# Discrete Mathematics #2

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# 1.6

## Problem 14

### A

p(x): x is a student in this class

q(x): x owns a red convertible

r(x): x has gotten at least one speeding ticket

| | |
|---|---|
| 1. p(Linda) | **premise** |
| 2. q(Linda) | **premise** |
| 3. p(Linda) $\wedge$ q(Linda) | **conjunction (1) (2)** |
| 4. $\forall x(q(x) \rightarrow r(x))$ | **premise** |
| 5. q(Linda) $\rightarrow$ r(Linda) | **universal instantiation (4)** |
| 6. r(Linda) | **modus ponens (2) (5)** |
| 7. p(Linda) $\wedge$ r(Linda) | **conjunction (1) (6)** |
| 8. $\exists x(p(x) \wedge r(x))$ | **existential generalization (7)** |

### B

p(x): x has taken a course in discrete mathematics

q(x): x can take a course in algorithms

| | |
|---|---|
| 1. p(5 roommates) | **premise** |
| 2. $\forall x(p(x)\text{->}q(x))$ | **premise** |
| 3. p(5 roommates)->q(5 roommates) | **universal instantiation (2)** |
| 4. q(5 roommates) | **Modus Ponens (1) (3)** |

## C

p(x): x is a movie produced by John Sayles

q(x): x is wonderful

| | |
|---|---|
| 1. ∀x(p(x)->q(x)) | **premise** |
| 2. p(coal miners)->q(coal miners) | **universal instantiation (1)** |
| 3. p(coal miners) | **premise** |
| 4. q(coal miners) | **Modus Ponens (2) (3)** |

## D

p(x): x is a student in this class

q(x): x has been to France

r(x): x has visited the Louvre

| | |
|---|---|
| 1. ∃x(p(x)∧q(x)) | **premise** |
| 2. ∀x(p(x)->r(x)) | **premise** |
| 3. p(c)∧q(c) | **Existential instantiation (1)** |
| 4. p(c)->r(x) | **Universal instantiation (2)** |
| 5. p(c) | **Simplification (3)** |
| 6. r(c) | **Modus Ponens (4) (5)** |

## Problem 16

### A

p(x): x is enrolled in the university

q(x): x has lived in a dormitory

| | |
|---|---|
| 1. $\forall x(p(x) \rightarrow q(x))$ | **premise** |
| 2. p(Mia) -> q(mia) | **universal instantiation (1)** |
| 3. ¬q(Mia) | **premise** |
| 4. ¬p(Mia) | **modus tollens (2) (3)** |

**∴ 'Mia is not enrolled in the university' is correct**

### B

p(x): x is a convertible car

q(x): x is fun to drive

| | |
|---|---|
| 1. $\forall x(p(x) \rightarrow q(x))$ | **premise** |
| 2. p(Issac) -> q(Isaac) | **universal instantiation (1)** |
| 3. ¬p(Issac) | **premise** |

Issac's car can be fun to drive even if it is not a convertible.

**∴ 'Isaac's car is not fun to drive' is incorrect**

### C

p(x): x is an action movie

q(x): Quincy likes x

1. $\forall x(p(x) \to q(x))$ **premise**

2. p(Eight Men Out) -> q(Eight Men Out) **universal instantiation (1)**

3. q(Eight Men Out) **premise**

Eight Men Out Can not be an action movie even if Quincy likes

∴ **'Eight Men Out is an action movie' is incorrect**'

**D**

p(x): x is a lobsterman

q(x): x sets at least a dozen traps

1. $\forall x(p(x) \to q(x))$ **premise**

2. p(Hamilton) -> q(Hamilton) **universal instantiation (1)**

3. p(Hamilton) **premise**

4. q(Hamilton) **Modus Ponens (2) (3)**

∴ **'Hamilton sets at least a dozen traps' is correct**

# 1.7

## Problem 4

even number: $2n, n \in \mathbb{Z}$

negative of an even number: $-2n$

$-2n = 2(-n), -n \in \mathbb{Z}$

$\therefore$ **The negative of an even number is an even number.**

## Problem 14

$x = \frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$ and because $x \neq 0, p \neq 0$

$\frac{1}{x} = \frac{q}{p}$ where $q, p \in \mathbb{Z}$ and $p \neq 0$

$\therefore \frac{1}{x}$ **is rational.**

## Problem 18

If $m$ and $n$ are odd, then $mn$ is odd where $m, n \in \mathbb{Z}$

$m = 2p + 1$

$n = 2q + 1$

$mn = (2p + 1)(2q + 1) = 4pq + 2p + 2q + 1 = 2(2pq + p + q) + 1$

$t = 2pq + p + q$

$mn = 2t + 1$

$\therefore mn$ is odd

**By contraposition, If $mn$ is even, then $m$ or $n$ is even.**

## Problem 22

n is not 0, so just devide by n.

$$P(n) : n^2 \geq n$$

$$: n \geq 1$$

$$P(1) : 1 \geq 1$$

**∴ P(1) is true**

# 1.8

## Problem 3

$P(n) : n^3 = 100, n \in \mathbb{Z}, n > 0$

$P(1) : 1^3 = 1$

$P(2) : 2^3 = 8$

$P(3) : 3^3 = 27$

$P(4) : 4^3 = 64$

$P(5) : 5^3 = 125$

100 is between 64 and 125 which is P(4) and P(5), but there is no integer that is between 4 and 5

**∴ 100 is not the cube of a positive integer.**

## Problem 25

- Harmonic Mean: $H = \frac{2xy}{(x+y)}$
- Geometric Mean: $G = \sqrt{xy}$

1. $x = 2, y = 8$

   $H = \frac{32}{10} = 3.2$

   $G = \sqrt{16} = 4$

2. $x = 3, y = 12$

   $H = \frac{72}{15} = 4.8$

   $G = \sqrt{36} = 6$

3. $x = 5, y = 20$

   $H = \frac{200}{25} = 8$

   $G = \sqrt{100} = 10$

Conjecture: $G > H$

Proof:

$(\sqrt{x} - \sqrt{y})^2 > 0$ for all distinct positive real numbers

$x - 2\sqrt{xy} + y > 0$

$x + y > 2\sqrt{xy}$

$1 > \frac{2\sqrt{xy}}{x+y}$

$\sqrt{xy} > \frac{2xy}{x+y}$

$G > H$

$\therefore G > H$