THE CAPACITY OF BLACK HOLES TO TRANSMIT QUANTUM INFORMATION

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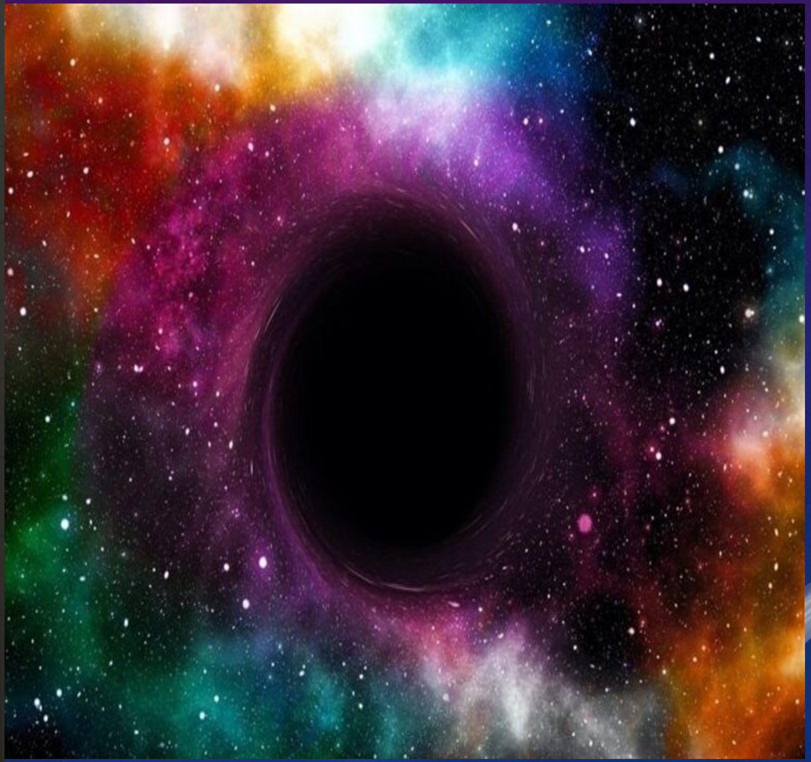
PH5842: ADVANCED TECHNIQUES IN QUANTUM INFORMATION AND QUANTUM COMPUTING

QUANTUM CHANNELS



QUANTUM CHANNEL

Suppose Iron Man prepare a state by applying random probabilities of unitary operator and send it to Spider man. Spider man can not prepare the state with accuracy. This is perfectly unitary operation and resulting state can be perfectly purified by appropriate ancilla.

Bekenstein speculated that black hole entropy is ensemble average over the many ways black hole can be prepared. Zurek described this as action of super operator which is now called CPTP map that is quantum channel. Therefore, mixed state after evaporation of black hole is not a failure of quantum mechanics.

ERROR CORRECTING CODE

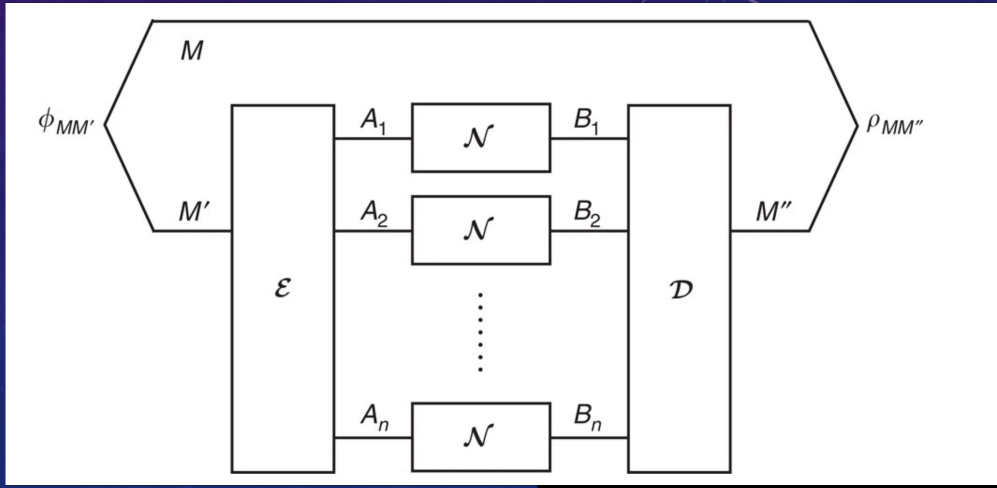
If the quantum channel, $\mathcal{N}$ is not too noisy then there exists

