

기공수 과제 #4

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4장 1절

8번 문제

$$f'(x) = 1 + \cos x$$

$$f''(x) = -\sin x$$

$$f'(x) \geq 0, \text{ 극값이 없음.}$$

$k\pi$ 에서 변곡점. ($k \in \mathbb{Z}$)

11번 문제

$y = \cos x \sqrt{\sin x}$ 의 최댓값과 최솟값을 구하여라.

$$y' = -\sin x \sqrt{\sin x} + \cos x \frac{1}{2\sqrt{\sin x}} \cos x$$

$$= \frac{\cos^2 x - 2\sin^2 x}{2\sqrt{\sin x}}$$

$$= \frac{1 - 3\sin^2 x}{2\sqrt{\sin x}}$$

$$\sin^2 x = \frac{1}{3}$$

$$\sin x = \frac{1}{\sqrt{3}} (\sin x \geq 0)$$

$$\sin x \text{가 } \frac{1}{\sqrt{3}} \text{이면 } \cos x \text{는 } \pm \sqrt{\frac{2}{3}}$$

$$\therefore \text{최댓값: } \sqrt{\frac{2}{3\sqrt{3}}}, \text{ 최솟값: } -\sqrt{\frac{2}{3\sqrt{3}}}$$

4장2절

2번 문제

1

$$\begin{aligned}\sin y &= \sqrt{1 - \cos^2 y} \\ &= \sqrt{1 - (\cos(\cos^{-1}))^2} \\ &= \sqrt{1 - x^2}\end{aligned}$$

2

$$\begin{aligned}\tan y &= \frac{\sin y}{\cos y} \\ &= \frac{\sqrt{1-x^2}}{\cos y} \\ &= \frac{\sqrt{1-x^2}}{x}, (x \neq 0)\end{aligned}$$

3

$$\begin{aligned}\sin 2y &= 2 \sin y \cos y \\ &= 2x\sqrt{1-x^2}\end{aligned}$$

4

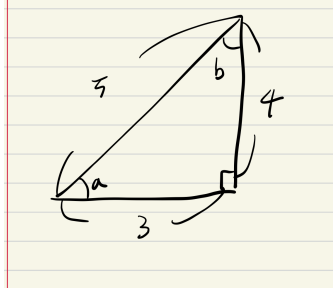
$$\begin{aligned}\cos 2y &= 2 \cos^2 y - 1 \\ &= 2x^2 - 1\end{aligned}$$

4번 문제

1.

$$\cos^{-1} \frac{3}{5} = a, \cos^{-1} \frac{4}{5} = b, 0 < a, b < \pi$$

$$\cos a = \frac{3}{5}, \cos b = \frac{4}{5}$$



위의 그림과 같이 표현할 수 있다.

$$\therefore \frac{\pi}{2}$$

2

$$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$$

$$\tan^{-1} 2 + \tan^{-1} 3$$

$$\tan^{-1} 2 = \theta_1, \tan^{-1} 3 = \theta_2, 0 < \theta_1 + \theta_2 < \pi$$

$$\tan(\theta_1 + \theta_2) = \frac{2+3}{1-2*3} = -1$$

$$\theta_1 + \theta_2 = \frac{3\pi}{4}$$

$$\therefore \frac{3\pi}{4}$$

3.

$$\sin \frac{2\pi}{3} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) = \frac{\pi}{3}$$

$$\therefore \frac{\pi}{3}$$

4.

$$\tan^{-1} x = \theta$$

$$\cos(\tan^{-1} x) = \cos \theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}, \cos \theta > 0$$

$$\begin{aligned}\cos \theta &= \frac{1}{\sec \theta} = \frac{1}{\sqrt{1+\tan^2 \theta}} \\ &= \frac{1}{\sqrt{1+x^2}} \\ &\therefore \frac{1}{\sqrt{1+x^2}}\end{aligned}$$

8번 문제

1.

$$\begin{aligned} & \frac{1}{\sqrt{x+1}\sqrt{x}}(\sqrt{x+1})\frac{dy}{dx} \\ &= \frac{1}{\sqrt{x+1}\sqrt{x}}\frac{1}{2\sqrt{x+1}} \\ &= \frac{1}{2(x+1)\sqrt{x}} \end{aligned}$$

2.

$$\begin{aligned} & \frac{1}{1+(\frac{x}{a})^2}\frac{1}{a} - \frac{1}{1+(\frac{a}{x})^2}\frac{a}{x^2} \\ &= \frac{a}{a^2+x^2} - \frac{a}{x^2+a^2} \\ &= 0 \end{aligned}$$

3.

$$\begin{aligned} & -\frac{1}{\sqrt{1-(x+1)^2}} = \frac{1}{\sqrt{1-(y-1)^2}}\frac{dy}{dx} \\ \frac{dy}{dx} &= -\frac{\sqrt{1-(y-1)^2}}{\sqrt{1-(x+1)^2}} \end{aligned}$$

4.

$$\begin{aligned} \frac{dy}{dt} &= -\frac{1}{1+(\frac{1}{t+1})^2}\frac{1}{(t+1)^2} \\ &= -\frac{1}{(t+1)^2+1} \\ 1 &= \frac{1}{1+(\frac{t}{t+1})^2}\frac{1}{(t+1)^2}\frac{dt}{dx} \\ 1 &= \frac{1}{(t+1)^2+t^2}\frac{dt}{dx} \\ (t+1)^2+t^2 &= \frac{dt}{dx} \\ \frac{dy}{dx} &= -\frac{(t+1)^2+t^2}{(t+1)^2+1} \end{aligned}$$

4장 3절

2번 문제

1.

$$8x \ln e = 1$$

$$8x = 1$$

$$x = \frac{1}{8}$$

2.

