Contd.

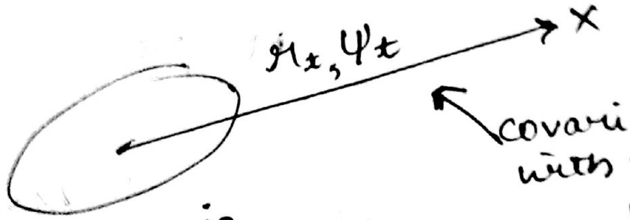
8/10/15

SLAM - Mapping & localization simultaneously

extension of ~~KF~~ EKF localization, EKF SLAM

~~Keep~~ stop the covariance from growing.

In SLAM, landmarks will have covariances



covariance associated with this

also there is a covariance associated with this.

$$\begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix}$$

NO help - as landmark provides help but your own origin is uncertain.

Each landmark has uncertainity (first landmark is good, uncorrupted, rest all corrupted, as covariance ↑↑)

$1k+1^{T}$

Gmapping - slightly diff. Slam algo (solved SLAM for indoor)

But still a challenge for outdoor (even if solns. are known good robot, good sensors, good [illegible] required)

The EKF SLAM Framework : Vehicle and Map states.

(Ch 7E)
E11

$$\mu_{t-1} = \begin{bmatrix} \overset{\text{state of vehicle}}{\mu_{v,\,t-1}} \\ \\ \underset{\text{state of map}}{\mu_{m,\,t-1}} \\ \\ \end{bmatrix}_{\cancel{(3+2N)\times 1}\;\; 3+(2N\times 1)} \quad (3+2N)\times 1$$

N - no. of landmarks

$$\mu_{v,\,t-1} = \begin{bmatrix} \mu_{x,\,t-1} \\ \mu_{y,\,t-1} \\ \mu_{z,\,t-1} \end{bmatrix} \qquad \mu_m = \begin{bmatrix} \mu_{m_1 x} \\ \mu_{m_1 y} \\ \mu_{m_2 x} \\ \mu_{m_2 y} \end{bmatrix}_{n=2}$$

centered at mean

Let ~~m=2~~ N=2

$$\underset{(3+2N)\times(3+2N)}{\Sigma_{t-1}} = \begin{bmatrix} \overset{3\times 3}{\Sigma_{vv}} & \overset{3\times 2N}{\Sigma_{vm}} \\ \underset{2N\times 3}{\Sigma_{mv}} & \underset{2N\times 2N}{\Sigma_{mm}} \end{bmatrix}$$

vm, mv, coz I have uncertainity in me, then estimates I makes on others I have uncertainity on them too.

But ~~then~~ there would not be any uncertainity b/w the two I judged.

$$F = \frac{\partial f}{\partial \mu_{t-1}} = \begin{bmatrix} \frac{\partial \hat{\mu}_{x,t}}{\partial \mu_{x,t-1}} & \frac{\partial \hat{\mu}_{x,t}}{\partial \mu_{\theta,t-1}} & \frac{\partial \mu_{x,t}}{\partial m_{2y}} \\ \vdots & & \\ \frac{\partial \mu_{m_2 y}}{\partial \mu_{x,t}} & \cdots & \frac{\partial \mu_{m_2 y}}{\partial \mu_{m_2 y}} \end{bmatrix}$$

Jacobian

estimate f upon the vehicle motion

?

$$\mu_t = \begin{bmatrix} \mu_{v,t-1} + \text{control} \\ \mu_{m_1 x} \\ \mu_{m_1 y} \\ \mu_{m_2 x} \\ \mu_{m_2 y} \\ \vdots \end{bmatrix}$$

$v \sin\phi$
$v \cos\phi$
ϕ

$$\mu_t = f(\mu_{t-1}, u_t)$$

jointly estimating states, since ~~my~~ state = my state + state of all others.

