Art Gallery Escape

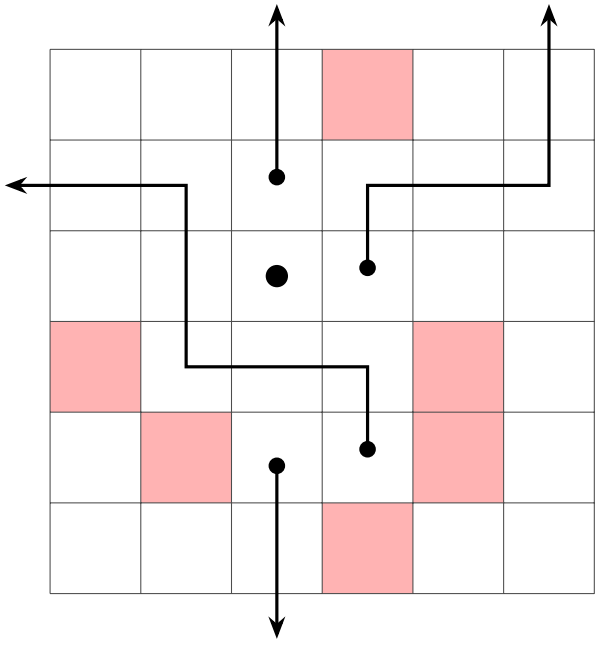
This is a **regular task**. You must submit a PDF, which can be produced using the LaTeX template on Moodle, exported from a word processor, hand-written or any other method.

There are k thieves in an art gallery, which is made up of rooms in an n by n grid. Each thief starts in a given *distinct* room, where they have just stolen an expensive artwork! As the ringleader, you now have to give instructions to the thieves *one at a time* to help them escape the gallery without getting caught.

Every room has a door on each of its four walls, allowing access to the next room in up to four directions. Every door on the perimeter of the gallery allows a person to exit the building.

Every room in the gallery contains a security camera, which starts recording a few seconds after it detects movement. As such, a single thief can pass through each room, but if a second thief enters the room they will get caught. Furthermore, there are some rooms containing particularly precious artworks which are protected by security guards. The thieves cannot enter these rooms at all, as they will be caught immediately.

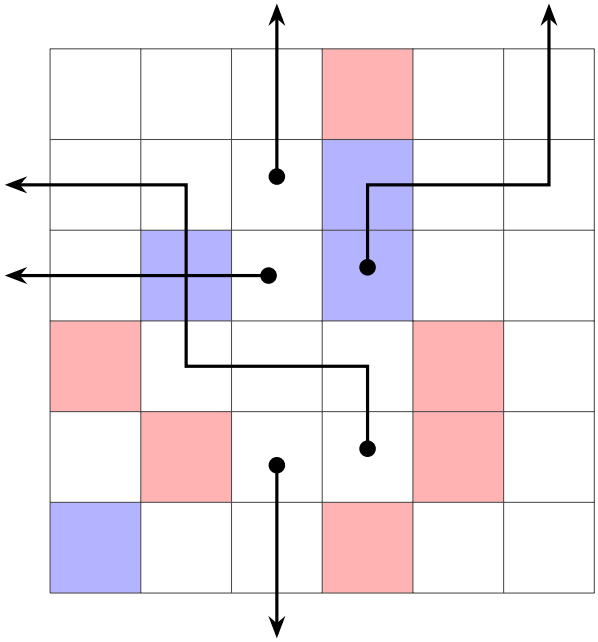
As the ringleader of this gang of thieves, your task is to determine how many thieves can escape without getting caught. For example, in the layout below, four of the five thieves can escape the gallery by taking the paths shown. The red squares have security guards.



(a) Given the starting coordinates of each thief and the coordinates of the rooms with security guards, design a flow network to find the number of thieves that can escape the gallery. Justify the correctness and time complexity of your algorithm.

(b) Now, suppose that due to budget constraints, there are rooms that do not have a security camera. Any number of thieves can pass through these rooms without getting caught. In the example below, two thieves escape through one of the unguarded rooms (in blue).

The safe rooms may appear anywhere in the building that is not protected by a security guard. This includes the perimeter, the rooms where the thieves start, and those adjacent to other safe rooms. State how you would modify your algorithm from part (a) to determine how many thieves can escape. Justify the correctness and time complexity of your approach.



Advice.

(a) You should design a flow network that solves the problem when an appropriate flow algorithm is used with your network as input.

Your flow network should be well-defined; that is, all edges (and their capacities) as well as all vertices should be constructed clearly.

When proving the correctness of your algorithm, you should show that every valid collection of thief paths corresponds to a valid flow, and that every valid flow corresponds to a valid collection of thief paths. You also need to conclude appropriately.

(b) You do not need to rewrite parts of part (a) if you are reusing those parts, and they are otherwise identical for your solution for part (b). You should still argue the correctness of any additions you make.

Expected length: Up to two pages.