

OR 과제 - 4

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5.3-1

a

$$\bullet x_B = [x_2, x_6, x_3]$$

$$\bullet c_B = [-1, 0, 2]$$

$$\bullet c_N = [1, 0, 0]$$

$$\bullet B^{-1} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

$$\bullet N = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\bullet b = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$$

$$\bullet \text{Reduced cost} = [-1, 0, 2] \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} - [1, 0, 0] = [2, 1, 1]$$

$$\bullet \text{resouce} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 14 \\ 5 \\ 11 \end{bmatrix}$$

$$\bullet B^{-1}N = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 3 \\ 2 & 0 & 1 \\ 4 & 1 & 2 \end{bmatrix}$$

$$\bullet c_B B^{-1}b = [-1, 0, 2] \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} = 5$$

위의 정보들을 바탕으로 표를 채우면 다음과 같다.

| | Z | x_1 | x_2 | x_3 | x_4 | x_5 | x_6 | RHS |
|-------|-----|-------|-------|-------|-------|-------|-------|-----|
| | 1 | 2 | 0 | 0 | 1 | 1 | 0 | 8 |
| x_2 | 0 | 5 | 1 | 0 | 1 | 3 | 0 | 14 |
| x_6 | 0 | 2 | 0 | 0 | 0 | 1 | 1 | 5 |
| x_3 | 0 | 4 | 0 | 1 | 1 | 2 | 0 | 11 |

b

1. $x_1 = 0$
2. $2x_1 - 2x_2 + 3x_3 = 5$
3. $x_1 + x_2 - x_3 = 3$

5.3-2

a

- $x_B = [x_2, x_4]$

- $c_B = [3, 2]$

- $c_N = [4, 1, 0, 0]$

- $B^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

- $N = \begin{bmatrix} 4 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix}$

- $b = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

- Reduced cost = $[3, 2] \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix} - [4, 1, 0, 0] = [3, 2, 1, 1]$

- resource = $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

- $B^{-1}N = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & 3 & -1 & 3 \end{bmatrix}$

- $c_B B^{-1}b = [3, 2] \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = 9$

위의 정보들을 바탕으로 표를 채우면 다음과 같다.

| | Z | x_1 | x_2 | x_3 | x_4 | x_5 | x_6 | RHS |
|-------|---|-------|-------|-------|-------|-------|-------|-----|
| | 1 | 3 | 0 | 2 | 0 | 1 | 1 | 9 |
| x_2 | 0 | 1 | 1 | -1 | 0 | 1 | -1 | 1 |
| x_4 | 0 | 2 | 0 | 3 | 1 | -1 | 2 | 3 |

b

1. $x_1 = 0$

2. $x_3 = 0$

3. $4x_1 + 2x_2 + x_3 + x_4 = 5$

4. $3x_1 + x_2 + 2x_3 + x_4 = 4$

5.3-3

- $x_B = [x_5, x_3, x_1]$

- $c_B = [0, 2, 6]$

- $c_N = [1, 0, 0]$

- $B^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix}$

- $N = \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

- $b = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

- Reduced cost = $[0, 2, 6] \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix} - [1, 0, 0] = [7, 2, 2]$

- resource = $\begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ 1 \end{bmatrix}$

- $B^{-1}N = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 1 & 2 \\ 4 & -2 & 4 \\ 0 & 1 & -1 \end{bmatrix}$

- $c_B B^{-1}b = [0, 2, 6] \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = 6$

위의 정보들을 바탕으로 표를 채우면 다음과 같다.

| | Z | x_1 | x_2 | x_3 | x_4 | x_5 | x_6 | RHS |
|-------|-----|-------|-------|-------|-------|-------|-------|-----|
| | 1 | 0 | 7 | 0 | 2 | 0 | 2 | 6 |
| x_5 | 0 | 0 | 4 | 0 | 1 | 1 | 2 | 7 |
| x_3 | 0 | 0 | 4 | 1 | -2 | 0 | 4 | 0 |
| x_1 | 0 | 1 | 0 | 0 | 1 | 0 | -1 | 1 |

