11 General Mathematics

PIVOT 4A LEARNER'S MATERIAL

General Mathematics
Functions
First Edition, 2020

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Published by the Department of Education
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Undersecretary: Diosdado M. San Antonio

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Senior High School

General Mathematics

Functions

Introductory Message

For the facilitator:

Welcome to Grade 11 General Mathematics Alternative Delivery Mode (ADM) Module on Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the learners to meet the standards set by the K to 12 Curriculum while overcoming the learners' personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help them acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to Grade 11 General Mathematics Alternative Delivery Mode (ADM) Module on Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you master the key concepts of functions specifically on representing functions in real life situations. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

1. recall the concepts of relations and functions;
2. define and explain functional relationship as a mathematical model of situation; and
3. represent real-life situations using functions, including piece-wise function.

What I Know

Before you proceed with this module, let's assess what you have already know about the lesson.

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What do you call a relation where each element in the domain is related to only one value in the range by some rules?
   a. Function
   b. Range
   c. Domain
   d. Independent

2. Which of the following relations is/are function/s?
   a. x = {(1,2), (3,4), (1,7), (5,1)}
   b. g = {(3,2), (2,1), (8,2), (5,7)}
   c. h = {(4,1), (2,3), (2, 6), (7, 2)}
   d. y = {(2,9), (3,4), (9,2), (6,7)}

3. In a relation, what do you call the set of x values or the input?
   a. Piecewise
   b. Range
   c. Domain
   d. Dependent

4. What is the range of the function shown by the diagram?
   a. R:{3, 2, 1}
   b. R:{a, b}
   c. R:{3, 2, 1, a, b}
   d. R:{all real numbers}

5. Which of the following tables represent a function?

a.

x	0	1	1	0
y	4	5	6	7

b.

x	-1	-1	3	0
y	0	-3	0	3

c.

x	1	2	1	-2
y	-1	-2	-2	-1

d.

x	0	-1	3	2
y	3	4	5	6

6. Which of the following real-life relationships represent a function?
   a. The rule which assigns to each person the name of his aunt.
   b. The rule which assigns to each person the name of his father.
   c. The rule which assigns to each cellular phone unit to its phone number.
   d. The rule which assigns to each person a name of his pet.

7. Which of the following relations is **NOT** a function?
   a. The rule which assigns a capital city to each province.
   b. The rule which assigns a President to each country.
   c. The rule which assigns religion to each person.
   d. The rule which assigns tourist spot to each province.

8. A person is earning ₱500.00 per day for doing a certain job. Which of the following expresses the total salary ***S*** as a function of the number ***n*** of days that the person works?
   a. $S(n) = 500 + n$
   b. $S(n) = \frac{500}{n}$
   c. $S(n) = 500n$
   d. $S(n) = 500 - n$

For number 9 - 10 use the problem below.
Johnny was paid a fixed rate of ₱ 100 a day for working in a Computer Shop and an additional ₱5.00 for every typing job he made.

9. How much would he pay for a 5 typing job he made for a day?
   a. ₱55.00
   b. ₱175.50
   c. ₱125.00
   d. ₱170.00

10. Find the fare function *f(x)* where x represents the number of typing job he made for the day.
   a. $f(x) = 100 + 5x$
   b. $f(x) = 100 - 5x$
   c. $f(x) = 100x$
   d. $f(x) = \frac{100}{5x}$

For number 11 - 12 use the problem below.
A jeepney ride in Lucena costs ₱ 9.00 for the first 4 kilometers, and each additional kilometers adds ₱0.75 to the fare. Use a piecewise function to represent the jeepney fare **F** in terms of the distance **d** in kilometers.

$$F(d) = \begin{cases} 11._____ \\ 12._____ \end{cases}$$

11.
   a. $F(d) = \{9\ if\ 0 > d \leq 4$
   b. $F(d) = \{9\ if\ 0 < d < 4$
   c. $F(d) = \{9\ if\ 0 \geq d \geq 4$
   d. $F(d) = \{9\ if\ 0 < d \leq 4$

12.
   a. $F(d) = \{9 + 0.75(n)\ if\ 0 > d \leq 4$
   b. $F(d) = \{(9\ + 0.75)\ if\ d > 4$
   c. $F(d) = \{(9 + 0.75)\ if\ d < 4$
   d. $F(d) = \{(9 + 0.75(n)\ if\ d > 4$

For number 13 - 15 use the problem below.
Under a certain Law, the first ₱30,000.00 of earnings are subjected to 12% tax, earning greater than ₱30,000.00 and up to ₱50,000.00 are subjected to 15% tax, and earnings greater than ₱50,000.00 are taxed at 20%. Write a piecewise function that models this situation.

$$t(x) = \begin{cases} 13._____ \\ 14._____ \\ 15._____ \end{cases}$$

13.

a. $t(x) = 0.12x \; if \; x \leq 30{,}000$
b. $t(x) = 0.12x \; if \; x < 30{,}000$
c. $t(x) = 0.12x \;\; if \; x > 30{,}000$
d. $t(x) = 0.12x \; if \; x \geq 30{,}000$

14.

a. $t(x) = 0.15x \; if \; 30{,}000 < x \geq 50{,}000$
b. $t(x) = 0.15x \; if \; 30{,}000 < x \leq 50{,}000$
c. $t(x) = 0.15x \; if \; 30{,}000 \leq x \geq 50{,}000$
d. $t(x) = 0.15x \; if \; 30{,}000 \geq x \geq 50{,}000$

15.

a. $t(x) = 0.20x \; if \; x \geq 50{,}000$
b. $t(x) = 0.20x \; if \; x \leq 50{,}000$
c. $t(x) = 0.20x \; if \; x > 50{,}000$
d. $t(x) = 0.20x \; if \; x < 50{,}000$

Lesson 1 Representing Real-Life Situations Using Functions

Welcome to the first lesson of your General Mathematics. This lesson will give you the practical application of functions in a real-life scenario including the piece-wise function. When you are in Grade 8, you already encountered relation and function. But in this module, let's take into a deeper sense on how this topic can be useful in our daily life. Are you all ready?

What's In

Before we proceed in representing real-life scenario using function, let's go back to where we start. What have you remembered about relations and functions?

A ***relation*** is any set of ordered pairs. The set of all first elements of the ordered pairs is called the *domain* of the relation, and the set of all second elements is called the *range*.

A ***function*** is a relation or rule of correspondence between two elements (domain and range) such that each element in the domain corresponds to exactly one element in the range.

To further understand function, let's study the following.

Given the following ordered pairs, which relations are functions?

A = {(1,2), (2,3), (3,4), (4,5)}

B = {(3,3), (4,4), (5,5), (6,6)}

C = {(1,0), (0, 1, (-1,0), (0,-1)}

D = {(a,b), (b, c), (c,d), (a,d)}

You are right! The relations A and B are functions because each element in the domain corresponds to a unique element in the range. Meanwhile, relations C and D are not functions because they contain ordered pairs with the same domain [C = (0,1) and (0,-1), D = (a,b) and (a,d)].

How about from the given table of values, which relation shows a function?

A.

x	1	2	3	4	5	6
y	2	4	6	8	10	12

B.

x	4	-3	1	2	5
y	-5	-2	-2	-2	0

C.

x	0	-1	4	2	-1
y	3	4	0	-1	1

That's right! A and B are functions since all the values of x corresponds to exactly one value of y. Unlike table C, where -1 corresponds to two values, 4 and 1.

We can also identify a function given a diagram. On the following mapping diagrams, which do you think represent functions?

Domain **Range**

A.

B.

C.

You are correct! The relations A and C are functions because each element in the domain corresponds to a unique element in the range. However, B is a mere relation and not function because there is a domain which corresponds to more than one range.

How about if the given are graphs of relations, can you identify which are functions? Do you still remember the vertical line test? Let's recall.

A relation between two sets of numbers can be illustrated by graph in the Cartesian plane, and that a function passes the **vertical line test**.

A graph of a relation is a function if any vertical line drawn passing through the graph intersects it at exactly one point.

Using the vertical line test, can you identify the graph/s of function?

A.

C.

B.

D.

Yes, that's right! A and C are graphs of functions while B and D are not because they do not pass the vertical line test.

In Mathematics, we can represent functions in different ways. It can be represented through words, tables, mappings, equations and graphs.

What's New

We said that for a relation to become a function, the value of the domain must correspond to a single value of the range. Let's read some of the conversations and determine if they can be classified as function or not

Scenario 1: June and Mae are in a long-time relationship until June realized that he wants to marry Mae.

Scenario 2: Kim is a naturally born Filipino but because of her eyes, many people confused if she is a Chinese. Let's see how she responds to her new classmates who are asking if she's a Chinese.

Scenario 3: As part of their requirements in Statistics class, Andrei made a survey on the religion of his classmates and here's what he found out.
Andrei: Good morning classmates, as our requirement in Statistics may I know your religion. This data will be part of my input in the survey that I am doing.
Ana 1: I am a Catholic.
Kevin: I am also a Catholic.
Sam: I am a member of the Iglesia ni Cristo.
Joey: I am a Born Again Christian.
Lanie: My family is a Muslim.
Jen: We are sacred a Catholic Family.
Andrei: Thank you classmates for your responses.

Reflect on this!

1. From the above conversations, which scenario/s do you think can be classified as function? _____

2. State the reason/s why or why not the above scenarios a function.

Scenario 1:

_____

_____

Scenario 2:

_____

_____

Scenario 3:

_____

_____

What is It

Functions as representations of real-life situations

Functions can often be used to model real-life situations. Identifying an appropriate functional model will lead to a better understanding of various phenomena.

The above scenarios are all examples of relations that show function. Monogamous marriage (e.g. Christian countries) is an example of function when there is faith and loyalty. Let say, June is the domain and Mae is the range, when there is faithfulness in their marriage, there will be one-to-one relationship - one domain to one range.

Nationality could also illustrate a function. We expect that at least a person has one nationality. Let say Kim is the domain and her nationality is the range, therefore there is a one-to-one relationship. Since Kim was born and live in the Philippines, she can never have multiple nationalities except Filipino. (Remember: Under RA 9225 only those naturally-born Filipinos who have become naturalized citizens of another country can have dual citizenship. This is not applicable to Kim since she was born in the Philippines and never a citizen of other country.)

Religion is also an example of function because a person can never have two religions. Inside the classroom, three classmates said that they are Catholic. This shows a many-to-one relationship. Classmates being the domain and religion being the range indicate that different values of domain can have one value of range. One-to-one relationship was also illustrated by the classmates who said that they are Born Again, Muslim and *Iglesia ni Cristo* - one student to one religion.

Can you cite other real-life situations that show functions?

The Function Machine

Function can be illustrated as a machine where there is the input and the output. When you put an object into a machine, you expect a product as output after the process being done by the machine. For example, when you put an orange fruit into a juicer, you expect an orange juice as the output and not a grape juice. Or you will never expect to have two kinds of juices - orange and grapes.

You have learned that function can be represented by equation. Since output (y) is dependent on input (x), we can say that y is a function of x. For example, if a function machine always adds three (3) to whatever you put in it. Therefore, we can derive an equation of x + 3 = y or f*(x)* = x+ 3 where f*(x)* = y.

Let's try the following real-life situation.

A. If height *(H)* is a function of age *(a)*, give a function H that can represent the height of a person in *a* age, if every year the height is added by 2 inches.

Solution:

Since every year the height is added by 2 inches, then the height function is $\boldsymbol{H(a) = 2 + a}$

B. If distance *(D)* is a function of time *(t)*, give a function D that can represent the distance a car travels in $\boldsymbol{t}$ time, if every hour the car travels 60 kilometers.

Solution:

Since every hour, the car travels 60 kilometers, therefore the distance function is given by $\boldsymbol{D(t) = 60t}$

C. Give a function *B* that can represent the amount of battery charge of a cellular phone in *h* hour, if 12% of battery was loss every hour.

Solution:

Since every hour losses 12% of the battery, then the amount of battery function is $\boldsymbol{B(h) = 100 - 0.12h}$

D. Squares of side x are cut from each corner of a 10 in x 8 in rectangle, so that its sides can be folded to make a box with no top. Define a function in terms of x that can represent the volume of the box.

Solution:

The length and width of the box are 10 - 2x and 8 - 2x, respectively. Its height is x. Thus, the volume of the box can be represented by the function.

$$\boldsymbol{V(x) = (10 - 2x)(8 - 2x)(x) = 80x - 36x^2 + 4x^3}$$

Piecewise Functions

There are functions that requires more than one formula in order to obtain the given output. There are instances when we need to describe situations in which a rule or relationship changes as the input value crosses certain boundaries. In this case, we need to apply the piecewise function.

A piecewise function is a function in which more than one formula is used to define the output. Each formula has its own domain, and the domain of the function is the union of all these smaller domains. We notate this idea like this:

$$f(x) = \begin{cases} \text{formula 1 if x is in domain 1} \\ \text{formula 2 if x is in domain 2} \\ \text{formula 3 if x is in domain 3} \end{cases}$$

Look at these examples!

A. A user is charged ₱250.00 monthly for a particular mobile plan, which includes 200 free text messages. Messages in excess of 200 are charged ₱1.00 each. Represent the monthly cost for text messaging using the function t(m), where m is the number of messages sent in a month.

Answer:

$$t(m) = \begin{cases} 250 \; if \; 0 < m \leq 200 \\ (250 + m) \; if \; m > 200 \end{cases}$$

For sending messages of not exceeding 200

In case the messages sent were more than 200

B. A certain chocolate bar costs ₱50.00 per piece. However, if you buy more than 5 pieces they will mark down the price to ₱48.00 per piece. Use a piecewise function to represent the cost in terms of the number of chocolate bars bought.

Answer:

$$f(n) = \begin{cases} 50 \; if \; 0 < n \leq 5 \\ (48n) \; if \; n > 5 \end{cases}$$

For buying 5 chocolate bars or less

For buying more than 5 chocolate bars

C. The cost of hiring a catering service to serve food for a party is ₱250.00 per head for 50 persons or less, ₱200.00 per head for 51 to 100 persons, and ₱150.00 per head for more than 100. Represent the total cost as a piecewise function of the number of attendees to the party.

Answer:

$$C(h) = \begin{cases} 250 \; if \; n \leq 50 \\ 200 \; if \; 51 \leq n \leq 100 \\ 150 \; if \; n > 100 \end{cases}$$

Cost for a service to at least 50 persons

Cost for a service to 51 to 100 persons

Cost for a service to more than 100 persons

What's More

Read each situation carefully to solve each problem. Write your answer on a separate sheet of your paper.

Independent Practice 1

1. A person is earning ₱750.00 per day to do a certain job. Express the total salary ***S*** as a function of the number ***n*** of days that the person works.

   Answer:

   S(n) = _____ (Hint: Think of the operation needed in order to obtain the total salary?)

2. Xandria rides through a jeepney which charges ₱ 8.00 for the first 4 kilometers and additional ₱0.50 for each additional kilometer. Express the jeepney fare (F) as function of the number of kilometers (d) that Xandria pays for the ride.

   Answer:

   F(d) = _____ (Hint: Aside from the usual fare charge, don't forget to include in the equation the additional fare charge for the exceeding distance)

Independent Assessment 1

1. A computer shop charges ₱15.00 in every hour of computer rental. Represent your computer rental fee **(R)** using the function R(t) where **t** is the number of hours you spent on the computer.

   Answer:

2. Squares of side **a** are cut from each corner of a 8 in x 6 in rectangle, so that its sides can be folded to make a box with no top. Represent a function in terms of **a** that can define the volume of the box.

   Answer:

Independent Practice 2

1. A tricycle ride costs ₱10.00 for the first 2 kilometers, and each additional kilometer adds ₱8.00 to the fare. Use a piecewise function to represent the tricycle fare in terms of the distance ***d*** in kilometers.

   Answer:

   $$\boldsymbol{C(d)} = \begin{cases} \mathbf{10}\ \boldsymbol{if}____ \\ (_____)\ \boldsymbol{if\ d} \geq \mathbf{3} \end{cases}$$

   (Fill in the missing terms to show the piecewise function of the problem)

3. A parking fee at SM Lucena costs ₱25.00 for the first two hours and an extra ₱5.00 for each hour of extension. If you park for more than twelve hours, you instead pay a flat rate of ₱100.00. Represent your parking fee using the function **p(t)** where ***t*** is the number of hours you parked in the mall.

   Answer:

   $$p(t) = \begin{cases} 25\ if_____ \\ (25 + 5t)\ if_____ \\ _____ift > 12 \end{cases}$$

   (Fill in the missing terms to show the piecewise function of the problem)

Independent Assessment 2

1. A van rental charges ₱5,500.00 flat rate for a whole-day tour in CALABARZON of 5 passengers and each additional passenger added ₱500.00 to the tour fare. Express a piecewise function to show to represent the van rental in terms number of passenger **n.**

   Answer:

2. An internet company charges ₱500.00 for the first 30 GB used in a month. Every exceeding GB will then cost ₱30.00 But if the costumer reach a total of 50 GB and above, a flat rate of ₱1,000.00 will be charged instead. Write a piecewise function C*(g)* that represents the charge according to GB used?

   Answer

What I Have Learned

A. Read and analyze the following statements. If you think the statement suggests an incorrect idea, rewrite it on the given space, otherwise leave it blank.

1. A relation is a set of ordered pairs where the first element is called the range while the second element is the domain.

_____
_____

2. A function can be classified as one-to-one correspondence, one-to-many correspondence and many-to-one correspondence.

_____
_____

3. In a function machine, the input represents the independent variable while the output is the dependent variable.

_____
_____

B. In three to five sentences, write the significance of function in showing real-life situations.

_____
_____
_____
_____
_____
.

C. In your own words, discuss when a piecewise function is being used.

_____
_____
_____
_____

What I Can Do

At home or in your community, look for the at least three (3) situations that could represent functions. From the identified situations, write a sample problem and its corresponding function equation.

Example:

Situation: The budget for food is a function of the number of family members.

Problem: Reyes family has Php ₱1,500.00 food budget for each member of their family in a month. Express the total food budget *(B)* as a function of number of family members *(n)* in one month.

Function: $B(x) = 1500x$

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following is **not true** about function?
   a. Function is composed of two quantities where one depends on the other.
   b. One-to-one correspondence is a function.
   c. Many-to-one correspondence is a function.
   d. One-to-many correspondence is a function.

2. In a relation, what do you call the y values or the output?
   a. Piecewise
   b. Range
   c. Domain
   d. Independent

3. Which of the following tables is **NOT** a representation of functions?
   a.

x	2	1	0	1
y	3	6	7	2

   b.

x	-2	-1	0	1
y	0	-3	0	3

c.

x	-1	-2	-3	-4
y	1	2	3	4

d.

x	0	2	4	6
y	6	5	4	3

4. In this table, what is the domain of the function?

x	1	2	3	4	5
y	a	b	c	d	e

a. D: {2, 4, 6, 8, 10}
b. D: {a, b, c, d, e}
c. D: {1, 2, 3, 4, 5}
d. y = {1, 2, 3, 4, 5, a, b, c, d}

5. Which of the following relations is/are function/s?
a. x = {(-1,2), (-3,4), (-1,7), (5,1)}
b. g = {(-3,2), (3,1), (-3,2), (5,7)}
c. h = {(6,1), (-2,3), (2, 6), (7, 2)}`
d. y = {(2,3), (3,2), (-2,3), (3, -2)}

6. Which of the following relations is/are function/s?
a. the rule which assigns to each person the name of his brother
b. the rule which assigns the name of teachers you have
c. the rule which assigns a pen and the color of its ink
d. the rule which assigns each person a surname

7. A person can encode 1000 words in every hour of typing job. Which of the following expresses the total words W as a function of the number n of hours that the person can encode?
a. $W(n) = 1000 + n$
b. $W(n) = \frac{1000}{n}$
c. $W(n) = 1000n$
d. $W(n) = 1000 - n$

8. Judy is earning ₱300.00 per day for cleaning the house of Mrs. Perez and additional ₱25.00 for an hour of taking care Mrs. Perez's child. Express the total salary (S) of Judy including the time (h) spent for taking care the child.
   a. $S(h) = 300 + 25h$
   b. $S(h) = 300 - 25h$
   c. $S(h) = 300(25h)$
   d. $S(h) = \frac{300}{25h}$

9. Which of the following functions define the volume of a cube?
   a. $V = 3s$, where s is the length of the edge
   b. $V = s^3$, where s is the length of the edge
   c. $V = 2s^3$, where s is the length of the edge
   d. $V = \frac{s}{3}$, where s is the length of the edge
10. Eighty meters of fencing is available to enclose the rectangular garden of Mang Gustin. Give a function A that can represent the area that can be enclosed in terms of x.
   a. $A(x) = 40x - x^2$
   b. $A(x) = 80x - x^2$
   c. $A(x) = 40x^2 - x$
   d. $A(x) = 80x^2 - x$

For number 11 - 12 use the problem below.

A user is charged ₱400.00 monthly for a Sun and Text mobile plan which include 500 free texts messages. Messages in excess of 500 is charged ₱1.00. Represent a monthly cost for the mobile plan using s(t) where t is the number of messages sent in a month.

$$s(t) = \begin{cases} 11._____ \\ 12._____ \end{cases}$$

11.
   a. $s(t) = \{400, if\ 0 < t \leq 500$
   b. $s(t) = \{400, if\ 0 < t \geq 500$
   c. $s(t) = \{400, if\ 0 < t < 500$
   d. $s(t) = \{400, if\ 0 > t > 500$

12.
   a. $s(t) = 400 + t, if\ t > 500$
   b. $s(t) = 400 + t, if\ t \leq 500$
   c. $s(t) = 400 + 2t, if\ t \geq 500$
   d. $s(t) = 400 + 2t, if t \leq 500$

For number 13 - 15 use the problem below.

Cotta National High School GPTA officers want to give t-shirts to all the students for the foundation day. They found a supplier that sells t-shirt for ₱200.00 per piece but can charge to ₱18,000.00 for a bulk order of 100 shirts and ₱175.00 for each excess t-shirt after that. Use a piecewise function to express the cost in terms of the number of t-shirt purchase

$$t(s) = \begin{cases} 13._____ \\ 14._____ \\ 15._____ \end{cases}$$

13.

a. $t(s) = \{200s, if\ 0 < s \leq 100$
b. $t(s) = \{200s, if\ 0 \geq s \leq 99$
c. $t(s) = \{200s, if\ 0 > s \leq 100$
d. $t(s) = \{200s, if\ 0 < s \leq 99$

14.

a. $t(s) = \{18{,}000, if\ s\ \geq 100$
b. $t(s) = \{18{,}000, if\ s > 100$
c. $t(s) = \{18{,}000, if\ s\ = 100$
d. $t(s) = \{18{,}000, if\ s\ < 100$

15.

a. $t(s) = \{18{,}000 + 175(s - 100), if\ s > 100$
b. $t(s) = \{18{,}000 + 175(s - 100), if\ s \geq 100$
c. $t(s) = \{18{,}000 + 175s, if\ s > 100$
d. $t(s) = \{18{,}000 + 175s, if\ s\ \leq 100$

Additional Activities

If you believe that you learned a lot from the module and you feel that you need more activities, well this part is for you.

Read and analyze each situation carefully and apply your learnings on representing real-life situations involving functions including piecewise.

1. Contaminated water is subjected to a cleaning process. The concentration of the pollutants is initially 5 mg per liter of water. If the cleaning process can reduce the pollutant by 10% each hour, define a function that can represent the concentration of pollutants in the water in terms of the number of hours that the cleaning process has taken place.

2. During typhoon Ambo, PAGASA tracks the amount of accumulating rainfall. For the first three hours of typhoon, the rain fell at a constant rate of 25mm per hour. The typhoon slows down for an hour and started again at a constant rate of 20 mm per hour for the next two hours. Write a piecewise function that models the amount of rainfall as function of time.

