The capacity of black holes to transmit quantum information*

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I. INTRODUCTION

In this report, quantum information theory is used to analyze how classical and quantum information interact with the black hole horizon and how much information can be recovered from the black hole. The black hole is described as an open quantum system whose purifying quantum system is not accessible. The black hole is described by a quantum channel, and its evolution is perfectly unitary.

Quantum Shannon theory is used to illustrate how no information can be hidden in the outgoing Hawking radiation, and to construct the quantum channel whose formalism goes beyond Hawking's standard results. The quantum channel is used to calculate the channel capacity, which depends on how well the black hole reflects the information. Two cases are considered: Perfectly absorbing and perfectly reflecting black holes. It is shown that in neither case is it possible to reconstruct the perfect state inside and outside the event horizon.

It is concluded that no quantum information can ever be extracted from Hawking radiation. It is argued that black holes hold quantum information as long as the black hole horizon is present. In other words, the information is not lost but remains encoded within the black hole.

The Unruh effect is also discussed, which concerns the radiation that an accelerated observer perceives when an observer at rest measures vacuum. The Unruh channel capacity is calculated using single-letter capacities. This channel has close resemblances to the physical process of black hole evaporation. Black hole channels are not the same as Unruh channels, but in the limit of perfectly reflecting black holes, the black hole channel and the Unruh channel coincide.

The project's introduction highlights the importance of quantum information theory in understanding the interaction between classical and quantum information and black holes. It also sheds light on the nature of black holes as open quantum systems, whose evolution is perfectly unitary, and the role of quantum channels in understanding the transmission of quantum information. The conclusion that black holes hold quantum information as long as the black hole horizon is present has significant implications for our understanding of black holes and the information paradox.

In summary, the project on the capacity of black holes to transmit quantum information is a fascinating and important field of study that combines concepts from quantum information theory and black hole physics. The use of quantum Shannon theory and quantum channels to analyze the interaction between classical and quantum information and black holes provides new insights into the nature of black holes as open quantum systems and the transmission of quantum information. The conclusion that black holes hold quantum information as long as the black hole horizon is present has significant implications for our understanding of black holes and the information paradox.

II. QUANTUM CHANNELS AND QUANTUM CAPACITY

The concept of quantum capacity is a fundamental concept in the field of quantum information theory. It measures the amount of information that can be transmitted through a quantum communication channel. The relevance of this concept to Hawking radiation, a phenomenon by which black holes emit radiation, has been a topic of much debate in the field of physics. In this analysis, quantum Shannon theory is used to show that no information can be hidden in the outgoing Hawking radiation. The formalism of the quantum channel constructed goes beyond Hawking's standard results and confirms the featurelessness of this radiation.

The analysis is based on a semi-classical framework that involves making some assumptions. First, the analysis considers macroscopic Schwarzschild black holes, where the effects of back reaction, or the influence of the black hole on the metric field surrounding it, are neglected. Second, the momentum conservation of the black hole is neglected. Finally, the black hole is assumed to emit scalar massless fields only, and gravitational redshift is neglected. The authors [1]claim that even if these assumptions are relaxed or other types of quantum fields are considered, their conclusion will not change qualitatively, as long as the super selection rules for higher-spin fields are taken into account.

The quantum capacity of a quantum channel $\mathcal{N}$ is calculated to determine how much information is encoded in Hawking radiation. Quantum channels are completely positive trace-preserving maps that represent the general noisy evolution of density operators. The quantum capacity is defined as the maximum ratio of the number of qubits that can be transmitted to the number of qubits generated by the encoder. If the quantum channel $\mathcal{N}$

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is not too noisy, then we can encode and decode it with two maps, an encoder ε and a decoder D. The composite channel $D \circ \mathcal{N}^{\otimes n} \circ \varepsilon$ is arbitrarily close to a noiseless map under a suitable norm called the Diamond norm[2]. This is equivalent to the error-correcting code in the asymptotic limit of many copies of the channel $\mathcal{N}$. Therefore, quantum capacity is a number between 0 and 1.

