Use of Fractional Analysis in Determining the Proper Interceptor Mix for the Multi-Mission Launcher (MML)

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ABSTRACT

This paper discusses a method by which Center for Army Analysis (CAA) was able to compute the set of cases for raid analyses — based on Extended Air Defense (EADSIM) model runs — that determined the best interceptor mix for Multi-Mission Launcher (MML) batteries deployed against certain threat fires in air defense operational scenarios. The discussion can be considered a practitioner's case study for how analyst insights into a particular problem domain, and into the modeling art itself, can be leveraged to cull an initial naïve (exhaustive) set of cases based upon initial primary constraints. In the CAA study in question, this yielded a reduction of the original case space by many orders of magnitude.

INTRODUCTION

The Multi-Mission Launcher (MML) is a mobile launcher platform which contains a missile rack and canisters that allow the launcher to launch different types of interceptors simultaneously. The interceptor types will have capability against cruise missiles (CMs) and rocket, artillery and mortar (RAM) rounds.

The Army Deputy Chief of Staff, G-8/FDG requested that Center for Army Analysis (CAA) provide insights into the best loadout of different interceptor types for a deployed MML battery. CAA determined a method to generate all the possible cases — of a particular threat (raid) against a particular defense laydown — to run in the Extended Air Defense Simulation (EADSIM) model. These EADSIM runs were then used to calculate the probability of kill (Pk) per case. The battery interceptor loadouts with the highest achieved protection level (APL) were deemed to be the best loadouts.

While the study also provided particular results and insights based upon the classified data to which CAA and the sponsor were privy, this unclassified paper will instead focus on discussing the method that CAA employed to design a set of model cases which answered the study sponsor's primary question with a timely response.

PRIMARY ASSUMPTIONS

The study had a couple of primary assumptions which were germane to the method. We assumed that all possible MML battery locations would be located where the threat types — in our case, RAM and cruise missiles — would be employed. We also assumed that no countermeasures would be utilized by the threat.

METHOD

The method for determining the study cases is outlined by the following paragraph, and uses the idea of achieved protection level (APL) — how well the defense performs against the threat (in the model).

First, we generated all the combinations of threat types, threat severity (quantities), and interceptor types and quantities as the initial space, or collection, of cases to be considered. Then, for each case we computed an upper bound on the APL — which we call the *expected APL* — and filtered out the cases in which the expected APL was strictly less than the desired protection level (DPL) — the DPL is how well we desire the defense to perform against the threat. These "winning" cases were then run in the EADSIM model to compute each case's APL; the MML battery interceptor mix which had the highest APL against a particular threat (types and quantities) was deemed optimal.

Achieved Protection Level (APL)

The achieved protection level (APL) is computed from the output of a simulation model, like EADSIM. It is a measure of how effectively the defense performed, in the model, against the incoming threat. If, for a particular threat raid played in the model, I is the number of incoming threat rounds (in this case, cruise missiles or RAM rounds) and L is the number of "leakers," i.e. threat rounds that the defense tried, but was not able, to eliminate, then the achieved protection level, **APL**, is given by

$$\mathbf{APL} = 1 - \frac{L}{I}.$$

Typically, I is computed from the threat rounds that the defense assesses to be a genuine threat, e.g. the threat round is going to impact an area that the defense has committed to defend. Conversely, this means that there could be some threat rounds among the incoming that are not counted in the computation of I, e.g. a threat round that is assessed to impact outside of the area of defense. However, in this study, the threat-round aim, flight, or guidance was sufficiently accurate as not to warrant such a discrimination, and hence all of the incoming threat rounds are counted in I.

Case Space Computation

The generation of the case space starts with considering the combinations due to applicable factors. These factors are threat and interceptor type and quantity.

The threat types for this study were **CM** and **RAM**. The threat severity (quantity) levels were classified as **VS** ("very small"), **S** ("small"), **M** ("medium"), **L** ("large"), **VL** ("very large"), and each indicate a quantity of "rounds" of either the **CM** or **RAM** type that make up a raid. (For this paper, we elided the threat type quantities actually used in the study and simply refer to their corresponding threat severity levels, due to the classification of the study.) A raid is denoted by these quantity levels, one for each threat type. For instance, a particular raid might be denoted as (**M**, **L**), meaning "**CM** at level **M** and **RAM** at level **L**." Hence, there are $5^2 = 25$ possible raids in our scheme.

To create a case, a raid is paired to a defense laydown. In our study, the laydown was a single MML battery, described by a specific mix of interceptors — by type and quantity — loaded on the launchers in the battery. We were charged to consider four different interceptor types. (We don't include the interceptor type names in this paper, but examples of the interceptor types that can be used on an MML launcher can be found in the open source, e.g. PEO MS (2013).)

However, there is a limit to what can be loaded on the launchers themselves, and this limit is principally dictated by munition weight; in our study we only considered the weight of each interceptor type and the maximum weight capacity of a launcher. Hence, computing the distinct instances of MML battery loadouts looks a lot like a construction for the multiple knapsack problem, where the launchers are the "knapsacks" and the various interceptors are the "items" — only that there is no optimization function involved — we are only to find the number of different ways of filling the knapsacks with the items, and we were able to accomplish this by generating the cases, then counting them. ("Filling" means adding items to the knapsack such that no remaining (outside) item could be added without exceeding the capacity of the knapsack.)

This can be computed with our present day desktops in short order, even with a brute force method. For each launcher (knapsack), the number of ways of filling that, with the weights and capacities in consideration for our study, was 528. That means that if we were to generate the different MML battery loadouts by choosing, for each of the battery's four launchers, a launcher loadout (among the 528), then we would have 3,275,282,340 different battery loadouts. (That is the number of combinations of 528 objects, chosen 4 at a time, with repetition.)