5.3-6

- 반복 1

- $\text{row}_0: \text{row}_0 + \frac{5}{2}\text{row}_2$

- $\text{row}_3: \text{row}_3 - 2\text{row}_2$

- 반복 2

- $\text{row}_0: \text{row}_0 + \frac{5}{2}\text{row}_2 + \text{row}_3 - 2\text{row}_2 = \text{row}_0 + \frac{3}{2}\text{row}_2 + \text{row}_3$

최종 $\text{row}_0 = \text{row}_0 + 0 * \text{row}_1 + \frac{3}{2}\text{row}_2 + \text{row}_3$

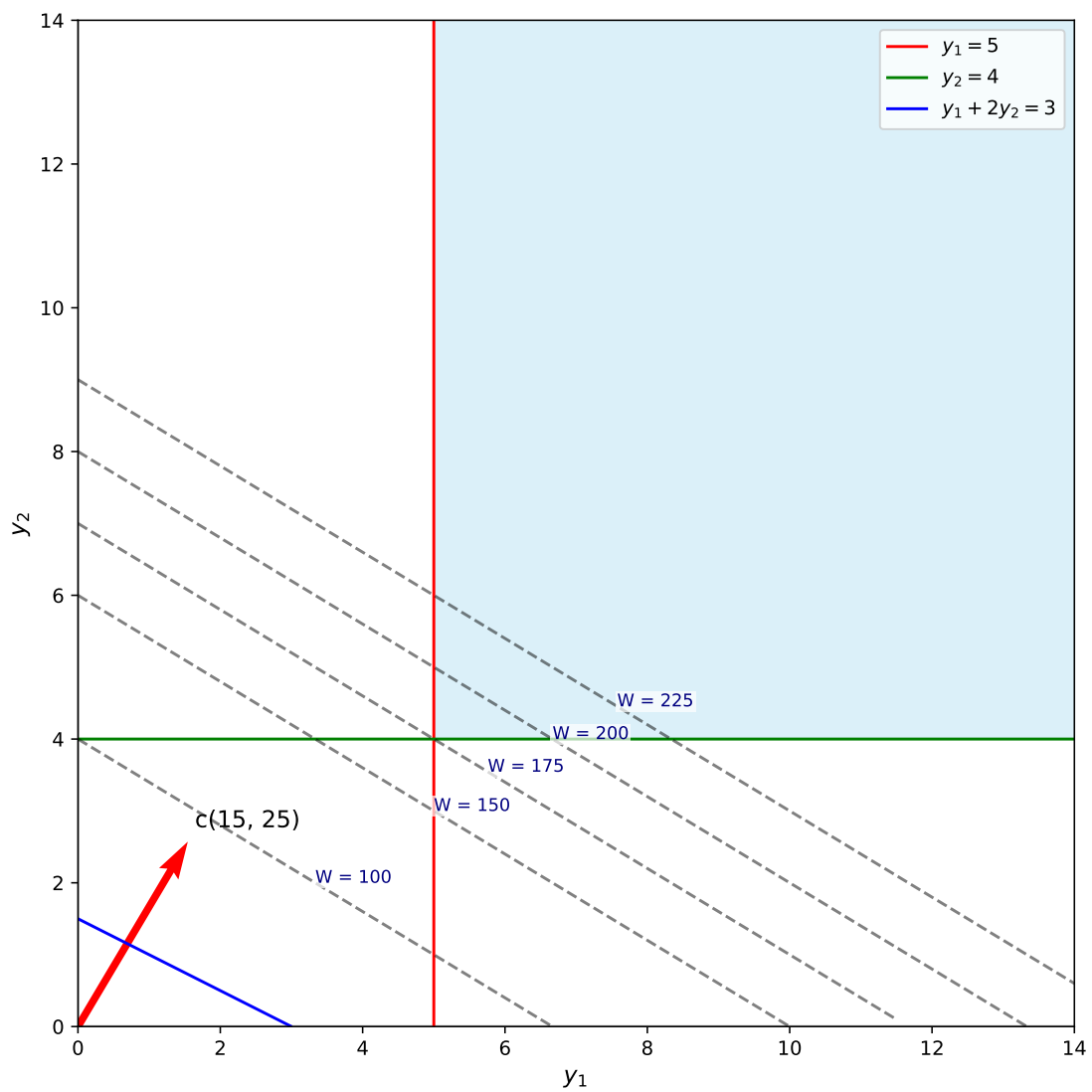
따라서 문제의 그림과 같은 형태가 나옴.