The dimension of Hilbert-space error-free transmission is approximately $2^{nQ(\mathcal{N})}$, where the exponent is the quantum capacity given by

$$Q(\mathcal{N}) = \lim_{n\to\infty} \frac{1}{n} Q^{(1)}(\mathcal{N}^{\otimes n}) \tag{1}$$

The quantity $Q^{(1)}(\mathcal{N}^{\otimes n})$ is called the optimised coherent information, defined by

$$Q^{(1)}(\mathcal{N}) = maxI_c(\mathcal{N}) = max[H(B) * \sigma - H(E) * \sigma] \tag{2}$$

$$\sigma_{RBE} = (id \otimes U_{\mathcal{N}})(\phi_{RA})$$

In figure 1, $U_{\mathcal{N}}$ is the isometric extension of the quantum channel $\mathcal{N}$. B refers to the Hilbert space of the receiver, while E refers to the (unmeasured) environment. R is the purifying system such that σ_{RBE} is a pure state. The maximization is performed over all possible entangled states ϕ_{RA}. The von Neumann entropy of the S_1 subsystem of an n-partite quantum state $\sigma_{S_1S_2S_3...}$ is denoted by $H(S_1)_\sigma$.

The calculation of quantum capacity is hard for arbitrary quantum channels. However, quantum capacity can be calculated for some classes of quantum channels by showing that the regularization in (1) is unnecessary. These channels are called "single-letter" quantum capacity formulas. Symmetric quantum channels are one such class whose quantum capacity is provably zero. This belongs to a broader class of channels called degradable channels, which play an important role in black hole quantum information theory. The quantum capacity of the Hawking radiation channel is given by the maximum ratio of the number of qubits that can be transmitted to the number of qubits generated by the encoder. This ratio is a measure of the efficiency of the channel, and it is bounded between 0 and 1. The quantum capacity of the Hawking radiation channel is zero, which means that no information can be transmitted through this channel. This result confirms the featurelessness of the Hawking radiation, which is a well-known but disputed statement in the field of physics.

It is important to note that the concept of quantum capacity has significant implications not only in the field of black hole physics but also in quantum communication and computing. The study of quantum capacity has led to the development of new error-correcting codes, which allow us to transmit information over long distances with high fidelity. It has also led to the development of new cryptographic protocols, which take advantage of the unique properties of quantum systems to provide unbreakable security.

Optimized coherent information is another concept that is discussed in this article. It is a measure of the amount of information that can be transmitted through a quantum channel. This quantity is defined as the maximum mutual information between the input and output of the channel, and it can be used to calculate the quantum capacity of the channel. The optimized coherent information of the Hawking radiation channel is also zero, which provides further evidence for the featurelessness of the radiation.

The concept of degradable channels is also discussed in this article. The Hawking radiation channel belongs to the class of channels called degradable channels, which have a zero quantum capacity. These channels play an important role in the study of black hole information loss, which is a long-standing problem in theoretical physics. The results of this article may have implications for the resolution of this problem, although further research is needed to fully understand the implications of this work.

A. Hawking Radiation

Hawking radiation is a theoretical idea initially presented by physicist Stephen Hawking in 1974. According to the hypothesis, quantum phenomena close to a black hole's event horizon cause it to spew radiation. The idea of virtual particles—pairs of particles that emerge and vanish in empty space—is the foundation for the theory of Hawking radiation. One of the particles may be pulled into the black hole while the other escapes when this occurs close to a black hole's event horizon. As a result, a small quantity of mass is lost by the black hole and transformed into energy and emmitted as radiation. For more understanding please refer to **??**.

The output of the channel described by the thermal hawking radiation is symmetric and as discussed in [3] we can say symmetric channels have zero quantum capacity. From this we lead to following conclusions stated in [1] for a quantum radiation to escape evaporating black hole, the radiation needs to be either

- display some form of inter-mode entanglement within the output multi-mode entangled state, or
- exhibit non-thermal corrections in the two-mode output state.

are necessary but not sufficient for non-zero quantum capacity.

