CSE426
Robot Kinematics

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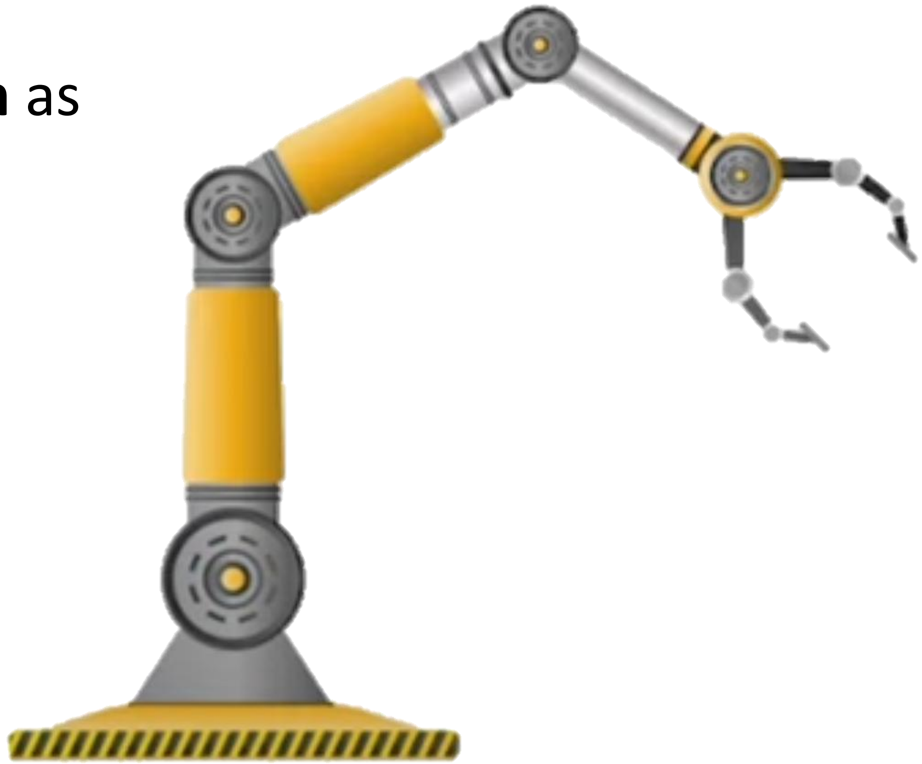
Robot kinematics deals with the analytical study of the **geometry of motion** of a manipulator with respect to **a fixed reference co-ordinate system** as a function of time **without regard to the force** that causes the motion



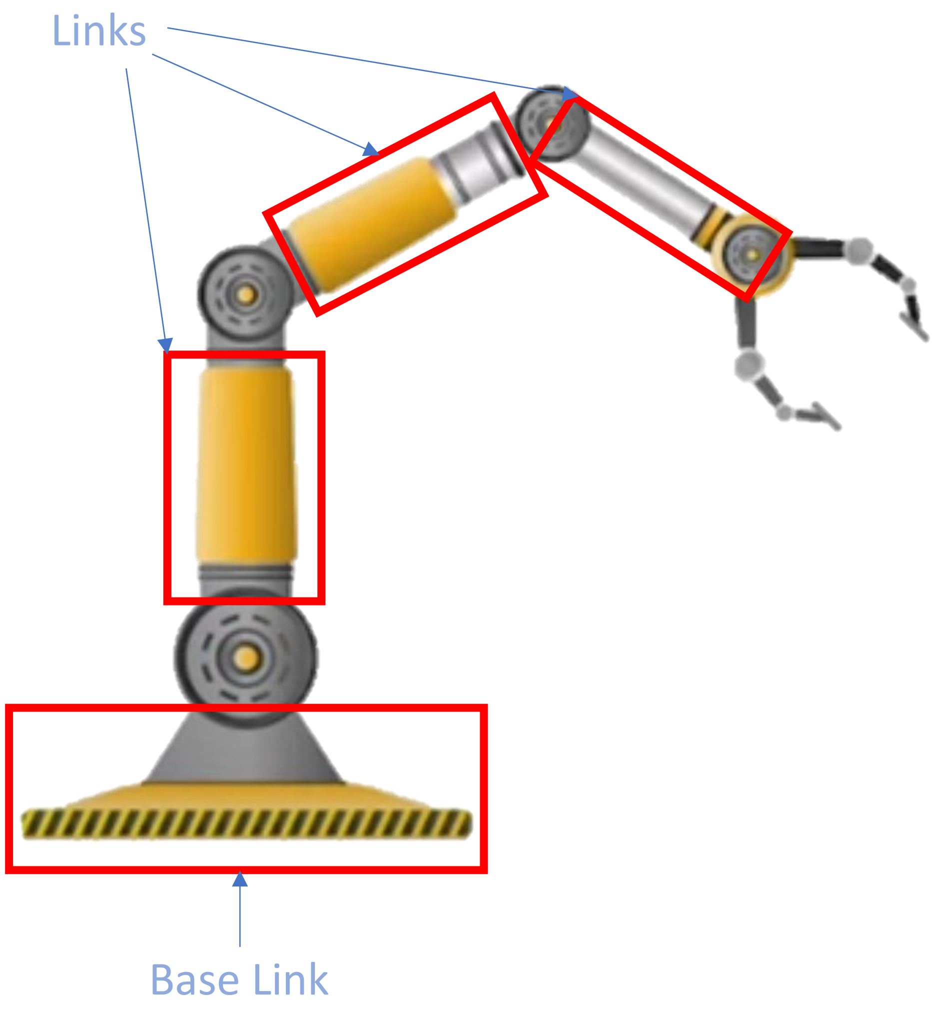
Example of robot manipulator: Robotic arm

Anatomy divided into base, links, joints and end-effector.

Kinematics deals with the relationships between joint coordinates, end-effector positions, and their velocities

Parts of a Robot Manipulator

- **Links:** The rigid segments between the joints.
- **Joints:** The movable connections between links, which can be rotational (revolute) or translational (prismatic).
- **End-Effector:** The tool or device at the end of the manipulator, designed to interact with the environment (e.g., grippers, welders, sensors).
- Actuators and Sensors



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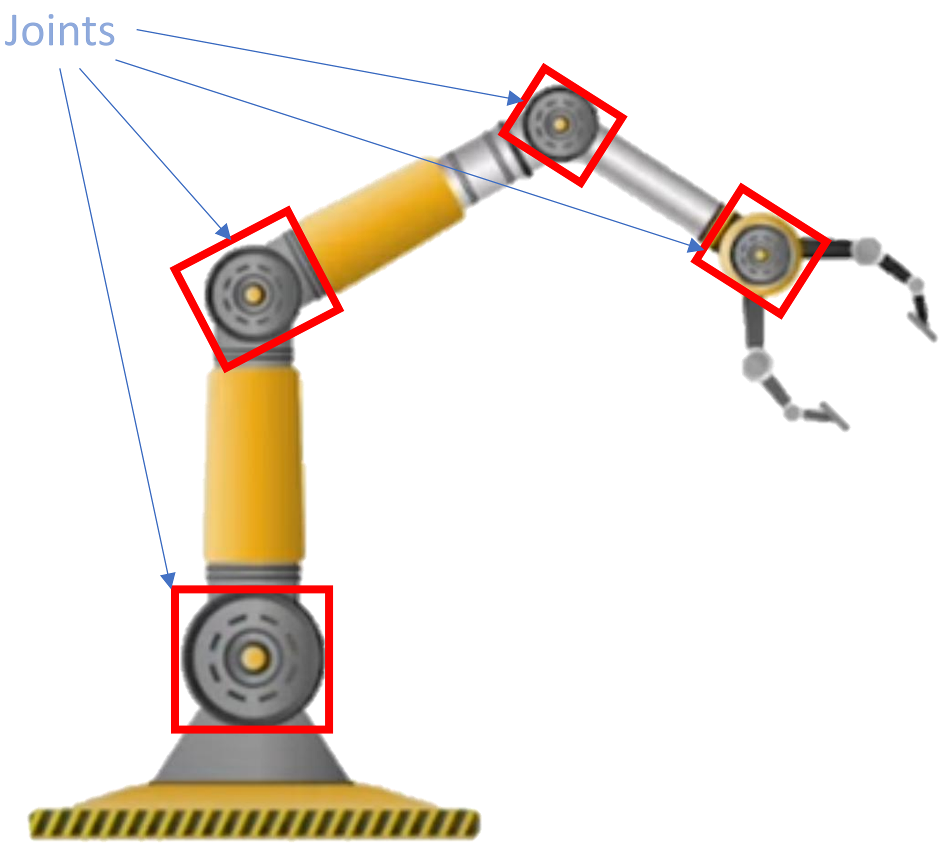
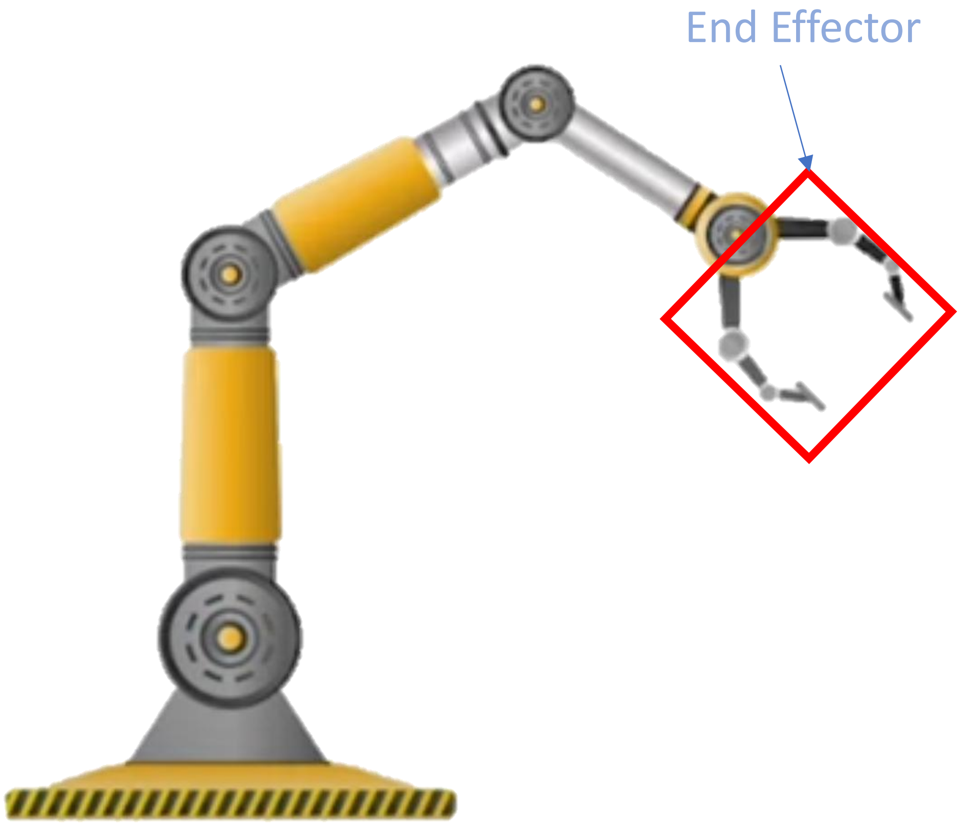