$$e^{2x} - 5 = 4e^x$$

$$e^{2x} - 4e^x - 5 = 0$$

$$(e^x - 5)(e^x + 1) = 0$$

$$e^x = 5$$

$$x = \ln 5$$

3.

$$1 + 2^x - 2^{2x} = 0$$

$$2^{2x} - 2^x - 1 = 0$$

$$2^x = \frac{1+\sqrt{5}}{2}$$

$$x = \log_2\left(\frac{1+\sqrt{5}}{2}\right)$$

$$x = \log_2(1 + \sqrt{5}) - 1$$

4.

$$3^{2x} - 3^{x+1} - 54 = 0$$

$$(3^x - 9)(3^x + 6) = 0$$

$$3^x = 9$$

$$x = 2$$

9번 문제

1.

$$f(x) = \ln x$$

$$f'(x) = \frac{1}{x}$$

$$f(1.1) \approx f(1) + f'(1)(0.1)$$

$$\therefore 0.1$$

2.

$$f(x) = \frac{1}{\ln x} = \ln x$$

$$f'(x) = -\frac{1}{x}$$

$$f(e - 0.01) \approx f(e) + f'(e)(-0.01)$$

$$\therefore 1 - \frac{0.01}{e}$$

3.

$$f(x) = \sin(\ln x)$$

$$f'(x) = \cos(\ln x) \frac{1}{x}$$

$$f(1.001) \approx f(1) + f'(1)(0.001)$$

$$\therefore 0.001$$

4장 4절

6번 문제

$$x < 2$$

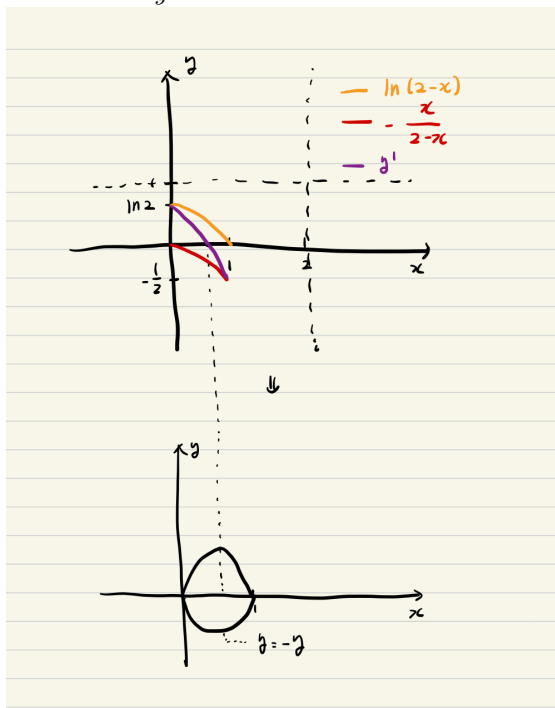
$$y^2 \geq 0, x \ln(2-x) \geq 0$$

$$\begin{cases} x \geq 0, \ln(2-x) \geq 0, 2-x \geq 1, \therefore 0 \leq x \leq 1 \\ x < 0, \ln(2-x) < 0, 2-x < 1, \therefore x > 1 \end{cases}$$

$$\therefore 0 \leq x \leq 1$$

$$2yy' = \ln(2-x) - \frac{x}{2-x}$$

$$y' = \frac{\ln(2-x) - \frac{x}{2-x}}{2y}$$



4장5절

8번 문제

1.

$$f'(x) = \cosh x + 2 \sinh 2x$$

2.

$$f'(x) = \cosh^2 x + \sinh^2 x$$

3.

$$f'(x) = 2(\tanh x + \operatorname{sech} x)(\operatorname{sech}^2 - \operatorname{sech} x \tanh x)$$

4.

$$f'(x) = \cosh(\sin x) \cos x$$

5.

$$f'(x) = \sinh \sqrt{x} \frac{1}{2\sqrt{x}}$$

6.

$$\begin{aligned} f'(x) &= 2 \sinh \sqrt{1-x^2} \cosh \sqrt{1-x^2} \frac{1}{2\sqrt{1-x^2}} (-2x) \\ &= -\sinh \sqrt{1-x^2} \cosh \sqrt{1-x^2} \frac{2x}{\sqrt{1-x^2}} \end{aligned}$$

7.

$$f'(x) = \sinh(\tanh x) \operatorname{sech}^2 x$$

8.

$$f'(x) = -\operatorname{sech}(\ln \sqrt{x}) \tanh(\ln \sqrt{x}) \frac{1}{2x}$$

4장 6절

5번 문제

1.

$$f'(x) = \frac{1}{\sqrt{x^2+1}}(\cosh^{-1} x) + (\sinh^{-1} x) \frac{1}{\sqrt{x^2-1}}$$

2.

$$f'(x) = \frac{6x^2}{\sqrt{4x^6-1}}$$

3.

$$f'(x) = \frac{\cosh x}{1-\sinh^2 x}$$

4.

$$f'(x) = \frac{1}{2\sqrt{\coth^{-1} x^2}} \frac{1}{1-x^4} 2x$$

5.

$$f'(x) = \frac{\frac{\sinh^{-1} x}{1-x^2} - \frac{\tanh^{-1} x}{\sqrt{x^2+1}}}{(\sinh^{-1} x)^2}$$

6.

$$f'(x) = \frac{1}{\tanh^{-1} x} \frac{1}{\sqrt{1-x^2}}$$

7.

$$f'(x) = e^{\cosh^{-1}(2x^3)} \frac{6x^2}{\sqrt{1-4x^6}}$$

8.

$$f'(x) = \frac{2x}{(x^2+1)(1-(\ln(x^2+1))^2)}$$