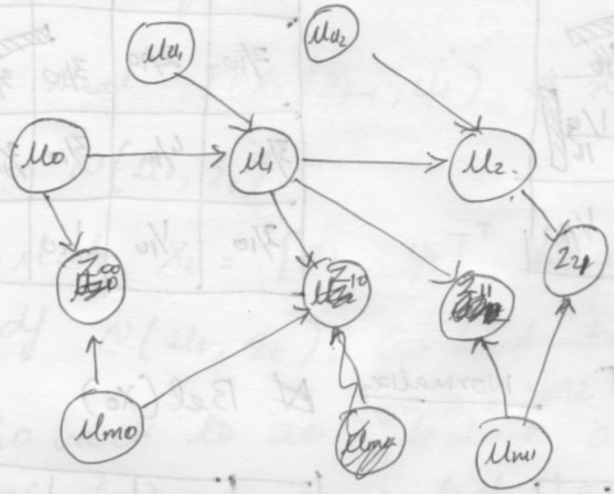
SAM:



Consider a 3 ~~state~~ vehicle/robot state, 2 landmark situation. We derive SAM equations for this

Z_{00} → landmark number
↳ time instant

~~Find argmax~~

Find $\mathcal{U}^*, \mathcal{U}_m^* = \underset{\mathcal{U}, \mathcal{U}_m}{\arg\max}\ P(\mathcal{U}, \mathcal{U}_m / \mathcal{U}_u, Z)$.

$$= \underset{\mathcal{U}, \mathcal{U}_m}{\arg\max}\ P(\mathcal{U}, \mathcal{U}_m, \mathcal{U}_u, Z)$$

$$= \alpha\ P(u_2 / u_1, u_{u1})\, P(z_{24} / u_{m1}, u_2)\, P(u_1 / u_0, u_{u1})\, P(z_{11} / u_{m1})\, P(z_{10} / u_{m0})\, P(u_0)\, P(z_{00} / u_{m0})$$

Actually you know how to compute all of the above term as they are just motion models / sensor model. Given those distributions it is merely a search in the space of

U's & U_m's to get the maximum value

$$U = \begin{bmatrix} U_0 \\ U_1 \\ U_2 \end{bmatrix}^T = \begin{bmatrix} U_{x0}, U_{y0}, U_{\theta 0}, \ldots, U_{\theta 2} \end{bmatrix}^T$$

$$U_m = \begin{bmatrix} U_{m0x} & U_{m0y} & U_{m1x} & U_{m1y} \end{bmatrix}^T$$

$$\cancel{P(U_t / U_{t-1}, U_{ut}) = P(U_t / \hat{U}_t)}$$

$$\cancel{\propto \exp [U_t - \hat{U}_t]^T \Sigma_t^{-1} [U_t -}$$

~~Now we follow Kaess derivation~~ closely by ~~generalizing for a time t~~

~~Consider $P(U_2 / U_{1})$ =~~

Consider $P(U_2 / U_1, U_{u2}) = P(U_2 / \hat{U}_2)$

$$\propto \exp\left(-\frac{1}{2}[U_2 - \hat{U}_2]^T \Sigma_{U_2}^{-1} [U_2 - \hat{U}_2]\right) \longrightarrow (1)$$

where $\hat{U}_2$ = predicted odometry reading

$= f(U_1, U_{u_2})$; U_2 = measured/obtained odometry

$$\Sigma_2 = F \Sigma_1 F^T + G \Sigma_{U_2} G^T.$$

Kaess write $[U_2 - \hat{U}_2]^T \Sigma_{U_2}^{-1} [U_2 - \hat{U}_2]$ as $\|U_2 - \hat{U}_2\|^2_{\Sigma_{U_2}}$

Hence (1) is $\propto \exp\left(-\frac{1}{2} \|U_2 - \hat{U}_2\|^2_{\Sigma_{U_2}}\right)$. $\longrightarrow (2)$.

~~8. Consider $P(z_{2p} / U_{m1}, U_2) = P(z_{2p} / \hat{z}_{2p})$~~

$$\cancel{\propto \exp\left(-\frac{1}{2} \| z_{2p} - \hat{z}_{2p} \|_H\right)}$$

where ~~$H = \frac{\partial h}{\partial}$~~

Why are we considering only Σ_{u_2} & not Σ_2.