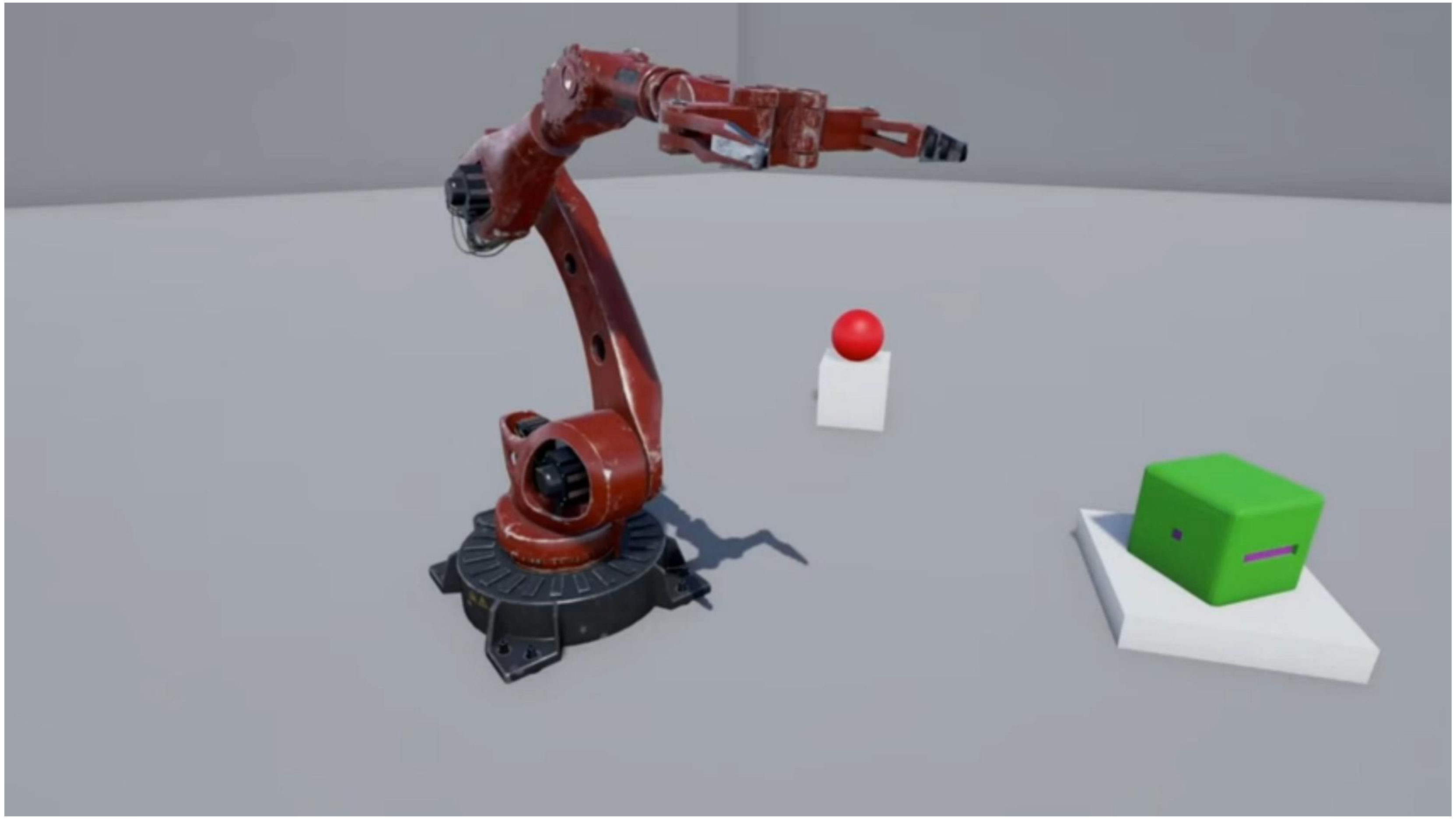
Parts of a Robot Manipulator

- **Links:** The ***rigid*** segments between the joints.
- **Joints:** The ***movable*** connections ***between links***, which can be rotational (revolute) or translational (prismatic).
- **End-Effector:** The tool at the end of the manipulator, designed to ***interact with the environment*** to achieve certain ***goals*** (e.g., grippers, welders, sensors).
- Actuators and Sensors



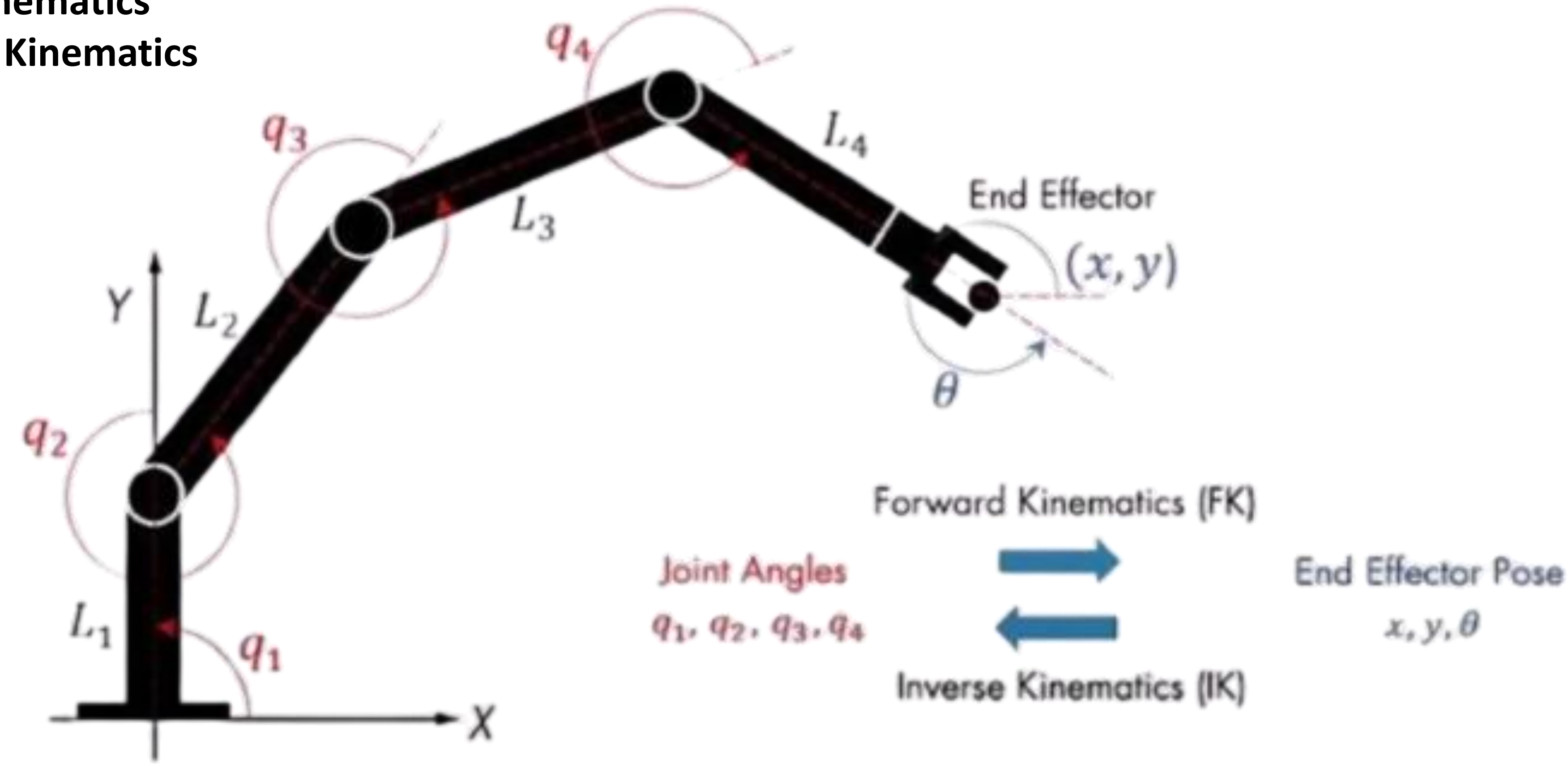
Pick and Place Scenario

- Use sensors to detect position of the object
- Calculate the object's coordinates
- Plan the optimal path to the object
- Calculate and actuate joint angles to reach the object
- Pick up the object and move it to the target location
- Position and release the object at the designated spot



Types of Kinematics

1. **Forward/Direct Kinematics**
2. **Backward/Inverse Kinematics**



Types of Kinematics

1. **Forward/Direct Kinematics** – We want to know the final position of the end effector due to the movement of the joints.

Joint angles $\Longrightarrow$ x, y, θ

2. **Backward/Inverse Kinematics** – Given the final position of the end effector, we want to know the angle of the joints.

x, y, θ $\Longrightarrow$ Joint angles

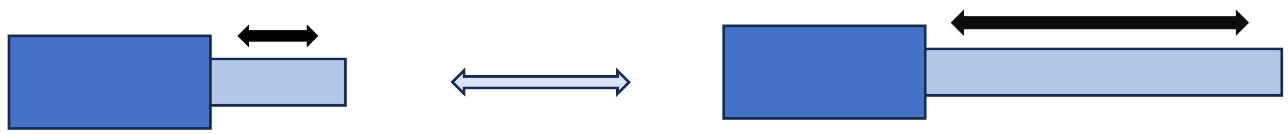
Types of Joints

Based on Mobility

- Active Joints – Can move, can change angles
- Passive Joints – Rigid. Cannot move, cannot change angles.