$F_{localization}$ same as k_f Jacobian you got when you did localization.

$$F_{slam} = \begin{bmatrix} F_{loc}^{3\times 3} & 0^{3\times 2N} \\ 0_{2N\times 3} & I_{2N\times 2N} \end{bmatrix}$$

$$G_{1\,ARM} = \frac{\partial f}{\partial u_{t-1}} = \begin{bmatrix} G_{LOC}\ 3\times 2 \\ O_{2N\times 2} \end{bmatrix}$$

$$\Sigma_t = F\,\Sigma_{t-1}\,F^T + G_1\,\Sigma_{u,t}\,G_1^T$$

$(3+2N, 3+2N)\times(3+2N, 3+2N) \times (3+2N, \cancel{3+2N})$ [3+2N] $+ (3+2N, 2)\times(2,2)\times(2, 3+2N)$

? $\hat{z}_t = h(\hat{u}_t)$

$$H = \frac{\partial h}{\partial \hat{u}_t} = \begin{bmatrix} \frac{\partial \hat{x}_{1,t}}{\partial \hat{u}_{x,t}} \end{bmatrix}$$

$H = (2M, 3+2N)$ — No. of landmarks

Batch

$(2, 3+2N)$

using deriv in loop batch? (or batch?) from (*)

* u_t will run over n nodes. sequential find

$$\frac{\partial h}{\partial \hat{u}_t} = \begin{bmatrix} \frac{\partial \hat{x}_{1,t}}{\partial \hat{u}_{x,t}} & \frac{\partial \hat{x}_{1,t}}{\partial \hat{u}_{y,t}} & \partial \hat{x} \\ \frac{\partial u}{\partial \hat{u}_{x,t}} \\ \partial u_{2,t} \end{bmatrix}$$

?

Batch (Book does it sequentially)

~~Bath~~ Batch. Sequential = ?

$$\mu_* = \hat{\mu}_* + k[z_t - \hat{z}_t]$$

$(3,2N) \times (2N,1)$

I can do with all the landmarks together, or single sequential

(Batch had it been 2, then sequential/single sequential

Batch mode works better

$$S = \underset{(2M,\ 3+2N) \times (3+2N,\ 3+2N) \times (3+2N,\ 2M) + (2M,2M)}{H \Sigma H^T + Q}$$

$$K = \hat{\Sigma}_t H^T S^{-1}$$

Write its dimensions

$$\Sigma_t = [I - KH]\hat{\Sigma}_*$$

covariance would not reduce much, coz map is not known.
~~get~~ (get to know good landmarks)

$$\underset{(3+2N,1)}{\mu_t} = \hat{\mu}_t + k[z - \hat{z}_t]$$

Augmenting state with a new landmark
(initially we started with 2, how to add 3rd)

What you do with covariance is tricky.
adding in μ_m is easy

Now $(3+2N+2,\ 3+2N+2)$
∴ appended 2 to the covariance matrix rows & columns

Who takes of care of correlation b/w landmarks?
↳ maths

$$\mu_{avg} = \begin{bmatrix} \mu_{v,t} \\ \mu_{m_1} \\ \mu_{m_2} \\ \mu_{m_3} \end{bmatrix}$$

augmenting third landmark

$$\mu_{m_3} = \begin{bmatrix} \mu_{m_3 x} \\ \mu_{m_3 y} \end{bmatrix}$$

$$\rightarrow f(\mu_t, r_{t3}, \psi_{t3})$$

this takes care of sensor noise, vehicle covariance & landmark covariance

$$= \begin{bmatrix} \mu_{x,t} + r_{t3}\cos(\mu_{\theta,t} + \psi_{t3}) \\ \mu_{y,t} + r_{t3}\sin(\mu_{\theta,t} + \psi_{t3}) \end{bmatrix}$$

$$Q = \begin{bmatrix} \sigma_{r_3}^2 & 0 \\ 0 & \sigma_{\psi_3}^2 \end{bmatrix}$$

$$\text{Initially } \Sigma_{avg,t} = \begin{bmatrix} \Sigma_v^{3\times 2N} & \Sigma_{vm}^{3\times 2N} & 0 \\ \Sigma_{mv} & \Sigma_{mm} & 0 \\ 0 & 0 & Q \end{bmatrix}_{(3+2N+2)\times(3+2N+2)}$$

