A unified picture of Balance puzzles and Group testing

Amrit Putcha, Siddhartha Chaganti

Indian Institute of Technology Madras

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Testing (or querying) is a process to reveal or enhance a particular hidden feature of a specimen.

Chennai: Six students test Covid-19 positive at Anna University

At least six students have tested positive for Covid-19 at Anna University in Chennai. More students, faculty and staff will undergo Covid-19 test to detect and stop the spread of coronavirus.

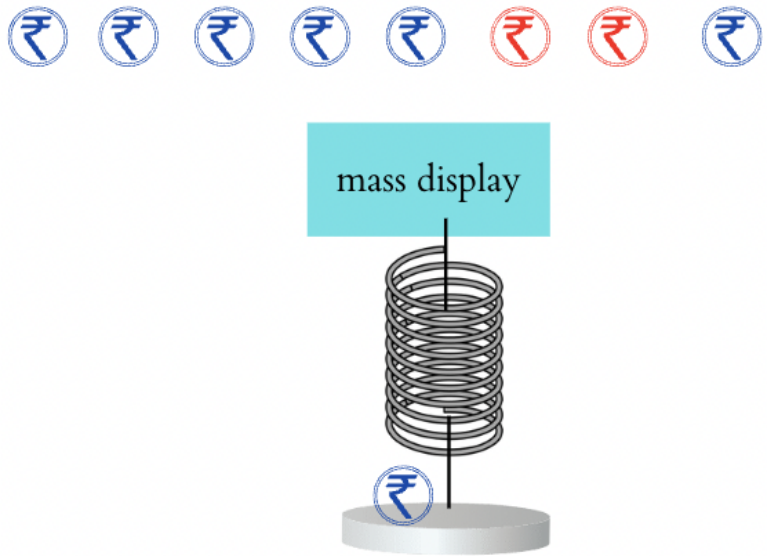
India Today / 24 May

Fake Indian Coins and Currencies - Part 2 Definitive Coin Fakes

A e.Book [Fake Coins of Republic India] of all the items is available as PDF in download section. This article is on Fake Indian C...

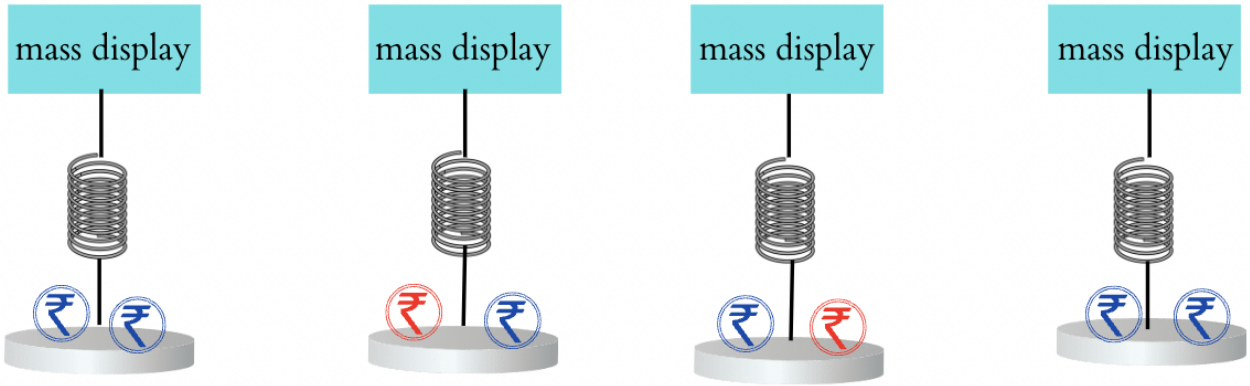
blogspot.com

Naive way would be to weigh each specimen individually. But we can do better.



