기공수 과제 #4

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4장 1절

8번 문제

$f'(x) = 1 + \cos x$

$f''(x) = -\sin x$

$f'(x) >= 0,$ 극값이 없음.

kπ 에서 변곡점. (k ∈ Z)

11번 문제

$y = \cos x \sqrt{\sin x}$의 최댓값과 최솟값을 구하여라.

$\sin x >= 0$ 이므로 $x \in [0 + 2\pi n, \pi + 2\pi n], n \in \mathbb{n}$

$y' = -\sin x \sqrt{\sin x} + \cos x \frac{1}{2\sqrt{\sin x}} \cos x$

$= \frac{\cos^2 x - 2\sin^2 x}{2\sqrt{\sin x}}$

$= \frac{1 - 3\sin^2 x}{2\sqrt{\sin x}}$

$\sin^2 x = \frac{1}{3}$

$\sin x = \pm \frac{1}{\sqrt{3}}$

여기서 어떻게 함?

4장2절

2번 문제

1

$\sin y$

$= \sqrt{1 - \cos^2 y}$

$= \sqrt{1 - (\cos(\cos^{-1}))^2}$

$= \sqrt{1 - x^2}$

2

$\tan y$

$= \frac{\sin y}{\cos y}$

$= \frac{\sqrt{1-x^2}}{\cos y}$
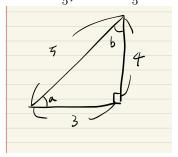
$= \frac{\sqrt{1-x^2}}{x}, (x \neq 0)$

3

$\sin 2y$

$= 2 \sin y \cos y$

$= 2\sqrt{1 - x^2}x$

4

$\cos 2y$

$= 2 \cos^2 y - 1$

$= 2x^2 - 1$

4번 문제

1.

$\cos^{-1}\frac{3}{5} = a, \cos^{-1}\frac{4}{5} = b, 0 <= a, b <= \pi$

$\cos a = \frac{3}{5}, \cos b = \frac{4}{5}$

위의 그림과 같이 표현할 수 있다.

$\therefore \frac{\pi}{2}$

2

$\tan^{-1} 2 + \tan^{-1} 3$

$\tan^{-1} 2 = \theta_1, \tan^{-1} 3 = \theta_2, -\pi < \theta_1 + \theta_2 < \pi$

$\tan(\theta_1 + \theta_2) = \frac{2+3}{1-2*3} = -1$

$\theta_1 + \theta_2 = -\frac{\pi}{4}, \frac{3\pi}{4}$

$\therefore -\frac{\pi}{4}, \frac{3\pi}{4}$

질문: 유튜브에서 $0 < \theta_1 + \theta_2 < \pi$로 알려주신거 같은데, tan^{-1}의 치역은 $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ 아닌가요? 그럼 $\pi < \theta_1 + \theta_2 < \pi$인 것 같은데 이 부분 이해가 잘 안갑니다.

3.

$sin\frac{2\pi}{3} = sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$sin^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$

$\therefore \frac{\pi}{3}$

4.

$\tan^{-1} x = \theta$

$\cos(\tan^{-1} x) = \cos\theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}, \cos\theta > 0$

$\cos\theta = \frac{1}{sec\theta} = \frac{1}{\sqrt{1+\tan^2\theta}}$

$= \frac{1}{\sqrt{1+x^2}}$

$\therefore \frac{1}{\sqrt{1+x^2}}$

8번 문제

1.

$$\frac{1}{\sqrt{x+1}\sqrt{x}}(\sqrt{x}+1)\frac{dy}{dx}$$
$$= \frac{1}{\sqrt{x+1}\sqrt{x}}\frac{1}{2\sqrt{x+1}}$$
$$= \frac{1}{2(x+1)\sqrt{x}}$$

2.

$$\frac{1}{1+(\frac{x}{a})^2}\frac{1}{a} - \frac{1}{1+(\frac{a}{x})^2}\frac{a}{x^2}$$
$$= \frac{a}{a^2+x^2} - \frac{a}{x^2+a^2}$$
$$= 0$$

3.

$$-\frac{1}{\sqrt{1-(x+1)^2}} = \frac{1}{\sqrt{1-(y-1)^2}}\frac{dy}{dx}$$
$$\frac{dy}{dx} = -\frac{\sqrt{1-(y-1)^2}}{\sqrt{1-(x+1)^2}}$$

