Cola Cables

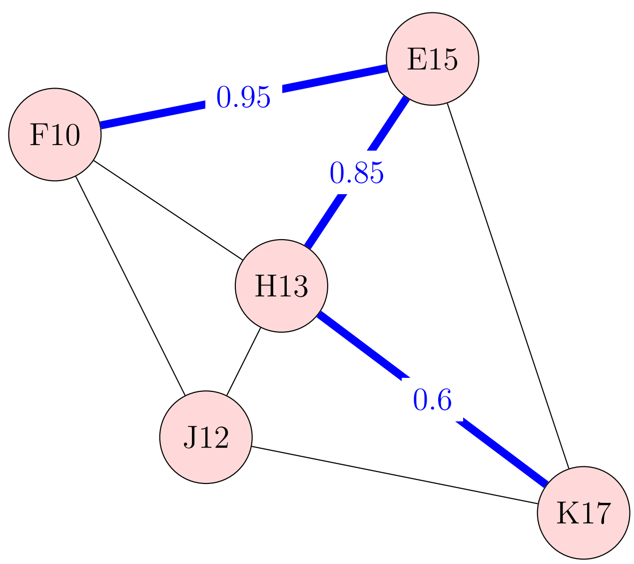
This is a **regular task**. You must submit a PDF, which can be produced using the LaTeX template on Moodle, exported from a word processor, hand-written or any other method.

Thomas is in charge of a distribution system for the Important Cryptograms and Cryptography Society (ICCSOC) at UNSW that connects many locations around the UNSW Kensington campus.

This involves n locations, and m transmission cables that connect a distinct pair of locations as a graph $G = (V, E)$. Assume that messages can be sent in either direction across each cable. Due to a spill of off-brand Cola, each of the cables has a probability p_i that a particular message will get through. You may assume that *all the probabilities are positive* (in particular, non-zero), so no cable is certain to fail. Further, you may assume that a path exists between any pair of distinct locations.

A message is received at its destination if and only if it is not blocked by one of the transmission cables along the way. Note that all probabilities are independent.

For example, in the diagram below, the probability that a message can travel from $F10$ to $E15$ is 0.95.



(a) Thomas, being the energetic encoder that he is, prefers to look at *information* receiving rather than probability. Information in this context is defined as, for an event A,

$$I(A) = -\log_2(p(A)),$$

where $p(A)$ is the probability that event A occurs.

Note: For those curious about this information quantity, it comes from a field of mathematics known as Information Theory. See here for the paper by Claude Elwood Shannon. This is explored in MATH3411: Information, codes and ciphers, among other courses offered at UNSW.

For example, the amount of information received for a message we are trying to send from $E15$ to $H13$ is

$$\begin{aligned} I(E15 \to H13) &= -\log_2 0.85 \\ &\approx 0.2345 \end{aligned} \tag{1}$$

whereas the information a receiving a message from $K17$ to $H13$ is

$$\begin{aligned} I(K17 \to H13) &= -\log_2 0.6 \\ &\approx 0.7370. \end{aligned} \tag{2}$$

Note that the information that we get from $K17$ to $H13$ is larger because there is a higher probability of the message being blocked so we are more surprised when we actually see something (as in it's more interesting in the intuitive sense).

Calculate the information corresponding to sending a message from $F10$ to $E15$.

(b) Find the probability that a message can be successfully delivered from $K17$ to $H13$ to $E15$ to $F10$ via the blue path seen in the above diagram and use this to find the information of this event.

(c) Compare your answer to (b) to the information found in (1), (2) and (a).

Hint: Recall properties of logarithms.

(d) Having been a part of ICCSOC for over 10 years, Thomas has always wanted to know the probability that a message will get through between any pair of distinct locations, assuming the best transmission strategy. That is, we are interested in the highest transmission probability between every pair of vertices, rather than the path itself.

By restating this problem in terms of information rather than probability, efficiently provide Thomas with the maximum probabilities between every pair of distinct locations by applying an established algorithm.

Advice.

(a) You just need to determine the information.

(b) You need to briefly explain your calculation.

(c) You need to briefly explain your calculation.

(d) You need to explain why your reduction (restating the problem) to your chosen established graph algorithm is valid and correct. You also need to analyse the time complexity of the algorithm in total.

Expected length:

- For (a), up to one sentence.
- For (b), up to one paragraph.
- For (c), up to one paragraph.
- For (d), up to half a page.