Discrete Mathematics #4

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8.2

Problem 4

a

$r^2 - r + 6 = 0$

$(r-3)(r+2) = 0$

$r = 3, -2$

$a_n = c_1 3^n + c_2(-2)^n$

$a_0 = 3 = c_1 + c_2$

$a_1 = 6 = 3c_1 - 2c_2$

$c_1 = 2, c_2 = 1$

$\therefore a_n = 2(3^n) + (-2)^n$

b

$r^2 - 7r + 10 = 0$

$(r-5)(r-2) = 0$

$r = 5, 2$

$a_n = c_1 5^n + c_2 2^n$

$a_0 = 2 = c_1 + c_2$

$a_1 = 1 = 5c_1 + 2c_2$

$c_1 = -1, c_2 = 3$

$\therefore a_n = -5^n + 3(2^n)$

c

$r^2 - 6r + 8 = 0$

$(r-4)(r-2) = 0$

$r = 4, 2$

$a_n = c_1 4^n + c_2 2^n$

$a_0 = 4 = c_1 + c_2$

$a_1 = 10 = 4c_1 + 2c_2$

$c_1 = 1, c_2 = 3$

$\therefore a_n = 4^n + 3(2^n)$

d

$r^2 - 2r + 1 = 0$

$(r-1)^2 = 0$

$r = 1$

$a_n = c_1 + c_2 n \, a_0 = 4 = c_1$

$a_1 = 1 = c_1 + c_2$

$c_1 = 4, c_2 = -3$

$\therefore a_n = 4 - 3n$

e

$$a_n = \begin{cases} 5 & \text{if n is even} \\ -1 & \text{if n is odd} \end{cases}$$

f

$r^2 + 6r + 9 = 0$

$(r+3)^2 = 0$

$r = -3$

$a_n = c_1(-3)^n + c_2 n(-3)^n$

$a_0 = 3 = c_1$

$a_1 = -3 = -3c_1 - 3c_2$

$c_1 = 3, c_2 = 0$

$\therefore a_n = 3(-3)^n$

g

$r^2 + 4r - 5 = 0$

$(r+5)(r-1) = 0$

$r = -5, 1$

$$a_n = c_1(-5)^n + c_2(1)^n$$
$$a_0 = 2 = c_1 + c_2$$
$$a_1 = 8 = -5c_1 + c_2$$
$$c_1 = -1, c_2 = 3$$
$$\therefore a_n = -(-5)^n + 3$$

8.3

$f(3^k) = 2^k f(1) + \Sigma_{i=0}^{k-1} 2^i 3^{2(k-i)}$

Problem 12

$f(n) = 2f(\frac{n}{3}) + 4$

$f(3^k) = 2f(3^{k-1}) + 4\; f(3^{k-1}) = 2f(3^{k-2}) + 4$

$f(3^k) = 2(2f(3^{k-2}) + 4) + 4 = 4f(3^{k-2}) + 12$

$f(3^k) = 2^k f(1) + 4\Sigma_{i=0}^{k-1} 2^i$

$f(3^k) = 2^k + 4(2^k - 1)$

$f(3^k) = 5(2^k) - 4$

Problem 13

$n = 3^k, k = \log_3 n$

$f(n) = 5(2^{\log_3 n}) - 4$

$O(2^{\log_3 n}) = O(n^{\log_3 2}) = O(n^{0.63})$

Problem 22

a

$f(16) = 2f(4) \log 16$

$f(4) = 2f(2) \log 4 = 2 * 1 * 2 = 4$

$f(16) = 2 * 4 * 4 = 32$

b

$f(n) = 2f(\sqrt{n}) \log n$

$n = 2^m, m = \log n$

$f(2^m) = 2f(2^{\frac{m}{2}}) \log 2^m$

$g(m) = f(2^m)$

$g(m) = 2g(\frac{m}{2})m$

$f(n) = O(m^2) = O((\log n)^2)$

8.4

Problem 24

a

$y_1 = x_1 - 3, y_1 \geq 0$
$G_1(z) = z^3(1 + z + z^2 + z^3 + ...) = \frac{z^3}{1-z}$
$G_2(z) = z(1 + z + z^2 + z^3 + z^4 + z^5) = z\frac{1-z^5}{1-z}$
$G_3(z) = 1 + z + z^2 + z^3 + z^4 = \frac{1-z^5}{1-z}$
$G_4(z) = z + z^2 + z^3 + ... = \frac{z}{1-z}$
$G(z) = G_1(z)G_2(z)G_3(z)G_4(z) = \frac{z^3}{1-z}\frac{z(1-z^5)}{1-z}\frac{1-z^5}{1-z}\frac{z}{1-z} = \frac{z^5(1-z^5)^2}{(1-z)^4}$

b

$G(z) = \frac{z^5(1-z^5)^2}{(1-z)^4} = \frac{z^5-2z^{10}+z^{15}}{(1-z)^4}$
$\frac{z^5}{(1-z)^4}, z^2$ from $\frac{1}{(1-z)^4}, \binom{5}{3} = 10$
$\frac{-2z^{10}}{(1-z)^4}, z^{-3}$ from $\frac{1}{(1-z)^4},$ none
$\frac{z^{15}}{(1-z)^4}, z^{-8}$ from $\frac{1}{(1-z)^4},$ none
$a_7 = 10$

Problem 34

$A(z) = a_0 + a_1z + a_2z^2 + ...$
$a_k = 7^ka_0, A(Z) = 5 + 5(7z) + 5(7z)^2 + ...$
$A(z) = 5(1 + 7z + 7^2z^2 + ...) = \frac{5}{1-7z}$
$a_k = 5(7^k)$

Problem 36

$A(z) = 3zA(z) + z\Sigma_{k=0}^{\infty}(4z)^k$
$A(z) = 3zA(z) + \frac{z}{1-4z}$
$A(z) = \frac{z}{(1-3z)(1-4z)}$