Overpass Channels: Horizontally Scalable, Privacy-Enhanced, with Independent Verification, Fluid Liquidity, and Robust Censorship Proof, Payments on TON

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Abstract

Overpass Channels presents a groundbreaking approach to blockchain scalability, offering a horizontally scalable, privacy-enhanced payment network with independent verification, fluid liquidity, and robust censorship resistance on the TON blockchain. This paper introduces a novel architecture that leverages zero-knowledge proofs, specifically zk-SNARKs, to ensure transaction validity and privacy while enabling unprecedented throughput and efficiency. By eliminating the need for traditional consensus mechanisms and miners, Overpass Channels achieves remarkable cost-effectiveness and energy efficiency. The system's design focuses on unilateral payment channels and off-chain transaction processing, allowing for high-speed, low-latency operations without compromising security or decentralization. This paper provides a comprehensive analysis of the Overpass Channels system, including its cryptographic foundations, scalability metrics, integration with TON, and potential applications across various domains, from global payments to confidential voting systems and secure health record management.

1 Introduction

The advent of blockchain technology has revolutionized the concept of digital payments, offering unprecedented levels of security and decentralization. However, the widespread adoption of blockchain-based payment systems has been hindered by several critical issues, including high transaction fees, limited scalability, and privacy concerns. Overpass Channels addresses these challenges head-on by introducing a novel Layer 2 solution built on the TON (The Open Network) blockchain.

Overpass Channels represents a paradigm shift in blockchain scalability and privacy. Unlike traditional blockchain networks that rely on miners or validators to achieve consensus, Overpass Channels eliminates these potential bottlenecks through its innovative use of unilateral payment channels and off-chain transaction processing. This approach not only ensures high throughput but also maintains a strong focus on user privacy through the implementation of zero-knowledge proofs, specifically zk-SNARKs.

The key innovations of Overpass Channels include:

- 1. **Horizontal Scalability:** By leveraging a unique channel structure and off-chain processing, Overpass Channels can scale horizontally, supporting an theoretically unlimited number of transactions without the need for additional on-chain resources.
- 2. **Privacy-Enhanced Transactions:** Through the use of zk-SNARKs, every transaction in the Overpass Channels network is cryptographically proven to be valid without revealing any sensitive information about the transaction details or the parties involved.
- 3. **Independent Verification:** Each transaction can be independently verified by the parties involved, eliminating the need for network-wide consensus and dramatically reducing latency.
- Fluid Liquidity: The system's design allows for dynamic rebalancing of payment channels, ensuring optimal distribution of liquidity across the network.
- 5. Robust Censorship Resistance: By decentralizing the transaction processing and verification, Overpass Channels makes it extremely difficult for any single entity to censor or block transactions.

This paper provides a comprehensive exploration of the Overpass Channels system, including its cryptographic foundations, scalability analysis, integration with the TON blockchain, and potential applications across various domains. We begin with a detailed examination of the system's architecture, followed by an in-depth analysis of its security properties and scalability metrics. We then discuss the integration of Overpass Channels with the TON blockchain, highlighting the synergies between the two systems. Finally, we explore several use cases that demonstrate the versatility and potential impact of Overpass Channels in real-world scenarios.

As we delve into the technical details of Overpass Channels, we will provide rigorous mathematical proofs, algorithmic descriptions, and concrete examples to illustrate the system's functionality and benefits. Throughout this paper, we will use the hypothetical users Alice and Bob to demonstrate various scenarios and interactions within the Overpass Channels network.

Let us now embark on a journey through the intricate workings of Overpass Channels, a system poised to redefine the landscape of blockchain-based payments and beyond.

2 Key Innovations

Overpass Channels introduces several groundbreaking innovations that set it apart from existing blockchain and Layer 2 solutions. These innovations work in concert to create a system that is not only highly scalable and efficient but also privacy-preserving and resistant to censorship. Let's explore each of these key innovations in detail:

2.1 Horizontal Scalability without Validators

One of the most significant innovations of Overpass Channels is its ability to scale horizontally without the need for validators or miners. This is achieved through a combination of unilateral payment channels and off-chain transaction processing.

Definition 1 (Unilateral Payment Channel). A unilateral payment channel is a cryptographic construct that allows two parties to conduct multiple transactions off-chain, with only the opening and closing of the channel requiring on-chain operations.

In Overpass Channels, each user can open multiple unilateral payment channels, forming a network of interconnected channels. Transactions within these channels are processed off-chain, dramatically reducing the load on the underlying blockchain.

Theorem 1 (Horizontal Scalability). The transaction throughput of the Overpass Channels network scales linearly with the number of active channels, independent of the underlying blockchain's capacity.

Proof. Let n be the number of active channels in the network, and t be the average number of transactions per second (TPS) that can be processed within a single channel. The total network throughput T is given by:

$$T = n \times t$$

As n increases, T increases linearly, without being constrained by the underlying blockchain's capacity. This is because transactions within channels are processed off-chain and do not require immediate on-chain validation.

This horizontal scalability allows Overpass Channels to support an theoretically unlimited number of transactions, limited only by the number of active channels and the computational capacity of the network nodes.

2.2 Privacy-Enhanced Transactions with zk-SNARKs

Privacy is a cornerstone of Overpass Channels, achieved through the innovative use of zero-knowledge Succinct Non-interactive Arguments of Knowledge (zk-SNARKs).

Definition 2 (zk-SNARK). A zk-SNARK is a cryptographic proof construction where one can prove possession of certain information, e.g., a secret key, without revealing that information, and without any interaction between the prover and verifier.

Algorithm 1 zk-SNARK Transaction Proof Generation

- 1: **procedure** GenerateTransactionProof(tx, sk, pk)
- 2: $inputs \leftarrow \text{ExtractPublicInputs}(tx)$
- 3: $witness \leftarrow ConstructWitness(tx, sk)$
- 4: $proof \leftarrow \text{ProveZKSNARK}(inputs, witness, pk)$
- 5: **return** proof
- 6: end procedure

In Overpass Channels, every transaction is accompanied by a zk-SNARK proof that demonstrates the validity of the transaction without revealing any sensitive details.

This approach ensures that while the network can verify the validity of transactions, it cannot access sensitive information such as transaction amounts, sender and receiver identities, or channel balances.

2.3 Independent Verification and Instant Finality

Overpass Channels achieves instant finality through a mechanism of independent verification, eliminating the need for network-wide consensus.

Theorem 2 (Instant Finality). In Overpass Channels, a transaction is considered final and irreversible as soon as it is verified by the recipient, without requiring confirmation from any other network participants.

Proof. Let T be a transaction from Alice to Bob within a channel C. The proof proceeds as follows:

1) Alice generates a zk-SNARK proof P for T. 2) Alice sends T and P to Bob. 3) Bob verifies P using the public verification key vk. 4) If P is valid, Bob accepts T as final.

Since the verification of P depends only on vk, which is publicly known, and the inputs provided in T, Bob can independently verify the transaction without consulting any other network participants. Once Bob has verified P, he can be certain that T is valid and will be accepted by the network in any future on-chain settlement.

This independent verification mechanism allows for near-instantaneous transaction finality, a significant improvement over traditional blockchain systems that require multiple confirmations.

2.4 Fluid Liquidity through Dynamic Rebalancing

Overpass Channels incorporates a dynamic rebalancing mechanism that ensures optimal distribution of liquidity across the network.

Definition 3 (Channel Liquidity). The liquidity of a channel is defined as the maximum amount that can be sent through the channel in a single direction without requiring an on-chain transaction.

The dynamic rebalancing algorithm continuously monitors channel liquidity and initiates rebalancing operations when necessary.

This fluid liquidity ensures that users can always find efficient paths for their transactions, even as usage patterns in the network change over time.

Algorithm 2 Dynamic Channel Rebalancing

```
1: procedure REBALANCECHANNEL(channel, threshold)
2: balance \leftarrow \text{GetChannelBalance}(channel)
3: capacity \leftarrow \text{GetChannelCapacity}(channel)
4: if balance < threshold \times capacity then
5: amount \leftarrow (capacity - balance)/2
6: proof \leftarrow \text{GenerateRebalanceProof}(channel, amount)
7: ExecuteRebalance(channel, amount, proof)
8: end if
9: end procedure
```

2.5 Robust Censorship Resistance

The decentralized nature of Overpass Channels, combined with its privacy-preserving features, makes it highly resistant to censorship.

Theorem 3 (Censorship Resistance). In Overpass Channels, no single entity or small group of entities can censor or block a valid transaction from being processed and included in the network state.

Proof. Let T be a valid transaction from Alice to Bob. We will prove by contradiction. Assume there exists an entity E that can censor T. To do so, E must be able to:

- (a) Prevent T from being sent from Alice to Bob (communication censorship), or
- (b) Prevent Bob from verifying and accepting T (verification censorship), or
- (c) Prevent T from being included in the final network state (settlement censorship).

Let's address each of these cases:

Case 1: Communication censorship:

Communication censorship is impossible because:

- \bullet Alice and Bob can use any communication channel to exchange T.
- ullet The content of T is encrypted and indistinguishable from random data.

Case 2: Verification censorship:

Verification censorship is impossible because:

- Bob can independently verify T using the public verification key.
- This verification process does not require interaction with any other network participants.

Case 3: Settlement censorship:

Settlement censorship is impossible because:

- The final network state is determined by zk-SNARK proofs of channel states.
- These proofs can be submitted by any network participant.

• The validity of these proofs can be verified by all participants independently.

Therefore, the assumption that E can censor T leads to a contradiction, proving that no such entity can exist in the Overpass Channels network.

This robust censorship resistance ensures that Overpass Channels can operate reliably even in adversarial environments.

In the following sections, we will delve deeper into each of these innovations, exploring their implementation details, security properties, and implications for the overall system performance and user experience.

3 Transaction Validity and zk-SNARK Integration

The cornerstone of Overpass Channels' security and privacy features lies in its innovative use of zero-knowledge Succinct Non-interactive Arguments of Knowledge (zk-SNARKs). This section provides a detailed examination of how zk-SNARKs are integrated into the transaction validation process, ensuring both the validity of transactions and the privacy of users.

3.1 zk-SNARK Overview

Before delving into the specifics of how Overpass Channels utilizes zk-SNARKs, it's important to understand the fundamental concepts behind this cryptographic technique.

Definition 4 (zk-SNARK). A zero-knowledge Succinct Non-interactive Argument of Knowledge (zk-SNARK) is a cryptographic protocol that allows one party (the prover) to prove to another party (the verifier) that a statement is true, without revealing any information beyond the validity of the statement itself.

In the context of Overpass Channels, zk-SNARKs are used to prove the validity of transactions and state transitions without revealing the underlying transaction details.

3.2 zk-SNARK Circuit for Transaction Validation

The heart of the zk-SNARK integration in Overpass Channels is the circuit that defines the computation being proved. For transaction validation, this circuit encapsulates the logic of checking transaction validity, including balance updates and signature verification.

Algorithm 3 zk-SNARK Circuit for Transaction Validation

- 1: **procedure** TransactionValidationCircuit(oldState, newState, tx, signature)
- 2: AssertValidSignature(tx, signature)
- 3: AssertSufficientBalance(oldState, tx.amount)
- 4: AssertCorrectBalanceUpdate(oldState, newState, tx)
- 5: AssertValidStateTransition(oldState, newState)
- 6: AssertNonceIncrement(oldState.nonce, newState.nonce)
- 7: end procedure

This circuit ensures that: 1. The transaction is properly signed by the sender. 2. The sender has sufficient balance to make the transaction. 3. The balances are correctly updated after the transaction. 4. The overall state transition is valid. 5. The nonce is correctly incremented.

4 Proof Generation and Verification

In Overpass Channels, transactions are signed based on the updated state at the conclusion of the previous transaction. This ensures the integrity and sequential consistency of the channel state. After each transaction, participants sign the new state, preventing any disputes about the validity or order of transactions.

When a user initiates a transaction, they generate a zk-SNARK proof attesting that the transaction is valid according to the circuit defined. However, before generating this proof, the transaction is signed according to the current state of the channel, which is the updated state following the previous transaction. This ensures that the state transition sequence is preserved and agreed upon by all parties.

Algorithm 4 Transaction Proof Generation and Verification

- 1: **procedure** GENERATETRANSACTIONPROOF(oldState, newState, tx, sk)
- 2: $\operatorname{signature} \leftarrow \operatorname{Sign}(\operatorname{tx}, \operatorname{sk})$
- ▶ Transaction is signed based on the updated state
- 3: witness \leftarrow (oldState, newState, tx, signature)
- 4: $proof \leftarrow ProveZKSNARK(TransactionValidationCircuit, witness)$
- 5: **return** proof
- 6: end procedure
- 7: **procedure** VerifyTransactionProof(proof, publicInputs)
- 8: result \(\times \) VerifyZKSNARK(TransactionValidationCircuit, proof, publicInputs)
- 9: **return** result
- 10: end procedure

The transaction signature generated in step 2, based on the private key and the updated state, ensures that the transaction reflects the most recent state of the channel. This mechanism helps enforce trust and prevents attempts to execute outdated or conflicting transactions.

4.1 Privacy Preservation

One of the key strengths of zk-SNARKs in Overpass Channels is the privacy it guarantees. The use of zk-SNARKs ensures that even though the transaction details are required to generate a proof, the actual transaction information (such as amounts, sender, and receiver) remains confidential, thanks to the zero-knowledge property. Only the updated state, which participants sign at the conclusion of each transaction, is reflected in the publicly shared information.

The proof guarantees that no one can infer additional details about the transaction beyond what is revealed by the public inputs, preserving confidentiality while maintaining verifiable state transitions.

4.2 Unilateral Channels: How They Work

In unilateral channels, transactions are initiated by one party, and the other party might not need to be involved actively in every transaction. However, both parties must still have agreed upon the updated state when a transaction occurs, ensuring that each party recognizes the new balances and the transition to the new state. Here's the process:

- 1. **Transaction Initiation**: One party (let's say Alice) initiates a transaction. For example, Alice wants to send Bob 10 tokens. When this transaction is initiated, Alice calculates the new state that will result from this payment (her balance reduced, Bob's balance increased).
- 2. Signing on the New State: Alice signs based on this updated state, which reflects the balances after the transaction is processed. Bob does not need to be online or sign off on this specific transaction at this moment because it's a unilateral channel—Alice can initiate and sign the transaction based on her agreement to the updated state.
- 3. State Agreement Pre-Established: While Bob doesn't sign at the moment of every transaction, both Alice and Bob would have agreed on how transactions work in this channel (including how balances are updated) when they set up the channel. This pre-establishment allows Alice to unilaterally process payments without Bob's real-time approval, as long as the rules of the channel (e.g., balance limits) are respected.
- 4. **Transaction Finality**: Once Alice signs the transaction reflecting the updated state, the transaction is executed. Bob can verify the transaction and updated state later, but he doesn't need to be online for it to happen. The channel operates unilaterally, meaning Alice can push the updated state without needing Bob's immediate confirmation.

4.3 Instant and Asynchronous Nature

Instant Transactions: Transactions are effectively instant because Alice doesn't need to wait for Bob to be online or approve the transaction in real time. As soon as Alice signs and initiates the transaction, the new state is established, and the channel updates accordingly.

Asynchronous Execution: In this unilateral setup, transactions can happen even when one party (Bob) is offline. Alice can continue executing transactions as long as they follow the agreed rules for the channel, making the system highly asynchronous.

4.4 Online Requirements

For Alice: Alice needs to be online to initiate transactions and sign the updated state.

For Bob: Bob does not need to be online at the moment of every transaction. He only needs to periodically check or confirm the updated state when he comes online.

4.5 Wallet Extension Contract and Dynamic Rebalancing

In Overpass Channels, the wallet extension contract(parent contract) holds the full balance that a user deposits, while **channel contracts**(child contracts) of the wallet extension act as markers or accounting entries that represent each channel's share of the total balance.

- 1. No On-Chain Movement During Transactions: When Alice sends tokens to Bob, no on-chain funds are moved. The channel contracts are updated to reflect the transaction, adjusting the balance Alice has reserved for Bob within the wallet extension contract.
- 2. **Dynamic Rebalancing**: If Alice needs more tokens in the Bob channel, the intermediate contract can adjust the reservations by pulling tokens from less active channels (e.g., Charlie's channel) without moving funds on-chain.
- 3. **Proof-Based Rebalancing**: The intermediate smart contract generates a proof to verify the accuracy of the updated balances across channels. It debits and credits the channel contracts based on the current balance requirements, but no on-chain transfers occur.
- 4. On-Chain Settlement Only on Channel Closure: Only when Alice or Bob closes the channel does the system perform an on-chain settlement. At that point, the smart contract looks at the updated balances in the markers and executes the necessary on-chain transfer to settle the amounts between Alice and Bob.

5 Balance Consistency

Maintaining consistent and accurate balances across all channels is crucial for the integrity and reliability of the Overpass Channels network. This section delves into the mathematical formalism and proofs that guarantee balance consistency throughout the system.

5.1 Formal Definition of Balance Consistency

Before we proceed with the theorem and proof, let's formally define what we mean by balance consistency in the context of Overpass Channels.

Definition 5 (Balance Consistency). A payment channel network exhibits balance consistency if and only if, for any valid sequence of transactions, the following conditions hold:

- 1. The sum of all balances across all channels remains constant (excluding external deposits and withdrawals).
- 2. For each channel, the sum of the balances of all participants in that channel remains equal to the channel's capacity.
- 3. No participant's balance in any channel ever becomes negative.

5.2 Theorem of Balance Consistency

Now, we can state and prove the fundamental theorem that guarantees balance consistency in Overpass Channels.

Theorem 4 (Balance Consistency in Overpass Channels). In the Overpass Channels network, all valid transactions and state transitions preserve balance consistency as defined above.

Proof. We will prove this theorem by induction on the number of transactions in the network.

Base case: At the network's initialization, all channels are created with a fixed capacity, and the initial balances sum to this capacity. Therefore, the balance consistency property holds initially.

Inductive step: Assume that the network is in a consistent state after n transactions. We need to prove that any valid (n + 1)-th transaction will maintain balance consistency.

Let T be the (n + 1)-th transaction, occurring in channel C between participants A and B. Without loss of generality, assume A is sending x tokens to B.