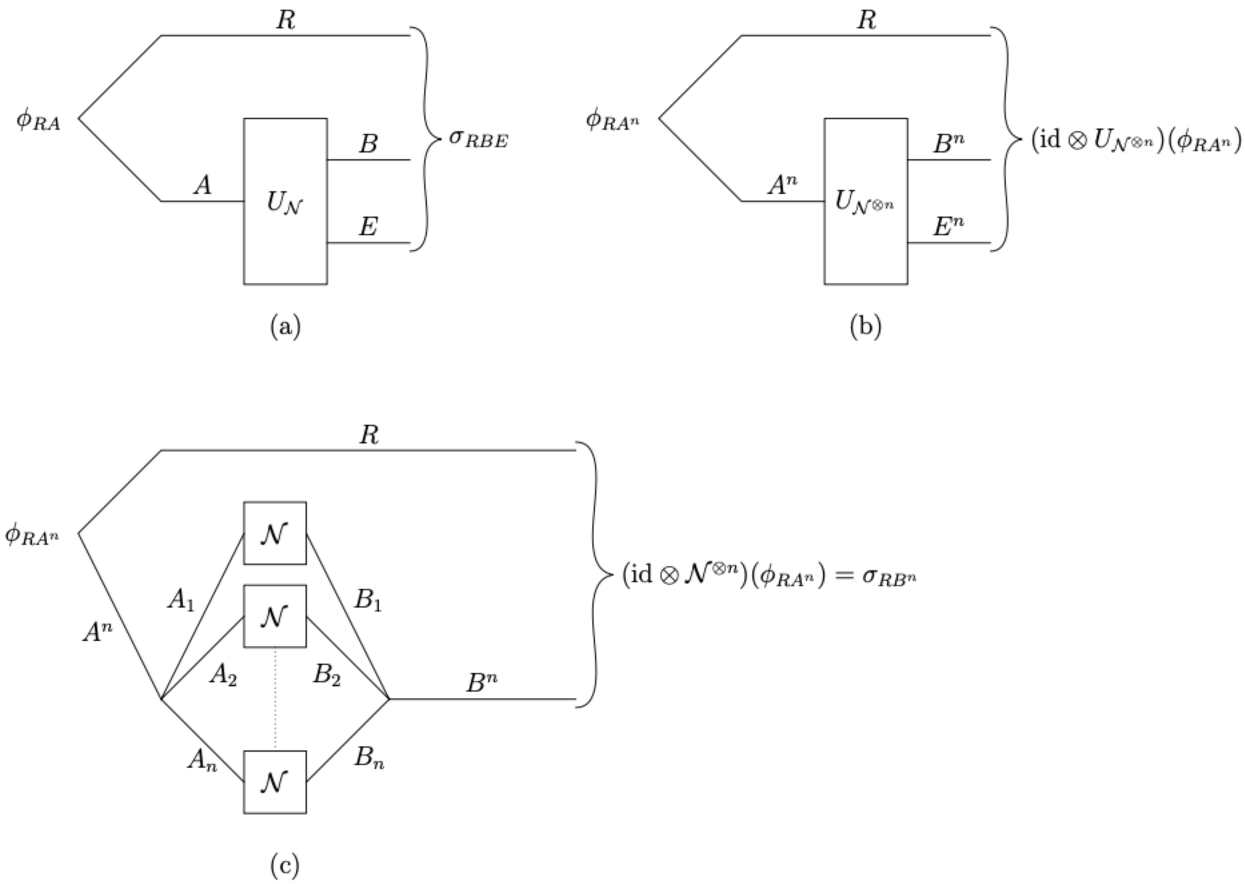

FIG. 1. Source:[1]

B. Black holes as quantum transmission channels

In the study of black holes, the concept of a quantum channel in the presence of these celestial bodies has been explored by previously by researchers Hayden and Preskill[4]. Two observers, Alice (the sender of quantum information) and Bob (the receiver), are considered in this scenario. Alice sends her quantum state into a Schwarzschild black hole, and Bob attempts to reconstitute the quantum states sent by Alice with the highest possible accuracy using a figure of merit known as quantum fidelity. If this is achievable, then it would demonstrate that quantum information processing by a black hole is unitary. If it is not possible, then quantum black holes hide quantum information while the black hole horizon is present, but this does not mean there is a violation of unitarity. The true effect of black holes on quantum information can only be determined if we can describe the entangled system after the black hole has fully evaporated.

In previous research[4, 5], it was discovered that the process of thermalization (also known as scrambling in high-energy terminology) appears to resemble a random code construction. This suggests that quantum information thrown into a black hole might be protected automatically. However, we do not necessarily assume that this procedure could preserve quantum information under all circumstances. The same randomization operation that generates a thermal state when applied to a vacuum may not produce a thermal state for an incoming n-photon state. Therefore, it is not immediately clear that randomization is a good encoder, especially in a more realistic, infinite-dimensional Hilbert space that we consider in our research. Therefore, quantum channel is considered for this operation.

Bekenstein[6] speculated that black hole entropy is ensemble average over the many ways black hole can be prepared. Zurek [7]described this as action of super operator which is now called CPTP map that is quantum channel. Therefore, mixed state after evaporation of black hole is not a failure of quantum mechanics.