Answer Key

What I Know

1. A
2. B
3. C
4. B
5. D
6. B
7. D
8. C
9. C
10. A
11. D
12. D
13. A
14. B
15. C

What's More

Independent Practice 1

1. $S(n) = 750n$
2. $F(d) = 8 + 0.50d$

Independent Assessment 1

1. $R(t) = 15t$
2. $V(a) = 48a - 28a^2 + 4a^3$

Independent Practice 2

1. $c(d) = \begin{cases} 10, if\ d \leq 2 \\ 10 + 8(d), if\ d \geq 3 \end{cases}$
2. $p(t) = \begin{cases} 25, if\ t \leq 2 \\ 25 + 5t, if\ 12 > t \geq 3 \\ 100, if\ t\ > 12 \end{cases}$

Independent Assessment2

1. $v(n) = \begin{cases} 5{,}500, if\ n \leq 5 \\ 5{,}500 + 500n, if\ n > 5 \end{cases}$
2. $C(g) = \begin{cases} 500, if\ 0 < g \leq 30 \\ 500 + 30g, if\ 50 > g \geq 31 \\ 1000, if\ g \geq 50 \end{cases}$

Assessment

1. D
2. B
3. A
4. C
5. C
6. D
7. C
8. A
9. B
10. A
11. A
12. B
13. D
14. C
15. A

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11 General Mathematics

General Mathematics
Evaluating Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Evaluating Functions

Introductory Message

For the facilitator:

Welcome to Grade 11 General Mathematics Alternative Delivery Mode (ADM) Module on Evaluating Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:
Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Evaluating Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you master the key concepts of functions specifically on evaluating functions. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

1. recall the process of substitution;
2. identify the various types of functions; and
3. evaluate functions.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following is a polynomial function?

   a. $f(x) = 2x^2 - 10x + 7$

   b. $g(x) = 4|x^2 - 3x + 8|$

   c. $p(x) = \|x^3 - 7\|$

   d. $s(x) = 2^m - 1$

2. What kind of function is being illustrated by $f(x) = |2x^3 - 3x| + 5$?

   a. Rational Function

   b. Constant Function

   c. Greatest Integer Function

   d. Absolute Value Function

3. Find the function value given $h(x) = 17 + 8x$ of $x = 4d$.

   a. $17 - 32d$

   b. $17 - 32d^2$

   c. $17 + 32d$

   d. $17 + 32d^2$

4. Which of the following shows a logarithmic function?

   a. $f(x) = 8x^3 + 8$

   b. $f(x) = \log_9 81$

   c. $f(x) = |3^x - 6|$

   d. $f(x) = \|x - 1\| - 8$

5. Find the function value given $h(x) = 7x - 11$, if $x = 8m + 3$.

   a. $56m + 10$

   b. $56m - 10$

   c. $56m^2 + 10$

   d. $56m^2 - 10$

6. Which of the following is the value of the function $f(x) = |3x^2 - 15x + 5| + 3$ given x = 3?

   a. 25

   b. 16

   c. 19

   d. 10

7. Evaluate the function $h(x) = \|x\| + 31$ given x = 2.5.

   a. 34

   b. -34

   c. -33

   d. 33

8. Give the value of the of the function $c(x) = 5x^3 - 18$ at $c(3)$.

   a. 117

   b. 27

   c. 153

   d. 63

9. Evaluate: $h(x) = 5x^2 - 8x + 12$ given x = 5.

a. 22
b. 145
c. 97
d. -3

10. Find the value of the function $h(x) = \sqrt{5x^2 - 4}$ if $x = 6$.

a. $\sqrt{80}$
b. $2\sqrt{19}$
c. $\sqrt{16}$
d. 4

11. Evaluate the function $f(x) = 3x^2 - 5x + 2$ given $x = 2x + 5$.

a. $12x^2 + 50x + 52$
b. $12x^2 + 65x + 77$
c. $12x^2 - 50x + 52$
d. $12x^2 - 65x + 77$

12. Given $h(x) = \dfrac{2x^2 - 5}{3}$, determine h(5).

a. -15
b. $-\dfrac{5}{3}$
c. 15
d. $\dfrac{5}{3}$

13. Evaluate the function $k(x) = 5^x$ if $x = \dfrac{2}{3}$.

a. $\sqrt[3]{5}$
b. $\sqrt{25}$
c. $\sqrt{5}$
d. $\sqrt[3]{25}$

14. Given $g(x) = \dfrac{2x^2 - 3x + 7}{3x - 4}$, determine $g(2)$.

a. $\dfrac{9}{2}$
b. $-\dfrac{9}{2}$
c. $\dfrac{8}{7}$
d. $-\dfrac{8}{7}$

15. For what values of x can we not evaluate the function $f(x) = \dfrac{3x + 7}{x^2 - 4}$?

a. ±4
b. ±3
c. ±2
d. ±1

Lesson 1 Evaluating Functions

Finding the value of "x" for most of the students is what Mathematics is all about. Sometimes, it seems to be a joke for the students to evaluate an expression, like what is shown by the illustration.

Find x.

Here it is!

If you want to learn how to find the value of "y", well then, you are in the right page. WELCOME to your second module!

What's In

Before we begin, let's go back to the time when you first encounter how to evaluate expressions.

Do you still remember?

Given the following expressions, find its value if x = 3.

1. $x-9$
2. $3x+7$
3. $x^2+4x-10$
4. $2x^2-6x+26$
5. $3x^2-6$

We have learned that, in an algebraic expression, letters can stand for numbers. And to find the value of the expression, there are two things that you have to do.

1. **Replace each letter in the expression with the assigned value.**
   First, replace each letter in the expression with the value that has been assigned to it. To make your calculations clear and avoid mistakes, always enclose the numbers you're substituting inside parentheses. The value that's given to a variable stays the same throughout the entire problem, even if the letter occurs more than once in the expression.

   However, since variables "vary", the value assigned to a particular variable can change from problem to problem, just not within a single problem.

2. **Perform the operations in the expression using the correct order of operations.**
   Once you've substituted the value for the letter, do the operations to find the value of the expression. Don't forget to use the correct order of operations: first do any operations involving exponents, then do multiplication and division, and finally do addition and subtraction!

If in the activity above, you do the same process in order to arrive with these answers, then, this module seems to be very easy to you.

Solutions:

Given the following expressions, find its value if x = 3.

1. $x-9$

$$= x-9$$
$$= (3)-9$$
$$= -6$$

Since x = 3, we just replaced x by 3 in the expression, then subtract by 9.

2. $3x+7$

$$= 3x+7$$
$$= 3(3)+7$$
$$= 9+7$$
$$= 16$$

Following the steps, we just replace x by 3, multiply it by the numerical coefficient 3, then add 7

3. $x^2 + 4x - 10$

$$= x^2 + 4x - 10$$
$$= (3)^2 + 4(3) - 10$$
$$= 9 + 12 - 10$$
$$= 11$$

After replacing x by 3, we get the squared of 3 which is 9, add it to the product of 4 and 3, then lastly, we subtracted 10 from its sum.

4. $2x^2 - 6x + 26$

$$= 2x^2 - 6x + 26$$
$$= 2(3)^2 - 6(3) + 26$$
$$= 18 - 18 + 26$$
$$= 26$$

Simply each term inside the parenthesis in order to arrive with 18 subtracted by 18 plus 26

5. $3x^3 - 6$

$$= 3x^3 - 6$$
$$= 3(3)^3 - 6$$
$$= 3(27) - 6$$
$$= 81 - 6$$
$$= 75$$

Get the cubed of 3 which is 27, then multiply it to 3 to get 81 then subtract 6

What's New

Types of Functions

Before you proceed to this module, try to look and analyze some of the common types of functions that you might encounter as you go on with this module.

Types of Function	Description	Example
Constant Function	A constant function is a function that has the same output value no matter what your input value is. Because of this, a constant function has the form $f(x) = b$, where b is a constant (a single value that does not change).	$y = 7$

Identity Function	The identity function is a function which returns the same value, which was used as its argument. In other words, the identity function is the function $f(x)=x$, for all values of x.	$f(2)=2$
Polynomial Function	A polynomial function is defined by $y=a^0+a^1x+a^2x^2+...+a^nx^n$, where n is a non-negative integer and a^0, a^1, a^2 ,..., n ∈ R.	
✓ Linear Function	The polynomial function with degree one. It is in the form $y=mx+b$	$y=2x+5$
✓ Quadratic Function	If the degree of the polynomial function is two, then it is a quadratic function. It is expressed as $y=ax^2+bx+c$, where a ≠ 0 and a, b, c are constant and x is a variable.	$y=3x^2+2x+5$
✓ Cubic Function	A cubic polynomial function is a polynomial of degree three and can be denoted by $f(x)=ax^3+bx^2+cx+d$, where a ≠ 0 and a, b, c, and d are constant & x is a variable.	$y=5x^3+3x^2+2x+5$
Power Function	A power function is a function in the form $y=ax^b$ where b is any real constant number. Many of our parent functions such as linear functions and quadratic functions are in fact power functions.	$f(x)=8x^5$
Rational Function	A rational function is any function which can be represented by a rational fraction say, $\frac{p(x)}{q(x)}$ in which numerator, p(x) and denominator, q(x) are polynomial functions of x, where q(x) ≠ 0.	$f(x)=\frac{x^2-3x+2}{x^2-4}$
Exponential function	These are functions of the form: $y=ab^x$, where x is in an exponent and a and b are constants. (Note that only b is raised to the power x; not a.) If the base b is greater than 1 then the result is exponential growth.	$y=2^x$
Logarithmic Function	Logarithmic functions are the inverses of exponential functions, and any exponential function can be expressed in logarithmic form. Logarithms are very useful in permitting us to work with very large numbers while manipulating numbers of a much more manageable size. It is written in the form $y=\log_b x \qquad x>0,\ where\ b>0\ and\ b\neq 1$	$y=\log_7 49$

Absolute Value Function	The absolute value of any number, c is represented in the form of |c|. If any function f: R→ R is defined by $f(x)=\|x\|$, it is known as absolute value function. For each non-negative value of x, f(x) = x and for each negative value of x, f(x) = -x, i.e., f(x) = {x, if x ≥ 0; – x, if x < 0.	$y=\|x-4\|+2$
Greatest Integer Function	If a function f: R→ R is defined by f(x) = [x], x ∈ X. It round-off to the real number to the integer less than the number. Suppose, the given interval is in the form of (k, k+1), the value of greatest integer function is k which is an integer.	$f(x)=\|\|x\|\|+1$ where $\|\|x\|\|$ is the greatest integer function

What is It

Evaluating function is the process of determining the value of the function at the number assigned to a given variable. Just like in evaluating algebraic expressions, to evaluate function you just need to a.) replace each letter in the expression with the assigned value and b.) perform the operations in the expression using the correct order of operations.

Look at these examples!

Example 1: Given $f(x)=2x-4$, find the value of the function if x = 3.

Solution:

$$f(3)=2(3)-4$$

$$f(3)=6-4$$

$$f(3)=2$$

- ✓ Substitute 3 for x in the function.
- ✓ Simplify the expression on the right side of the equation.

Answer: Given $f(x)=2x-4$, $f(3)=2$

Example 2: Given $g(x) = 3x^2 + 7$, find $g(-3)$.

Solution:

$$g(-3) = 3(-3)^2 + 7$$
$$g(-3) = 3(9) + 7$$
$$g(-3) = 27 + 7$$
$$g(-3) = 34$$

- ✓ Substitute -3 for x in the function.
- ✓ Simplify the expression on the right side of the equation.

Answer: Given $g(x) = 3x^2 + 7$, $g(-3) = 34$

Example 3: Given $p(x) = 3x^2 + 5x - 2$, find $p(0)$ and $p(-1)$.

Solution:

$$p(0) = 3(0)^2 + 5(0) - 2$$
$$p(0) = 3(0) + 0 - 2$$
$$p(0) = 0 + 0 - 2$$
$$p(0) = -2$$

$$p(0) = 3(-1)^2 + 5(-1) - 2$$
$$p(0) = 3(1) - 5 - 2$$
$$p(0) = 3 - 5 - 2$$
$$p(0) = -4$$

Treat each of these like two separate problems. In each case, you substitute the value in for *x* and simplify. Start with *x* = 0, then x=-1.

Answer: Given $p(x) = 3x^2 + 5x - 2$, $p(0) = -2$, $p(-1) = -4$

Example 4: Given $f(x) = 5x + 1$, find $f(h+1)$.

Solution:

$$f(h+1) = 5(h+1) + 1$$
$$f(h+1) = 5h + 5 + 1$$
$$f(h+1) = 5h + 6$$

- ✓ This time, you substitute (h + 1) into the equation for *x*.
- ✓ Use the distributive property on the right side, and then combine like terms to simplify.

Answer: Given $f(x) = 5x + 1$, $f(h+1) = 5h + 6$

Example 5: Given $g(x) = \sqrt{3x - 2}$, find $g(9)$.

Solution:

$$g(9) = \sqrt{3(9) - 2}$$
$$g(9) = \sqrt{27 - 2}$$
$$g(9) = \sqrt{25}$$
$$g(9) = 5$$

- ✓ Substitute 9 for x in the function.
- ✓ Simplify the expression on the right side of the equation.

Answer: Given $g(x) = \sqrt{3x - 2}$, $g(9) = 5$

Example 6: Given $h(x)=\dfrac{4x+8}{2x-4}$, find the value of function if $x=-5$

Solution:

$$h(-5)=\frac{4(-5)+8}{2(-5)-4}$$

$$h(-5)=\frac{-20+8}{-10-4}$$

$$h(-5)=\frac{-12}{-14}$$

$$h(-5)=\frac{6}{7}$$

- ✓ Substitute -5 for x in the function.
- ✓ Simplify the expression on the right side of the equation. (recall the concepts of integers and simplifying fractions)

Answer: Given $h(x)=\dfrac{4x+8}{2x-4}$, $h(-5)=\dfrac{6}{7}$

Example 7: Evaluate $f(x)=2^x$ if $x=\dfrac{3}{2}$.

Solution:

$$f\left(\frac{3}{2}\right)=2^{\frac{3}{2}}$$

$$f\left(\frac{3}{2}\right)=\sqrt{2^3}$$

$$f\left(\frac{3}{2}\right)=\sqrt{8}$$

$$f\left(\frac{3}{2}\right)=\sqrt{4\bullet 2}$$

$$f\left(\frac{3}{2}\right)=2\sqrt{2}$$

- ✓ Substitute $\dfrac{3}{2}$ for x in the function.
- ✓ Simplify the expression on the right side of the equation. (get the cubed of 2 which is 8, then simplify)

Answer: Given $f(x)=2^x$, $f\left(\dfrac{3}{2}\right)=2\sqrt{2}$

Example 8: Evaluate the function $h(x) = \lfloor x \rfloor + 2$ where $\lfloor x \rfloor$ is the greatest integer function given $x = 2.4$.

Solution:

$$h(2.4) = \lfloor 2.4 \rfloor + 2$$
$$h(2.4) = 2 + 2$$
$$h(2.4) = 4$$

- ✓ Substitute 2.4 for x in the function.
- ✓ Simplify the expression on the right side of the equation. (remember that in greatest integer function, value was rounded-off to the real number to the integer less than the number)

Answer: Given $h(x) = \lfloor x \rfloor + 2$, $h(2.4) = 4$

Example 9:Evaluate the function $f(x) = |x - 8|$ where $|x - 8|$ means the absolute value of $x - 8$ if $x = 3$.

Solution:

$$f(3) = |3 - 8|$$
$$f(3) = |-5|$$
$$f(3) = 5$$

- ✓ Substitute 3 for x in the function.
- ✓ Simplify the expression on the right side of the equation. (remember that any number in the absolute value sign is always positive)

Answer: Given $f(x) = |x - 8|$, $f(3) = 5$

Example 10: Evaluate the function $f(x) = x^2 - 2x + 2$ at $f(2x - 3)$.

Solution:

$$f(2x-3) = (2x-3)^2 - 2(2x-3) + 2$$
$$f(2x-3) = (4x^2 - 12x + 9) - 4x + 6 + 2$$
$$f(2x-3) = 4x^2 - 12x + 9 - 4x + 6 + 2$$
$$f(2x-3) = 4x^2 - 12x - 4x + 9 + 6 + 2$$
$$f(2x-3) = 4x^2 - 16x + 17$$

- ✓ Substitute $2x - 3$ for x in the function.
- ✓ Simplify the expression on the right side of the equation.

What's More

Your Turn!

Independent Practice 1: Fill Me

Evaluate the following functions by filling up the missing parts of the solution.

1. $f(x) = 3x - 5$, find $f(2)$

   Solution:

   $f(2) =$ _____

   $f(2) = 6 - 5$

   $f(2) =$ _____

2. $g(x) = 3|2x|$, find *g(6)*

   Solution:

   $g(6) =$ _____

   $g(6) = 3|12|$

   $g(6) =$ _____

3. $k(a) = |a| - 2$, find $k(-9)$

   Solution:

   $k(-9) =$ _____

   $k(-9) = 9 - 2$

   $k(-9) =$ _____

4. $p(a) = -4a - 2$, find $p(2a)$

   Solution:

   $p(2a) =$ _____

   $p(2a) =$ _____

5. $g(t) = t^2 - 2$, find $g(-2)$

   Solution:

   $g(-2) =$ _____

   $g(-2) =$ _____

   $g(-2) =$ _____

Independent Assessment 1: Evaluate!

Evaluate the following functions. Write your answer and complete solution on separate paper.

1. Given $w(n) = n - 1$, find the value of the function if w = -1.

2. Given $f(x) = \|x\| - 3$, find $f(9.3)$.

3. Evaluate the function $w(x) = |-2x + 3|$ if x = -1.

4. Evaluate: $f(x) = -x - 1$, find $f(a^2)$

5. Given $f(x) = 4x - 5$, find $f(2x+3)$

Independent Practice 2: TRUE or SOLVE!

Analyze the following functions by evaluating its value. Write **TRUE** of the indicated answer and solution is correct, if not, rewrite the solution to arrive with the correct answer on the space provided.

1. Evaluate $f(t) = 2t - 3$; $f(t^2)$

Solution:

$$f(t^2) = 2(t^2) - 3$$
$$f(t^2) = 2t^2 - 3$$

Answer:

2. Given the function $g(x) = \sqrt{5x - 13}$, find $g(9)$.

Solution:

$$g(9) = \sqrt{5(9) - 13}$$
$$g(9) = \sqrt{45 - 13}$$
$$g(9) = \sqrt{32}$$
$$g(9) = 16\sqrt{2}$$

Answer:

3. Given the function $f(x)=\dfrac{5x-7}{3x-2}$, find the value of the function if $x=-3$.

Solution:

$$f(-3)=\frac{5(-3)-7}{3(-3)-2}$$

$$f(-3)=\frac{-15-7}{-9-2}$$

$$f(-3)=\frac{22}{11}$$

$$f(-3)=-2$$

Answer:

4. Evaluate the function $f(x)=x^2-3x+5$ at $f(3x-1)$.

Solution:

$$f(3x-1)=(3x-1)^2-3x+5$$

$$f(3x-1)=9x^2-6x+1-3x+5$$

$$f(3x-1)=9x^2-9x+6$$

Answer:

5. Evaluate: $g(x)=3^x$ if $x=\dfrac{4}{3}$

Solution:

$$g\left(\frac{4}{3}\right)=3^{\frac{4}{3}}$$

$$g\left(\frac{4}{3}\right)=\sqrt[3]{3^4}$$

$$g\left(\frac{4}{3}\right)=\sqrt[3]{81}$$

$$g\left(\frac{4}{3}\right)=\sqrt[3]{27\bullet 3}$$

$$g\left(\frac{4}{3}\right)=3\sqrt[3]{3}$$

Answer:

Independent Assessment 2: Find my Value!

Evaluate the following functions. Write your solution on a separate paper.

1. $g(x)=5x-7$; $g(x^2+1)$

   Answer: _____

2. $h(t)=\sqrt{x^2+2x+4}$; $h(2)$

   Answer: _____

3. $k(x) = \dfrac{3x^2 - 1}{2x + 4}$; $k(-3)$

Answer: _____

4. $f(x) = 2x^2 + 5x - 9$; $f(5x - 2)$

Answer: _____

5. $g(p) = 4^x$; $x = \dfrac{3}{2}$

Answer: _____

What I Have Learned

A. Complete the following statements to show how you understood the different types of functions. Answer using your own words,

1. A polynomial function is _____
_____.

2. An exponential function _____
_____.

3. A rational function _____
_____.

4. An absolute value function _____
_____.

5. A greatest integer function _____
_____.

B. Fill in the blanks to show how we evaluate functions.

Evaluating function is the process of _____ of the function at the _____ assigned to a given variable. Just like in evaluating algebraic expressions, to evaluate function you just need to _____ in the expression with the assigned value, then _____ in the expression using the correct order of operations. Don't forget to _____ your answer.

What I Can Do

In this part of the module, you will apply your knowledge on evaluating functions in solving real-life situations. Write your complete answer on the given space.

1. Mark charges ₱100.00 for an encoding work. In addition, he charges ₱5.00 per page of printed output.

   a. Find a function f(x) where x represents the number page of printed out.

   b. How much will Mark charge for 55-page encoding and printing work?

2. Under certain circumstances, a virus spreads according to the function:

$$P(t) = \frac{1}{1+15(2.1)^{-0.3t}}$$

Where where P(t) is the proportion of the population that has the virus (t) days after the acquisition of virus started. Find p(4) and p(10), and interpret the results.

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following is **not** a polynomial function?

   a. $f(x) = 2x - 10$

   b. $g(x) = 4x^2 - 3x + 8$

   c. $p(x) = x^3 - 7$

   d. $s(x) = |3x - 4| - 9$

2. What kind of function is being illustrated by $f(x) = \frac{3x-11}{x+7}$?

   a. Rational Function

   b. Constant Function

c. Greatest Integer Function

d. Absolute Value Function

3. Find the function value given $h(x) = 9 - 5x$ of $x = 3m$.

a. $9 - 15m$

b. $9 - 15m^2$

c. $9 + 15m$

d. $9 + 15m^2$

4. Which of the following shows an exponential function?

a. $f(x) = 3x + 8$

b. $f(x) = 2x^3 - 7$

c. $f(x) = 3^x - 6$

d. $f(x) = \|x\| - 8$

5. Find the function value given $h(x) = 3x - 8$, if $x = 9a + 1$.

a. $27a + 5$

b. $27a - 5$

c. $18a + 11$

d. $18a - 11$

6. Which of the following is the value of the function $f(x) = |4x^2 - 8| + 2$ given x = 2?

a. 8

b. 9

c. 10

d. 11

7. Evaluate the function $h(x) = \|x\| - 11$ given x = 3.5.

a. -8

b. 8

c. -9

d. 9

8. Give the value of the of the function $c(x) = 3x^2 - 36$ at $c(5)$.

a. -21

b. 14

c. 111

d. 39

9. Evaluate: $h(x) = 5x^3 - 3x \ + 9$ given x = 3.

a. 45

b. 63

c. 135

d. 153

10. Find the value of the function $f(x) = \sqrt{2x^2 + 3}$ if $x = 6$.

a. $\sqrt{75}$

b. $5\sqrt{3}$

c. $\sqrt{15}$

d. $2\sqrt{3}$

11. Evaluate the function $f(x) = 2x^2 - 3x + 1$ given $x = 3x - 5$.

a. $f(3x-5) = 18x^2 - 69x + 66$

b. $f(3x-5) = 18x^2 - 63x + 51$

c. $f(3x-5) = 18x^2 + 69x - 66$

d. $f(3x-5) = 18x^2 + 63x - 51$

12. Given g(x) = $\dfrac{x^2 - 3}{2}$, determine g(5).

a. 11

b. $\dfrac{7}{2}$

c. -11

d. $-\dfrac{7}{2}$

13. Evaluate the function $g(x) = 3^x$ if $x = \frac{5}{3}$.

a. $\sqrt[3]{243}$

b. $\sqrt{243}$

c. $9\sqrt{3}$

d. $3\sqrt[3]{9}$

14. Given $g(x) = \frac{x^2 - 2x + 5}{x + 3}$, determine $g(4)$.

a. $\frac{5}{7}$

b. $-\frac{5}{7}$

c. $\frac{13}{7}$

d. $-\frac{13}{7}$

15. For what values of x can we not evaluate the function $f(x) = \frac{x + 4}{x^2 - 9}$?

a. ±4

b. ±3

c. ±2

d. ±1

Additional Activities

Difference Quotient

$\frac{f(x+h)-f(x)}{h}$ this quantity is called difference quotient. Specifically, the difference quotient is used in the discussion of the rate of change, a fundamental concept in calculus.