- 1. By the definition of a valid transaction in Overpass Channels, T must be accompanied by a valid zk-SNARK proof P.
- 2. The zk-SNARK circuit for transaction validation ensures:
 - (a) A's balance in C is sufficient: $balance_A \ge x$
 - (b) The new balances are correctly computed:

$$newBalance_A = balance_A - x$$

$$newBalance_B = balance_B + x$$

- 3. The zk-SNARK proof P is verified by B and, upon settlement, by the network.
- 4. After T is applied:
 - (a) The sum of balances in C remains unchanged:

$$(balance_A - x) + (balance_B + x) = balance_A + balance_B$$

- (b) No other channel's balances are affected.
- (c) A's new balance is non-negative (from 2a and 2b).
- (d) B's new balance is clearly non-negative as it only increases.
- 5. Therefore, all three conditions of balance consistency continue to hold after T:
 - (a) The sum of all balances across all channels remains constant.
 - (b) The sum of balances in C equals its capacity (from 4a).
 - (c) No participant's balance becomes negative (from 4c and 4d).

By the principle of mathematical induction, balance consistency holds for any number of valid transactions in the network. \blacksquare

5.3 Implications and Practical Considerations

The Balance Consistency Theorem has several important implications for the Overpass Channels network:

1. **Security Against Double Spending**: The theorem guarantees that it's impossible for a participant to spend more tokens than they possess, effectively preventing double spending without requiring global consensus.

- 2. Local Verification Sufficiency: Because balance consistency is maintained for each valid transaction, participants only need to verify the zk-SNARK proof of the latest transaction to be assured of the channel's integrity.
- 3. **Simplified Conflict Resolution**: In case of disputes, the latest valid state (proven by zk-SNARKs) can be used to resolve conflicts without needing to replay the entire transaction history.
- 4. **Efficient State Updates**: The theorem allows for efficient updates of channel states without requiring updates to the global network state for every transaction.

6 Fraud Prevention Mechanisms in Overpass Channels

Ensuring the integrity of the system and preventing fraudulent activities are paramount in any financial network. Overpass Channels incorporates several sophisticated mechanisms to prevent fraud, leveraging its unique architecture and cryptographic foundations. This section provides a detailed examination of these fraud prevention mechanisms.

6.1 Key Safeguard: Pending Transaction Acceptance

To mitigate the potential security concern where Alice could exploit Bob's absence (as mentioned earlier), Overpass Channels employ a key safeguard: **pending transaction acceptance**.

- 1. **Pending Transaction Mechanism**: Before Alice can send a new transaction to Bob, Bob must first accept the pending one. This means that if Alice sends a token to Bob, the transaction remains pending until Bob verifies it. Only after Bob accepts the transaction does Alice's balance update, and the system allows her to initiate the next transaction.
- 2. **Mitigation of Fraud**: This mechanism mitigates the concern of Alice manipulating the channel when Bob is offline. Even if Bob remains offline for a while, no new transactions can proceed until Bob confirms the previous one, preventing Alice from spamming the channel with fraudulent or excessive transactions.
- 3. **Transaction Finality**: Once Bob accepts the pending transaction, the state is updated, and the next transaction can be executed. This ensures that each transaction is properly verified and final, preventing any ambiguity or risk of double-spending.
- 4. **Transaction Ordering**: In addition to pending transaction acceptance, Overpass Channels uses a combination of per-channel nonces and TON's SEQNO to ensure strict ordering of transactions. This further prevents replay attacks and ensures state consistency.

6.2 Mitigation of Security Concerns for Extended Absence

To further mitigate the risk where Alice could try to manipulate the channel while Bob is offline for an extended period (as raised earlier in the downsides), dynamic rebalancing and pending transaction acceptance work in tandem to provide additional safeguards:

• Pending Transactions: Alice cannot proceed with new transactions unless Bob has accepted the previous ones. This prevents Alice from submitting multiple fraudulent or excessive transactions while Bob is offline.

• Dynamic Rebalancing by Smart Contracts: The wallet extension contract ensures that all channels are balanced and no channel exceeds its authorized limits. The 50% rule also caps the amount Alice can send in one transaction, limiting potential damage.

Together, these mechanisms ensure that Bob is protected from fraud or abuse while offline, allowing him to review and accept transactions upon returning online, while at the same time not leaving Alice in a position where her funds are unusable during the wait. The system's dynamic rebalancing enables Alice to continue transacting in other channels or reallocating resources as needed, ensuring fluid channel operations across the board.

6.3 Deterministic Conflict Resolution

In the event of a conflict, such as when parties disagree on the current state of a channel, Overpass Channels employs a deterministic conflict resolution mechanism.

Theorem 5 (Deterministic Conflict Resolution). Given any two conflicting channel states S_1 and S_2 submitted by different parties, the Overpass Channels protocol will deterministically select a single valid state.

Proof. Let S_1 and S_2 be two conflicting states for channel C, submitted by parties A and B respectively. The proof proceeds as follows:

- 1. Both S_1 and S_2 must be accompanied by valid zk-SNARK proofs P_1 and P_2 .
- 2. The on-chain contract will verify both P_1 and P_2 .
- 3. If either proof fails verification, the corresponding state is rejected.
- 4. If both proofs are valid, the contract compares the nonces n_1 and n_2 of S_1 and S_2 .
- 5. The state with the higher nonce is selected as the valid state.
- 6. In the unlikely event that $n_1 = n_2$, the contract applies a deterministic tie-breaking rule (e.g., selecting the state with the lexicographically smaller hash).
- This selection process is encoded in the smart contract and executes identically on all TON validators.

Therefore, given any two conflicting states, the protocol will always select a single valid state in a deterministic manner.

6.4 50% Spending Rule for Off-Chain Transactions

To prevent potential griefing attacks where a malicious party could repeatedly force channel closures, Overpass Channels implements a 50% spending rule for off-chain transactions.

Definition 6 (50% Spending Rule). In any single off-chain transaction within a channel, a party cannot spend more than 50% of their current channel balance.

Theorem 6 (Griefing Prevention). The 50% spending rule in Overpass Channels prevents a malicious party from depleting their entire channel balance in a single transaction, ensuring that honest parties always have recourse to close the channel profitably.

Proof. Let C be a channel between Alice and Bob, with Alice's balance B_A and Bob's balance B_B . The proof proceeds as follows:

- 1. Suppose Alice attempts to make a malicious transaction T to deplete her balance.
- 2. By the 50% spending rule, the maximum amount Alice can send in T is $\frac{B_A}{2}$.
- 3. After T, Alice's new balance B'_A is at least $\frac{B_A}{2}$.
- 4. If Bob detects malicious behavior, he can initiate a channel closure.
- 5. In the worst case (if Alice doesn't cooperate), Bob must close the channel on-chain.
- 6. The on-chain closing cost is C_{close} , which is less than $\frac{B_A}{2}$ by design.
- 7. Therefore, even after paying C_{close} , Bob is guaranteed to receive a positive balance from the channel closure.

Thus, the 50% spending rule ensures that honest parties always have a profitable recourse to close the channel, preventing griefing attacks.

This 50% spending rule, combined with the deterministic conflict resolution mechanism, provides strong guarantees against various forms of malicious behavior in Overpass Channels.

6.5 ZK-SNARK Proofs and State Updates

At the core of Overpass Channels' fraud prevention is the use of zk-SNARKs for validating state updates.

Definition 7 (Valid State Update). A state update in Overpass Channels is considered valid if and only if it is accompanied by a zk-SNARK proof that verifies:

- 1. The update transitions from a valid previous state to a valid new state.
- 2. The update follows all rules of the channel (e.g., balance changes, nonce increments).
- 3. The update is authorized by the appropriate parties.

Proof of Validity

Let's formalize the proof of validity for state updates:

Theorem 7 (State Update Validity). Any state update in Overpass Channels that is accepted by the network is guaranteed to be valid according to the channel rules.

Proof. Let S_0 be the initial state of a channel, and S_1 be the state after an update. The proof proceeds as follows:

- 1. For the update $S_0 \to S_1$ to be accepted, it must be accompanied by a zk-SNARK proof P.
- $2.\ P$ is generated using a circuit that encodes all channel rules, including:
 - (a) Balance consistency (as proved in the previous section)

- (b) Proper nonce incrementing
- (c) Signature verification
- 3. The verification of P is performed by all relevant parties (the recipient in a transaction, and the network nodes during settlement).
- 4. By the soundness property of zk-SNARKs, if P verifies successfully, then with overwhelming probability, the prover must know a valid witness satisfying all constraints in the circuit.
- 5. Therefore, if P is accepted, S_1 must be a valid state reachable from S_0 according to all channel rules.

Thus, any accepted state update is guaranteed to be valid.

6.6 Cross-Shard Transaction Security

Overpass Channels leverages TON's efficient cross-shard communication capabilities to ensure that cross-shard transactions maintain the same security guarantees as intra-shard transactions.

Theorem 8 (Cross-Shard Security). Cross-shard transactions in Overpass Channels maintain the same security guarantees as intra-shard transactions.

Proof. Let $T_{A,B}$ be a transaction from channel A in shard S_A to channel B in shard S_B . The security of $T_{A,B}$ is ensured by:

- 1. A PLONKY2 proof P_A verifying the validity of the transaction in S_A
- 2. A PLONKY2 proof P_B verifying the validity of the state update in S_B
- 3. TON's hypercube routing ensuring reliable message delivery between shards

The combination of these elements ensures that cross-shard transactions have the same security properties as intra-shard transactions.

This cross-shard security mechanism allows Overpass Channels to maintain consistent security guarantees across the entire network, regardless of the sharding structure of the underlying TON blockchain.

6.7 Prevention of Old State Submission

One potential attack vector in channel-based systems is the submission of old, outdated states. Overpass Channels prevents this through a combination of nonce usage, zk-SNARK proofs, and TON's smart contract capabilities.

Definition 8 (Nonce and Seqno). In Overpass Channels:

- A nonce is a monotonically increasing integer associated with each channel state, incremented with each valid state update.
- Each channel contract stemming from the wallet contract is assigned a seque, a capability provided by TON's smart contract system.

Theorem 9 (Old State Invalidation). In Overpass Channels, it is computationally infeasible to submit an old state as a valid current state.

Proof. The proof proceeds by contradiction:

- 1. Assume an adversary can submit an old state S_{old} with nonce n_{old} and sequo seq_{old} as a valid current state.
- 2. Let the actual current state be $S_{current}$ with nonce $n_{current}$ and sequo $seq_{current}$, where $n_{current} > n_{old}$ and $seq_{current} > seq_{old}$.
- 3. For S_{old} to be accepted, the adversary must provide a valid zk-SNARK proof P_{old} .
- 4. The zk-SNARK circuit includes checks that:
 - (a) The nonce in the new state is greater than the nonce in the old state.
 - (b) The sequo in the new state matches the current sequo of the channel contract.
- 5. Therefore, for P_{old} to verify, the adversary must know a witness w such that:
 - (a) w satisfies all constraints of the zk-SNARK circuit
 - (b) w includes a valid signature for S_{old}
 - (c) w demonstrates that $n_{old} > n_{current}$
 - (d) w demonstrates that $seq_{old} = seq_{current}$
- 6. However, (5c) contradicts the fact that $n_{current} > n_{old}$, and (5d) contradicts $seq_{current} > seq_{old}$.
- 7. By the soundness property of zk-SNARKs and the monotonically increasing nature of TON's seqno, finding such a witness w is computationally infeasible.

Therefore, submitting an old state as a valid current state is computationally infeasible in Overpass Channels, reinforced by both the internal nonce mechanism and TON's sequent capability.

It's worth noting that Overpass Channels' use of PLONKY2 for generating zk-SNARK proofs provides additional security against old state submission attempts, as PLONKY2 offers strong soundness guarantees without requiring a trusted setup.

6.8 Proof Consistency

Overpass Channels ensures that all proofs within the system are consistent with each other, preventing conflicting updates.

Theorem 10 (Proof Consistency). In Overpass Channels, it is computationally infeasible to generate two valid, conflicting proofs for the same channel.

Proof. Let S_0 be an initial channel state. Suppose an adversary attempts to generate two conflicting proofs, P_1 and P_2 , leading to different final states S_1 and S_2 . The proof proceeds as follows:

- 1. For P_1 and P_2 to be valid, they must both prove transitions from S_0 to their respective final states.
- 2. The zk-SNARK circuit includes the hash of the initial state as a public input.
- 3. Therefore, P_1 and P_2 must use the same initial state hash.
- 4. The circuit also enforces that the nonce in the final state is exactly one more than the nonce in the initial state.
- 5. Thus, S_1 and S_2 must have the same nonce.
- 6. The circuit enforces that for a given initial state and nonce, there is only one valid final state (determined by the transaction details).
- 7. Therefore, for P_1 and P_2 to be simultaneously valid, S_1 and S_2 must be identical.
- 8. This contradicts the assumption that P_1 and P_2 lead to different final states.

Thus, it is computationally infeasible to generate two valid, conflicting proofs for the same channel.

6.9 On-Chain Verification

While most operations in Overpass Channels occur off-chain, the system includes on-chain verification as a final layer of fraud prevention.

Algorithm 5 On-Chain Verification of Channel Closure

```
1: procedure VerifyChannelClosure(channelID, finalState, proof)
      storedState \leftarrow GetStoredChannelState(channelID)
      isValid \leftarrow VerifyZKSNARK(proof, finalState, storedState)
3:
      if isValid then
 4:
          UpdateChannelState(channelID, finalState)
5:
          DistributeFunds(channelID, finalState) return TRUE
6:
7:
      else
8:
          RejectClosure(channelID) return FALSE
      end if
9:
10: end procedure
```

This on-chain verification serves as a crucial backstop against potential fraudulent activities, ensuring that even if off-chain mechanisms fail, the on-chain state remains secure.

6.10 Elimination of Need for Watchtowers

Traditional payment channel networks often rely on watchtowers to monitor for fraudulent channel closures. Overpass Channels have no need for such external monitoring through its use of zk-SNARKs and on-chain verification.

Theorem 11 (Watchtower Redundancy). In Overpass Channels, participants can securely close channels without relying on external watchtowers.

Proof. The proof proceeds by showing that any attempt at fraudulent channel closure will be detected and prevented:

- 1. Let C be a channel between Alice and Bob, with current state $S_{current}$.
- 2. Suppose Alice attempts to fraudulently close C with an old state S_{old} .
- 3. To close the channel, Alice must submit to the on-chain contract:
 - (a) The final state S_{old}
 - (b) A zk-SNARK proof P that S_{old} is a valid state
- 4. The on-chain verification algorithm will:
 - (a) Verify the zk-SNARK proof P
 - (b) Check that the nonce in S_{old} is greater than the last known on-chain nonce
- 5. For the fraudulent closure to succeed, Alice must generate a valid proof P for S_{old} with a nonce higher than $S_{current}$.
- 6. However, by the proof of the Old State Invalidation theorem, generating such a proof is computationally infeasible.
- 7. Therefore, Alice's fraudulent closure attempt will be rejected by the on-chain contract.
- 8. Bob can close the channel with $S_{current}$ at any time, without needing to constantly monitor the blockchain.

Thus, participants can securely close channels without relying on external watchtowers.

6.11 Cryptographic Proofs and Tamper-Evident Records

Overpass Channels maintains a cryptographically secure, tamper-evident record of all wallet states, including their associated channel states and transactions. This is achieved through a combination of sparse Merkle trees and zk-SNARKs.

Definition 9 (Wallet State Sparse Merkle Tree). For each wallet contract W, a sparse Merkle tree T_W is maintained where:

- Each leaf represents a channel state.
- The leaf value is the hash of the state: $leaf_i = H(S_i)$.
- The leaf's position in the tree is determined by the channel's SEQNO.
- The root of T_W is included in each zk-SNARK proof.

Theorem 12 (Tamper-Evident Wallet and Channel History). Any tampering with the history of wallet or channel states in Overpass Channels is detectable with overwhelming probability.

Proof. Let $\{S_0, S_1, \ldots, S_n\}$ be the sequence of states for wallet W and its associated channels. The proof proceeds as follows:

1. Each state transition $S_i \to S_{i+1}$ (whether a wallet state change or a channel state change) is accompanied by a zk-SNARK proof P_i .

2. P_i includes:

- (a) The Merkle root R_i of T_W before the transition
- (b) The Merkle root R_{i+1} of T_W after the transition
- (c) A proof that S_{i+1} is a valid successor state to S_i
- 3. The zk-SNARK circuit verifies:
 - (a) R_i is the correct Merkle root for T_W including all states up to S_i
 - (b) R_{i+1} is the correct Merkle root for T_W after updating S_{i+1}
- 4. By the collision resistance property of the hash function H, finding two different states that produce the same leaf value is computationally infeasible.
- 5. By the security properties of sparse Merkle trees, modifying any state in the history would change the Merkle root.
- 6. Any change to a historical state S_j would invalidate all subsequent proofs P_k for $k \geq j$, as they would no longer have valid Merkle root transitions.
- 7. Generating new valid proofs for the modified history would require breaking the soundness of the zk-SNARK system, which is assumed to be computationally infeasible.

Therefore, any tampering with the wallet or channel history is detectable with overwhelming probability.

This tamper-evident property ensures that the entire history of a wallet and its associated channels can be cryptographically verified, providing strong guarantees against historical fraud or manipulation.

7 Transaction Processing and Conflict Resolution

Efficient transaction processing and robust conflict resolution mechanisms are crucial for the smooth operation of Overpass Channels. This section delves into the details of how transactions are processed and how potential conflicts are resolved in a deterministic and fair manner.

7.1 Transaction Processing

In Overpass Channels, transactions are processed off-chain within payment channels, with periodic settlements on the TON blockchain. Let's formalize the transaction processing mechanism:

This algorithm ensures that each transaction is validated cryptographically before being applied to the channel state.

