OR 과제 - 6

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6.7-2

최종 해법은 다음과 같다.

	Z	x_1	x_2	x_3	x_4	x_5	RHS
	1	0	0	2	5	0	100
x_2	0	-1	1	3	1	0	20
x_5	0	16	0	-2	-4	1	10

a

- $B^{-1}b_{new} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 30 \\ 90 \end{bmatrix} = \begin{bmatrix} 30 \\ -30 \end{bmatrix}$
- $Z_{new} = [5, 0] \begin{bmatrix} 30 \\ -30 \end{bmatrix} = 150.$

(0) $Z + 2x_3 + 5x_4 = 150$
(1) $-x_1 + x_2 + 3x_3 + x_4 = 30$
(2) $16x_1 - 2x_3 - 4x_4 + x_5 = -30$

- 기저해: (0, 30, 0, 0, -30)
- $x_5 < 0$ (infeasible)
- Z-행의 비기저 변수 x_1, x_3, x_4의 계수는 각각 $0, 2, 5$. bfs였다면 최적 조건을 만족

b

- $B^{-1}b_{new} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 70 \end{bmatrix} = \begin{bmatrix} 20 \\ -10 \end{bmatrix}$
- $Z_{new} = [5, 0] \begin{bmatrix} 20 \\ -10 \end{bmatrix} = 100.$

(0) $Z + 2x_3 + 5x_4 = 100$
(1) $-x_1 + x_2 + 3x_3 + x_4 = 20$
(2) $16x_1 - 2x_3 - 4x_4 + x_5 = -10$

- 기저해: (0, 20, 0, 0, -10). infeasible
- rc: (0, 0, 2, 5, 0). bfs였다면 최적

c

- $B^{-1}b_{new} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 100 \end{bmatrix} = \begin{bmatrix} 10 \\ 60 \end{bmatrix}$
- $Z_{new} = [5, 0] \begin{bmatrix} 10 \\ 60 \end{bmatrix} = 50.$

(0) $Z + 2x_3 + 5x_4 = 50$
(1) $-x_1 + x_2 + 3x_3 + x_4 = 10$
(2) $16x_1 - 2x_3 - 4x_4 + x_5 = 60$

- 기저해: (0, 10, 0, 0, 60). feasible
- rc: 0, 0, 2, 5, 0. 최적

d

- origin: $z - c_3 = 2, z = 15$
- new: $z - c_3 = 15 - 80 = -65$

(0) $Z - 65x_3 + 5x_4 = 100$
(1) $-x_1 + x_2 + 3x_3 + x_4 = 20$
(2) $16x_1 - 2x_3 - 4x_4 + x_5 = 10$

- 기저해: (0, 20, 0, 0, 10). feasible
- rc: (0, 0, 7, 5, 0). 최적

e

- x_1의 rc: $c_B B^{-1} A_{.1} - c_1 = [5, 0] \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} - (-2) = 2$
- $N_{.1}$: $B^{-1} A_{.1} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$

(0) $Z + 2x_1 + 2x_3 + 5x_4 = 100$
(1) $x_2 + 3x_3 + x_4 = 20$
(2) $5x_1 - 2x_3 - 4x_4 + x_5 = 10$

- 기저해: (0, 20, 0, 0, 10). feasible
- rc: (2, 0, 7, 5, 0). 최적

f

- $B = \begin{bmatrix} 2 & 0 \\ 5 & 1 \end{bmatrix}$
- $B^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix}$
- $c_B = [6, 0]$
- $B^{-1}b = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 90 \end{bmatrix} = \begin{bmatrix} 10 \\ 40 \end{bmatrix}$
- $Z = [6, 0] \begin{bmatrix} 10 \\ 40 \end{bmatrix} = 60.$
- x_1의 rc: $c_B B^{-1} A_{\cdot 1} - c_1 = [6, 0] \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 12 \end{bmatrix} - (-5) = 2$
- $N_{\cdot 1}$: $B^{-1} A_{\cdot 1} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 12 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{29}{2} \end{bmatrix}$
- x_3의 rc: $c_B B^{-1} A_{\cdot 3} - c_3 = [6, 0] \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 10 \end{bmatrix} - 13 = -4$
- $N_{\cdot 3}$: $B^{-1} A_{\cdot 3} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 10 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ \frac{5}{2} \end{bmatrix}$
- x_4의 rc: $c_B B^{-1} A_{\cdot 4} - c_4 = [6, 0] \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 0 = 3$
- $N_{\cdot 4}$: $B^{-1} A_{\cdot 4} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{5}{2} \end{bmatrix}$

(0) $Z + 2x_1 - 4x_3 + 3x_4 = 60$
(1) $-\frac{1}{2}x_1 + x_2 + \frac{3}{2}x_3 + \frac{1}{2}x_4 = 10$
(2) $\frac{29}{2}x_1 + \frac{5}{2}x_3 - \frac{5}{2}x_4 + x_5 = 40$

- 기저해: (0, 10, 0, 0, 40). feasible
- rc: (2, 0, -4, 3, 0). 최적 아님.

g

- $N_{\cdot 6}$: $B^{-1} A_{\cdot 6} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$