Group Testing

1



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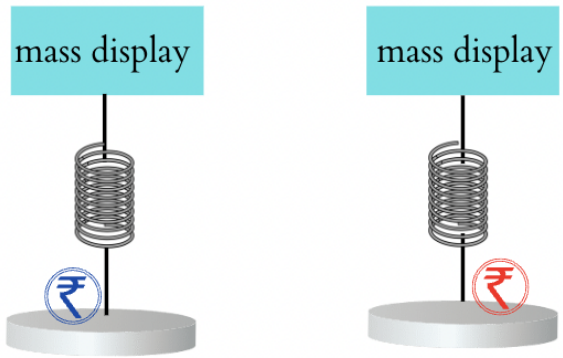
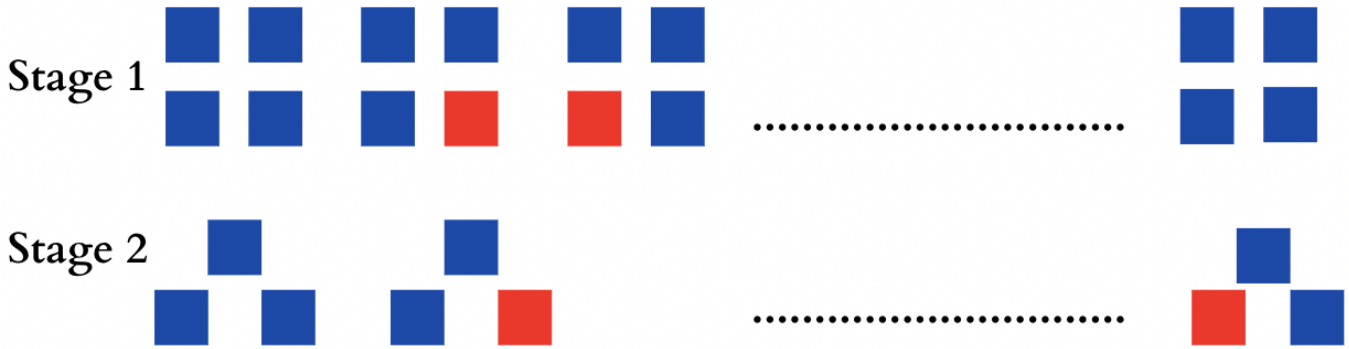


Li's S stage algorithm

- For S=1, the number of queries required are N, where N is the total number of specimens.
- For S=2, g1 groups of size k1 in stage 1 and g2 groups of size k2 in stage 2 and so on.



The Math

Calculations show that for S=2, the tests in the worst case scenario come out to be

$$t_2 = g_2 + g_1 \leq \frac{n}{k_1} + dk_1$$

Upper bound is minimized by $k_1 = \sqrt{\frac{n}{d}}$

$$\text{This gives us } g_1 = \sqrt{nd} \& t_2 \leq 2\sqrt{nd}$$

Similar calculations are performed for S stages and the maximum number of tests comes out to be

$$t_s \leq sd\left(\frac{n}{d}\right)^{\frac{1}{s}} \leq ed\ln\left(\frac{n}{d}\right)$$

Cases of Concern

The pooling strategy discussed may not be optimal when the prevalence rate 'p' is above certain threshold. p = d/N should be less than 1.

Thus for S=2, p should be less than 0.25, otherwise the number of tests become more than the population size, and the activity becomes useless.

Similarly, a threshold value for the prevalence rate can be calculated for S stages and it comes out to satisfy

$$-p\ln(p) \geq \frac{1}{e}$$

Problem Statement

N similar specimens from suspected patients and a test for the presence of the infection, d are infected.

