

OR 과제 - 4

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2025-04-10

5.3-1

a

$$\cdot x_B = [x_2, x_6, x_3]$$

$$\cdot c_B = [-1, 0, 2]$$

$$\cdot c_N = [1, 0, 0]$$

$$\cdot B^{-1} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

$$\cdot N = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\cdot b = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$$

$$\cdot \text{Reduced cost} = [-1, 0, 2] \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} - [1, 0, 0] = [2, 1, 1]$$

$$\cdot \text{resource} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 14 \\ 5 \\ 11 \end{bmatrix}$$

$$\cdot B^{-1}N = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 3 \\ 2 & 0 & 1 \\ 4 & 1 & 2 \end{bmatrix}$$

$$\cdot c_B B^{-1}b = [-1, 0, 2] \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} = 5$$

위의 정보들을 바탕으로 표를 채우면 다음과 같다.

	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
	1	2	0	0	1	1	0	8
x_2	0	5	1	0	1	3	0	14
x_6	0	2	0	0	0	1	1	5
x_3	0	4	0	1	1	2	0	11

b

1. $x_1 = 0$
2. $2x_1 - 2x_2 + 3x_3 = 5$
3. $x_1 + x_2 - x_3 = 3$

5.3-2

a

- $x_B = [x_2, x_4]$
- $c_B = [3, 2]$
- $c_N = [4, 1, 0, 0]$
- $B^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$
- $N = \begin{bmatrix} 4 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix}$
- $b = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$
- Reduced cost = $[3, 2] \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix} - [4, 1, 0, 0] = [3, 2, 1, 1]$
- resource = $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$
- $B^{-1}N = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & 3 & -1 & 3 \end{bmatrix}$
- $c_B B^{-1} b = [3, 2] \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = 9$

위의 정보들을 바탕으로 표를 채우면 다음과 같다.

	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
	1	3	0	2	0	1	1	9
x_2	0	1	1	-1	0	1	-1	1
x_4	0	2	0	3	1	-1	2	3

b

1. $x_1 = 0$
2. $x_3 = 0$
3. $4x_1 + 2x_2 + x_3 + x_4 = 5$

$$4. \quad 3x_1 + x_2 + 2x_3 + x_4 = 4$$

5.3-3

- $x_B = [x_5, x_3, x_1]$

- $c_B = [0, 2, 6]$

- $c_N = [1, 0, 0]$

- $B^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix}$

- $N = \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

- $b = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

- Reduced cost = $[0, 2, 6] \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix} - [1, 0, 0] = [7, 2, 2]$

- resource = $\begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ 1 \end{bmatrix}$

- $B^{-1}N = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 1 & 2 \\ 4 & -2 & 4 \\ 0 & 1 & -1 \end{bmatrix}$

- $c_B B^{-1}b = [0, 2, 6] \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = 6$

위의 정보들을 바탕으로 표를 채우면 다음과 같다.

	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
	1	0	7	0	2	0	2	6
x_5	0	0	4	0	1	1	2	7
x_3	0	0	4	1	-2	0	4	0
x_1	0	1	0	0	1	0	-1	1

5.3-6

- 반복 1

- $\text{row}_0: \text{row}_0 + \frac{5}{2}\text{row}_2$

- $\text{row}_3: \text{row}_3 - 2\text{row}_2$

- 반복 2

- $\text{row}_0: \text{row}_0 + \frac{5}{2}\text{row}_2 + \text{row}_3 - 2\text{row}_2 = \text{row}_0 + \frac{3}{2}\text{row}_2 + \text{row}_3$

최종 $\text{row}_0 = \text{row}_0 + 0 * \text{row}_1 + \frac{3}{2}\text{row}_2 + \text{row}_3$

따라서 문제의 그림과 같은 형태가 나옴.