- x_6의 rc: $(5,0)\begin{bmatrix}3\\-7\end{bmatrix}-10=5.$

(0) $Z+2x_3+5x_4+5x_6=100$

(1) $-x_1+x_2+3x_3+x_4+3x_6=20$

(2) $16x_1-2x_3-4x_4+x_5-7x_6=10$

- 기저해: (0, 20, 0, 0, 10, 0). feasible
- rc: (0, 0, 2, 5, 0, 5). 최적.

h

	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
	1	0	0	2	5	0	0	100
x_2	0	-1	1	3	1	0	0	20
x_5	0	16	0	-2	-4	1	0	10
x_6	0	2	3	5	0	0	1	50

	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
	1	0	0	2	5	0	0	100
x_2	0	-1	1	3	1	0	0	20
x_5	0	16	0	-2	-4	1	0	10
x_6	0	5	0	-4	-3	0	1	-10

(0) $Z+2x_3+5x_4=100$

(1) $-x_1+x_2+3x_3+x_4=20$

(2) $16x_1-2x_3-4x_4+x_5=10$

(3) $5x_1-4x_3-3x_4+x_6=-10$

- 기저해: (0, 20, 0, 0, 10, -10). infeasible
- rc: (0, 0, 2, 5, 0). bfs면 최적.

i

- $B=\begin{bmatrix}1&0\\5&1\end{bmatrix}$
- $B^{-1}=\begin{bmatrix}1&0\\-5&1\end{bmatrix}$
- $c_B=[5,0]$

- $B^{-1}b = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 100 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$
- $Z = [5, 0] \begin{bmatrix} 20 \\ 0 \end{bmatrix} = 100$
- x_1의 rc: $c_B B^{-1} A_{.1} - c_1 = [5, 0] \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 10 \end{bmatrix} - (-5) = 0$
- $N_{.1}$: $B^{-1} A_{.1} = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 10 \end{bmatrix} = \begin{bmatrix} -1 \\ 15 \end{bmatrix}$
- x_3의 rc: $c_B B^{-1} A_{.3} - c_3 = [6, 0] \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 10 \end{bmatrix} - 13 = 2$
- $N_{.3}$: $B^{-1} A_{.3} = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 10 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$
- x_4의 rc: $c_B B^{-1} A_{.4} - c_4 = [6, 0] \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 0 = 5$
- $N_{.4}$: $B^{-1} A_{.4} = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$

(0) $Z + 0x_1 + 2x_3 + 5x_4 = 100$
(1) $x_2 - x_1 + 3x_3 + x_4 = 20$
(2) $x_5 + 15x_1 - 5x_3 - 5x_4 = 0$

- 기저해: (0, 20, 0, 0, 0). feasible
- rc: (0, 0, 2, 5, 0). 최적.

7.1-4

a

쌍대문제

$$
\begin{aligned}
\text{Minimize} \quad & W = 40y_1 + 20y_2 + 90y_3 \\
\text{Subject to} \quad & 3y_1 + y_2 + 5y_3 \geq 5 \\
& y_1 + y_2 + 3y_3 \geq 10 \\
& y_1, y_2, y_3 \geq 0
\end{aligned}
$$

iteration 1

	Z	x_1	x_2	x_3	x_4	x_5	RHS
	1	-5	-10	0	0	0	0
x_3	0	3	1	1	0	0	40
x_4	0	1	1	0	1	0	20
x_5	0	5	3	0	0	1	90

- bfs: (0, 0, 40, 20, 90)
- 상보기저해: (0, 0, 0, -5, -10)

iteration 2

	Z	x_1	x_2	x_3	x_4	x_5	RHS
	1	5	0	0	10	0	200
x_3	0	2	0	1	-1	0	20
x_2	0	1	1	0	1	0	20
x_5	0	2	0	0	-3	1	30

- bfs: (0, 20, 20, 0, 30)
- 상보기저해: (0, 10, 0, 5, 0)

b

$$\begin{aligned} \text{Minimize} \quad & W = 40y_1 + 20y_2 + 90y_3 \\ \text{Subject to} \quad & -3y_1 - y_2 - 5y_3 \leq -5 \\ & -y_1 - y_2 - 3y_3 \leq -10 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$

$$\begin{aligned} \text{Maximize} \quad & -W = -40y_1 - 20y_2 - 90y_3 \\ \text{Subject to} \quad & -3y_1 - y_2 - 5y_3 + y_4 = -5 \\ & -y_1 - y_2 - 3y_3 + y_5 = -10 \\ & y_1, y_2, y_3, y_4, y_5 \geq 0 \end{aligned}$$

iteration 1

	$-W$	y_1	y_2	y_3	y_4	y_5	RHS
	1	40	20	90	0	0	0
y_4	0	-3	-1	-5	1	0	-5
y_5	0	-1	-1	-3	0	1	-10

