

## Geese in a Trenchcoat II

This is a **regular task**. You must submit a PDF, which can be produced using the L<sup>A</sup>T<sub>E</sub>X template on Moodle, exported from a word processor, hand-written or any other method.

Recall the example problem *Geese in a Trenchcoat* from the first lecture.

Consider the following alternative algorithm for the same problem, which uses a “two pointers” method.

Sort the geese from left to right in ascending order of height, using merge sort. Put a red hat on the shortest goose and a blue hat on the tallest goose. Then:

- if the hatted geese have total height less than  $H$ , move the red hat one space to the right;
- if the hatted geese have total height greater than  $H$ , move the blue hat one space to the left, and
- if the hatted geese have total height equal to  $H$ , report this pair as a solution.

If a single goose ever has both hats, report that there is no solution.

Justify why the algorithm is correct, and analyse its time complexity.

**Hint:** When we remove a hat (say the red hat) from a particular goose, what are we saying about that goose, and how do we know that conclusion to be true?

### Advice.

- Explain why, if there is a solution, then it is found by this algorithm.
  - You’ll need to justify that each move of the hats is valid, i.e. doesn’t lose any solutions.
- Explain why, if there is no solution, then the algorithm reports that there is no solution. This part should be easy!
- To analyse the time complexity, consider the sorting as well as the part with the hats.
  - How many times can each hat move?

**Expected length:** Up to half a page.