Based on type of movement (applicable for active joints)

- Prismatic Joints (P-joint) – Allows linear/sliding motion along a single axis (1DoF)

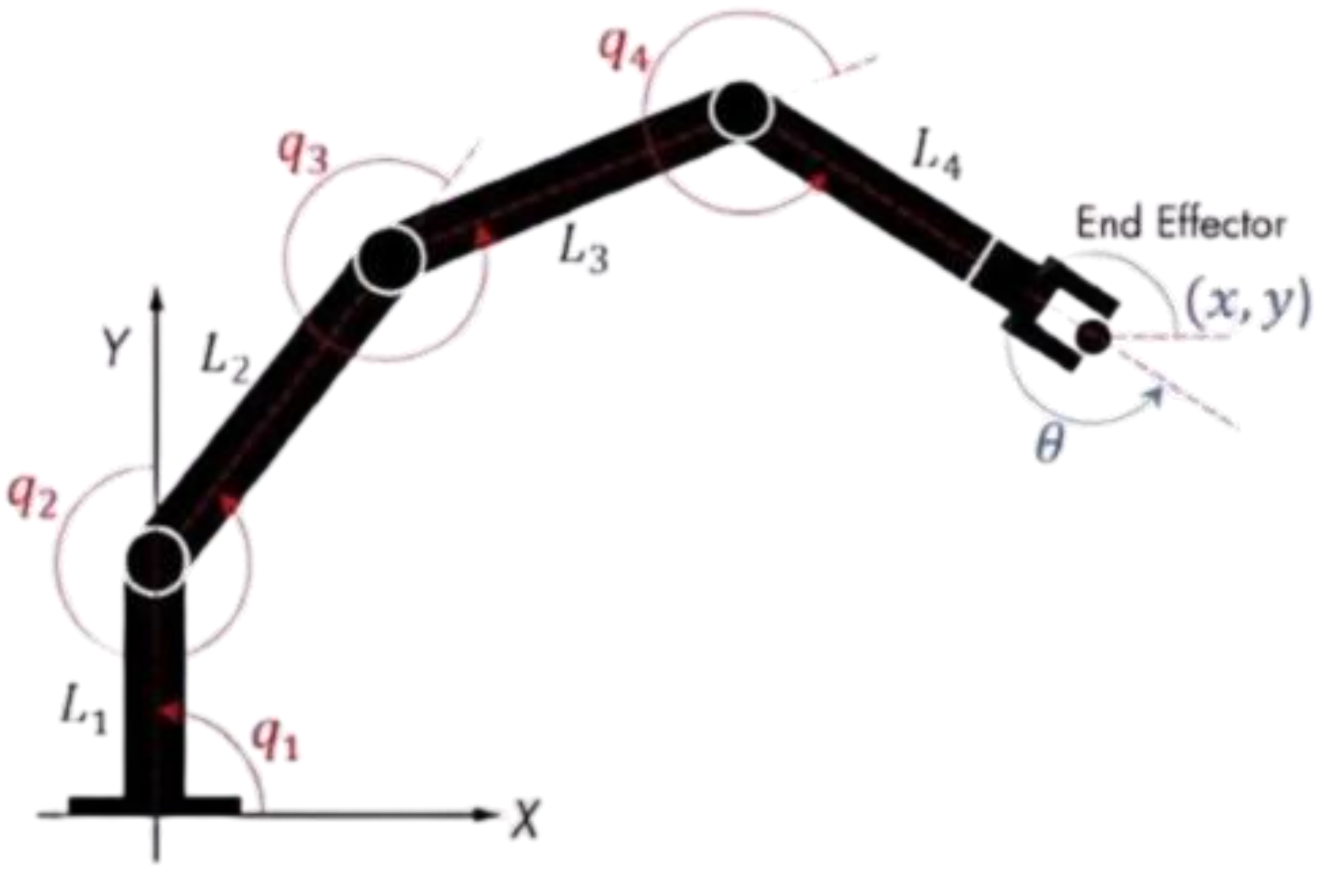


- Revolute Joint (R-Joint) – Allows rotational motion around a single axis (1 DoF)

DoF (Degree of Freedom) – No. of active joints

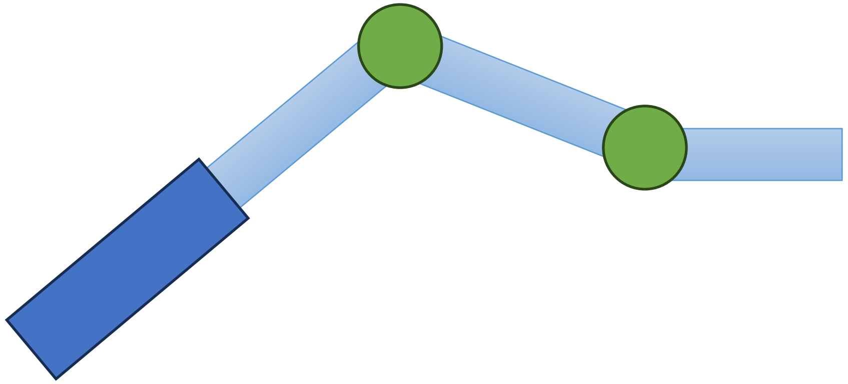
Types:

- General purpose robots – 6 DoF
- Redundant Robots – more than 6 DoF
- Deficient Robots – less than 6 Dof