Q needs to blend itself, as its uncertainity is dependend on previous m_1 & m_2 also. coz my uncertainity was depended on m_1 & m_2 (gaussian takes care of it)

$$\Sigma_t = F_{avg} \, \Sigma_{avg,t} \, F_{avg}$$

$$F_{avg} = \frac{\partial f_{avg}}{\partial \mu_{avg}} = \begin{bmatrix} I_v & 0 & 0 \\ 0 & I_m & 0 \\ \frac{\partial f_{avg}}{\partial \mu_{v,t}} & 0 & \frac{\partial f_{avg}}{\partial Z} \end{bmatrix}$$

Jacobian of augmentation

transformation of gaussian probabilities from one space to another

new landmarks get co-related with all old landmarks

where $f_{avg}(\mu_t, \mu_{m3}) = \begin{bmatrix} \mu_{v\,t} \\ \mu_m \\ \mu_{m3} \end{bmatrix}$

$\mu_{v\,t}$ → most recent vehicle state

μ_m → most recent old landmarks

μ_{m3} → most recent added landmarks

taken derivative w.r.t f_{avg}

$$Z = \begin{bmatrix} r_{t3} \\ \psi_{t,3} \end{bmatrix}$$

had Im been first landmark added.

$$\frac{\partial f_{avg}}{\partial \mu_{avg}} = \begin{bmatrix} \boxed{} \end{bmatrix}$$

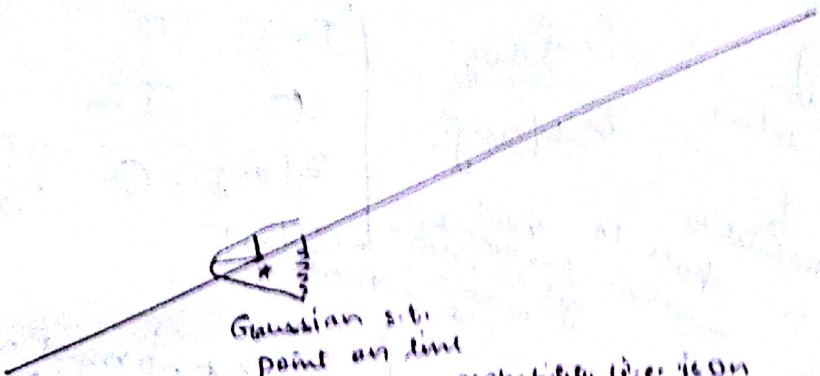
?

Q - says how much they are related.

new estimate - I make from my position, but then my uncertainity affects others.

26/10/15

Standard Line Fitting (Least Squares)



Gaussian s.t. point on line has max probability (i.e. is on mean)

* max this

$\equiv$

$\sum$ minimise this

$$\prod_{i=1}^{n} P(y_i | x_i)$$

max log

||

min (− log)

maximise likelihood of this, for what likelihood would this be maximum

m & c ~~c or c~~ (slope & intercept) → only these two to be estimated

$$\frac{\partial}{\partial u} [\underset{n\times 2}{A} \underset{2\times 1}{u} - b]^T \cdot [Au - b]$$

$$v = Au - b$$

$$\frac{\partial J}{\partial u} = \frac{\partial J}{\partial v} \frac{\partial v}{\partial u}$$

$$= \frac{\partial v^T v}{\partial v} \qquad \frac{\partial (Au - b)}{\partial u}$$

$$= 2 v^T A$$

$$= 2 [Au - b]^T A$$

[illegible] $[A^TA]^{-1}A^T$

System identification (SI)

How we estimate model

LS - more common method for SI used estimating model parameters

$$K = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$ by SI

In SLAM we assume model is known.
I have already calibrated my motion model.
I know M & C, estimate y & x.

Smoothing and mapping: model is known.

