

Chocolate Bar

This is a **regular task**. You must submit a PDF, which can be produced using the L^AT_EX template on Moodle, exported from a word processor, hand-written or any other method.

Charlotte looks after children at a nursery. She has to split up a giant Toblerone chocolate bar amongst the children. The bar is made up of n pieces in a line, the i th of which contains $T[i]$ clumps of nougat. Charlotte wants to split the bar into contiguous segments to give to the children. A child will only be happy if their segment has at least k clumps of nougat in it. A child can only receive one contiguous segment.

For example, if $A = [2, 4, 1, 2, 3, 1, 1, 2, 1]$ and $k = 4$, then Charlotte can make three children happy. One way to achieve this is to give out the segments according to the diagram below, where each colour (red, blue and green) indicates the segment given to each child, and white pieces are unused.



This way, the three children get five, five and four clumps of nougat respectively, meeting the requirement. This is not the only way to make three children happy, but it is not possible to make at least four children happy.

- Design an efficient algorithm that determines the maximum number of children that Charlotte can make happy.
- Prove the correctness of your algorithm using *greedy stays ahead* or an *exchange argument*.

Hint: Given an instance of the problem, in what sense might each stage of your algorithm be ‘ahead’?

Advice.

- Design your algorithm, and justify its time complexity. You don’t have to prove its correctness until the next part! You should use the writing conventions from lectures.
- All the information you need is in the task.

Expected length:

- For (a), up to a short paragraph
- For (b), up to 3 paragraphs