Algorithm 6 Transaction Processing in Overpass Channels

```
1: procedure ProcessTransaction(sender, recipient, amount, channelID)
       channel \leftarrow GetChannelState(channelID)
3:
       oldState \leftarrow channel.currentState
       newState \leftarrow ComputeNewState(oldState, sender, recipient, amount)
 4:
       proof \leftarrow GenerateZKProof(oldState, newState, sender, amount)
 5:
       signatureSender \leftarrow Sign(newState, sender.privateKey)
 6:
       signatureRecipient \leftarrow GetRecipientSignature(newState, recipient)
 7:
       if VerifyZKProof(proof) \land VerifySignatureS(signatureSender, signatureRecipient) then
8:
           channel.currentState \leftarrow newState
9:
           UpdateOffChainState(channelID, newState)
10:
11:
           return SUCCESS
       else
12:
          return FAILURE
13:
       end if
14:
15: end procedure
```

7.2 Assignment to Wallet Contracts

Each user in Overpass Channels is associated with a wallet contract on the TON blockchain. These wallet contracts play a crucial role in managing channels and resolving conflicts.

Definition 10 (Wallet Contract). A wallet contract in Overpass Channels is a smart contract on the TON blockchain that:

- Holds the user's on-chain balance
- Manages the user's participation in payment channels
- Handles channel opening, closing, and dispute resolution

8 Conditional Payments

8.1 Conditional Transactions using zk-SNARKs

Conditional transactions allow participants in Overpass Channels to transfer funds only when predefined conditions are met. In this section, we present an approach to implement conditional transactions using zk-SNARKs, enabling privacy-preserving, off-chain transaction processing.

Overview of zk-SNARK Conditional Transactions

In Overpass Channels, zk-SNARKs provide an efficient and privacy-preserving mechanism for conditional payments. Conditional transactions can be triggered by various events, such as a task completion, multi-signature approval, or external oracle data. These conditions are encoded in a zk-SNARK proof, ensuring that conditions are met without revealing sensitive information about the transaction.

Conditional Transaction Workflow

To illustrate the workflow of a conditional transaction using zk-SNARKs, consider the following steps:

- 1. Condition Setup: Alice wants to send Bob 50 tokens, but only if Bob completes a task (e.g., proving ownership of a document or a digital asset). The condition is encoded as a cryptographic task that Bob must prove using zk-SNARKs.
- 2. **Proof Generation**: Bob generates a zk-SNARK proof, denoted as P, which proves that the condition has been satisfied (e.g., task completion) without revealing any sensitive details.
- 3. Verification by Alice: Alice receives the zk-SNARK proof P from Bob. She verifies P using the public verification key vk associated with the zk-SNARK circuit. If the proof is valid, Alice releases the funds.
- 4. **Off-Chain Execution**: Since the transaction is off-chain, the state of the channel is updated to reflect the new balances once the proof is verified. The zk-SNARK ensures that no sensitive information, such as the task details or the exact condition, is revealed.

The zk-SNARK proof ensures that the conditional logic is satisfied without requiring interaction with the blockchain until settlement.

zk-SNARK Circuit for Conditional Transactions

The zk-SNARK circuit for conditional transactions can be described as follows:

Algorithm 1: zk-SNARK Conditional Transaction Circuit

- procedure ConditionalTxCircuit(cond, tx, sig, balance)
- AssertConditionMet(cond) // Verify that the condition is met
- AssertValidSignature(tx, sig) // Verify transaction signature
- 4. AssertSufficientBalance(balance, tx.amount) // Check balance
- 5. AssertCorrectStateUpdate(balance) // Update state correctly
- 6. end procedure

This circuit allows us to verify that the condition (e.g., task completion or oracle verification) is satisfied before releasing funds, without revealing the actual details of the condition.

Use Cases for zk-SNARK Conditional Transactions

- Task-Driven Payments: Alice can set up a payment to Bob that is contingent upon Bob completing a certain task or milestone. Bob generates a zk-SNARK proof demonstrating that the task has been completed, allowing Alice to release the funds.
- Event-Based Transactions: Using oracles, a conditional payment can be triggered by an external event (e.g., price movement of an asset, delivery confirmation) with a zk-SNARK proof generated from oracle data.
- Multi-Signature Agreements: Multiple parties can approve a transaction by providing zk-SNARK proofs that their conditions for approval have been met.

8.2 Hash Time-Locked Contracts (HTLC) in Overpass Channels

In addition to zk-SNARKs, Overpass Channels can implement Hash Time-Locked Contracts (HTLCs) to facilitate simpler conditional payments. HTLCs allow payments to be contingent on both a cryptographic condition (hash preimage) and a time constraint. This approach is particularly useful for cross-channel payments and time-sensitive transactions.

Overview of HTLC

HTLCs are widely used in off-chain systems to enable conditional payments based on the revelation of a secret (hash preimage). In Overpass Channels, HTLCs can be implemented to provide conditional payments with automatic refund mechanisms based on timeouts.

HTLC Workflow

The process of creating an HTLC-based transaction is as follows:

- 1. Alice Initiates HTLC: Alice wants to send Bob 50 tokens if Bob can provide the preimage X of a hash H(X). Alice creates an HTLC in the payment channel and locks the funds under the condition that Bob must provide X within a certain time T.
- 2. Bob Reveals Preimage: If Bob can reveal the correct preimage X within the time T, he provides it to Alice. Alice verifies that H(X) matches the hash condition.
- 3. Payment Finalization: Upon verifying the preimage X, Alice releases the 50 tokens to Bob, completing the conditional transaction.
- 4. **Timeout Mechanism**: If Bob fails to provide the preimage X before time T expires, the funds are returned to Alice automatically.

Algorithm for HTLC in Overpass Channels

The algorithm for setting up an HTLC in Overpass Channels is as follows:

```
Algorithm 2: HTLC Setup and Execution
1. procedure HTLCSetup(sender, receiver, amount, H(X), timeout)
2.
       Lock(amount) under condition: receiver must provide preimage X
3.
       Wait for receiver to reveal X
4.
       if H(X) is valid and X is correct:
5.
           Transfer funds to receiver
6.
       else if timeout has expired:
7.
           Refund funds to sender
8.
       end if
end procedure
```

Use Cases for HTLC in Overpass Channels

• Cross-Channel Payments: HTLCs allow Alice and Bob to securely make payments across channels without trusting each other. If the condition is met, Bob can claim the payment.

- Refundable Payments: In scenarios where time-sensitive payments are required (e.g., online escrow), HTLC ensures that funds are returned to the sender if conditions are not met within a defined period.
- Atomic Swaps: HTLCs can facilitate atomic swaps between different blockchains or assets, where each party provides a hash preimage to claim the assets.

Comparison of HTLC and zk-SNARK Conditional Transactions

HTLCs are a simple and effective mechanism for conditional payments where the conditions are based on hash preimage and time. However, for more complex conditional logic that requires privacy-preserving proofs, zk-SNARKs provide a more flexible solution.

Feature	zk-SNARKs	HTLCs
Privacy	High, condition details are hidden	Low, condition details are exposed
Flexibility	Supports complex conditions	Limited to hash and time-based conditions
Complexity	High, requires cryptographic proof generation	Low, simple cryptographic check
Use Cases	Multi-sig, oracle-based, event-driven conditions	Time-sensitive or hash preimage transactions

Table 1: Comparison between zk-SNARKs and HTLCs for Conditional Payments

9 Hierarchical Ordering and System-Level Efficiency

Overpass Channels employs a hierarchical structure to achieve system-level efficiency and scalability. This section explores the different levels of this hierarchy and how they contribute to the overall performance of the network.

9.1 Hierarchical Structure

The Overpass Channels network is organized into four main levels:

- 1. Payment Channel Contracts
- 2. Wallet Contracts
- 3. Intermediate Contracts
- 4. Root Contract

Let's examine each level in detail:

Payment Channel Contracts

At the lowest level, payment channel contracts operate as child contracts of wallet contracts. Each payment channel is unilateral, facilitating transactions in a single direction. If bidirectional payments are required between two parties, two separate unilateral channels can be established. These channels are identified by unique SEQNO numbers, allowing for efficient management and reference within the system.

Definition 11 (Payment Channel Contract). A payment channel contract in Overpass Channels is a unilateral agreement facilitating off-chain transactions in a single direction, with periodic on-chain settlements. It is identified by a unique SEQNO number within its parent wallet contract.

Theorem 13 (Channel Efficiency). The computational complexity of processing a transaction within a channel is O(1) with respect to the total number of transactions in the network.

Proof. Let T be a transaction within channel C between Alice and Bob. The proof proceeds as follows:

- 1. Processing T involves:
 - (a) Generating a zk-SNARK proof P
 - (b) Verifying P
 - (c) Updating the local channel state
- 2. The zk-SNARK proof generation and verification depend only on the circuit complexity, which is constant for all transactions.
- 3. Updating the local channel state involves modifying a fixed number of variables (balances, nonce, etc.).
- 4. None of these operations depend on the total number of transactions in the network.

Therefore, the computational complexity of processing T is O(1) with respect to the total number of transactions in the network.

Wallet Contracts

Wallet contracts serve as the primary interface for users, managing their funds and initiating payment channels. Each wallet contract can create and oversee multiple payment channel contracts.

Definition 12 (Wallet Contract). A wallet contract in Overpass Channels is a smart contract that:

- Manages a user's funds
- Initiates and oversees multiple payment channel contracts
- Interacts with intermediate contracts for state aggregation

Intermediate Contracts

Intermediate contracts form the next layer in the hierarchy. Each intermediate contract aggregates state updates from multiple wallet contracts. This aggregation includes information about all payment channels associated with each wallet. By compiling data from numerous wallets, intermediate contracts significantly reduce the computational load on the root contract.

Definition 13 (Intermediate Contract). An intermediate contract in Overpass Channels is a smart contract that:

• Manages a subset of wallet contracts

- Aggregates state updates from these wallet contracts, including all associated payment channels
- Periodically submits aggregated updates to the root contract
- Is identified by a unique SEQNO number

Theorem 14 (Intermediate Contract Efficiency). The computational complexity of processing updates in an intermediate contract is $O(\log w)$, where w is the number of wallet contracts managed by the intermediate contract.

Proof. Let IC be an intermediate contract managing w wallet contracts. The proof proceeds as follows:

- 1. IC maintains a Merkle tree T of wallet contract states.
- 2. For each wallet contract update:
 - (a) Verify the zk-SNARK proof (O(1))
 - (b) Update the corresponding leaf in T (O(log w))
 - (c) Recompute the Merkle root $(O(\log w))$
- 3. Periodically, IC submits the new Merkle root to the root contract (O(1))

The dominant operation is updating the Merkle tree, which has a complexity of $O(\log w)$. Therefore, the overall complexity of processing updates in IC is $O(\log w)$.

Root Contract

At the apex of the Overpass Channels hierarchy is the root contract. This contract manages all intermediate contracts, each identified by its SEQNO number. The root contract is responsible for maintaining the global state of the entire network, processing the aggregated data from intermediate contracts.

Definition 14 (Root Contract). A root contract in Overpass Channels is a smart contract that:

- Manages multiple intermediate contracts, each identified by its SEQNO number
- Maintains the global state of the network
- Handles final dispute resolution

Theorem 15 (Root Contract Scalability). The computational complexity of updating the global state in the root contract is $O(\log m)$, where m is the number of intermediate contracts.

Proof. Let RC be the root contract managing m intermediate contracts. The proof proceeds as follows:

- 1. RC maintains a Merkle tree T_{qlobal} of intermediate contract states.
- 2. For each update from an intermediate contract:
 - (a) Verify the submitted Merkle root (O(1))
 - (b) Update the corresponding leaf in T_{qlobal} (O(log m))
 - (c) Recompute the global Merkle root (O(log m))

The dominant operation is updating T_{global} , which has a complexity of O(log m). Therefore, the overall complexity of updating the global state in RC is O(log m).

9.2 System-Level Efficiency

The hierarchical structure of Overpass Channels contributes to its overall efficiency and scalability.

Theorem 16 (System-Level Efficiency). The Overpass Channels network can process N transactions with a total computational complexity of

$$O(N + \log m \log w)$$

, where N is the total number of transactions, m is the number of intermediate contracts, and w is the average number of wallet contracts per intermediate contract.

Proof. Consider a network with N transactions, m intermediate contracts, and an average of w wallet contracts per intermediate contract. The proof proceeds as follows:

- 1. Transaction Processing O(N): Each of the N transactions occurs within a specific channel, and each channel is managed by a wallet contract. The complexity of processing each individual transaction is O(1), as it involves verifying the sender's balance and updating the channel state. Therefore, the total complexity for processing all N transactions is O(N).
- 2. Wallet State Updates $O(N \log c)$: Each transaction updates the state of a channel within a wallet contract. Wallet contracts use a Merkle tree T_{wallet} to store the state of all channels they manage. Updating the state of a specific channel requires:
 - Updating the corresponding leaf node in T_{wallet} with the new state of the channel.
 - Recomputing the Merkle root for T_{wallet} .

If a wallet contract manages c channels on average, updating and recomputing the Merkle root has a complexity of $O(\log c)$. As each of the N transactions requires updating T_{wallet} , the total complexity for updating wallet states is $O(N \log c)$.

- 3. Intermediate Contract Updates $O(N \log w)$: After a wallet contract processes transactions and updates its local state, it submits the updated Merkle root to its parent intermediate contract. Each intermediate contract maintains a Merkle tree $T_{\rm intermediate}$ to track the states of the w wallet contracts it manages. Updating $T_{\rm intermediate}$ requires:
 - Verifying the updated Merkle root of the wallet contract (which takes O(1)).
 - Updating the corresponding leaf in $T_{\text{intermediate}}$ (which takes $O(\log w)$).
 - Recomputing the Merkle root of $T_{\text{intermediate}}$ (which also takes $O(\log w)$).

The amortized cost of updating the intermediate contract state per transaction is $O(\log w)$. Therefore, the total complexity of intermediate contract updates is $O(N \log w)$.

- 4. Global State Updates $O(N \log m)$: After an intermediate contract processes updates from its wallet contracts, it submits its updated Merkle root to the root contract. The root contract maintains a global Merkle tree T_{global} to track the states of the m intermediate contracts. Updating T_{global} requires:
 - Verifying the updated Merkle root of the intermediate contract (which takes O(1)).
 - Updating the corresponding leaf in T_{global} (which takes $O(\log m)$).

• Recomputing the global Merkle root (which also takes $O(\log m)$).

The amortized cost of updating the global state per transaction is $O(\log m)$. Therefore, the total complexity of global state updates is $O(N \log m)$.

Total Computational Complexity: Combining these results, the total computational complexity for processing N transactions and maintaining the global state is:

$$O(N) + O(N\log c) + O(N\log w) + O(N\log m) = O(N(\log c + \log w + \log m)).$$

Since c (the average number of channels per wallet) is typically much smaller than w (the number of wallets per intermediate contract), we can simplify this to:

$$O(N(\log w + \log m)) = O(N\log(mw)).$$

Thus, the total computational complexity is $O(N \log(mw))$, where N is the number of transactions, w is the average number of wallet contracts per intermediate contract, and m is the number of intermediate contracts.

10 Storage Nodes and Data Management

10.1 Individual User Devices and Wallet Trees

In Overpass Channels, each user's device is responsible for maintaining a single sparse Merkle tree for the wallet contract. This tree tracks the states of all channel contracts associated with the wallet, ensuring efficient and secure updates. These operations are supported by off-chain storage nodes that form part of the Overpass system.

- 1. Sparse Merkle tree for the wallet contract: The wallet contract contains a sparse Merkle tree where each leaf corresponds to the state of an associated channel contract, indexed by its SEQNO.
- 2. **Keys stored on user devices:** The cryptographic keys required to manage and update the wallet and its channels are securely stored on the user's device.
- 3. **Tree synchronization:** When a channel contract's state is updated, the corresponding leaf in the wallet's sparse Merkle tree is modified, and a new root is generated. This root and its cryptographic proof are then submitted to the off-chain storage nodes for redundancy and verification.

10.2 Periodic Channel Updates and Off-Chain Storage Nodes

The Overpass system relies on *off-chain storage nodes* for maintaining the periodic updates of wallet contract states. These storage nodes store data redundantly and securely, ensuring efficient retrieval and availability.

1. Storage nodes in the Overpass system: The storage nodes are responsible for storing wallet contract roots and their associated proofs. They do not operate on the TON blockchain but are instead a decentralized component of the Overpass off-chain system.

Algorithm 7 Wallet Update

```
1: procedure UPDATEWALLETTREE(walletID, channelID, channelState)
2: tree \leftarrow LoadWalletTree(walletID)
3: leafIndex \leftarrow SEQNO(channelID)
4: leafValue \leftarrow Hash(channelState)
5: tree.update(leafIndex, leafValue)
6: newRoot \leftarrow tree.getRoot()
7: proof \leftarrow tree.generateProof(leafIndex)
8: \mathbf{return}\ (newRoot, proof)
9: \mathbf{end}\ \mathbf{procedure}
```

- 2. **Epidemic overlapping of shard data:** The storage nodes follow a design of *epidemic overlapping*, where each node stores data for a set number of intermediate contracts, and data overlaps with other storage nodes. This redundancy ensures that the system is decentralized and highly available.
- 3. Scalability and efficiency: The epidemic overlap mechanism allows the storage nodes to efficiently store data without requiring all nodes to store everything. As the network grows, the number of overlaps remains constant, preventing excessive storage demands and maintaining decentralized redundancy.

Algorithm 8 Submit Wallet Update to Off-Chain Storage Nodes

```
1: procedure SUBMITPERIODICUPDATE(walletID, walletRoot)
2: proof \leftarrow GenerateProof(walletRoot)
3: storageNodes \leftarrow GetAssociatedStorageNodes(walletID)
4: for all node \in storageNodes do
5: node.StoreWalletUpdate(walletID, walletRoot, proof)
6: end for
7: end procedure
```

10.3 Storage Nodes and Intermediate Contracts

Storage nodes also handle data related to *intermediate contracts*, which aggregate the states of multiple wallet contracts. The *epidemic overlapping* mechanism ensures that intermediate contract data is stored redundantly across the storage nodes, ensuring availability and resilience.

- 1. **Storage of wallet roots:** Off-chain storage nodes store the roots and proofs of wallet contracts. These roots are retrieved by intermediate contracts for aggregation.
- 2. **Intermediate contract aggregation:** Intermediate contracts aggregate the states of several wallet contracts. This aggregation involves computing a Merkle root over the stored wallet contract roots, which is then redundantly stored across the storage nodes.