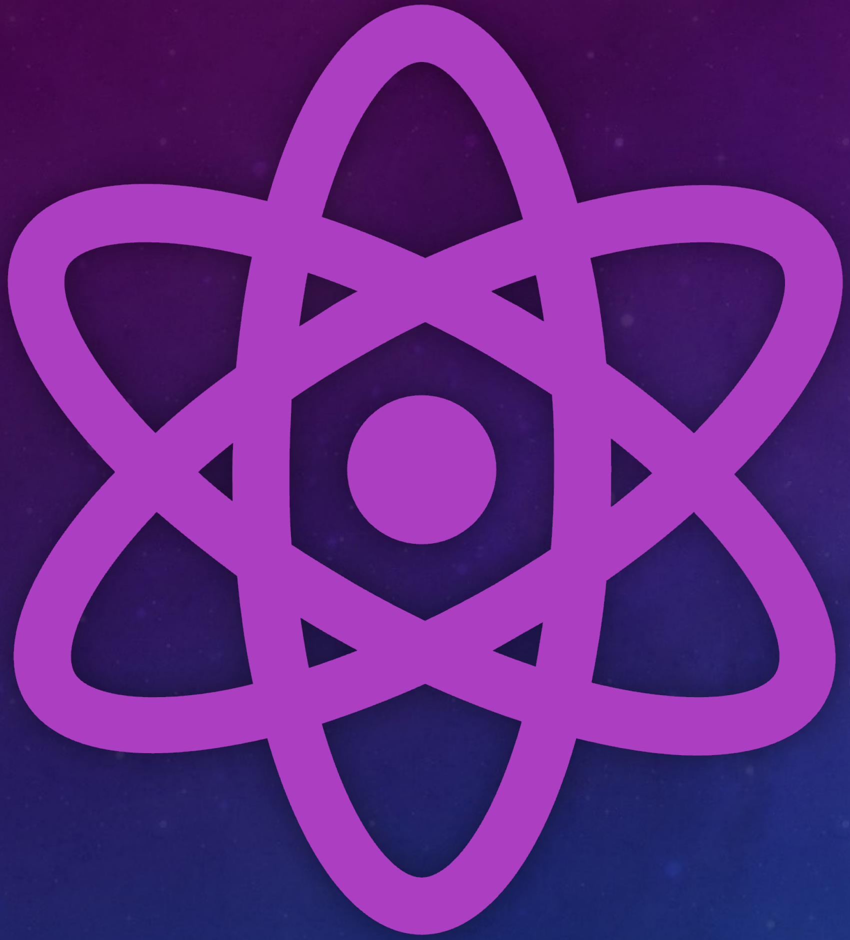
an Encoder, $\mathcal{E}$

a Decoder, $\mathcal{D}$

such that the composite channel $\mathcal{E} \circ \mathcal{N} \circ \mathcal{D}$

is arbitrarily close to a noiseless map under suitable norm called Diamond norm.