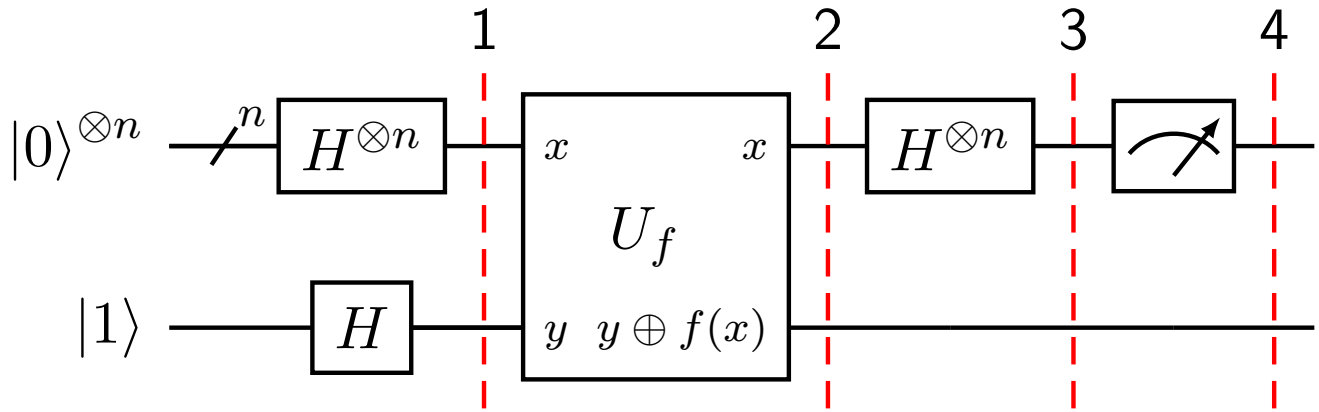
What is the maximum tests required by the Li's pooling strategy considering 'g' pools of size 'k' ?

Deutsch-Josza Algorithm

Given a boolean function f, which takes as input a string of bits, and returns either 0 or 1,

$$f : \{x_1, x_2 \dots\} \rightarrow 0 \text{ or } 1, \text{ where } x_n \text{ is 0 or 1.}$$

The property of the given Boolean function is that it is guaranteed to either be balanced or constant. Our task is to determine whether the given function is balanced or constant.

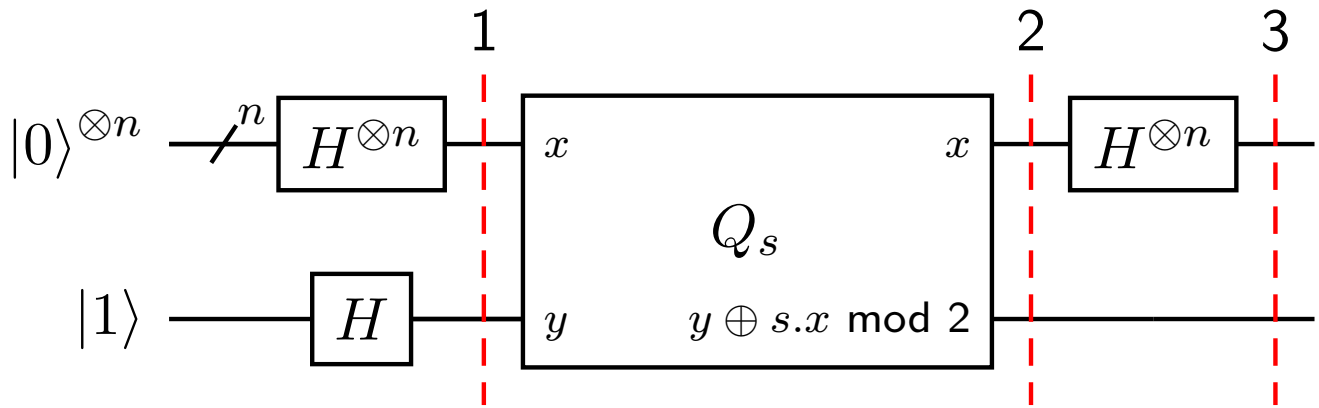


Bernstein-Vazirani Algorithm

Given a boolean function f, which takes as input a string of bits, and returns either 0 or 1,