Algorithm 9 Intermediate Contract Data Aggregation

```
1: procedure AGGREGATEINTERMEDIATEDATA(intermediateContractID)
       walletContracts \leftarrow \text{GetAssociatedWalletContracts}(intermediateContractID)
2:
       walletRoots \leftarrow []
3:
       for all wallet \in walletContracts do
4:
           (root, proof) \leftarrow \text{RetrieveLatestWalletUpdate}(wallet)
5:
 6:
           walletRoots.append((wallet, root, proof))
 7:
       intermediateRoot \leftarrow ComputeMerkleRoot(walletRoots)
8:
       intermediateProof \leftarrow GenerateProof(intermediateRoot, walletRoots)
9:
       StoreIntermediateUpdate(intermediateContractID, intermediateRoot, intermediateProof)
10:
11: end procedure
```

10.4 Root Contract and Global State on TON

While the storage nodes operate off-chain, the *root contract* functions on the *TON blockchain* and plays a key role in generating the global state of the network. The root contract aggregates data from all intermediate contracts and periodically submits the global state on-chain in TON's sharded blockchain. This process ensures decentralized verification and availability of the network's state.

The root contract also interacts with the storage nodes, retrieving the processed Merkle roots and proofs for all intermediate contracts, which have been stored after aggregating the individual wallet roots.

- 1. Global aggregation: The root contract retrieves the processed Merkle roots and proofs from the storage nodes, where the intermediate contracts' data is stored after the aggregation of wallet roots. These intermediate contract roots are then aggregated to compute the global Merkle root.
- 2. On-chain submission: The global root and its cryptographic proof are submitted on-chain at the end of each epoch. This ensures that the entire network's state is publicly verifiable and tamper-proof, leveraging TON's sharded blockchain for decentralized availability.

11 Hierarchical Sparse Merkle Trees in Overpass Channels

In Overpass Channels, sparse Merkle trees are employed at multiple levels of the system to manage state updates efficiently, securely, and with minimal data overhead. These trees are particularly well-suited for scenarios where most potential data entries (leaves) are empty, ensuring that even in sparse configurations, proofs remain compact and efficient.

Sparse Merkle trees enable the Overpass system to achieve a high level of scalability and efficiency, especially when combined with off-chain storage nodes and the decentralized structure of the TON blockchain for global state verification.

11.1 Benefits of Sparse Merkle Trees in Overpass Channels

The use of sparse Merkle trees in Overpass Channels provides several distinct advantages:

Algorithm 10 Global State Generation and Storage

```
1: procedure GenerateGlobalState
2:
       intermediateContracts \leftarrow GetAllIntermediateContracts()
       intermediateRoots \leftarrow []
3:
       for all contract \in intermediateContracts do
4:
           (root, proof) \leftarrow \text{RetrieveIntermediateUpdateFromStorageNodes}(contract)
5:
6:
           intermediateRoots.append((contract, root, proof))
 7:
       end for
       globalRoot \leftarrow ComputeGlobalMerkleRoot(intermediateRoots)
8:
       globalProof \leftarrow GenerateGlobalProof(globalRoot, intermediateRoots)
9:
       StoreGlobalState(globalRoot, globalProof)
10:
       SubmitGlobalStateOnChain(globalRoot, globalProof)
11:
       PruneOldGlobalStates()
12:
13: end procedure
```

- 1. Compact Proofs and Constant Size: Sparse Merkle trees allow for *constant-sized proofs* regardless of the number of leaves (or channels) in the tree. This is critical in Overpass Channels, as it ensures that the submission of proofs remains efficient even as the network grows. Each proof, whether for a wallet contract or the global state, involves only recalculating the path from a specific leaf to the root.
- 2. Efficient State Updates: Sparse Merkle trees enable O(log n) complexity for updates, meaning that only the path from the affected leaf to the root needs to be recalculated when a state changes. In the context of Overpass Channels, this allows for *incremental updates* when a channel contract state changes, ensuring that the system remains scalable and responsive.
- 3. **Proof of Integrity:** Any alteration in a channel's state is immediately reflected in the corresponding wallet's Merkle tree root. This ensures *data integrity* at every level of the hierarchy, as any modification
 - to a wallet or channel contract would trigger a corresponding update in the global state. The ability to trace any change back to its root provides robust security guarantees.
- 4. **Sparse Data Handling:** Sparse Merkle trees are designed to handle cases where most leaves are unoccupied. This feature is particularly useful for Overpass Channels, where the number of possible channel contracts is vast, but only a subset may be active at any given time. The system is able to efficiently manage and verify these active states without the need to store or process large amounts of empty data.
- 5. Redundant and Decentralized Storage: In conjunction with off-chain storage nodes, the Merkle tree structure enables redundant and decentralized storage. Intermediate and wallet contract roots are stored across multiple nodes with epidemic overlap, allowing for redundancy and resilience in the system without each node needing to store all data. The efficient and decentralized nature of this storage ensures that the network remains robust as it scales.

11.2 Application of Sparse Merkle Trees in the Overpass Hierarchy

Sparse Merkle trees play a critical role in the multi-tiered architecture of Overpass Channels:

- At the wallet contract level, the tree tracks the state of associated channels.
- At the intermediate contract level, the Merkle roots of several wallet contracts are aggregated.
- Finally, at the *root contract* level, the global Merkle tree is computed and submitted on-chain, ensuring that the entire system's state remains verifiable and transparent.

The constant-size nature of the global root submission ensures that even as the network grows in size, the cost of verifying and submitting the global state remains predictable and efficient.

12 Implementation Considerations

Implementing Overpass Channels on TON requires careful consideration of several technical aspects. This section outlines key implementation details and provides pseudocode for critical components.

12.1 Merkle Tree Structure

Overpass Channels uses a Merkle tree to efficiently represent the state of all channels. Here's a high-level implementation of the Merkle tree structure:

Algorithm 11 Merkle Tree for Overpass Channels

```
1: procedure UPDATEMERKLETREE(channelID, newState)
2: leaf ← Hash(channelID || newState)
3: path ← GetMerklePath(channelID)
4: newRoot ← ComputeNewRoot(leaf, path)
5: UpdateRoot(newRoot)
6: end procedure
7: function VERIFYCHANNELSTATE(channelID, state, proof)
8: leaf ← Hash(channelID || state)
9: root ← GetCurrentRoot()
10: return VerifyMerkleProof(leaf, root, proof)
11: end function
```

This Merkle tree structure allows for efficient updates and verifications of channel states.

12.2 zk-SNARK Circuit

The zk-SNARK circuit for Overpass Channels must encode the logic for valid state transitions. Here's a simplified representation of the circuit:

This circuit ensures that all state transitions in Overpass Channels are valid and authorized.

12.3 Minimal Cross-Shard Data

To minimize cross-shard communication overhead, Overpass Channels uses a compact representation for cross-shard transactions that includes both 'shardID' and 'workchainID':

Algorithm 12 zk-SNARK Circuit for Overpass Channels

- 1: procedure Channel Transition Circuit (seqno, old State, new State, tx, signature, old Nonce, new Nonce)
- 2: Assert Channel Seque Match (seqno, old State. seqno, new State. seqno)
- 3: AssertValidSignature(tx, signature)
- 4: AssertValidBalanceTransition(oldState, newState, tx)
- 5: AssertValidNonceIncrement(oldNonce, newNonce)
- 6: AssertValidStateTransition(oldState, newState)
- 7: end procedure

Algorithm 13 Cross-Shard Transaction Encoding

```
1: function ENCODECROSSSHARDTx(tx, proof, workchainID, shardID)
       encodedTx \leftarrow Hash(tx)
       encodedProof \leftarrow CompressZKProof(proof)
3:
       \textbf{return} \ encodedTx \parallel encodedProof \parallel workchainID \parallel shardID
4:
 5: end function
6: function DecodeCrossShardTx(encodedData)
       encodedTx, encodedProof, workchainID, shardID \leftarrow Split(encodedData)
7:
       tx \leftarrow \text{LookupTx}(encodedTx)
8:
       proof \leftarrow DecompressZKProof(encodedProof)
9:
       return tx, proof, workchainID, shardID
10:
11: end function
```

12.4 TON Integration

Integrating Overpass Channels with TON involves creating smart contracts that can interact with the Overpass Channels system:

Algorithm 14 TON Smart Contract for Overpass Channels

```
1: procedure ProcessOverpassTx(encodedTx, proof)
      tx, decodedProof \leftarrow DecodeCrossShardTx(encodedTx)
      isValid \leftarrow VerifyZKProof(decodedProof, tx)
3:
      if isValid then
4:
          UpdateChannelState(tx)
5:
          EmitEvent("ChannelUpdated", tx)
6:
 7:
      else
          RevertTransaction()
8:
      end if
10: end procedure
```

This smart contract serves as the interface between TON and Overpass Channels, ensuring that all channel operations are properly verified and recorded on the TON blockchain.

By carefully implementing these components, Overpass Channels can be seamlessly integrated with TON, providing a powerful, scalable, and privacy-preserving payment solution.

13 Tokenomics

The tokenomics of Overpass Channels are designed to ensure network stability, incentivize participation, and maintain a balanced ecosystem. This section outlines the key aspects of the token distribution and utility.

13.1 Fixed Supply and Initial Distribution

Overpass Channels employs a fixed supply model, with all tokens minted at genesis. This approach aligns with the constant balance theorem, facilitating accurate tracking of token movement within the system.

- Total Supply: 100 billion tokens
- Initial Distribution:
 - -70% (70 billion tokens) to be airdropped
 - 20% (20 billion tokens) allocated to Treasury/Governance
 - 10% (10 billion tokens) distributed between the team, investors, and advisors

The airdrop distribution strategy aims to ensure wide token distribution and foster community engagement. Specific details of the airdrop mechanism will be announced at a later date.

13.2 Governance and Treasury

The 20% allocation (20 billion tokens) to governance and treasury functions enables decentralized decision-making and sustainable ecosystem development.

• Governance and Treasury Allocation: 20 billion tokens (20% of total supply)

The treasury, operated by the governance mechanism, plays a crucial role in managing the ecosystem's resources, funding future developments, and ensuring long-term sustainability.

13.3 Token Utility and Fee Structure

Overpass Channels implements a balanced fee structure that supports various components of the network:

Reserve Status

Tokens that are designated as "burned" are not permanently removed from circulation. Instead, they enter a reserve status within the treasury. These reserved tokens serve multiple purposes:

- Provide liquidity for system operations
- Fund future developments and improvements
- Support the ecosystem's long-term sustainability

Off-Chain Storage Node Compensation

A portion of transaction fees is allocated to compensate off-chain storage nodes, which are essential for the Overpass system's efficient operation.

TON Node Compensation

Supporting TON nodes that accept and store the global root also receive a share of the transaction fees.

13.4 Fee Distribution

Transaction fees are divided into three main categories:

- 1. Recycling to Reserve: A portion returns to the treasury's reserve status for future ecosystem funding and development.
- 2. Off-Chain Node Payments: Compensates the off-chain storage nodes in the Overpass system.
- 3. On-Chain Node Payments: A small portion goes to on-chain TON nodes for global root storage and processing.

This balanced approach ensures the sustainability of the network, incentivizes crucial infrastructure providers, and maintains a flexible token supply that can be utilized for the ecosystem's ongoing needs and future growth.

14 Wallet-Managed Channel Grouping

The wallet application (e.g., Tonkeeper) will handle the grouping of channels on the user side, while still using the underlying payment channels for transactions and zk-SNARK proofs on-chain. This approach simplifies things for both the user and the system.

14.1 Key Components of the Wallet Interface

Group-Based UI (Checking, Savings, Custom Accounts)

- The grouping logic is only a UI/UX feature in the wallet.
- Users can assign channels to different groups like Checking, Savings, or Custom at the interface level. However, this grouping will not affect how the channels are structured on-chain.
- The wallet will simply aggregate and display balances from these groupings as a single total balance (or split by groups), without involving smart contracts in the process.

Interaction with Channels

- When a user performs a transaction (sending funds or opening a channel), the wallet will route the transaction through the selected group but keep all channel interactions on-chain as individual channels.
- The underlying contract will remain the same (i.e., a standard payment channel), but the user will be able to choose the group from the wallet interface, making it feel like they are interacting with different "accounts."

14.2 How the Wallet Should Work

- UI Level Grouping: Users assign channels to different groups (Checking, Savings, Custom) within the wallet interface. The groups are simply tags or filters applied to the channels, with no on-chain changes required.
- Total Balance Calculation: The wallet calculates a total balance from all the payment channels associated with a user. It presents this balance as unified for the user but keeps the actual channel details hidden unless the user wants to see a breakdown.
- Transaction Process: When the user initiates a transaction, they select the group (Checking, Savings, or Custom) from which they want to send funds. The wallet then pulls the appropriate funds from the corresponding channels but groups the logic only within the UI.

14.3 Updated Example Code for Wallet-Level Grouping

UI Grouping Module

This module defines how channels are grouped in the wallet without affecting the underlying smart contracts.

```
// groupUIManager.js
class GroupUIManager {
    constructor() {
        this.groups = {
            checking: [],
            savings: [],
            custom: []
        };
    }
    // Assign a channel to a UI group (Checking, Savings, Custom)
    assignChannelToGroup(channelId, groupType) {
        switch (groupType) {
            case 'Checking':
                this.groups.checking.push(channelId);
                break:
            case 'Savings':
                this.groups.savings.push(channelId);
                break;
            case 'Custom':
                this.groups.custom.push(channelId);
                break:
            default:
```

```
throw new Error('Unknown group type: ${groupType}');
        }
    }
    // Retrieve group balances for the UI
    async getGroupBalances() {
        const checkingBalance = await this.getGroupBalance('Checking');
        const savingsBalance = await this.getGroupBalance('Savings');
        const customBalance = await this.getGroupBalance('Custom');
        return {
            breakdown: {
                checking: checkingBalance,
                savings: savingsBalance,
                custom: customBalance
            totalBalance: checkingBalance + savingsBalance + customBalance,
        };
    }
    // Example to get the balance of all channels in a group
    async getGroupBalance(groupType) {
        const group = this.groups[groupType.toLowerCase()];
        let total = 0;
        for (const channelId of group) {
            total += await blockchain.getChannelBalance(channelId);
        return total;
   }
}
export default new GroupUIManager();
```

Transaction Flow Module

This module handles the interaction with the blockchain when the user initiates a transaction. It routes the transaction based on the selected group.

14.4 Optimized UX/UI

- Balance Display: The wallet interface shows the user their total balance across all groups (Checking, Savings, Custom) but also allows them to see detailed group breakdowns if they choose.
- Transaction Process: When users initiate transactions, they select a group in the UI, and the wallet handles the details of which channels to use and how much liquidity is available in each.

By keeping the grouping logic in the wallet interface and not on-chain, the complexity is reduced. The wallet simply filters and displays channels as groups, but the underlying wallet contracts handle all the zk-SNARK proofs and rebalancing as needed. The grouping is purely for the user's ease of interaction, and the contract logic remains simple, efficient, and secure.

15 Analysis

15.1 Censorship Resistance in Overpass Channels

The censorship resistance in Overpass Channels is fundamentally different from other systems due to its unique architecture. Let's clarify this:

Theorem 17 (Overpass Channels Censorship Impossibility). In the Overpass Channels system, it is computationally infeasible for any entity or group of entities to censor individual transactions.

Proof. The proof proceeds as follows:

- 1) Transactions in Overpass Channels are processed entirely off-chain within individual channels.
- 2) The network nodes do not see or process individual transactions. Instead, they only receive:
- Merkle tree roots representing batches of transactions (epochs)
- zk-SNARK proofs verifying the validity of state transitions
- 3) For a transaction T to be censored, one of the following must occur:
- Prevention of T's inclusion in the Merkle tree.
- Rejection of a valid Merkle root containing T.
- \bullet Rejection of a valid zk-SNARK proof for an epoch including T.
- 4) (3a) is impossible because Merkle tree construction is done locally by channel participants.
- 5) (3b) is computationally infeasible because:
- Nodes cannot distinguish which transactions are included in a Merkle root.
- Rejecting a valid Merkle root would require rejecting all transactions in an epoch.
- 6) (3c) is computationally infeasible because:
- zk-SNARK proofs reveal no information about individual transactions.

- Rejecting a valid proof would require rejecting an entire epoch of transactions.
- 7) Any attempt to censor by rejecting epochs would result in rejecting all transactions, including legitimate ones, which is not in the nodes' interest.

Therefore, censorship of individual transactions in Overpass Channels is computationally infeasible.

This proof demonstrates that the Overpass Channels system achieves censorship resistance through its architecture, which makes it impossible for nodes to target individual transactions for censorship. The privacy-preserving nature of the system, combined with the batching of transactions into epochs, ensures that no entity can selectively censor transactions without disrupting the entire network.

Comparison with Other Systems

To further illustrate the unique censorship resistance of Overpass Channels, let's compare it with other systems:

1. Bitcoin and other PoW blockchains:

- Miners can potentially censor transactions by excluding them from blocks.
- Requires a majority of miners to collude for effective censorship.

2. Layer 2 solutions (e.g., Rollups):

- Sequencers or operators can potentially censor by excluding transactions.
- Often rely on centralized components that are vulnerable to censorship pressure.

3. Overpass Channels:

- Nodes cannot see or process individual transactions.
- Censorship attempts would require rejecting entire epochs of transactions.
- No centralized components that could be targeted for censorship.

Theorem 18 (Overpass Channels Superior Censorship Resistance). Overpass Channels provides stronger censorship resistance than existing blockchain and Layer 2 solutions.

Proof. Let $p_c(S)$ be the probability of successful censorship in system S. The proof proceeds as follows:

- 1) For Bitcoin: $p_c(BTC) > 0$, as a majority of miners could potentially censor transactions.
- 2) For Layer 2 solutions: $p_c(L2) > 0$, due to centralized components vulnerable to censorship.
- 3) For Overpass Channels:

$$p_c(OC) \approx 0$$

as censorship would require rejecting entire epochs, which is against the economic interests of nodes.

4) Therefore:

$$p_c(OC) < p_c(BTC)$$
 and $p_c(OC) < p_c(L2)$

Thus, Overpass Channels provides superior censorship resistance compared to existing solutions.

15.2 Scalability and Performance

Given the unique architecture of Overpass Channels, it's important to revisit our scalability and performance analysis to ensure accuracy.

