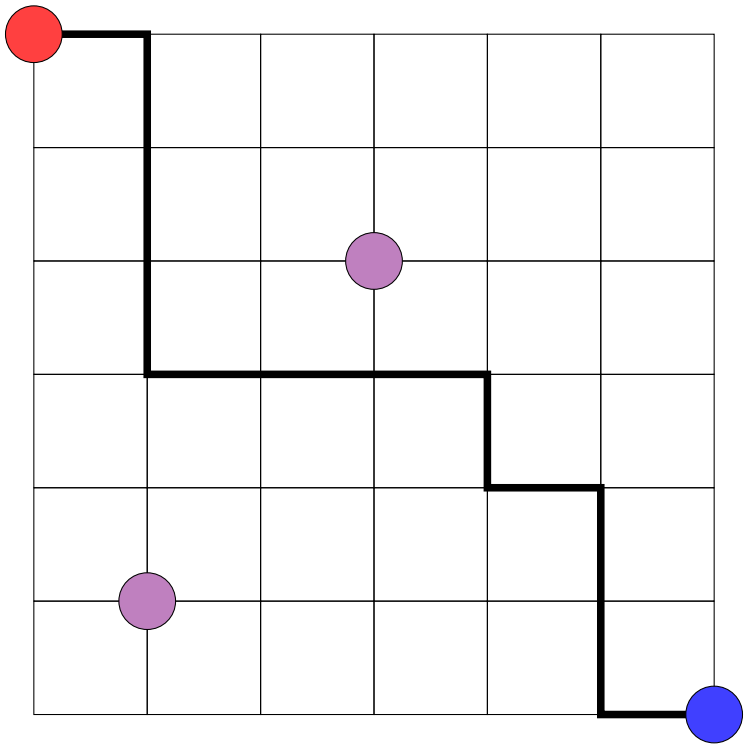
Slide To The Bottom Right

This is a **regular task**. You must submit a PDF, which can be produced using the LaTeX template on Moodle, exported from a word processor, hand-written or any other method.

Suppose we have a grid G with n rows and m columns and a set S of points in this grid. A path is *valid* if it starts from the top-leftmost point (1, 1) and finishes at the bottom-rightmost point (m, n), and the only moves are to step one unit to the right or one unit down.

A path is *unblocked* if it is both valid and does not go through any points in S. See the diagram below: $(1, 1)$, (m, n) are labelled red and blue respectively; elements of S are labelled violet. The thick black outline is an example of an unblocked path, as it does not go through any violet points and uses only rightwards and downwards moves.



Design and analyse an efficient algorithm that outputs the number of unblocked paths.

Advice. Your solution should include:

- A clear subproblem definition.
- Base cases for your subproblem definition.
- A well-defined recurrence with respect to your subproblem definition and base cases.
- Some output that solves the original problem, as a function of the results generated by your recurrence.
- A correct order of computation, with respect to your recurrence.
- Time complexity analysis for your algorithm.
- Justification that your algorithm solves the problem correctly, with specific reference to the correctness of the base case(s), recurrence and overall answer.

Consider the case where $(1, 1) \in S$. How many paths are there in that case?

Expected length: Up to half a page.