- bfs: (0, 0, 0, -5, -10)

iteration 2

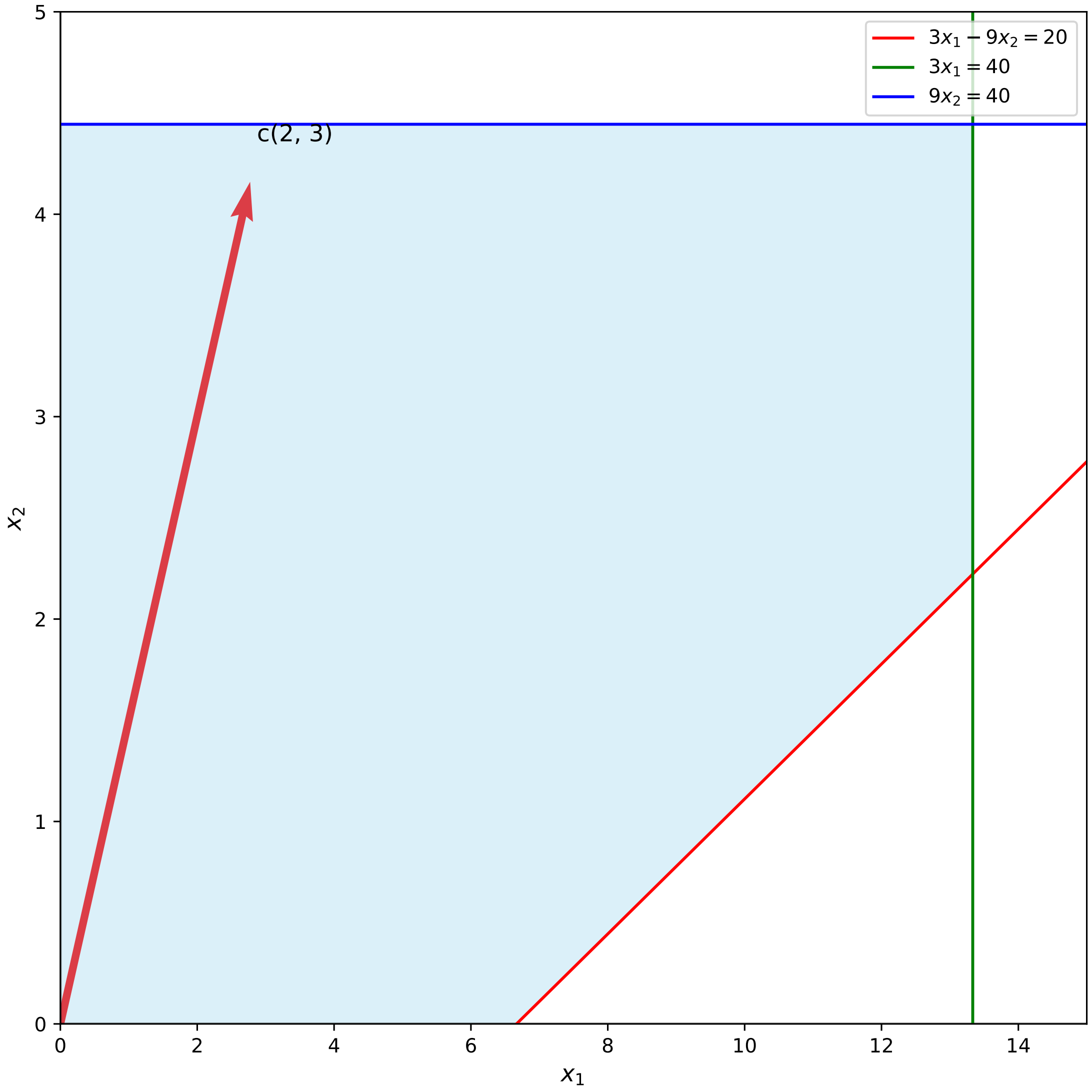
	$-W$	y_1	y_2	y_3	y_4	y_5	RHS
	1	20	0	30	0	20	-200
y_4	0	-2	0	-2	1	-1	5
y_2	0	1	1	3	0	-1	10

- bfs: (0, 10, 0, 5, 0)

7.3-1

$$\begin{aligned} \text{Maximize} \quad & Z = 2x_1 + 3x_2 \\ \text{Subject to} \quad & 3x_1 - 9x_2 \leq 20 \\ & 0 \leq x_1 \leq \frac{40}{3}, 0 \leq x_2 \leq \frac{40}{9} \end{aligned}$$

a



- bfs: $(\frac{40}{3}, \frac{40}{9})$
- obj: 40

b

$$\begin{aligned} \text{Maximize} \quad & Z - 2x_1 - 3x_2 = 0 \\ \text{Subject to} \quad & x_3 = 20 - 3x_1 + 9x_2 \\ & 0 \le x_1 \le \frac{40}{3}, 0 \le x_2 \le \frac{40}{9}, x_3 \ge 0 \end{aligned}$$

- x_2 enter

$$\begin{aligned} Z - 2x_1 - 3x_2 = 0 \\ x_3 = 20 + 9x_2 \quad \dots \quad x_2 \le \frac{40}{9} \end{aligned}$$