6.1-5

a

$$\begin{aligned} \text{Maximize } & W = 15y_1 + 25y_2 \\ \text{Subject to } & y_1 \geq 5 \\ & y_2 \geq 4 \\ & y_1 + 2y_2 \geq 3 \end{aligned}$$

b



- 최적해는 175.

- 이때 $y_1 = 5$ 이고 $y_2 = 4$ 이다.

C

자동으로 풀라는 말은 프로그램을 쓰라는 말로 해석해서 gurobi를 사용했습니다.

```
from gurobipy import *

model = Model("ex6.1-5")
model.setParam(GRB.Param.OutputFlag, 0)

x1 = model.addVar(vtype=GRB.CONTINUOUS, name="x1")
x2 = model.addVar(vtype=GRB.CONTINUOUS, name="x2")
x3 = model.addVar(vtype=GRB.CONTINUOUS, name="x3")

model.setObjective(5 * x1 + 4 * x2 + 3 * x3, GRB.MAXIMIZE)

model.addConstr(x1 + x3 <= 15)
model.addConstr(x2 + 2 * x3 <= 25)

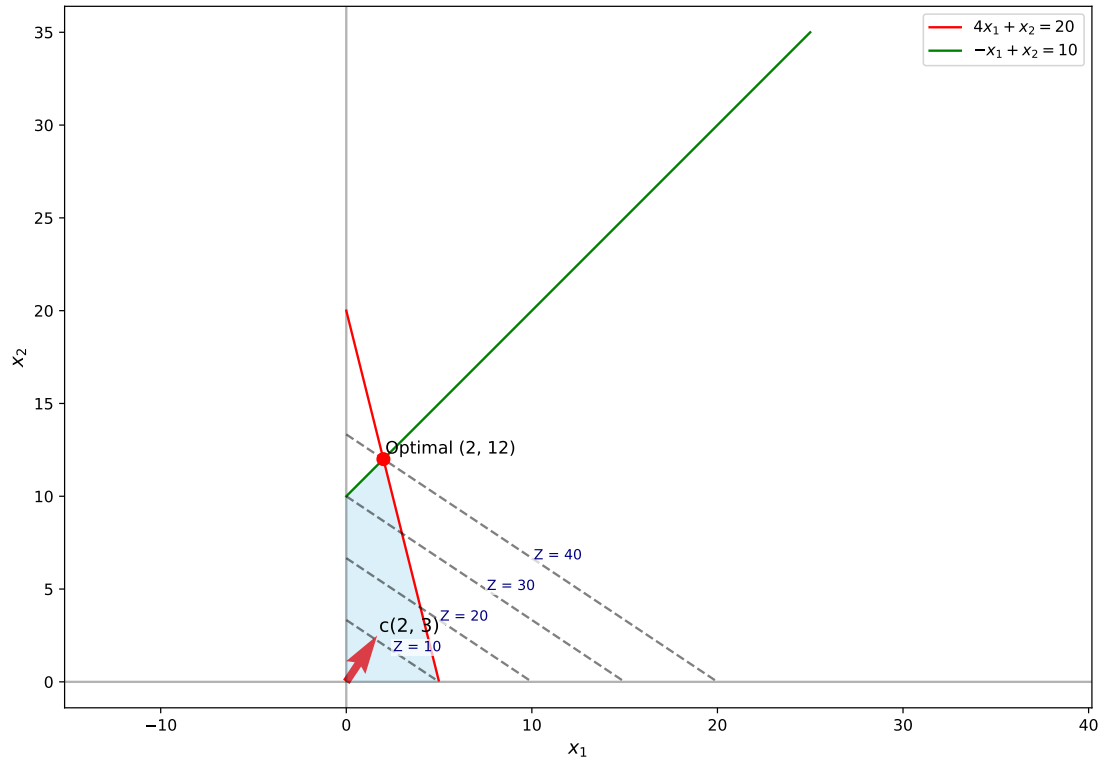
model.optimize()

for var in model.getVars():
    print(f"{var.varName}: {var.x}")
print("Answer is ", model.objVal)
```

```
x1: 15.0
x2: 25.0
x3: 0.0
Answer is 175.0
```

