OR 과제 - 4

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5.3-1

a

- $x_B = [x_2, x_6, x_3]$
- $c_B = [-1, 0, 2]$
- $c_N = [1, 0, 0]$
- $B^{-1} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$
- $N = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$
- $b = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$
- Reduced cost $= [-1, 0, 2] \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} - [1, 0, 0] = [2, 1, 1]$
- resouce $= \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 14 \\ 5 \\ 11 \end{bmatrix}$
- $B^{-1}N = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 3 \\ 2 & 0 & 1 \\ 4 & 1 & 2 \end{bmatrix}$
- $c_B B^{-1} b = [-1, 0, 2] \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} = 5$

위의 정보들을 바탕으로 표를 채우면 다음과 같다.

	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
	1	2	0	0	1	1	0	8
x_2	0	5	1	0	1	3	0	14
x_6	0	2	0	0	0	1	1	5
x_3	0	4	0	1	1	2	0	11

b

1. $x_1 = 0$
2. $2x_1 - 2x_2 + 3x_3 = 5$
3. $x_1 + x_2 - x_3 = 3$

5.3-2

a

- $x_B = [x_2, x_4]$

- $c_B = [3, 2]$

- $c_N = [4, 1, 0, 0]$

- $B^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

- $N = \begin{bmatrix} 4 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix}$

- $b = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

- Reduced cost $= [3, 2] \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix} - [4, 1, 0, 0] = [3, 2, 1, 1]$

- resouce $= \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

- $B^{-1} N = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & 3 & -1 & 3 \end{bmatrix}$

- $c_B B^{-1} b = [3, 2] \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = 9$

위의 정보들을 바탕으로 표를 채우면 다음과 같다.

	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
	1	3	0	2	0	1	1	9
x_2	0	1	1	-1	0	1	-1	1
x_4	0	2	0	3	1	-1	2	3

b

1. $x_1 = 0$
2. $x_3 = 0$
3. $4x_1 + 2x_2 + x_3 + x_4 = 5$

4. $3x_1 + x_2 + 2x_3 + x_4 = 4$

5.3-3

- $x_B = [x_5, x_3, x_1]$

- $c_B = [0, 2, 6]$

- $c_N = [1, 0, 0]$

- $B^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix}$

- $N = \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

- $b = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

- Reduced cost $= [0, 2, 6] \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix} - [1, 0, 0] = [7, 2, 2]$

- resouce $= \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ 1 \end{bmatrix}$

- $B^{-1}N = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 1 & 2 \\ 4 & -2 & 4 \\ 0 & 1 & -1 \end{bmatrix}$

- $c_B B^{-1} b = [0, 2, 6] \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = 6$

위의 정보들을 바탕으로 표를 채우면 다음과 같다.

	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
	1	0	7	0	2	0	2	6
x_5	0	0	4	0	1	1	2	7
x_3	0	0	4	1	-2	0	4	0
x_1	0	1	0	0	1	0	-1	1

5.3-6

- 반복 1
    - row_0: $row_0 + \frac{5}{2}row_2$
    - row_3: $row_3 - 2row_2$
- 반복 2
    - row_0: $row_0 + \frac{5}{2}row_2 + row_3 - 2row_2 = row_0 + \frac{3}{2}row_2 + row_3$

최종 $row_0 = row_0 + 0 * row_1 + \frac{3}{2}row_2 + row_3$

따라서 문제의 그림과 같은 형태가 나옴.