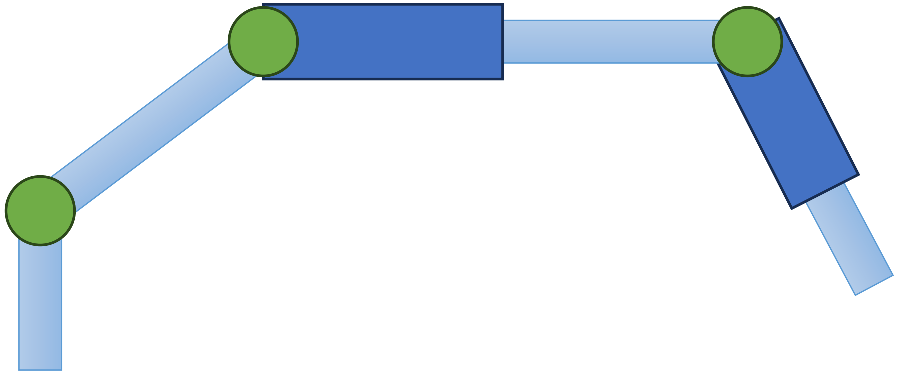


Exercise

1. DoF = ?

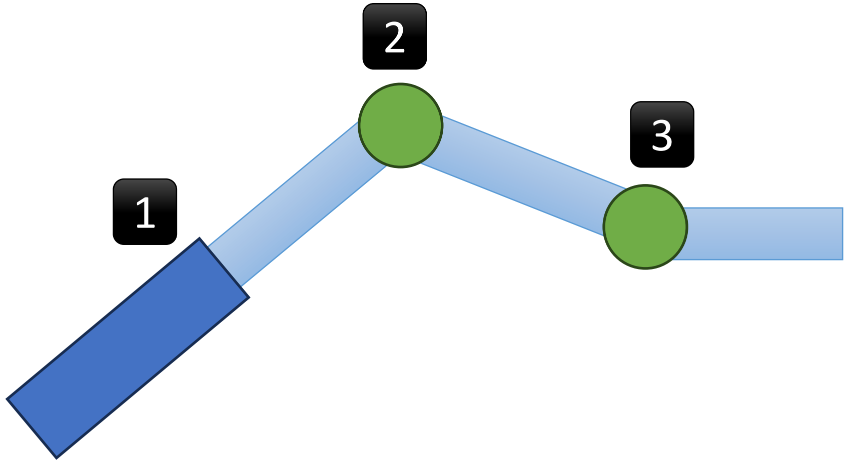


2. DoF = ?

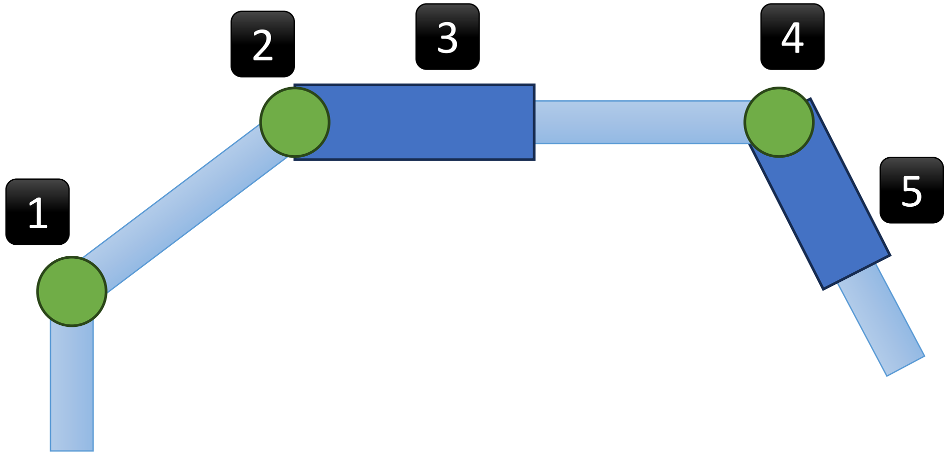


Solution

1. DoF = 3



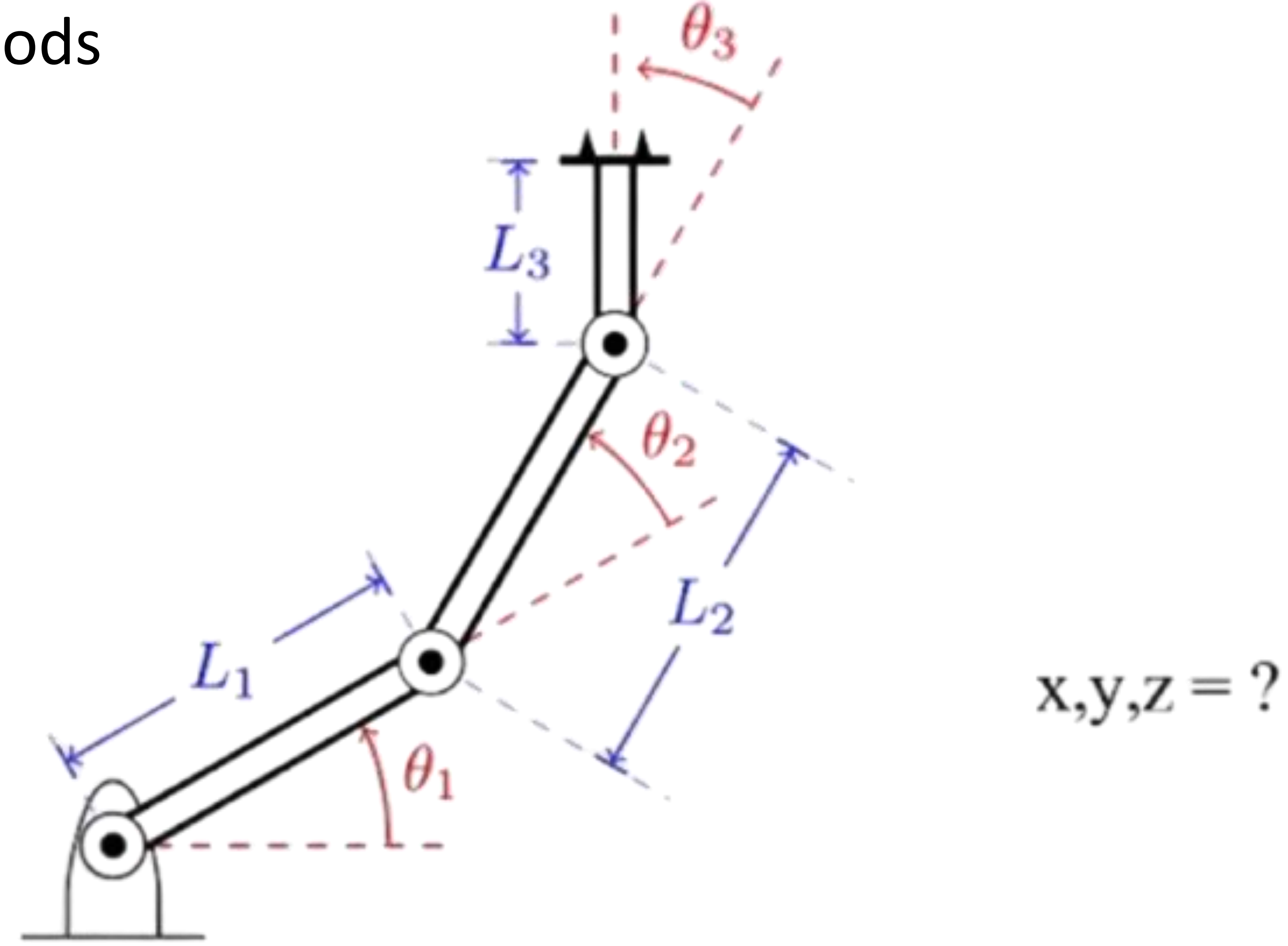
2. DoF = 5



Kinematic Problem Solving Methods

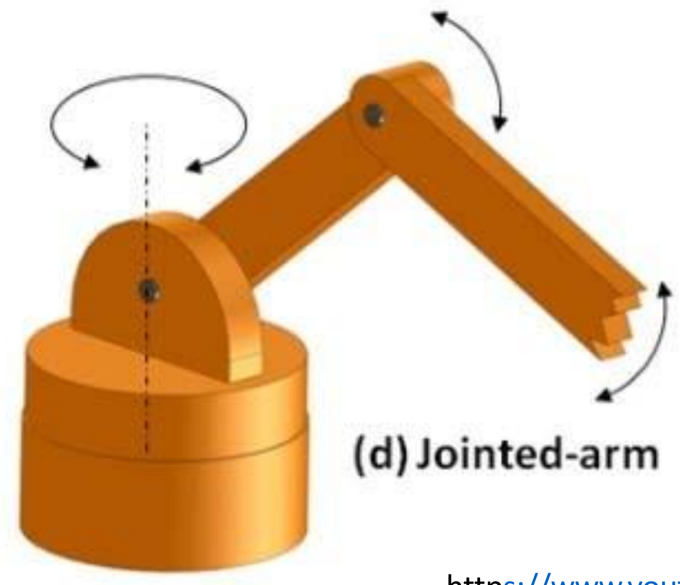
1. Geometrically
   - Using Trigonometry
   - Simple, easy to use
   - Suitable for 2-3 DoFs
   - Very complex for >3 DoFs

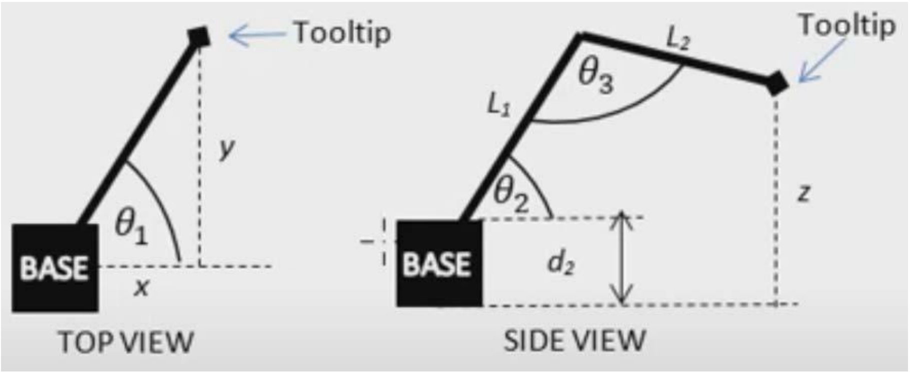
2. Using Link Parameter
   - Using D-H Parameter
   - Simple, robust process
   - Common procedure for all cases
   - Can solve for all DoFs in the same way



Forward and Backward Kinematics on Manipulator

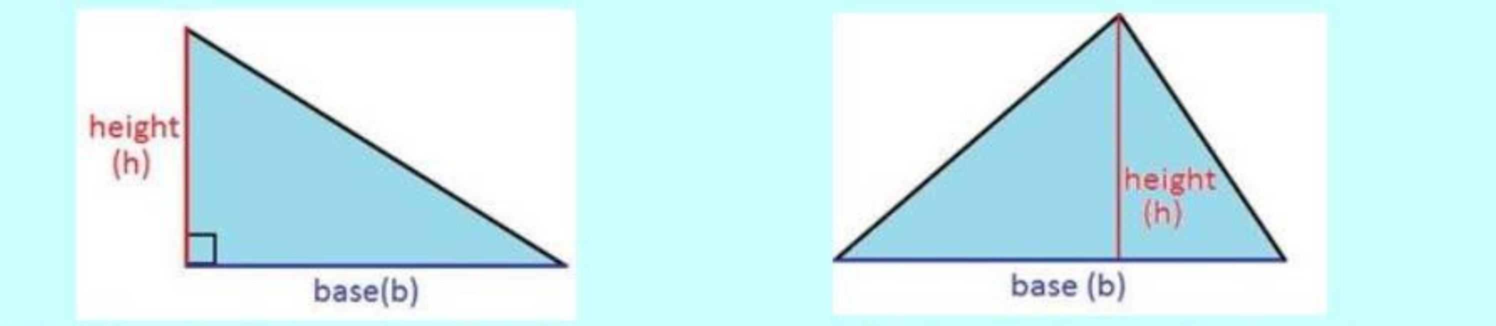
Forward Kinematics





https://www.youtube.com/watch?v=NRgNDlVtmz0

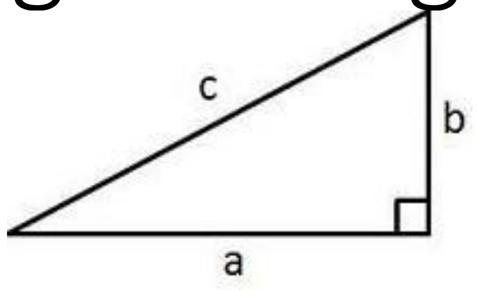
Area of a triangle

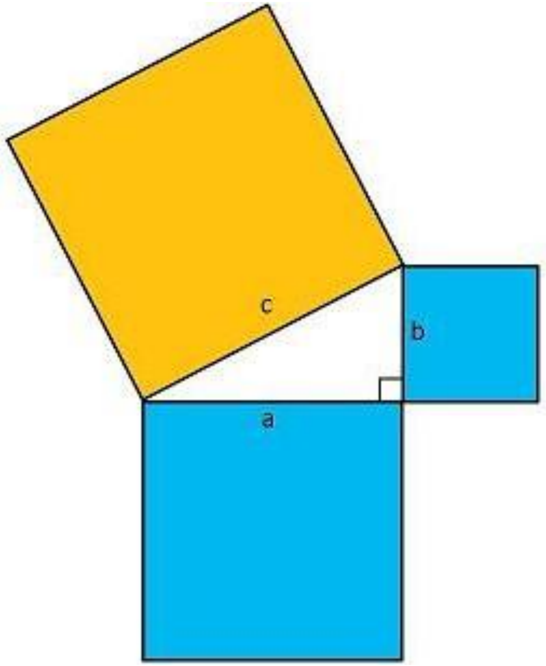


A right triangle showing height and base. A non-right triangle showing height and base.

- $area\ of\ a\ triangle = \frac{1}{2}\ X\ base\ X\ height$

Pythagoras' Theorem for right triangle

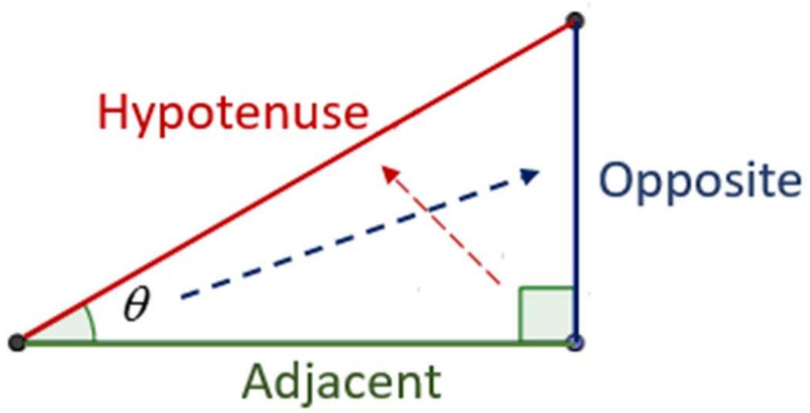




- $c^2 = a^2 + b^2$

Basic Trigonometric Functions

SOHCAHTOA



SOH $\quad \text{Sin}\,\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$

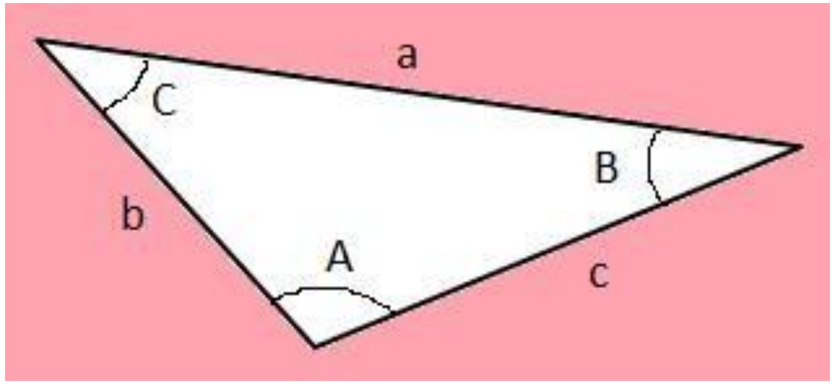
CAH $\quad \text{Cos}\,\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA $\quad \text{Tan}\,\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