4.

$$\frac{dy}{dt} = -\frac{1}{1+(\frac{1}{t+1})^2}\frac{1}{(t+1)^2}$$
$$= -\frac{1}{(t+1)^2+1}$$
$$1 = \frac{1}{1+(\frac{t}{t+1})^2}\frac{1}{(t+1)^2}\frac{dt}{dx}$$
$$1 = \frac{1}{(t+1)^2+t^2}\frac{dt}{dx}$$
$$(t+1)^2 + t^2 = \frac{dt}{dx}$$
$$\frac{dy}{dx} = -\frac{(t+1)^2+t^2}{(t+1)^2+1}$$

4장 3절

2번 문제

1.

$8x \ln e = 1$

$8x = 1$

$x = \frac{1}{8}$

2.

$e^{2x} - 5 = 4e^x$

$e^{2x} - 4e^x - 5 = 0$

$(e^x - 5)(e^x + 1) = 0$

$e^x = 5$

$x = \ln 5$

3.

$1 + 2^x - 2^{2x} = 0$

$2^{2x} - 2^x - 1 = 0$

$2^x = \frac{1+\sqrt{5}}{2}$

$x = \log_2\left(\frac{1+\sqrt{5}}{2}\right) \; x = \log_2(1 + \sqrt{5}) - 1$

$x = \frac{\ln(1+\sqrt{5})}{\ln 2} - 1$

4.

$3^{2x} - 3^{x+1} - 54 = 0$

$(3^x - 9)(3^x + 6) = 0$

$3^x = 9$

$x = 2$

9번 문제

1.

$f(x) = \ln x$

$f'(x) = \frac{1}{x}$

$f(1.1) \approx f(1) + f'(1)(0.1)$

∴ 0.1

2.

$f(x) = \frac{1}{\ln x} = \ln x$

$f'(x) = -\frac{1}{x}$

$f(e - 0.01) \approx f(e) + f'(e)(-0.01)$

$∴ 1 - \frac{0.01}{e}$

3.

$f(x) = \sin(\ln x)$

$f'(x) = \cos(\ln x)\frac{1}{x}$

$f(1.001) \approx f(1) + f'(1)(0.001)$

∴ 0.001

4장 4절

6번 문제

곡선$y^2 = x \ln{(2-x)}$의 개략적인 형태를 그려라

$x < 2$

$2yy' = \ln(2-x) - \frac{x}{2-x}$

$y' = \frac{\ln(2-x) - \frac{x}{2-x}}{2y}$

4장5절

8번 문제

1.

$$f'(x) = \cosh x + 2\sinh 2x$$

2.

$$f'(x) = \cosh^2 x + \sinh^2 x$$

3.

$$f'(x) = 2(\tanh x + sechx)(sech^2 - sechx \tanh x)$$

4.

$$f'(x) = \cosh(\sin x)\cos x$$

5.

$$f'(x) = \sinh \sqrt{x}\frac{1}{2\sqrt{x}}$$

6.

$$f'(x) = 2\sinh \sqrt{1-x^2}\cosh \sqrt{1-x^2}(-2x)$$

7.

$$f'(x) = \sinh(\tanh x)sech^2 x$$

8.

$$f'(x) = -sech(\ln \sqrt{x})\tanh(\ln \sqrt{x})\frac{1}{2x}$$

$$f'(x) = -sech(\ln \sqrt{x})\tanh(\ln \sqrt{x})\frac{1}{2x} \qquad 12$$

4장 6절

5번 문제

1.

$$f'(x) = \frac{1}{\sqrt{x^2+1}}(\cosh^{-1} x) + (\sinh^{-1} x)\frac{1}{\sqrt{1-x^2}}$$

2.

$$f'(x) = \frac{1}{\sqrt{1-4x^6}}6x^2$$

3.

$$f'(x) = \frac{1}{1-\sinh^2 x}\frac{1}{\sqrt{x^2+1}}$$

4.

$$f'(x) = \frac{1}{2\sqrt{\coth^{-1}x^2}}\frac{1}{1-x^4}2x$$

5.

$f'(x) = $

6.

7.

$$f'(x) = \cosh^{-1}(2x^3)e^{\cosh^- 1(2x^3)}\frac{1}{\sqrt{1-4x^6}}6x^2$$

8.

$f'(x) =