Literally 1984

This is a **regular task**. You must submit a PDF, which can be produced using the LaTeX template on Moodle, exported from a word processor, hand-written or any other method.

The city of UNSW has decided to partition High Street into voting districts. There are n houses on High Street, living on consecutive addresses labelled $1, 2, \ldots, n$. Each voting district is an interval of addresses $i, i+1, \ldots, j$ with $1 \leq i < j \leq n$. The city has enforced two constraints:

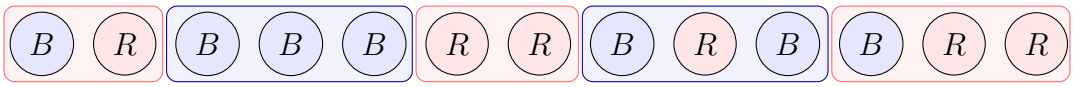
(1) Each house on High Street must belong to *exactly* one voting district.

(2) The number of houses in each district must lie between k and $2k$, where k is a parameter.

In each election, houses can vote for either Oceania or Eurasia but they can only vote for one party. Since the majority of houses in the city of UNSW support Oceania, the city deems a district *favourable* if more than half of the houses in the district support Oceania; otherwise, the district is *unfavourable*. Of course, the city has a complete record of all of the votes in the election.

The next election is coming up and the city has decided to ask you, as the budding algorithmist, to help assign districts. You are given a boolean array Vote$[1..n]$, where Vote$[i] = 1$ if resident i voted for Oceania and Vote$[i] = 0$ if resident i voted for Eurasia. You are also given an integer k. Describe an $O(nk^2)$ algorithm that returns the largest number of favourable districts in any legal partition of the houses.

For example, consider the following legal partition of $n = 13$ houses with $k = 2$ (therefore, each district can be assigned 2 to 4 houses). Houses who voted for Oceania are coloured blue and the houses who voted for Eurasia are coloured red. There are two favourable districts (coloured blue) and three unfavourable districts (coloured red).



A legal partition of houses to five districts. The first district is unfavourable, the second district is favourable, the third district is unfavourable, the fourth district is favourable, and the fifth district is unfavourable.

Note. *You may assume that a legal partition always exists.*

Advice. Your solution should include:

- A clear subproblem definition.
- Base cases for your subproblem definition.
- A well-defined recurrence with respect to your subproblem definition and base cases.
- Some output that solves the original problem, as a function of the results generated by your recurrence.
- A correct order of computation, with respect to your recurrence.
- Time complexity analysis for your algorithm.
- Justification that your algorithm solves the problem correctly, with specific reference to the correctness of the base case(s), recurrence and overall answer.

Expected length: Up to two pages.