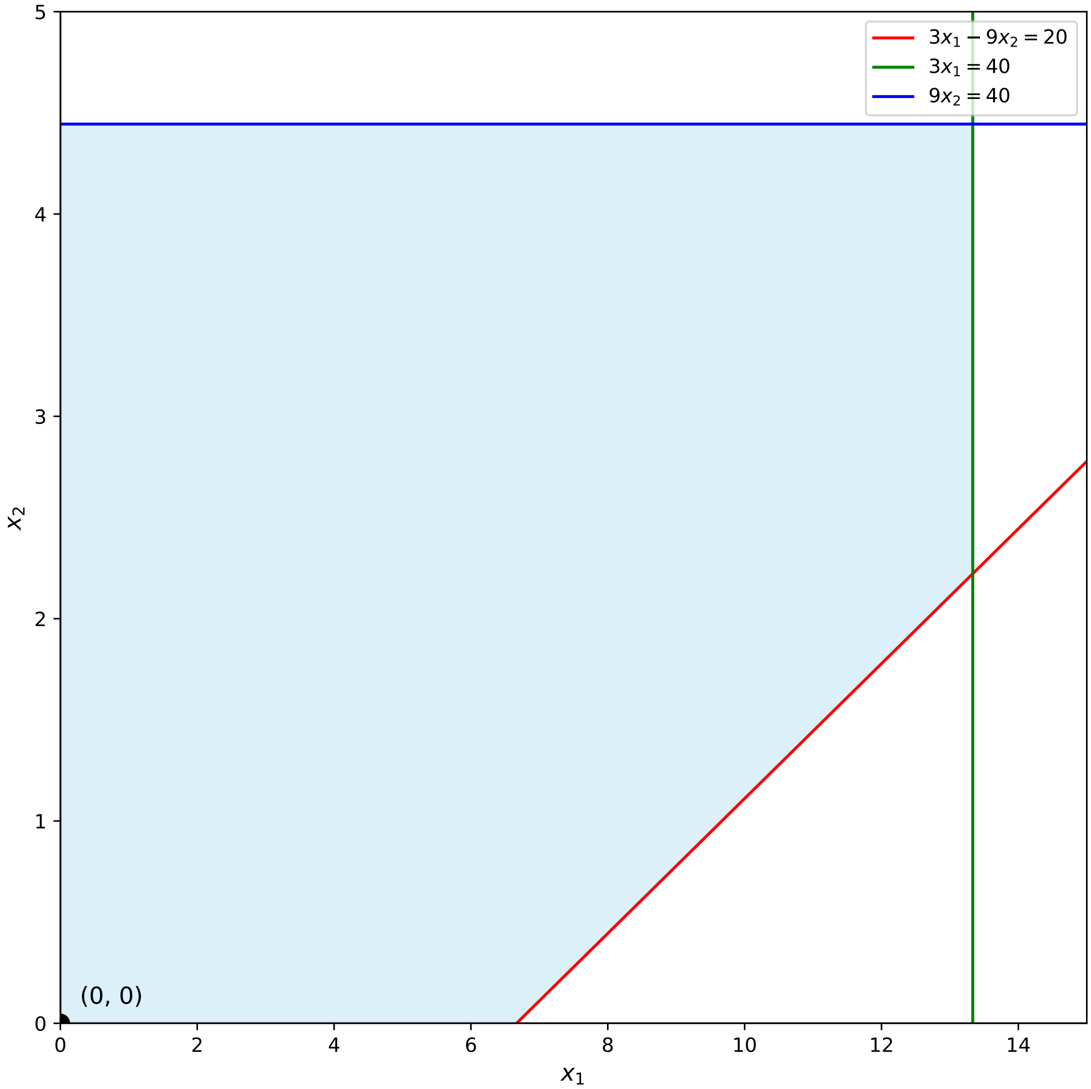
$$\begin{aligned} x_2 = \frac{40}{9} - y_2 \\ x_3 = 60 - 3x_1 - 9y_2 \quad \dots \quad x_1 \le \frac{40}{3} \\ Z - 2x_1 + 3y_2 = \frac{40}{3} \end{aligned}$$