6.1-5

a

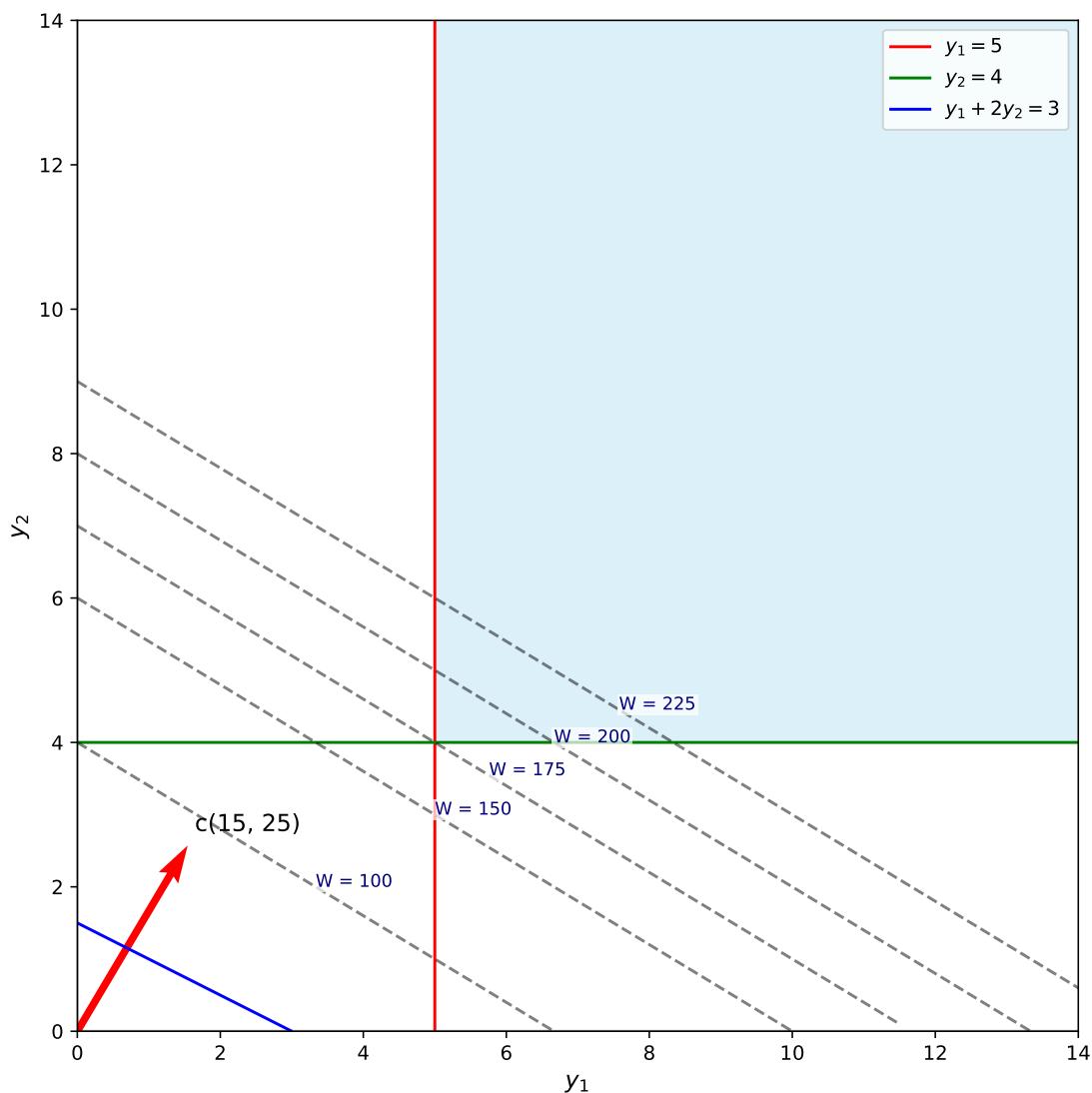
$$\text{Maximize } W = 15y_1 + 25y_2$$

$$\text{Subject to } y_1 \geq 5$$

$$y_2 \geq 4$$

$$y_1 + 2y_2 \geq 3$$

b



- 최적해는 175.

- 0|때 $y_1 = 5$ 이고 $y_2 = 4$ 이다.