Example: Find the difference quotient for each of the following function.

A. *f(x) = 4x - 2*

B. *f(x) = x²*

Solution:

A. *f(x) = 4x - 2*

$$f(x+h)=4(x+h)-2=4x+4h-2$$
$$\frac{f(x+h)-f(x)}{h}=\frac{4x+4h-2-(4x-2)}{h}$$
$$=\frac{4x+4h-2-4x+2)}{h}$$
$$=\frac{4h}{h}$$
$$=4$$

B. *f(x) = x²*

$$f(x+h)=(x+h)^2=x^2+2hx+h^2$$
$$\frac{f(x+h)-f(x)}{h}=\frac{x^2+2hx+h^2-(x)^2}{h}$$
$$=\frac{x^2+2hx+h^2-(x)^2}{h}$$
$$=\frac{2hx+h^2}{h}$$
$$=2x+h$$

YOUR TURN!

Find the value of $\frac{f(x+h)-f(x)}{h}$, h ≠ 0 for each of the following function.

1. $f(x)=3x+4$
2. $g(x)=x^2+3$

Answer Key

What I Know

1. A
2. D
3. C
4. B
5. A
6. B
7. D
8. A
9. C
10. B
11. A
12. C
13. D
14. A
15. C

What's More

Independent Practice 1

1. $f(2) = 3(2) - 5$
   $f(2) = 6 - 5$
   $f(2) = 1$
2. $g(6) = 3|2 \bullet 6|$
   $g(6) = 3|12|$
   $g(6) = 36$
3. $k(-9) = |-9|$
   $k(-9) = 9 - 2$
   $k(-9) = 7$
4. $p(2a) = -4(2a) - 2$
   $p(2a) = -8a - 2$
   $p(2a) = -2(a - 1)$
5. $g(-2) = (-2)^2$
   $g(-2) = 4 - 2$
   $g(-2) = 2$

Assessment

1. D
2. A
3. A
4. C
5. B
6. C
7. A
8. D
9. C
10. B
11. A
12. A
13. D
14. C
15. B

Independent Assessment 1

1. -2
2. 6
3. 5
4. $-a^2 - 1$
5. $8x + 7$

Independent Practice 2

1. **TRUE**
2. $4\sqrt{2}$
3. **2**
4. $9x^2 - 15x + 9$
5. **TRUE**

Independent Assessment 2

1. $5x^2 - 2$
2. $2\sqrt{3}$
3. **-13**
4. $50x^2 - 15x - 11$
5. 8

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11 General Mathematics

General Mathematics
Representing Real-Life Situations Using Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics
Operations on Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics Alternative Delivery Mode (ADM) Module on Operations on Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Alternative Delivery Mode (ADM) Module on Operations on Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer What I Know before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

In this module, the different operations on functions were discussed. Examples were provided for you to be able to learn the five (5) operations: addition, subtraction, multiplication, division and composition of functions. Aside from algebraic solutions, these examples were illustrated, represented in tables and/or mapping diagram for better understanding of the concepts. Activities were provided to enhance your learning. Finally, your task is to answer a guided real-world example that involves operations on functions.

After going through this module, you are expected to:

1. define operations on functions
2. identify the different operations on functions.
3. perform addition, subtraction, multiplication, division, and composition of functions

What I Know

Direction. Write the letter of the correct answer on a separate sheet of paper.

1. The statement "$p(x) - q(x)$ is the same as $q(x) - p(x)$", $p(x) \neq q(x)$ is _____.
   a. always true b. never true c. sometimes true d. invalid

2. Given $h(x) = 2x^2 - 7x$ and $r(x) = x^2 + x - 1$, find $(h + r)(x)$.
   a. $2x^2 - 1$ b. $3x^2 + 6x - 1$ c. $3x^4 - 6x^2 - 1$ d. $3x^2 - 6x - 1$

3. Given: $f(a) = 2a + 1$ and $g(a) = 3a - 3$. Find $f(a) + g(a)$
   a. $5a - 2$ b. $-5a + 2$ c. $-2a + 1$ d. $-6a - 1$

4. $g(x) = 2x - 4$ and $h(x) = 2x - 7$ Find $(g + h)(3)$.
   a. -7 b. 1 c.-1 d. 8

5. $f(x) = 6x^2 + 7x + 2$ and $g(x) = 5x^2 - x - 1$, find $(f - g)(x)$.
   a. $x^2 + 8x + 3$ b. $5x^2 + 8x - 1$ c. $x^2 + 6x - 1$ d. $x^2 + 8x - 1$

6. $f(x) = x - 8$ and $g(x) = x + 3$, Find $f(x) \bullet g(x)$
   a. $x^2 + 24$ b. $x^2 - 5x + 24$ c. $x^2 - 5x - 24$ d. $x^2 + 5x + 24$

7. If $p(x) = x - 1$ and $q(x) = x - 1$, what is $p(x) \bullet q(x)$
   a. $x^2 + 1$ b. $x^2 + 2x - 1$ c. $x^2 - 2x + 1$ d. $x^2 - 1$

8. Given $h(x) = x - 6$ *and* $s(x) = x^2 - 13x + 42$. Find $\frac{h}{s}(x)$.
   a. $\frac{1}{x-7}$ b. $x - 7$ c. $\frac{x-6}{x-7}$ d. $x - 6$

9. $g(x) = 6x - 7$ and $h(x) = 5x - 1$, Find $g(h(x))$
   a. $-9x + 11$ b. $9x^2 + 4x$ c.$30x + 13$ d. $30x - 13$

10. If $j(x) = \sqrt{x + 6}$ and $k(x) = 9 - x$. Find $j(k(-1))$
    a. $9 - \sqrt{5}$ b. $\sqrt{14}$ c. 16 d. 4

For numbers 11-13, refer to figure below

11. Evaluate $p(5)$

a. 0 b. 3 c. 2 d. 7

12. Find $q(p(0))$

a. -3 b. 1 c. -3 d. -5

13. Find $(q \circ p)(3)$

a. 3 b. 5 c. 7 d. -1

For numbers 14-15, refer to the table of values below

$m(x) = 3x - 5$ $n(x) = x^2 - 2x + 1$

x	0	1	2	3	4	5	6	7	8
$m(x)$	-5	-2	1	4	7	10	13	16	19
$n(x)$	1	0	1	4	9	16	25	36	49

14. Find $\frac{m}{n}(7)$

a. $\frac{4}{9}$ b. $\frac{9}{4}$ c. 1 d. 0

15. Find $(n \circ m)(4)$

a. 9 b. 16 c. 19 d. 36

Lesson 1 Operation on Functions

Operations on functions are similar to operations on numbers. Adding, subtracting and multiplying two or more functions together will result in another function. Dividing two functions together will also result in another function if the denominator or divisor is not the zero function. Lastly, composing two or more functions will also produce another function.

The following are prerequisite skills before moving through this module:

- Rules for adding, subtracting, multiplying and dividing fractions and algebraic expressions, real numbers (especially fractions and integers).
- Evaluating a function.

A short activity was provided here for you to help in recalling these competencies. If you feel that you are able to perform those, you may skip the activity below. Enjoy!

What's In

SECRET MESSAGE

Direction. Answer each question by matching column A with column B. Write the letter of the correct answer at the blank before each number. Decode the secret message below using the letters of the answers.

Column A

_____1. Find the LCD of $\frac{1}{3}$ and $\frac{2}{7}$.

_____2. Find the LCD of $\frac{3}{x-2}$ and $\frac{1}{x+3}$

_____3. Find the sum of $\frac{1}{3}$ and $\frac{2}{7}$.

_____4. Find the sum of $\frac{2}{x}+\frac{5}{x}$

_____5. Find the product of $\frac{3}{8}$ and $\frac{12}{5}$.

_____6. Find the sum of $\frac{3}{x-2}$ and $\frac{1}{x+3}$

For numbers 7-14, find the factors.

_____7. x^2+x-12

_____8. x^2-5x-6

_____9. x^2+6x+5

Column B

A. $(x+4)(x-3)$

C. $\frac{4x+7}{x^2+x-6}$

D. $\frac{(x-3)(x+5)}{(x-6)(x+3)}$

E. $(x-2)(x+3)$ or x^2+x-6

G. $\frac{x+4}{x+2}$

H. $(x+1)(x-6)$

I. $\frac{13}{21}$

L. $(x-4(x-3)$

M. -5

_____10. $x^2 + 7x + 12$	N. 21
_____11. $x^2 - 7x + 12$	O. $(x-5)(x-3)$
_____12. $x^2 - 5x - 14$	R. $(x+4)(x+3)$
_____13. $x^2 - 8x + 15$	S. $(x-7)(x-5)$
_____14. $x^2 - 12x + 35$	T. $\frac{9}{10}$
_____15. Find the product of $\frac{x^2+x-12}{x^2-5x-6}$ and $\frac{x^2+6x+5}{x^2+7x+12}$.	U. $(x-7(x+2)$
_____16. Divide $\frac{x^2+x-12}{x^2-5x-14}$ by $\frac{x^2-8x+15}{x^2-12x+35}$	W. $\frac{7}{x}$
_____17. In the function $f(x) = 4 - x^2$, $find\ f(-3)$	Y. $(x+5)(x+1)$

Secret Message:

4	2	11	6	13	17	2	5	13	14	2	1	3	13	10			
8	3	16	8	14	6	8	13	13	11	3	17						
16	11	7	15	9	13	12	7	10	2	8	2	10	2				

What's New

SAVE FOR A CAUSE

Thru inspiration instilled by their parents and realization brought by Covid-19 pandemic experience, Neah and Neoh, both Senior High School students decided to save money for a charity cause. Neah has a piggy bank with ₱10.00 initial coins inside. She then decided to save ₱5.00 daily out of her allowance. Meanwhile, Neoh who also has a piggy bank with ₱5.00 initial coin inside decided to save ₱3.00 daily. Given the above situation, answer the following questions:

a. How much money will be saved by Neah and Neah after 30 days? after 365 days or 1 year? their combined savings for one year?
b. Is the combined savings enough for a charity donation? Why?
c. What values were manifested by the two senior high school students?
d. Will you do the same thing these students did? What are the other ways that you can help less fortunate people?
e. Do you agree with the statement of Pope John Paul II said that "Nobody is so poor he has nothing to give, and nobody is so rich he has nothing to receive"? Justify your answer.
f. What functions can represent the amount of their savings in terms of number of days?

What is It

In the previous modules, you learned to represent real life situations to functions and evaluate a function at a certain value. The scenario presented above is an example of real world problems involving functions. This involves two functions representing the savings of the two senior high school students.

Below is the representation of two functions represented by a piggy bank:

Neah	Neoh	Combined
$f(x) = 5x + 10$	$g(x) = 3x + 5$	$h(x) = 8x + 15$

Suppose that we combine the piggy banks of the two students, the resulting is another piggy bank. It's just like adding two functions will result to another function.

Definition. Let f and g be functions.

1. Their sum, denoted by $f + g$, is the function denoted by $(f + g)(x) = f(x) + g(x)$.
2. Their difference, denoted by $f - g$, is the function denoted by $(f - g)(x) = f(x) - g(x)$.
3. Their product, denoted by $f \bullet g$, is the function denoted by $(f \bullet g)(x) = f(x) \bullet g(x)$.
4. Their quotient, denoted by f/g, is the function denoted by $(f/g)(x) = f(x)/g(x)$, excluding the values of x where $g(x) = 0$.
5. The composite function denoted by $(f \circ g)(x) = f(g(x))$. The process of obtaining a composite function is called function composition.

Example 1. Given the functions:

$f(x) = x + 5$ $\quad$ $g(x) = 2x - 1$ $\quad$ $h(x) = 2x^2 + 9x - 5$

Determine the following functions:

a. $(f + g)(x)$
b. $(f - g)(x)$
c. $(f \bullet g)(x)$
d. $(\frac{h}{g})(x)$
e. $(f + g)(3)$
f. $(f - g)(3)$
g. $(f \bullet g)(3)$
h. $(\frac{h}{g})(3)$

Solution:

$a.\ (f+g)(x) = f(x) + g(x)$	definition of addition of functions
$= (x+5) + (2x-1)$	replace f(x) and g(x) by the given values
$= 3x + 4$	combine like terms

b. $(f-g)(x) = f(x) - g(x)$	definition of subtraction of functions
$= (x+5) - (2x-1)$	replace f(x) and g(x) by the given values
$= x + 5 - 2x + 1$	distribute the negative sign
$= -x + 6$	combine like terms

c. $(f \bullet g)(x) = f(x) \bullet g(x)$	definition of multiplication of functions
$= (x+5) \bullet (2x-1)$	replace f(x) and g(x) by the given values
$= 2x^2 + 9x - 5$	multiply the binomials

d. $\left(\frac{h}{g}\right)(x) = \frac{h(x)}{g(x)}$	definition of division of functions
$= \frac{2x^2+9x-5}{2x-1}$	replace h(x) and g(x) by the given values
$= \frac{(x+5)(2x-1)}{2x-1}$	factor the numerator
$= \frac{(x+5)\cancel{(2x-1)}}{\cancel{2x-1}}$	cancel out common factors
$= x + 5$	

e. $(f+g)(3) = f(3) + g(3)$

Solve for $f(3)$ and $g(3)$ separately:

$f(x) = x + 5$	$g(x) = 2x - 1$
$f(3) = 3 + 5$	$g(3) = 2(3) - 1$
$= 8$	$= 5$

$$\therefore f(3) + g(3) = 8 + 5 = 13$$

Alternative solution:

We know that $(f+g)(3)$ means evaluating the function $(f+g)$ at 3.

$(f+g)(x) = 3x + 4$	resulted function from item a
$(f+g)(3) = 3(3) + 4$	replace x by 3
$= 9 + 4$	multiply
$= 13$	add

For item f *to* h we will use the values of $f(3) = 8$ *and* $g(3) = 5$

f. $(f-g)(3) = f(3) - g(3)$	definition of subtraction of functions
$= 8 - 5$	replace f(3) and g(3) by the given values
$= 3$	subtract

Alternative solution:

$(f-g)(x) = -x+6$	resulted function from item b
$(f-g)(3) = -3+6$	replace x by 3
$= 3$	simplify

g. $(f \bullet g)(3) = f(3) \bullet g(3)$ — definition of multiplication of functions

$= 8 \bullet 5$ — replace f(3) and g(3) by the given values

$= 40$ — multiply

Alternative solution:

$(f \bullet g)(x) = 2x^2 + 9x - 5$	resulted function from item c
$(f \bullet g)(3) = 2(3)^2 + 9(3) - 5$	replace x by 3
$= 2(9) + 27 - 5$	square and multiply
$= 18 + 27 - 5$	multiply
$= 40$	simplify

h. $\left(\frac{h}{g}\right)(3) = \frac{h(3)}{g(3)}$

Solve for $h(3)$ and $g(3)$ separately:

$h(x) = 2x^2 + 9x - 5$

$h(3) = 2(3)^2 + 9(3) - 5$

$= 18 + 27 - 5$

$= 40$

$g(x) = 2x - 1$

$g(3) = 2(3) - 1$

$= 5$

$$\therefore \left(\frac{h}{g}\right)(3) = \frac{h(3)}{g(3)} = \frac{40}{5} = 8$$

Alternative solution:

$\left(\frac{h}{g}\right)(x) = x + 5$	resulted function from item d
$\left(\frac{h}{g}\right)(x) = 3 + 5$	replace x by 3
$= 8$	simplify

Can you follow with what has been discussed from the above examples? Notice that addition, subtraction, multiplication, and division can be both performs on real numbers and functions.

The illustrations below might help you to better understand the concepts on function operations.

In the illustrations, the numbers above are the inputs which are all 3 while below the function machine are the outputs. The first two functions are the functions to be added, subtracted, multiplied and divided while the rightmost function is the resulting function.

Addition

Subtraction

Multiplication

Division

Notes to the Teacher

Give emphasis to the students that performing operations on two or more functions results to a new function. The function $(f+g)(x)$ is a new function resulted from adding $f(x)$ and $g(x)$. The new function can now be used to evaluate $(f+g)(3)$ and it will be the same as adding $f(3)$ and $g(3)$.

Composition of functions:

In composition of functions, we will have a lot of substitutions. You learned in previous lesson that to evaluate a function, you will just substitute a certain number in all of the variables in the given function. Similarly, if a function is substituted to all variables in another function, you are performing a composition of functions to create another function. Some authors call this operation as "function of functions".

Example 2. Given $f(x) = x^2 + 5x + 6$, and $h(x) = x + 2$

Find the following:

a. $(f \circ h)(x)$
b. $(f \circ h)(4)$
c. $(h \circ f)(x)$

Solution.

a. $(f \circ h)(x) = f(h(x))$	definition of function composition
$= f(x+2)$	replace h(x) by x+2
Since $f(x) = x^2 + 5x + 6$	given
$f(x+2) = (x+2)^2 + 5(x+2) + 6$	replace x by x+2
$= x^2 + 4x + 4 + 5x + 10 + 6$	perform the operations
$= x^2 + 9x + 20$	combine similar terms

Composition of function is putting a function inside another function. See below figure for illustration.

b. $(f \circ h)(4) = f(h(4))$

Step 1. Evaluate $h(4)$

$$h(x) = x + 2$$
$$h(4) = 4 + 2$$
$$= 6$$

Step 2. Evaluate $f(6)$

$$f(x) = x^2 + 5x + 6$$
$$f(6) = 6^2 + 5(6) + 6$$
$$= 36 + 30 + 6$$
$$= 72$$

$$(f \circ h)(4) = f(h((4))$$
$$= f(6)$$
$$\therefore \quad = 72$$

To evaluate composition of function, always start with the inside function (from right to left). In this case, we first evaluated $h(4)$ and then substituted the resulted value to $f(x)$.

Alternative solution:

$(f \circ h)(x)) = f(h(x))$	definition of function composition
$f(h(x)) = x^2 + 9x + 20,$	from item a
$(f \circ h)(4)) = 4^2 + 9(4) + 20$	replace all x's by 4
$= 16 + 36 + 20$	perform the indicated operations
$= 72$	simplify

A mapping diagram can also help you to visualize the concept of evaluating a function composition.

From the definition of function composition, $(f \circ h)(4) = f(h((4))$. Looking at the mapping diagram for values and working from right to left, $h(4) = 6$. Substituting 6 to $h(4)$ we have $f(6)$. From the diagram, $f(6)$ is equal to 72. Therefore, $(f \circ h)(4) = f(h((4)) = 72$. In the diagram, the first function $h(x)$ served as the inside function while the second function $f(x)$ is the outside function.

A table of values is another way to represent a function. The mapping diagram above has a corresponding table of values below:

$h(x) = x + 2$ $\qquad$ $f(x) = x^2 + 5x + 6$

x	1	3	4	6
$h(x)$	3	5	6	8
$f(x)$	12	30	42	72

$(f \circ h)(4) = f(h((4))$	definition of composition of functions
$= f(6)$	substitute h(4) by 6
$= 72$	from the table
c. $(h \circ f)(x) = h(f(x))$	definition of composition of functions
$= h(x^2 + 5x + 6)$,	substitute f(x) by $x^2 + 5x + 6$, given
Since $h(x) = x + 2$	given
$h(x^2 + 5x + 6) = x^2 + 5x + 6 + 2$	substitute x by $x^2 + 5x + 6$
$= x^2 + 5x + 8$	combine similar terms

Notes to the Teacher

The functions $(f \circ h)(x)$ and $(h \circ f)(x)$ are generally not the same as we see in the previous examples. It only means that order of functions counts in composition of function operation. There are special cases where they will be the same; this is when the two functions are inverses. Graphing and finding the domain and range of algebraic operations is not covered by this module but this is an interesting activity that can be used as enrichment once this module was mastered.

What's More

Activity 1:

MATCHING FUNCTIONS

Direction: Match column A with column B by writing the letter of the correct answer on the blank before each number

Given:

$a(x) = x + 2$
$b(x) = 5x - 3$
$c(x) = \dfrac{x+5}{x-7}$
$d(x) = \sqrt{x+5}$
$e(x) = \dfrac{3}{x-7}$

Column A	Column B
_____1. $(a+b)(x)$	a. $\frac{3}{x+5}$
_____2. $(a \bullet b)(x)$	b. ± 3
_____3. $(d \circ a)(x)$	c. -7
_____4. $\left(\frac{e}{c}\right)(x)$	d. $-\frac{4}{5}$
_____5. $(c-e)(x)$	e. $\sqrt{x+7}$
_____6. $(a+b)(-1)$	f. $\frac{x+2}{x-7}$
_____7. $(a \bullet b)(0)$	g. $6x-1$
_____8. $(d \circ a)(2)$	h. 1
_____9. $\left(\frac{e}{c}\right)(-2)$	i. -6
_____10. $(c-e)(2)$	j. $5x^2+7x-6$

Activity 2:

LET'S SIMPLIFY

A. Let $p(x) = 2x^2 + 5x - 3, m(x) = 2x - 1, and\ h(x) = \frac{x+1}{x-2}$ Find:
   1. $(m-p)(x)$
   2. $p(5) + m(3) - h(1)$
   3. $\frac{m(x)}{p(x)}$
   4. $p(x+1)$
   5. $p(3) - 3(m(2)$

B. Given the following:

- $m(x) = 5x - 3$
- $n(x) = x + 4$
- $c(x) = 5x^2 + 17x - 12$
- $t(x) = \frac{x-5}{x+2}$

Determine the following functions.

1. $(m + n)(x)$
2. $(m \cdot n)(x)$
3. $(n - c)(x)$
4. $(c/m)(x)$
5. $(m \circ n)(x)$
6. $(n \circ c)(-3)$
7. $n(m(m(2)))$

C. Given the functions $g(x) = x^2 - 4$ and $h(x) = x + 2$, Express the following as the sum, difference, product, or quotient of the functions above.

1. $p(x) = x - 2$
2. $r(x) = x^2 + x - 2$
3. $s(x) = x^3 + 2x^2 - 4x - 8$
4. $t(x) = -x^2 + x + 6$

D. Answer the following:

1. Given $h(x) = 3x^2 + 2x - 4, what\ is\ h(x - 3)$?

2. Given $n(x) = x + 5\ and\ p(x) = x^2 + 3x - 10, find$:
   a. $(n - p)(x) + 3p(x)$
   b. $\frac{n(x)}{p(x)}$
   c. $(p \circ n)(x)$
3. Let $m(x) = \sqrt{x + 3}, n(x) = x^3 - 4, and\ p(x) = 9x - 5, find\ \big(m \circ (n - p)\big)(3)$.
4. Given $w(x) = 3x - 2$, $v(x) = 2x + 7$ and $k(x) = -6x - 7$, find $(w - v - k)(2)$
5. If $s(x) = 3x - 2$ and $r(x) = \frac{2}{x+5}$, find $2(s + r)(x)$
6. Given $a(x) = 4x + 2, b(x) = \frac{3}{2}x, and\ c(x) = x - 5, find\ (a \bullet b \bullet c)(x)$

What I Have Learned

Complete the worksheet below with what have you learned regarding operations on functions. Write your own definition and steps on performing each functions operation. You may give your own example to better illustrate your point.

Addition	Subtraction	Multiplication	Division	Composition

What I Can Do

Direction: Read and understand the situation below, then answer the questions that follow.

The outbreak of coronavirus disease 2019 (COVID-19) has created a global health crisis that has had a deep impact on the way we perceive our world and our everyday lives, (https://www.frontiersin.org). Philippines, one of the high-risk countries of this pandemic has recorded high cases of the disease. As a student, you realize that Mathematics can be a tool to better assess the situation and formulate strategic plan to control the disease.