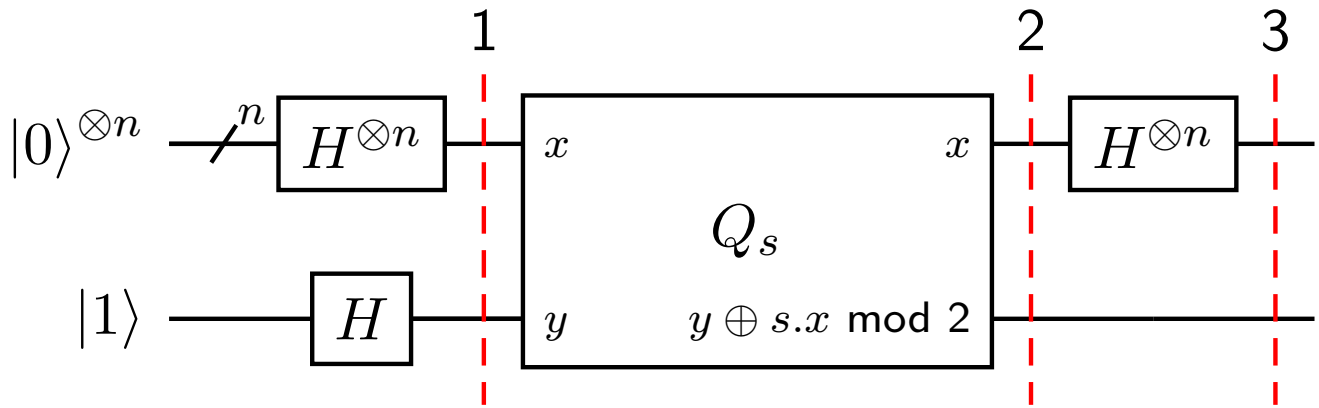
$$f : \{x_1, x_2 \dots\} \rightarrow 0 \text{ or } 1, \text{ where } x_n \text{ is 0 or 1.}$$

the function is guaranteed to return the bitwise product of the input with some string,s . In other words, given an input x, $f(x) = s.x \mod 2$. We are expected to find s.



Quantum Solution to Bernstein-Vazirani Algorithm

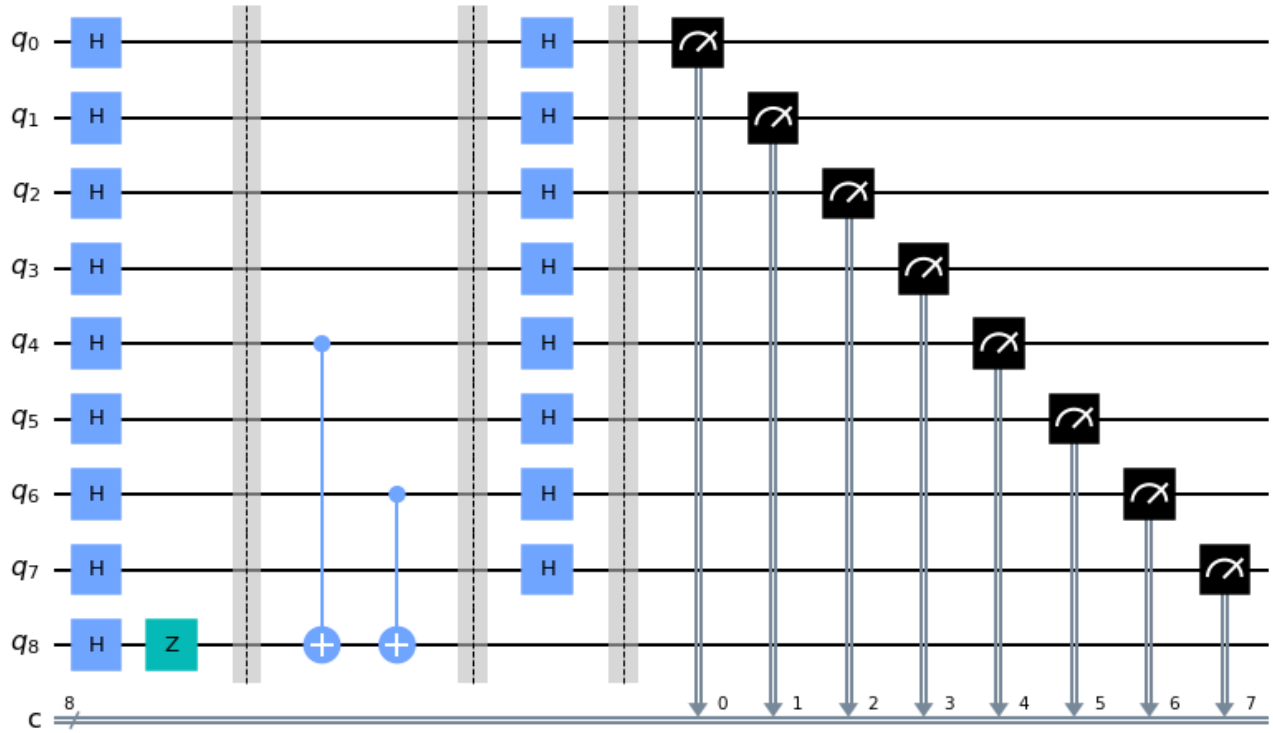
1. Initialize the qubits to the $|0\rangle^{\otimes n}$ state, and the output qubit to $|-\rangle$ state.
2. Apply Hadamard gates to input register.
3. Query the oracle.
4. Apply the Hadamard gates to the input register.
5. Measure.