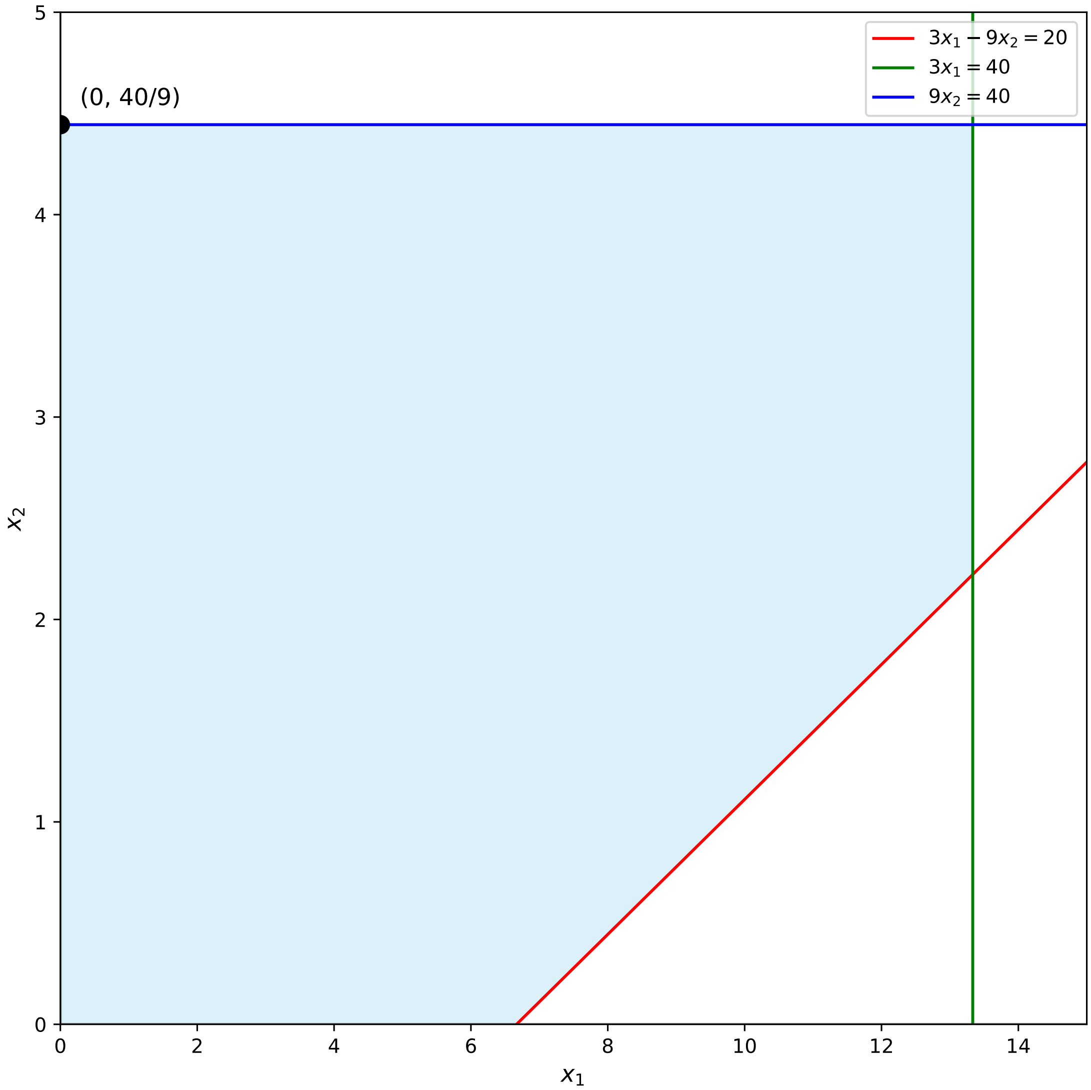
- x_1 enter

$$\begin{aligned} x_1 = \frac{40}{3} - y_1 \\ x_3 = 20 + 3y_1 - 9y_2 \\ Z + 2y_1 + 3y_2 = 40 \end{aligned}$$

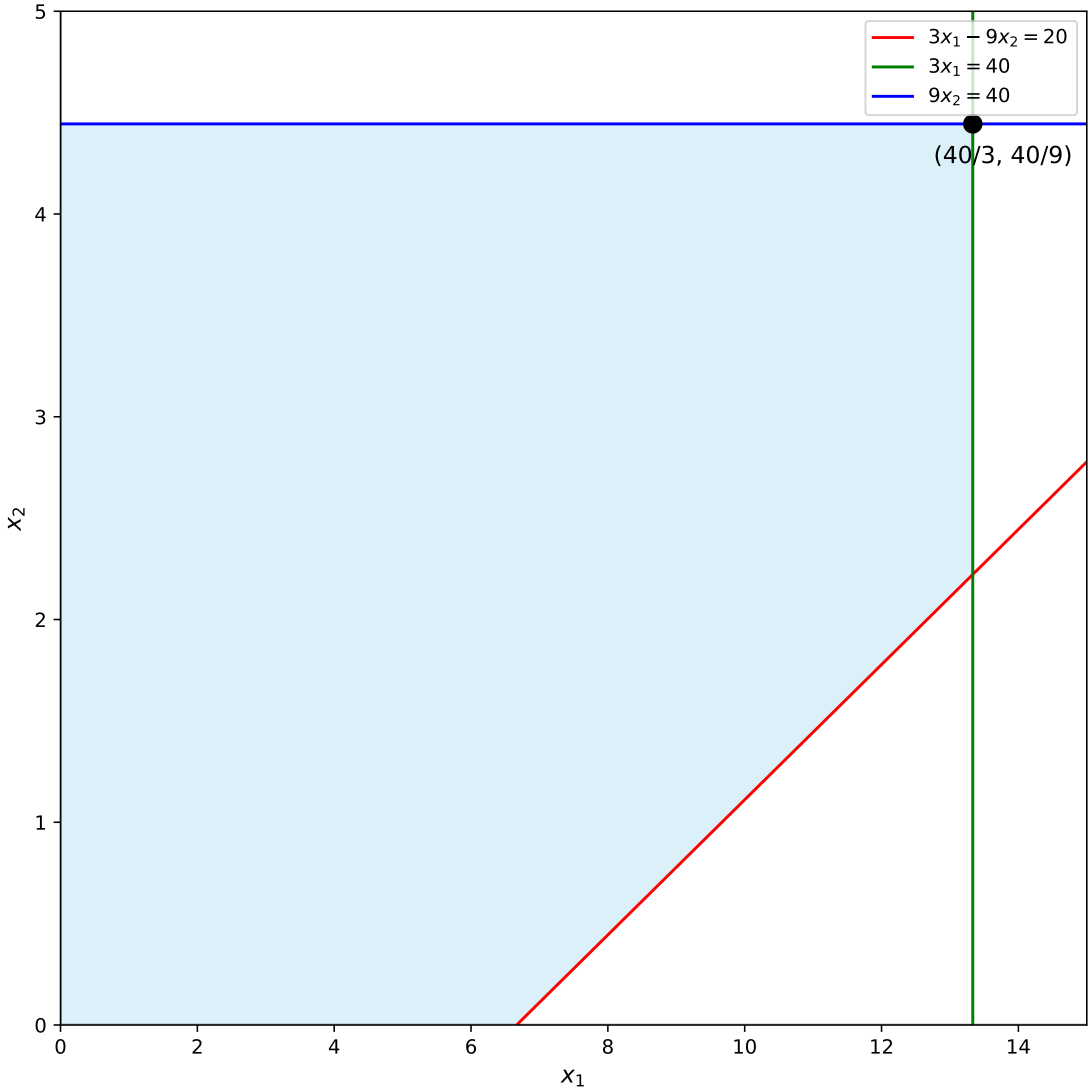
- bfs$(x_1, x_2, x_3, y_1, y_2)$: $(\frac{40}{3}, \frac{40}{9}, 20, 0, 0)$
- obj: = 40

