

# Discrete Mathematics #2

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# 1.6

## Problem 14

### A

$p(x)$ : x is a student in this class

$q(x)$ : x owns a red convertible

$r(x)$ : x has gotten at least one speeding ticket

- |  |                                       |
|--|---------------------------------------|
| 1. $p(\text{Linda})$                             | <b>premise</b>                        |
| 2. $q(\text{Linda})$                             | <b>premise</b>                        |
| 3. $p(\text{Linda}) \wedge q(\text{Linda})$      | <b>conjunction (1) (2)</b>            |
| 4. $\forall x(q(x) \rightarrow r(x))$            | <b>premise</b>                        |
| 5. $q(\text{Linda}) \rightarrow r(\text{Linda})$ | <b>universal instantiation (4)</b>    |
| 6. $r(\text{Linda})$                             | <b>modus ponens (2) (5)</b>           |
| 7. $p(\text{Linda}) \wedge r(\text{Linda})$      | <b>conjunction (1) (6)</b>            |
| 8. $\exists x(p(x) \wedge r(x))$                 | <b>existential generalization (7)</b> |

### B

$p(x)$ : x has taken a course in discrete mathematics

$q(x)$ : x can take a course in algorithms

- |  |                                    |
|--|------------------------------------|
| 1. $p(5 \text{ roommates})$                                    | <b>premise</b>                     |
| 2. $\forall x(p(x) \rightarrow q(x))$                          | <b>premise</b>                     |
| 3. $p(5 \text{ roommates}) \rightarrow q(5 \text{ roommates})$ | <b>universal instantiation (2)</b> |
| 4. $q(5 \text{ roommates})$                                    | <b>Modus Ponens (1) (3)</b>        |

## C

$p(x)$ : x is a movie produced by John Sayles

$q(x)$ : x is wonderful

- |  |                                    |
|--|------------------------------------|
| 1. $\forall x(p(x) \rightarrow q(x))$                        | <b>premise</b>                     |
| 2. $p(\text{coal miners}) \rightarrow q(\text{coal miners})$ | <b>universal instantiation (1)</b> |
| 3. $p(\text{coal miners})$                                   | <b>premise</b>                     |
| 4. $q(\text{coal miners})$                                   | <b>Modus Ponens (2) (3)</b>        |

## D

$p(x)$ : x is a student in this class

$q(x)$ : x has been to France

$r(x)$ : x has visited the Louvre

- |                                       |                                      |
|---------------------------------------|--------------------------------------|
| 1. $\exists x(p(x) \wedge q(x))$      | <b>premise</b>                       |
| 2. $\forall x(p(x) \rightarrow r(x))$ | <b>premise</b>                       |
| 3. $p(c) \wedge q(c)$                 | <b>Existential instantiation (1)</b> |
| 4. $p(c) \rightarrow r(c)$            | <b>Universal instantiation (2)</b>   |
| 5. $p(c)$                             | <b>Simplification (3)</b>            |
| 6. $r(c)$                             | <b>Modus Ponens (4) (5)</b>          |

## Problem 16

### A

$p(x)$ : x is enrolled in the university

$q(x)$ : x has lived in a dormitory

- |  |                             |
|--|-----------------------------|
| 1. $\forall x(p(x) \rightarrow q(x))$        | premise                     |
| 2. $p(\text{Mia}) \rightarrow q(\text{mia})$ | universal instantiation (1) |
| 3. $\neg q(\text{Mia})$                      | premise                     |
| 4. $\neg p(\text{Mia})$                      | modus tollens (2) (3)       |
- $\therefore$  'Mia is not enrolled in the university' is correct**

### B

$p(x)$ : x is a convertible car

$q(x)$ : x is fun to drive

- |  |                             |
|--|-----------------------------|
| 1. $\forall x(p(x) \rightarrow q(x))$            | premise                     |
| 2. $p(\text{Issac}) \rightarrow q(\text{Isaac})$ | universal instantiation (1) |
| 3. $\neg p(\text{Issac})$                        | premise                     |

Issac's car can be fun to drive even if it is not a convertible.