Suppose that in a certain part of the country, the following data have been recorded.

d	0	1	2	3	4	5	6	7	8
$I(d)$	3	5	9	12	18	25	35	47	82

Where $I(d)$ is the function of the number of people who got infected in d days

The number of recoveries was also recorded in the following table as the function $R(i)$ where R as the number of recoveries is dependent to number of infected (I).

I	3	5	9	12	18	25	35	47	82
$R(I)$	0	1	2	5	7	9	12	18	25

a. Evaluate the following and then interpret your answer.
   1. $R(I(3))$
   2. $R(I(8))$
   3. $I(R(18))$

b. The number of deaths (M) was also dependent on the number of infected (I). Complete the table with your own number of deaths values for the given number of infected.

I	3	5	9	12	18	25	35	47	82
$M(I)$	0	0	1	1	1	2	3	4	6

   Evaluate the following and then interpret your answer.
   1. $M(I(1))$
   2. $M(I(4))$
   3. $I(M(12))$

c. What can you conclude about the data presented?
d. What can you suggest to the government to solve the problem?

Assessment

Direction. Write the letter of the correct answer on a separate answer sheet.

1. The following are notations for composite functions EXCEPT,
   a. $h(p(x))$ b. $f(x)g(x)$ c. $(s \circ t)(x)$ d. $f(g(x))$

2. Find $h(3) + d(2)$ $if\ h(x) = x - 1\ and\ d(x) = 7x + 3$
   b. 2 b. 5 c. 14 d. 19

3. $t(x) = -x^2 + 7x + 1\ and\ r(x) = 5x^2 - 2x + 8, find\ (t - r)(2)$.
   a. 18 b. -18 c. -13 d. 13

4. $f(x) = 4x + 2\ and\ g(x) = 3x - 1, find\ (f - g)(4)$.
   a. 0 b. -9 c. 7 d. -8

5. $If\ g(x) = x - 4\ and\ f(x) = x + 5\ Find\ f(x) \bullet g(x)$
   a. $x^2 + x + 20$ c. $x^2 - x - 20$
   b. $x^2 - x + 20$ d. $x^2 + x - 20$

6. Given $h(n) = \frac{n+6}{n-4}$ and $p(k) = \frac{n+6}{n^2+4n-32}$. Find $\frac{h}{p}(k)$.
   a. $\frac{1}{n+8}$ b. $n - 8$ c. $\frac{1}{n-8}$ d. $n + 8$

7. If $f(x) = 18x^2$ and $t(x) = 8x$, find $\frac{f}{t}(x)$.
   a. $\frac{9x}{4}$ b. $\frac{4x}{9}$ c. $\frac{4}{9x}$ d. $\frac{9}{4x}$

8. When $f(x) = 3x - 5$ and $g(x) = 2x^2 - 5$, find $f(g(x))$.
   a. $x^2 + 2x + 3$ b. $6x^2 - 20$ c. $6x^2 + 20$ d. $2x^2 + 6$

9. $r(x) = x + 5$ and $q(x) = 2x^2 - 5$, Find $q(r(-2))$
   a. 8 b. -8 c. 13 d. -13

10. Let $f(x) = 3x + 8$ and $g(x) = x - 2$. Find $f(g(x))$.
    a. $2x + 3$ b. $2x - 3$ c. $4x + 1$ d. $3x + 2$

For numbers 11-13, refer to the figure below:

11. Evaluate $r(2)$

a. -11 b. -3 c. 5 d. 11

12. Find $s(r(7))$

a. 7 b. 1 c. -1 d. -7

13. Find $(s \circ r)(1)$

a. -3 b. 3 c. 5 d. -5

For numbers 14-15, refer to the table of values below

$t(x) = 2x + 1$ $k(x) = 2x^2 - 7x - 5$

x	0	1	2	3	4	5	6	7	8
$t(x)$	1	3	5	7	9	11	13	15	17
$k(x)$	-5	-10	-11	-8	-1	10	25	44	67

14. Find $(k - t)(4)$

a. 8 b. -8 c. 10 d. -10

15. Find $(k \circ t)(2)$

a. 10 b. -10 c. -5 d. -1

Additional Activities

PUNCH D LINE

Direction: Find out some of favorite punch lines by answering operations on functions problems below. Phrases of punch lines were coded by the letters of the correct answers. Write the punch lines on the lines provided.

Given:

$f(x) = 2x - 1$ $\quad$ $g(x) = |3x - 4|$ $\quad$ $h(x) = \frac{x}{2}$

$r(x) = x + 3$ $\quad$ $s(x) = x^2 - 4x - 21$

Column A	Column B
_____1. $f(0) =$	A. -11
_____2. $g(3) =$	B. 2
_____3. $s(-1) =$	C. $3x + 2$
_____4. $h(0) =$	D. $x - 7$
_____5. $(f + r)(x) =$	E. $-x + 4$
_____6. $(f + r)(3) =$	F. 0
_____7. $(r - f)(x) =$	G. $2x^2 + 5x - 3$
_____8. $(r - f)(2) =$	H. 6
_____9. $(f \bullet r)(x) =$	I. -16
_____10. $(f \bullet r)(1) =$	J. $2x + 2$
_____11. $\frac{s}{r}(x) =$	K. 5
_____12. $\frac{s}{r}(-4) =$	L. 1
_____13. $(r \circ f)(x) =$	M. 11
_____14. $(r \circ f)(2) =$	N. -1
_____15. $(g \circ f)(1) =$	O. 4

Code:

tingnan mo ako	K	ang laman ng utak ko?	J
para may attachment lagi tayo	L	buhay nga pero patay	I
ang parents ko	E	Hindi lahat ng buhay ay buhay	N
na ako sa'yo	O	Di mo pa nga ako binabato	B
Masasabi mo bang bobo ako?	D	na patay naman sa'yo	F
Kasi, botong-boto sayo	M	Tatakbo ka ba sa eleksyon?	C
Kung ikaw lamang	A	pero tinamaan	G
Sana naging email na lang ako	H		

Punch lines:

(1-4) _____

(5-7) _____

(8-10) _____

(11-13) _____

(14-15) _____

Answer Key

What I know

1. b	6. c	11. c
2. d	7. c	12. d
3. a	8. a	13. c
4. b	9. d	14. a
5. a	10. d	15. D

What's In

1. N	6. C	11. L	16. G
2. E	7. A	12. U	17. M
3. I	8. H	13. O	
4. W	9. Y	14. S	
5. T	10. R	15. D	

Secret Message:

WELCOME TO SENIOR
HIGH SCHOOL IM GLAD
YOU ARE HERE

What's New

a. After 30 days:
Neah has ₱160 and Neoh has ₱95

After 365 days or 1 year:
Neah has ₱1835 and Neoh has ₱1100

Their combined savings for 1 year is ₱2935

b. Answers may vary
c. Answers may vary
d. Answers may vary
e. Answers may vary
f. Let x = number of days
$f(x)$ = amount of savings of Neah
$g(x)$ = amount of savings of Neoh

$f(x) = 5x + 10$
$g(x) = 3x + 5$

What's More

Activity 1: Matching Functions

1. g	6. c
2. j	7. i
3. e	8. b
4. a	9. h
5. f	10. d

Activity 2: Let's Simplify

A.

1. $(m - p)(x) = -2x^2 - 3x + 2$
2. $p(5) + m(3) - h(1) = 79$
3. $\frac{m(x)}{p(x)} = \frac{1}{x+3}$
4. $p(x + 1) = 2x^2 + 9x + 4$
5. $p(3) - 3(m(2)) = 21$

B.

1. $(m + n)(x) = 6x + 1$
2. $(m \cdot n)(x) = 5x^2 + 17x - 12$
3. $(n - c)(x) = -5x^2 - 16x + 16$
4. $(c/m)(x) = x + 4$
5. $(m \circ n)(x) = 5x + 17$
6. $(n \circ c)(-3) = -14$
7. $n\big(m(m(2))\big) = 38$

C.

1. $p(x) = \frac{g(x)}{h(x)}$
2. $r(x) = g(x) + h(x)$
3. $s(x) = g(x) \bullet h(x)$
4. $t(x) = h(x) - g(x)$

D.

1. $3x^2 - 16x + 17$
2. a. $2x^2 + 7x - 15$
   b. $\frac{1}{x-2}$
   c. $x^2 + 13x + 30$
3. 2
4. 12
5. $\frac{6x^2+26x-16}{x+5}$
6. $6x^3 - 27x^2 - 15x$

What I can Do

a. 1. $R(I(3)) = R(12) = 5$
On the third day, there were 12 infected and 5 recovered people
2. $R(I(8)) = R(82) = 25$
On 8th day, there were 82 people infected and 25 recovered people.
3. $I(R(18)) = I(7) = 47$
Although we can evaluate the composition of function here, this value does not make sense. I(d) is the function of days, but 7 in I(7) means number of recovered people.

b. Answers may vary
1. M(I(1))=M(5)=0
On the first day, there were 5 infected and no death.
2. M(I(4))=M(18)=1
On the fourth day, there were 18 infected and 1 death
3. I(M(12))=I(1)=5
Although we can evaluate the composition of function here, this value does not make sense. I(d) is the function of days, but 1 in I(1) means number of deaths.

c. Answers may vary

d. Answers may vary

Assessment

1. b	6. d	11. b
2. d	7. a	12. b
3. c	8. b	13. c
4. c	9. c	14. d
5. d	10. d	15. A

Additional Activities
Punch d line

1. N	6. M	11. D
2. K	7. E	12. A
3. I	8. B	13. J
4. F	9. G	14. H
5. C	10. O	15. L

Punch lines:

1. Hindi lahat ng buhay ay buhay, tingnan mo ako, buhay nga pero patay na patay naman sa'yo.
2. Tatakbo ka ba sa eleksyon? Kasi, botong-boto sayo ang parents ko.
3. Di mo pa nga ako binabato pero tinamaan na ako sa'yo.
4. Masasabi mo bang bobo ako? Kung ikaw lamang ang laman ng utak ko?
5. Sana naging email na lang ako para may attachment lagi tayo.

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11 General Mathematics

General Mathematics
Solving Real- Life Problems Involving Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Solving Real-Life Problems Involving Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics Alternative Delivery Mode (ADM) Module on Solving Real-Life Problems Involving Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Alternative Delivery Mode (ADM) Module on Solving Real-Life Problems Involving Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written to help you solve problems involving functions bearing in mind that you already know how to represent real – life situation using functions including piece-wise functions, evaluate functions and perform operations on functions. These skills will aid you in attaining success on this module.

Solving problems involving functions is essential in predicting values that will help in decision making process. This module covers varied situations that can be seen in real life such as travel fares, monthly bills sales and the like. It is hoped that upon exploring this learning kit you will find the eager and enthusiasm in completing the task required. Best of luck!

After going through this module, you are expected to:

1. represent situations as functions and evaluate functions to determine the required quantity
2. apply concepts learned in solving real-life problems involving functions
3. solve problems involving functions

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Given $h(x) = \frac{5x-6}{3}$, determine $h(3)$
   a. -3
   b. 3
   c. 1
   d. -1

2. Let $f(x) = 3x^2 - x + 5$, find $f(x+1)$
   a. $3x^2 + 5x + 5$
   b. $3x^2 + 5x + 7$
   c. $3x^2 - x + 9$
   d. $3x^2 + 5x + 9$

3. Which of the following expresses the total earnings *(E)* as a function of the number (*n*) of days if a laborer works and earning ₱400.00 per day?
   a. E*(n)* = 400 + n
   b. E*(n)* = 400 ÷ n
   c. E*(n)* = 400(n)
   d. E*(n)* = 400 – n

4. If the temperature in degrees Celsius inside the Earth is represented by T*(d)* = 10*d* + 20 where *(d)* is the depth in kilometers, what is the temperature inside the Earth in 10 kilometers?
   a. 40℃
   b. 50℃
   c. 120℃
   d. 180℃

5. Express the perimeter *P* of a square with side x as a function of its area
   a. $A = \frac{P^2}{16}$
   b. $A = 16P^2$
   c. $A = \frac{P^2}{4}$
   d. $A = \frac{16}{P^2}$

For numbers 6 – 7 use the problem below:

Cotta National High School has 1,200 students enrolled in 2016 and 1,500 students in 2019. The student population P grows as a linear function of time *(t)*, where *t* is the number of years after 2016.

6. Which of the following functions represents the student population that relates to time *t*?
   a. P*(t)*= 100t + 1,200
   b. P*(t)* = 1,200t -100
   c. P(*t)* = 1,200t + 100
   d. P*(t)* = 100t – 1,200

7. How many students will be enrolled in Cotta National High School by 2020?
   a. 1,800
   b. 1,700
   c. 1,600
   d. 1,650

For numbers 8 – 10 use the problem below:

A proposed tricycle fare would charge ₱20.00 for the first 5 km of travel and ₱0.75 for each additional kilometer over the proposed fare.

8. Find the fare function *f(x)* where *x* represents the number of kilometers travelled.
   a. *f(x)* = 20 + 0.75x
   b. *f(x)* = 20 - 0.75x
   c. *f(x)* =16.25 + 0.75x
   d. *f(x)* = 6.25 - 0.75x

9. How much is the proposed fare for distance of 3 km?
   a. ₱4.00
   b. ₱8.00
   c. ₱12.00
   d. ₱20.00

10. Find the proposed fare for distance of 55 km.
    a. ₱57.50
    b. ₱91.25
    c. ₱60.50
    d. ₱105.75

11. The cost of producing *x* tools by a B&G Corp. is given by *C(x)*=₱1,200.00*(x)* + ₱5,500.00. How much is 100 tools?
    a. ₱6,700.00
    b. ₱12,550.00
    c. ₱125,500.00
    d. ₱551,200.00

For numbers 12 - 15 use the problem below:

Mark charges ₱100.00 for an encoding work. In addition, he charges ₱5.00 per page of printed output.

12. Find a function f*(x)* where x represents the number of pages of printed out.
    a. $f(x) = 100 + 5x$
    b. $f(x) = 100 - 5x$
    c. $f(x) = 100x + 5$
    d. $f(x) = 100x - 5$

13. How much will Mark charge for 55-page encoding and printing work?
    a. ₱275.00
    b. ₱175.00
    c. ₱375.00
    d. ₱475.00

14. How many pages were printed if Mark received a payment of ₱600.00?
    a. 100 pages
    b. 80 pages
    c. 60 pages
    d. 50 pages

15. If Mark offers a promo to loyal costumer that the first 20 pages of the printed output will be free of charge, how much will he charge to a loyal customer who printed 70 pages of output?
    a. ₱250.00
    b. ₱50.00
    c. ₱350.00
    d. ₱450.00

Lesson 1 Solving Real-Life Problems Involving Functions

Majority of the problems we encounter in real life situation involve relationship between two quantities where one quantity depends on another. For example, personnel in Department of Health observes the number of persons infected by a particular virus in a certain community increases with time. In finding out the exact function relating to the number of persons infected to time, modelling can be used. Once the model is determined solving and predicting the properties of the subject being studied can be done. At this point we will focus on solving in order for us to predict answer to existing problems.

What's In

YES I CAN!

Listed below are the skills and competencies you should possess before proceeding to this lesson. Read the statements and assess yourself whether you agree or disagree with the statements.

Statement	Agree	Disagree
1. I can carefully read and analyze a given problem		
2. I can determine the given and the facts required in a problem		
3. I can represent real – life situation using function, including piece –wise function		
4. I can perform operations on functions		
5. I can evaluate functions		

- If you agree with all the statements that means you are very much ready with this module, however if there are some statements where you disagree that means you need to have a quick review of the previous lesson that will aid you in gaining success in this lesson.
  Let us take a quick tour to what you learn in the previous modules

1. Being able to understand a problem presented is the first thing that we learn on how to solve problems in Mathematics. Careful reading leads you to thorough analysis in the identification of given facts and in determining the required or the unknown quantity. Precise label of the known and unknown quantities will help you set up a direction towards the solution.

2. A function is a rule of correspondence between two non-empty sets, such that to each element of the first set called domain, there corresponds one and only one element of the second set called range.

3. Functions are used to model real life situations and in representing real – life situations the quantity of one variable depends or corresponds to or mapped onto another quantity.

4. Piece-wise function are functions that may be represented by a combination or of equations.

5. If a function f is defined by $y = f(x)$ and an independent variable x is found by substituting x into the function rule then it undergoes a process of evaluating function. Moreover, you also studied fundamental operation can be applied to two or more functions to form a new function. Such operations are addition, subtraction, multiplication and division.

Consider the examples below and reflect if you are confident enough to proceed

1. Write a function C that represent the cost of buying x facemask, if a facemask cost ₱65.00

$C(x) = 65x$

2. A commuter pays ₱ 9.00 for a jeepney fare for the first 5 km and an additional ₱ 0.75 for every succeeding distance d in kilometer. Represent the situation as function

$F(d) = 9$, if $0<d\leq 5$

$F(d) = 9 + 0.75(d)$, if $d>5$

3. If $f(x) = x + 6$, evaluate: a. $f(4)$ b. $f(-2)$ c. $f(-x)$

Solution**:** a. $f(x) = x + 6$ b. $f(x) = x + 6$ c. $f(x) = x + 6$

$f(4) = 4 + 6$ $f(-2) = (-2) + 6$ $f(-x) = -x + 6$

$f(4) = 10$ $f(-2) = 4$

4.Le $f(x) = x + 3$t and $g(x) = x - 2$. Find a. $f(3) + g(-2)$ b. $f(4) - g(0)$

c. $f(x) \cdot g(x)$ d. $\frac{f(9)}{g(8)}$

Solution:

a. $f(x) = x + 3$ $\quad$ $g(x) = x - 2$

$f(3) = 3 + 3$ $\quad$ $g(-2) = (-2) - 2$

$f(3) = 6$ $\quad$ $g(-2) = -4$

$$f(3) + g(-2) = 6 + (-4)$$

$$f(3) + g(-2) = 2$$

b. $f(x) = x + 3$ $\quad$ $g(x) = x - 2$

$f(4) = 4 + 3$ $\quad$ $g(0) = 0 - 2$

$f(4) = 7$ $\quad$ $g(0) = -2$

$$f(4) - g(0) = 7 - (-2)$$

$$f(4) - g(0) = 9$$

c. $f(x) = x + 3$ $\quad$ $g(x) = x - 2$

$$f(x) \cdot g(x) = (x + 3)(x - 2)$$

$$f(x) \cdot g(x) = x^2 + x - 6$$

d. $f(x) = x + 3$ $\quad$ $g(x) = x - 2$

$f(9) = 9 + 3$ $\quad$ $g(8) = 8 - 2$

$f(9) = 12$ $\quad$ $g(8) = 6$

$$\frac{f(9)}{g(8)} = \frac{12}{6} = 2$$

At this point you may now proceed to the next section of this module!

Notes to the Teacher

The teacher may also point out the importance of the concept of zero of linear function in solving problems involving functions. The zero of a linear function *f(x) is the real number a such that f(a)=0.* This suggest that the zero of linear function is found by equating it to zero and then solving the resulting equation for x. This will be used in the latter example in the module.

What's New

JEEPNEY OR TRICYCLE?

Read and analyze the problem below.

Miguel is a senior high school student who commutes from home to school which is 15 km apart. There are two modes of transportation the first one is through jeep and the other one is through tricycle. In riding a jeepney the fare charge ₱9.00 for the first 4 km travel and ₱0.75 for each additional kilometer. Meanwhile in riding a tricycle the fare would be ₱10.00 for the first km travel and ₱1 for each additional kilometer.

Will you help Miguel analyze his situation?

Questions

1. If you are Miguel and decided to ride in a jeepney, how much will be your fare? _____
   Hint: You may use the table below to compute the fare

No. of km	0-4	5	6	7	8	9	10	15
Amount charge	9	9+ 0.75	9.75+ 0.75					

2. If you decided to ride in a tricycle how much will be your fare? _____
   Hint: You may use the strategy in no. 2

3. What characteristics does Miguel possess if he chose to ride a jeepney?
   _____

4. Is there any advantage in riding a jeepney instead of tricycle? Or riding a tricycle instead of jeepney? What would it be?
   _____

5. If you are Miguel which between the two modes of transportation will you choose? Why? _____

What is It

Decision making is always part of our lives, from the moment we wake- up we start to decide the proper action to undertake be it minor or major decisions. In the problem presented one of the factors that Miguel can use in making decision about the dilemma he is facing is the cost of the fare in jeepney and in tricycle. If he will ride a jeepney he will only pay ₱17.25 however if he will ride a tricycle, he needs to pay ₱24.00 So it will be more economical if he chose to ride a jeepney. However, the cost of the fare is just one of the factors. There are times that convenience is also considered in choosing the mode of transportation since it is not crowded and you can reach your destination faster. Therefore, in deciding the mode of transportation the priority of the commuters whether to be more economical or to meet convenience is considered.

In the previous problem we determine the cost of the fare by using a table wherein we repeatedly add the fare charge per kilometer. Thus, this type of problem can be solved using functions, and at this point let us determine how we are going to do that.

Example no. 1

LET'S TRAVEL

A proposed Light Rail Transit System Line 1 (LRT-1) fare would charge ₱18.00 for the first four stations and ₱5.00 for each additional station over the proposed fare.

a. Find the fare function $f(x)$ where x represents the number of stations traveled
b. Find the proposed fare for 15 stations
c. Find the proposed fare for 20 stations

To solve problems that involve functions you can employ George Polya's 4-step rule.

George Polya's 4 – Step Rule

1. **Explore.** This step involves careful reading, analyzing, identifying the given and unknown facts in the problem and expressing the unknown in terms of variables.
2. **Plan**. In this step writing an equation that describes the relationships between or among the variables is involve.
3. Solve. This step requires working out with the written equation and other number relations to determine the required quantities that answer the question in the problem.
4. Check. The final step that employ the use of other approaches to examine the appropriateness of the answer.

Solution

a. **Explore.** Since the first step involves analysis and proper labeling of the known and unknown facts we will let x = number of stations traveled. There are also some conditions that was set in the problem such as the cost of fare which is set up to 4 stations only thus we can represent $x - 4$ = number of stations traveled over and above 4 stations

Plan. In writing an equation that will represent the relationship between the known and unknown quantities, since we know that if we travelled up to 4 stations we must pay P18, we can represent it as
$f(x) = 18$ for $0 < x \leq 4$

However, if we travelled more than 4 stations the cost of the fare have different method of computation so we need to consider it. Since the cost of every station after the 4th station is ₱5.00 we will now obtain
$f(x) = 18 + 5(x - 4)$

Now simplifying the equation will lead us to:
$f(x) = 18 + 5x - 20$
$f(x) = 5x - 2$

At this point we can say that the fare function is $f(x) = 5x - 2$

b. **Solve.** To find the fare charge for 15 stations the fare function $f(x) = 5x - 2$ will be used and 15 will be substituted to the function
$f(15) = 5(15) - 2$
$= 73$
By evaluating the function we obtained $f(15) = 73$

Check. To check whether we arrived at the correct solution you can use table or graph.
Thus. the proposed fare for 15-station travel is *₱73.00*

c. $f(20) = 5(20) - 2$
$= 98$

The proposed fare for 20 – station travel is *₱98*

Example no. 2

BINGE WATCH

Lucena Network charges ₱450.00 monthly cable connection fee plus ₱130.00 for each hour of pay-per-view (PPV) event regardless of a full hour or a fraction of an hour.

a. Find payment function *f(x)* where *x* represents the number of PPV hours.
b. What is the monthly bill of a customer who watched 25 hours of PPV events?
c. What is the monthly bill of a customer who watched 0.5 hour of PPV events?