c





(0, 40/9)
$3x_1 - 9x_2 = 20$
$3x_1 = 40$
$9x_2 = 40$
x_1
x_2
0
2
4
6
8
10
12
14
0
1
2
3
4
5


$3x_1 - 9x_2 = 20$
$3x_1 = 40$
$9x_2 = 40$
(40/3, 40/9)
x_1
x_2

8.2-7

a

초기 수송표

	목적지 1	목적지 2	목적지 3	목적지 4	공급
근원지 1	3	7	6	4	**5**
근원지 2	2	4	3	8	**2**
근원지 3	4	3	8	5	**3**
수요	**3**	**3**	**2**	**2**	

iteration

	목적지 1	목적지 2	목적지 3	목적지 4	공급
근원지 1	**3** ($x_{11} = 3$)	7	6	4	2
근원지 2	2	4	3	8	2
근원지 3	4	3	8	5	3
수요	0	**3**	**2**	**2**	

	목적지 1	목적지 2	목적지 3	목적지 4	공급
근원지 1	**3** ($x_{11} = 3$)	**2** ($x_{12} = 2$)	6	4	0
근원지 2	2	4	3	8	**2**
근원지 3	4	3	8	5	**3**
수요	0	1	**2**	**2**	

	목적지 1	목적지 2	목적지 3	목적지 4	공급
근원지 1	**3** ($x_{11} = 3$)	**2** ($x_{12} = 2$)	6	4	0
근원지 2	2	**1** ($x_{22} = 1$)	3	8	1
근원지 3	4	3	8	5	**3**
수요	0	0	**2**	**2**	

	목적지 1	목적지 2	목적지 3	목적지 4	공급
근원지 1	**3** ($x_{11} = 3$)	**2** ($x_{12} = 2$)	6	4	0
근원지 2	2	**1** ($x_{22} = 1$)	**1** ($x_{23} = 1$)	8	0
근원지 3	4	3	8	5	**3**
수요	0	0	1	**2**	

	목적지 1	목적지 2	목적지 3	목적지 4	공급
근원지 1	**3** ($x_{11} = 3$)	**2** ($x_{12} = 2$)	6	4	0
근원지 2	2	**1** ($x_{22} = 1$)	**1** ($x_{23} = 1$)	8	0
근원지 3	4	3	**1** ($x_{33} = 1$)	5	2
수요	0	0	0	**2**	

	목적지 1	목적지 2	목적지 3	목적지 4	공급
근원지 1	**3** ($x_{11} = 3$)	**2** ($x_{12} = 2$)	6	4	0
근원지 2	2	**1** ($x_{22} = 1$)	**1** ($x_{23} = 1$)	8	0
근원지 3	4	3	**1** ($x_{33} = 1$)	**2** ($x_{34} = 2$)	0
수요	0	0	0	0	

초기 기저 가능해

- $x_{11} = 3$ (비용: $3 \times 3 = 9$)
- $x_{12} = 2$ (비용: $7 \times 2 = 14$)
- $x_{22} = 1$ (비용: $4 \times 1 = 4$)
- $x_{23} = 1$ (비용: $3 \times 1 = 3$)
- $x_{33} = 1$ (비용: $8 \times 1 = 8$)
- $x_{34} = 2$ (비용: $5 \times 2 = 10$)
- 다른 모든 $x_{ij} = 0$.

초기해의 총 수송 비용

$$Z = (3 \times 3) + (7 \times 2) + (4 \times 1) + (3 \times 1) + (8 \times 1) + (5 \times 2)$$

$$Z = 9 + 14 + 4 + 3 + 8 + 10 = 48$$

iteration 1

기저 변수

- $u_1 = 0$ 으로 설정.