**$\therefore$  'Isaac's car is not fun to drive' is incorrect**

### C

$p(x)$ : x is an action movie

$q(x)$ : Quincy likes x

- |  |                                    |
|--|------------------------------------|
| 1. $\forall x(p(x) \rightarrow q(x))$                            | <b>premise</b>                     |
| 2. $p(\text{Eight Men Out}) \rightarrow q(\text{Eight Men Out})$ | <b>universal instantiation (1)</b> |
| 3. $q(\text{Eight Men Out})$                                     | <b>premise</b>                     |

Eight Men Out Can not be an action movie even if Quincy likes

**$\therefore$  'Eight Men Out is an action movie' is incorrect**

## **D**

$p(x)$ : x is a lobsterman

$q(x)$ : x sets at least a dozen traps

- |  |                                    |
|--|------------------------------------|
| 1. $\forall x(p(x) \rightarrow q(x))$                  | <b>premise</b>                     |
| 2. $p(\text{Hamilton}) \rightarrow q(\text{Hamilton})$ | <b>universal instantiation (1)</b> |
| 3. $p(\text{Hamilton})$                                | <b>premise</b>                     |
| 4. $q(\text{Hamilton})$                                | <b>Modus Ponens (2) (3)</b>        |

**$\therefore$  'Hamilton sets at least a dozen traps' is correct**

## 1.7

### Problem 4

even number:  $2n, n \in \mathbb{Z}$

negative of an even number:  $-2n$

$$-2n = 2(-n), -n \in \mathbb{Z}$$

**$\therefore$  The negative of an even number is an even number.**

## Problem 14

$x = \frac{p}{q}$  where  $p, q \in \mathbb{Z}$  and  $q \neq 0$  and because  $x \neq 0, p \neq 0$

$\frac{1}{x} = \frac{q}{p}$  where  $q, p \in \mathbb{Z}$  and  $p \neq 0$

$\therefore \frac{1}{x}$  is rational.

## Problem 18

If  $m$  and  $n$  are odd, then  $mn$  is odd where  $m, n \in \mathbb{Z}$

$$m = 2p + 1$$

$$n = 2q + 1$$

$$mn = (2p + 1)(2q + 1) = 4pq + 2p + 2q + 1 = 2(2pq + p + q) + 1$$

$$t = 2pq + p + q$$

$$mn = 2t + 1$$

$\therefore mn$  is odd

**By contraposition, If  $mn$  is even, then  $m$  or  $n$  is even.**



## Problem 22

$n$  is not 0, so just divide by  $n$ .

$$P(n) : n^2 \geq n$$

$$: n \geq 1$$

$$P(1) : 1 \geq 1$$

**$\therefore P(1)$  is true**

## 1.8

### Problem 3

$$P(n) : n^3 = 100, n \in \mathbb{Z}, n > 0$$

$$P(1) : 1^3 = 1$$

$$P(2) : 2^3 = 8$$

$$P(3) : 3^3 = 27$$

$$P(4) : 4^3 = 64$$

$$P(5) : 5^3 = 125$$

100 is between 64 and 125 which is  $P(4)$  and  $P(5)$ , but there is no integer that is between 4 and 5

**$\therefore$  100 is not the cube of a positive integer.**

## Problem 25

- Harmonic Mean:  $H = \frac{2xy}{(x+y)}$
- Geometric Mean:  $G = \sqrt{xy}$

1.  $x = 2, y = 8$

$$H = \frac{32}{10} = 3.2$$

$$G = \sqrt{16} = 4$$

2.  $x = 3, y = 12$

$$H = \frac{72}{15} = 4.8$$

$$G = \sqrt{36} = 6$$

3.  $x = 5, y = 20$

$$H = \frac{200}{25} = 8$$

$$G = \sqrt{100} = 10$$

Conjecture:  $G > H$

Proof:

$(\sqrt{x} - \sqrt{y})^2 > 0$  for all distinct positive real numbers

$$x - 2\sqrt{xy} + y > 0$$

$$x + y > 2\sqrt{xy}$$

$$1 > \frac{2\sqrt{xy}}{x+y}$$

$$\sqrt{xy} > \frac{2xy}{x+y}$$

$$G > H$$

$$\therefore G > H$$