1) Because $P(\mu_2 / \mu_1, u_2)$ is a function of only control noise Σ_{u_2} ~~as we~~ and it is only the belief $\hat{Bel}(x_t) = N(\hat{\mu}_t, \hat{\Sigma}_t)$
$= \sum P(x_t / x_{t-1}, u_t) . Bel(x_{t-1})$ is a function of the state covariance Σ_t

2) Another ~~th~~ way of looking at it is that the optimization routine searches over all possible μ_1 and for every such μ_1 it considers it only need to consider control noise for that ~~μ_1~~ μ_1's evolution to μ_2

Consider $P(z_{21} / \mu_2, \mu_{m1}) = P(z_{21} / \hat{z}_{21})$

$$\propto \exp\left(-\frac{1}{2} \| z_{21} - \hat{z}_{21} \|^2_{Q_{21}}\right) \longrightarrow (3)$$

where Q_{21} is the measurement noise covariance of the 1st landmark observed from second state / or second time sample.

$$\hat{z}_{21} = h(\hat{\mu}_2, \hat{\mu}_{m1}) \longrightarrow (4)$$

From (5) take Taylor serie of ~~$($~~$\|\mu_i - f(\mu_{i-1}, u_i)\|^2_{\Sigma_{ui}}$ to get

$$\mu_i^0 + \frac{\partial \mu_i}{\partial \mu_i} \delta \mu_i \Big|_{\mu_i^0} - \left[f(\mu_{i-1}^0) + \frac{\partial f(\mu_{i-1}^0, u_i)}{\partial \mu_{i-1}} \Big|_{\mu_{i-1}^0} \right]$$

$$= \mu_i^0 + \delta \mu_i - f(\mu_{i-1}^0) - \cancel{F} F \delta \mu_{i-1} \longrightarrow (6)$$

Now $\mathcal{U}^*, \mathcal{U}_m^* = \underset{\mathcal{U}, \mathcal{U}_m}{\arg\max}\, P(\mathcal{U}, \mathcal{U}_m \mid \mathcal{U}_u, Z)$

$$= \underset{\mathcal{U}, \mathcal{U}_m}{\arg\min} \left(-\log P(\mathcal{U}, \mathcal{U}_m, \mathcal{U}_u, Z) \right)$$

$$\cancel{= \sum_{i=1}^{2} \frac{1}{2}}$$

$$= \underset{\mathcal{U}, \mathcal{U}_m}{\arg\min} \left\{ \sum_{i=1}^{2} \|\mathcal{U}_i - \hat{\mathcal{U}}_i\|^2_{\Sigma_{u_i}} + \sum_{i,k} \|z_{ik} - \hat{z}_{ik}\|^2_{Q_{ik}} \right\} \longleftrightarrow (5).$$

Only for those Z_k's seen from $\mathcal{U}_i$

We now write $\mathcal{U}_i = \mathcal{U}_i^0 + \delta \mathcal{U}_i$ as a perturbation around the obtained odometry.

$\hat{\mathcal{U}}_i = f(\mathcal{U}_{i-1}, u_i)$ is written as

$$f(\mathcal{U}^0_{i-1}) + \frac{\partial f(\mathcal{U}_{i-1}, u_i)}{\partial \mathcal{U}_{i-1}} \delta \mathcal{U}_{i-1} \Big|_{\mathcal{U}^0_{i-1}} = f(\mathcal{U}^0_{i-1}) + F \delta \mathcal{U}_{i-1} \quad \cancel{\longleftarrow (6)}.$$

$\|$ly we find Taylor serie $\cancel{z_{ik} = z^0_{ik} +}$ for $h(\mathcal{U}_i, \mathcal{U}_{mk}) - Z_{ik}$.