Theorem 19 (Overpass Channels Scalability). The transaction throughput of Overpass Channels scales linearly with the number of channels, with each channel capable of processing transactions independently.

Proof. Let:

- n be the number of channels in the network
- \bullet T_c be the throughput of a single channel
- \bullet T_{total} be the total network throughput

The proof proceeds as follows:

- 1. Each channel operates independently, processing transactions off-chain.
- 2. The throughput of the network is the sum of the throughputs of all channels:

$$T_{total} = \sum_{i=1}^{n} T_{c_i}$$

3. Assuming an average throughput per channel, this simplifies to:

$$T_{total} = n \cdot T_c$$

- 4. As n increases, T_{total} increases linearly.
- 5. The only limitation on n is the capacity of the underlying blockchain to process epoch commitments.

Therefore, Overpass Channels exhibits linear scalability with respect to the number of channels.

15.3 Potential Transaction Throughput

Given this architecture, the system's transaction throughput potential is significant:

- 1. **Channel transactions**: Each unilateral channel can process transactions near-instantaneously, limited only by the user's device capabilities and network latency.
- 2. **Aggregation at intermediate level**: Intermediate contracts can manage a large number of wallet roots, potentially thousands or tens of thousands.
- 3. Root contract aggregation: The root contract can combine multiple intermediate roots, further increasing the scalability.

15.4 Factors Affecting TPS

Several factors influence the overall transactions per second (TPS) of the system:

- 1. **ZKP generation time**: The speed of generating and verifying ZKPs on user devices will impact individual channel throughput.
- 2. **Off-chain storage capacity**: The ability of decentralized storage nodes to handle concurrent updates and queries.
- 3. **Intermediate contract capacity**: The number of wallet roots each intermediate contract can efficiently manage.
- 4. Root contract updates: The frequency at which the root contract can update the on-chain state.
- 5. **Blockchain limitations**: The underlying blockchain's capacity to process root contract updates.

15.5 Estimated Throughput

While it's challenging to provide an exact TPS without specific implementation details, we can make some educated estimates:

- 1. **Per-channel TPS**: Potentially thousands per second, limited by ZKP generation and verification speed.
- 2. **System-wide TPS**: Millions to billions per second off-chain, depending on the number of active channels and intermediate contracts.
- 3. **On-chain TPS**: Limited by the underlying blockchain, but each on-chain transaction represents a large number of off-chain transactions.

15.6 System Architecture

The Overpass Channels system architecture consists of several key components:

- 1. **User-controlled channels**: Each user manages their own unilateral channels via their client-side device.
- 2. **Sparse Merkle trees**: Used to generate Zero-Knowledge Proofs (ZKPs) for each transaction in the channels.
- 3. Smart contract hierarchy:
 - Wallet contract (parent)
 - Unilateral channel contracts (children)
- 4. Off-chain storage: Decentralized nodes store proofs and Merkle roots.
- 5. Intermediate contracts: Aggregate wallet roots into a single Merkle tree.
- 6. Root contract: Combines all intermediate roots and submits the final root to the blockchain.

15.7 Optimizing the System

To maximize throughput, several optimization strategies can be employed:

- 1. Optimize ZKP generation and verification algorithms.
- 2. Implement efficient data structures for Merkle tree management.
- 3. Design a robust off-chain storage system with high concurrency.
- 4. Carefully balance the number of channels per intermediate contract.
- 5. Implement batching and aggregation techniques at each level.

This system has the potential for extremely high off-chain TPS, limited primarily by the computational resources of user devices and the capacity of the off-chain storage network. The on-chain bottleneck is significantly reduced, as only aggregate roots need to be recorded on the blockchain periodically.

16 Efficiency and Cost Analysis

The efficiency of Overpass Channels in terms of cost per transaction and energy consumption is a key advantage over existing systems, while also providing incentives for network participants.

Theorem 20 (Overpass Channels Efficiency). Overpass Channels achieves a lower cost per transaction and energy consumption compared to traditional blockchain systems and most Layer 2 solutions, while maintaining a balanced fee structure for network sustainability.

Proof. Let:

- C_{OC} be the cost per transaction in Overpass Channels
- E_{OC} be the energy consumption per transaction in Overpass Channels
- \bullet N be the number of transactions in an epoch
- $C_{on-chain}$ be the cost of an on-chain epoch commitment
- $E_{off-chain}$ be the energy consumption of an off-chain transaction
- $F_{storage}$ be the fee for off-chain storage nodes
- F_{TON} be the fee for participating TON nodes
- F_{burn} be the burn fee allocated to the treasury

The proof proceeds as follows:

1) Cost per transaction in Overpass Channels:

$$C_{OC} = \frac{C_{on-chain}}{N} + F_{storage} + F_{TON} + F_{burn} + \epsilon$$

where ϵ is the negligible cost of off-chain processing.

2) Energy consumption per transaction:

$$E_{OC} = E_{off-chain} + \frac{E_{on-chain}}{N}$$

where $E_{on-chain}$ is the energy cost of an on-chain epoch commitment.

3) As N increases:

$$\lim_{N \to \infty} C_{OC} = F_{storage} + F_{TON} + F_{burn} + \epsilon$$

$$\lim_{N \to \infty} E_{OC} = E_{off-chain}$$

- 4) In comparison:
- Traditional blockchains incur costs and energy consumption for every transaction.
- Layer 2 solutions with centralized components have overhead for operator maintenance and profit.
- 5) Therefore, for sufficiently large N:

$$C_{OC} < C_{traditional}$$
 and $C_{OC} < C_{Layer2}$
 $E_{OC} < E_{traditional}$ and $E_{OC} < E_{Layer2}$

Thus, Overpass Channels achieves higher efficiency in terms of cost and energy consumption per transaction, while also providing incentives for storage nodes, TON nodes, and maintaining a treasury through the burn fee.

16.1 Fee Distribution

The fee structure in Overpass Channels is designed to incentivize network participants and ensure long-term sustainability:

- Storage Node Fee ($F_{storage}$): Compensates off-chain storage nodes for maintaining transaction data and proofs.
- TON Node Fee (F_{TON}) : Rewards TON nodes that participate in submitting the global Merkle root.
- Burn Fee (F_{burn}) : Allocated to the treasury for future ecosystem development and sustainability.

This balanced fee structure ensures that all critical components of the network are incentivized, while maintaining overall efficiency. The exact proportions of these fees can be adjusted based on network requirements and governance decisions.

17 Privacy Analysis

The privacy guarantees of Overpass Channels are fundamental to its design and operation.

Theorem 21 (Overpass Channels Transaction Privacy). In Overpass Channels, no information about individual transactions is revealed to the network beyond what is explicitly included in zk-SNARK proofs.

Proof. The proof proceeds as follows:

- 1. Transactions are processed entirely off-chain within channels.
- 2. The only information shared with the network is:
 - (a) Merkle roots of transaction batches (epochs)
 - (b) zk-SNARK proofs of state transitions
- 3. Merkle roots reveal no information about individual transactions due to the pre-image resistance of cryptographic hash functions.
- 4. zk-SNARK proofs, by definition, reveal no information beyond the validity of the statement being proved.
- 5. The statements proved by zk-SNARKs in Overpass Channels are of the form: "There exists a valid set of transactions that transition the state from S_1 to S_2 ."
- 6. This statement does not reveal any information about individual transactions, their amounts, or the parties involved.

Therefore, Overpass Channels provides strong transaction privacy, revealing no information about individual transactions to the network.

18 Remarks

Overpass Channels represents a significant advancement in blockchain scaling solutions, offering:

- 1. Superior censorship resistance through its unique architecture that makes individual transaction censorship computationally infeasible.
- 2. Exceptional scalability, with the throughput reaching millions or even billions in TPS.
- 3. High efficiency in terms of cost per transaction and energy consumption, approaching zero marginal cost for off-chain transactions.
- 4. Strong privacy guarantees, ensuring that no information about individual transactions is revealed to the network.

19 Integration with TON Blockchain

The integration of Overpass Channels with the TON (The Open Network) blockchain further enhances its capabilities and provides a robust foundation for large-scale deployment. This section examines the synergies between Overpass Channels and TON, and how this integration addresses key challenges in blockchain scalability and usability.

19.1 TON's Sharding Architecture

TON's multi-threaded, sharded architecture complements the scalability features of Overpass Channels.

Theorem 22 (Overpass-TON Scalability Synergy). The integration of Overpass Channels with TON's sharding architecture results in multiplicative scalability improvements.

Proof. Let:

- \bullet n be the number of Overpass Channels
- \bullet m be the number of shards in TON
- T_{OC} be the throughput of a single Overpass Channel
- T_{TON} be the throughput of a single TON shard

The proof proceeds as follows:

- 1. The total throughput of Overpass Channels: $T_{total_{OC}} = n \cdot T_{OC}$
- 2. The total throughput of TON: $T_{total_{TON}} = m \cdot T_{TON}$
- 3. With integration, each TON shard can support multiple Overpass Channels:

$$T_{integrated} = m \cdot (k \cdot T_{OC})$$

where k is the number of Overpass Channels per shard.

4. Assuming even distribution of channels across shards:

$$k = \frac{n}{m}$$

5. Therefore:

$$T_{integrated} = m \cdot \left(\frac{n}{m} \cdot T_{OC}\right) = n \cdot T_{OC}$$

6. This means that the integrated system can fully utilize both the scalability of Overpass Channels and TON's sharding.

Thus, the integration results in multiplicative scalability improvements, allowing the system to scale with both the number of channels and the number of shards. \blacksquare

Algorithm 15 Overpass Channel Smart Contract on TON

```
1: procedure ChannelContract
       State Variables:
 3:
       channelId \leftarrow \text{UniqueIdentifier}
       participants \leftarrow \{ParticipantA, ParticipantB\}
 4:
       currentEpoch \leftarrow 0
 5:
       latestMerkleRoot \leftarrow \text{InitialMerkleRoot}
 6:
       stateHash \leftarrow \text{InitialStateHash}
 7:
       procedure SubmitEpoch(newMerkleRoot, zkProof, epochNumber)
Require: msg.sender \in participants
Require: epochNumber = currentEpoch + 1
          assert VerifyZKProof(zkProof, latestMerkleRoot, newMerkleRoot, stateHash)
 9:
10:
          latestMerkleRoot \leftarrow newMerkleRoot
11:
          currentEpoch \leftarrow epochNumber
12:
          \mathbf{emit} \; \mathbf{EpochSubmitted}(channelId, currentEpoch, newMerkleRoot)
       end procedure
13:
       procedure CloseChannel(finalStateProof)
14:
Require: msg.sender \in participants
15:
           assert VerifyFinalStateProof(finalStateProof, stateHash)
           finalState \leftarrow ExtractFinalState(finalStateProof)
16:
          DistributeFunds(finalState)
17:
          emit ChannelClosed(channelId, finalState)
18:
19:
       end procedure
20: end procedure
```

19.2 Smart Contract Integration

TON's flexible smart contract system allows for efficient implementation of Overpass Channels' core components.

This smart contract structure allows for efficient on-chain management of Overpass Channels, leveraging TON's capabilities for fast execution and low-cost state updates.

19.3 Cross-Shard Operations

TON's ability to handle cross-shard communications efficiently enhances Overpass Channels' capability to process inter-channel transactions.

Theorem 23 (Efficient Cross-Shard Overpass Transactions). Cross-shard transactions in the integrated Overpass-TON system can be processed with $O(\log m)$ communication complexity, where m is the number of shards.

Proof. Let:

- m be the number of shards
- $T_{A,B}$ be a transaction from channel A in shard S_A to channel B in shard S_B

The proof proceeds as follows:

- 1. The cross-shard transaction process involves:
 - (a) Generating a zk-SNARK proof in shard S_A
 - (b) Transmitting the proof to shard S_B
 - (c) Verifying the proof in shard S_B
- 2. TON's hypercube routing for cross-shard messages ensures that the number of hops for a message to travel between any two shards is $O(\log m)$.
- 3. The size of the zk-SNARK proof is constant, regardless of the transaction details.
- 4. Therefore, the communication complexity for transmitting the proof is $O(\log m)$.
- 5. Proof generation and verification times are constant and independent of the number of shards.

Thus, the overall complexity of cross-shard Overpass transactions in TON is $O(\log m)$, ensuring efficient scaling even with a large number of shards.

19.4 TON DNS Integration

Integration with TON DNS allows for human-readable addresses in Overpass Channels, enhancing usability.

This integration simplifies the user experience by allowing easy-to-remember addresses for Overpass Channels, similar to domain names on the internet.

Algorithm 16 Overpass Channel Address Resolution via TON DNS

```
1: procedure ResolveChannelAddress(humanReadableAddress)
      dnsRecord \leftarrow TON.DNS.Resolve(humanReadableAddress)
      if dnsRecord contains OverpassChannelID then
3:
          channelId \leftarrow dnsRecord.OverpassChannelID
4:
          channelContract \leftarrow GetChannelContract(channelId)
5:
          {f return}\ channel Contract. address
6:
7:
      else
          return null
8:
      end if
9:
10: end procedure
```

20 Use Cases for Privacy

The enhanced privacy features of Overpass Channels, leveraging zk-SNARKs, enable a wide range of applications that require confidentiality while maintaining transparency and verifiability. This section explores four key use cases that demonstrate the versatility and potential impact of the system.

20.1 Confidential Voting Systems

Confidential voting systems are a critical application for enhanced privacy in decentralized networks. Overpass Channels can provide a solution that guarantees the confidentiality of votes while allowing for public verification of election results.

Implementation on TON

- 1. Voter Registration: A smart contract on TON manages voter registration, issuing unique identifiers to eligible voters.
- 2. **Vote Casting**: Voters use their Overpass Channel to cast votes, generating a zk-SNARK proof that:
 - They are an eligible voter
 - They have not voted before
 - Their vote is for a valid candidate

The proof is submitted to a voting contract on TON.

- 3. **Vote Tallying**: The voting contract aggregates votes without revealing individual choices. A final zk-SNARK proof is generated to prove the correctness of the tally.
- 4. **Result Verification**: Anyone can verify the final tally by checking the zk-SNARK proof on the TON blockchain.

20.2 Private Asset Transfers

Private asset transfers are another significant use case for enhanced privacy in a decentralized network. Overpass Channels enable confidential transfers while maintaining the integrity and verifiability of transactions.

Implementation on TON

- 1. Asset Tokenization: Assets are represented as tokens on the TON blockchain.
- 2. **Private Transfers**: Users transfer tokens through Overpass Channels, generating zk-SNARK proofs that:
 - The sender owns the tokens
 - The transfer amount is valid (non-negative and within the sender's balance)
 - The receiver is a valid recipient
- 3. On-chain Settlement: Periodically, the aggregate state of transfers is settled on-chain through a smart contract, updating token balances without revealing individual transaction details.
- 4. **Audit Capability**: While individual transactions are private, users can generate proofs of their transaction history for auditing purposes when required.

20.3 Secure Health Records Management

Secure health records management is an essential use case where sensitive health information must be kept confidential while ensuring that authorized parties can verify the records.

Implementation on TON

- 1. **Record Storage**: Health records are stored off-chain, with hashes of the records stored in Overpass Channels.
- 2. Access Control: Smart contracts on TON manage access rights to health records.
- 3. Data Sharing: When sharing data, a zk-SNARK proof is generated to prove:
 - The data belongs to the claimed patient
 - The recipient has the right to access the data
 - The data has not been tampered with
- 4. **Verification**: Authorized parties can verify the authenticity and integrity of the records on the TON blockchain without accessing the actual content.

20.4 Global Payment System

A global payment system is perhaps the most scalable and impactful use case for Overpass Channels, providing sufficient privacy to protect user transactions while ensuring transparency and scalability to facilitate mass adoption.

Implementation on TON

- 1. **Multi-Currency Support**: The system supports multiple currencies, with exchange rates managed by oracle contracts on TON.
- 2. **Cross-Border Transfers**: Users can make cross-border payments through Overpass Channels, with zk-SNARKs proving:
 - The sender has sufficient funds
 - The currency conversion is accurate
 - Compliance with relevant regulations (e.g., AML checks)
- Liquidity Pools: Smart contracts on TON manage liquidity pools to facilitate instant currency conversions.
- 4. **Regulatory Compliance**: The system includes built-in compliance features, such as generating confidential reports for regulators without compromising user privacy.
- 5. **Integration with Traditional Finance**: Bridge contracts on TON facilitate interaction with traditional financial systems, allowing for deposits and withdrawals while maintaining privacy for on-network transactions.

These use cases illustrate how Overpass Channels can achieve a balance between privacy and transparency, facilitating mass adoption while maintaining the necessary confidentiality. By leveraging zk-SNARKs, the system provides enhanced privacy and scalability, making it suitable for a wide range of applications in various sectors.

21 Future Directions and Challenges

While Overpass Channels on TON presents a powerful solution for scalable, private, and censorship-resistant payments, several areas warrant further research and development:

21.1 Post-Quantum Security

As quantum computing advances, ensuring the long-term security of Overpass Channels becomes crucial.

Theorem 24 (Post-Quantum Vulnerability). The current zk-SNARK implementations used in Overpass Channels are vulnerable to quantum attacks.

- *Proof.* 1. Current zk-SNARK constructions rely on discrete logarithm or elliptic curve discrete logarithm problems.
 - 2. Shor's algorithm, when implemented on a sufficiently powerful quantum computer, can solve these problems in polynomial time.
 - 3. Therefore, a quantum computer could potentially break the security of current zk-SNARK implementations.

Future research should focus on developing and implementing post-quantum secure zero-knowledge proof systems that maintain the efficiency and privacy guarantees of current zk-SNARKs.

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21.2 Privacy-Preserving Analytics

While Overpass Channels provide strong transaction privacy, this can make it challenging to gather network-wide analytics, which can be crucial for optimizing performance and detecting potential issues.

Theorem 25 (Privacy-Analytics Tradeoff). There exists a fundamental tradeoff between transaction privacy and the ability to perform network-wide analytics in Overpass Channels.

Research into privacy-preserving data analysis techniques, such as secure multi-party computation or differential privacy, could help address this challenge without compromising the core privacy guarantees of the system.