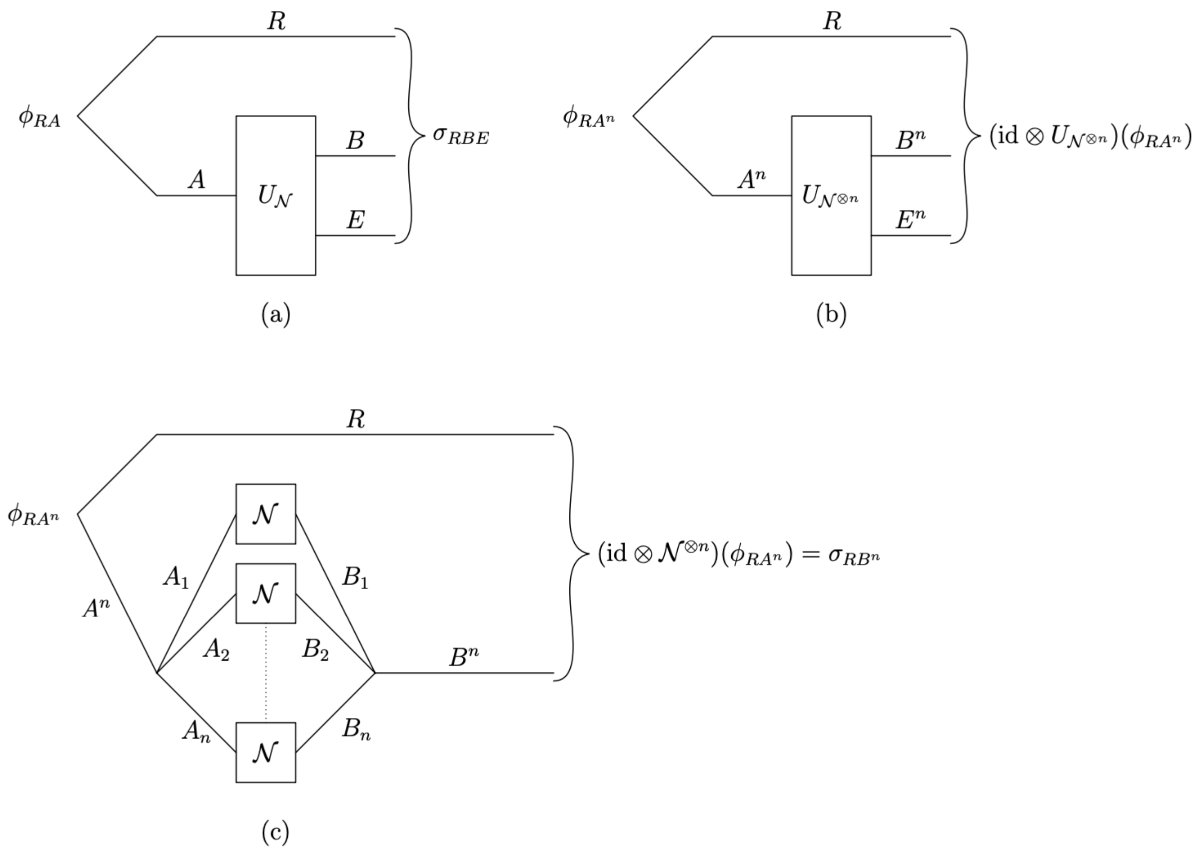


QUANTUM CAPACITY

Quantum capacity is defined as the maximum of the ratio of the number qubits we wish to send to the number of qubits generated by the encoder to make sure the system can be faithfully decodable by the receiver. Therefore, it is a number between 0 and 1.

QUANTUM CAPACITY

$$Q(\mathcal{N}) = \lim_{n\to\infty} \frac{1}{n} Q^{(1)}(\mathcal{N}^{(\otimes n)})$$



$$Q^{(1)}(\mathcal{N}) \stackrel{\text{df}}{=} \max_{\phi_{RA}} I_c(\mathcal{N}) = \max_{\phi_{RA}} \left[H(B)_\sigma - H(E)_\sigma\right].$$

SINGLE LETTER, SYMMETRIC, AND DEGRADABLE QUANTUM CHANNELS

The quantum capacity of certain class of channels can be calculated by showing the regularization is unnecessary these channels are called Single letter quantum capacity formulas.