6.1-5

a

$$\text{Maximize} \quad W = 15y_1 + 25y_2$$
$$\text{Subject to} \quad y_1 \geq 5$$
$$y_2 \geq 4$$
$$y_1 + 2y_2 \geq 3$$

b
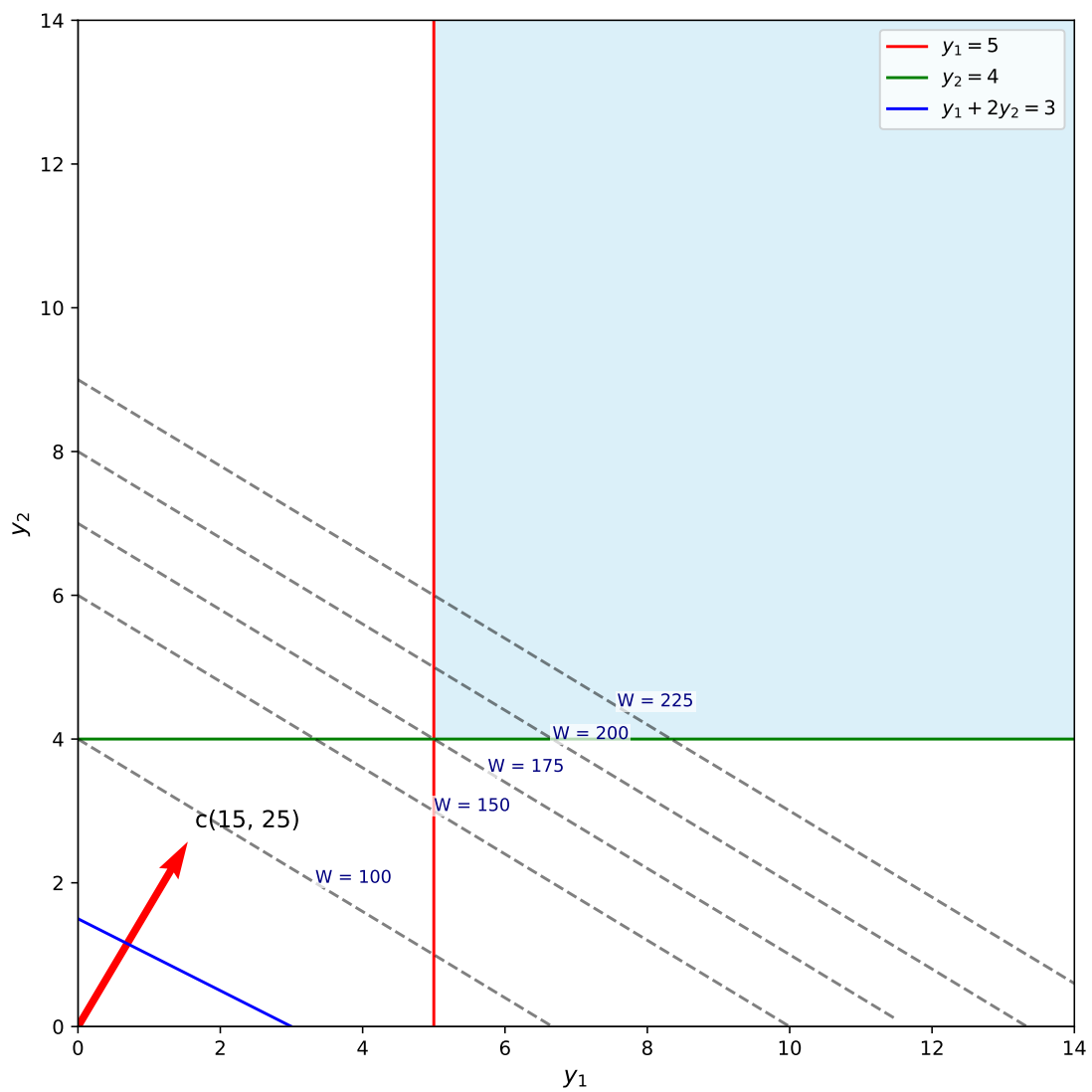
- 최적해는 175.

- 이때 $y_1 = 5$이고 $y_2 = 4$이다.