22 Conclusion

Overpass Channels, integrated with the TON blockchain, represents a significant advancement in blockchain scalability, privacy, and censorship resistance. By leveraging zk-SNARKs, off-chain processing, and TON's sharding architecture, it offers a solution that can potentially scale to global payment network levels while maintaining strong security and privacy guarantees.

The system's ability to process millions, if not billions, of transactions per second, with linear scalability as nodes are added, positions it as a highly competitive solution in the blockchain space. Its unique architecture makes censorship of individual transactions computationally infeasible, providing robust protection against potential attacks or regulatory pressures.

Furthermore, the integration with TON's flexible smart contract system, efficient cross-shard communication, and user-friendly features like DNS integration creates a comprehensive ecosystem for next-generation decentralized applications.

As the blockchain industry continues to evolve, solutions like Overpass Channels on TON pave the way for a future where decentralized, private, and efficient global payment systems become a reality, potentially revolutionizing financial interactions on a global scale.

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23 Appendix

1 Circuits

The Overpass Channels system uses hierarchical circuits to manage transactions, state updates, and global consistency. These circuits are organized into individual channels, intermediate contracts, and root contracts. Below is an explanation of how individual channels work, along with the related algorithms.

Channel 1: From Party A to Party B

In this section, we will describe the setup of a unilateral channel from Party A to Party B, followed by the process of executing a transaction and verifying the result.

Unilateral Channel Setup The setup for a unilateral payment channel involves locking the funds and verifying the signatures from both parties. The following algorithm explains how the setup works.

```
Algorithm 1: Unilateral Channel Setup
```

- Inputs: Funding Party (A), Receiving Party (B), Initial Balance (B_AB1), Channel ID (id_AB1), Signatures (\sigma_A, \sigma_B)
- 2. Initialize channel with balances:

```
B_A = B_AB1, B_B = 0, nonce = 0
```

3. PoseidonHash the channel ID and initial balance:

```
h_channel1 = PoseidonHash(id_AB1, B_AB1)
```

4. Verify signatures from both parties:

```
SignatureVerify(\sigma_A, A)
```

- SignatureVerify(\sigma_B, B)
- 5. Generate recursive zk-SNARK proof \pi_setup1
- 6. Output: zk-SNARK result C_setup1

In this process:

- 1. Initializes the payment channel with the balance from Party A.
- 2. Hashes the channel's initial data using the Poseidon hashing algorithm.
- 3. Verifies that both parties have signed and acknowledged the setup.

Transaction Execution Once the channel is set up, Party A can send funds to Party B through a transaction. The following algorithm describes the transaction process.

```
B'_A = B_A - \Delta_1
B'_B = B_B + \Delta_1
4. Verify balance conservation:
B'_A + B'_B = B_A + B_B
5. Ensure 50\% spending rule:
```

 $\label{eq:local_bound} $$ \Delta_1 \leq \frac{B_A}{2} $$$

6. Increment nonce:

```
n'_1 = n_1 + 1
```

- 7. Verify signature from A: SignatureVerify(\sigma_A, A)
- 8. Generate zk-SNARK proof \pi_transaction1
- 9. Output: zk-SNARK result C_transaction1

This algorithm handles the transaction between Party A and Party B, ensuring that:

- 1. The current state is hashed to preserve consistency of the channel data.
- 2. The balance conservation rule and the 50% spending rule are both followed.
- 3. The sender's signature is verified before generating the zk-SNARK proof.

Transaction Acceptance Once Party A has executed the transaction, Party B must accept it. The following algorithm explains the acceptance procedure.

- 4. Update the channel contract with the new state
- 5. Remove pending status from the transaction
- 6. Output: Channel state is updated and transaction is finalized

During transaction acceptance:

- 1. The zk-SNARK proof generated by Party A is verified.
- 2. Party B's signature on the new state is validated.
- 3. The channel contract is updated, and the transaction is finalized.

1.1 Channel State Verification for Both Channels

Both channels independently verify their states using Merkle proofs to ensure their integrity and consistency. Each channel maintains a Merkle root that reflects the current state of the balances and transactions.

Closure for Channel 1 $(A \rightarrow B)$

Algorithm: Closure for Channel 1

- 1. Inputs:
 - Channel Identifier (id_AB1)
 - Final Balances (B_A, B_B)
 - Final Signatures (\sigma_A, \sigma_B)
 - Merkle Proof for Channel State (\pi_Merkle1)
- 2. PoseidonHash the final state:

h_final1 = PoseidonHash(id_AB1, B_A, B_B)

- 3. Verify that the final balances match the expected total:
 - $B_A + B_B$ must equal the total recorded balance.
- 4. Verify signatures from both parties:

SignatureVerify(\sigma_A, A)

SignatureVerify(\sigma_B, B)

- 5. Verify the Merkle proof \pi_Merkle1 to ensure the correct state closure.
- 6. Generate zk-SNARK proof \pi_closure1 for closing the channel.
- 7. Output: zk-SNARK result C_closure1

In this process:

- 1. The final state of the channel is hashed using the Poseidon algorithm, ensuring that the balances and channel identifier are encoded securely.
- 2. The balance verification ensures that the final balances match the expected total, preventing any discrepancy before closure.
- 3. Both parties' signatures are verified, ensuring mutual agreement on the channel closure.
- 4. The Merkle proof confirms that the state being closed is the correct and most up-to-date state.
- 5. Finally, a zk-SNARK proof is generated, which will confirm the validity of the closure.

1.2 Intermediate Contract Circuits

Rebalancing Circuit

Algorithm: Rebalancing Circuit

- 1. Inputs:
 - Channel Identifier (id)
 - Current Balance Distribution (B_A, B_B)
 - Desired Balance Distribution (B'_A, B'_B)
 - Rebalancing Amount (\Delta_rebalance)
 - Signatures (\sigma_A, \sigma_B)
 - Current Shard Merkle Root (R_M_shard)
- 2. PoseidonHash the balances before and after rebalancing:

h_before = PoseidonHash(id, B_A, B_B)

h_after = PoseidonHash(id, B'_A, B'_B)

3. Verify balance conservation:

```
B_A + B_B = B'_A + B'_B
```

4. Calculate rebalancing amount:

```
\Delta = B'_A - B_A = B'_B - B_B
```

- 5. Update the shard's Merkle tree with the new state:
 - R_M_shard_new = UpdateMerkleRoot(R_M_shard, h_after)
- 6. Generate zk-SNARK proof \pi_rebalance showing rebalancing and agreement.
- 7. Output: zk-SNARK proof \pi_rebalance and updated Merkle root R_M_shard_new.

In this process:

- 1. The balances are hashed using Poseidon before and after rebalancing to ensure integrity.
- 2. Balance conservation is checked to guarantee that the total funds are preserved during rebalancing.
- 3. The rebalancing amount is calculated to determine how much to adjust the balances.
- 4. The shard's Merkle tree is updated to reflect the new state after rebalancing.
- 5. A zk-SNARK proof is generated, confirming that the rebalancing occurred as expected.

Cross-Channel Communication Circuit

Both channels independently verify their states using Merkle proofs to ensure their integrity and consistency. Each channel maintains a Merkle root that reflects the current state of the balances and transactions.

Algorithm: Cross-Channel Communication Circuit

- 1. Inputs:
 - Source Channel State (S_source)
 - Destination Channel State (S_destination)
 - Transaction Amount (\Delta)
 - Merkle Proofs for Both Channels (\pi_Merkle_source, \pi_Merkle_destination)
 - Current Shard Merkle Root (R_M_shard)
- 2. Verify Merkle proofs of both source and destination states:

VerifyMerkleProof(R_M_shard, \pi_Merkle_source, S_source)

VerifyMerkleProof(R_M_shard, \pi_Merkle_destination, S_destination)

3. Ensure sufficient funds in source channel:

B_source \geq \Delta

4. Update balances:

B_source_new = B_source - \Delta

B_destination_new = B_destination + \Delta

- 5. Generate zk-SNARK proof \pi_cross_channel, verifying transaction validity.
- 6. Compute new shard Merkle root:
 - R_M_shard_new = UpdateMerkleRoot(R_M_shard, S_source_new, S_destination_new)
- 7. Output: zk-SNARK proof \pi_cross_channel and updated Merkle root R_M_shard_new

In this process:

- 1. The Merkle proofs for the source and destination channels are verified to ensure inclusion in the current shard state.
- 2. The system checks if the source channel has enough balance to execute the transaction.
- 3. Balances in both the source and destination channels are updated accordingly.
- 4. A zk-SNARK proof is generated to confirm the transaction's validity.
- 5. Finally, the Merkle root is updated to reflect the new state of both channels.

Shard State Submission Circuit

This circuit handles the submission of the shard's updated state to the root contract, ensuring all updates and transactions are reflected in the global state.

```
Algorithm: Shard State Submission Circuit

1. Inputs:

- Current Shard Merkle Root (R_M_shard)

- New Shard Merkle Root (R_M_shard_new)

- Set of Aggregated Update Proofs (\pi_aggregation1, \pi_aggregation2, ...)

- Cross-Channel Transaction Proofs (\pi_cross_channel1, \pi_cross_channel2, ...)

2. Verify all aggregated update proofs:
```

For each \pi_aggregation_i:
 VerifyProof(\pi_aggregation_i)

3. Verify all cross-channel transaction proofs:

```
For each \pi_cross_channel_j:
    VerifyProof(\pi_cross_channel_j)
```

- 4. Recalculate the shard's Merkle root based on all updates:
 - R_M_shard_final = RecalculateMerkleRoot(R_M_shard, all updates)
- 5. Generate zk-SNARK proof \pi_shard_update, verifying the correct shard state update.
- 6. Output: zk-SNARK proof \pi_shard_update and final shard Merkle root R_M_shard_final

In this process:

- 1. Aggregated update proofs are verified to ensure all individual state transitions are valid.
- 2. Cross-channel transaction proofs are also verified for integrity.
- 3. The shard's Merkle root is recalculated based on all the updates and transactions within the shard.
- 4. A zk-SNARK proof is generated to confirm the correct update of the shard's state.
- 5. Finally, the final Merkle root and proof are submitted to the root contract.

1.3 Summary of Intermediate Contract Circuit Responsibilities

- Rebalancing Circuit: Manages liquidity by rebalancing channel balances and recalculating Merkle roots.
- 2. **State Aggregation Circuit**: Aggregates multiple channel updates in a shard into one efficient zk-SNARK proof.
- 3. Cross-Channel Communication Circuit: Facilitates transactions between channels within the same shard.
- 4. Shard State Submission Circuit: Consolidates all shard updates and prepares the final state for submission to the root contract.

Global State Update Circuit

This circuit ensures that all shard updates are aggregated and the global state is updated consistently across the entire network.

```
Algorithm: Global State Update Circuit
1. Inputs:
  - Current Global Merkle Root (R_M_global)
  - Set of Shard Merkle Roots (R_M_shard1, R_M_shard2, ...)
  - Set of Shard Update Proofs (\pi_shard_update1, \pi_shard_update2, ...)
2. Merkle Tree Verification:
  For each shard i:
      h_i = PoseidonHash(R_M_shardi)
       VerifyMerkleProof(R_M_global, h_i, proof_i)
3. Balance Consistency:
  Verify total balance consistency:
       sum(B_shardi) = B_total
  Ensure no overflows:
       forall i, B_shardi < 2^64
4. Recursive zk-SNARK Proof Generation:
   \pi_global_update = Plonky2Generate(\pi_shard_update1, \pi_shard_update2, ...)
5. Global Merkle Root Update:
  R_M_global_new = UpdateGlobalRoot(R_M_shard1, R_M_shard2, ...)
6. Output: zk-SNARK proof \pi_global_update and updated global Merkle root R_M_global_new
```

- In this process:
- 1. Each shard's Merkle root is verified and included in the global Merkle tree.
- 2. The total balance across all shards is checked to ensure consistency, and overflows are prevented.
- 3. A zk-SNARK proof is generated to validate the state updates across all shards.
- 4. The global Merkle root is updated to reflect the new state of the network.

Cross-Shard Transaction Verification Circuit

This circuit processes transactions between shards, ensuring balances are updated in both the source and destination shards.

```
Algorithm: Cross-Shard Transaction Verification Circuit
  - Current Global Merkle Root (R_M_global)
  - Source Shard Merkle Root (R_M_source)
  - Destination Shard Merkle Root (R_M_destination)
  - Cross-Shard Transaction Proof (\pi_cross_shard)
   - Transaction Details (sender, receiver, amount)
2. Merkle Tree Verification:
  Hash source and destination shards:
      h source = PoseidonHash(R M source)
      h_dest = PoseidonHash(R_M_destination)
  Verify Merkle proofs:
      VerifyMerkleProof(R_M_global, h_source, proof_source)
       VerifyMerkleProof(R_M_global, h_dest, proof_dest)
3. Balance Validation:
   Check sender balance:
       B_sender >= amount
  Update balances:
       B_sender_new = B_sender - amount
       B_receiver_new = B_receiver + amount
4. Recursive zk-SNARK Proof Generation:
   \pi_cross_shard = Plonky2Generate(transaction_details, R_M_source, R_M_destination)
5. Global Merkle Root Update:
  R_M_global_new = UpdateGlobalRoot(R_M_global, R_M_source_new, R_M_destination_new)
```

- In this process:
- 1. The source and destination shard states are verified through their respective Merkle proofs.

6. Output: zk-SNARK proof \pi_cross_shard and updated global Merkle root R_M_global_new

- 2. The sender's balance is validated, ensuring they have sufficient funds for the transaction.
- 3. The balances for both the sender and receiver are updated.
- 4. A zk-SNARK proof is generated to confirm the validity of the cross-shard transaction.
- 5. The global Merkle root is updated to reflect the changes in both shards.

Global State Consistency Check Circuit

This circuit checks the consistency of the global state across all shards, ensuring that no shard has an inconsistent state and preventing double-spending.

```
Algorithm: Global State Consistency Check Circuit
1. Inputs:
   - Current Global Merkle Root (R_M_global)
  - Set of Channel States (S1, S2, ..., SN)
   - Merkle Proofs for Each Channel State (\pi_Merkle_channel1, \pi_Merkle_channel2, ...)
2. Merkle Tree Verification:
  For each channel state Si:
       h_i = PoseidonHash(S_i)
       VerifyMerkleProof(R_M_global, h_i, \pi_Merkle_channeli)
3. State Validation:
  For each channel i:
       B_i >= 0
       B_i < 2^64
       SequenceNumber_i >= PreviousSequenceNumber_i
4. Recursive Proof Generation:
   \pi_global_consistency = Plonky2Generate(S1, S2, ..., SN, R_M_global)
5. Output: zk-SNARK proof \pi_global_consistency and Boolean result (IsConsistent)
```

In this process:

- 1. Each channel state is verified using its Merkle proof and checked for inclusion in the global Merkle tree.
- 2. Balances and sequence numbers are validated to ensure no channel is in an inconsistent state.
- 3. A zk-SNARK proof is generated to verify the overall global state consistency.
- 4. A Boolean output indicates whether the global state is consistent.

Epoch Transition Circuit

This circuit handles periodic state transitions by processing all transactions within an epoch and updating the global state.

```
Algorithm: Epoch Transition Circuit
1. Inputs:
    - Current Epoch Number (E)
    - Current Global Merkle Root (R_M_global)
    - Set of Transactions for Current Epoch (T1, T2, ..., TM)
    - Set of Transaction Proofs (\pi1, \pi2, ..., \piM)
2. Merkle Tree Verification:
    For each transaction Ti:
        h_i = PoseidonHash(T_i)
        VerifyMerkleProof(R_M_global, h_i, proof_i)
3. Transaction Validation:
    For each transaction Ti:
        B_sender >= amount_i
        B_sender_new = B_sender - amount_i
```

```
B_receiver_new = B_receiver + amount_i
4. zk-SNARK Proof Generation:
  \pi_epoch = Plonky2Generate(T1, T2, ..., TM, \pi1, \pi2, ..., \piM)
5. Global State Update:
  R_M_global_new = UpdateGlobalRoot(R_M_global, T1, T2, ..., TM)
  Increment epoch: E_new = E + 1
```

6. Output: zk-SNARK proof \pi_epoch, updated global Merkle root R_M_global_new, new epoch number E_1

In this process:

- 1. Each transaction in the current epoch is verified and included in the global state.
- 2. The sender's and receiver's balances are updated according to each transaction.
- 3. A zk-SNARK proof is generated to validate the epoch's transactions.
- 4. The global Merkle root is updated, and the epoch number is incremented.

Conditional Payments Circuit

Objective: Facilitate conditional payments between parties, where the payment is released only when certain conditions, such as the revelation of a preimage or the completion of a task, are met. This leverages zk-SNARKs to verify the conditions off-chain while maintaining on-chain trustlessness.

```
Algorithm: Conditional Payments Circuit
1. Inputs:
  - Sender (S)
  - Receiver (R)
  - Payment Amount (P)
  - Condition (C), e.g., a preimage X such that H(X) = H_target
  - zk-SNARK Proof (\pi_condition) for condition fulfillment
2. Hash Condition Verification:
  Verify that the condition holds:
      h_condition = PoseidonHash(H_target, X)
      Assert h_condition == H_target
3. Balance Update (Goldilocks):
  Check if the sender has enough funds:
      B_S >= P
  Update balances:
      B_S_{new} = B_S - P
      B_R_new = B_R + P
4. Recursive zk-SNARK Proof Generation:
   \pi_conditional_payment = Plonky2Generate(C, \pi_condition, B_S, B_R)
5. Output: zk-SNARK proof \pi_conditional_payment and updated balances (B_S_new, B_R_new)
```

In this process:

1. The condition for the payment, such as revealing a correct preimage, is verified using the Poseidon hashing algorithm.

- 2. The sender's balance is checked to ensure sufficient funds for the payment.
- 3. If the condition is met, the balances for both the sender and receiver are updated.
- 4. A zk-SNARK proof is generated to verify the correct execution of the conditional payment.

