Discrete Mathematics #3

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8.3

Problem 29

$f(n) = af(\frac{n}{b}) + cn^d$

$f(\frac{n}{b}) = af(\frac{n}{b^2}) + c(\frac{n}{b})^d$

$f(n) = a(af(\frac{n}{b^2}) + c(\frac{n}{b})^d) + cn^d$

$= a^2 f(\frac{n}{b^2}) + ac(\frac{n}{b})^d + cn^d$

$f(\frac{n}{b^2}) = af(\frac{n}{b^3}) + c(\frac{n}{b^2})^d$

$f(n) = a^3 f(\frac{n}{b^3}) + a^2 c(\frac{n}{b^2})^d + ac(\frac{n}{b})^d + cn^d$

$\therefore f(n) = a^k f(1) + c\sum_{i=0}^{k-1} a^i (\frac{n}{b^i})^d$

$a = b^d, a^i(\frac{n}{b^i})^d = n^d$

$f(n) = a^k f(1) + cn^d \sum_{i=0}^{k-1} 1$

$= a^k f(1) + cn^d k$

$n = b^k, k = \log_b n$

$a^k = a^{\log_b n} = n^{\log_b a}, \log_b a = d$

$f(n) = n^d f(1) + cn^d \log_b n$

Problem 30

$f(n) = n^d f(1) + cn^d \log_b n$

$n^d f(1) = O(n^d)$

$cn^d \log_b n = cn^d \frac{\log n}{\log b} = \frac{c}{\log b} n^d \log n = O(n^d \log n)$

n^d보다 $n^d \log n$이 더 빠르게 증가하므로

$\therefore f(n) = O(n^d \log n)$

Problem 31

Problem 29와 마찬가지로

$f(n) = a^k f(1) + c\sum_{i=0}^{k-1} a^i (\frac{n}{b^i})^d$

$f(n) = a^k f(1) + cn^d \sum_{i=0}^{k-1} (ab^{-d})^i$

$= a^k f(1) + cn^d \frac{1-(ab^{-d})^k}{1-ab^{-d}}$

$n = b^k, k = \log_b n$

$f(n) = a^k f(1) + cn^d \frac{1-(ab^{-d})^{\log_b n}}{1-ab^{-d}}$

$= n^{\log_b a} f(1) + \frac{cn^d}{1-ab^{-d}} - \frac{cn^d (ab^{-d})^{\log_b n}}{1-ab^{-d}}$

$= n^{\log_b a} f(1) + n^d \frac{cb^d}{b^d - a} + \frac{cb^d n^d (ab^{-d})^{\log_b n}}{a-b^d}$

$= n^{\log_b a} f(1) + n^d \frac{cb^d}{b^d - a} + \frac{cb^d n^d n^{\log_b ab^{-d}}}{a-b^d}$

$= n^{\log_b a} f(1) + n^d \frac{cb^d}{b^d - a} + \frac{cb^d n^{d + \log_b a - d}}{a-b^d}$

$= n^{\log_b a} f(1) + n^d \frac{cb^d}{b^d - a} + \frac{cb^d n^{\log_b a}}{a-b^d}$

$= n^{\log_b a} (f(1) + \frac{cb^d}{a-b^d}) + n^d (\frac{cb^d}{b^d - a})$

$C_1 = \frac{b^d c}{b^d - a}, C_2 = f(1) + \frac{b^d c}{a - b^d}$라고 할 때,

$f(n) = C_1 n^d + C_2 n^{\log_b a}$

Problem 32

$f(n) = C_1 n^d + C_2 n^{\log_b a}$

$a < b^d, \log_b a < d$

n^d이 $n^{\log_b a}$보다 빠르게 증가

$\therefore f(n) = O(n^d)$

Problem 33

Problem 32의 과정에서 $a > b^d$인 경우로 생각하면

$n^{\log_b a}$이 n^d보다 빠르게 증가한다는 사실을 알 수 있다

$\therefore f(n) = O(n^{\log_b a})$