- $x_{11} : u_1 + v_1 = c_{11} \Rightarrow 0 + v_1 = 3 \Rightarrow v_1 = 3$
- $x_{12} : u_1 + v_2 = c_{12} \Rightarrow 0 + v_2 = 7 \Rightarrow v_2 = 7$
- $x_{22} : u_2 + v_2 = c_{22} \Rightarrow u_2 + 7 = 4 \Rightarrow u_2 = -3$
- $x_{23} : u_2 + v_3 = c_{23} \Rightarrow -3 + v_3 = 3 \Rightarrow v_3 = 6$
- $x_{33} : u_3 + v_3 = c_{33} \Rightarrow u_3 + 6 = 8 \Rightarrow u_3 = 2$
- $x_{34} : u_3 + v_4 = c_{34} \Rightarrow 2 + v_4 = 5 \Rightarrow v_4 = 3$

$\therefore u_1 = 0, u_2 = -3, u_3 = 2, v_1 = 3, v_2 = 7, v_3 = 6, v_4 = 3.$

비기저 변수 제약식

- $x_{13} : c_{13} - u_1 - v_3 = 6 - 0 - 6 = 0$
- $x_{14} : c_{14} - u_1 - v_4 = 4 - 0 - 3 = 1$
- $x_{21} : c_{21} - u_2 - v_1 = 2 - (-3) - 3 = 2 - (-3 + 3) = 2$
- $x_{24} : c_{24} - u_2 - v_4 = 8 - (-3) - 3 = 8 - (-3 + 3) = 8$
- $x_{31} : c_{31} - u_3 - v_1 = 4 - 2 - 3 = -1$
- $x_{32} : c_{32} - u_3 - v_2 = 3 - 2 - 7 = -6$
- enter: x_{32}
- 경로: $x_{32} \to x_{33} \to x_{23} \to x_{22} \to x_{32}$
- $\theta = \text{min}(x_{33}, x_{22}) = \text{min}(1, 1) = 1.$

할당량

- $x_{32} = 0 + \theta = 1$
- $x_{33} = 1 - \theta = 0$
- $x_{23} = 1 + \theta = 2$
- $x_{22} = 1 - \theta = 0$ (탈락 변수)
- 기저해: $x_{11} = 3, x_{12} = 2, x_{23} = 2, x_{32} = 1, x_{33} = 0, x_{34} = 2$
- $Z = (3 \times 3) + (7 \times 2) + (3 \times 2) + (3 \times 1) + (8 \times 0) + (5 \times 2) = 42.$

iteration 2

기저 변수

- $u_1 = 0$
- $x_{11} : u_1 + v_1 = c_{11} \Rightarrow 0 + v_1 = 3 \Rightarrow v_1 = 3$
- $x_{12} : u_1 + v_2 = c_{12} \Rightarrow 0 + v_2 = 7 \Rightarrow v_2 = 7$

- $x_{32} : u_3 + v_2 = c_{32} \Rightarrow u_3 + 7 = 3 \Rightarrow u_3 = -4$
- $x_{33} : u_3 + v_3 = c_{33} \Rightarrow -4 + v_3 = 8 \Rightarrow v_3 = 12$
- $x_{34} : u_3 + v_4 = c_{34} \Rightarrow -4 + v_4 = 5 \Rightarrow v_4 = 9$
- $x_{23} : u_2 + v_3 = c_{23} \Rightarrow u_2 + 12 = 3 \Rightarrow u_2 = -9$

$\therefore u_1 = 0, u_2 = -9, u_3 = -4, v_1 = 3, v_2 = 7, v_3 = 12, v_4 = 9.$

비기저 변수의 제약식

- $x_{13} : k_{13} = c_{13} - u_1 - v_3 = 6 - 0 - 12 = -6$
- $x_{14} : k_{14} = c_{14} - u_1 - v_4 = 4 - 0 - 9 = -5$
- $x_{21} : k_{21} = c_{21} - u_2 - v_1 = 2 - (-9) - 3 = 2 + 9 - 3 = 8$
- $x_{22} : k_{22} = c_{22} - u_2 - v_2 = 4 - (-9) - 7 = 4 + 9 - 7 = 6$
- $x_{24} : k_{24} = c_{24} - u_2 - v_4 = 8 - (-9) - 9 = 8 + 9 - 9 = 8$
- $x_{31} : k_{31} = c_{31} - u_3 - v_1 = 4 - (-4) - 3 = 4 + 4 - 3 = 5$
- enter: x_{13}
- 경로: $(1,3) \xrightarrow{+} (3,3) \xrightarrow{-} (3,2) \xrightarrow{+} (1,2) \xrightarrow{-} (1,3)$
- $\theta = \text{min}(x_{33}, x_{12}) = \text{min}(0, 2) = 0.$

할당량

- $x_{13} = 0 + 0 = 0$
- $x_{33} = 0 - 0 = 0$ (탈락 변수)
- $x_{32} = 1 + 0 = 1$
- $x_{12} = 2 - 0 = 2$
- 기저해: $x_{11} = 3, x_{12} = 2, x_{13} = 0, x_{23} = 2, x_{32} = 1, x_{34} = 2.$
- $Z = (3 \times 3) + (7 \times 2) + (6 \times 0) + (3 \times 2) + (3 \times 1) + (5 \times 2) = 9 + 14 + 0 + 6 + 3 + 10 = 42.$

iteration 3

기저 변수

- $u_1 = 0$ 으로 설정
- $x_{11} : u_1 + v_1 = c_{11} \Rightarrow 0 + v_1 = 3 \Rightarrow v_1 = 3$
- $x_{12} : u_1 + v_2 = c_{12} \Rightarrow 0 + v_2 = 7 \Rightarrow v_2 = 7$
- $x_{13} : u_1 + v_3 = c_{13} \Rightarrow 0 + v_3 = 6 \Rightarrow v_3 = 6$

