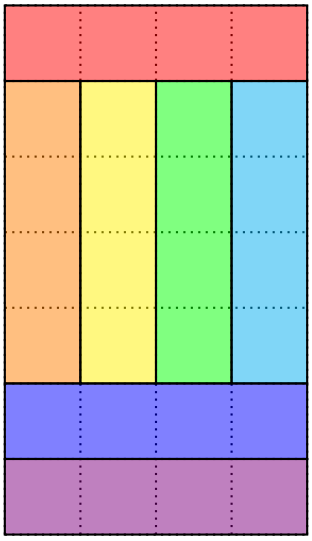
Career Change

This is a **regular task**. You must submit a PDF, which can be produced using the LaTeX template on Moodle, exported from a word processor, hand-written or any other method.

Having recently become the first person to beat the original NES version of Tetris, you've turned your interests to something much more interesting: dynamic programming! Of course, having loved Tetris for many years you only use your newfound skills to answer questions about Tetris that seem cool.

Recently, you've been thinking about how many configurations of a Tetris board there could be. After many hours working on the problem, you've determined that this is too difficult to answer, so you've decided to instead tackle a smaller version of the problem, limiting yourself only to I-pieces (which you know are 1×4 tiles) in a board with n rows and 4 columns, where $n \geq 1$. Having spent so long playing Tetris, you also know that you can rotate I-pieces, so they can be either 4×1 or 1×4.

An example of filling a board with I-pieces, and $n = 7$, is below:



The colours are only to make boundaries between the I-pieces clear, and do not have any other significance. Recolouring the pieces does not constitute a new arrangement.

The total number of possible ways to fill a board with 7 rows and 4 columns with I-pieces is 5.

Design an algorithm which determines the total number of ways to completely fill an $n \times 4$ board with I-pieces, and runs in $O(n)$ time.

Advice. Your solution should include:

- A clear subproblem definition.
- Base cases for your subproblem definition.
- A well-defined recurrence with respect to your subproblem definition and base cases.
- Some output that solves the original problem, as a function of the results generated by your recurrence.
- A correct order of computation, with respect to your recurrence.
- Time complexity analysis for your algorithm.
- Justification that your algorithm solves the problem correctly, with specific reference to the correctness of the base case(s), recurrence and overall answer.

Expected length: Up to a page.