HTLC (Hashed Time-Lock Contract) Circuit

Objective: Implement a time-locked conditional payment system where the receiver can claim funds only by revealing the correct preimage within a specified time period. If the receiver fails to do so, the funds are returned to the sender.

```
Algorithm: HTLC Circuit
1. Inputs:
  - Sender (S)
  - Receiver (R)
  - Payment Amount (P)
  - Hash Condition H_target
  - Timeout (T)
   - zk-SNARK Proof (\pi_htlc)
2. Hash Condition Verification:
  Verify the preimage:
      Assert PoseidonHash(X) == H_target
3. Timeout Check:
   If timeout T has expired, revert the payment:
      If current_time > T, refund P to S
4. Balance Update (Goldilocks):
   If condition is met and within time limit:
      B_S_new = B_S - P
      B_R_new = B_R + P
  If timeout:
      B_S_{new} = B_S
      B_R_new = B_R
5. Recursive zk-SNARK Proof Generation:
   \pi_htlc = Plonky2Generate(H_target, X, T, current_time)
6. Output: zk-SNARK proof \pi_htlc and updated balances
```

In this process:

- 1. The receiver must reveal the correct preimage that hashes to the target hash using Poseidon hashing.
- 2. The circuit checks whether the timeout has been reached before processing the payment.
- 3. If the condition is met within the time limit, the balances are updated; otherwise, the funds are returned to the sender.
- 4. A zk-SNARK proof is generated to confirm the correct execution of the HTLC.

2 Decentralized Exchange (DEX) on Overpass

The Overpass Channels-based decentralized exchange (DEX) employs a hybrid architecture that integrates both on-chain and off-chain components to ensure scalability, efficiency, and security. By leveraging zk-SNARKs for every individual action and Sparse Merkle Trees (SMTs) for state tracking, the system allows for private and secure off-chain trading while maintaining cryptographic proofs for all operations. This architecture does not rely on traditional consensus mechanisms or rollups; instead, it uses overpass zk-SNARKs for each state transition, eliminating the need for trust between participants and allowing instant validation of actions.

2.1 Overview of the System Components

The system consists of three primary components:

- On-Chain Hub Contract: Manages on-chain operations like liquidity deposits, channel opening/closing, and final settlement.
- Off-Chain Router: Handles off-chain operations, including order book management, order matching, and state tracking using zk-SNARKs. The router is also responsible for managing its own Sparse Merkle Tree (SMT) to store off-chain state updates.
- **zk-SNARK Circuits**: Used for every state transition in the system, ensuring that all actions (e.g., placing orders, executing trades, updating balances) are cryptographically secure.

This section explores how each component interacts, how zk-SNARKs and SMTs are used for security, and how the off-chain router functions as an intermediary for state management.

2.2 On-Chain Hub Contract

The on-chain hub contract serves as the core on-chain component, ensuring that liquidity providers (LPs) can deposit assets, traders can open and close payment channels, and all state changes are cryptographically verified before being settled on-chain. The hub contract interacts with the off-chain router to finalize state transitions after they have been validated using zk-SNARKs.

Key Functions of the On-Chain Hub Contract:

- Liquidity Management: LPs can deposit assets directly into the hub contract to provide liquidity to the DEX. These assets are stored securely on-chain and can be withdrawn when an LP chooses to exit the pool.
- Payment Channel Opening and Closing: Traders open payment channels with the hub contract, allowing them to engage in off-chain trading. When a trader decides to settle their channel, the hub contract ensures that the final off-chain state is reflected on-chain.
- Final Settlement: The hub contract verifies zk-SNARK proofs submitted by the router, ensuring that the final off-chain balances and trades are correctly reflected on-chain when channels are closed.

The hub contract operates primarily as a verifier of zk-SNARK proofs, minimizing the need for direct on-chain execution of logic, and enabling low-cost, fast transactions.

2.3 Off-Chain Router

The off-chain router is the central component for managing all off-chain operations within the DEX. It handles the interaction between users, the order book, and liquidity providers. The router manages the majority of the logic for the DEX, including:

- **Order Management and Matching**: All buy and sell orders are submitted to the off-chain router. The router maintains the order book, matches orders, and ensures that all orders meet the necessary conditions before being added to the book.
- **State Management with Sparse Merkle Trees**: The router maintains its own Sparse Merkle Tree (SMT), which tracks every state transition, including order placements, order matches, and user balance updates. This SMT allows the router to efficiently manage and verify the validity of all state transitions.
- **Interaction with zk-SNARK Circuits**: Every action processed by the router is accompanied by a zk-SNARK proof, which verifies the correctness of the action (e.g., verifying that a buyer has sufficient funds to place an order). These proofs ensure the correctness of each state transition.

The router is responsible for tracking and validating the entire state of the off-chain order book, balances, and trades, ensuring that only valid and cryptographically proven actions are processed.

Router's Role in State Transitions and Conditional Proofs

In the Overpass Channels system, the router is responsible for processing state transitions using **zk-SNARK proofs** to ensure all conditions are met before an action is accepted. These actions include:

- **Placing Orders**: When a user places a buy or sell order, the router triggers a zk-SNARK operation to validate the user's available balance (for buy orders) or assets (for sell orders). This validation ensures that the order is valid before it is added to the order book.
- **Order Matching**: Once orders are matched by the router, it verifies that both the buyer and seller meet the required conditions (e.g., the buyer has sufficient funds, the seller has the assets to sell) before executing the trade off-chain.
- **Trade Execution**: After a match is confirmed, the router updates the off-chain balances of both parties and records the state transition in its dedicated SMT. This ensures that the correct balances are maintained off-chain and that the entire process is verifiable via zk-SNARK proofs.

Pseudocode for Order Placement and Execution:

The following pseudocode outlines how the router processes an order and verifies its validity using zk-SNARKs:

Algorithm: Router's Order Placement with zk-SNARK Validation

1. procedure SubmitOrder(User, OrderType, Amount, Price)

```
2.
       if OrderType == 'Buy' then
3.
           zkProof = zkSNARK_Validate_BuyOrder(User, Amount, Price)
4.
           if zkProof is valid then
5.
               AddOrderToOrderBook(User, 'Buy', Amount, Price)
6.
               UpdateRouterSMT(User, 'Buy', Amount, Price)
7.
           else
8.
               RejectOrder(User) # Insufficient funds or invalid conditions
9.
           end if
10.
       else if OrderType == 'Sell' then
11.
           zkProof = zkSNARK_Validate_SellOrder(User, Amount, Price)
12.
           if zkProof is valid then
               AddOrderToOrderBook(User, 'Sell', Amount, Price)
13.
14.
               UpdateRouterSMT(User, 'Sell', Amount, Price)
15.
           else
16.
               RejectOrder(User) # Insufficient assets or invalid conditions
17.
           end if
18.
       end if
19. end procedure
```

In this process: - The router validates all order conditions via zk-SNARK proofs before an order is added to the order book. - Once validated, the router updates its Sparse Merkle Tree (SMT) with the new state, ensuring that each state transition is cryptographically secure and stored for later verification.

2.4 Sparse Merkle Trees for State Tracking

Sparse Merkle Trees (SMTs) provide an efficient way to store and verify state transitions within the system. The off-chain router maintains its own SMT dedicated to tracking all state updates, including order placements, matches, and balance adjustments. Each state change is recorded in the SMT, allowing the system to generate **Merkle proofs** to validate the current state when required.

Structure of the Router's Sparse Merkle Tree

The router's SMT stores state updates in a tree-like structure where each leaf node represents a specific state change (e.g., an order placed, a trade executed). The Merkle root represents the cumulative state of all off-chain actions up to a specific point in time.

Each leaf in the tree contains a hashed value representing:

- **Order Information**: Details about the buy or sell order, including user ID, token type, amount, and price.
- \bullet **Balance Updates**: Changes to a user's balance after an order is placed or executed.
- **Trade Completion**: Final state of a trade, including the new balances of both the buyer and seller.

Sparse Merkle Tree Example:

```
Merkle Root \leftarrow H(H(Order 1, Order 2), H(Order 3, Order 4))
```

The root of the tree is updated every time a new state transition occurs, ensuring that the entire history of state transitions is securely stored and verifiable.

State Transition Verification via Merkle Proofs

When the router needs to verify a state transition (e.g., during settlement or upon request), it provides a **Merkle proof** that proves the current state of the system is valid based on prior updates. This proof includes the following components:

- The **Merkle root** representing the latest state of the SMT.
- The **Merkle path** leading to the specific state (e.g., an order or balance) that needs to be verified.
- The **leaf value** corresponding to the state transition being verified.

The **on-chain hub contract** can verify this Merkle proof to ensure that the state transition is valid before updating on-chain balances. This reduces the need for large-scale on-chain storage and ensures that the system remains efficient.

Pseudocode for Merkle Proof Verification:

Algorithm: Verifying a State Transition with Merkle Proof

```
1. procedure VerifyMerkleProof(MerkleRoot, MerklePath, LeafValue)
2.
       currentHash = LeafValue
3.
       for each node in MerklePath do
4.
           currentHash = Hash(currentHash, node)
5.
6.
       if currentHash == MerkleRoot then
7.
           return True # State transition is valid
8.
       else
9.
           return False # Invalid state transition
10.
       end if
11. end procedure
```

2.5 zk-SNARK Integration for Validating Every Action

The Off-Chain Router ensures that every action taken by users is valid by generating zk-SNARK proofs for each operation. Whether a user is placing a buy or sell order, or performing a trade, these operations are validated off-chain with zk-SNARKs before being reflected in the Sparse Merkle Tree (SMT). This validation process ensures that each action meets the necessary conditions (such as sufficient funds or valid asset ownership), making the DEX system robust, secure, and resistant to fraudulent activity.

zk-SNARK Generation for State Transitions

For every state transition, the router generates a zk-SNARK proof to validate the following:

- **For Buy Orders**: The buyer has sufficient funds locked in their payment channel to cover the order. The proof verifies that the balance exists off-chain and that it has been properly locked.
- **For Sell Orders**: The seller has the corresponding assets available to fulfill the sell order. This proof validates the presence of the assets in the seller's off-chain balance.
- **For Order Matching**: The router ensures that the matched buy and sell orders meet all predefined conditions, such as price compatibility and quantity availability.

These proofs are generated using the zk-SNARK framework, ensuring that every action performed off-chain is verifiable without requiring consensus or extensive on-chain verification. Once the zk-SNARK proof is generated, the router updates its Sparse Merkle Tree (SMT) with the new state, cryptographically ensuring that the state change is valid and secure.

Pseudocode for zk-SNARK Proof Generation in a Trade:

Algorithm: zk-SNARK Proof for State Transition in a Trade

```
1. procedure Generate_ZKProof(OrderType, User, Amount, Token)
2.
       if OrderType == 'Buy' then
3.
           Check user has sufficient funds locked
           Create proof zkProof = Prove(User.Balance >= Amount)
4.
       else if OrderType == 'Sell' then
5.
6.
           Check user has sufficient assets to sell
           Create proof zkProof = Prove(User.Asset >= Amount)
7.
8.
       end if
9.
       return zkProof
10. end procedure
```

Once the zk-SNARK proof is successfully generated, it guarantees that the user's action is valid, allowing the system to proceed with order placement or execution.

2.6 Trade Execution Process

When the router matches a buy and sell order, it executes the trade off-chain by updating both parties' balances. This process is done in conjunction with zk-SNARK proofs to ensure that the conditions for both parties (buyer and seller) are met. The steps for executing a trade are as follows:

Off-Chain Trade Execution

Once the orders are matched, the router performs the following actions:

1. **Generate zk-SNARK Proofs for Buyer and Seller**: - The router generates zk-SNARK proofs to verify that the buyer has sufficient funds and the seller has the required assets to fulfill the trade. If both proofs are valid, the trade proceeds.

2. **Update Router's Sparse Merkle Tree**: - After the trade is validated by zk-SNARK proofs, the router updates the SMT to reflect the new state: - The buyer's balance is reduced by the amount they paid (e.g., USDT). - The seller's asset balance is reduced by the quantity sold (e.g., ETH). - Both new balances are reflected in the SMT, and the Merkle root is updated accordingly.

Pseudocode for Off-Chain Trade Execution:

Algorithm: Off-Chain Trade Execution

```
    procedure ExecuteTrade(BuyOrder, SellOrder)

2.
       # Validate both orders using zk-SNARKs
       buyerProof = Generate_ZKProof('Buy', BuyOrder.User, BuyOrder.Amount, Token)
3.
       sellerProof = Generate_ZKProof('Sell', SellOrder.User, SellOrder.Amount, Token)
4.
5.
       if buyerProof is valid and sellerProof is valid then
6.
           # Update balances in the router's Sparse Merkle Tree (SMT)
7.
           UpdateRouterSMT(BuyOrder.User, 'Subtract', BuyOrder.Amount)
           UpdateRouterSMT(SellOrder.User, 'Subtract', SellOrder.Amount)
8.
9.
           UpdateRouterSMT(BuyOrder.User, 'Add', SellOrder.Amount) # Buyer gets tokens
10.
           UpdateRouterSMT(SellOrder.User, 'Add', BuyOrder.Amount) # Seller gets funds
           return Success
11.
12.
       else
           return Failure # Invalid trade conditions
13
14.
       end if
15. end procedure
```

- 3. **Distribute Liquidity Provider Rewards**: Liquidity providers (LPs) are rewarded based on the trade volume. The router uses the SMT to keep track of liquidity utilization and calculates rewards based on the LP's contribution. LP rewards are recorded as state changes in the SMT and distributed accordingly.
- 4. **Final State in the Router's SMT**: The final post-trade state is stored in the router's Sparse Merkle Tree. This state includes the updated balances of both the buyer and seller, as well as the LP rewards. The Merkle root is updated to reflect the latest state.

2.7 Sparse Merkle Trees for Order and Balance Tracking

The router leverages Sparse Merkle Trees (SMTs) to efficiently track all off-chain actions, including orders, trades, and balance updates. Each state transition is stored as a node in the SMT, and every action (whether it be placing an order or fulfilling a trade) results in an update to the tree.

Router's SMT Structure

The router's SMT tracks all state transitions, ensuring that each state change is cryptographically secure and can be verified using Merkle proofs. The structure of the SMT can be described as follows:

• **Leaves**: Each leaf node in the SMT represents a specific state transition, such as a balance update, an order placed in the order book, or a completed trade.

- **Branches**: The branches of the SMT represent the intermediary states of multiple leaves. The hashes of these branches allow for efficient verification of the entire state.
- **Root**: The Merkle root summarizes the state of all active orders, user balances, and liquidity data. The root is updated after each state transition and can be used for final verification when settling on-chain.

Example of Router's SMT for a Trade:

Merkle Root \leftarrow H(H(Buy Order, Sell Order), H(Balance A, Balance B))

The Merkle root represents the final, cumulative state of all orders and balances in the DEX system. Each new trade or balance update results in the Merkle root being recalculated to reflect the latest state.

Merkle Proofs for Verifying Trades

When a trade is executed or a balance is updated, the router provides a Merkle proof to verify the correctness of the state transition. The Merkle proof consists of:

- **Merkle root**: The latest root of the SMT, which summarizes the entire state of the system.
- **Merkle path**: The set of hashes from the leaf node to the root, proving that a specific state change (e.g., a trade or balance update) is valid.
- **Leaf node**: The specific node representing the state change (e.g., a user's balance after a trade).

The router uses this proof to confirm that a state transition is valid without needing to reveal the entire state tree. The on-chain hub contract can verify the Merkle proof to ensure that the off-chain state is valid before updating on-chain balances.

2.8 On-Chain Final Settlement

When users want to settle their balances on-chain or close their payment channels, the router submits the final off-chain state to the **on-chain hub contract**. The router provides both a **Merkle proof** (to validate the state transition in the SMT) and a **zk-SNARK proof** (to validate that all actions leading to the final state were correct). This two-pronged approach ensures that the state transitions were valid without requiring extensive on-chain computations.

Closing a Payment Channel

When a user decides to close their payment channel with the DEX, the following process occurs:

• **Provide zk-SNARK and Merkle Proofs**: The router submits both the zk-SNARK proof (proving that the state transitions were valid) and the Merkle proof (proving that the current state reflects the final balance) to the on-chain hub contract.

- **Verify Proofs On-Chain**: The hub contract verifies the zk-SNARK proof to ensure that all off-chain state transitions (e.g., order placements, trade executions) were valid. It also verifies the Merkle proof to confirm that the current state is correctly reflected in the router's SMT.
- **Finalize Settlement**: Once both proofs are verified, the hub contract updates the user's on-chain balance based on the final state of their off-chain payment channel.

This process ensures that final settlement is secure, efficient, and cryptographically proven. Pseudocode for On-Chain Settlement:

Algorithm: On-Chain Final Settlement

```
    procedure FinalizeSettlement(User, zkProof, MerkleProof)

       # Verify the zk-SNARK proof for all state transitions
3.
       if VerifyZKProof(zkProof) == True then
4.
           # Verify the Merkle proof to confirm the final state
5.
           if VerifyMerkleProof(MerkleProof) == True then
               # Update on-chain balances based on final off-chain state
6.
7.
               UpdateOnChainBalance(User, FinalState)
               return Success
8.
9.
           else
               return Failure # Invalid Merkle proof
10.
11.
           end if
12.
       else
           return Failure # Invalid zk-SNARK proof
13.
14.
       end if
15. end procedure
```

In this process, the system ensures that every off-chain action leading to the final on-chain settlement is valid and secure, eliminating the risk of fraud or incorrect state transitions.

2.9 Liquidity Provision and LP Rewards Distribution

Liquidity provision is a key component of the decentralized exchange (DEX) on Overpass Channels. Liquidity providers (LPs) deposit assets into the system, enabling traders to execute orders by leveraging the available liquidity in the pools. The rewards for providing liquidity are distributed based on the trading activity and the LP's contribution to the liquidity pool.

Depositing Liquidity to the Hub Contract

Liquidity providers deposit assets directly into the **on-chain hub contract**, which manages the liquidity pools for different trading pairs (e.g., ETH/USDT, BTC/ETH). Each LP is issued **LP tokens** representing their share of the pool, which entitles them to a proportional share of the trading fees and rewards.

The process of depositing liquidity works as follows:

1. LP Deposits Assets On-Chain: An LP deposits a certain amount of assets (e.g., 100 ETH) into the hub contract.

- 2. LP Tokens Minted: In return, the LP receives LP tokens that represent their proportional share in the liquidity pool. These tokens can be used to track the LP's stake in the pool and earn rewards based on trading activity.
- 3. **Hub Contract Updates Liquidity Pools**: The hub contract updates the corresponding liquidity pool for the specific trading pair (e.g., ETH/USDT) to reflect the newly added liquidity.

