①

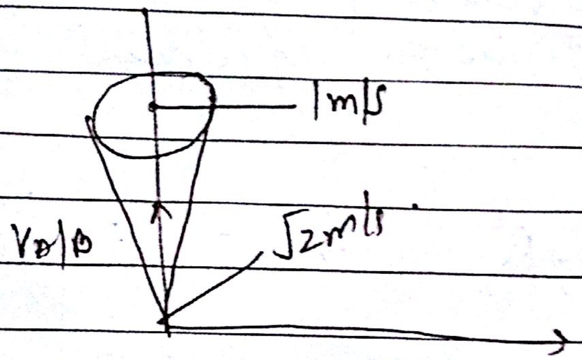
Mobile Robotics

Danish Sodhi
201331202

① (i) relative velocity $V_{1/2}$

$$V_A = \hat{i} + \hat{j}$$
$$V_B = \hat{i}$$
$$V_{A/B} = V_A - V_B = \hat{j}$$

(ii) Collision cone



Collision cone is the shaded region.
$V_{A/B}$ is in shaded region, so collision will occur

(iii) $$|\vec{r}|^2 = \frac{\vec{r} \cdot \vec{V}_{A/B}}{|V_{1/2}|^2} \geq R^2$$

$$|\vec{r}| = (0-0)\hat{i} + (0-3)\hat{j} = -3\hat{j}$$
$$V_{1/2} = a\hat{i} + b\hat{j}$$

$$9 - \frac{(\cdot 3\hat{j})(a\hat{i} + b\hat{j})}{a^2 + b^2} \geq 0.25$$

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$$\Rightarrow 2.916 \geq \frac{b}{a^2+b^2}$$

assume $b = 2$, $a = 1$ or $b = 3$, $a = 2$

$$2.916 \geq \frac{2}{4+1} = 0.4$$

$\therefore$ relative velocity can be $\hat{i} + 2\hat{j}$ or $2\hat{i} + 3\hat{j}$ to avoid collision

(d) possible relative velocity for which we can avoid collision without changing direction

$$|r|^2 - \frac{\hat{r}\,\vec{v}_{A/B}}{|v_{A/B}|^2} \geq R^2 \qquad v_{A/B} = (s-1)\hat{i} + s\hat{j}$$

$$a - \frac{(-3\hat{j})\left((s-1)\hat{i} + s\hat{j}\right)}{(s-1)^2 + s^2} \geq 0.25$$

$$2.916 \geq \frac{-s}{(s-1)^2 + s^2}$$

let $s = 3$

$$2.916 \geq \frac{-3}{4+9}$$

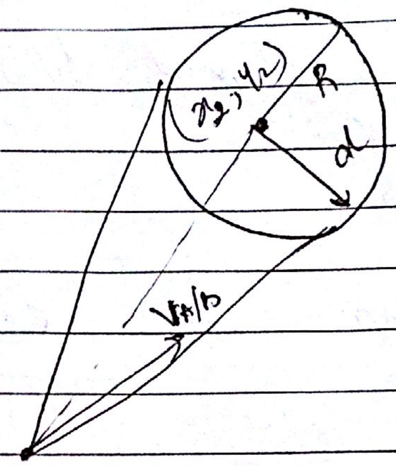
$$v_{A'B} = 2\hat{i} + 3\hat{j}$$

or let 5 = 5

$2.916 \geq \frac{-5}{16+25}$

$V_{AB} = 4\hat{i} + 5\hat{j}$

(2)



$$d^2 = |\vec{r}|^2 - \frac{\vec{r} \cdot \vec{V}_{1/2}}{|V_{1/2}|^2} \geq R^2$$

This equation says that object A should not move in collision cone, If it there will be a collision between A and B. And we need to avoid it
It can be a grazing condition therefore there is equal to sign.

So we basically convert velocity of A & B to relative velocity $V_{A/B}$ and consider radius of B $= r_A + r_B$ and then apply

pythagoras theorem, such that
relative velocity on the hypotenuse or
away from it.

$\therefore \vec{r}$ is the distance b/w A & B

$R = r_A + r_B$

Ie the hypotenuse is $\dfrac{\vec{r} \cdot V_{1/2}}{|V_{1/2}|^2}$

So by following this inequality
that base is greater than R,
then we can avoid collision.