In notes uploaded: Least square on u_1 & u_2 control

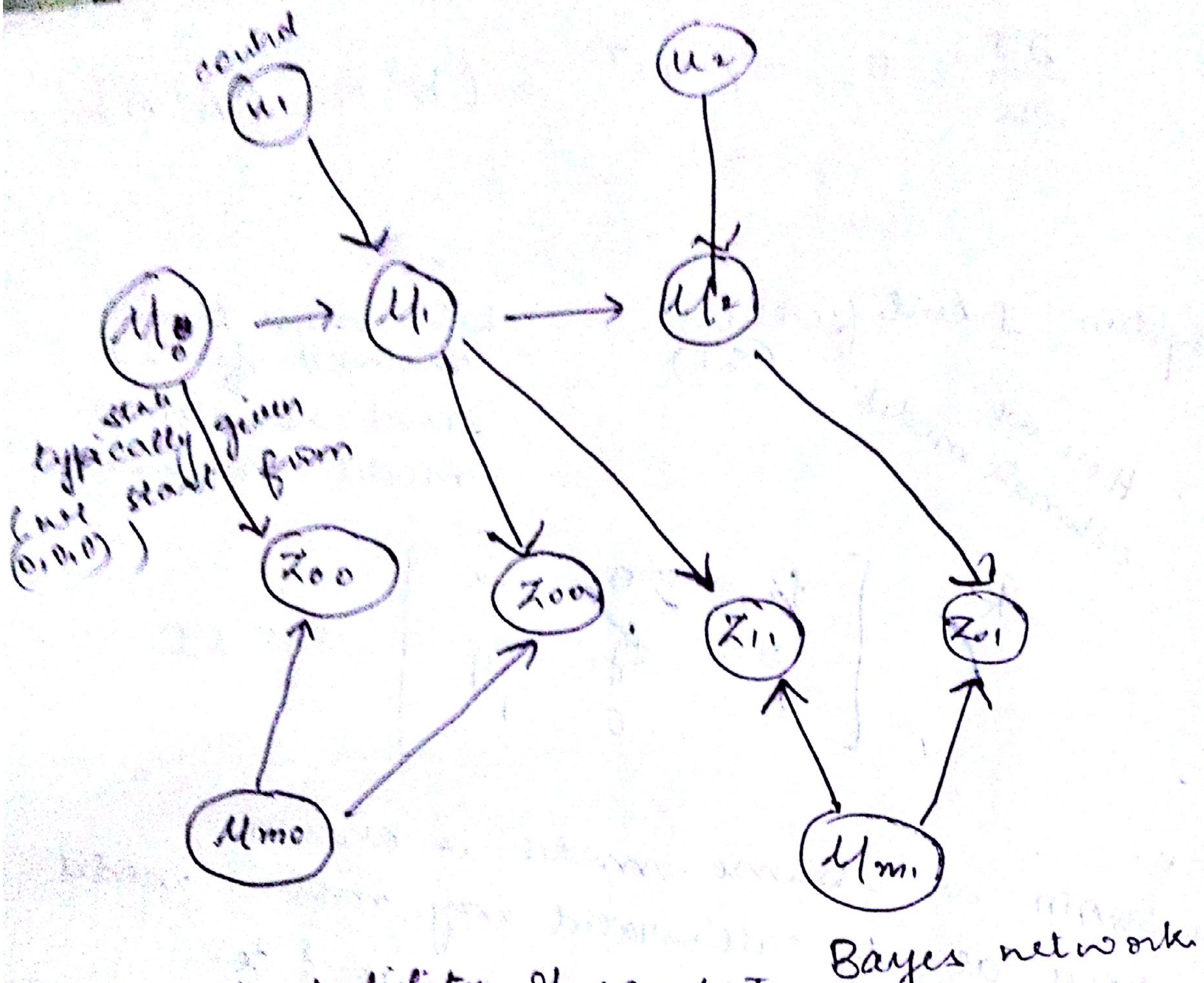
estimate u_1 by [illegible]
u_2 by [illegible]

Transform Bayes Network to Least Square

Maximum aposteriori estimate.

