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$$H_M = \sum_{u \in V} X_u \prod_{w \in N(u)} W_w$$

$$H_p = -\sum_u Z_u$$

starting state: $|1^{\otimes n}\rangle$

one-application of unitary.

$$|\beta,\gamma\rangle = e^{-i\beta H_M} e^{-i\gamma H_p} |1^{\otimes n}\rangle$$

Expectation value of H_p = cost

$$\langle \beta,\gamma | H_p | \beta,\gamma\rangle = \langle 1^{\otimes n}| e^{i\gamma H_p} e^{i\beta H_M} H_p e^{-i\beta H_M} e^{-i\gamma H_p} |1^{\otimes n}\rangle$$

$$\boxed{\because H_p |1^{\otimes n}\rangle = n|1^{\otimes n}\rangle \Rightarrow e^{-i\gamma H_p}|1^{\otimes n}\rangle = e^{-i\gamma n}|1^{\otimes n}\rangle \Rightarrow \langle 1^{\otimes n}| e^{i\gamma H_p} = \langle 1^{\otimes n}| e^{i\gamma n}}$$

$$\langle \beta,\gamma | H_p | \beta,\gamma\rangle = \langle 1^{\otimes n}| \underbrace{\cancel{e^{i\gamma n}}}_{\text{number}} \underbrace{e^{i\beta H_M} H_p e^{-i\beta H_M}}_{\text{operators}} \underbrace{\cancel{e^{-i\gamma n}}}_{\text{number}} |1^{\otimes n}\rangle$$

$$F(\beta) = \langle 1^{\otimes n}| e^{i\beta H_M} H_p e^{-i\beta H_M} |1^{\otimes n}\rangle$$

$$\boxed{\because e^{-i\beta H_M} = I - i\beta H_M}$$

$$e^{+i\beta H_M} H_p e^{-i\beta H_M} = (I + i\beta H_M) H_p (I - i\beta H_M)$$

$$= H_p + i\beta(H_M H_p - H_p H_M) + \beta^2 H_M H_p H_M$$

$$\boxed{F(\beta) = \langle 1^{\otimes 3}| H_p |1^{\otimes 3}\rangle + i\beta \langle 1^{\otimes 3}| H_M H_p - H_p H_M |1^{\otimes 3}\rangle + \beta^2 \langle 1^{\otimes 3}| H_M H_p H_M |1^{\otimes 3}\rangle}$$