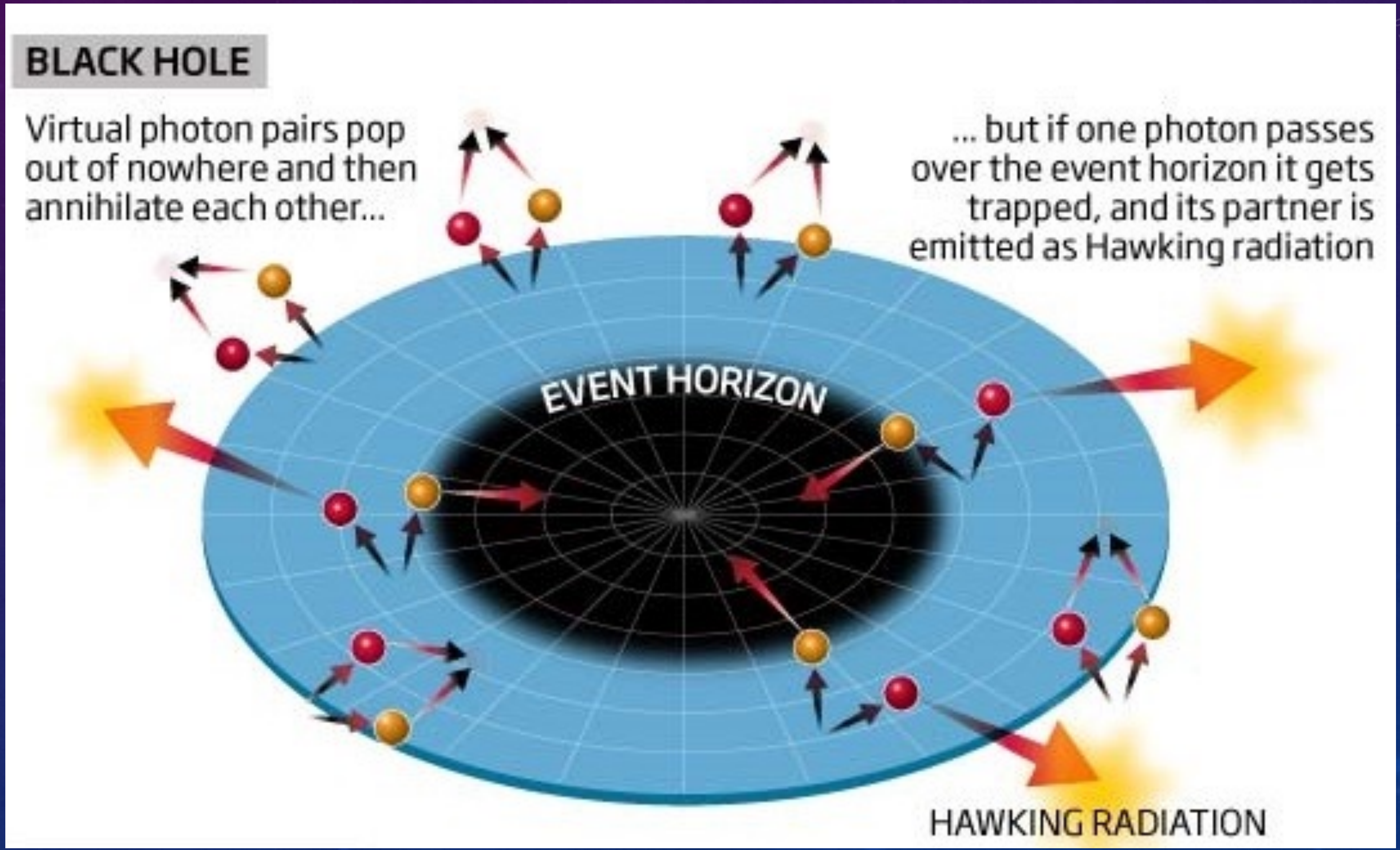
Symmetric quantum channels are one such class whose quantum capacity as proved by Smith et al. is zero.

Symmetric quantum channels belong to a broader class of quantum channels called Degradable channels which play important role in black hole Quantum Information Theory.