Our outlook for getting the model runs completed in time turned out not to be so dim, however, since we repeated the "knapsack filling" computation alternatively, using a simplifying assumption: we treated the battery as an atomic unit, i.e. one big knapsack whose capacity is simply the sum of the capacities of its launchers. When this is admitted, the number of battery loadouts becomes 26,400. The drastic reduction signifies how much repetition in launcher effectiveness was carried in the previous computation of loadouts (involving the individual launcher capacities); however, there might be some cases of the 26,400 which would have exceeded at least one of the launcher capacities in the previous computation, since the "wasted capacities" of each launcher could have been added up in this "rollup case" to admit at least an extra interceptor that would not have been allowed due to individual launcher capacity constraints.

The number of cases was still too large for us to have time to setup and execute the model runs; so we came upon a way to prune the case space further.

Expected APL

We discovered that we could further prune (fractionalize) the case space if we considered how well any battery loadout could expect to do in the best case. When the APL is computed via the model runs, the input Pks of the various interceptor types against threat types are played in the dynamic of the model, which includes many mitigating factors with respect to the outcome, or output, Pk of the battery. These model mitigating factors include probability of detection and radar resource limits. These factors only serve to reduce the output Pk, with respect to the input Pks. But this means that if we were to build an expected value model that only considered the input Pks for the interceptor to threat type pairings — meaning that the mitigating factors of the APL-computing model are not even played — then the ratio of the expected number of kills (computed by the expected value model) to the number of incoming threat rounds would have to be an upper bound on the APL. In other words, the APL could do no better than this ratio, which we call the *expected APL*.

Finally, if the expected APL for a particular case was strictly less than the DPL (which for our study was chosen to be 0.70), then this case was pruned from the case space because we could not hope to achieve the DPL in the model runs. The salient condition of these pruned cases is APL < expected APL < DPL, and these cases are characterized by a threat case (raid) that overwhelms the defense.

Fractional Factorial Design

We then employed another simplifying assumption that pruned the case space even further: we assumed that only two interceptor types would be employed in a battery. This is not unreasonable since two of the four interceptor types in question only had anti-cruise missile (or unmanned aerial vehicle (UAV)) capability while the remaining two interceptors had both counter-RAM (C-RAM) and anti-CM capabilities. During the detailed discussion with the study sponsor, the MML battery Program Manager, it was deemed unrealistic to loadout a single MML battery with more than two types of interceptors. This was based on detailed consideration of all possible future MML battery locations. The decision to study two-interceptor-type mixes for MML batteries was mainly driven by the types and quantities of threats to be expected at each of these future MML battery locations, while also taking into account the other Air and Missile Defense (AMD) systems deployed in conjunction with the MML battery, such as the Patriot AMD system. For example, one potential MML battery location involved the use of the MML battery for full 360-degree CM coverage working with existing Patriot AMD batteries. In this case, the best mix for the MML battery consisted of a long-range anti-CM interceptor with a short-range anti-CM interceptor, a two-interceptor-type mix. In another potential MML battery location, the main mission for the MML battery would be C-RAM, with a secondary mission of anti-UAV. Thus, for this location, a C-RAM interceptor along with a short-range anti-CM interceptor was required, another two-interceptor-type mix.

Thus, the study sponsor requested CAA to proceed determining the optimum mix of interceptors per MML battery loadout with the cases involving only pairs of interceptor types. To insure that all possible combinations of the two-interceptor mixes were modeled within EADSIM, we employed a 2nd order fractional factorial design with the interceptor-type pairs being the two variables considered. Since it was determined, with the sponsor, that the minimum quantity per interceptor type that could be provided per MML battery was 10, increments of ten were used for the quantity of each interceptor per MML battery loadout (i.e., 10, 20, 30 , 40 and 50). For each of the cases, then, the factorial design yielded 25 different loadout combinations. Finally, the APL was calculated for each of the 25 loadout pairings using the EADSIM model output. The loadout pairing with the highest APL was chosen to be the optimal loadout pairing for each case.

CONCLUSION

Using the method just described, we were able to deliver a quick turnaround analysis that was based on reasonable simplifying assumptions. This is in keeping with the ethos of simulation modeling in general, that the model rests on assumptions that are reasonable, but that have to be admitted to satisfy the strictures of the study resources limitations. This is an example of the part of modeling we usually call "the art." While these techniques rely on underlying reasonable arguments, they are not particularly systematized and require some insight into the domain, or modeling itself, on the part of the analyst.

Another way we could have gotten an edge on the time requirement would have been to improve the process by which we build, or setup, the EADSIM cases, or to improve the throughput of the EADSIM model replications by, for instance, better parallelizing the execution of the replications. There is the likely possibility that such process improvements could have been leveraged to relax or remove some of the simplifying assumptions on which we had to rely.

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DESCRIPTORS

Air and missile defense, cruise missile, CM, rocket, artillery, mortar, RAM, fractional analysis, Multi-Mission Launcher, MML, experiment design, achieved protection level, APL, desired protection level, DPL.

ACRONYMS

AMD — Air and Missile Defense.

APL — achieved protection level.

CAA — Center for Army Analysis.

CM — cruise missile.

C-RAM — counter-RAM.

DPL — desired protection level.

EADSIM — Extended Air Defense Simulation.

MML — Multi-Mission Launcher.

Pk — probability of kill.

RAM — rocket, artillery and mortar.

UAV — unmanned aerial vehicle.