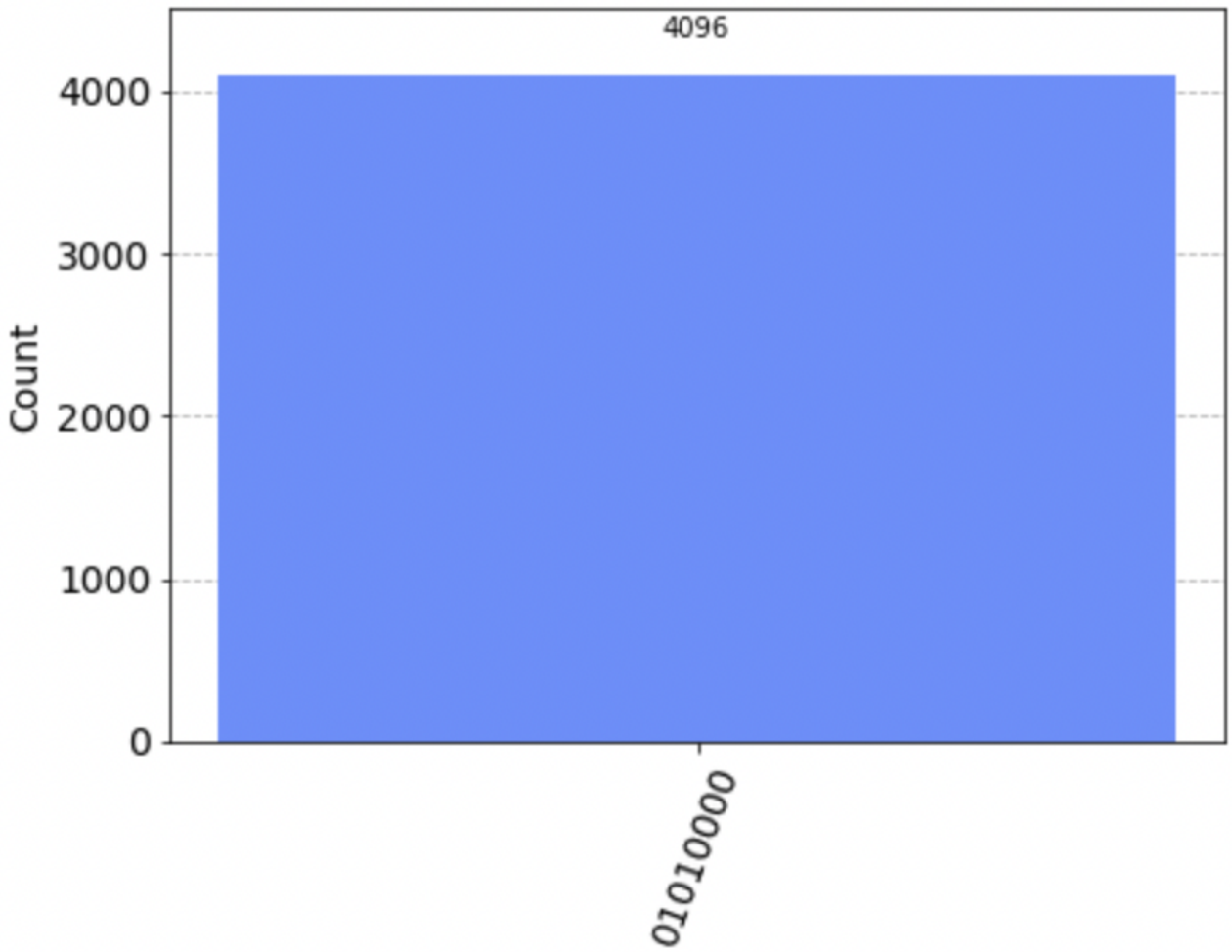
RTPCR/ Counterfeit Coin problem as Bernstein Vazirani problem