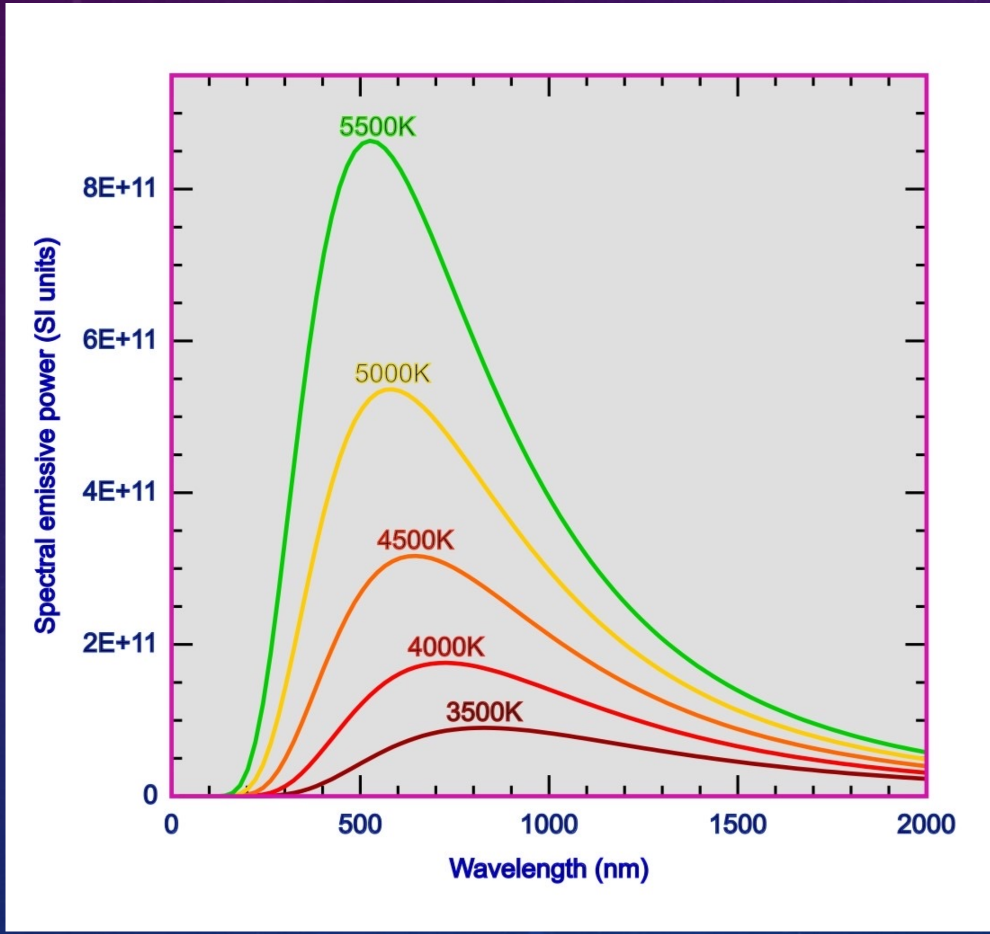
BLACK HOLE

A black hole refers to a region in space where the gravitational pull is incredibly strong, to the point where even light cannot escape. Such intense gravity results from a massive amount of matter compressed into an extremely small space. This occurrence often takes place when a star is in its final stages of life.

HAWKING RADIATION



THERMAL BLACK BODY RADIATION



$$r_s = \frac{2GM}{c^2}$$

$$T = \frac{\hbar c^3}{8\pi M G k_B}$$

HAWKING RADIATION

- a_k annihilates the receiver's vacuum. (B)
- b_k annihilates vacuum states beyond the black hole horizon. (E)
- r_ω is related to mode frequency (ω) and surface gravity $\kappa = \frac{1}{2\,M}$ by $\exp\left(-\pi\frac{\omega}{\kappa}\right) = r_\omega$

$$V(r_\omega) = \prod_\omega \exp\left[r_\omega(a_k^\dagger b_{-k}^\dagger - a_k b_{-k})\right]$$

$$V(r_\omega)\,|\mathrm{vac}\rangle = \prod_\omega \frac{1}{\cosh r_\omega} \sum_{n=0}^{\infty} \tanh^n r_\omega\, |n\rangle_B\, |n\rangle_E = \prod_\omega \sigma_{\omega,BE}$$

NO QUANTUM INFORMATION IN HAWKING RADIATION

$$\prod_{\omega} \varrho^{\text{th}}_{\omega,B} \overset{\text{df}}{=} \prod_{\omega} \left(1 - e^{-\frac{2\pi\omega}{\kappa}}\right) \sum_{n=0}^{\infty} e^{-\frac{2\pi n\omega}{\kappa}} |n\rangle_B \langle n| = \prod_{\omega} \frac{1}{\cosh^2 r_\omega} \sum_{n=0}^{\infty} \tanh^{2n} r_\omega |n\rangle_B \langle n|$$

$$\varrho^{\text{th}}_{\omega,B} = \varrho^{\text{th}}_{\omega,E}.$$

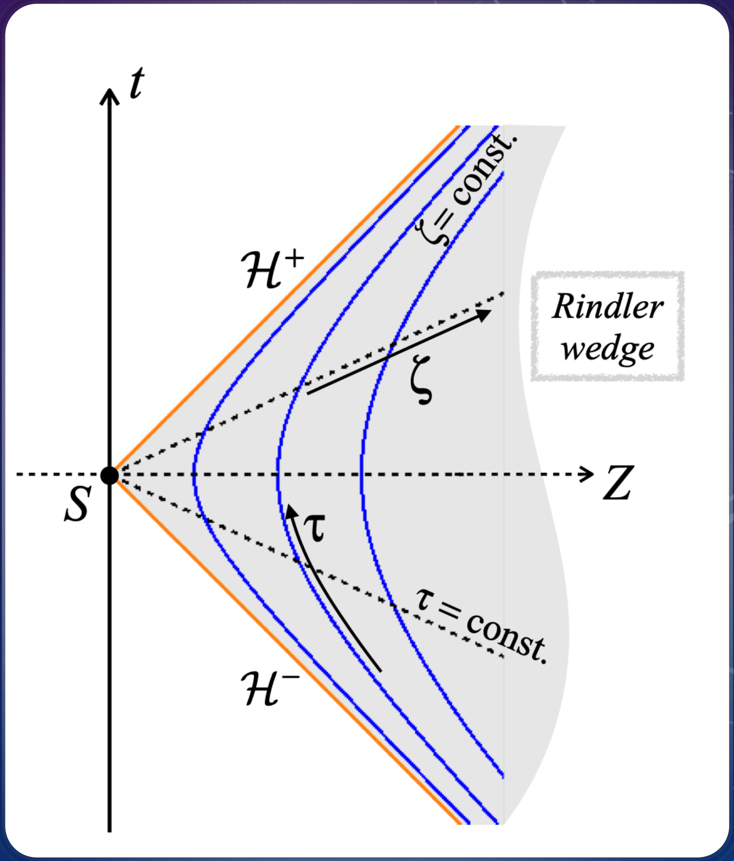
Symmetric and Quantum capacity is zero.