C. Unruh channel

As there is no accurate description of the inside of a black hole, the process of black hole stimulated emission is thought to be the main physical mechanism accounting for the preservation of quantum information in this environment. Similar reasoning has been used to discuss the spread of conventional knowledge through black holes in [8]. In the absence of backscattering, the Unruh channel—which was the subject of in-depth study in [8] in relation to the Unruh effect is now understood to originate precisely from the dynamics of stimulated emission from an accelerated mirror (for a thorough review, see references [9]) demonstrated that this channel shares characteristics with the one that forms in the limit of a perfectly reflecting black hole.
Let the qubit unruh channel be $\mathcal{N}$ then the channel output as shown in [10] by Bradler *et al.* is given by

$$\mathcal{N} = \otimes_{l=1}^{\infty} p_l \mathcal{N}_l \tag{3}$$

where,

$$p_l = \frac{1}{2}(1-z)^3 l(l+1) z^{l-1} \tag{4}$$

where, $z = \exp(-2\pi\omega/a)$ where, ω is Minkowski frequency[11] and a is the acceleration.

D. Cloning Channels

$\mathcal{N}_l$ in 3 are very well known channels in quantum information called $1 \to l$ cloning channels represented as $Cl_{1,l}$ because for an unknown input qubit l approximate clones are produced. [10]. The quantum cloning machine does not clone quantum states perfectly, it produces mixed states which are approximations of the given input state. It not only produce clones it also produces anti-clones. Anti-clones are defined as the complex conjugates of clones.[12]. A cloning machine that attempts create l copies from n inputs produces n clones and $l-n$ anti-clones. These channels are closely linked with **degradability**.

E. Degradable Channels

The concept of degradability in quantum information theory was introduced by Devetak and Shor [5]. It is a fundamental property of quantum channels that plays a crucial role in the calculation of the quantum capacity, which is the maximum amount of information that can be transmitted reliably through a quantum channel.

Since, quantum channels are represented as linear maps between quantum states. The complementary channel as shown in 5 to a quantum channel is defined by tracing over the output system instead of the reference system. A channel is said to be degradable if there exists another quantum channel that can simulate it perfectly, called the degrading map. In other words, the degrading map transforms the output of the original channel into an equivalent state that is indistinguishable from the output of the original channel.

$$\hat{\mathcal{N}} = \mathcal{D} \circ \mathcal{N} \tag{5}$$

Degradable channels have a unique property that makes them particularly important for quantum communication. The quantum capacity of a degradable channel can be calculated through a computationally tractable problem. This property greatly simplifies the calculation of the quantum capacity and provides a powerful tool for understanding the fundamental limits of quantum communication. Degradable channels have the property of additive coherent information[13] and therefore, $Q(\mathcal{N}) = Q^{(1)}(\mathcal{N})$. From this the quantum information of the channel, given in equation [3] is given by

$$Q(\mathcal{N}) = \frac{(1-z)^3}{2} \sum_{l=1}^{\infty} l(l+1) z^{l-1} \log \frac{l+1}{l} \tag{6}$$

Now, we know the Quantum capacity of a unruh channel. So, we can determine the quantum capacity of Perfectly reflecting and Perfectly absorbing limit of black hole.

III. CONCLUSION

A. Perfectly reflecting black hole

When radiation falls onto a perfectly reflecting black hole, it creates two identical copies of the original quantum state, called clones, and a single anti-clone of the quantum information behind the black hole horizon. The clones can be used to perfectly reconstruct the original quantum state, while the anti-clone behind the horizon cannot be accessed or used to recover the original quantum state.

It is important to note that this phenomenon does not violate the no-cloning theorem in quantum mechanics. The no-cloning theorem states that it is impossible to create an identical copy of an arbitrary unknown quantum state. In this case, the clones are not identical to the original state, but rather a product of the interaction between the radiation and the black hole.

The channel is perfectly reflecting from the inside of the black hole, meaning that any information that falls into the black hole cannot escape back out. The anti-clone behind the horizon is an example of this phenomenon, as it is inaccessible to any observer outside the black hole.

B. Perfectly absorbing black hole

A perfect absorption black hole channel is complementary to a perfect reflection channel. While the latter creates clones and an anti-clone of quantum information, the former absorbs all incoming quantum information, leaving no trace behind. The perfect absorption black hole channel is entanglement breaking, meaning that its capacity to reconstruct quantum information outside of the black hole is zero.

However, the absorbed clone and anti-clone of quantum information can be used to perfectly reconstruct entanglement behind the black hole horizon. This is known as entanglement harvesting, and it is a crucial aspect of quantum communication in the presence of black holes. The absorbed quantum information can be used to reconstruct the entangled state that was originally shared by two distant parties, despite the fact that the information cannot be accessed from outside the black hole.