$$\cancel{Z_{ik}}\ h\cancel{(\mathcal{U}_i, \mathcal{U}_{mk})} \approx \cancel{\frac{\partial h}{\partial \mathcal{U}_i}}$$

$$= h(\mathcal{U}_i^0, \mathcal{U}^0_{mk}) + \frac{\partial h}{\partial \mathcal{U}_i} \delta \mathcal{U}_i \Big|_{\mathcal{U}_i^0} + \frac{\partial h}{\partial \mathcal{U}_{mk}} \delta \mathcal{U}_{mk} \Big|_{\mathcal{U}^0_{mk}} - Z_{ik}$$

$$= h(\mathcal{U}_i^0, \mathcal{U}^0_{mk}) + H_{ik} \delta \mathcal{U}_i + J_{ik} \delta \mathcal{U}_{mk} - Z_{ik} \longrightarrow (7).$$

$$\cancel{\text{Now } \mathcal{U}_i - f(\mathcal{U}_{i-1}, u_i)}$$

$$\cancel{= \mathcal{U}_i + \delta \mathcal{U}_i - f(\mathcal{U}^0_{i-1}, u_i) - F \delta \mathcal{U}_{i-1}}$$

$$\cancel{= \delta \mathcal{U}_i - F \delta \mathcal{U}_{i-1}}$$

~~Now $f(\mathcal{U}_{i-1}, \mathcal{U}_i) - \mathcal{U}_i$~~

Now $f(\mathcal{U}_{i-1}, u_i) - \mathcal{U}_i$

$$= f(\mathcal{U}^0_{i-1}, u_i) + F\delta\mathcal{U}_{i-1} - \mathcal{U}^0_i - \delta\mathcal{U}_i$$

$$= -a_i + F\delta\mathcal{U}_{i-1} + G\,\delta\mathcal{U}_i \longrightarrow (8)\text{, where } G = -I_{3\times3}$$

(odometry prediction error)

and $h(\mathcal{U}_i, \mathcal{U}_{mk}) - Z_{ik}$

$$= h(\mathcal{U}^0_i, \mathcal{U}^0_{mk}) - Z_{ik} + H_{ik}\delta\mathcal{U}_i + J_k\,\delta\mathcal{U}_{mk}$$

$$= H_{ik}\delta\mathcal{U}_i + J_k\delta\mathcal{U}_{mk} - C_k \longrightarrow 9$$

↳ measurement error.

∴ We want to find

$$\underset{\mathcal{U}, \mathcal{U}_m}{\operatorname{argmin}} \left\{ \sum_{i=1}^{2} \| F\delta\mathcal{U}_{i-1} + G\,\delta\mathcal{U}_i - a_i \|^2_{\Sigma_{ui}} + \sum_{i,k} \right.$$

$$\sum_{i,k} \| H_{ik}\delta\mathcal{U}_i + J_k\,\delta\mathcal{U}_{mk} - C_k \|^2_{\Sigma_{ik}} \longrightarrow (10)$$

We write the $F\delta\mathcal{U}_{i-1} + G\,\delta\mathcal{U}_i - a_i$ as.

$$\underbrace{\begin{bmatrix} \overset{3\times3}{F} & \overset{3\times3}{G} \end{bmatrix}}_{3\times6} \underbrace{\begin{bmatrix} \delta\mathcal{U}_{i-1} \; {\scriptstyle 3\times1} \\ \delta\mathcal{U}_i \; {\scriptstyle 3\times1} \end{bmatrix}}_{6\times1}{\scriptstyle 6\times1} - \underset{3\times1}{a_i}$$

$$= Ax - b$$

∴ First term in (10) is of the form

$$[Ax - b]^T [\Sigma_{ui}]^{-1} [Ax - b]$$

Let $\Sigma_{ui}^{-1} = \gamma$ and since γ is a symmetric matrix then $[\gamma] = [\gamma]^{1/2}[\gamma]^{T/2}$, where $\gamma^{1/2}$ is the square root matrix of γ.

~~<u>Square root matrix</u>: R is said to be square root matrix of A, where $A = VDV^T$, then~~

~~$R = VD^{1/2}V^T$~~

~~$RR = [VD^{1/2}V^T][VD^{1/2}V^T] = VD^{1/2}D^{1/2}V^T$~~ $= VDV^T = A$.

How do you find the square root of a matrix?