BLACK HOLES AS QUANTUM TRANSMISSION CHANNELS

- Hayden and Preskill proposed that black holes can serve as quantum transmission channels, allowing for the transfer of quantum states from one point to another through the black hole's event horizon.
- If Spider man can accurately reconstruct Iron man's quantum state from the radiation collected at future null infinity, it suggests that black hole information processing is unitary, although if this is not possible, it does not violate unitarity.
- The process of thermalization acts as a form of protection for quantum information, but it may not be applicable to all states, especially those with infinite dimensions. Therefore, the authors suggest using encoding as a quantum channel to protect against the loss of quantum information in black holes.

UNRUH EFFECT

- The Unruh effect is a quantum field theory phenomenon that predicts that an observer accelerating in a vacuum will perceive the vacuum as a thermal bath of particles. This effect was first proposed by physicist William Unruh in 1976.
- The Unruh effect is based on the idea that a vacuum is not empty but rather contains virtual particles that arise due to quantum fluctuations. In the presence of an accelerating observer, these virtual particles are no longer symmetrically distributed and thus give rise to a thermal spectrum of real particles that the observer perceives as radiation.



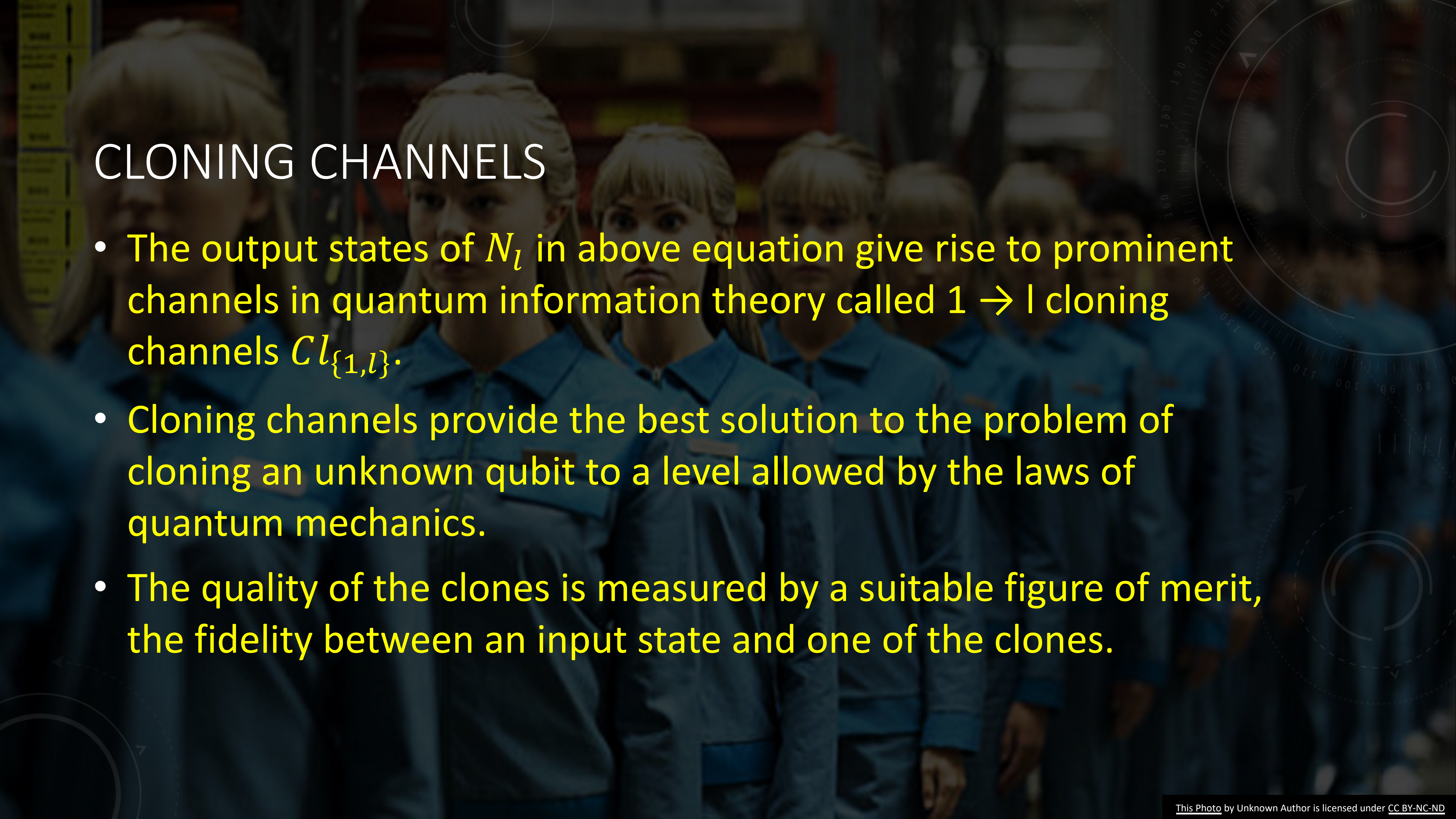
UNRUH CHANNEL

$$V_\Omega^{A\to BE}(|n\rangle_A) = \frac{1}{\cosh^{1+n} r_\Omega} \sum_{m=0}^{\infty} \binom{n+m}{n}^{1/2} \tanh^m r_\Omega \, |n+m\rangle_B \, |m\rangle_E,$$

$$\mathcal{N} = \bigoplus_{\ell=1}^{\infty} p_\ell \mathcal{N}_\ell,$$

$$p_\ell = \frac{1}{2}(1-z)^3 \ell(\ell+1) z^{\ell-1}$$

The limit where the black hole is perfectly reflecting incoming radiation, the black hole channel exactly coincides with the Unruh channel.



CLONING CHANNELS

- The output states of N_l in above equation give rise to prominent channels in quantum information theory called 1 → l cloning channels $Cl_{\{1,l\}}$.