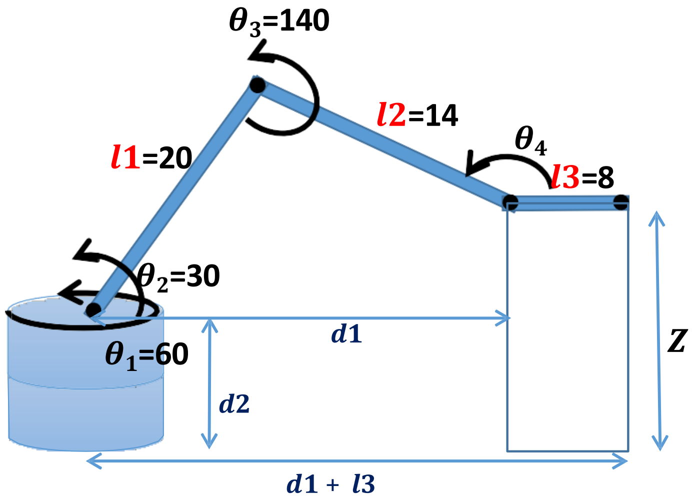
The Sine and Cosine Rules

- *Sine Rule:*
- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

- *Cosine Rule:*
- $a^2 = b^2 + c^2 - 2bcCosA$
- *or*
- $CosA = \frac{b^2+c^2-a^2}{2bc}$



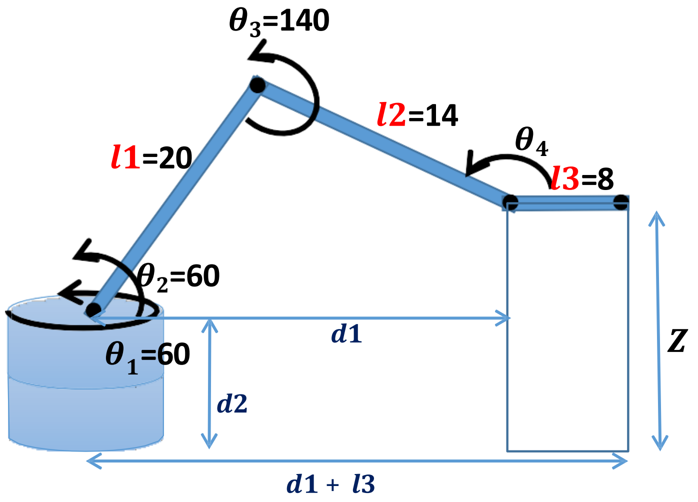
4 DOF Arm Calculation



- $l1$=Length of first arm
- $l2$=Length of 2nd Arm
- $l3$=Length of 3rd Arm (end effector)
- $d2$=height of base

- θ_1 = Angle of base rotation
- θ_2 = Angle of first arm from horizon
- θ_3 = Angle between 1st and 2nd arm
- θ_4 = Angle between 2nd arm and end effector

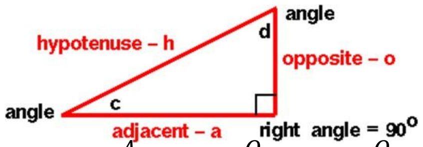
Example with a Value



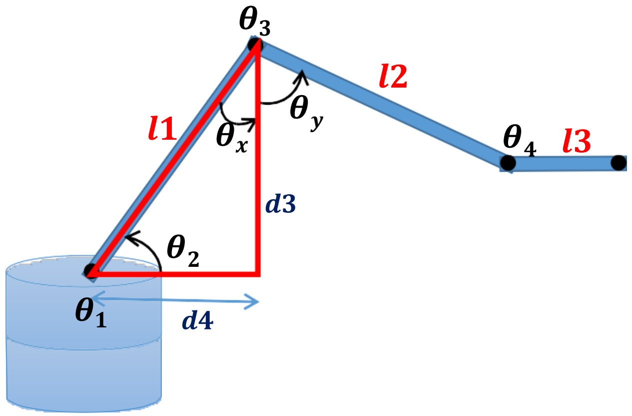
- *l1=20”*
- *l2=14”*
- *l3=8”*
- *d2=18”*

- $\theta_1 = 50$
- $\theta_2 = 60$
- $\theta_3 = 95$

Forward Kinematics Calculation

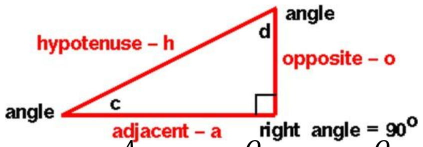


$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$



- $If\ we\ draw\ a\ right\ angle\ triangle\ with\ first\ arm,$
- θ_x $is\ the\ angle\ with\ opposit\ —d3$
- θ_y $is\ the\ angle\ with\ Adjacent\ —d4$
- $So, \theta_3 = \theta_x + \theta_y$
- $Here\ l1\ is\ hypotenus$
- $So, sin\theta = \frac{0}{H};\ sin\theta_2 = \frac{d3}{l1}$
- $d3 = sin\theta_2 * l1$
- $cos\theta = \frac{A}{H};\ cos\theta_2 = \frac{d4}{l1}$
- $d4 = cos\theta_2 * l1$

Example continuation…



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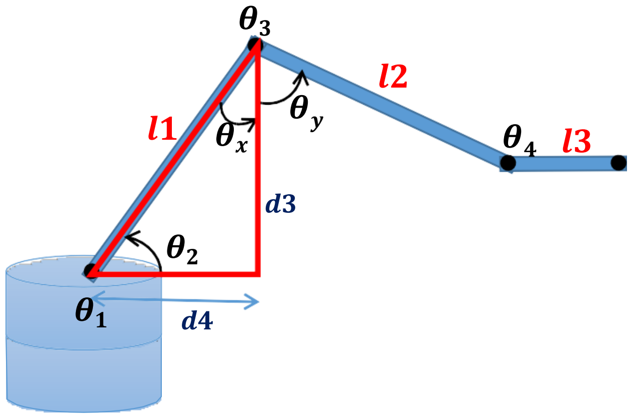
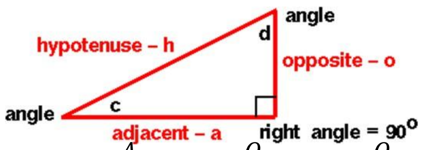


- $\theta_x = 180 - 90 - \theta_2$
- $\theta_x = 180 - 90 - 60 = 30$
- $\theta_3 = \theta_x + \theta_y$
- $\theta_y = (\theta_3 - \theta_x)$
- $\theta_y = 95 - 30 = 65$
- $d3 = sin\theta_2 * l1$
- $d3 = sin60 * l1$
- $d3 = 0.866 * 20 = 17.32$
- $d4 = cos\theta_2 * l1$
- $d4 = cos60 * l1$
- $d4 = .5 * 20 = 10$

l1=20”
l2=14”
l3=8”
d2=18”
$\theta_1 = 50$
$\theta_2 = 60$
$\theta_3 = 95$

Forward Kinematics Calculation



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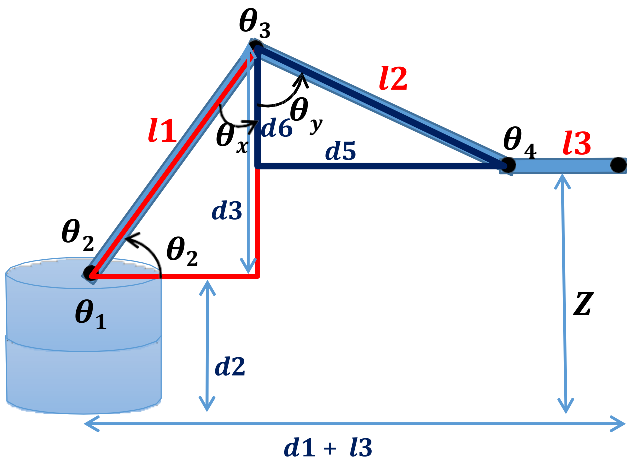
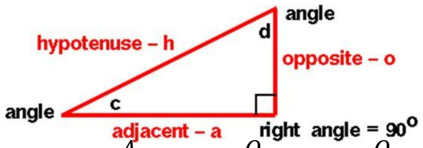


- $If\ we\ draw\ a\ right\ angle\ triangle\ with\ 2nd\ arm\ l2$
- $\theta_y\ is\ the\ angle\ with\ opposit\ —d6$
- $l2\ is\ hypotenus$
- $So,\ cos\theta = \frac{\text{A}}{\text{H}};\ cos\theta_y = \frac{\text{d6}}{\text{l2}};$
- $d6 = cos\theta_y * l2;$
- $sin\theta = \frac{\text{0}}{\text{H}};\ sin\theta_y = \frac{\text{d5}}{\text{l2}};$
- $d5 = sin\theta_y * l2$

Example continuation…



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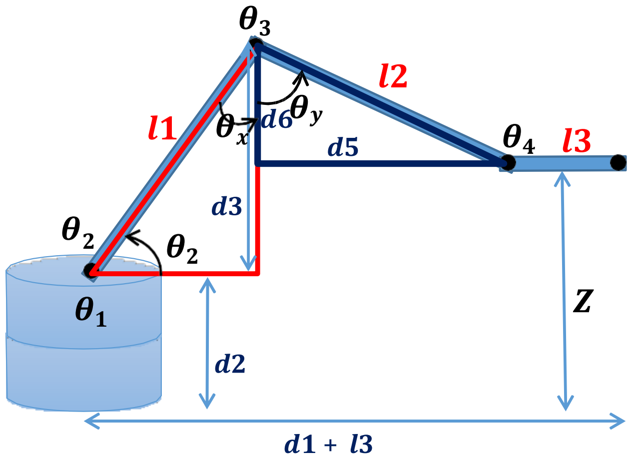
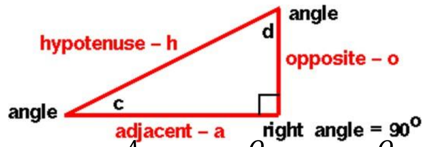


- $d6 = cos\theta_y * l2;$
- $d6 = cos65 * l2;$
- $d6 = 0.4226 * 14 = 5.91$
- $d5 = sin\theta_y * l2$
- $d5 = sin65 * l2$
- $d5 = .9 * 14$
- $d5 = 12.69$

l1=20”
l2=14”
l3=8”
d2=18”
$\theta_1 = 50$
$\theta_2 = 60$
$\theta_3 = 95$
$\theta_x = 30$
$\theta_y = 65$
$d3 = 17.32$
$d4 = 10$

