e,phi) > 1) { e := e / gcd(e,phi); }
v := e;
s := v^(-1);
```

[2] A. Menezes, P. van Oorschot, S. Vanstone: *Handbook of Applied Cryptology*, CRC Press, 1997, p. 412

Guillou-Quisquater Protocol[2] 3

> *2. Selection of per-user parameters.*
>
> (a) Each entity A is given a unique identity I_A, from which (the *redundant identity*) $J_A = f(I_A)$, satisfying $1 < I_A < n$, is derived using a known redundancy function f […]
>
> (b) T gives A the secret (*accreditation data*) $s_A = (J_A)^{-s} \bmod n$.

```
IA, JA: Integer;
sA: Z(n);
JA := f(IA); // function f assumed available
sA := JA^(-s);
```

[2] A. Menezes, P. van Oorschot, S. Vanstone: *Handbook of Applied Cryptology*, CRC Press, 1997, p. 412

Guillou-Quisquater Protocol[2] 4

3. *Protocol messages*. Each of t rounds has three messages as follows (often $t = 1$).

$A \rightarrow B$:	I_A, $x = r^v \bmod n$.	(1)
$A \leftarrow B$:	e (where $1 \leq e \leq v$);	(2)
$A \rightarrow B$:	$y = r \cdot s_A^e \bmod n$	(3)

4. *Protocol actions*. A proves its identity to B by t executions of the following; B accepts the identity only if all t executions are successful.
   (a) A selects a random secret integer r (the *commitment*), $1 \leq r \leq n - 1$, and computes (the *witness*) $x = r^v \bmod n$.
   (b) A sends to B the pair of integers (I_A, x).
   (c) B selects and sends to A a random integer e (the *challenge*), $1 \leq e \leq v$.
   (d) A computes and sends to B (the response) $y = r \cdot s_A^e \bmod n$.
   (e) B receives y, constructs J_A from I_A using f (see above), computes $z = J_A^e \cdot y^v \bmod n$, and accepts A's proof of identity if both $z = x$ and $z \neq 0$. (The latter precludes an adversary succeeding by choosing $r = 0$).

[2] A. Menezes, P. van Oorschot, S. Vanstone: *Handbook of Applied Cryptology*, CRC Press, 1997, p. 412

Guillou-Quisquater Protocol[2] 5

```
t: Integer;
r: Integer;
x,y,z: Z(n);
success: Boolean;
XR: RandomGenerator(1:n-1);      // for message (1)
XE: RandomGenerator(1:n-2);      // for message (2)
t := 10; // run 10 rounds
success := true;   // no rounds failed so far…
for i = 1 to t {
      r := XR;     // get a random value
      x := r^v;
      /* sending (IA,x) requires no action… */
      e := 1 + XE MOD int(v);    // random challenge
      y := r * sA^e;      /* compute the response */
      z := JA^e*y^v; /* check the acceptance condition */
      if (z!=x OR z==0) { success := false; }
}
// Boolean variable "success" contains the decision
```

[2] A. Menezes, P. van Oorschot, S. Vanstone: *Handbook of Applied Cryptology*, CRC Press, 1997, p. 412

Three-Party Diffie-Hellman Protocol[3] 1

Input: The public parameters $(G,\oplus,H,\boxplus,P,e)$ with a bilinear map e.
Output: the joint key $K \in H$.

```
1.  $a_A \in_R \mathbb{N}$
2.  $P_A \leftarrow [a_A]P$
3.  send $P_A$ to B, C
4.  receive $P_B, P_C$ from B, C
5.  $K \leftarrow [a_A](e(P_B, P_C))$
```

- The scalar multiplication $[a]P$ can be written plainly as `a*P`.
- The group $(G,\oplus)$ is the EC group (declared below). The group $(H,\boxplus)$ is the target group of the pairing (the result type of ***`TLPairing`***)

```
const P: EC(GF(127,[x^2+1]), a4 := [5])
      := << [121] , [95] >>;
const g: Integer := 64;    // order of the EC subgroup
                           // generated by P
```

[3] Cohen, H. & Frey, G. (Eds.) Handbook of elliptic and hyperelliptic curve cryptography Handbook of elliptic and hyperelliptic curve cryptography, CRC Press, 2005

Three-Party Diffie-Hellman Protocol[3] 2