- Cloning channels provide the best solution to the problem of cloning an unknown qubit to a level allowed by the laws of quantum mechanics.
- The quality of the clones is measured by a suitable figure of merit, the fidelity between an input state and one of the clones.

CLONING CHANNELS

Clones and anti-clones are distinguished in the literature, with anti-clones being the complex conjugate of the clones and representing the best possible approximation of the orthogonal complement of a given pure state.

A cloning machine that attempts to create l copies from n inputs creates n (approximate) clones and l − n anti-clones.

Before studying the structure of these channels in more detail, the concept of "degradability" needs to be discussed.

DEGRADABILITY

- The concept of degradability was introduced by Devetak and Shor.
- The complementary channel to a quantum channel is defined by tracing over the output system instead of the reference system $\widehat{\mathcal{N}} = Tr_B\left[\, V_{\Omega}^{A\to BE}\right]$.
- A channel is degradable if there exists another quantum channel such that. $\widehat{\mathcal{N}} = D \circ \mathcal{N}$, D is called a degrading map.
- For degradable channels, the quantum capacity can be calculated through a computationally tractable problem.

DEGRADABILITY

- Quantum capacities satisfy a certain entropic inequality that makes it possible to prove the inequality for the optimized coherent information.

$$Q^{(1)}(\mathcal{N} \otimes \mathcal{N}) \leq 2Q^{(1)}(\mathcal{N})$$

- The quantum capacity of a probabilistic mixture of quantum channels is a probabilistic mixture of quantum capacities.

$$Q(\mathcal{N}) = \frac{(1-z)^3}{2} \sum_{\ell=1}^{\infty} \ell(\ell+1) z^{\ell-1} \log_2 \frac{\ell+1}{\ell}.$$

CONJUGATE DEGRADABILITY

Conjugate degradable channels are channels degradable up to complex conjugation, and additivity of the optimized coherent information holds for them as well.

Cloning channels are both degradable and conjugate-degradable, but it is not clear whether there exist conjugate-degradable channels that are not degradable.