Forward Kinematics Calculation



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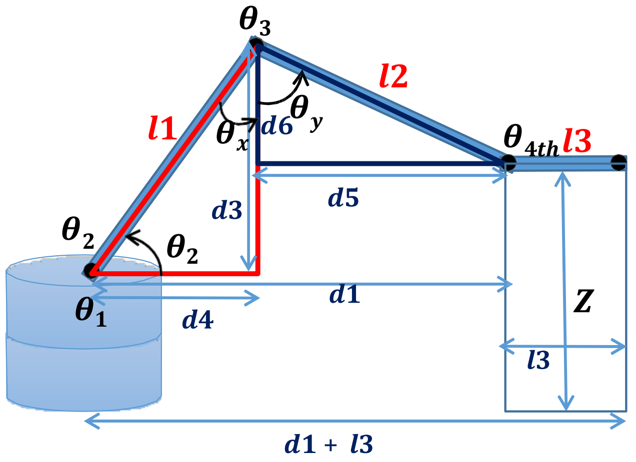
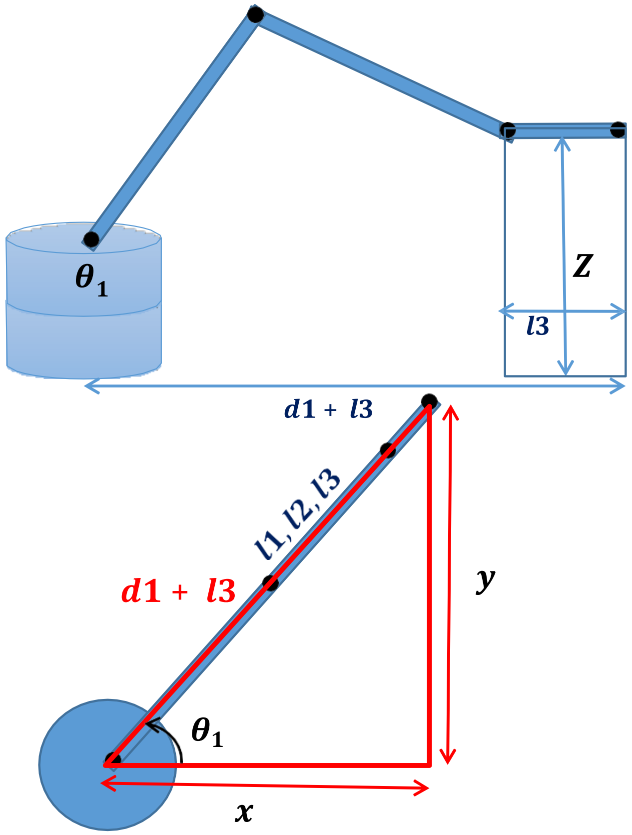
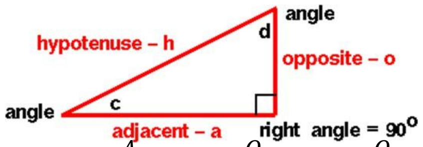


- $Z = d2 + (d3 - d6)$
- $d1 = d4 + d5$

- *So, d1+l3 is the current length of arm from base*
- *Z is the height of arm endpoint*

Now transform from top view

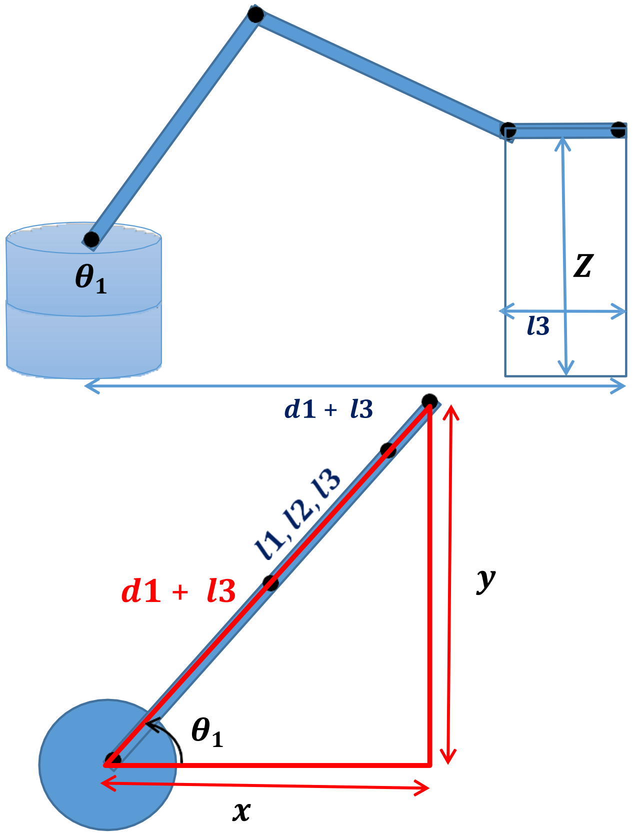


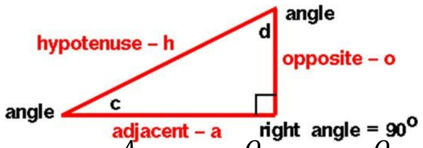


$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$

- $cos\theta_1 = \frac{\mathrm{x}}{\mathrm{d1+l3}};$
- $x = cos\theta_1 * (d1 + l3)$
- $sin\theta_1 = \frac{\mathrm{y}}{\mathrm{d1}};$
- $y = sin\theta_1 * (d1 + l3)$
- $Position\ is: (x, y, z)$

Example continuation…



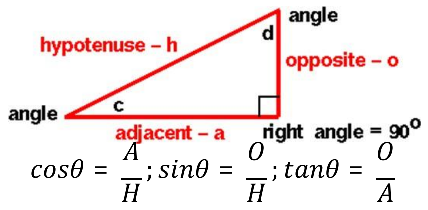


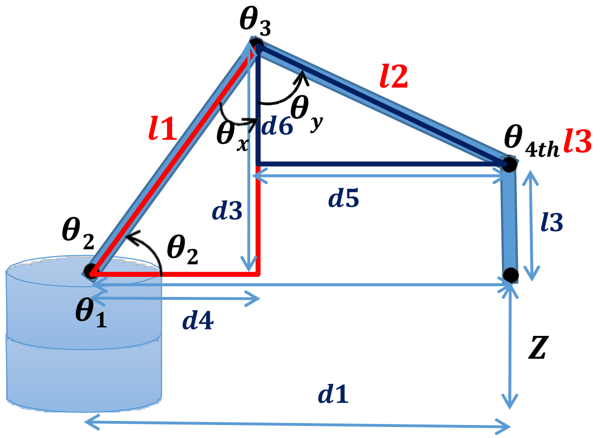
$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$

- $Z = d2 + (d3 — d6)$
- $Z = 18 + 17.32 — 5.916 = 29.4$
- $d1 = d4 + d5 = 10 + 12.69 = 22.69$
- $x = cos\theta_1 * (d1 + l3)$
- $x = cos50 * (22.69 + 8) = 19.73$
- $y = sin50 * (22.69 + 8) = 23.51$
- *Position is*:
- $(x, y, z) = (19.73, 23.51, 29.4)$

l1=20"
l2=14"
l3=8"
d2=18"
$\theta_1 = 50$
$\theta_2 = 60$
$\theta_3 = 95$
$\theta_x = 30$
$\theta_y = 65$
$d3 = 17.32$
$d4 = 10$
$d6 = 5.91$
$d5 = 12.69$

Forward Kinematics Calculation

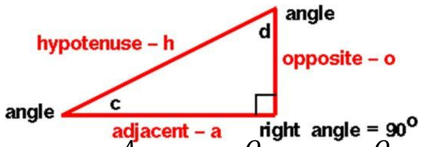




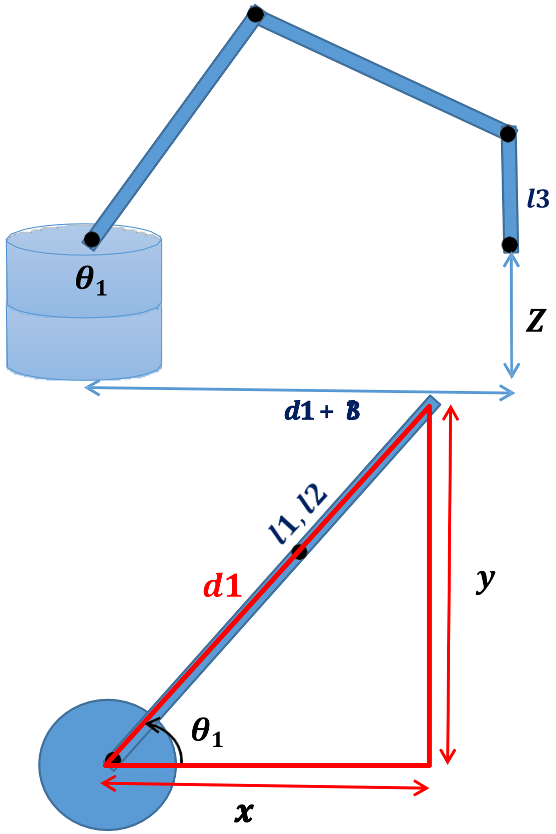
- $Z = d2 + (d3 - d6) - l3$
- $d1 = d4 + d5$

- *So, d1 is the current length of arm from base*
- *Z is the height of arm endpoint*

Now transform from top view

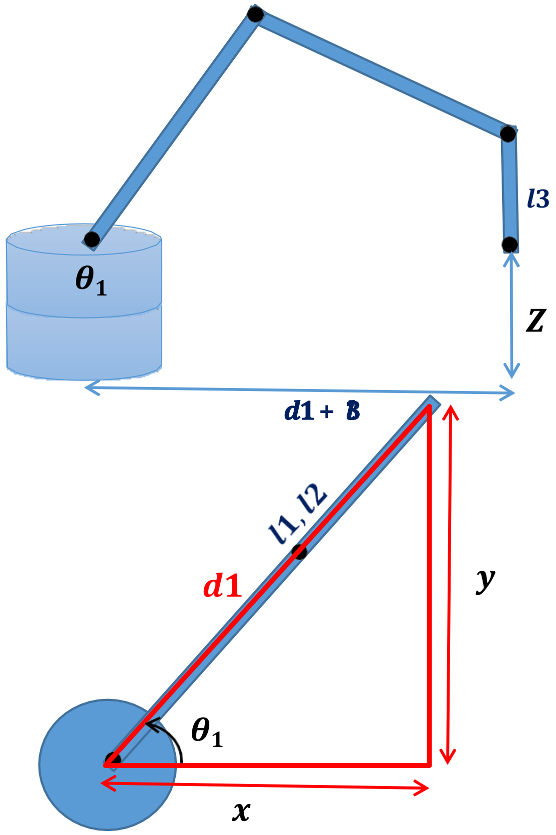


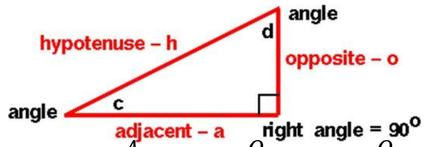
$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$