```
Input: The public parameters $(G,\oplus,H,\boxplus,P,e)$ with a bilinear map $e$.
Output: the joint key $K \in H$.
    1. $a_A \in_R \mathbb{N}$
    2. $P_A \leftarrow [a_A]P$
    3. send $P_A$ to B, C
    4. receive $P_B, P_C$ from B, C
    5. $K \leftarrow [a_A]\big(e(P_B, P_C)\big)$
```

- For a nontrivial pairing, we require a distortion map for this group

```
// Distorsion Map
function distorsion(e : EC) : EC {
    x,y: BaseGF(e); //coordinates lie in the EC‘s base field
    res: SameAs(e); //the result is on the same curve as e
    << x,y >> := e; //extract the point coordinates
    res := << -x, [x]*y >>;
    return res;
}
```

[3] Cohen, H. & Frey, G. (Eds.) Handbook of elliptic and hyperelliptic curve cryptography Handbook of elliptic and hyperelliptic curve cryptography, CRC Press, 2005

Three-Party Diffie-Hellman Protocol[3] 3

```
Input: The public parameters $(G,\oplus,H,\boxplus,P,e)$ with a bilinear map $e$.
Output: the joint key $K \in H$.
    1.  $a_A \in_R \mathbb{N}$
    2.  $P_A \leftarrow [a_A]P$
    3.  send $P_A$ to B, C
    4.  receive $P_B, P_C$ from B, C
    5.  $K \leftarrow [a_A](e(P_B, P_C))$
```

- Declaration of variables and the protocol (next slide)

```
rng: RandomGenerator(1:g-1);
aA,aB,aC: Integer;
Pa,Pb,Pc: SameAs(P);
Ka,Kb,Kc: BaseGF(P);
```

[3] Cohen, H. & Frey, G. (Eds.) Handbook of elliptic and hyperelliptic curve cryptography Handbook of elliptic and hyperelliptic curve cryptography, CRC Press, 2005

Three-Party Diffie-Hellman Protocol[3] 4

```
Input: The public parameters $(G,\oplus,H,\boxplus,P,e)$ with a bilinear map $e$.
Output: the joint key $K \in H$.
    1.  $a_A \in_R \mathbb{N}$
    2.  $P_A \leftarrow [a_A]P$
    3.  send $P_A$ to $B$, $C$
    4.  receive $P_B, P_C$ from $B$, $C$
    5.  $K \leftarrow [a_A]\big(e(P_B, P_C)\big)$
```

```
aA := rng; aB := rng; aC := rng; // random values
Pa := aA*P; Pb := aB*P; Pc := aC*P;     // partial keys
Ka := TLPairing(Pa,distorsion(Pb))^aC;
Kb := TLPairing(Pb,distorsion(Pc))^aA;
Kc := TLPairing(Pa,distorsion(Pc))^aB;
// output (for verification)
println(str(Ka) + " == " + str(Kb) + " == " + str(Kc));
```

[3] Cohen, H. & Frey, G. (Eds.) Handbook of elliptic and hyperelliptic curve cryptography Handbook of elliptic and hyperelliptic curve cryptography, CRC Press, 2005

Open Issues and Known Bugs

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Open Issues (for future versions)

- This is an (incomprehensive) list of features that would be nice to have in future versions.
- All of these are open for implementation in a software practical, practice semester or master thesis
  (please send inquiries to stefan.rass@aau.at)

- API extensions to FFAPL, including (but not limited to):
  - built-in functions for AES encryption and decryption
  - a way to define **`Record`** as parameter and return type for functions
  - …whatever else you may propose as useful…
- IDE extensions to SUNSET
  - Digital signatures for code
  - customizable API restrictions
  - …whatever else you may propose as useful…

Known Bugs and Limitations (as of version 2.1)

- Comparison to point at infinity in elliptic curves fails if `<<PAI>>` is used directly as a literal:
  - `if (P == <<PAI>>)  // this fails (without error msg; internal exception)`
  - Workaround: use a variable of the same type as P and assign it the point at infinity as a value:
    ```
    neutral_element := <<PAI>>;
    if (P == neutral_element) // this works
    ```
- Elliptic curve arithmetic (especially random points) works reasonably efficient only for small parameter settings
  Workaround: …unless you are willing to wait really long for the arithmetic to complete, you should do prototyping and demonstrations with small paramters with $\leq$ 10 bits

  …whatever you find buggy, please report by email to stefan.rass@aau.at $\rightarrow$ thank you!