The covariance matrix and its inverse are symmetric. A <u>symmetric matrix</u>, ~~is dio~~ B is <u>diagonalizable</u> as

$\rightarrow$ $B = VDV^{-1}$, where D is a diagonal matrix

$\rightarrow$ The diagonal entries of D are ~~eva~~ eiginvalues of B.

$\rightarrow$ Column vectors of V are the eigin ~~value~~ vector of B.

$\rightarrow$ The <u>column vectors are orthonormal</u>

Let S be the sqrt matrix of D, ~~then~~ obtained by taking square root of diagonal entries of D

Then let $R = VSV^T$, then

$$RR = VSV^TVSV^T = VSSV^T = VDV^T = B$$

(Since V is an orthonormal matrix

$VV^T = VV^{-1} = I$ & $V^T = V^{-1}$).

Then R is the sqrt matrix of B or $R = B^{1/2}$

Then $\cancel{[Ax-b]^T \Sigma_{u_i}^{-1} [Ax-b]}$

$$\cancel{= [Ax-b]^T \gamma [Ax-b]}$$

$$\cancel{= [Ax-b]^T \gamma^{1/2}}$$

Also $RR^T = \cancel{VD}\, VSV^T[VSV^T]^T = VDV^T = B$

Not only $RR = B$, $RR^T = B$, when B is ~~diagonal~~ symmetric.

Now $[Ax-b]^T \Sigma_{u_i}^{-1} [Ax-b]$

$$= [Ax-b]^T \gamma [Ax-b]$$

γ is symmetric & $\gamma = RR^T$

$$= [Ax-b]^T RR^T [Ax-b] =$$

$$= \|R^T [Ax-b]\|^2 = \|\gamma^{T/2} [Ax-b]\|^2 \longrightarrow (11)$$

$$= \|\gamma^{T/2} e\|^2$$

Second term on (10) is written similarly as $\|\eta^{T/2} [Cy-d]\|^2 \longrightarrow (12)$

$$= \|\eta^{T/2} f\|^2$$

(11) + (12) is of the form

$$[\gamma^{T/2} e]^T [\gamma^{T/2} e] + [\eta^{T/2} f]^T [\eta^{T/2} f]$$

or $[Re]^T [Re] + [Sf]^T [Sf]$

$$= \left[\begin{bmatrix} R & 0 \\ 0 & S \end{bmatrix} \begin{bmatrix} e \\ f \end{bmatrix}\right]^T \left[\begin{bmatrix} R & 0 \\ 0 & S \end{bmatrix} \begin{bmatrix} e \\ f \end{bmatrix}\right] \longrightarrow (13)$$

Also $RR^T = V\Sigma V^T [V \Theta^{1}]^T = V\Sigma V^T = B$

Note: ... $RR^T = B$, ... $R^T R = B$, when B is symmetric

New $[Ax-b]^T \Sigma_{\omega}^{-1} [Ax-b]$

$$= [Ax-b]^T Y [Ax-b]$$

Y is symmetric & $Y = RR^T$

$$= [Ax-b]^T R R^T [Ax-b]$$

$$= \| R^T [Ax-b] \|^2 = \| Y^{T/2} [Ax-b] \|^2 \longrightarrow (11)$$

$$= \| Y^{T/2} e \|^2$$

Second term in (10) is written

similarly as $\| \Omega^{T/2} [Cx-d] \|^2 \longrightarrow (12)$

$$= \| \Omega^{T/2} f \|^2$$

(11) + (12) is of the form

$$[Y^{T/2} e]^T [Y^{T/2} e] + [\Omega^{T/2} f]^T [\Omega^{T/2} f]$$

or $[Re]^T [Re] + [Sf]^T [Sf]$

$$= \left[\begin{bmatrix} R & 0 \\ 0 & S \end{bmatrix} \begin{bmatrix} e \\ f \end{bmatrix} \right]^T \left[\begin{bmatrix} R & 0 \\ 0 & S \end{bmatrix} \begin{bmatrix} e \\ f \end{bmatrix} \right] \longrightarrow (13)$$

Hence all the summation terms in (10) can be lumped into a minimizatio of the form

$$\theta^* = \arg\min_{\theta} \| A\theta - b \|^2$$

θ is solved by SVD or QR etc.

How to solve for θ incrementally
- ISAM?