C

자동으로 풀라는 말은 프로그램을 쓰라는 말로 해석해서 gurobi를 사용했습니다.

```python
from gurobipy import *

model = Model("ex6.1-5")
model.setParam(GRB.Param.OutputFlag, 0)

x1 = model.addVar(vtype=GRB.CONTINUOUS, name="x1")
x2 = model.addVar(vtype=GRB.CONTINUOUS, name="x2")
x3 = model.addVar(vtype=GRB.CONTINUOUS, name="x3")

model.setObjective(5 * x1 + 4 * x2 + 3 * x3, GRB.MAXIMIZE)

model.addConstr(x1 + x3 <= 15)
model.addConstr(x2 + 2 * x3 <= 25)

model.optimize()

for var in model.getVars():
    print(f"{var.varName}: {var.x}")
print("Answer is ", model.objVal)
```

```
Restricted license - for non-production use only - expires 2026-11-23
x1: 15.0
x2: 25.0
x3: 0.0
Answer is  175.0
```
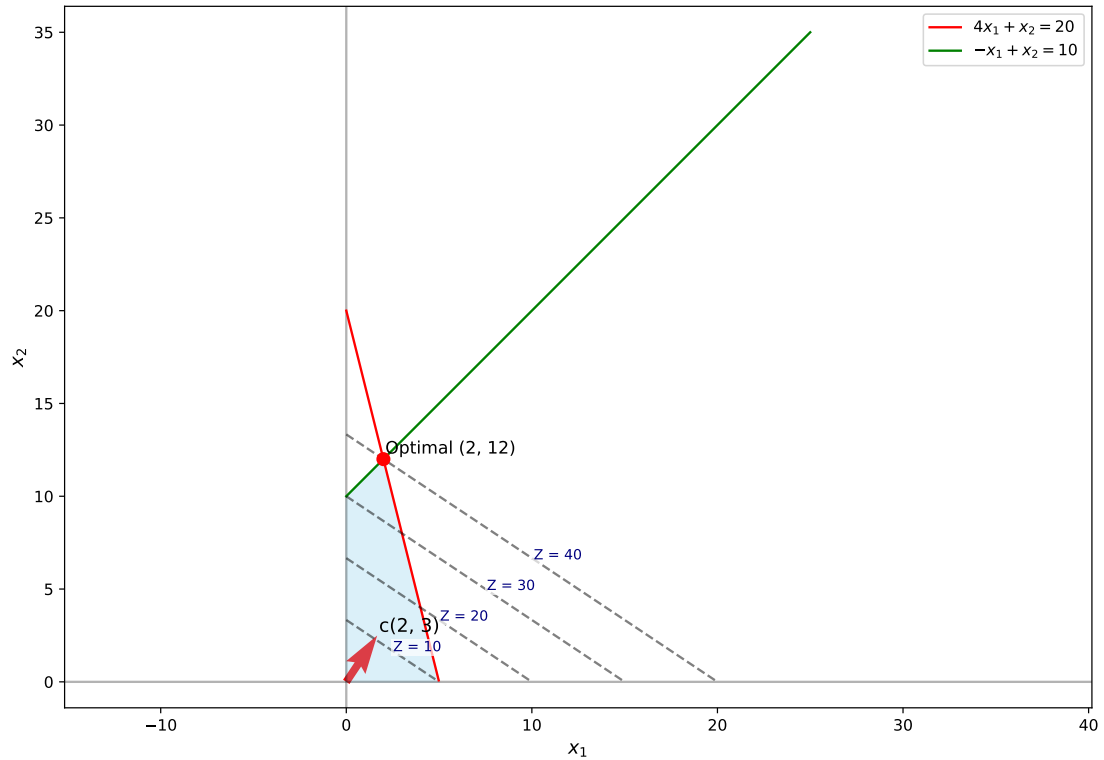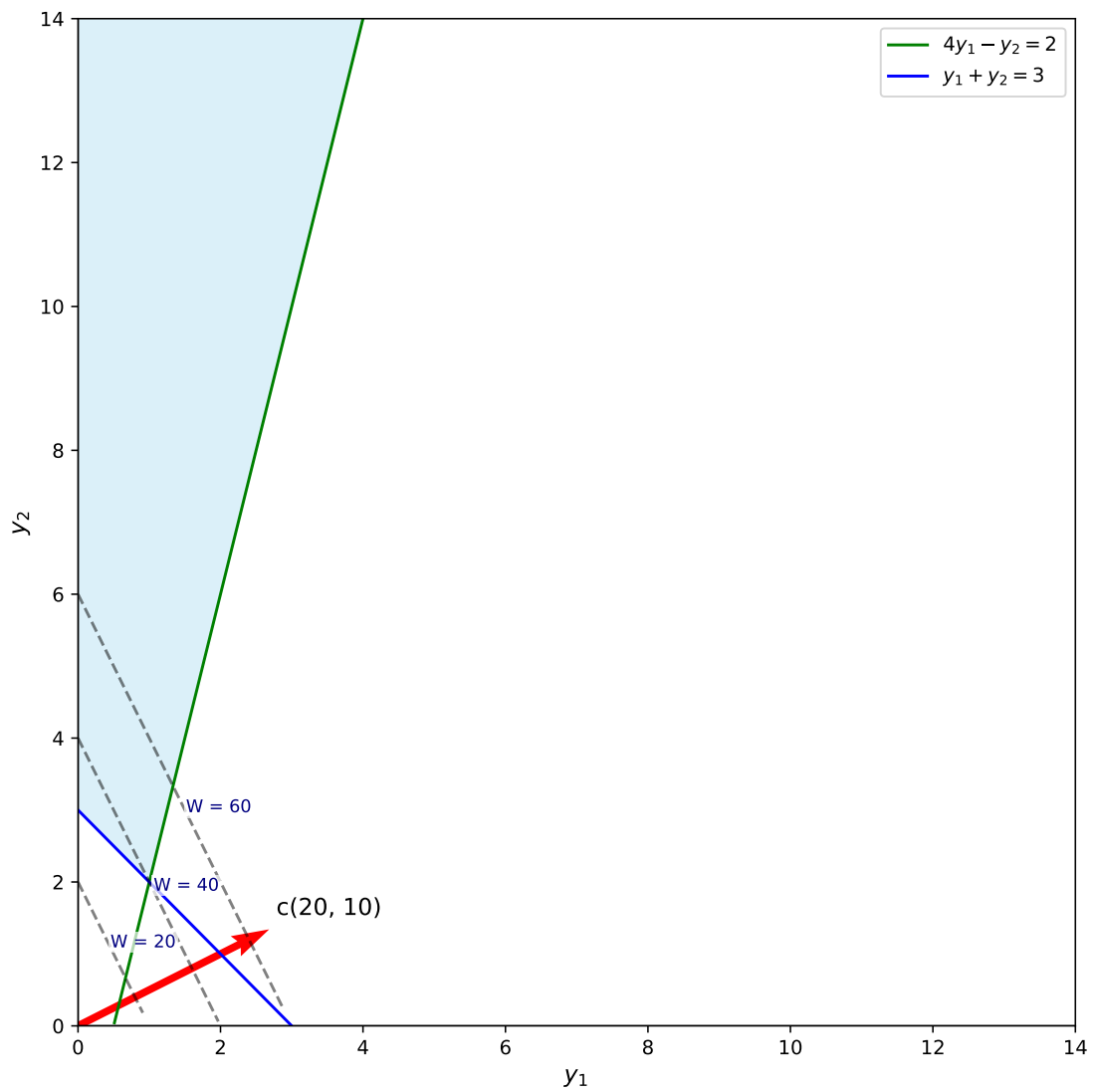
6.1-7

a

- 문제에서는 가능해들을 가지지 못함을 보이라 했는데 가능해를 가진다.

b

$$\text{Minimize} \quad W = 20y_1 + 10y_2$$
$$\text{Subject to} \quad 4y_1 - y_2 \geq 2$$
$$y_1 + y_2 \geq 3$$

c

- 최적해는 40.