Z_i O
landmark O from states

Solving whole SLAM problem as optimisation problem as least sq. non linear, but we linearize by Taylor expansion



Joint probability of complete map and be factorised (given the Bayes Net)

$P(\mu_t \mid \mu_{t-1}, u_t)$ Prediction model of KF

$= f(\mu_{t-1}, u_t)$

~~$P(\mu_t$~~ $\mid z_t$

$P(\mu_t, \mu_{mj} \mid z_{tj})$ ↙ measurement will transform to this form

$P(z_{tj} \mid \mu_t, \mu_{mj})$

μ^* — robot state

μ_m^* — map states

state — robot's state + map's state (we want to know complete)

$$\underset{\mu_1, \mu_2, \mu_{m0}, \mu_{m1}, \mu_{m2}}{\operatorname{argmax}} P(\mu_2, \mu_1, \mu_0, z_{00}, z_{10}, z_{11}, z_{21}, \mu_{m0}, \mu_{m1}, \mu_{m2}, \cancel{\mu_{m0}}\ \mu_1, \mu_2)$$

= We take each of the random variable & find out on what each of them is ~~dependent~~ conditioned on.

$$= P(\underbrace{\mu_2 \mid \mu_1, \mu_0}_{\text{motion model}}) P(\mu_1 \mid \mu_1, \mu_0) P(\underbrace{z_{21} \mid \mu_2, \mu_{m1}}_{\text{measurement model}})$$

$$= P(z_{11} \mid \mu_1, \mu_{m1}) P(z_{10} \mid \mu_1, \mu_{m0}) P(z_{00} \mid \mu_{m0}, \mu_0)$$

$$= P(\mu_0) P(\mu_1) P(\mu_2) P(\mu_{m0}) P(\mu_{m1}) \rightarrow \eta$$

Least squares itself can be posed as Bayes Networks.

maximise states x, y given measurements.

$$\underset{c_0^*, c_1^*}{\operatorname{argmax}} P(y_0 \mid x_0) P(y_1 \mid x_1) \ldots P(y_n \mid x_n)$$

$\mu_{t+1} = f(\mu_t, u_t) \rightarrow$ EKF prediction equation

$z_{t+1} = h(\mu_{t+1}, m) \rightarrow$ EKF measurement equation covariance coming only in control states

look at all possible previous states

optimisation - searching over the space of u_{m_1} u_{m_2} u_0

Non linear least squares

$$\sum_{i=1}^{n} ||y_i - f(x_i)||^2$$

non linear function

$u_i \rightarrow$ odometry

$f(u_{i+1}) \rightarrow$ motion model

Multivariante optimisation

$$\underbrace{u_i - f(u_i)}_{\text{odometry error}}$$

Newton Raphson

29/10/18

$$\sum \|Ax - b\|^2_{\Sigma_i^{-1}} + \sum \|CY - d\|^2_{Q_i^{-1}}$$

$$= \sum \|A\theta - b\|^2$$

$A\theta = b$

$A^T A\theta = A^T b$

$\theta = (A^T A)^{-1} A^T b$

This is what I do not want to do.

$\Rightarrow$ QR decomposition

$$[Ax - b]^T \Sigma_i^{-1} [Ax - b]$$

$$\Sigma_{ui}^{-1} = Y$$

$$[Y] = [Y^{1/2}][Y^{1/2}]^{-1}$$

positive definite of symmetric Square Root Decomposition

$B = VDV^{-1}$ ★ (See various decomposition methods of matrices)

—— x ——

$$\text{Min } \|AS - b\|^2$$

$$A = \underset{m\times m}{Q} \; R_{m \times n}$$

orthogonal ~~orthonormal~~

$$R \rightarrow \begin{bmatrix} R \\ 0 \end{bmatrix}$$

$R \rightarrow (n \times n)$ Diagonal matrix

$0 \rightarrow (m-n) \times n$ matrix of zeroes

$$\|AS - b\|^2$$

$$= \left\|Q \begin{bmatrix} R \\ 0 \end{bmatrix} S - b\right\|^2$$

$$= \left[Q \begin{bmatrix} R \\ 0 \end{bmatrix} \delta - b \right]^T \left[Q \begin{bmatrix} R \\ 0 \end{bmatrix} \delta - b \right]$$

$$= \underbrace{\left[Q \begin{bmatrix} R \\ 0 \end{bmatrix} \delta - b \right]^T}_{1 \times m} \underbrace{Q Q^T}_{m \times m} \underbrace{\left[Q \begin{bmatrix} R \\ 0 \end{bmatrix} \delta - b \right]}_{m \times 1}$$