- $x_{23} : u_2 + v_3 = c_{23} \Rightarrow u_2 + 6 = 3 \Rightarrow u_2 = -3$
- $x_{32} : u_3 + v_2 = c_{32} \Rightarrow u_3 + 7 = 3 \Rightarrow u_3 = -4$
- $x_{34} : u_3 + v_4 = c_{34} \Rightarrow -4 + v_4 = 5 \Rightarrow v_4 = 9$

$\therefore u_1 = 0, u_2 = -3, u_3 = -4, v_1 = 3, v_2 = 7, v_3 = 6, v_4 = 9.$

비기저 변수 제약식

- $x_{14} : k_{14} = c_{14} - u_1 - v_4 = 4 - 0 - 9 = -5$
- $x_{21} : k_{21} = c_{21} - u_2 - v_1 = 2 - (-3) - 3 = 2 + 3 - 3 = 2$
- $x_{22} : k_{22} = c_{22} - u_2 - v_2 = 4 - (-3) - 7 = 4 + 3 - 7 = 0$
- $x_{24} : k_{24} = c_{24} - u_2 - v_4 = 8 - (-3) - 9 = 8 + 3 - 9 = 2$
- $x_{31} : k_{31} = c_{31} - u_3 - v_1 = 4 - (-4) - 3 = 4 + 4 - 3 = 5$
- $x_{33} : k_{33} = c_{33} - u_3 - v_3 = 8 - (-4) - 6 = 8 + 4 - 6 = 6$
- enter: x_{14}
- 경로: $(1,4) \xrightarrow{+} (3,4) \xrightarrow{-} (3,2) \xrightarrow{+} (1,2) \xrightarrow{-} (1,4)$
- $\theta = \text{min}(x_{34}, x_{12}) = \text{min}(2,2) = 2.$

할당량

- $x_{14} = 0 + 2 = 2$
- $x_{34} = 2 - 2 = 0$ (탈락 변수)
- $x_{32} = 1 + 2 = 3$
- $x_{12} = 2 - 2 = 0$
- 기저해: $x_{11} = 3, x_{12} = 0, x_{13} = 0, x_{14} = 2, x_{23} = 2, x_{32} = 3.$
- $Z = (3 \times 3) + (7 \times 0) + (6 \times 0) + (4 \times 2) + (3 \times 2) + (3 \times 3) = 32$

iteration 4

기저 변수

- $u_1 = 0$ 으로 설정.
- $x_{11} : u_1 + v_1 = c_{11} \Rightarrow 0 + v_1 = 3 \Rightarrow v_1 = 3$
- $x_{12} : u_1 + v_2 = c_{12} \Rightarrow 0 + v_2 = 7 \Rightarrow v_2 = 7$
- $x_{13} : u_1 + v_3 = c_{13} \Rightarrow 0 + v_3 = 6 \Rightarrow v_3 = 6$
- $x_{14} : u_1 + v_4 = c_{14} \Rightarrow 0 + v_4 = 4 \Rightarrow v_4 = 4$
- $x_{23} : u_2 + v_3 = c_{23} \Rightarrow u_2 + 6 = 3 \Rightarrow u_2 = -3$

- $x_{32} : u_3 + v_2 = c_{32} \Rightarrow u_3 + 7 = 3 \Rightarrow u_3 = -4$

$\therefore u_1 = 0, u_2 = -3, u_3 = -4, v_1 = 3, v_2 = 7, v_3 = 6, v_4 = 4.$

비기저 변수 제약식

- $x_{21} : k_{21} = c_{21} - u_2 - v_1 = 2 - (-3) - 3 = 2$
- $x_{22} : k_{22} = c_{22} - u_2 - v_2 = 4 - (-3) - 7 = 0$
- $x_{24} : k_{24} = c_{24} - u_2 - v_4 = 8 - (-3) - 4 = 7$
- $x_{31} : k_{31} = c_{31} - u_3 - v_1 = 4 - (-4) - 3 = 5$
- $x_{33} : k_{33} = c_{33} - u_3 - v_3 = 8 - (-4) - 6 = 6$
- $x_{34} : k_{34} = c_{34} - u_3 - v_4 = 5 - (-4) - 4 = 5$

모든 $k_{ij} \geq 0$ 이므로, 현재 해가 최적.

- $x_{11} = 3$
- $x_{12} = 0$
- $x_{13} = 0$
- $x_{14} = 2$
- $x_{23} = 2$
- $x_{32} = 3$
- 다른 모든 $x_{ij} = 0.$

$Z = 9 + 8 + 6 + 9 = 32.$

8.4-5

피할당인	과업 1	과업 2	과업 3	과업 4
A	4	1	0	1
B	1	3	4	0
C	3	2	1	3
D	2	2	3	0

피할당인	과업 1	과업 2	과업 3	과업 4
A	4	1	0	1
B	1	3	4	0
C	2	1	0	2
D	2	2	3	0

피할당인	과업 1	과업 2	과업 3	과업 4
A	3	0	0	1
B	0	2	4	0
C	1	0	0	2
D	1	1	3	0

최적 할당 및 총비용:

- A -> 과업 2
- B -> 과업 1
- C -> 과업 3
- D -> 과업 4

Z = 1 + 1 + 1 + 0 = 3