PERFECTLY REFLECTING BLACK HOLE

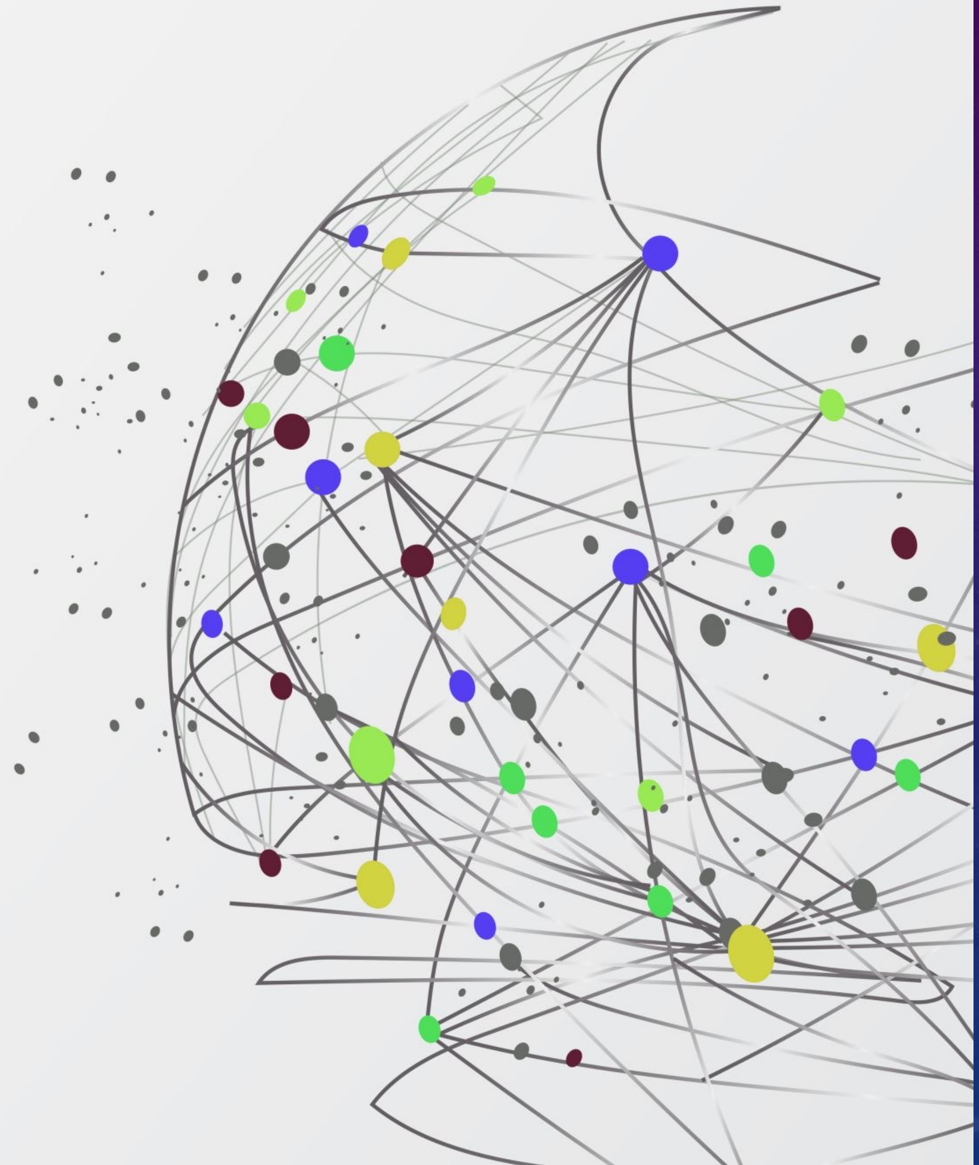
1. Radiation creates two clones and a single anti-clone of quantum information when impinging on a perfectly reflecting black hole.

2. The clones can be used to perfectly reconstruct the original quantum state, while the anti-clone behind the horizon cannot.

3. The channel is perfectly reflecting from the inside of the black hole, and the no-cloning theorem remains inviolate.

PERFECTLY ABSORBING BLACK HOLE

1. Perfect absorption black hole channel is complementary to perfect reflection channel.
2. Capacity to reconstruct quantum information outside of black hole is zero, channel is entanglement breaking.
3. Absorbed clone and anti-clone can be used to perfectly reconstruct entanglement behind the horizon.



THANK YOU