Pseudocode for Liquidity Deposit:

Algorithm: Liquidity Deposit

- 1. procedure DepositLiquidity(LP, Token, Amount)
- 2. # LP deposits assets into the on-chain hub contract
- 3. MintLPTokens(LP, Token, Amount) # Issue LP tokens to the liquidity provider
- 4. UpdateLiquidityPool(Token, Amount) # Update the liquidity pool state
- 5. return Success
- 6. end procedure

Once the liquidity is deposited, the LP is eligible to earn rewards based on the trading volume in the pool.

Off-Chain Liquidity Utilization

Once liquidity is deposited into the hub contract, the router manages its utilization in off-chain trades. The router keeps track of which trades are consuming liquidity from specific pools and ensures that LPs are compensated for providing liquidity.

- **Liquidity Tracking in SMT**: Each time liquidity is used to fulfill a trade, the router updates its Sparse Merkle Tree (SMT) to reflect the amount of liquidity consumed. This ensures that the off-chain state accurately tracks liquidity usage.
- **LP Rewards Calculation**: As trades are executed off-chain, the router calculates the trading fees and rewards for each LP based on their contribution to the liquidity pool.
- **Final State Recorded in SMT**: The final state of liquidity utilization is recorded in the router's SMT, ensuring that the rewards distribution can be verified using Merkle proofs.

LP Rewards Distribution Based on Usage

LPs are rewarded based on the trading activity that occurs within the liquidity pool. The more trades that are executed using the liquidity provided by the LP, the higher the rewards they earn. The process works as follows:

- 1. **Trade Execution**: When a trade is executed off-chain, the router calculates the trading fees based on the volume of the trade.
- 2. **Proportional Reward Allocation**: The fees generated from the trade are distributed to LPs based on their contribution to the liquidity pool. LPs that have provided more liquidity receive a larger share of the fees.

3. **Update in the SMT**: The router updates the SMT to record the rewards distribution, ensuring that the rewards are properly reflected in the final state.

Pseudocode for LP Rewards Distribution:

Algorithm: LP Rewards Distribution

```
1. procedure DistributeLPRewards(TradeVolume, Pool, FeePercentage)
2.
       # Calculate total fees for the trade
3.
       TotalFees = TradeVolume * FeePercentage
4.
       for each LP in Pool do
5.
           # Calculate LP's share based on their contribution
6.
           LpShare = LP.Contribution / Pool.TotalLiquidity
           # Distribute fees to the LP
7.
8.
           LpReward = TotalFees * LpShare
           UpdateRouterSMT(LP, 'Add', LpReward) # Update SMT with the LP reward
9.
10.
       end for
11.
       return Success
12. end procedure
```

Once the rewards are calculated and distributed, the LPs can claim their rewards either off-chain by keeping the LP tokens or on-chain by closing their channel with the hub contract.

2.10 Order Lifecycle with zk-SNARK Validations and SMT Updates

The lifecycle of an order on the DEX follows a highly secure and efficient process, with every action verified through zk-SNARK proofs and reflected in the router's Sparse Merkle Tree. This ensures that only valid orders are processed, trades are executed correctly, and state transitions are cryptographically secured.

Order Placement and Validation

When a user places an order (either buy or sell), the router validates the order using a zk-SNARK proof to ensure that all conditions are met. This includes verifying that the user has the required balance or assets to fulfill the order.

Steps in Order Placement:

- 1. **User Submits Order**: The user submits a buy or sell order to the off-chain router, specifying the asset, price, and quantity.
- 2. **zk-SNARK Validation**: The router triggers a zk-SNARK circuit to validate the order. For buy orders, it verifies that the user has sufficient funds locked in their payment channel. For sell orders, it checks that the user has enough of the asset to sell.
- 3. **Order Added to Order Book**: Once validated, the router adds the order to the off-chain order book and updates its Sparse Merkle Tree (SMT) to reflect the new state.

Pseudocode for Order Placement with zk-SNARK Validation:

Algorithm: Order Placement

```
    procedure SubmitOrder(User, OrderType, Amount, Price)

       if OrderType == 'Buy' then
З.
           zkProof = zkSNARK_Validate_BuyOrder(User, Amount, Price)
4.
           if zkProof is valid then
               AddOrderToOrderBook(User, 'Buy', Amount, Price)
5.
6.
               UpdateRouterSMT(User, 'Buy', Amount, Price)
7.
           else
8.
               RejectOrder(User) # Insufficient funds or invalid conditions
9.
           end if
10.
       else if OrderType == 'Sell' then
11.
           zkProof = zkSNARK_Validate_SellOrder(User, Amount, Price)
12.
           if zkProof is valid then
               AddOrderToOrderBook(User, 'Sell', Amount, Price)
13.
               UpdateRouterSMT(User, 'Sell', Amount, Price)
14.
15.
16.
               RejectOrder(User) # Insufficient assets or invalid conditions
17.
           end if
18.
       end if
19. end procedure
```

This process ensures that only valid orders are placed and prevents spam or invalid orders from entering the system.

Order Matching and Trade Execution

Once an order is placed and added to the order book, the router constantly checks for matching orders. When a buy order and sell order meet the necessary conditions (price, quantity, etc.), the router executes the trade off-chain, updating the balances of both parties.

Steps in Order Matching and Trade Execution:

- 1. **Order Matching**: The router scans the order book for matching orders. A buy order is matched with a sell order if the price and quantity are compatible.
- 2. **Trade Validation with zk-SNARKs**: Before executing the trade, the router generates zk-SNARK proofs for both the buyer and seller to ensure that both parties can fulfill the trade (e.g., buyer has enough funds, seller has enough assets).
- 3. **Trade Execution**: Once validated, the router updates the off-chain balances of both parties and records the trade in its SMT.

Pseudocode for Order Matching and Trade Execution:

Algorithm: Order Matching and Trade Execution

- 1. procedure MatchOrders()
- 2. for each BuyOrder in BuyOrderBook do

```
3.
           for each SellOrder in SellOrderBook do
4.
               if BuyOrder.Price >= SellOrder.Price and
                  BuyOrder.Amount >= SellOrder.Amount then
5.
6.
                   # Validate both orders using zk-SNARKs
                   buyerProof = zkSNARK_Validate_BuyOrder(BuyOrder.User, BuyOrder.Amount)
7.
8.
                   sellerProof = zkSNARK_Validate_SellOrder(SellOrder.User, SellOrder.Amount)
9.
                   if buyerProof is valid and sellerProof is valid then
                       ExecuteTrade(BuyOrder, SellOrder)
10.
                       UpdateRouterSMT(BuyOrder.User, 'Subtract', BuyOrder.Amount)
11.
                       UpdateRouterSMT(SellOrder.User, 'Subtract', SellOrder.Amount)
12.
                       UpdateRouterSMT(BuyOrder.User, 'Add', SellOrder.Amount)
13.
                       UpdateRouterSMT(SellOrder.User, 'Add', BuyOrder.Amount)
14.
15.
                       RemoveMatchedOrders(BuyOrder, SellOrder)
16.
                   end if
17.
               end if
18.
           end for
19.
       end for
20. end procedure
```

This process ensures that trades are executed only when both orders are valid and all conditions are met. The off-chain router takes care of updating the balances and recording the trade in its Sparse Merkle Tree.

Trade Finalization and Settlement

After the trade is executed and recorded in the router's SMT, the users may choose to settle onchain. The final state is submitted to the on-chain hub contract, along with zk-SNARK and Merkle proofs, to finalize the trade and update the users' on-chain balances.

Steps in Trade Finalization:

- 1. **Final State Submission**: When a user closes their payment channel, the router provides the final state of their off-chain balance, along with zk-SNARK and Merkle proofs.
- 2. **On-Chain Verification**: The on-chain hub contract verifies both the zk-SNARK proof (ensuring that the off-chain actions were valid) and the Merkle proof (ensuring that the final balance is accurate).
- 3. **Final Settlement**: Once verified, the hub contract updates the user's on-chain balance based on the final off-chain state.

Pseudocode for Final Settlement:

Algorithm: Final Settlement and Balance Update

```
    procedure SettleOnChain(User, zkProof, MerkleProof)
    if VerifyZKProof(zkProof) == True then
    if VerifyMerkleProof(MerkleProof) == True then
    UpdateOnChainBalance(User, FinalState)
```

```
5. return Success
6. else
7. return Failure # Invalid Merkle proof
8. end if
9. else
10. return Failure # Invalid zk-SNARK proof
11. end if
12. end procedure
```

This ensures that the final on-chain balance reflects all valid off-chain actions, creating a secure and efficient settlement process.

3 Security and Efficiency through zk-SNARKs and SMTs

The use of zk-SNARKs and Sparse Merkle Trees ensures the security, privacy, and efficiency of the Overpass Channels DEX. Every action is cryptographically proven, ensuring that only valid trades are executed, liquidity providers are compensated accurately, and users' final balances are securely settled on-chain.

3.1 Scalability with Off-Chain Processing

By performing most operations off-chain, the DEX minimizes the load on the underlying blockchain. This allows the system to scale to handle a high volume of trades without being constrained by on-chain throughput. The use of zk-SNARK proofs ensures that all off-chain actions are secure and verifiable, while the Sparse Merkle Tree structure enables efficient tracking and verification of state transitions.

3.2 Privacy through zk-SNARKs

zk-SNARKs enable private transactions by allowing the router to validate actions without revealing sensitive information, such as the exact amount of a user's balance or the details of a trade. This ensures that users' financial privacy is preserved, even as their actions are verified cryptographically.

3.3 Security through Merkle Proofs

The use of Sparse Merkle Trees allows for efficient verification of the system's state. Every state transition (whether it be an order placement, trade execution, or liquidity update) is recorded in the SMT, ensuring that the entire history of actions can be cryptographically proven. This reduces the risk of fraud or manipulation and ensures that final on-chain settlements are based on a valid, verifiable off-chain state.

4 Advanced Order Types and Centralized Exchange-like Experience

One of the goals of the Overpass Channels DEX architecture is to provide a user experience that rivals that of centralized exchanges (CEXs) while maintaining the security, privacy, and trustless

nature of a decentralized system. To achieve this, the DEX supports a variety of advanced order types, fast execution, and an order book that feels as responsive as its centralized counterparts.

4.1 Order Types in the DEX

The Overpass Channels DEX supports multiple order types, enabling users to execute complex trading strategies. These order types are implemented off-chain via the router, which handles the order matching and execution while ensuring that every action is validated via zk-SNARKs.

Market Orders

A market order allows a trader to buy or sell an asset immediately at the best available price. This order type does not specify a price; instead, it is executed against the current available orders in the order book.

Steps in Market Order Execution:

- 1. **Order Submission**: A trader submits a market order (buy or sell) to the router.
- 2. **Order Matching**: The router immediately matches the market order with the best available limit orders in the order book.
- 3. **zk-SNARK Validation**: The router verifies the trader's balance via a zk-SNARK proof before executing the trade.
- 4. **Execution**: The trade is executed at the best available price, and the corresponding balances are updated in the Sparse Merkle Tree (SMT).

Pseudocode for Market Order Execution:

Algorithm: Market Order Execution

```
    procedure SubmitMarketOrder(User, OrderType, Amount)

       # Fetch best available order from the order book
3.
       BestOrder = FetchBestAvailableOrder(OrderType)
       if BestOrder is found then
5.
           # Validate market order using zk-SNARK
6.
           zkProof = zkSNARK_ValidateOrder(User, Amount)
7.
           if zkProof is valid then
8.
               # Execute market order at the best price
               ExecuteTrade(User, BestOrder, Amount)
9.
               UpdateRouterSMT(User, 'Subtract', Amount)
10.
               UpdateRouterSMT(BestOrder.User, 'Add', Amount)
11.
12.
               return Success
13.
           else
               return Failure # Invalid balance or conditions
14.
15.
           end if
16.
       else
17.
           return Failure # No matching orders available
18.
       end if
19. end procedure
```

Limit Orders

A limit order allows traders to specify the price at which they want to buy or sell an asset. The trade will only be executed if the market price reaches the specified limit price or better.

Steps in Limit Order Placement and Execution:

- 1. **Order Submission**: A trader submits a limit order specifying the price and amount.
- 2. **zk-SNARK Validation**: The router generates a zk-SNARK proof to validate the trader's balance or assets required for the order.
- 3. **Order Added to Order Book**: If validated, the limit order is added to the order book.
- 4. **Order Matching**: The router continuously monitors the order book to check if there is a match with an incoming buy or sell order that meets the limit price.
- 5. **Execution**: When the conditions are met (e.g., the market price matches or exceeds the limit price), the router executes the trade, and the balances are updated in the SMT.

Pseudocode for Limit Order Execution:

Algorithm: Limit Order Execution

```
    procedure SubmitLimitOrder(User, OrderType, Amount, LimitPrice)

2.
       # Validate the limit order using zk-SNARK
3.
       zkProof = zkSNARK_ValidateLimitOrder(User, Amount, LimitPrice)
       if zkProof is valid then
4.
           # Add limit order to the order book
5.
           AddOrderToOrderBook(User, OrderType, Amount, LimitPrice)
6.
           UpdateRouterSMT(User, 'Lock', Amount) # Lock funds for the order
7.
           return Success
8.
9.
       else
10.
           return Failure # Invalid balance or conditions
11.
       end if
12. end procedure
```

Stop-Loss Orders

A stop-loss order allows traders to set a predefined price at which an asset should be sold to limit losses. The order is triggered when the asset's market price falls to the stop price.

Steps in Stop-Loss Order Execution:

- 1. **Order Submission**: The trader submits a stop-loss order with a specified stop price and amount.
- 2. **zk-SNARK Validation**: The router verifies the stop-loss order using a zk-SNARK proof to ensure that the user has sufficient assets to sell.
- 3. **Order Triggering**: Once the asset's market price reaches the stop price, the router triggers the stop-loss order and converts it into a market order.

4. **Execution**: The market order is executed, and the trader's assets are sold at the best available price.

Pseudocode for Stop-Loss Order Execution:

Algorithm: Stop-Loss Order Execution

```
    procedure SubmitStopLossOrder(User, Amount, StopPrice)

       # Validate the stop-loss order using zk-SNARK
2.
3.
       zkProof = zkSNARK_ValidateStopLoss(User, Amount, StopPrice)
4.
       if zkProof is valid then
           AddStopLossOrder(User, Amount, StopPrice)
5.
           UpdateRouterSMT(User, 'Lock', Amount) # Lock assets for stop-loss
6.
7.
           return Success
8.
       else
           return Failure # Invalid balance or conditions
9.
10.
       end if
11. end procedure
12. procedure TriggerStopLossOrder(User, Amount, StopPrice)
       if MarketPrice <= StopPrice then
           ConvertStopLossToMarketOrder(User, Amount)
14.
15.
           SubmitMarketOrder(User, 'Sell', Amount)
16.
       end if
17. end procedure
```

Stop-Limit Orders

A stop-limit order combines the functionality of a stop-loss order and a limit order. Once the stop price is reached, the order becomes a limit order, and the trade will only be executed at the limit price or better.

Steps in Stop-Limit Order Execution:

- 1. **Order Submission**: The trader submits a stop-limit order specifying the stop price, limit price, and amount.
- 2. **zk-SNARK Validation**: The router validates the order and the trader's balance using a zk-SNARK proof.
- 3. **Order Triggering**: When the market price reaches the stop price, the order is converted into a limit order.
- 4. **Execution**: The limit order is executed once the market price meets or exceeds the limit price.

Pseudocode for Stop-Limit Order Execution:

Algorithm: Stop-Limit Order Execution

```
    procedure SubmitStopLimitOrder(User, Amount, StopPrice, LimitPrice)

       # Validate the stop-limit order using zk-SNARK
       zkProof = zkSNARK_ValidateStopLimit(User, Amount, StopPrice, LimitPrice)
3.
4.
       if zkProof is valid then
           AddStopLimitOrder(User, Amount, StopPrice, LimitPrice)
5.
           UpdateRouterSMT(User, 'Lock', Amount) # Lock assets for stop-limit
6.
7.
           return Success
8.
       else
9.
           return Failure # Invalid balance or conditions
10.
       end if
11. end procedure
12. procedure TriggerStopLimitOrder(User, Amount, StopPrice, LimitPrice)
       if MarketPrice <= StopPrice then
13.
14.
           ConvertStopLimitToLimitOrder(User, Amount, LimitPrice)
15.
           SubmitLimitOrder(User, 'Sell', Amount, LimitPrice)
16.
       end if
17. end procedure
```

4.2 Order Book Management and User Experience

The order book in the Overpass Channels DEX is designed to provide a fast, responsive experience similar to that of centralized exchanges. The order book is managed off-chain by the router and continuously updated as new orders are placed, matched, or cancelled.

Centralized Exchange-like Speed

The DEX's off-chain architecture allows for near-instantaneous order placement and matching, providing a user experience that feels as fast as using a centralized exchange. The key to achieving this speed is by performing the order matching and validation off-chain, using zk-SNARKs for verification while keeping on-chain interactions minimal.

Features of CEX-like Speed and Experience:

- **Instant Order Book Updates**: Since the order book is managed off-chain, new orders appear in the order book instantly once validated.
- **Fast Order Matching**: The off-chain router constantly scans for matching orders, ensuring that trades are executed as quickly as possible once the conditions are met.
- **Secure Execution with zk-SNARKs**: Every order and trade is validated securely using zk-SNARK proofs, ensuring that the system remains decentralized and trustless, even while delivering a centralized exchange-like experience.

4.3 Liquidity, Depth, and Advanced Trading Strategies

To further replicate the feel of a centralized exchange, the DEX supports deep liquidity pools and allows for advanced trading strategies. With LPs providing liquidity across multiple trading pairs,

traders can execute large orders without causing significant slippage, similar to what they would expect on a traditional exchange.

Features for Advanced Trading Strategies:

- **High Liquidity**: LPs deposit assets into the on-chain hub contract, creating deep liquidity pools for major trading pairs.
- **Low Slippage**: The deep liquidity pools reduce slippage, enabling large trades to be executed without significantly affecting the market price.
- **Advanced Order Types**: The DEX supports various order types (market, limit, stop-loss, stop-limit), allowing traders to implement sophisticated trading strategies.

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