Solution:

a. ₱450.00 = fixed monthly cable connection fee
   Let *x* = number of PPV hours in a month
   ₱130.00(x) = amount of PPV payment in a specific hour

   The payment function is f(x) = 450 + 130(x).

b. The monthly bill of a customer who watched 25 hours PPV events can be represented by $24 < x \leq 25$.

$$f(x) = 450 + 130(x).$$

$$f(25) = 450 + 130(25)$$

$$= 450 + 3{,}250$$

$$= 3{,}750$$

The monthly bill of a customer who watched 25 hours of PPV event is ₱3,750.00

c. The monthly bill of a customer who watched 0.5 hour PPV events can be represented by $0 < x \leq 1$ and since the problem states that regardless of a full hour or a fraction of an hour the additional charge will be made on hourly basis only, thus the value of *x* will be 1

$$f(x) = \text{Php } 450.00 + \text{Php } 130.00(x).$$

$$f(1) = \text{Php } 450.00 + \text{Php } 130.00(1)$$

$$= \text{Php } 450.00 + \text{Php } 130.00$$

$$= \text{Php } 580.00$$

What's More

Read each situation carefully to solve each problem. Write your answer on a separate sheet of your paper.

Independent Practice 1

Business As Usual

Bakers' Club is trying to raise funds by selling premium chocolate chip cookies in a school fair. The variable cost to make each cookie is ₱15.00 and it is being sold for ₱25.00 So far the organization has already shelled out ₱790.00 for the cookie sale

a. Find the profit function *P(x)* where *x* represents the number of cookies sold
Hint: Profit = Total Revenue – Total Cost
Total Revenue = Price per unit x quantity sold
Total Cost = Total variable cost + fixed cost

_____

b. If 146 cookies were sold, how much is the total profit?

_____

c. How many cookies must be made and sold to break even?
Hint: Break even point is the zero of P(x)

_____

d. How many cookie should be sold to gain a profit of ₱250.00?

_____

Independent Assessment 1

Baker's Nook

Elisha just opened a bakery along Macalintal Avenue which sells fresh doughnuts. The selling price is ₱20.00 per doughnut and the cost of making it is ₱8.00 The daily operating expense is ₱600.00.

a. Find the profit function P*(x)* where *x* represent the number of doughnuts sold.
b. If 100 doughnuts were sold, what is the total profit?
c. How many doughnuts must be made and sold to break even?
d. How many doughnuts should be sold to gain a profit of ₱600.00?

Independent Practice 2

Hello!

A certain cellphone company offers a plan that costs ₱1,200.00 per month. The plan includes 180 free minutes of call and charges ₱7.00 for each additional minute of usage.

a. Find the monthly cost function *C(x)* where *x* represents the number of minutes used.
Hint: Monthly Cost = Plan Cost + Additional Charge per Minute
_____

b. How much is the monthly cost incurred if the owner used 180 minutes of call?_____

c. How much is the monthly cost of the plan if the owner used 300 minutes of call?_____

Independent Assessment 2

Connected!

CATV Lucena costs ₱1,500.00 a month which also includes 15 GB of data monthly. It charges ₱50.00 for each additional gigabytes usage. Find the monthly cost incurred if the owner used 45 GB of data in a month.

What I Have Learned

According to Alice Hoffman every problem has a solution. In finding the solution one important aspect to consider is the "how" or the process of finding it. In solving problems involving functions there are different process that we can employ to attain the solution.

In three to five sentences write the process that you follow in solving problems involving functions.

_____

_____

_____

_____

_____

_____

What I Can Do

You wanted to join a booth fair, and you are aiming to get a profit that is twice as your capital. Your starting capital is ₱15,000.00. Make a financial plan for the booth that you will set up and the product that you will sell. You may use the sample plan below:

FINANCIAL PLAN

Product: _____

Description of product: _____

Goal: _____

Capital: ₱15,000.00

Fixed Cost (Labor, Machineries, Expenses for the booth etc): _____

Variable Cost (Materials, Ingredients, etc): _____

Profit function: _____

Prove that profit function will yield an amount that is twice the capital

_____

_____

Rubrics for the Task

Categories	Excellent 3	Fair 2	Poor 1
Budgeting	Excellent understanding in creating a plan for spending the money	Some understanding in creating a plan for spending the money	Little to no understanding in creating a plan for spending the money
Planning	The goal set is achievable and realistic	The goal set is hard to achieve	The goal set is not achievable and not realistic
Accuracy of Solution	The computation in obtaining the desired profit using the profit function is correct	The computation in obtaining the desired profit using the profit function has flaws	There is no attempt in computing the desired profit using the profit function

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

For numbers 1- 5 use the problem below:

Emmanuel decided to put up a candy shop that sells a dairy chocolate that cost ₱135.00 per pack. The cost of making the chocolate is ₱90.00 and the operating expense is ₱4,500.00

1. Which of the following pertains to the profit function?
   a. $P(x) = 45x - 4{,}500$
   b. $P(x) = 225x - 4{,}500$
   c. $P(x) = 45x + 4{,}500$
   d. $P(x) = 225x + 4{,}500$

2. How many packs of dairy chocolate must be sold to break even?
   a. 50
   b. 100
   c. 150
   d. 200

3. Which value of x will make Emmanuel's candy shop suffer loss for selling packs of chocolates?
   a. $X > 100$
   b. $x \leq 100$
   c. $x < 100$
   d. $x \geq 100$

4. How many chocolate bars must be sold if Emmanuel wanted to earn a profit of ₱6,750.00?
   a. 100
   b. 150
   c. 250
   d. 350

5. How much is the gain if Emmanuel sold 350 packs of chocolates?
   a. ₱6,750.00
   b. ₱9,750.00
   c. ₱11.250.00
   d. ₱15,250.00

For numbers 6- 10 use the problem below

Mariel wanted to avail a cellphone plan that offers a monthly fee of ₱2,500.00. It includes 240 minutes of call and charges ₱7.50 for each additional minute of usage.

6. Which of the following pertains to the monthly cost function?
   a. $C(x) = 2{,}500 - 1800x$
   b. $C(x) = 2{,}500 + 1800x$
   c. $C(x) = 700 + 7.50x$
   d. $C(x) = 700 - 7.50x$

7. What value of x will not require any additional charge in her monthly bill?
   a. $x > 240$
   b. $x < 240$
   c. $x \geq 240$
   d. $x \leq 240$

8. How many additional minutes of call did she make, if she paid ₱2,800.00 in her monthly bill?
   a. 20 minutes
   b. 40 minutes
   c. 60 minutes
   d. 80 minutes

9. How much is her monthly cost incurred if she made an additional usage of 20 minutes of call?
   a. ₱2,500.00
   b. ₱2,600.00
   c. ₱2,650.00
   d. ₱3,350.00

10. How much will she need to pay from using a total of 350 minutes of call in one month?
    a. ₱950.00
    b. ₱2,610.00
    c. ₱3,325.00
    d. ₱4,325.00

For numbers 11- 15 use the problem below

A local cable network charges ₱950.00 monthly connection fee plus ₱100.00 for each hour of pay-per-view (PPV) event regardless of a full hour or a fraction of an hour.

11. Which of the following pertains to the payment function suggested in the problem?
    a. $f(x) = 100x + 950$
    b. $f(x) = 100x - 950$
    c. $f(x) = 950x + 100$
    d. $f(x) = 950x - 100$

12. What is the monthly bill of a customer who watched 20 hours of PPV events?
    a. ₱2,950.00
    b. ₱3,950.00
    c. ₱4,950.00
    d. ₱5,950.00

13. How much is the monthly bill of a customer who watched 0.5 hours of PPV events?
    a. ₱950.00
    b. ₱1,050.00
    c. ₱2,050.00
    d. ₱3,050.00

14. What will be the monthly bill of a customer who watched 12.3 hours of PPV events?
    a. ₱1,250.00
    b. ₱2,250.00
    c. ₱3,250.00
    d. ₱4,250.00

15. How many hours did a customer watched PPV events if the monthly payment is ₱1,450.00?
    a. 2 hours
    b. 3 hours
    c. 4 hours
    d. 5 hours

Additional Activities

To practice your skills in solving problems involving functions the exercises below is for you.

Read and solve the problem.

Mall Goers

1. Inter Global Mall charges ₱30.00 for the first hour or a fraction of an hour for the parking fee. An additional ₱15.00 is charged for every additional hour of parking. The parking area operates from 7am to 12 midnight everyday.

a. Write a function rule for the problem

b. How much will be charged to the car owner if he parked his car from 7am to 3pm?

c. How much will be charged to a car owner who parked his car from 9am to 11:30pm?

Geometry

A man with 200 ft. of fencing material wishes to fence off an area in the shape of a rectangle. What should be the dimensions of the area if the enclosed space is to be as large as possible? What is the largest area?
Hint: $A = lw$, $P = 2l + 2w$

Answer Key

What I Know

1. B
2. B
3. C
4. C
5. A
6. A
7. C
8. C
9. D
10. A
11. C
12. A
13. C
14. A
15. C

What's More

Independent Practice 1
a. P*(x)* = 10x – 790
b. ₱670.00
c. 79 cookies
d. 104 cookies
Independent Assessment 1
a. P(*x)* = 12x – 600
b.₱600.00
c. 50 doughnuts
d. 100 doughnuts
Independent Practice 2
a. C*(x)* = 7x – 60
b. ₱1,200.00
c. ₱2,040.00
Independent Assessment 2
₱3,000.00

Assessment

1. A
2. B
3. C
4. C
5. C
6. C
7. D
8. B
9. C
10. C
11. A
12. A
13. B
14. B
15. D

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11 General Mathematics

General Mathematics
Rational Functions, Equations and Inequalities
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Rational Functions, Equations and Inequalities

Introductory Message

For the facilitator:

Welcome to the General Mathematics Alternative Delivery Mode (ADM) Module on Rational Functions, Equations and Inequalities!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Alternative Delivery Mode (ADM) Module on Rational Functions, Equations and Inequalities!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed to help learners gain understanding about rational functions. It is composed of two lessons. The first lesson tackles about representing real life situations using rational functions, and the second lesson will delve about distinguishing rational function, rational equation and rational inequality. It is assumed that the learners already grasp full understanding with functions which was found on the previous modules.

The first part of this module covers varied situations that can be seen in real life such as budgeting distance and concentration of medicine in the blood while the second lesson will proceed to deeper portion or rational sentences. It is hoped that upon exploring this learning kit you will find the eagerness and enthusiasm in completing the task required. Best of luck!

After going through this module, you are expected to:

1. represent real – life situations using rational functions
2. distinguishes rational function, rational equation and rational inequality

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. A truck that delivers essentials in remote areas can travel 85 kilometers. Which of the following expresses the velocity v as a function of travel time t in hours?
   a. $v(t) = \frac{85}{t}$
   b. $v(t) = \frac{t}{85}$
   c. $t(v) = \frac{85}{v}$
   d. $t(v) = \frac{v}{85}$

2. If the truck in problem no. 1 was delayed by 4 hours due to the checkpoints that it passed through what will be the time t as a function of velocity v in km/hr?
   a. $v(t) = \frac{85}{t}+4$
   b. $v(t) = \frac{t}{85}+4$
   c. $t(v) = \frac{85}{v} + 4$
   d. $t(v) = \frac{v}{85} + 4$

3. As a cure to the epidemic that spread in the whole country, the Department of Health (DOH) released a new drug that is subject for experimentation, supposed that $c(t) = \frac{2t}{t+2}$(in mg/mL)represents the concentration of a drug in a patient's blood stream in t hours, how concentrated is the drug after 2 hours of administration?
   a. 4mg/mL
   b. 3mg/mL
   c. 2mg/mL
   d. 1mg/mL

4. If the distance from Manila to Lucena is approximately 140 kilometers, which of the following pertains to the function *(s)*, where s is the speed of travel that describes the time it takes to drive from Manila to Baguio?
   a. $t(s) = \frac{140}{s}$
   b. $t(s) = \frac{s}{140}$
   c. $s(t) = \frac{140}{t}$
   d. $s(t) = \frac{t}{140}$

For numbers 5-6 use the problem below:

Due to the Enhanced Community Quarantine, Banawe Footspa temporarily stopped its operation and to help the employees the owner decided to split evenly its total revenue of ₱65,000.00

5. If the number of employees is represented by x, which function represents the amount each received?
   a. $f(x) = 65000x$
   b. $f(x) = x + 65000$
   c. $f(x) = \frac{65000}{x}$
   d. $f(x) = x - 65000$

6. If the owner held a fund raising activity that aimed to help the employees and collected ₱5000.00 per employee, which of the following represents the total amount an employee will receive?
   a. $f(x) = 65000x + 5000$
   b. $f(x) = x + 65000 + 5000$
   c. $f(x) = \frac{65000}{x} + 5000$
   d. $f(x) = x - 65000 + 5000$

For numbers 7-8, refer to problem below:

Due to the inclement weather the plane slows down the regular flying rate which results to additional 2 hours in covering a 4000-km distance to its regular time.

7. Write a function that expresses the time t as a function of regular rate r in travelling.
   a. $t(r) = \frac{4000}{r}$
   b. $t(r) = \frac{r}{4000}$
   c. $t(r) = \frac{4000+2}{r}$
   d. $t(r) = \frac{4000}{r+2}$

8. What function expresses the time as a function of rate during inclement weather in travelling?
   a. $t(r) = \frac{4000}{r}+2$
   b. $t(r) = \frac{r}{4000}+2$
   c. $t(r) = \frac{4000+2}{r}$
   d. $t(r) = \frac{4000}{r+2}$

9. Which of the following is a rational function?
   a. $f(x) = 2x^2 - 7$
   b. $f(x) = \frac{4x-10}{x-1}$
   c. $x + 2 \geq \frac{x+3}{x-2}$
   d. $\frac{x-8}{2x} = 12$

10. How will you classify $y = \frac{x^2-9}{x+3}$?
   a. Rational Equation
   b. Rational Inequality
   c. Rational Function
   d. Rational Expression

11. What symbol must be placed in the blank to make the sentence rational equation:
____ $= \frac{x+4}{3}$
   a. *f(x)*
   b. y
   c. $\leq$
   d. 6

12. Which of the following is considered rational inequality?
   a. $x + \sqrt{3} \leq 5$
   b. $f(x) = \frac{x+5}{4}$
   c. $5 \geq \frac{x+5}{4}$
   d. $x + 2 \approx \frac{x+5}{4}$

13. Which of the following is considered rational equation?
   a. $4 + 5 = 9$
   b. $\frac{x^2+5}{x+1}$
   c. $2 = \frac{\sqrt{3x}+1}{x+2}$
   d. $2 = \frac{3x+1}{x+2}$

14. In the equation: $\frac{x+3}{x+2} = x + 5,$ what symbol must be replaced with 5 to make the equation a rational function?
   a. y
   b. $\sqrt{5}$
   c. $\leq$
   d. 5x

15. What symbol is present in the equation $y = \frac{\sqrt{x^2+3}}{x+2}$ for not considering it as rational function?
   a. y
   b. =
   c. $x+2$
   d. $\sqrt{x^2 + 3}$

Lesson 1 Representing Real – life Situations Using Rational Functions

Rational functions can model a number of real-life situations. One particular example is the help that is extended by the government to the citizen during the time of pandemic. Majority of our fellow citizens experienced hardship and required help coming from the government. As a response, they provided a particular amount to a certain percentage of the population that can be represented as rational function to determine how much either in cash or kind an individual may receive. However, it is not enough that only the government will take part to solve this crisis everyone can be part of the solution if we played our role properly. Real-life situations that involve rational functions is mostly seen in economics and science however other disciplines also incorporate this concept. If you wonder how rational function can help, you can explore this module.

What's In

ADMISSION CARD

Listed below are the skills and competencies you should possess before proceeding to this lesson. Read the statements and assess yourself about your level of understanding by answering yes if you agree and no if otherwise.

Statement	Yes	No
1. I can represent real-life situations using function		
2. I can recognize polynomial functions		

- If your answer to all the items is yes then you are confident to proceed to the next lesson.
- If you answered no to any of the statements there is a need for you to have a quick review on the following:

Functions are used to model real life situations and in representing real – life situations the quantity of one variable depends or corresponds to or mapped onto another quantity.

Consider the examples below and reflect if you are confident enough to proceed.

1. Write a function C that represent the cost of buying alcohol a, if an alcohol costs ₱155.00

$$C(a) = 155a$$

2. A commuter pays ₱20.00 for a tricycle ride for the first 5 km and an additional ₱ 0.75 for every succeeding distance d in kilometer. Represent the situation as function

$$F(d) = 20, \text{ if } 0<d\leq 5$$
$$F(d) = 20 + 0.75(d), \text{ if } d>5$$

Let n be a nonnegative integer, and let $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ be real numbers with $a_n \neq 0$. The function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ is called a polynomial function of x with degree n. The coefficient a_n is called the leading coefficient, and a_0 is the constant.

Here are the examples of polynomial functions of particular degree together with their names:

Polynomial	Degree	Special Name
$f(x) = 3$	0	Constant Function
$f(x) = -2x + 1$	1	Linear Function
$f(x) = 3x^2 - 5x + 2$	2	Quadratic Function
$f(x) = 4x^3 + 2x - 7$	3	Cubic Function

At this point you may now proceed to the next section of this module!

Notes to the Teacher

The teacher may reiterate the formula in finding the distance which is $d = rt$ and the other relationships derived from this formula such $r = \frac{d}{t}$ and $t = \frac{d}{r}$. These relationships will be helpful in meeting the expected outcome of this module.

What's New

Read and analyze the comics below.

LOVE IN TIMES OF COVID?

Questions

1. How much is the total amount of money the barangay can use for its relief operations?_____

2. What is the concern of one the Barangay Kagawad regarding the total number of families who will benefit with the relief? Do you think it is valid? Why?
_____
_____

3. Suppose the officials conducted a survey for 4 days to determine the actual number of families residing in the barangay and the secretary constructed a table to keep track of the data. Complete the table below:

Day	0	1	2	3	4
Number of Families	850	855	882	910	931
Amount of relief each family will receive	₱520.00				

4. Create a model or equation that will represent to the amount of relief each family will receive bearing in mind that the number of families will vary.

   _____

5. After resolving the amount of relief each family may receive, what other problem may arise in the current situation?

   _____

6. If you are one of the residents of that barangay what will be your participation to help the officials?

   _____

What is It

In the previous activity we see a real- life scenario, which can be analyzed mathematically. To be able to determine the amount of relief that will be distributed to 850 families we first add the donations and the total budget of the barangay. The total donations obtained is ₱95,000 while the budget of the barangay is ₱347,000 which gives a total of ₱442,000. This amount will be divided by 850 families to obtain ₱520.00 However there are cases that one quantity varies that in this case it is the total number of families. Due to unavoidable circumstances such as being stranded and change of residency we cannot control that variable. As a result, another computation was made. After the first day of survey it was determined that there are 855 families living in the barangay, then the amount of relief each family will receive will be ₱516.96. After the second day it was determined that there were 882 families living in the barangay then the amount of relief will be changed to ₱501.13. After the third day the amount of the relief will be ₱485.71 and after the fourth day it will become ₱474.76 However, we can also use a model that will represent real – life situations. In this module you will learn how to represent real-life situations using rational functions.

Definition of Rational Function

A rational function, *r(x)* is a function of the form

$$r(x) = \frac{p(x)}{q(x)}$$

where *p(x)* and *q(x)* are polynomial functions and $q(x) \neq 0$

The domain of *r(x)* is a set of real numbers such that *q(x)* is not zero.

The following are examples of rational functions:

1. $r(x) = \frac{x^3-1}{x+1}, x \neq 1$; Both numerator and denominator are polynomial functions, denominator has restriction because it should not be equal to zero

2. $f(x) = \frac{1}{x}, x \neq 0$; The numerator 1 is a polynomial function with a degree 0, the denominator is a polynomial function and it must not be equal to 0

There are different scenarios or real-world relationships that can be modeled by rational functions, let us take the following examples:

1. The Local Government Unit allotted a budget of ₱100,000.00 for the feeding program in the Day Care Center. The amount will be divided equally to all the pupils in the Day Care Center. Write an equation showing the relationship of the allotted amount per pupil represented by *f(x)* versus the total number of children represented by x

   Showing the relationship in tabular form we will arrived at

No. of children (x)	10	20	50	100	200
Allocated amount per child	₱10,000	₱5000	₱2000	₱1000	₱500

   Notice that as the number of children increase the amount allocated per child decrease.

   In writing a representation we will arrived at $f(x) = \frac{100000}{x}$

2. Suppose a benefactor wants to supplement the budget allotted for each child by donating additional ₱650.00 per child. If h(x) represents the new amount allotted per child, construct a function representing the relationship.
   Using the table we used earlier:

No. of children (x)	10	20	50	100	200
Allocated amount per child	₱10,000.00 +₱650.00	₱5000.00 +₱650.00	₱2000.00 +₱650.00	₱1000.00 +₱650.00	₱500.00 +₱650.00

Thus, the representation of the rational function is $g(x) = \frac{100000}{x} + 650$

3. A car is to travel a distance of 70 kilometers. Express the velocity (v) as a function of travel time (t) in hours.

Let us first show the relationship using a table. Remember that as time increases in travelling the velocity or the speed of a car will decrease

Time (hours)	1	2	3	5	10
Velocity (km/hr)	70	35	23.33	14	7

Thus, the function $v(t) = \frac{70}{t}$ can represent v as a function of t

What's More

Read each situation carefully to solve each problem. Write your answer on a separate sheet of your paper.

Independent Practice 1

School is Cool

During the first quarter of the school year the officers –elect of the Supreme Student Government decided to divide their budget evenly to the different committees. If their budget is ₱35,000 construct a function M which would give the amount of money each of the n number of committees would receive.

a. You may construct a table to aid you in determining the relationships between quantities

Number of Committees	2	4	6	8
Amount allocated for each committee				

b. Write the rational function that represents the situation

_____

Independent Assessment 1

Pastry Corner

Manuel has 10 cups of flour to be used in baking cakes, he wanted to split it evenly among the containers that he will use so that he can adjust the measurements of other ingredients. Construct a function C which would give the number of cups of flour each of the number of containers n will have.

Independent Practice 2

Medicine Dosage

Let $C(t) = \frac{3t}{t+6}$ be the function that describes the concentration of a certain medication in the bloodstream overtime t. If 4 hours have passed after the medicine was intake, how concentrated is it in the blood?

a. What is the rational function that serves as the model?

_____

b. How are you going to determine the concentration of the medicine given the rational function and the number of hours?

_____

_____

Independent Assessment 2

How Long

The distance between the school and your home is 5 kilometers. Express velocity (v) as a function of travel time (t) in hours

What I Have Learned

3-2-1 Action

What are the three things that help you in representing real –life situations to rational function?

1. _____

2. _____

3. _____

What are the two questions that you want to ask to clarify the process of translating real – life situations to rational functions

1._____

2. _____

Share one tip or suggestion on how others can represent real-life situations using rational functions

1. _____

What I Can Do

You conducted an outreach activity to help the needy in your community and have solicited ₱52,000.00. You wanted to propose a plan on how to equally divide the money and the possible relief goods that will be included. If you will make a proposal what plan will you do? Show your plan by filling up the form below:

OUTREACH PROPOSAL

Total amount Solicited: _____

Rational Function Model:_____

Possible number of beneficiaries, amount allocated and relief included

	Option 1	Option 2	Option 3
Number of Beneficiaries			
Amount allocated per beneficiary			
Relief Goods included and breakdown cost of each goods			

Rubrics for the Task

Categories	Excellent 3	Fair 2	Poor 1
Budgeting	Excellent understanding in creating a plan for spending the money	Some understanding in creating a plan for spending the money	Little to no understanding in creating a plan for spending the money
Planning	The goal set is achievable and realistic	The goal set is hard to achieve	The goal set is not achievable and not realistic
Accuracy of Solution	The computation in obtaining the desired profit using the profit function is correct	The computation in obtaining the desired profit using the profit function has flaws	There is no attempt in computing the desired profit using the profit function

Lesson 2 Rational Functions, Equations and Inequalities

It was defined in the previous lesson that rational functions are expressed as a ratio of two polynomials P and Q. The value of rational functions is defined for all real numbers x, except for the value of x that makes the denominator zero. There are different relationships between rational expressions. It may involve inequality, equality and functions and that is what we are going to dig deeper in this lesson

What's In

Admission Card

Listed below are the skills and competencies you should possess before proceeding to this lesson. Read the statements and assess yourself about your level of understanding by answering yes if you agree and no if otherwise.

