

# I Can See You

This is a **regular task**. You must submit a PDF, which can be produced using the  $\text{\LaTeX}$  template on Moodle, exported from a word processor, hand-written or any other method.

You are given a directed graph  $G(V, E)$  where each vertex  $v \in V$  has a positive weight, denoted  $w(v)$ . A set  $S \subseteq V$  is said to be all-seeing if each vertex of the graph is reachable from at least one member of  $S$ . The weight of a subset  $S$  is the total weight of all its vertices. Consider the graph pictured below, with weights not shown.

```
graph LR; a((a)) --> b((b)); b --> c((c)); b --> d((d)); c --> e((e)); d --> e; e --> f((f)); e --> g((g)); f --> f
```

We can verify that  $\{a, c, e\}$  is all-seeing. However,  $\{a, c\}$  is *not* all-seeing, as vertices  $e, f$  and  $g$  are not reachable from either  $a$  or  $c$ .

vertex	$a$	$b$	$c$	$d$	$e$	$f$	$g$
reachable from	$a$	$a$	$a, c, e$	$a, e$	$e$	$e$	$e$

Design and analyse an efficient algorithm that finds the minimum weight all-seeing subset.

**Advice.**

- There are two key components to the proof. Both are implied in the problem statement, and the definition of an all-seeing set.

**Expected length:** Up to two-thirds of a page.