C

자동으로 풀라는 말은 프로그램을 쓰라는 말로 해석해서 gurobi를 사용했습니다.

```
from gurobipy import *

model = Model("ex6.1-5")
model.setParam(GRB.Param.OutputFlag, 0)

x1 = model.addVar(vtype=GRB.CONTINUOUS, name="x1")
x2 = model.addVar(vtype=GRB.CONTINUOUS, name="x2")
x3 = model.addVar(vtype=GRB.CONTINUOUS, name="x3")

model.setObjective(5 * x1 + 4 * x2 + 3 * x3, GRB.MAXIMIZE)

model.addConstr(x1 + x3 <= 15)
model.addConstr(x2 + 2 * x3 <= 25)

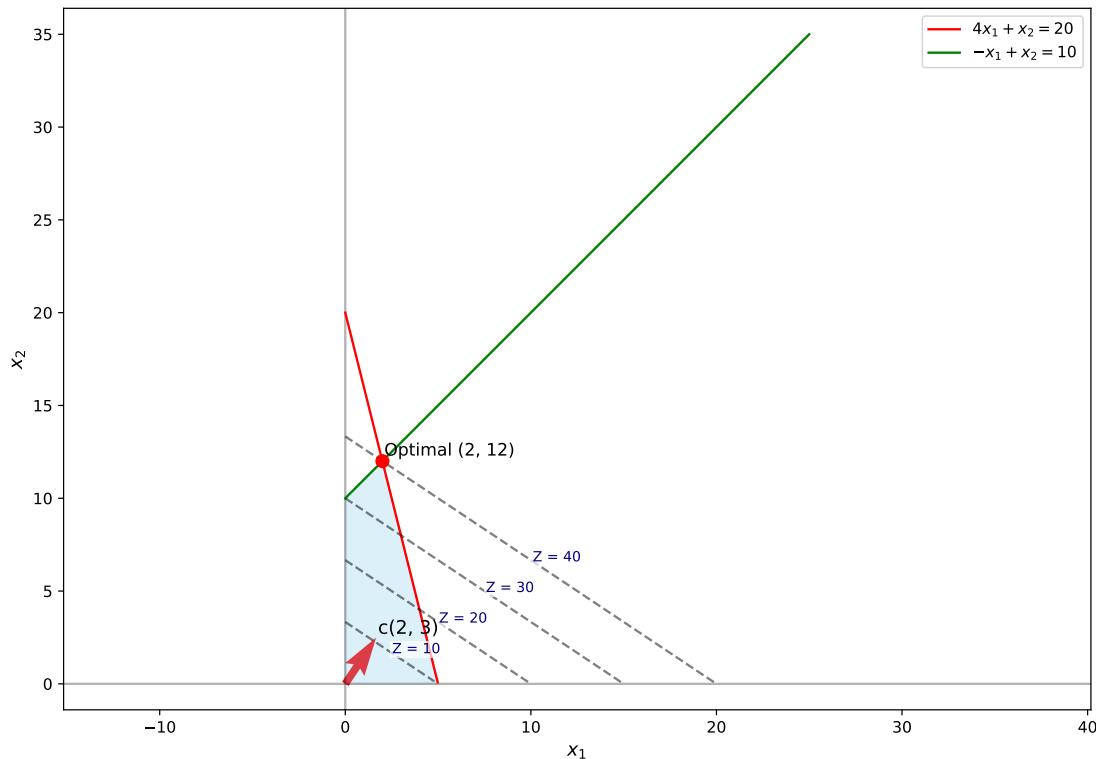
model.optimize()

for var in model.getVars():
    print(f"{var.varName}: {var.x}")
print("Answer is ", model.objVal)
```

```
Restricted license - for non-production use only - expires 2026-11-23
x1: 15.0
x2: 25.0
x3: 0.0
Answer is 175.0
```

6.1-7

a



- 문제에서는 가능해들을 가지지 못함을 보이라 했는데 가능해를 가진다.

