기공수 과제 #4

20192208 김형훈

2024-10-14

4장 1절

8번 문제

$f'(x) = 1 + \cos x$

$f''(x) = -\sin x$

$f'(x) >= 0$, 극값이 없음.

kπ 에서 변곡점. (k ∈ Z)

11번 문제

$y = \cos x\sqrt{\sin x}$**의 최댓값과 최솟값을 구하여라.**

$y' = -\sin x\sqrt{\sin x} + \cos x\frac{1}{2\sqrt{\sin x}}\cos x$

$= \frac{\cos^2 x - 2\sin^2 x}{2\sqrt{\sin x}}$

$= \frac{1-3\sin^2 x}{2\sqrt{\sin x}}$

$\sin^2 x = \frac{1}{3}$

$\sin x = \frac{1}{\sqrt{3}}(\sin x >= 0)$

$\sin x$가 $\frac{1}{\sqrt{3}}$이면 $\cos x$는 $\pm\sqrt{\frac{2}{3}}$

∴ 최댓값: $\sqrt{\frac{2}{3\sqrt{3}}}$**, 최솟값:** $-\sqrt{\frac{2}{3\sqrt{3}}}$

4장2절

2번 문제

1

sin y

= $\sqrt{1-\cos^2 y}$

= $\sqrt{1-(\cos(\cos^{-1}))^2}$

= $\sqrt{1-x^2}$

2

tan y

= $\frac{\sin y}{\cos y}$

= $\frac{\sqrt{1-x^2}}{\cos y}$

= $\frac{\sqrt{1-x^2}}{x}$, $(x \neq 0)$

3

sin $2y$

= $2 \sin y \cos y$

= $2x\sqrt{1-x^2}$

4

cos $2y$

= $2\cos^2 y - 1$

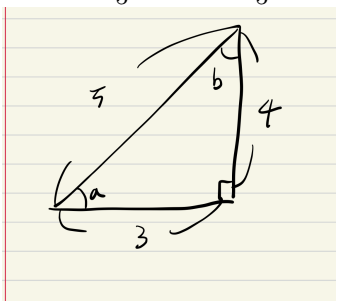
= $2x^2 - 1$

4번 문제

1.

$\cos^{-1}\frac{3}{5}=a, \cos^{-1}\frac{4}{5}=b, 0<=a,b<=\pi$

$\cos a=\frac{3}{5}, \cos b=\frac{4}{5}$



위의 그림과 같이 표현할 수 있다.

$\therefore \frac{\pi}{2}$

2

$-\frac{\pi}{2}<\tan^{-1}x<\frac{\pi}{2}$

$\tan^{-1}2+\tan^{-1}3$

$\tan^{-1}2=\theta_1, \tan^{-1}3=\theta_2, 0<\theta_1+\theta_2<\pi$

$\tan(\theta_1+\theta_2)=\frac{2+3}{1-2*3}=-1$

$\theta_1+\theta_2=\frac{3\pi}{4}$

$\therefore \frac{3\pi}{4}$

3.

$sin\frac{2\pi}{3}=sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

$sin^{-1}(\frac{\sqrt{3}}{2})=\frac{\pi}{3}$

$\therefore \frac{\pi}{3}$

4.

$\tan^{-1}x=\theta$

$\cos(\tan^{-1}x)=\cos\theta, -\frac{\pi}{2}<\theta<\frac{\pi}{2}, \cos\theta>0$

$\cos\theta = \frac{1}{sec\theta} = \frac{1}{\sqrt{1+\tan^2\theta}}$

$= \frac{1}{\sqrt{1+x^2}}$

$\therefore \frac{1}{\sqrt{1+x^2}}$

8번 문제

1.

$\frac{1}{\sqrt{x+1}\sqrt{x}}(\sqrt{x+1})\frac{dy}{dx}$
$= \frac{1}{\sqrt{x+1}\sqrt{x}}\frac{1}{2\sqrt{x+1}}$
$= \frac{1}{2(x+1)\sqrt{x}}$

2.

$\frac{1}{1+(\frac{x}{a})^2}\frac{1}{a} - \frac{1}{1+(\frac{a}{x})^2}\frac{a}{x^2}$
$= \frac{a}{a^2+x^2} - \frac{a}{x^2+a^2}$
= 0

3.

$-\frac{1}{\sqrt{1-(x+1)^2}} = \frac{1}{\sqrt{1-(y-1)^2}}\frac{dy}{dx}$
$\frac{dy}{dx} = -\frac{\sqrt{1-(y-1)^2}}{\sqrt{1-(x+1)^2}}$

4.

$\frac{dy}{dt} = -\frac{1}{1+(\frac{1}{t+1})^2}\frac{1}{(t+1)^2}$
$= -\frac{1}{(t+1)^2+1}$
$1 = \frac{1}{1+(\frac{t}{t+1})^2}\frac{1}{(t+1)^2}\frac{dt}{dx}$
$1 = \frac{1}{(t+1)^2+t^2}\frac{dt}{dx}$
$(t+1)^2 + t^2 = \frac{dt}{dx}$
$\frac{dy}{dx} = -\frac{(t+1)^2+t^2}{(t+1)^2+1}$

4장 3절

2번 문제

1.

$8x \ln e = 1$

$8x = 1$

$x = \frac{1}{8}$

2.