- The word sample is used to represent RT-PCR sample and Coin.
- If a sample is tested then we represent the qubit respective to it as $|1\rangle$ else $|0\rangle$.
- The oracle contains wether the sample is different from others or not. (which is unknown to us.)
- By above method we can clearly convert our problem of sample testing to Bernstein Vazirani problem.

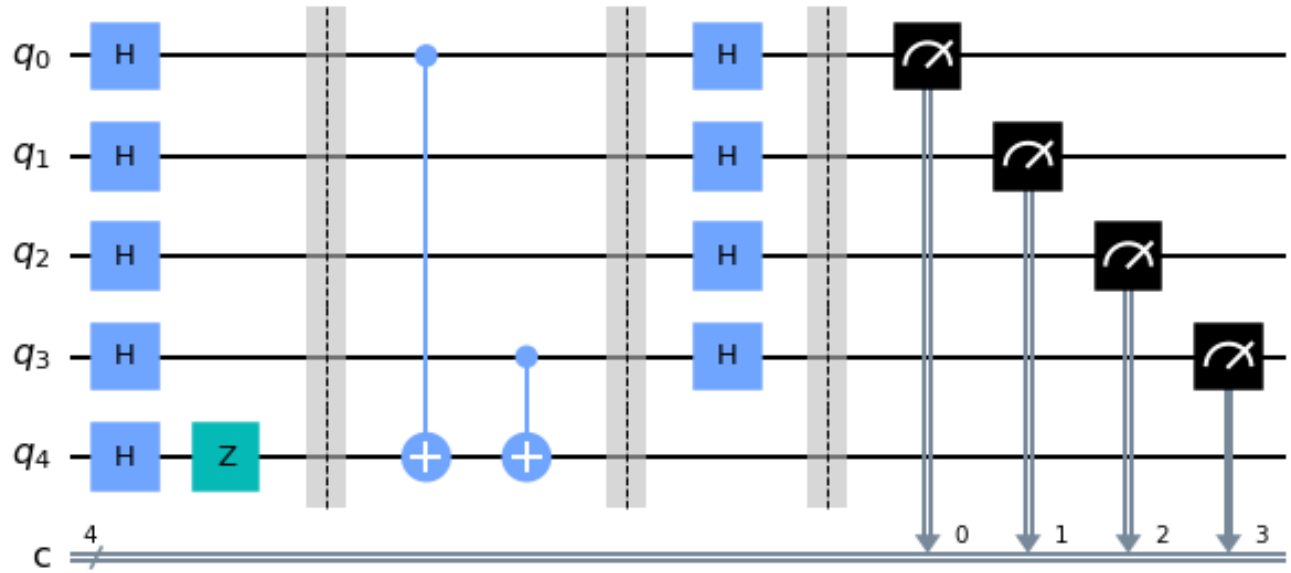
Circuit for Sample testing with sample size 8



