Assignment-1

Mobile Robotics

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Ans1. Robot is heading along the x-axis of frame $G_1$.

a) for first 2 seconds, $v = 2\,m/s$, $\omega = \pi/10$ rad/s.

b) for next 2 sec., $v = 2\,m/s$, $\omega = -\pi/10$ rad/s

c) for next 2 sec, $v = 3\,m/s$ $\omega = 0$

d) for next 2 sec, $v = 2\,m/s$ $\omega = \pi/5$ rad/s

the homogeneous transform with respect to the frame of reference $G_1$ is given by,

$$X_{G_1} = \begin{bmatrix} x_R^{G_1} \\ y_R^{G_1} \\ \theta_R^{G_1} \\ 1 \end{bmatrix} = T_R^{G_1} \begin{bmatrix} x_R \\ y_R \\ \theta_R \\ 1 \end{bmatrix}$$

for multiple transform

$$X_{G_1} = \begin{bmatrix} T_{R_1}^{G_1} & T_{R_2}^{G_1} & \dots & T_{R_n}^{G_1} \end{bmatrix} \begin{bmatrix} x_{R_n} \\ y_{R_n} \\ \theta_{R_n} \\ 1 \end{bmatrix}$$

where $T_R^{G_1} = \begin{bmatrix} \cos\theta_c & -\sin\theta_c & 0 & t_{Rx} \\ \sin\theta_c & \cos\theta_c & 0 & t_{Ry} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

here, for first 2 seconds,

$$T_{R_1}^{G_1} = \begin{bmatrix} \cos(\omega t) & -\sin\omega t & 0 & x_1 \\ \sin\omega t & \cos\omega t & 0 & y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $\omega = \pi/10$ & $t = 2$, & $x_1 = \int_0^2 v\cos\omega t\,dt = \frac{20}{\pi}\sin\frac{\pi}{5}$

& $y_1 = \int_0^2 v\sin\omega t\,dt = \frac{20}{\pi}\left(1 - \cos\frac{\pi}{5}\right)$

$$\therefore T_{R_1}^{0} = \begin{bmatrix} \cos\frac{\pi}{5} & -\sin\frac{\pi}{5} & 0 & \frac{20}{\pi}\sin\frac{\pi}{5} \\ \sin\frac{\pi}{5} & \cos\frac{\pi}{5} & 0 & \frac{20}{\pi}(1-\cos\frac{\pi}{5}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

for next two sec. $T_{R_2}^{0}$, $\omega t = -\frac{\pi}{5}$, $x_2 = \frac{20}{\pi}\sin\frac{\pi}{5}$ $y_2 = -\frac{20}{\pi}(1-\cos\frac{\pi}{5})$

$T_{R_3}^{0}$, $\omega t = 0$, $x_3 = 6$, $y_3 = 0$

~~$T_{R_1}^{0} \Rightarrow \omega t = 0$~~

the last matrix $\begin{bmatrix} x_R \\ y_R \\ \theta_R \\ 1 \end{bmatrix} \Rightarrow$ $x_R \Rightarrow x$ pt. w.r.to $4^{th}$ frame $\Rightarrow \frac{15}{\pi}\sin 2\frac{\pi}{5}$

$y_R = \frac{15}{\pi}(1-\cos\frac{2\pi}{5})$

$\theta_R = \frac{2\pi}{5}$ here $\omega t = \frac{2\pi}{5}$, $v = 3\,m/s$

$$\therefore X_G = \begin{bmatrix} \cos\frac{\pi}{5} & -\sin\frac{\pi}{5} & 0 & \frac{20}{\pi}\sin\frac{\pi}{5} \\ \sin\frac{\pi}{5} & -\cos\frac{\pi}{5} & 0 & \frac{20}{\pi}(1-\cos\frac{\pi}{5}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{5} & \sin\frac{\pi}{5} & 0 & \frac{20}{\pi}\sin\frac{\pi}{5} \\ -\sin\frac{\pi}{5} & \cos\frac{\pi}{5} & 0 & -\frac{20}{\pi}(1-\cos\frac{\pi}{5}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X_G = \begin{bmatrix} 1 & 0 & 0 & \frac{40}{\pi}\sin\frac{\pi}{5} \\ 0 & 1 & 0 & \frac{40}{\pi}(1-\cos\frac{\pi}{5}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{15}{\pi}\sin 2\frac{\pi}{5} \\ \frac{15}{\pi}(1-\cos 2\frac{\pi}{5}) \\ \frac{2\pi}{5} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{15}{\pi}\sin 2\frac{\pi}{5} \\ \frac{15}{\pi}(1-\cos\frac{2\pi}{5}) \\ \frac{2\pi}{5} \\ 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 6+\frac{40}{\pi}\sin\frac{\pi}{5} \\ 0 & 1 & 0 & \frac{40}{\pi}(1-\cos\frac{\pi}{5}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{15}{\pi}\sin 2\frac{\pi}{5} \\ \frac{15}{\pi}(1-\cos 2\frac{\pi}{5}) \\ \frac{2\pi}{5} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{15}{\pi}\sin 2\frac{\pi}{5} + \frac{40}{\pi}\sin\frac{\pi}{5} + 6 \\ \frac{15}{\pi}(1-\cos\frac{2\pi}{5}) + \frac{40}{\pi}(1-\cos\frac{\pi}{5}) \\ \frac{2\pi}{5} \\ 1 \end{bmatrix}$$

$\therefore x_{R_G} = \frac{15}{\pi}\sin\frac{2\pi}{5} + 6 + \frac{40}{\pi}\sin\frac{\pi}{5} = 18.024$

$y_{R_G} = \frac{15}{\pi}(1-\cos\frac{2\pi}{5}) + \frac{40}{\pi}(1-\cos\frac{\pi}{5}) = 5.730$

$\theta_{R_G} = \frac{2\pi}{5} = 72°$