$$= \left[Q^T Q \begin{bmatrix} R \\ 0 \end{bmatrix} \delta - b \right]^T \left[Q^T \begin{bmatrix} R \\ 0 \end{bmatrix} \delta - b \right]$$

$$= \left\| \underbrace{Q^T Q}_{I} \begin{bmatrix} R \\ 0 \end{bmatrix} \delta - Q^T b \right\|^2$$

$$= \left\| \begin{bmatrix} R \\ 0 \end{bmatrix} \delta - \begin{bmatrix} d \\ e \end{bmatrix} \right\|^2$$

$$= \| R\delta - d \|^2 + \| e \|^2$$

$$R\delta^* = d$$

R – upper triangular matrix

can solve for δ without taking inverse [illegible] formed it as least squares problem

5/11/15

$Q^T = G_1 \; \ldots \; G_n$ seq. of givens matrix each of these are orthogonal

—— x ——

13/11/15

Kidnap Robot (Placed at a location robot does not have a motion model)

$$\underbrace{Bel(X_t)}_{\text{RV Posterior}} = N(\mu_t, \Sigma_t)$$

$$= \underbrace{P(z_t \mid x_t)}_{\text{Likelihood Distribution}} \underbrace{Bel(\hat{x}_t)}_{\text{Prior}}$$

$\hat{Bel}(x_0)$

1/16	1/16	1/16	1/16
1/16	1/16	1/16	1/16
1/16	1/16	1/16	1/16
1/16	1/16	1/16	1/16

equal probability of my being here in the sample space.

$P(z_0 \mid x_0)$ ← we need to define

$\hat{x}_0 = [\hat{O}_1, \ldots \hat{O}_8]^T$ 8 sensors

prediction of 8 readings

$z = [O_1 \; - - - - \; O_8]^T$

the 8 readings from sensors

$$P(z_0 \mid x_0) = \text{some loss function}$$

$$= \eta \prod_{i=0}^{8} \exp\left(\frac{(O_i - \hat{O}_i)^2}{2\sigma^2} \right)$$

$\hat{O}_i$ – predicted measurement of i^{th} sensor at some $x_0 = x_{0i}$

1/10	1/10	1/10	1/10
2/10	2/10	3/10	3/10
3/10	4/10	3/10	2/10
2/10	1/10	1/10	1/10

$$P(z_0 \mid x_0) = P(O_1 \; - - - \; O_8 \mid x_0)$$

$$= P(O_1 / x_0) \; P(O_2 / x_0) \; .. \; P(O_8 / x_0)$$

$P(z_0 / x_0) \; Bel(\hat{x}_0)$

1/160	1/160	1/160	1/160
2/160	2/160	3/160	3/160
3/160	4/160	3/160	2/160
2/160	1/160	1/160	1/160

This is not a probability distribution since sum is not 1 so we normalize it.

Normalize

1/30	1/30	1/30	1/30
2/30	2/30	3/30	3/30
3/30	4/30	3/30	1/30
2/30	1/30	1/30	1/30

Bel (x_0)
posterior has more meaning
as it says the ~~ro~~ robot
is most likely here.

16/11/15

early 2nd / 3rd ch Probabilistic Robot

In slides eg:
Initial prior - not gaussian
(that's y cant apply
Kalman filter).
becoz of which non
parametric models come into play

$$Bel(x_t) = P(z_t | x_t)\ \hat{Bel}(x_t)$$ - see derivation

Measurement model

$$Bel(x_t) = P(X_t | z_t, u_t, z_{1,t-1}, u_{1,t-1})$$ Measurement update in EKF

point by point multiplication
convolution of Image pixels with map (grid)

$$\hat{Bel}(x_t) = \sum P(x_t \mid x_{t-1}, u_t)\, Bel(x_{t-1})$$

Bcoz of this we can think it as convolution.