6.1-7

a

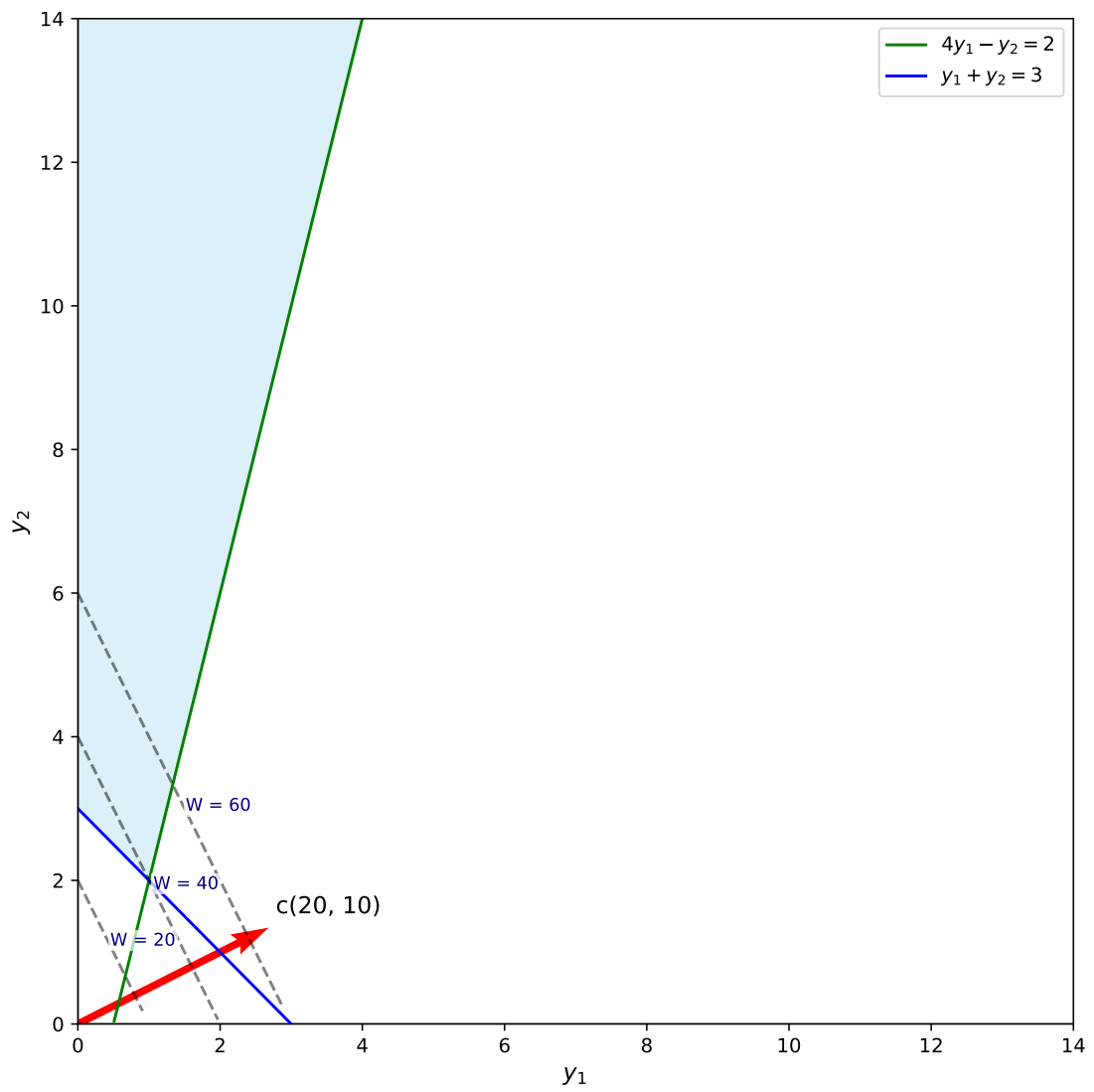


- 문제에서는 가능해들을 가지지 못함을 보이려 했는데 가능해를 가진다.

b

$$\begin{aligned} &\text{Minimize } W = 20y_1 + 10y_2 \\ &\text{Subject to } 4y_1 - y_2 \geq 2 \\ &\quad \quad \quad y_1 + y_2 \geq 3 \end{aligned}$$

c



- 최적해는 40.