Statement	Yes	No
1. I can describe a rational expression		
3. I can distinguish whether an expression is rational or not		

- If your answer to all the items is yes then you are confident to proceed to the next lesson.
- If you answered no to any of the statements, there is a need for you to have a quick review on the following:

  A rational expression can be described as a ratio or quotient of two polynomials. Let us look at the examples:

Consider the following algebraic expressions, determine whether they are rational or not and state the reason.

1. $\frac{3x^2-5x+2}{x+1}$; Rational expression because it is a ratio of two polynomials
2. $\frac{2}{x-5}$; Rational expression because 1 and *x-5* are polynomials
3. $\frac{\sqrt{x-4}}{2x+1}$; Not a rational expression since the numerator is not a polynomial
4. $x+5$; Rational expression because the numerator *x+5* and denominator 1 are polynomials

What's New

Read and analyze the advertisement below

Wanted!!!

Call for Applicants for Contract of Service Workers

For the purpose of 2020 census of population and housing, Philippine Statistics Authority is hiring enumerators.

Any male or female
Age - at least 18 years old and at most 45 years old
Educational Attainment – at least SHS graduate
Salary - ₱700.00 per day
Physically fit for field work
Interested applicants must submit application letter and Personal Data Sheet (PDS) to PSA- Quezon

Questions

1. What is the advertisement all about?
   _____
2. What job is offered by PSA? _____
3. Who can apply for the position?
   _____
   _____
4. How will you translate the required age mathematically?
   _____
5. Represent the salary *(S)* that will be received as a function of number of days used for work *(n).*
   _____
6. What are the mathematical symbols that you used in answering questions numbers 4 and 5?
   _____
7. Do you think the age requirement for the job is fair enough or is there a bias? Why do you say so?
   _____
8. Why do you think having a census is important?
   _____

What is It

The job advertisement shows the need to conduct a census. Census is important because it will serve as the basis for planning for the future such as public safety, infrastructures like hospital and schools and improving homes in the neighborhood. This also serves as the basis in predicting the number of people who need help in times of crisis. However, in finding enumerators who will conduct the census, there are different qualifications needed and some of those are the age requirement and educational attainment, to represent it mathematically the use of inequality symbol was employed. However, the salary was also mentioned and to be able to represent it, equality symbol is used. At this section we will compare rational expressions using equality and inequality symbols, thus we will further classify the differences among rational equation, rational inequality and rational function.

To determine the difference among rational function, rational equation and rational inequality study the table below:

	Rational Equation	Rational Inequality	Rational Function
Definition	An equation involving rational expression	An inequality involving rational expressions	A function of the form $f(x) = \frac{p(x)}{q(x)}$ where *p(x)* and *q(x)* are polynomial functions and *q(x)* is not the zero function
Example	$\frac{x+4}{x-1} = \frac{1}{5}$	$\frac{x-2}{5} > 3$	$f(x) = \frac{x^2+6x+8}{x+4}$

Additional examples:

Determine whether the given sentence is a rational equation, a rational function, a rational inequality or none of these.

1. $\frac{x+5}{x-1} = y$; This is an example of rational function because the symbol y is also a representation of function of x or *f(x)*

2. $\frac{\sqrt{2}}{x+1} \leq 3$; None of these because $\frac{\sqrt{2}}{x+1}$ is not a rational expression

What's More

Independent Practice 1

Determine whether the given is a rational function, rational equation, rational inequality or none of these.

1. $\frac{1+x}{x-2} = 4$ _____

2. $5x \geq \frac{2}{2x-1}$ _____

3. $f(x) = \frac{x^2-7}{x+2} - 3$ _____

4. $\frac{x+2}{x-2} = y + 3$; **Hint y is represented by *f(x)*** _____

5. $\frac{x+1}{2} < \sqrt{x+3}$ _____

Independent Assessment 1

Determine whether the given is a rational function, a rational equation, a rational inequality or none of these

1. $y = 3x^2 - x - 1$
2. $\frac{3}{x} - 3 = \frac{2x}{2x+1}$
3. $\sqrt{x+5} = 2$
4. $\frac{x+5}{x-5} = x^2$
5. $6x - \frac{x}{3} \leq 2$

Independent Practice 2

Read the statements carefully and choose the letter that corresponds to the correct answer.

1. How do you classify $\frac{x+3}{x-3} = 3y$?

   a. Rational Equation

   b. Rational Function

   c. Rational Inequality

   d. None of these

2. Which of the following is an example of rational function?

A. $\frac{2x}{x^2} - 3 = \frac{22}{x+1}$

B. $\sqrt{x+2} = f(x)$

c. $\frac{2x}{x^2} - 3 > \frac{22}{x+1}$

d. $y = 2x + 3$

3. What symbol will be replaced with the equal sign in the equation $\frac{2x+5}{3x-5} = 2x^2$ to make it an inequality?

a. ~

b. ≈

c.>

d. ≡

Independent Assessment 2

Read the statements carefully and choose the letter that corresponds to the correct answer.

1. How do you classify $\frac{3x^2}{3x} \approx 3y$?

a. Rational Equation

b. Rational Function

c. Rational Inequality

d. None of these

2. Which of the following is an example of rational function?

a. $5x^3 = \frac{2}{x+4}$

b. $\sqrt{2x^2} = f(x)$

c. $\frac{2x}{x^2} - 3 > \frac{22}{x+1}$

d. $x + y = 2x + 3$

3. What symbol will be replaced with the " ≤ " sign in $\frac{2x+5}{3x-5} \leq 2x^2$ to make it a rational equation?

a. ~

b. ≈

c.=

d. ≡

What I Have Learned

3-2-1 Action

What are the distinct features of rational function, rational equation and rational inequality?

1. Rational Function

_____

_____

2. Rational Equation

_____

_____

3. Rational Inequality

_____

_____

What are the differences of the following:

1. Rational Function and Rational Equation

_____

_____

2. Rational Equation and Rational Inequalities

_____

_____

Share one tip or suggestion on how to identify whether the given is a rational function, rational equation or rational inequality

1. _____

_____

_____

What I Can Do

You are analyzing your bills in MERALCO for the consecutive months. Using your bill at home express the amount per kwh *(A)* as a function of kilowatt per hour *(k)* consumed for the last two consecutive months. Then create austerity plan for the next month. (austerity- measures taken to reduce spending)

Austerity Plan

Bill for the last two consecutive months:

_____

_____

Create a rational inequality showing the relationship between the bill in the last consecutive months:

_____

List down the austerity measures to be undertaken to reduce the electricity consumption:

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Rubrics for the Task

Categories	Excellent 3	Fair 2	Poor 1
Solution Process	Complete and appropriate solution process	An appropriate solution process that is partially complete	Needed extensive guidance to work on the problem
Planning	The goal set is achievable and realistic	The goal set is hard to achieve	The goal set is not achievable and not realistic
Accuracy of Solution	The computation in obtaining the desired profit using the profit function is correct	The computation in obtaining the desired profit using the profit function has flaws	There is no attempt in computing the desired profit using the profit function

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

For numbers 1-2 refer to problem below:

Due to Typhoon Rosing the bus slows down the regular trip rate which results to additional 2 hours in covering a 140-km distance to its regular time.

1. Write a function that expresses the time t as a function of regular rate r in travelling.

   a. $t(r) = \frac{140}{r}$

   b. $t(r) = \frac{r}{140}$

   c. $t(r) = \frac{140+2}{r}$

   d. $t(r) = \frac{140}{r+2}$

2. What function expresses the time as a function of rate during the typhoon?
   a. $t(r) = \frac{140}{r}+2$
   b. $t(r) = \frac{r}{140}+2$
   c. $t(r) = \frac{140+2}{r}$
   d. $t(r) = \frac{140}{r+2}$
3. Which of the following is a rational function?
   a. $f(x) = \sqrt{5}$
   b. $f(x) = \frac{2x-5}{x-1}$
   c. $x + 4 \geq \frac{x+2}{x-1}$
   d. $\frac{x-8}{4x} = 20$
4. How will you classify $y = \frac{x^2-16}{x+4}$?
   a. Rational Equation
   b. Rational Inequality
   c. Rational Function
   d. Rational Expression
5. What symbol must be placed in the blank to make the sentence rational equation:

   ____ $= \frac{2x+5}{8}$
   a. *f(x)*
   b. y
   c. $\leq$
   d. 3

6. Which of the following is considered rational inequality
   a. $\sqrt{5} \leq 5$
   b. $y = \frac{x+15}{3}$
   c. $8 \geq \frac{2x+15}{14}$
   d. $x + 2 \approx \frac{x+5}{4}$

7. Which of the following is considered rational equation?
   a. 5x+ 8
   b. $\frac{x^2+25}{x+5}$
   c. $6 = \frac{\sqrt{3x}+1}{x+2}$
   d. $3 = \frac{4x+1}{2x+2}$
8. In the equation: $\frac{2x+3}{2x+2} = x + 8,$ what symbol must be replaced with 8 to make the equation rational function?
   a. y
   b. $\sqrt{5}$
   c. $\leq$
   d. 5x

9. What symbol is present in the equation $y = \frac{\sqrt{3x^2+3}}{2x+2}$ for not considering it as rational function?
   a. y
   b. =
   c. $2x+2$
   d. $\sqrt{3x^2 + 3}$

10. A delivery track that will bring cargo will travel 80 kilometers. Which of the following expresses the velocity v as a function of travel time t in hours?
   a. $v(t) = \frac{80}{t}$
   b. $v(t) = \frac{t}{80}$
   c. $t(v) = \frac{80}{v}$
   d. $t(v) = \frac{v}{80}$

11. If the truck in problem no. 1 was delayed by 4 hours due to the checkpoints that it passed through what will be the time t as a function of velocity v in km/hr?
   a. $v(t) = \frac{80}{t}$+4
   b. $v(t) = \frac{t}{80}$+4
   c. $t(v) = \frac{80}{v}$+4
   d. $t(v) = \frac{v}{80} + 4$

12. As a cure to the epidemic that spread in the whole country the Department of Health (DOH) released a new drug that is subject for experimentation, supposed that $c(t) = \frac{2t}{t+2}$(in mg/mL)represents the concentration of a drug in a patient's blood stream in t hours, how concentrated is the drug after 4 hours of administration?
    a. 4.67mg/mL
    b. 3.33mg/mL
    c. 2.67mg/mL
    d. 1.33mg/mL
13. If the distance from Manila to Batangas is approximately 109 kilometers, which of the following pertains to the function *(s)*, where *s* is the speed of travel that describes the time it takes to drive from Manila to Batangas?
    a. $t(s) = \frac{109}{s}$
    b. $t(s) = \frac{s}{109}$
    c. $s(t) = \frac{109}{t}$
    d. $s(t) = \frac{t}{109}$

For numbers 14-15 use the problem below:

Due to the Enhanced Community Quarantine, Toy's for Her and Him temporarily stopped its operation and to help the employees the owner decided to split evenly its total revenue of ₱45, 000.00

14. If the number of employees is represented by x , which function represents the amount each received?
    a. $f(x) = 45000x$
    b. $f(x) = x + 45000$
    c. $f(x) = \frac{45000}{x}$
    d. $f(x) = x - 45000$
15. If the owner held a fund-raising activity that aimed to help her employees and collected ₱1500.00 per employee, which of the following represents the total amount an employee will receive?
    a. $f(x) = 45000x + 1500$
    b. $f(x) = x + 45000 + 1500$
    c. $f(x) = \frac{45000}{x} + 1500$
    d. $f(x) = x - 45000 + 1500$

Additional Activities

To strengthen your skills in determining rational functions, rational equations and rational inequality, construct five examples of each category.

Rational Equation	Rational Function	Rational Inequality
1. _____	1. _____	1. _____
2. _____	2. _____	2. _____
3. _____	3. _____	3. _____
4. _____	4. _____	4. _____
5. _____	5. _____	5. _____

Answer Key

What I Know

1. A
2. C
3. D
4. C
5. C
6. C
7. A
8. A
9. B
10. C
11. D
12. C
13. D
14. A
15. D

What's More

Lesson 1

Independent Practice 1

a.

2	4	6	8
17500	8750	5833.33	4375

b. $M(n) = \frac{35000}{n}$

Independent Assessment 1

$$C(n) = \frac{10}{n}$$

Independent Practice 2

a. $C(t) = \frac{3t}{t+6}$

b. Substitute 4 to the function; C(4)=1.2

Independent Assessment 2

$$v(t) = \frac{5}{t}$$

Lesson 2

Independent Practice 1

1. Rational Equation
2. Rational Inequality
3. Rational Function
4. Rational Function
5. None of These

Independent Assessment 1

1.Rational Function
2. Rational Equation
3. None of These
4. Rational Equation
5. Rational Inequality

Independent Practice 2

1. B 2. D 3. C

Independent Assessment 2

1. D 2. D 3. C

Assessment

1. A
2. A
3. B
4. C
5. D
6. C
7. D
8. A
9. D
10. A
11. C
12. D
13. C
14. C
15. C

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11 General Mathematics

General Mathematics
Solving Rational Equations and Inequalities
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Solving Rational Equations and Inequalities

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Rational Equations and Inequalities!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Rational Equations and Inequalities!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled into process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written for learners like you to determine a method and set of steps for solving rational equations and inequalities. Learners like you can also explore and develop new methods that you have synthesized and apply these techniques for performing operations with rational expressions.

In this module, you will able to explain the appropriate methods in solving rational equations and inequalities you used. You will also be able to check and explain extraneous solutions.

After going through this module, you are expected to:

1. Apply appropriate methods in solving rational equations and inequalities.
2. Solve rational equations and inequalities using algebraic techniques for simplifying and manipulating of expressions.
3. Determine whether the solutions found are acceptable for the problem by checking the solutions.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following expressions is an equality between two expressions containing one or more variables?
   a. rational function
   b. rational equation
   c. rational inequality
   d. irrational expression

2. What do you call a root obtained in the process of solving an equation which looks correct but after analyzing it turns out as incorrect?
   a. extraneous solution
   b. rational expression
   c. least common denominator
   d. quotient

3. What do you call an inequality which involves one or more rational expressions?
   a. rational function
   b. rational equation
   c. rational inequality
   d. irrational expression

4. What is the usual technique to a solve rational equation?
    a. multiply both sides of the equation by its greatest common factor
    b. multiply both sides of the equation by its least common denominator
    c. multiply both sides of the equation by its inverse factor
    d. multiply both sides of the equation by its greatest common denominator

For items 5-9: Refer to the rational equation below.

$$\frac{x}{3} + \frac{1}{4} = \frac{x}{2}$$

5. What is the LCD of the denominator 3, 4 and 2?
    a. 3
    b. 6
    c. 8
    d. 12

6. What property will be used if you multiply the LCD on both sides of the equation?
    a. Distributive Property
    b. Associative Property
    c. Commutative Property
    d. Additive Property

7. What will be the new form of the equation after applying the property and simplifying?
    a. $4x + 3 = 6x$
    b. $3x + 4 = 2x$
    c. $6x + 4 = 3x$
    d. $12x + 3 = 12x$

8. What will be the solution on the given rational equation?
    a. $\frac{2}{3}$
    b. $\frac{3}{2}$
    c. 2
    d. 3

9. How will you check if your solution is correct?
    a. by eliminating the rational expressions.
    b. by dividing both sides of the equation by LCD.
    c. by applying Commutative Property.
    d. by substituting the answer or solution in the original equation.

10. If by solving a rational equation you obtain a number that makes an expression in the equation undefined, then what will you do?
    a. Accept even if it is untrue value.
    b. Do not reject since it will satisfy the equation in a long run.
    c. The number is not a real solution then discard it.
    d. Continue the solution even if it will give undefined answer.

11. Which of the following is NOT an inequality sign?
    a. ≤
    b. √
    c. ≥
    d. <

12. Express the graph of solution set into interval notation.

    a. $\{x \mid -3 \leq x < 1\}$
    b. $\{x \mid -3 \leq x \leq 1\}$
    c. $\{x \mid \ 3 < x \leq 1\}$
    d. $\{x \mid \ 3 \leq x < 1\}$

13. Below are the steps in solving rational inequality EXCEPT
    a. Put the inequality in general form.
    b. Set the numerator and denominator equal to one and solve.
    c. Plot the critical values on a number line, breaking the number line into intervals and take a test number from each interval by substituting into the original inequality.
    d. Determine if the endpoints of the intervals in the solution should be included in the intervals.

14. Solve for the solutions of the rational inequality $\frac{(x+3)}{(x-2)} \leq 1$.
    a. [∞, 2)
    b. (∞, 2]
    c. (-∞, 2)
    d. [-∞, 2)

15. How will you know that the critical points for item no. 14 will satisfy the inequality?
    a. If it makes a true statement, then the interval from which it came is not in the solution.
    b. If it makes a false statement, then the interval from which it came is in the solution.
    c. If it makes a true statement, then the interval from which it came is in the solution.
    d. If it makes a false statement, then the interval from which it came is either in the solution or not.

Lesson 1 Solving Rational Equations and Inequalities

In this lesson, you shall explore more about solving rational equations and inequalities by carefully studying the step by step methods of solutions. You will first start from the easiest procedures in solving this type of equation and as you progress you will gain learn more techniques and concepts that will help you to solve more complex problems related to this topic. Exercises will range from the simplest problems to the most complex.

At this point, students like you have already solved a variety of equations, including linear and quadratic equations from the previous grade level. Rational equations and inequalities follow the sequence of solving problems by combining the concepts used in solving both linear and quadratic equations. Students will be assessed using both formative and summative assessments along the way to best evaluate your progress.

What's In

Let's Review!

How do you solve algebraic expressions? What are the different properties you need to apply to solve problems involving rational equations and inequalities?

For you to begin, you need to recall some properties and processes to simplify rational expressions by answering the following problems below. Write your answer inside the box.

1. Simplify the given rational expression: $\frac{x-2}{x^2-4}$

2. Multiply the given rational expressions: $\frac{3x+1}{x^2-1} \cdot \frac{x+1}{3x^2+x}$

3. Find the sum of given rational expressions with like denominators: $\frac{5x-1}{x-8} + \frac{3x+4}{x-8}$

4. Find the difference of the given rational expressions with unlike denominators:

$$\frac{6}{x^2-4}-\frac{2}{x^2-5x+6}$$

Let's check if you have made it! You can also write your solution on the prepared box to compare the techniques you apply.

1. To simplify the rational expression you can do the following steps.

Steps in simplifying rational expression	$\frac{x-2}{x^2-4}$	
1. Factor the denominator of the rational expression.	$\frac{x-2}{(x-2)(x+2)}$	Write your previous solution here for comparison.
2. Cancel the common factor.	$\frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)}$	
3. Write the simplified rational expression.	$\frac{1}{x+2}$	

2. To multiply rational expressions you can do the following steps.

Steps in multiplying rational expressions	$\frac{3x+1}{x^2-1}\cdot\frac{x+1}{3x^2+x}$	
1. Factor out all possible common factors.	$\frac{3x+1}{(x+1)(x-1)}\cdot\frac{x+1}{x(3x+1)}$	Write your previous solution here for comparison.
2. Multiply the numerators and denominators.	$\frac{(3x+1)(x+1)}{(x+1)(x-1)(x)(3x+1)}$	
3. Cancel out all common factors.	$\frac{(\cancel{3x+1})(\cancel{x+1})}{(\cancel{x+1})(x-1)(x)(\cancel{3x+1})}$	
4. Write the simplified rational expression.	$\frac{1}{x(x-1)}$	

3. To add and subtract rational expressions with like denominators you can do the following steps.

Steps in addition or subtraction of rational expressions with like denominators	$\frac{5x-1}{x-8}+\frac{3x+4}{x-8}$	
1. the numerators of both expressions and keeping the common denominator.	$\frac{5x-1+3x+4}{x-8}$	Write your previous solution here for comparison.
2. Combine like terms in the numerator.	$\frac{5x+3x+4-1}{x-8}$	
3. Write the simplified rational expression.	$\frac{8x+3}{x-8}$	

4. To add and subtract rational expressions with unlike denominators you can do the following steps.

Steps in adding or subtracting rational expressions with unlike denominators	$\frac{6}{x^2-4}-\frac{2}{x^2-5x+6}$	
1. Factor the denominator of each fraction to help find the LCD.	$\frac{6}{(x-2)(x+2)}-\frac{2}{(x-2)(x-3)}$	Write your previous solution here for comparison.
2. Find the least common denominator (LCD).	$LCD: (x-2)(x+2)(x-3)$	
3. Multiply each expression by its LCD	$\frac{6(LCD)}{(x-2)(x+2)}-\frac{2(LCD)}{(x-2)(x-3)}$	
4. Write the simplified expression.	$6(x-3)-2(x+2)$	
5. Let the simplified expression as the numerator and the LCD as the denominator of the new fraction	$\frac{6x-18-2x-4}{(x-2)(x+2)(x-3)}$	
6. Combine like terms and reduce the rational expression	$\frac{4x-22}{(x-2)(x+2)(x-3)}$	

if you can. In this case, the rational expression cannot be simplified.		

How was the activity? Did you answer all the reviewed items correctly? Great! If you did, then you can now move forward on the next stage of this topic and I am confident that it will be very easy for you to understand the lesson.

Notes to the Teacher

Please remind our students that learning mathematics is a linear process wherein the math skills and knowledge from the previous modules and grade level will be used throughout this topic. For example, if the students have not mastered arithmetic properties and processes then they will have difficulty with the current topic because it requires all of these prerequisite skills. Therefore, it will be necessary to go back, review previous topics and problem-solving skill before they can continue. Inspire our students that learning is not always onward and upward, sometimes we have to take a glimpse of the past before we can move forward.

What's New

Follow Me Activity

Solving Rational Equations and Inequalities

Before you proceed on the lesson proper try to answer the rational equation and inequality using guided procedure. You can synthesize your own steps in solving the problem. You can refer to previous activities if you are having difficulty processing arithmetic properties. Hope you enjoy answering before you continue to the next part of the discussion.

