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MR Assignment 3

classmate

Date _____

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$$\mu_{x,t=0}$$

$$\mu_{y,t+1}$$

$$\mu_{y,t=0}$$

$$\sigma_x^2 = 0$$

$$\mu_{y,t}$$

$$\sigma_y^2 = 0$$

$$\mu_{\phi,t=0}$$

$$T=1$$

$$\phi=30$$

$$\mu_{x,t}$$

$$\mu_{x,t+1}$$

$$\begin{bmatrix} \hat{\mu}_{x,t+1} \\ \hat{\mu}_{y,t+1} \\ \hat{\mu}_{\phi,t+1} \end{bmatrix} = \begin{bmatrix} \mu_{x,t} \\ \mu_{y,t} \\ \mu_{\phi,t} \end{bmatrix} + \begin{bmatrix} T \cos(\mu_{\phi,t} + \phi) \\ T \sin(\mu_{\phi,t} + \phi) \\ \phi \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \cos(0+30^\circ) \\ \sin(0+30^\circ) \\ 30^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 3/2 \\ 0.5 \\ 30^\circ \end{bmatrix}$$

$$\Sigma_t = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

because there is no uncertainty in initial pos

$$\Sigma_{t+1} = \begin{bmatrix} \sigma_T^2 & 0 \\ 0 & \sigma_\phi^2 \end{bmatrix} = \begin{bmatrix} 0.2 & 0 \\ 0 & 4 \end{bmatrix}$$

$$G = \frac{S_b}{S_{u_t}} = \begin{bmatrix} \cos(\mu_{\phi,t} + \phi) & -T \sin(\mu_{\phi,t} + \phi) \\ \sin(\mu_{\phi,t} + \phi) & T \cos(\mu_{\phi,t} + \phi) \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(0+30^\circ) & -\sin(0+30^\circ) \\ \sin(0+30^\circ) & \cos(0+30^\circ) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \\ 0 & 1 \end{bmatrix}$$

$$\hat{\Sigma}_{t+1} = F \Sigma_t F^T + G \Sigma_u G^T$$

$$= 0 + \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.85 & -1.688 & -2 \\ -1.688 & 3.05 & 3.46 \\ -2 & 3.46 & 4 \end{bmatrix}$$

$$\text{and } \hat{u}_t = \begin{bmatrix} 0.866 \\ 0.5 \\ \pi/6 \end{bmatrix}$$

$$\hat{\Sigma}_{t+1}^{-1} = \begin{bmatrix} -0.532 & 0.391 & -0.604 \\ 0.39 & 17.22 & -14.704 \\ -0.604 & -14.704 & 12.662 \end{bmatrix}$$

Manhattan's distance =

$$\begin{bmatrix} 0.6 - 0.866 \\ 0.4 - 0.5 \\ \frac{\pi}{6} - \frac{\pi}{6} \end{bmatrix}^T \begin{bmatrix} -0.532 & 0.391 & -0.604 \\ 0.391 & 17.225 & -14.704 \\ -0.604 & -14.704 & 12.662 \end{bmatrix} \begin{bmatrix} -0.246 \\ -0.246 \\ 0 \end{bmatrix}$$

$$= 0.1554 \text{ (Matlab)}$$

$$P(x) = \frac{1}{2\pi \sqrt{\det \Sigma}} \exp \left(-\frac{1}{2} (\text{Mahalanobis}) \right)$$

$$= \frac{1}{2\pi (2.32)} (0.469)$$

$$= 0.0314$$