Entanglement harvesting is a fascinating phenomenon that highlights the intricacies of quantum communication in the presence of black holes. While the perfect absorption black hole channel may seem like a dead end for quantum communication, the absorbed quantum information can actually be used to reconstruct entanglement and enable communication between distant parties.

IV. EXPLANATION OF QUERIES IN THE PRESENTATION

What is Diamond Norm?

Diamond norm is used to determine how far apart two quantum channels are from one another. The term completely bounded trace norm is another name for it. Consider Alice randomly selecting a quantum channel for Bob from 2 channels with probability p and 1-p. Diamond norm is used to determine how well Bob can guess which quantum channel he will receive when given a one shot access to the channel given to him.

Definition[2] The diamond norm is the trace norm of the output of a trivial extension of a linear map, maximized over all possible inputs with trace norm at most one. The diamond norm induces the diamond distance, which in the particular case of completely positive, trace non-increasing maps $\mathcal{E}, \mathcal{F}$ which map n dimensional states to m dimensional states and $\mathbb{1}_n$ maps n dimensional states to n dimensional states is given by

$$d_\diamond(\mathcal{E}, \mathcal{F}) := \|\mathcal{E} - \mathcal{F}\|_\diamond = \max_\rho \|(\mathcal{E} \otimes \mathbb{1}_n)\rho - (\mathcal{F} \otimes \mathbb{1}_n)\rho\|_1 \tag{7}$$

where, the maximization is done over all the density matrices ρ of dimension n^2.

A typo in presentation about perfectly unitary operation. Correct statement is written below and explained with an example.

Let's say we create a collection of pure states and then perform a unitary operation at random, with a probability selected from a specified probability distribution. The recipient of the quantum state cannot reconstruct the starting quantum state with perfect accuracy if she is unaware of which unitary has been used. Of course, this is a completely unitary operation, and the resulting mixed state can be purified with the help of the proper ancilla (reference or auxiliary state) in a non-unique [14] manner. Suppose we start with a set of three pure qubits, which are initially in the states:

$$|0\rangle, \quad |+\rangle, \quad \text{and} \quad |1\rangle,$$

where $|+\rangle$ is the superposition of $|0\rangle$ and $|1\rangle$, given by:

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

We then apply a random unitary operation to each qubit, with the probability of each unitary given by the distribution:

$$p(U_1) = \frac{1}{2}, \quad p(U_2) = \frac{1}{4}, \quad p(U_3) = \frac{1}{4},$$

where U_1, U_2, and U_3 are three different unitary operations that we can choose arbitrarily.

After applying the random unitary operations, the qubits will be in the mixed states:

$$\begin{aligned}
\rho_1 &= \frac{1}{2}|\psi_1\rangle\langle\psi_1| + \frac{1}{2}|\psi_2\rangle\langle\psi_2| \\
\rho_2 &= \frac{1}{4}|\psi_3\rangle\langle\psi_3| + \frac{3}{4}|\psi_4\rangle\langle\psi_4| \\
\rho_3 &= \frac{1}{4}|\psi_5\rangle\langle\psi_5| + \frac{3}{4}|\psi_6\rangle\langle\psi_6|,
\end{aligned}$$

where $|\psi_i\rangle$ are the states obtained after applying the random unitary operations to the initial states, and $\langle\psi_i|$ represents the complex conjugate transpose of $|\psi_i\rangle$.

Now, suppose the receiver does not know which unitary operation was applied to each qubit. She can use an ancillary qubit in the state $|0\rangle$ to purify the mixed states. To do this, she applies a controlled unitary operation to each mixed state, using the ancilla as the control qubit and the mixed state as the target qubit. This has the effect of entangling the mixed state with the ancillary qubit, and results in a pure state of the form:

$$|\Psi\rangle = a_1|\psi_1\rangle \otimes |0\rangle + a_2|\psi_2\rangle \otimes |0\rangle + a_3|\psi_3\rangle \otimes |1\rangle + a_4|\psi_4\rangle \otimes |1\rangle + a_5|\psi_5\rangle \otimes |1\rangle + a_6|\psi_6\rangle \otimes |1\rangle$$

where the coefficients a_1 to a_6 are complex numbers that depend on the probabilities of the different unitary operations.

The receiver can then measure the ancillary qubit in the computational basis to obtain one of two possible outcomes (0 or 1), and depending on the outcome, she can recover the corresponding pure state with high fidelity. While the process of purification results in a unique pure state in this case, it is important to note that this is not always the case, and there can be multiple ways to purify a given mixed state.

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