Discrete Mathematics #2

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1.6

Problem 14

A

p(x): x is a student in this class

q(x): x owns a red convertible

r(x): x has gotten at least one speeding ticket

1. p(Linda)	**premise**
2. q(Linda)	**premise**
3. p(Linda) $\wedge$ q(Linda)	**conjunction (1) (2)**
4. $\forall x(q(x) \rightarrow r(x))$	**premise**
5. q(Linda) $\rightarrow$ r(Linda)	**universal instantiation (4)**
6. r(Linda)	**modus ponens (2) (5)**
7. p(Linda) $\wedge$ r(Linda)	**conjunction (1) (6)**
8. $\exists x(p(x) \wedge r(x))$	**existential generalization (7)**

B

p(x): x has taken a course in discrete mathematics

q(x): x can take a course in algorithms

1. p(5 roommates)	**premise**
2. $\forall$x(p(x)->q(x))	**premise**
3. p(5 roommates)->q(5 roommates)	**universal instantiation (2)**
4. q(5 roommates)	**Modus Ponens (1) (3)**

C

p(x): x is a movie produced by John Sayles

q(x): x is wonderful

1. $\forall x(p(x)\text{->}q(x))$	**premise**
2. p(coal miners)->q(coal miners)	**universal instantiation (1)**
3. p(coal miners)	**premise**
4. q(coal miners)	**Modus Ponens (2) (3)**

D

p(x): x is a student in this class

q(x): x has been to France

r(x): x has visited the Louvre

1. $\exists x(p(x)\wedge q(x))$	**premise**
2. $\forall x(p(x)\text{->}r(x))$	**premise**
3. $p(c)\wedge q(c)$	**Existential instantiation (1)**
4. p(c)->r(x)	**Universal instantiation (2)**
5. p(c)	**Simplification (3)**
6. r(c)	**Modus Ponens (4) (5)**

Problem 16

A

p(x): x is enrolled in the university

q(x): x has lived in a dormitory

1. $\forall x(p(x) \rightarrow q(x))$	**premise**
2. p(Mia) -> q(mia)	**universal instantiation (1)**
3. ¬q(Mia)	**premise**
4. ¬p(Mia)	**modus tollens (2) (3)**

∴ 'Mia is not enrolled in the university' is correct

B

p(x): x is a convertible car

q(x): x is fun to drive

1. $\forall x(p(x) \rightarrow q(x))$	**premise**
2. p(Issac) -> q(Isaac)	**universal instantiation (1)**
3. ¬p(Issac)	**premise**

Issac's car can be fun to drive even if it is not a convertible.

∴ 'Isaac's car is not fun to drive' is incorrect

C

p(x): x is an action movie

q(x): Quincy likes x

1. ∀x(p(x) -> q(x)) **premise**
2. p(Eight Men Out) -> q(Eight Men Out) **universal instantiation (1)**
3. q(Eight Men Out) **premise**

Eight Men Out Can not be an action movie even if Quincy likes

∴ **'Eight Men Out is an action movie' is incorrect**'

D

p(x): x is a lobsterman

q(x): x sets at least a dozen traps

1. ∀x(p(x) -> q(x)) **premise**
2. p(Hamilton) -> q(Hamilton) **universal instantiation (1)**
3. p(Hamilton) **premise**
4. q(Hamilton) **Modus Ponens (2) (3)**

∴ **'Hamilton sets at least a dozen traps' is correct**

1.7

Problem 4

even number: $2n, n \in \mathbb{Z}$

negative of an even number: $-2n$

$-2n = 2(-n), -n \in \mathbb{Z}$

$\therefore$ **The negative of an even number is an even number.**

Problem 14

$x = \frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$ and because $x \neq 0, p \neq 0$

$\frac{1}{x} = \frac{q}{p}$ where $q, p \in \mathbb{Z}$ and $p \neq 0$

$\therefore \frac{1}{x}$ **is rational.**

Problem 18

If m and n are odd, then mn is odd where $m, n \in \mathbb{Z}$

$m = 2p + 1$

$n = 2q + 1$

$mn = (2p + 1)(2q + 1) = 4pq + 2p + 2q + 1 = 2(2pq + p + q) + 1$

$t = 2pq + p + q$

$mn = 2t + 1$

$\therefore mn$ is odd

By contraposition, If mn is even, then m or n is even.

Problem 22

n is not 0, so just devide by n.

$$P(n) : n^2 \geq n$$

$$: n \geq 1$$

$$P(1) : 1 \geq 1$$

∴ P(1) is true

1.8

Problem 3

$P(n) : n^3 = 100, n \in \mathbb{Z}, n > 0$

$P(1) : 1^3 = 1$

$P(2) : 2^3 = 8$

$P(3) : 3^3 = 27$

$P(4) : 4^3 = 64$

$P(5) : 5^3 = 125$

100 is between 64 and 125 which is P(4) and P(5), but there is no integer that is between 4 and 5

∴ 100 is not the cube of a positive integer.

Problem 25

- Harmonic Mean: $H = \frac{2xy}{(x+y)}$
- Geometric Mean: $G = \sqrt{xy}$

1. $x = 2, y = 8$

   $H = \frac{32}{10} = 3.2$

   $G = \sqrt{16} = 4$

2. $x = 3, y = 12$

   $H = \frac{72}{15} = 4.8$

   $G = \sqrt{36} = 6$

3. $x = 5, y = 20$

   $H = \frac{200}{25} = 8$

   $G = \sqrt{100} = 10$

Conjecture: $G > H$

Proof:

$(\sqrt{x} - \sqrt{y})^2 > 0$ for all distinct positive real numbers

$x - 2\sqrt{xy} + y > 0$

$x + y > 2\sqrt{xy}$

$1 > \frac{2\sqrt{xy}}{x+y}$

$\sqrt{xy} > \frac{2xy}{x+y}$

$G > H$

$\therefore G > H$