Motion model

$\cancel{P(x_t)}$

$$\hat{Bel}(x_t) = P(x_t \mid u_t, z_{1,t-1}, u_{1,t-1})$$

analogous to Prediction Equation in EKF

$N(\hat{\mu}_t, \hat{\Sigma}_t)$ (after motion)

$Bel(x_t) \rightarrow N(\mu_t, \Sigma_t)$ after measurement

$$\hat{\Sigma}_t = F\, \Sigma_{t-1} F^T + G\, \Sigma_{u_t} G^T$$

parametric way

Lets do in Non parametric way:

$Bel(x_{t-1})$

$\Sigma_{t-1} \leftarrow$

0	0	0	0	0	0	0
0	(1,1) 0.1	0.2	0.1	0	0	0
0	0.2	2 0.3	0.1 (2,4)	0	(3,6)	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

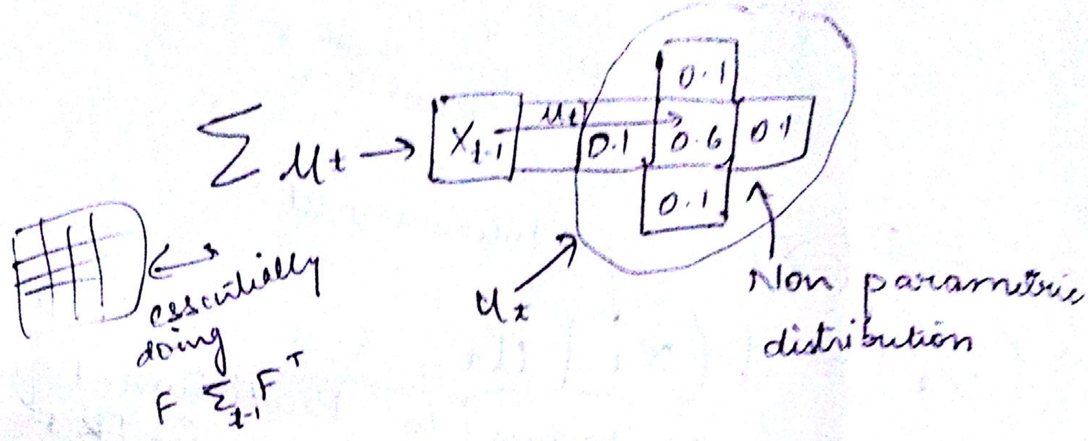
Row indexing → 0, Column → 1

x & y localization (had been x, y, θ then 3 dimensions)

u_t = move to the right by 3 cells

Scanned by CamScanner

$P(x_t \mid x_{t-1}, u_t)$



$\hat{Bel}(X_t = 2,6) = P(X_t = 2,6 \mid X_{t-1} = 2,3, u_t) \times$
$Bel(X_{t-1} = 2,3)$
$+$
$P(X_t = 2,6 \mid X_{t-1} = 2,2, u_t)$
$Bel(X_{t-1} = 2,2) +$

$P(x_t = 2,6 \mid X_{t-1} = 2,4, u_t) Bel(X_{t-1} = 2,4) +$
$P(\underbrace{x_t = 2,6 \mid x_{t-1} = 1,3}_{0.1}, u_t) \; Bel(x_{t-1} = 1,3)$
$+ P(x_t = 2,6 \mid x_{t-1} = 3,3) \; Bel(X_{t-1} = 3,3)$
$+ P(x_t = 2,6 \mid x = 0,0,0)$
$Bel(x_t = 0,0,0)$

$= 0.6(0.3) + (0.1)(0.2) + (0.1)(0.1) + (0.1)(0.2)$
$+ (0.1)(0) + 0(0)(0.1) \ldots$