1. Solve example 2 of the rational equation by following the given steps.

Rational Equation	Example 1 $\frac{x-3}{x^2-25}+\frac{1}{x+5}=\frac{1}{(x-5)}$	Example 2 $\frac{2}{x^2-1}-\frac{1}{x-1}=\frac{1}{2}$
1. Find the Least Common Denominator (LCD).	LCD: $(x+5)(x-5)$	
2. Multiply both sides of the equation by its the LCD.	$(x+5)(x-5)[\frac{x-3}{x^2-25}+\frac{1}{x+5}=\frac{1}{(x-5)}]$	
3. Apply the Distributive Property and then simplify.	$(x-3)+1(x-5)=1(x+5)$ $x-3+x-5=x+5$ simplify: $2x-8=x+5$ $2x-x=8+5$ $x=13$	
4. Find all the possible values of x.	$x=13$	
5. Check each value by substituting into original equation and reject any extraneous root/s	Checking: $\frac{x-3}{x^2-25}+\frac{1}{x+5}=\frac{1}{(x-5)}$ $\frac{13-3}{13^2-25}+\frac{1}{13+5}=\frac{1}{(13-5)}$ $\frac{10}{169-25}+\frac{1}{18}=\frac{1}{8}$ $\frac{10}{144}+\frac{1}{18}=\frac{1}{8}$ $\frac{10+8}{144}=\frac{1}{8}$ $\frac{1}{8}=\frac{1}{8}$ ✓ Note: No extraneous root	

2. Solve example 2 of rational inequality. You can refer to example 1 for the guided steps.

Rational Inequality	Example 1 $\frac{3}{x-2}\leq -1$	Example 2 $\frac{3x+1}{x-1}\geq 2$
1. Put the rational inequality in general form. $\frac{R(x)}{Q(x)}>0$ where > can be replaced by $<, \leq$ *and* $\geq$	$\frac{3}{x-2}+1\leq 0$	

2. Write the inequality into a single rational expression on the left side. (You can refer to the review section for solving unlike denominators)	$\frac{3+1(x-2)}{x-2} \leq 0$ $\frac{x+1}{x-2} \leq 0$	
3. Set the numerator and denominator equal to zero and solve. The values you get are called critical values.	Numerator: $x+1=0$ $x=-1$ Denominator: $x-2=0$ $x=2$	
4. Plot the critical values on a number line, breaking the number line into intervals.		
5. Substitute critical values to the inequality to determine if the endpoints of the intervals in the solution should be included or not.	$\frac{3}{x-2} \leq -1$ when $x=-1$ $\frac{3}{-1-2} \leq -1$ $\frac{3}{-3} \leq -1$ $-1 \leq -1$ ✓ ($x=-1$ is included in the solution) when $x=2$ $\frac{3}{2-2} \leq -1$ $\frac{3}{0} \leq -1$ $undefined \leq -1$ X ($x=2$ is not included in the solution)	
5. Select test values in each interval and substitute those values into the inequality. Note: If the test value makes the inequality true, then the entire interval is a solution to the inequality. If the test value makes the inequality false, then the entire interval is not a solution to the inequality.	 when $x=-2$ $\frac{3}{-2-2} \leq -1$ $\frac{3}{-4} \leq -1$ X $false$ when $x=0$ $\frac{3}{0-2} \leq -1$ $\frac{3}{-2} \leq -1$ ✓$true$	

	when $x = 3$ $\frac{3}{3-2} \leq -1$ $3 \leq -1 \quad X \ false$	
6. Use interval notation or set notation to write the final answer.	$[-1,2)$	

How do you find the activity? Have you enjoyed it? Did you follow the steps correctly? The activity tells you about solving rational equations and inequalities. Yes, you read it right. You almost got it!

Let's check if your answers are correct and which process did you find it difficult. I hope you enjoyed answering by your own.

What is It

Rational equation is an equation containing at least one rational expression with a polynomial in the numerator and denominator. It can be used to solve a variety of problems that involve rates, times and work. Using rational expressions and equations it can help us to answer questions about how to combine workers or machines to complete a job on schedule.
Let us use the previous activity to discuss and deepen your knowledge and skills in solving rational equation. The first thing to be in your mind in solving rational equation is to eliminate all the fractions.

Let us solve

$$\frac{2}{x^2-1} - \frac{1}{x-1} = \frac{1}{2}$$

Step 1. You need to find the Least Common Denominator (LCD).
The LCD of the given fractions is $2(x-1)(x+1)$
Step 2. You need to multiply LCD to both sides of the equation to eliminate the fractions. You can also apply cross multiplication if and only if you have one fraction equal to one fraction, that is, if the fractions are proportional. In this case you cannot use the cross multiplication unless you simplify the left equation into a single fraction.

$$2(x-1)(x+1)\left[\frac{2}{x^2-1} - \frac{1}{x-1} = \frac{1}{2}\right]$$

Step 3. You simplify the resulting equation using the distributive property and then combine all like terms.

$$2(2) - 2(x+1) = (x-1)(x+1)$$
$$4 - 2x - 2 = x^2 - 1$$
$$x^2 + 2x - 3 = 0$$

Step 4. You need to solve the simplified equation to find the value/s of x. In this case, we need to get the equation equal to zero and solve by factoring.

$$x^2 + 2x - 3 = 0$$
$$(x+3)(x-1) = 0$$
$$x + 3 = 0 \; or \; x - 1 = 0$$
$$x = -3 \; or \; x = 1$$

So possible solutions are -3 and 1.

Step 5. Finally, you can now check each solution by substituting in the original equation and reject any extraneous root/s (which do not satisfy the equation).

$$\frac{2}{x^2-1} - \frac{1}{x-1} = \frac{1}{2}$$

When $x = -3$

$$\frac{2}{(-3)^2-1} - \frac{1}{(-3)-1} = \frac{1}{2}$$
$$\frac{2}{8} + \frac{1}{4} = \frac{1}{2}$$
$$\frac{1}{2} = \frac{1}{2} \quad \checkmark$$

When $x = 1$

$$\frac{2}{(1)^2-1} - \frac{1}{(1)-1} = \frac{1}{2}$$
$$\frac{2}{0} - \frac{1}{0} = \frac{1}{2}$$
$$0 = \frac{1}{2} \quad X$$

In this case, $x = -3$ is the only solution. That's why it is always important to check all solutions in the original equations. You may find that they yield untrue statements or produce undefined expressions.

Rational inequality is an inequality which contains one or more rational expressions. It can be used in engineering and production quality assurance as well as in businesses to control inventory, plan production lines, produce pricing models, and for shipping/warehousing goods and materials.

Solving an inequality is much like solving a rational equation except that there are additional steps that focus on illustrating the solution set of an inequality on a number line.

Let us use problem number 2 in the previous activity to discuss and deepen your knowledge and skills in solving rational inequality.

$$\frac{3x+1}{x-1} \geq 2$$

Step 1. Put the rational inequality in the general form where > can be replaced by $<, \leq and \geq$.

$$\frac{R(x)}{Q(x)} > 0$$

$$\frac{3x+1}{x-1} - 2 \geq 0$$

Step 2. Write the inequality into a single rational expression on the left-hand side.

$$\frac{3x+1-2(x-1)}{x-1} \geq 0$$

$$\frac{x+3}{x-1} \geq 0$$

Note: Remember that one side must always be zero and the other side is always a single fraction, so simplify the fractions if there is more than one fraction.

Step 3. Set the numerator and denominator equal to zero and solve. The values you get are called **critical values**.

Numerator: $x + 3 = 0$

$x = -3$

Denominator: $x - 1 = 0$

$x = 1$

Step 4. Plot the critical values on a number line, breaking the number line into intervals.

Step 5. Substitute critical values to the inequality to determine if the endpoints of the intervals in the solution should be included or not.

$$\frac{3x+1}{x-1} \geq 2$$

when $x = -3$

$$\frac{3(-3)+1}{(-3)-1} \geq 2$$

$$\frac{-8}{-4} \geq 2$$

$2 \geq 2$ ✓ ($x = -3$ is included in the solution)

when $x = 1$

$$\frac{3(1)+1}{(1)-1} \geq 2$$

$$\frac{4}{0} \geq 2$$

$undefined \geq 2$ X ($x = 1$ is not included in the solution)

See the illustration below.

Step 6. Select test values in each interval and substitute those values into the inequality.

$$\frac{3x+1}{x-1} \geq 2$$

when $x = -5$

$$\frac{3(-5)+1}{(-5)-1} \geq 2$$

$$\frac{-14}{-6} \geq 2$$

$$\frac{7}{3} \; or \; 2.33 \; \geq 2 \; (x = -5 \text{ TRUE})$$

when $x = -1$

$$\frac{3(-1)+1}{(-1)-1} \geq 2$$

$$\frac{-2}{-2} \geq 2$$

$$1 \; \geq 2 \quad (x = -1 \text{ FALSE})$$

when $x = 3$

$$\frac{3(3)+1}{(3)-1} \geq 2$$

$$\frac{10}{2} \geq 2$$

$$5 \; \geq 2 \quad (x = 5 \text{ TRUE})$$

Note:

a. If the test value makes the inequality **TRUE**, then the entire interval is a **solution** to the inequality.

b. If the test value makes the inequality **FALSE**, then the entire interval is **not a solution** to the inequality.

Step 7. Use interval notation to write the final answer.

$$\boxed{(-\infty, -3] \cup (1, \infty)}$$

Let's learn more!

Solve each rational equation and inequality.

1. $\dfrac{4x+1}{x+1} - 3 = \dfrac{12}{x^2-1}$

2. $\dfrac{2x-8}{x-2} \geq 0$

Solution:

Rational Equation	$\dfrac{4x+1}{x+1} - 3 = \dfrac{12}{x^2-1}$
1. Find the Least Common Denominator (LCD).	LCD: $(x+1)(x-1)$
2. Multiply both sides of the equation by its the LCD.	$(x+1)(x-1)[\dfrac{4x+1}{x+1} - 3 = \dfrac{12}{x^2-1}]$
3. Apply the Distributive Property and then simplify.	$(x-1)(4x+1) - 3(x+1)(x-1) = 12$ simplify: $4x^2 - 3x - 1 - 3x^2 + 3 = 12$ $x^2 - 3x + 2 = 12$ $x^2 - 3x - 10 = 0$ Factor $(x-5)(x+2) = 0$
4. Find all the possible values of x.	$x - 5 = 0$ $\quad x = 5$ $x + 2 = 0$ $\quad x = -2$
5. Check each value by substituting into original equation and reject any extraneous root/s	Checking: $\dfrac{4x+1}{x+1} - 3 = \dfrac{12}{x^2-1}$ when $x = 5$ $\dfrac{4(5)+1}{5+1} - 3 = \dfrac{12}{5^2-1}$ $\dfrac{21}{6} - 3 = \dfrac{12}{24}$

	$\frac{3}{6} = \frac{12}{24}$ $\frac{1}{2} = \frac{1}{2}$ ✓ when $x = -2$ $\frac{4(-2)+1}{(-2)+1} - 3 = \frac{12}{(-2)^2 - 1}$ $\frac{-7}{-1} - 3 = \frac{12}{3}$ $4 = 4$ ✓ Note: No extraneous root

Rational Inequality	$\frac{2x-8}{x-2} \geq 0$
1. Put the rational inequality in general form. $\frac{R(x)}{Q(x)} > 0$ where > can be replaced by $<, \leq$ and $\geq$	This inequality is already in general form. We are all set to go.
2. Write the inequality into a single rational expression on the left side. (You can refer to the review section for solving unlike denominators)	This inequality is already in a single rational expression wherein 0 is on one side. $\frac{2x-8}{x-2} \geq 0$
3. Set the numerator and denominator equal to zero and solve. The values you get are called critical values.	Numerator: $2x - 8 = 0$ $2x = 8$ $x = 4$ Denominator: $x - 2 = 0$ $x = 2$
4. Plot the critical values on a number line, breaking the number line into intervals.	
5. Substitute critical values to the inequality to determine if the endpoints of the intervals in the solution should be included or not.	$\frac{2x-8}{x-2} \geq 0$ when $x = 2$ $\frac{2(2)-8}{2-2} \geq 0$

	$\frac{-4}{0} \geq 0$ $undefined \geq 0 \quad X$ ($x = 2$ is not included in the solution) when $x = 4$ $\frac{2(4)-8}{4-2} \geq 0$ $\frac{0}{2} \geq 0$ $0 \geq 0$ ✓ ($x = 4$ is included in the solution)
5. Select test values in each interval and substitute those values into the inequality. Note: If the test value makes the inequality true, then the entire interval is a solution to the inequality. If the test value makes the inequality false, then the entire interval is not a solution to the inequality.	 $\frac{2x-8}{x-2} \geq 0$ when $x = 1$ $\frac{2(1)-8}{1-2} \geq 0$ $\frac{-6}{-1} \geq 0$ $6 \geq 0$ ✓ $true$ when $x = 3$ $\frac{2(3)-8}{3-2} \geq 0$ $\frac{-2}{1} \geq 0$ $-2 \geq 0 \quad X \quad false$ when $x = 5$ $\frac{2(5)-8}{5-2} \geq 0$ $\frac{2}{3} \geq 0$ ✓ $true$
6. Use interval notation or set notation to write the final answer.	$(-\infty, 2) \cup [4, \infty)$

What's More

Activity 1.

Solve the following rational equations and inequalities using the guided procedure on the table below.

1.

Rational Equation	$\frac{x-2}{x^2-4}+\frac{1}{x+2}=\frac{1}{x-2}$
1. Find the Least Common Denominator (LCD).	
2. Multiply both sides of the equation by its the LCD.	
3. Apply the Distributive Property and then simplify.	
4. Find all the possible values of x.	$x=6$
5. Check each value by substituting into the original equation and reject any extraneous root/s	

2.

Rational Equation	$\frac{x-6}{x^2-4x-12}+\frac{2}{x+2}=\frac{1}{x-6}$
1. Find the Least Common Denominator (LCD).	
2. Multiply both sides of the equation by its the LCD.	
3. Apply the Distributive Property and then simplify.	

4. Find all the possible values of x.	$x = 10$
5. Check each value by substituting into original equation and reject any extraneous root/s	

3.

Rational Inequality	$\frac{2(x-4)}{x} < -4$
1. Put the rational inequality in general form. $\frac{R(x)}{Q(x)} > 0$ where > can be replaced by $<, \leq$ and $\geq$	
2. Write the inequality into a single rational expression on the left side. (You can refer to the review section for solving unlike denominators)	
3. Set the numerator and denominator equal to zero and solve. The values you get are called critical values.	
4. Plot the critical values on a number line, breaking the number line into intervals.	
5. Substitute critical values to the inequality to determine if the endpoints of the intervals in the solution should be included or not.	
5. Select test values in each interval and substitute those values into the inequality. Note: If the test value makes the inequality true, then the entire interval is a solution to the inequality. If the test value makes the inequality false, then the entire interval is not a solution to the inequality.	
6. Use interval notation or set notation to write the final answer.	$(0, \frac{3}{4})$

4.

Rational Inequality	$\frac{x^2+x-6}{x^2-3x-4} \leq 0$
1. Put the rational inequality in general form. $\frac{R(x)}{Q(x)} > 0$ where > can be replaced by $<, \leq$ and $\geq$	
2. Write the inequality into a single rational expression on the left side. (You can refer to the review section for solving unlike denominators)	
3. Set the numerator and denominator equal to zero and solve. The values you get are called critical values.	
4. Plot the critical values on a number line, breaking the number line into intervals.	
5. Substitute critical values to the inequality to determine if the endpoints of the intervals in the solution should be included or not.	
5. Select test values in each interval and substitute those values into the inequality. Note: If the test value makes the inequality true, then the entire interval is a solution to the inequality. If the test value makes the inequality false, then the entire interval is not a solution to the inequality.	
6. Use interval notation or set notation to write the final answer.	$[-3,-1) \cup [2,4)$

Activity 2

Solve each problem below and choose the letter that corresponds to the solution to each problem. Place the correct answer in the corresponding lines.

What did the bible verses 1John 4:7-21 is all about?

_____	_____	_____	_____	_____	_____	_____	_____	_____
1	2	3	4	5	6	7	8	9

1. $\frac{x+2}{3} = \frac{2x-4}{2}$
2. $\frac{7}{4x} - \frac{3}{x^2} = \frac{1}{2x^2}$
3. $\frac{2x}{x+1} + \frac{5}{2x} = 2$
4. $\frac{x^2-1}{x-3} = \frac{8}{x-3}$
5. $\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2-8x+12}$
6. $\frac{5x}{x-1} < 4$
7. $\frac{x}{x-2} - 7 = \frac{2}{x-2}$
8. $\frac{x^2+x-12}{x-1} \leq 0$
9. $\frac{3x+1}{x-2} \geq 5$

A. -3
C. -1 and 6
D. -5
E. $(2, \frac{11}{2}]$
G. 4
I. 3
L. (-4, 1)
N. -3 and 3
O. 2
S. -1
V. $(-\infty, -4] \cup (1, 3]$
Y. $(-\infty, -4) \cup [1, 3)$

What I Have Learned

Complete the following statements by writing the correct word or words and formulas.

1. A _____ is an equation containing at least one rational expression with a polynomial in the numerator and denominator. 2. To determine if the endpoints of the intervals in the solution should be included or not you need to _____ the critical values to the inequality.
3. In order to get the critical values you need to set _____ and _____ equal to zero.
4. The first step in solving rational inequality is to put the inequality in general form where in one side must always be _____ and the other side is in a _____ fraction.

5. If the test value makes the inequality _____, then the entire interval is a solution to the inequality.

Topics for Discussion

1. Explain the similarities and differences in solving between rational equations and inequalities?

2. Why do extraneous solutions sometimes occur and don't work in the original form of the equation?

What I Can Do

Read the situation carefully and solve the problem.

The new COVID-19 testing facility in Lucena City is operating with two laboratory technicians. Technician A takes 2 hours to finish 50 samples of specimens from CoVID-19 patients. Technician B takes 3 hours to finish 45 samples of specimens from COVID-19 patients. Working together, how long should it take them to finish 150 samples of specimens from COVID-19 patients?

Hint:
Think about how many samples of specimens each technician can finish in one hour. This is their testing rate.

Assessment

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. It is an equation containing at least one fraction whose numerator and denominator are polynomials.
   a. rational function
   b. rational equation
   c. rational inequality
   d. irrational equation

2. The usual technique to eliminate denominator in solving a rational equation is to multiply both sides of the equation by its
   a. inverse factor
   b. greatest common factor
   c. least common denominator
   d. greatest common denominator

3. An inequality which involves one or more rational expressions is called
   a. rational function
   b. rational equation
   c. rational inequality
   d. irrational equation

4. You can only use cross multiplication in solving rational equation if and only if you have one fraction equal to one fraction, that is, if the fractions are _____.
   a. negative
   b. positive
   c. inequal
   d. proportional

5. If the test value makes the inequality _____, then the entire interval is not a solution to the inequality.
   a. true
   b. false
   c. proportional
   d. reciprocal

For items 6-7: Refer to the rational equation below.

$$\frac{5}{2x-4}+\frac{2}{x+3}=\frac{3}{x-2}$$

6. To solve the equation, we multiply both sides by
   a. $x-2$
   b. $x+3$
   c. $(x+2)(x-3)$
   d. $(x-2)(x+3)$

7. Which of the following will be the solution to the given rational equation?
   a. $\frac{11}{3}$
   b. $\frac{3}{11}$
   c. $-\frac{11}{3}$
   d. $-\frac{3}{11}$

For items 8-10: Refer to the rational inequality below.

$$\frac{x+12}{x+2}\leq 2$$

8. What are the critical values in the given rational inequality?
   a. $x = -2 \; and \; 8$
   b. $x = -2 \; and \; -8$
   c. $x = -2 \; and \; 12$
   d. $x = -2 \; and \; -12$

9. Which of the critical value or values is/are included as endpoints of the intervals?
   a. -2
   b. 2
   c. -8
   d. 8

10. Which of the following is the solution in the given inequality?
   a. $(-\infty, -2) \cup (8, \infty)$
   b. $(-\infty, -2] \cup [8, \infty)$
   c. $(-\infty, -2) \cup [8, \infty)$
   d. $(-\infty, -2) \cup [-8, \infty)$

For items 11-13, solve for the solutions of the following rational equations.

11. $\frac{2}{x+2} + \frac{1}{x-2} = \frac{3}{x}$
   a. 6
   b. -6
   c. 8
   d. -8

12. $\frac{8}{x^2} + 1 = \frac{9}{x}$
   a. $-1 \; and - 8$
   b. $1 \; and \; 8$
   c. $-1 \; and \; 8$
   d. $1 \; and - 8$

13. $\frac{1}{x^2} - 16 = 0$
   a. ± 1
   b. ± 2
   c. $\pm \frac{1}{2}$
   d. $\pm \frac{1}{4}$

For items 14-15, solve for the solutions of the following rational equations.

14. $\frac{5}{x-3} > \frac{3}{x+1}$

a. $(-\infty, -7) \cup (-1, \infty)$
b. $(-\infty, -1) \cup (3, \infty)$
c. $(-7, -1) \cup (3, \infty)$
d. $(-7, -1] \cup [3, \infty)$

15. $\frac{(x-3)(x+2)}{x-1} \leq 0$

a. $(-\infty, -2) \cup (1,3]$
b. $(-\infty, -2] \cup (1,3]$
c. $(-\infty, -2] \cup [1,3)$
d. $(-\infty, -2) \cup [1,3)$

Additional Activities

Practice Worksheet: Solving Rational Equations and Inequalities

Solve each equation. Check extraneous solutions for rational equations. Write your answer in interval notation for rational inequalities.

LEVEL 1	
1. $\frac{8}{x+1} = \frac{4}{3}$	2. $2x + 3 = \frac{x}{4}$
3. $\frac{x-4}{x+5} \leq 0$	4. $\frac{x+3}{3x-6} > 0$

5. $$\frac{4}{x}+\frac{1}{3x}=9$$	6. $$\frac{1}{x^2-4}\leq 0$$

LEVEL 2	
7. $$\frac{20}{x}-\frac{20}{x-2}=\frac{4}{x}$$	8. $$\frac{2}{x^2-x}=\frac{1}{x-1}$$
9. $$\frac{x-9}{3x+2}\geq 3$$	10. $$\frac{x+32}{x+6}\leq 6$$
11. $$\frac{4}{x}+\frac{1}{x^2}=\frac{1}{5x^2}$$	12. $$1+\frac{2}{x+1}<\frac{2}{x}$$

LEVEL 3	
13. $$\frac{3x}{x+1}=\frac{12}{x^2-1}+2$$	14. $$\frac{2}{4x^2-9}+\frac{1}{2x-3}=\frac{3}{2x+3}$$
15. $$\frac{(x^2+1)(x-2)}{(x-1)(x+1)}\geq 0$$	16. $$\frac{(x+7)(x-3)}{(x-5)^2}>0$$
17. $$\frac{x-3}{2x+10}+2x-12=\frac{x^2+3x-18}{2x+10}$$	18. $$\frac{12x^3+16x^2-3x-4}{8x^3+12x^2+10x+15}<0$$

Answer Key

What I Know

1. B
2. A
3. C
4. B
5. D
6. A
7. A
8. B
9. D
10. C
11. B
12. A
13. B
14. D
15. C

What's More

1. G
2. O
3. D
4. I
5. S
6. L
7. O
8. V
9. E

What I Have Learned

1. Extraneous solutions
2. Substitute
3. Numerator, denominator
4. zero, single
5. true

Assessment

1. B
2. C
3. C
4. D
5. B
6. D
7. A
8. A
9. D
10. C
11. A
12. B
13. D
14. C
15. A

What I Can Do

It will take 3 hours and 45 minutes for the two laboratory technicians to finish 150 samples of specimens.

Additional Activities

1. $x = 5$
2. $x = -\frac{12}{7}$
3. $(-5,4]$
4. $(-\infty,-3) \cup (3,\infty)$
5. $x = \frac{13}{27}$
6. $(-2,2)$
7. $x = -8$
8. $x = 2$
9. $\left[-\frac{13}{5}, -\frac{2}{3}\right)$
10. $(-\infty,-6) \cup [-\frac{4}{5},\infty)$
11. $x = -\frac{1}{5}$
12. $(-2,-1) \cup (0,1)$
13. $x = -2 \text{ and } 5$
14. $x = \frac{7}{2}$
15. $(-1,1) \cup [2,\infty)$
16. $(-\infty,-7) \cup (3,5) \cup (5,\infty)$
17. $x = 7$
18. $\left(-\frac{3}{2},-\frac{4}{3}\right) \cup (-\frac{1}{2},\frac{1}{2})$

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11 General Mathematics

General Mathematics
Representations of Rational Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Representations of Rational Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics 11_Alternative Delivery Mode (ADM) Module on Representations of Rational Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics 11_Alternative Delivery Mode (ADM) Module on Representations of Rational Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you represents rational function through table of values, graphs and equations. The scope of this module permits it to be used in many different learning situations. This module will guide you on how to see the essence of rational functions which we think has no value in real life, but in reality, it is in everything that we do.

In this module, you will learn to represent a rational function in three different ways. It is important that you apply the skills you have learned on how to represent a function in the previous module. Good luck!

After going through this module, you are expected to:

Represents rational function using:

a. table of values
b. graphs
c. equations

What I Know

Before studying this module, let us assess on what you already know about this topic.