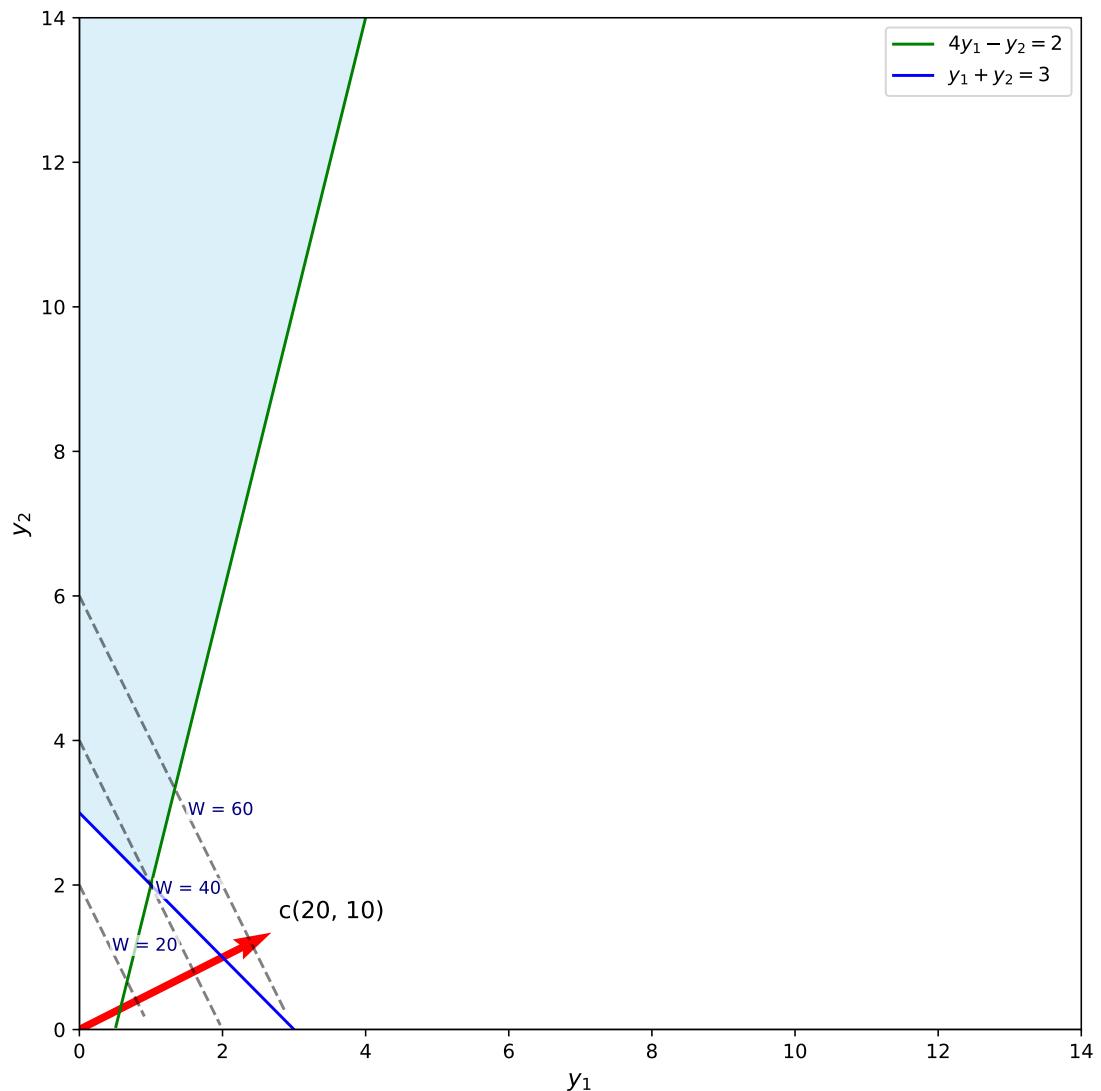
b

$$\text{Minimize } W = 20y_1 + 10y_2$$

$$\text{Subject to } 4y_1 - y_2 \geq 2$$

$$y_1 + y_2 \geq 3$$

C



- 최적해는 40.