$P(x_t \mid x_{t-1}, u_t)$ — Generative Model

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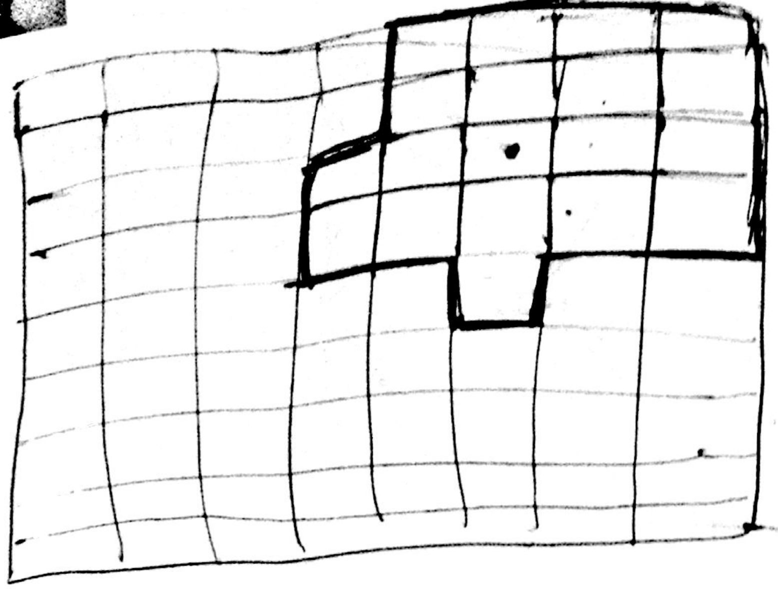


$$\Sigma_t = \underbrace{[I - KH]\hat{\Sigma}_t}_{\text{always less than } \hat{\Sigma}_t}$$

Derivation

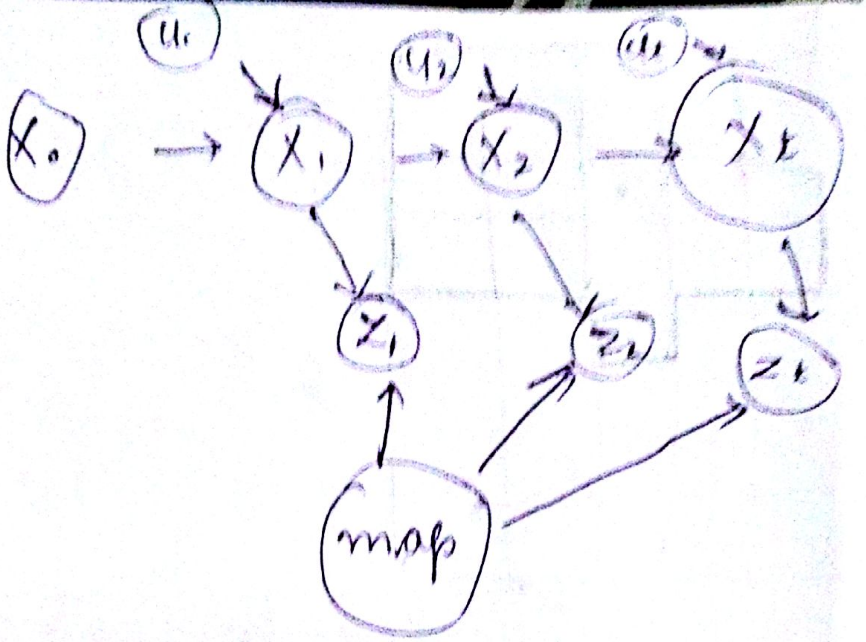
$$Bel(\hat{X}_t) = p(x_t \mid z_{1,t-1}, u_{1:t})$$

$$Bel(X_t) = P(x_t \mid z_{1:t}, u_{1:t})$$

$$\hat{z}_{t+1} = h(\hat{u}_{t+1})$$

$$\hat{u}_{t+1} = f(u_t, u_{t+1})$$

$$\sum \| u_{t+1} - f(u_t, u_{t+1}) \|^2 + \| z_{t+1} - h(u_{t+1}, u) \|^2$$



$$\hat{Bel}(x_t) = P(x_t \mid z_{1:t-1}, u_{1:t})$$

posterior

z_t - measurement at time t

$$P(\underset{A}{x_t} \mid \underset{B}{z_t}, \underbrace{z_{1:t-1}, u_{1:t}}_{C})$$

$$= \frac{P(z_t \mid x_t, z_{1:t-1}, u_{1:t})\, P(x_t \mid z_{1:t-1}, u_{1:t})}{P(z_t \mid z_{1:t-1}, u_{1:t})}$$

Markov assumption

$$= \eta\, P(z_t \mid x_t)\, P(x_t \mid z_{1:t-1}, u_{1:t})$$

Prior before last set of sensor measurements

$$= \eta\, P(z_t \mid x_t)\, \hat{Bel}(x_t)$$