$e^{2x} - 5 = 4e^x$

$e^{2x} - 4e^x - 5 = 0$

$(e^x - 5)(e^x + 1) = 0$

$e^x = 5$

$x = \ln 5$

3.

$1 + 2^x - 2^{2x} = 0$

$2^{2x} - 2^x - 1 = 0$

$2^x = \frac{1+\sqrt{5}}{2}$

$x = \log_2(\frac{1+\sqrt{5}}{2})$

$x = \log_2(1 + \sqrt{5}) - 1$

4.

$3^{2x} - 3^{x+1} - 54 = 0$

$(3^x - 9)(3^x + 6) = 0$

$3^x = 9$

$x = 2$

9번 문제

1.

$f(x) = \ln x$

$f'(x) = \frac{1}{x}$

$f(1.1) \approx f(1) + f'(1)(0.1)$

∴ 0.1

2.

$f(x) = \frac{1}{\ln x} = \ln x$

$f'(x) = -\frac{1}{x}$

$f(e - 0.01) \approx f(e) + f'(e)(-0.01)$

$\therefore 1 - \frac{0.01}{e}$

3.

$f(x) = \sin(\ln x)$

$f'(x) = \cos(\ln x)\frac{1}{x}$

$f(1.001) \approx f(1) + f'(1)(0.001)$

∴ 0.001

4장 4절

6번 문제

$x < 2$

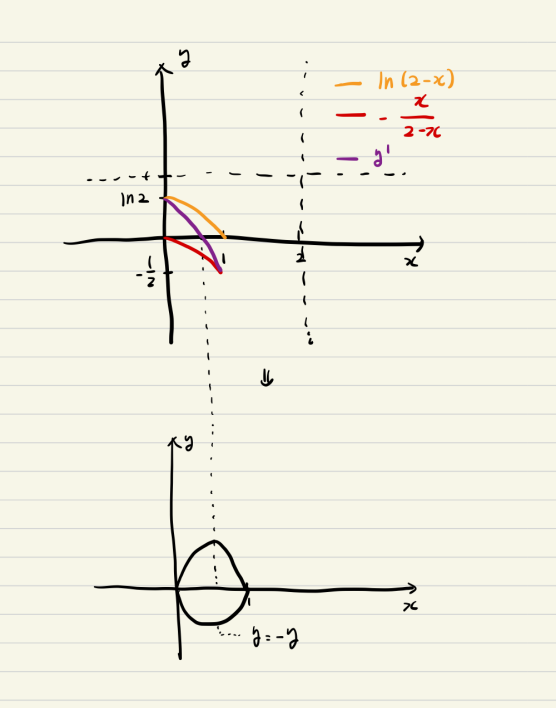
$y^2 >= 0, x\ln(2-x) >= 0$

$$\begin{cases} x \geq 0, \ln(2-x) \geq 0, 2-x \geq 1, \therefore 0 \leq x \leq 1 \\ x < 0, \ln(2-x) < 0, 2-x < 1, \therefore x > 1 \end{cases}$$

$\therefore 0 \leq x \leq 1$

$2yy' = \ln(2-x) - \frac{x}{2-x}$

$y' = \frac{\ln(2-x) - \frac{x}{2-x}}{2y}$



4장5절

8번 문제

1.

$f'(x) = \cosh x + 2\sinh 2x$

2.

$f'(x) = \cosh^2 x + \sinh^2 x$

3.

$f'(x) = 2(\tanh x + sechx)(sech^2 - sechx \tanh x)$

4.

$f'(x) = \cosh(\sin x)\cos x$

5.

$f'(x) = \sinh \sqrt{x}\frac{1}{2\sqrt{x}}$

6.

$f'(x) = 2\sinh \sqrt{1-x^2}\cosh \sqrt{1-x^2}\frac{1}{2\sqrt{1-x^2}}(-2x)$
$= -\sinh \sqrt{1-x^2}\cosh \sqrt{1-x^2}\frac{2x}{\sqrt{1-x^2}}$

7.

$f'(x) = \sinh(\tanh x)sech^2 x$

8.

$f'(x) = -sech(\ln \sqrt{x}) \tanh(\ln \sqrt{x}) \frac{1}{2x}$

4장 6절

5번 문제

1.

$$f'(x) = \frac{1}{\sqrt{x^2+1}}(\cosh^{-1} x) + (\sinh^{-1} x)\frac{1}{\sqrt{x^2-1}}$$

2.

$$f'(x) = \frac{6x^2}{\sqrt{4x^6-1}}$$

3.

$$f'(x) = \frac{\cosh x}{1-\sinh^2 x}$$

4.

$$f'(x) = \frac{1}{2\sqrt{coth^{-1}x^2}}\frac{1}{1-x^4}2x$$

5.

$$f'(x) = \frac{\frac{\sinh^{-1} x}{1-x^2} - \frac{\tanh^{-1} x}{\sqrt{x^2+1}}}{(\sinh^{-1} x)^2}$$

6.

$$f'(x) = \frac{1}{\tanh^{-1} x}\frac{1}{\sqrt{1-x^2}}$$

7.

$$f'(x) = e^{\cosh^{-} 1(2x^3)}\frac{6x^2}{\sqrt{1-4x^6}}$$

8.

$f'(x) = \frac{2x}{(x^2+1)(1-(\ln(x^2+1))^2)}$