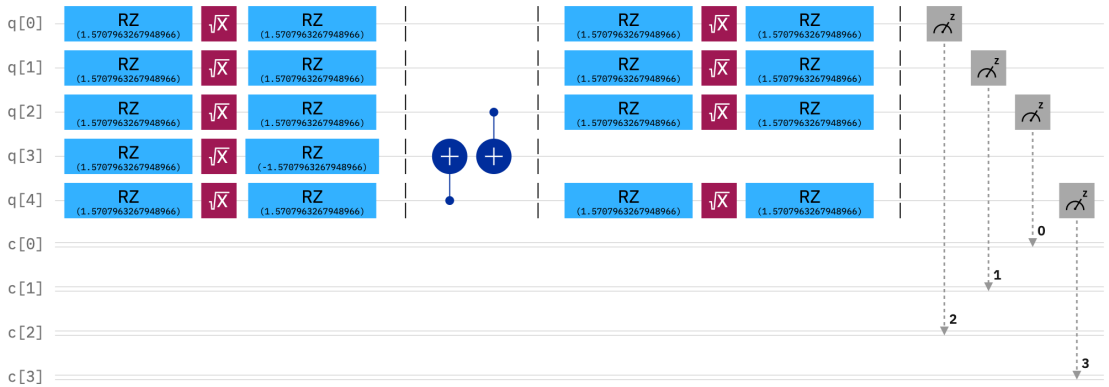
Result on qsam simulator for sample size of 8



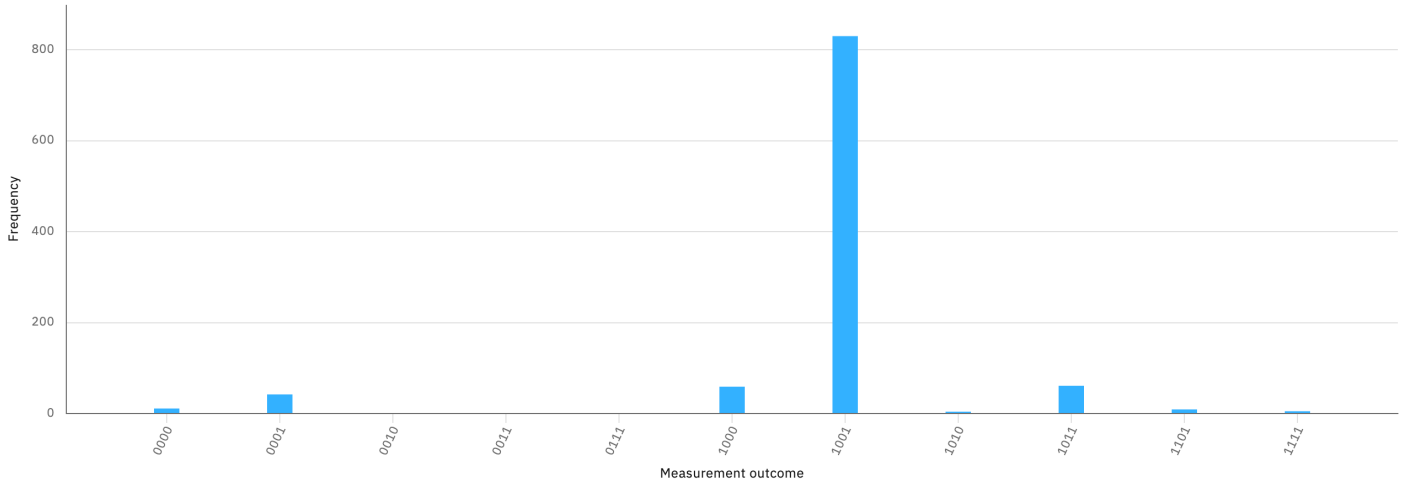
Circuit for Sample testing with sample size 5



Transpiled circuit



Result ibmq machine for sample size 5



Future scope

In the past Iwama et.al4 has considered solving the balance puzzle using quantum mechanics. They have aptly named it the "quantum counterfeit coin" puzzle. They however in their article consider a beam balance rather than a spring balance. Due to this they consider a query trit (ternary bit) string rather than a query bit string. A single trit is considered to be of a value either '-1','0' or '1'. -1 representing placing the coin on left pan, 1 representing placing on the right pan and 0 placing it nowhere. Moreover, the Hamming weight of the query string in their case has to be even (i.e. the number of trits in the trit string different than 0 has to be even).

References

- A unified picture of Balance puzzles and Group testing : Some lessons from quantum mechanics for the pandemic. Chetan Waghela.
- Kazuo Iwama, Harumichi Nishimura, Rudy Raymond, and Junichi Teruyama
  Quantum Counterfeit Coin Problems.
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