- $Z = d2 + (d3 - d6) - l3$
- $cos\theta_1 = \frac{\mathrm{x}}{\mathrm{d1}}$;
- $x = cos\theta_1 * d1$
- $sin\theta_1 = \frac{\mathrm{y}}{\mathrm{d1}}$;
- $y = sin\theta_1 * d1$
- $Position\ is: (x, y, z)$

Now transform from top view



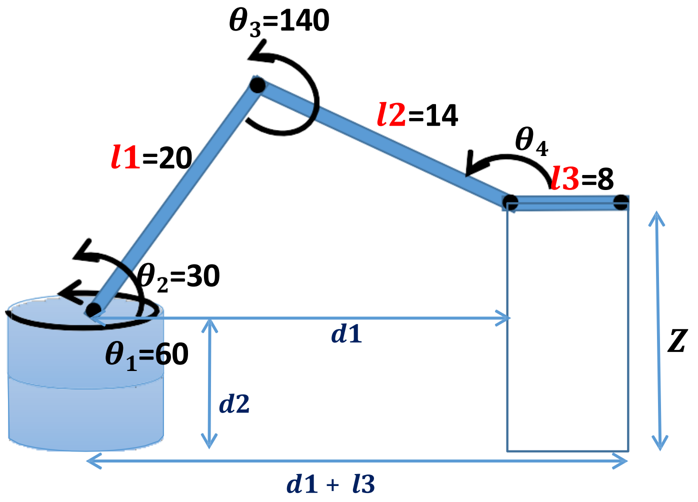


$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$

- $Z = d2 + (d3 - d6) - l3$
- $Z = 18 + 17.32 - 5.916 - 8 = 21.4$
- $d1 = d4 + d5 = 10 + 12.69 = 22.69$
- $x = cos\theta_1 * d1$
- $x = cos50 * 22.69 = 14.58$
- $y = sin\theta_1 * d1$
- $y = sin50 * 22.69 = 17.38$
- $Position\ is$:
- $(x, y, z) = (14.58, 17.38, 21.4)$

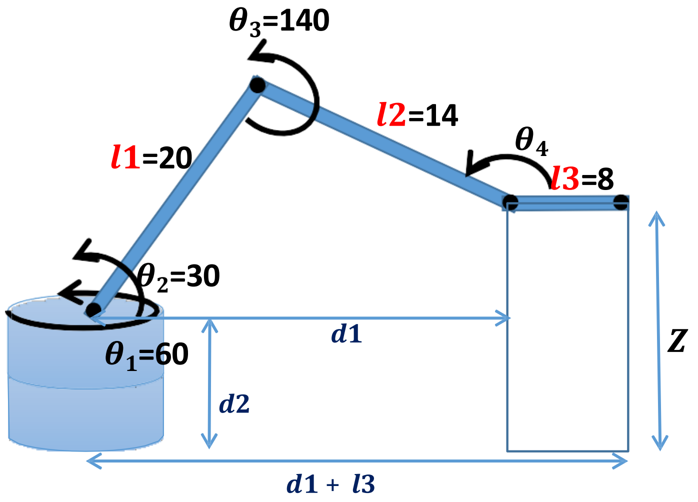
$l1 = 20''$
$l2 = 14''$
$l3 = 8''$
$d2 = 18''$
$\theta_1 = 50$
$\theta_2 = 60$
$\theta_3 = 95$
$\theta_x = 30$
$\theta_y = 65$
$d3 = 17.32$
$d4 = 10$
$d6 = 5.91$
$d5 = 12.69$

Reverse Kinematics



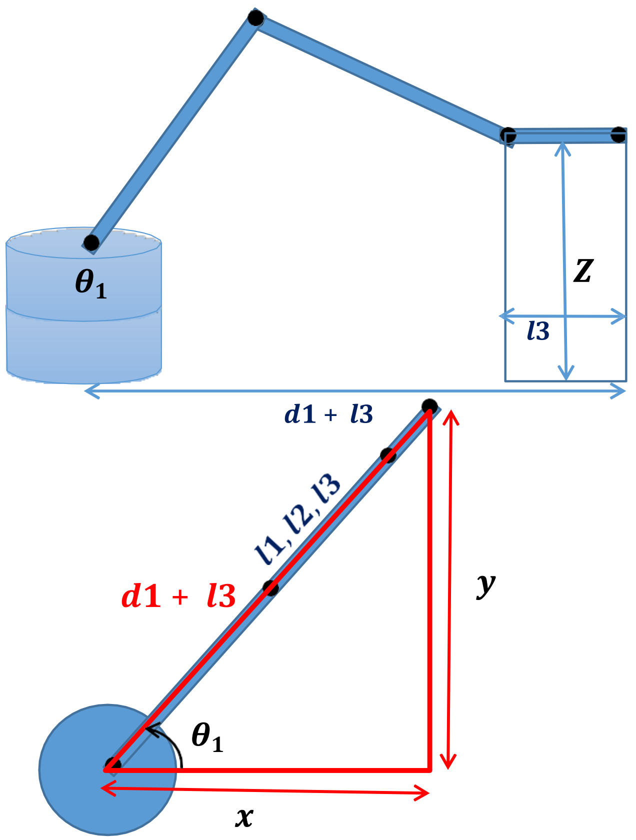
- $x =$ *Position on front*
- $y =$ *Position on left or right*
- $z =$ *Position on height*
- $l1 =$ *Length og first arm*
- $l2 =$ *Length of second arm*
- $l3 =$ *Length of third arm*
- $d2 =$ *Height of base from ground*

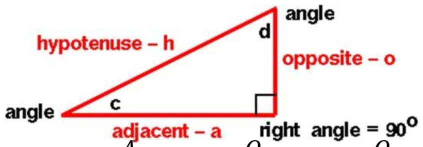
Reverse Kinematics Example value



- $x = 20$
- $y = 25$
- $z = 30$
- $l1 = 20$
- $l2 = 14$
- $l3 = 8$
- $d2 = 18$

Reverse Kinematics

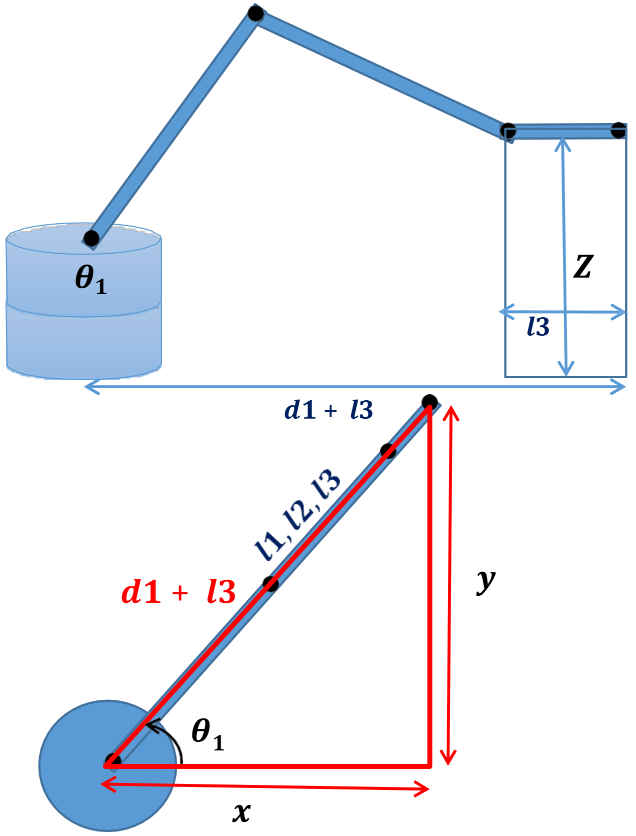


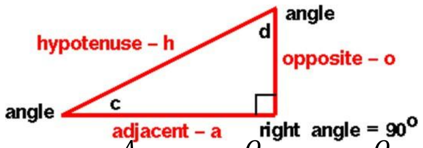


$$\cos\theta = \frac{A}{H};\ \sin\theta = \frac{O}{H};\ \tan\theta = \frac{O}{A}$$

- $(d1 + l3)^2 = x^2 + y^2$
- $d1 = \sqrt{x^2 + y^2} - l3$
- $Cos\theta_1 = \frac{x}{d1+l3}$
- $\theta_1 = Cos^{-1}\left(\frac{x}{d1+l3}\right)$

Example continuation…



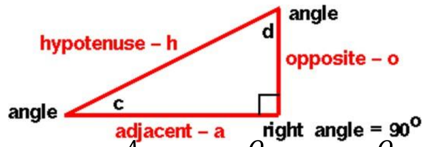


$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$

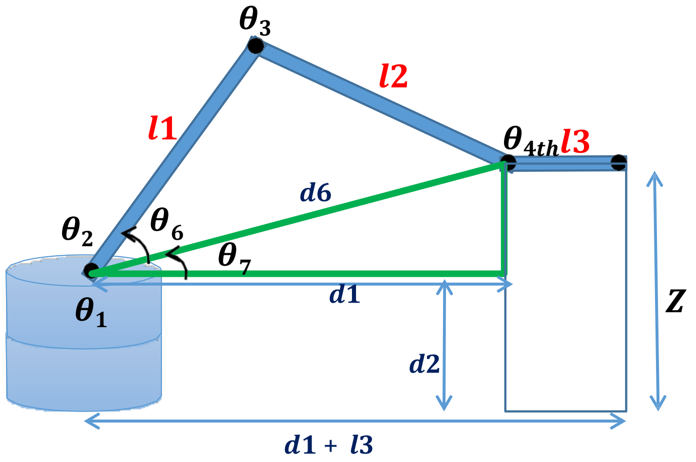
- $d1 = \sqrt{x^2 + y^2} - l3$
- $d1 = \sqrt{19.73^2 + 23.51^2} - 8$
- $d1 = 22.692$
- $\theta_1 = Cos^{-1}\left(\frac{\mathrm{x}}{\mathrm{d1+l3}}\right)$
- $\theta_1 = Cos^{-1}\left(\frac{19.73}{22.69+8}\right)$
- $\theta_1 = 50$

