

Discrete Mathematics #2

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1.6

Problem 14

A

$p(x)$: x is a student in this class

$q(x)$: x owns a red convertible

$r(x)$: x has gotten at least one speeding ticket

- | | |
|--|---------------------------------------|
| 1. $p(\text{Linda})$ | premise |
| 2. $q(\text{Linda})$ | premise |
| 3. $p(\text{Linda}) \wedge q(\text{Linda})$ | conjunction (1) (2) |
| 4. $\forall x(q(x) \rightarrow r(x))$ | premise |
| 5. $q(\text{Linda}) \rightarrow r(\text{Linda})$ | universal instantiation (4) |
| 6. $r(\text{Linda})$ | modus ponens (2) (5) |
| 7. $p(\text{Linda}) \wedge r(\text{Linda})$ | conjunction (1) (6) |
| 8. $\exists x(p(x) \wedge r(x))$ | existential generalization (7) |

B

$p(x)$: x has taken a course in discrete mathematics

$q(x)$: x can take a course in algorithms

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|--|------------------------------------|
| 1. $p(5 \text{ roommates})$ | premise |
| 2. $\forall x(p(x) \rightarrow q(x))$ | premise |
| 3. $p(5 \text{ roommates}) \rightarrow q(5 \text{ roommates})$ | universal instantiation (2) |
| 4. $q(5 \text{ roommates})$ | Modus Ponens (1) (3) |

C

$p(x)$: x is a movie produced by John Sayles

$q(x)$: x is wonderful

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|--|------------------------------------|
| 1. $\forall x(p(x) \rightarrow q(x))$ | premise |
| 2. $p(\text{coal miners}) \rightarrow q(\text{coal miners})$ | universal instantiation (1) |
| 3. $p(\text{coal miners})$ | premise |
| 4. $q(\text{coal miners})$ | Modus Ponens (2) (3) |

D

$p(x)$: x is a student in this class

$q(x)$: x has been to France

$r(x)$: x has visited the Louvre

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|---------------------------------------|--------------------------------------|
| 1. $\exists x(p(x) \wedge q(x))$ | premise |
| 2. $\forall x(p(x) \rightarrow r(x))$ | premise |
| 3. $p(c) \wedge q(c)$ | Existential instantiation (1) |
| 4. $p(c) \rightarrow r(c)$ | Universal instantiation (2) |
| 5. $p(c)$ | Simplification (3) |
| 6. $r(c)$ | Modus Ponens (4) (5) |

Problem 16

A

$p(x)$: x is enrolled in the university

$q(x)$: x has lived in a dormitory

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|--|-----------------------------|
| 1. $\forall x(p(x) \rightarrow q(x))$ | premise |
| 2. $p(\text{Mia}) \rightarrow q(\text{mia})$ | universal instantiation (1) |
| 3. $\neg q(\text{Mia})$ | premise |
| 4. $\neg p(\text{Mia})$ | modus tollens (2) (3) |
- \therefore 'Mia is not enrolled in the university' is correct

B

$p(x)$: x is a convertible car

$q(x)$: x is fun to drive

- | | |
|--|-----------------------------|
| 1. $\forall x(p(x) \rightarrow q(x))$ | premise |
| 2. $p(\text{Issac}) \rightarrow q(\text{Isaac})$ | universal instantiation (1) |
| 3. $\neg p(\text{Issac})$ | premise |

Issac's car can be fun to drive even if it is not a convertible.

\therefore 'Isaac's car is not fun to drive' is incorrect

C

$p(x)$: x is an action movie

$q(x)$: Quincy likes x

- | | |
|--|------------------------------------|
| 1. $\forall x(p(x) \rightarrow q(x))$ | premise |
| 2. $p(\text{Eight Men Out}) \rightarrow q(\text{Eight Men Out})$ | universal instantiation (1) |
| 3. $q(\text{Eight Men Out})$ | premise |

Eight Men Out Can not be an action movie even if Quincy likes

\therefore 'Eight Men Out is an action movie' is incorrect

D

$p(x)$: x is a lobsterman

$q(x)$: x sets at least a dozen traps

- | | |
|--|------------------------------------|
| 1. $\forall x(p(x) \rightarrow q(x))$ | premise |
| 2. $p(\text{Hamilton}) \rightarrow q(\text{Hamilton})$ | universal instantiation (1) |
| 3. $p(\text{Hamilton})$ | premise |
| 4. $q(\text{Hamilton})$ | Modus Ponens (2) (3) |

\therefore 'Hamilton sets at least a dozen traps' is correct

1.7

Problem 4

even number: $2n, n \in \mathbb{Z}$

negative of an even number: $-2n$

$$-2n = 2(-n), -n \in \mathbb{Z}$$

\therefore The negative of an even number is an even number.

Problem 14

$x = \frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$ and because $x \neq 0, p \neq 0$

$\frac{1}{x} = \frac{q}{p}$ where $q, p \in \mathbb{Z}$ and $p \neq 0$

$\therefore \frac{1}{x}$ is rational.

Problem 18

If m and n are odd, then mn is odd where $m, n \in \mathbb{Z}$

$$m = 2p + 1$$

$$n = 2q + 1$$

$$mn = (2p + 1)(2q + 1) = 4pq + 2p + 2q + 1 = 2(2pq + p + q) + 1$$

$$t = 2pq + p + q$$

$$mn = 2t + 1$$

$\therefore mn$ is odd

By contraposition, If mn is even, then m or n is even.

Problem 22

n is not 0, so just divide by n .

$$P(n) : n^2 \geq n$$

$$: n \geq 1$$

$$P(1) : 1 \geq 1$$

$\therefore P(1)$ is true

1.8

Problem 3

$$P(n) : n^3 = 100, n \in \mathbb{Z}, n > 0$$

$$P(1) : 1^3 = 1$$

$$P(2) : 2^3 = 8$$

$$P(3) : 3^3 = 27$$

$$P(4) : 4^3 = 64$$

$$P(5) : 5^3 = 125$$

100 is between 64 and 125 which is $P(4)$ and $P(5)$, but there is no integer that is between 4 and 5

\therefore 100 is not the cube of a positive integer.

Problem 25

- Harmonic Mean: $H = \frac{2xy}{(x+y)}$
- Geometric Mean: $G = \sqrt{xy}$

1. $x = 2, y = 8$

$$H = \frac{32}{10} = 3.2$$

$$G = \sqrt{16} = 4$$

2. $x = 3, y = 12$

$$H = \frac{72}{15} = 4.8$$

$$G = \sqrt{36} = 6$$

3. $x = 5, y = 20$

$$H = \frac{200}{25} = 8$$

$$G = \sqrt{100} = 10$$

Conjecture: $G > H$

Proof:

$(\sqrt{x} - \sqrt{y})^2 > 0$ for all distinct positive real numbers

$$x - 2\sqrt{xy} + y > 0$$

$$x + y > 2\sqrt{xy}$$

$$1 > \frac{2\sqrt{xy}}{x+y}$$

$$\sqrt{xy} > \frac{2xy}{x+y}$$

$$G > H$$

$$\therefore G > H$$