$x = 19.73,$
$y = 23.51,$
$z = 29.4$
$l1 = 20$
$l2 = 14$
$l3 = 8$
$d2 = 18$

Reverse Kinematics

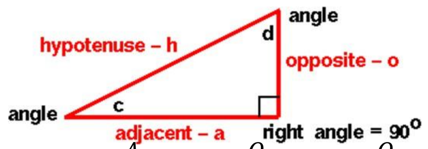


$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$

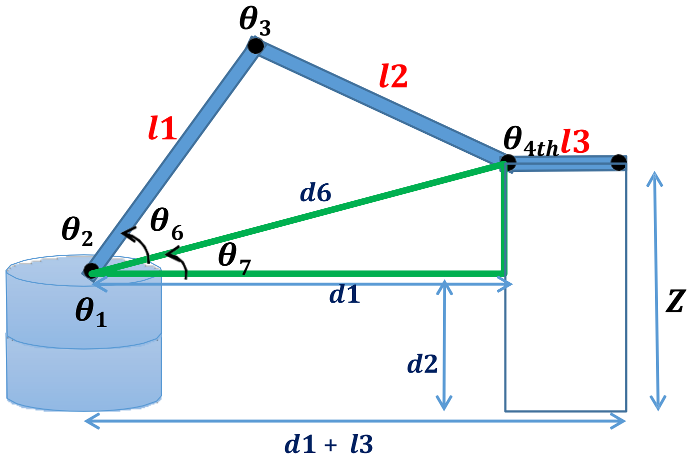


- $d6^2 = d1^2 + (z - d2)^2$
- $d6 = \sqrt{d1^2 + (z - d2)^2}$
- $Cos\theta_7 = \frac{d1}{d6};$
- $\theta_7 = Cos^{-1}(\frac{d1}{d6})$

Example continuation…



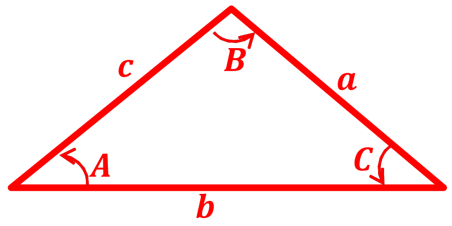
$$\cos\theta = \frac{A}{H}; \sin\theta = \frac{O}{H}; \tan\theta = \frac{O}{A}$$

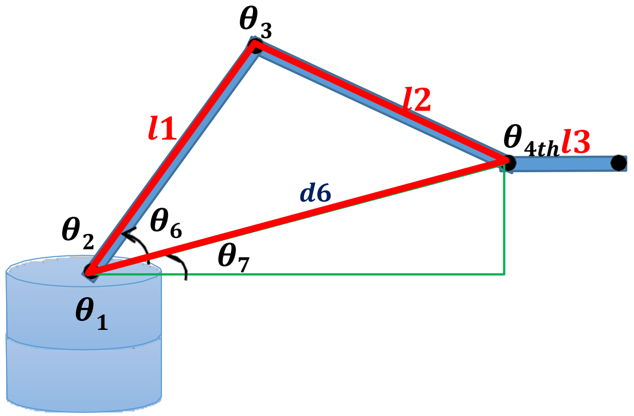


- $d6 = \sqrt{d1^2 + (z - d2)^2}$
- $d6 = \sqrt{22.69^2 + (29.4 - 18)^2}$
- $d6 = 25.393$
- $\theta_7 = Cos^{-1}(\frac{d1}{d6})$
- $\theta_7 = Cos^{-1}(\frac{22.692}{25.393})$
- $\theta_7 = 26.67$

$x = 19.73,$
$y = 23.51,$
$z = 29.4$
$l1 = 20$
$l2 = 14$
$l3 = 8$
$d2 = 18$
$d1 = 22.692$
$\theta_1 = 50$

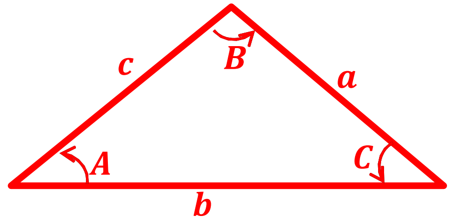
Reverse Kinematics



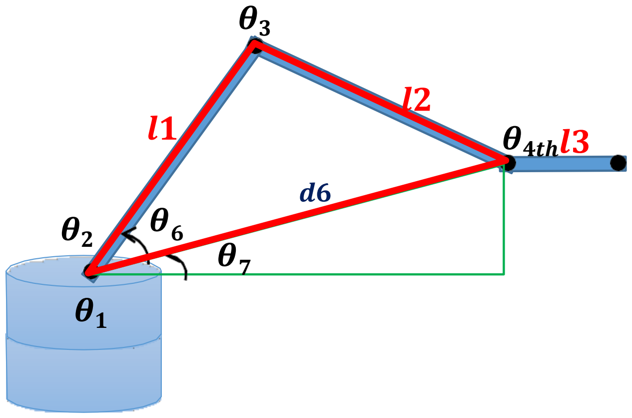


- $b^2 = a^2 + c^2 - 2acCosB$
- $Cos\theta_3 = \left(\frac{l1^2 + l2^2 - d6^2}{2*l1*l2}\right);$
- $\theta_3 = Cos^{-1}\left(\frac{l1^2 + l2^2 - d6^2}{2*l1*l2}\right)$
- $Cos\theta_6 = \left(\frac{l1^2 + d6^2 - l2^2}{2*l1*d6}\right);$
- $\theta_6 = Cos^{-1}\left(\frac{l1^2 + d6^2 - l2^2}{2*l1*d6}\right)$
- $\theta_4 = \theta_2 + \theta_3$

Example continuation...

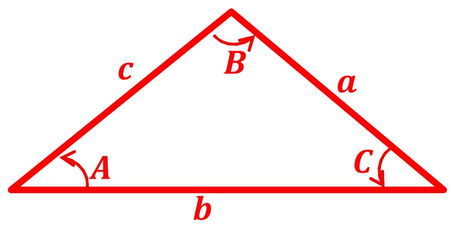


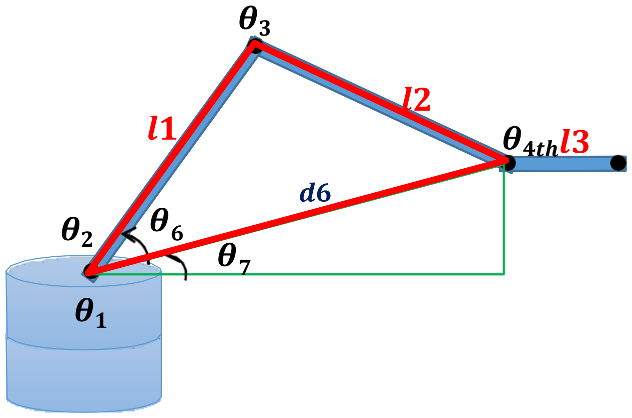
- $\theta_3 = Cos^{-1}\left(\frac{l1^2+l2^2-d6^2}{2*l1*l2}\right)$
- $\theta_3 = Cos^{-1}\left(\frac{20^2+14^2-25.39^2}{2*20*14}\right)$
- $\theta_3 = 95$



$x = 19.73,$
$y = 23.51,$
$z = 29.4$
$l1 = 20$
$l2 = 14$
$l3 = 8$
$d2 = 18$
$d1 = 22.692$
$\theta_1 = 50$
$d6 = 25.393$
$\theta_7 = 26.67$

Example continuation…

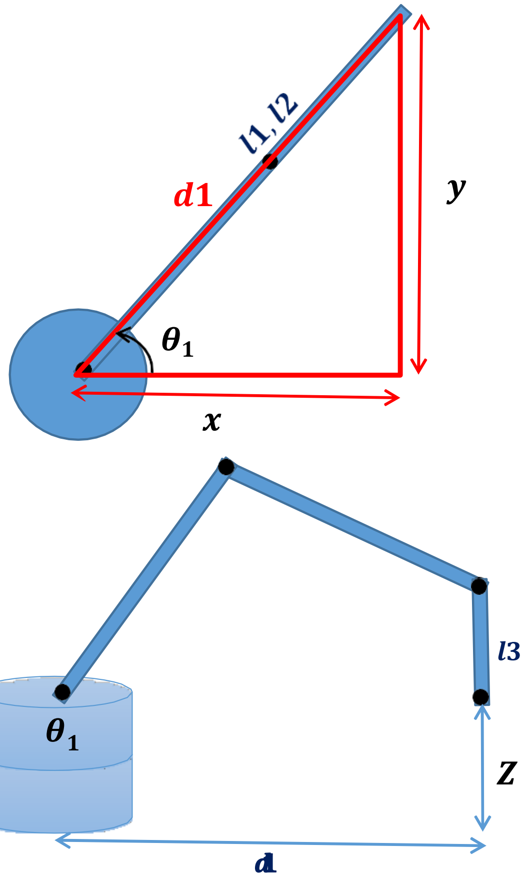




- $\theta_6 = Cos^{-1}\left(\frac{l1^2+d6^2-l2^2}{2*l1*d6}\right)$
- $\theta_6 = Cos^{-1}\left(\frac{20^2+25.39^2+14^2}{2*20*25.39}\right)$
- $\theta_6 = 33.31$
- $\theta_2 = 33.31 + 26.67$
- $\theta_2 = 59.98 \approx 60$
- $(\theta_1, \theta_2, \theta_3 \theta_4)$
- $(50, 60, 95)$

$x = 19.73,$
$y = 23.51,$
$z = 29.4$
$l1 = 20$
$l2 = 14$
$l3 = 8$
$d2 = 18$
$d1 = 22.692$
$\theta_1 = 50$
$d6 = 25.393$
$\theta_7 = 26.67$

Reverse Kinematics



- $d1 = \sqrt{x^2 + y^2}$
- $\theta_1 = Cos^{-1}(\frac{x}{d1})$
- $d6 = \sqrt{d1^2 + (z - d2)^2}$
- $\theta_7 = Cos^{-1}(\frac{d1}{d6})$ $l1^2+l2^2-$
- $\theta_3 = Cos^{-1}(\frac{d6^2}{2*l1*l2})$
- $\theta_6 = Cos^{-1}(\frac{l1^2+d6^2-l2^2}{2*l1*d6})$
- $\theta_2 = \theta_6 + \theta_7$
- $\theta_4 = \theta_2 + \theta_3 + 90$
- $(\theta_1, \theta_2, \theta_3, \theta_4)$