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which family of function does the graph on the right belongs to?

a. Trigonometric
b. Logarithmic
c. Exponential
d. Rational

2. Complete the table using the equation f(x) = $\frac{x-1}{x+1}$

x	1	2	3	4
f*(x)*	0	$\frac{1}{3}$	$\frac{1}{2}$	?

a. $\frac{3}{5}$
b. $\frac{5}{3}$
c. 3
d. 5

3. Which of the following table represents the function $y = \frac{2x+1}{x}$?

a.

x	1	2	3	4
y	-1	-2	-3	-4

b.

x	1	2	3	4
y	1	2	3	4

c.

x	1	2	3	4
y	3	$\frac{5}{2}$	$\frac{7}{3}$	$\frac{9}{4}$

d.

x	1	2	3	4
y	-3	$-\frac{5}{2}$	$-\frac{7}{3}$	$-\frac{9}{4}$

4. From the question in number 3, which represents the graph of the function $f(x) = \frac{2x+1}{x}$?

a.

c.

b.

d.

5. Rational expression can be described as a function where either the numerator, denominator or both have variable on it. Below are examples of rational expression EXCEPT

a. $\frac{3x^3+3x+2}{2x+4}$

b. $\frac{x^2+4x-2}{4}$

c. $\frac{\frac{2}{3x}}{x-3}$

d. $\frac{\sqrt{x+1}}{3x}$

5. Reynolds manufactures aluminum cans in the shape of a cylinder with a capacity of 500 cubic centimeters ($\frac{1}{2}$liters). The top and bottom of the can are made with special aluminum alloy that costs 0.05 cents per square centimeter. The sides of the can are made of material that cost 0.02 cents per square centimeter. Express the cost of the material for the can as a function of the radius r of the can.

a. $0.10\pi r^2+0.04\pi r^2$

b. $0.10\pi r^2-0.04\pi r^2$

c. $0.10\pi r+0.04\pi$r

d. $0.10\pi r-0.04\pi$r

6. Which of the following the graph of $f(x) = \frac{x}{x-2}$ using the values x= -2, -1, 0, 1, 2?

a.

c.

b.

d.

7. Using the rational function $g(x) = \frac{x^2-1}{x+1}$, what is the value of g(x) when x=4? (simplify the function first)

a. 3
b. -3
c. 5
d. -5

For numbers 8-10, consider this situation

A cylindrical soft drink can is to be constructed so that it will have a volume of 21.6 cubic inches.

8. Write the total surface area A of the can as a function of r, where r is the radius of the can in inches.

a. $A = \frac{r+2}{r+3}$
b. $A = \frac{r-2}{r+3}$
c. $A = \frac{r+2}{r-3}$
d. $A = \frac{r-2}{r-3}$

9. What should be the value of the function when values of x approaches to 3?

a. 5
b. $\frac{5}{6}$
c. 6
d. $\frac{6}{5}$

10. What is the graph of the function using x= 1, 2, 3, 4?

a.

c.

b.

d.

For numbers 11-13, refer to the problem below.

Lina is doing mathematics tutorial for a summer job. For every tutorial, she charges an initial fee of ₱500.00 per month, plus a constant fee of ₱200.00 for each hour of tutorial.

11. Which of the following equation best describes Lina's fee for each of her tutorials?

a. $fee = 500 + 200x$
b. $fee = 500x + 200$
c. $fee = 500 - 200x$
d. $fee = 500x - 200$

12. Which of the following table best represents Lina's fee for each of her tutorials?

a.

No of hours	1	2	3	4
Fee	200	400	600	800

b.

No of hours	1	2	3	4
Fee	700	1200	1700	2200

c.

No of hours	1	2	3	4
Fee	700	1400	2100	2800

d.

No of hours	1	2	3	4
Fee	700	900	1100	1300

13. If Lina spends 15hours on a student and another 13 hours for another student in a month, how much will she earn? Write an equation that will suit best the situation

a. $fee = [500 - 200(15)] + [500 - 200(13)]$
b. $fee = [500 + 200(15)] + [500 + 200(13)]$
c. $fee = [500(15) + 200] + [500(13) + 200]$
d. $fee = [500(15) - 200] + [500(13) - 200]$

For numbers 14 and 15, refer to the problem below.

There are 1,200 freshmen and 1,500 sophomores at SSG Election Meeting de Avance at noon. After 12 p.m., 20 freshmen arrive at the gymnasium every five minutes while 15 sophomores leave the gymnasium.

14. Which equation best describes the total number of students who attended the SSG Election Meeting de Avance?

a. $y = [1200x + 20] + [1500x - 15]$
b. $y = [1200 + 20x] + [1500 - 15x]$
c. $y = [1200x - 20] + [1500x + 15]$
d. $y = [1200 - 20x] + [1500 + 15x]$

15. Simplifying the answer in number 14, we can get

a. $y = 2700x + 5$
b. $y = 2700 + 5x$
c. $y = 2700x - 5$
d. $y = 2700 - 5x$

Lesson 1 Representations of Rational Functions

This lesson is about representations of rational function in different ways. We will deal with the application of rational functions that may involve the number of persons who can do a task in a certain amount of time. We can handle these applications involving work in a manner similar to the method we used to solve distance, speed, and time problems.

What's In

Let us recall on how to represent a polynomial function problem through table, graphs, and equation.

Example: A siomai vendor can wrap 4 dumplings every minute. If he wraps a total of 32 dumplings, how much time did he spent wrapping? Use the following to justify your answer:

a. table of Values
b. graph
c. equation

Solution:

For A, we will complete the table using the given information provided in the problem. Since a vendor can wrap 4 dumplings per minute, we will add 4 dumplings for every minute until we reach 32 dumplings.

Number of dumplings wrapped	4	8	12	16	20	24	28	32
Minutes	1	2	3	4	5	6	7	8

Using this table, we can say that the vendor was able to wrap 32 dumplings in 8 minutes.

For B, let us use the data we have in A and plot these points in a Cartesian plane.

For C, we can solve the problem by formulating a formula where we divide the total number of dumplings made by the number of dumplings per minute. In symbol,

$$T = \frac{D}{N}$$

Where;

T= Time in Minutes

D= Total number of Dumplings made

N= Number of Dumplings made per minute

Notes to the Teacher

Tell the students to use the proper scaling and labeling. It is important that the graphical representations are neatly made especially if not using a graphing paper.

Teacher can also introduce using tools such as GeoGebra, Desmos, and other applications.

What's New

Life is a Beach!

Pueblo por la Playa is a 12.5 hectare Mexican-inspired exclusive leisure club nestled off the calm, clear waters of Pagbilao Quezon. The "Pueblo" offers the total leisure and recreation experience for the entire family. Since it is an exclusive resort, it has a membership fee. Pueblo Por La Playa charges a ₱300,000.00 annual fee, then ₱700.00 for each day you stay there. Find the average cost per day to stay in the resort in 5, 10, 15 and up to 30 days. Graph the function to show whether it forms a straight line or a curve.

a. Define a formula for the average cost for every 5 days to stay in the resort f(x). Hint: Since the problem ask for the average cost, use the formula in getting an average

b. Based from the situation above, complete the following table to show the average cost every 5 days.

X	0	5	10	15	20	25	30
Y	0						

Hint: Substitute the value of x in your equation

c. Plot the following points on the cartesian plane

To graph, simply plot the points and connect it by a smooth curve line.

What is It

The problem presented above is an example of Rational Function. To solve the problem, let us answer each question one by one. Below is the definition of a Rational Function.

> Definition
>
> Rational function is written in the form of $f(x) = \frac{p(x)}{q(x)}$. It should follow the following conditions; namely:
>
> 1. Both p(x) and q(x) are polynomial functions wherein it has no negative and fractional exponents.
> 2. The denominator or q(x) should not be equal to 0.
> 3. The domain of all values of x where q(x) ≠ 0.

a. Define a formula for the average cost for every 5 days to stay in the resort f(x). To define the formula, use the formula in getting the average cost.

Let the function be f*(x)*. We can use the formula of getting an average. Average problems use the formula $A = \frac{X}{N}$, where A= Average, X= cost, and s= number of days

Let f(x) represents the average cost per day and x represent the number in days. Note that ₱300,000.00 is a fixed price you need to pay plus the ₱700.00 per day divided by the number of days (*x)*. We will have,

$$f(x) = \frac{300000 + 700(x)}{x}$$

Observe that it is similar to the structure of our original formula. Note that you will be using a formula depending on the classification of problems given to you.

b. For every 5 day stay in the resort, create a table of values showing the average cost.

Solution: Make a table of values with *x*-values at 0, 5, 10, 15, 20, 25, 30.

X	0	5	10	15	20	25	30
Y	0	20,965	41,930	62,895	83,860	104,825	125,790

From the table, we can observe that the average cost of stay decreases as the time increases. We can use a graph to determine if the points of this function follow a curve or a line

c. Graph the following points in the Cartesian plane.

.

By connecting the lines, we can clearly see that it follows a curve, thus a Rational Function.

Example 2:

$$f(x) = \frac{x}{x+2}$$

a. Since we already have an equation, we can skip this part. Proceed with the table of values

b. Construct table of values from -2 to 2. We can substitute each values on the equation to complete the table. We will get,

X	-2	-1	0	1	2
f(x)	Und	-1	0	0.33	0.5

We can observe that the value of f(x) is undefined in when x= -2. It is because when you substitute -2 in the function it will have an answer of zero whereas in the definition of rational function, we cannot have a denominator equal to zero.

c. Plot the points in the Cartesian plane and determine whether the points form a smooth curve or a straight line.

It can be observed that the function formed a curve.

What's More

In the following activities, read each situation carefully to solve each problem. Write your answer on a separate sheet of paper.

Practice Activity 1

Represent this rational equation through table and graph. Identify whether the graph forms a straight line or a curve.

$$f(x) = \frac{x+3}{x+1}$$

a. Since we already have an equation, we can skip this part. Proceed with letter b. Note that when given with a word problem, you cannot skip this part.

b. Construct table of values from -2 to 2 .
Hint: Substitute the value of x to obtain f(x).

x	-2	-1	0	1	2
f(x)	-1	undefined	3	2	$\frac{5}{3}$

The function is undefined when x= -1 since it makes the denominator zero.

c. Plot the points in the Cartesian plane and determine whether the points form a smooth curve or a straight line.

By plotting the points obtained in B we can get,

Independent Assessment 1

In order to join a voice lesson class, you pay a ₱1,500.00 pesos fee, then ₱500.00 for each class you go to. What is the average cost per class? Graph the function to show whether it forms a straight line or a curve.

Independent Assessment 2

Represent this rational equation through table and graph. Identify whether the graph forms a straight line or a curve.

$$\boldsymbol{P(x) = \frac{x^2 + 1}{x + 1}}$$

What I Have Learned

In your own words, how do you represent rational function using

a. Equation

_____

_____

_____

b. Graph

_____

_____

_____

c. Table of values

_____

_____

_____

What I Can Do

Read and analyze the situation below then answer the question given.

Rational function is one of the functions that is underappreciated because they say we cannot use it in real life. No. Rational function can be seen in most of our daily activities, we just didn't know it.

Basketball League

In an inter-barangay basketball league, the team from Hermana Fausta has won 12 out of 25 games, a winning percentage of 48%. We have seen that they need to win 8 games consecutively to raise their percentage to 60%. What will their winning percentage if they win 10, 20, 30, 50, 100 games? Can they reach a 100% winning percentage? (hint: try to substitute 300 games). Write your interpretation on the space provided.

_____

_____

_____

_____

_____

Here are the steps to solve the problem and the rubric that will guide you in giving the correct representations to the problem.

Steps in Problem Solving	Possible Highest Points	Your Score
1. Give the Appropriate model or equation	3 points	
2. Create a table of Values	3 points	
3. Graph	4 points	
4. Essay	5 points	
Total	15 points	

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. It is in the form of f(x) = $\frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomial functions and q(x) is not equal to zero.
   a. Rational Equality
   b. Rational Inequality
   c. Rational Function
   d. None of these

For questions 2 and 3, refer to this situation.

Martha has won 19 out of 28 tennis matches this season.

2. Which equation models suggest how many more games she needs to win to average 75% wins over loses?
   a. $\frac{19}{28+x} = 0.75$
   b. $\frac{19+x}{28+x} = 0.75$
   c. $\frac{19}{28+x} = 75$
   d. $\frac{19+x}{18+x} = 75$

3. In order to get a college tennis scholarship, Martha needs to bring her winning average to 80%. What is the number of matches she needs to win given that she already won 19 out of 28?
   a. 3
   b. 4
   c. 17
   d. 22

For questions 4-6, refer to this situation.
Joel is working on his chemistry project and he has 300mL of 12% acid solution.

4. If he needed to decrease the acidity of the solution, which of the following is correct function that would show the new acidity of the solution given x mL of water added?

a. $f(x) = \frac{36}{300+x}$
b. $f(x) = \frac{0.36}{300+x}$
c. $f(x) = \frac{12}{300+x}$
d. $f(x) = \frac{0.12}{300+x}$

5. If Joel decided to decrease the acidity of the solution by adding 15 more than at every interval, which table of values is correct?

a.

X	1	2	3	4
f(*x*) in %	11.43%	10.91%	10.43%	10%

b.

X	1	2	3	4
f(*x*) in %	0.1143%	0.1091%	0.1043%	0.10%

c.

X	1	2	3	4
f(*x*) in %	3.81%	3.64%	3.48%	3.33%

d.

X	1	2	3	4
f(*x*) in %	3.81%	3.64%	3.48%	3.33%

6. Which graph shows the decrease of acidity in Joel's solution?

a.

c.

b.

d.

7. Which of the following is the correct table of values of the rational function $f(x) = \frac{X}{X+1}$?

a.

x	-1	0	1
Y	Und	0	0.5

b.

X	-1	0	1
Y	0.5	und	0.5

c.

x	-1	0	1
y	-0.5	0	und

d.

x	-1	0	1
y	-0.5	Und	-0.5

8. When is the graph of the function undefined in a certain value of x?

a. When the value of the numerator is zero.

b. When the value of the denominator is zero.

c. When the value of the function is zero.

d. None of the above.

9. Which equations satisfies the table of values below?

X	-2	-1	0	1	2
Y	-1	Und	3	2	1.67

a. $y = \frac{x+1}{x+3}$

b. $y = \frac{x+3}{x+1}$

c. $y = \frac{x-3}{x+1}$

d. $y = \frac{x-1}{x+3}$

10. Which table of values satisfies the graph presented on the right side?

a.

X	-2	-1	0	1	2
Y	0	0.5	1	1.5	2

b.

X	-2	-1	0	1	2
Y	-2	-1	0	1	2

c.

X	-2	-1	0	1	2
Y	0	-0.5	-1	-1.5	-2

d.

X	-2	-1	0	1	2
Y	2	1	0	-1	-2

11. Using values from -10 to 10 with an interval of 5. Which of the following best describes the table of values of the function $g(x) = 2x^3 + 4x - 19$?

a.

X	-10	-5	0	5	10
g(x)	2021	251	-19	-289	-2059

b.

X	-10	-5	0	5	10
g(x)	-2059	-289	-19	251	2021

c.

X	-10	-5	0	5	10
g(x)	2059	289	19	-251	-2021

d.

X	-10	-5	0	5	10
g(x)	-2021	-251	19	289	2059

12. In a Bread and Pastry class, a certain recipe calls for 3 kgs of sugar for every 6 kgs of flour. If 60 kgs of this sweet has to be prepared, how much sugar is required? Which equation satisfies the problem?

a. $x = \frac{60+3}{6(3)}$

b. $x = \frac{6(3)}{60+3}$

c. $x = \frac{60(3)}{6+3}$

d. $x = \frac{6+3}{60(3)}$

13. How many kilograms of sugar is needed for 90 kilograms of sweets?

a. 20
b. 25
c. 30
d. 35

For questions number 14 and 15, refer to the problem below.
In a business math class, the Teacher Alex assigned his students a business project. For the business to be established, a certain establishment needs to pay for a semestral fee (5 months) of ₱50.00 pesos and a weekly tax of ₱10.00 which the proceeds will go to their Christmas Party expenses.

14. What is the average amount collected per group in his class? Formulate an equation for this.

a. $f(x) = \frac{50-10x}{x}$

b $f(x) = \frac{x}{50+10x}$

c. $f(x) = \frac{50+10x}{x}$

d. $f(x) = \frac{x}{50-10x}$

15. How much will be collected in each group for a period of 13weeks?
   a. ₱170.00
   b. ₱180.00
   c. ₱190.00
   d. ₱200.00

Additional Activities

Do the following to enhance your learning.

1. Construct a table of values of the following functions using the interval of -5 to 5.

   a. $g(x) = \frac{x^3+3x-5}{x^2}$ b. $j(t) = \frac{1}{t^2-2t+1}$

2. Using the data from the table of values, plot the points on the cartesian plane and connect the points of

   a. g*(x)* b. j*(t)*

An application of rational functions may involve the number of persons who can do a task in a certain amount of time. We can handle these applications involving work in a manner similar to the method we used to solve distance, speed, and time problems. Work = Rate x Time. Suppose you can finish a report in 2 hours. Your classmate can finish the same report in 4 hours. How long will it take to finish the report if both of you work together? We have a saying that "Two heads are better than one", would you rather work alone or with a team? Why?

_____

_____

_____

Answer Key

What I Know

1. D
2. A
3. C
4. B
5. D
6. C
7. B
8. A
9. B
10. B
11. A
12. D
13. C
14. B
15. B

What's More

Independent Activity 1

A. $f(x) = \frac{1500+500x}{x}$

B.

x	1	2	3	4	5
f(x)	2000	1250	1000	875	800

C.

Independent Activity 2

A.

X	-2	-1	0	1	2
f(x)	-6	Und	1	1	1.67

B.

Assessment

1. C
2. B
3. C
4. A
5. A
6. C
7. A
8. B
9. B
10. A
11. B
12. C
13. C
14. D
15. B

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Graphing tools

https://www.geogebra.org/graphing?lang=en

https://www.desmos.com/calculator

For inquiries or feedback, please write or call:

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11 General Mathematics

General Mathematics
The Domain and Range of a Rational Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

The Domain and Range of a Rational Functions

Introductory Message

For the facilitator:

Welcome to General Mathematics Alternative Delivery Mode (ADM) Module on The Domain and Range of a Rational Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to General Mathematics Alternative Delivery Mode (ADM) Module on The Domain and Range of a Rational Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you master the domain and range of a rational function. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

1. define domain and range;
2. find the domain and range of a rational function

What I Know

Direction: Read and analyze each item carefully. Encircle the letter that corresponds to your answer for each statement.

1. In the coordinate system, the x-axis is called _____.

   a. abscissa c. quadrant

   b. ordinate d. slope

2. In the coordinate system, the y-axis is called _____.

   a. abscissa c. quadrant

   b. ordinate d. slope

3. In the linear form $y = 2x + 3$, which is the independent variable?

   a. b c. x

   b. m d. y

4. In the linear form $y = 3x - 4$, which is the dependent variable?

   a. b c. x

   b. m d. y

5. In writing sets, the format $\{x \in R \mid x \neq 2\}$ is called a _____

   a. enumeration c. set-builder notation

   b. roster form d. interval notation

6. In a set of ordered pairs (1,2), (2,3), (3,4), (4,5), (5,6), the domain D = _____

a. {1,2,3,4,5} c. {6,5,4,3,2}

b. {2,3,4,5,6} d. {1,2,3,4,5,6}

7. In a set of ordered pairs (1,3), (2,5), (3,7), (4,9), (5,11), the range R = _____

a. {1,2,3,4,5} c. {5,4,3,2,1}

b. {3,5,7,9,11} d. {1,2,3,4,5,7,9,11}

8. Some values for the Domain of the rational function f(x) = $3x^2 - 5$ are 1, 2, 3, 4, and 5. Find the Range corresponding to each value. _____

a. {-2,7,22,43,70} c. {5,4,3,2,1}

b. {-5,-2,7,22,43,70} d. {1,2,3,4,5}

9. Some values for the Range of the rational function f(x) = $3x^2 - 5$ are 22, 70, and 295. Find the Domain corresponding to each value. _____

a. {-5,-3,3,5,10} c. {3,5,10}

b. {-3,-5,-10} d. {±3, ±5, ±10}

10. Find the domain and range of the linear function f(x) = $2x + 4$.D _____ R _____

a. D {x ϵ R| x ≠ -4};R {y ϵ R| y ≠ 4}
b. D {x|x ϵ R};R {y|y ϵ R}
c. D {x ϵ R| x ≠ 0};R {y ϵ R| y ≠ 0}
d. D {x|x ϵ N};R {y|y ϵ N}

11. Find the domain and range of the quadratic function f (x) = $x^2 + 3x + 4$. D _____ R _____

a. D {x ϵ R| x ≠ 4};R {y ϵ R| y ≠ -4}
b. D {x|x ϵ N};R {y|y ϵ N}
c. D {x ϵ R| x ≠ -3};R {y ϵ R| y ≠ 3}
d. D {x|x ϵ R};R {y|y ϵ R}

12. Find the domain and range of the rational function f(x) = $\frac{22+x}{25+x}$ D _____R _____

a. D(-∞, -22) U (-22, ∞);R(-∞, 1) U (1, ∞).
b. D(-∞, -25) U (-25, ∞);R(-∞, -25) U (-25, ∞).
c. D(-∞, -25) U (-25, ∞);R(-∞, 1) U (1, ∞).
d. D(-∞, -22) U (-25, ∞);R(-∞, -25) U (-22, ∞).

13. Find the domain and range of the rational function f(x) = $\frac{x^2-x-6}{x+2}$ D_____R _____

a. D(-∞, -2) U (-2, ∞);R (-∞, -2) U (-2, ∞).
b. D(-∞, -2) U (-2, ∞);R (-∞, -3) U (-3, ∞).
c. D(-∞, -2) U (-2, ∞);R (-∞, 3) U (3, ∞).
d. D(-∞, -2) U (-2, ∞);R (-∞, -5) U (-5, ∞).

14. Find the domain of the rational function f(x) = $\frac{3x^2-8x-3}{2x^2+7x-4}$ D _____

a. D(-∞, -4) U (-4, 2) U (2, ∞).

b. D(-∞, -4) U (-4, 1/2) U (1/2, ∞).

c. D(-∞, -1/2) U (-1/2, 4) U (4, ∞).

d. D(-∞, -4) U (-4, ∞)

15. A cellphone provider offers a new phone for ₱7,000.00 plus ₱1,299.00 monthly plan. What would be the average cost after 12 months?

a. ₱1,799.00
b. ₱1,882.00
c. ₱7,799.00
d. ₱8,299.00

Lesson 1 The Domain and Range of a Rational Functions

Introduction

To be able to understand the domain and range of a rational function, let us see the real-life application of a rational function in this situation:

Average Grade Problem

Let's say you are taking an exam in your General Mathematics subject. You knew that you already have 22 correct answers out of 25 questions. Twenty-two out of 25 questions had already an 88% rating. Now you told yourself, "I must have a final grade of 90%." The question is how many more correct answers to additional questions do you need, to get your desired final grade of 90? How about if you desire a grade of 89 or 95?

In this module, you will know the answer to that real-life problem. But first, you will have to learn the definition of the domain and range of a rational function. Subsequently, along your study, you will learn how to find the domain and range and apply your knowledge of it in solving real-life problems involving a rational function.

What's In

In your previous lesson, you learned how to represent rational functions in the form of table of values, graphs, and equations.

As a review, ready yourself in doing this first drill.

Let us have an equation y = 3x + 2. When x = 6, you know that y = 20. Now, find the value of y on the following equations when x = -4.

a) 2y = 4x – 6

b) y = $\frac{7x+4}{3x}$

c) y = $\frac{x^2+x-4}{x+4}$

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

What's New

Now imagine that you are ninety-six kilometers (96 km.) away from Manila. You are planning to visit your grandmother who will be celebrating her birthday. Your father has allowed you to drive his car so that you will arrive at the party on time. Assuming there will be no traffic during that day, you have resolved to arrive at the party from 1 to 2 hours. So, at what speed are you going to travel to arrive in a certain period of time?

For your Activity 1, construct a table of values that would represent the given problem. Subsequently, plot the values obtained on a Cartesian plane. You may use paper and pencil or any applicable graphing apps such as MS Excel, GeoGebra, or Desmos. You may use the table below as your reference.

Table 1

t in hours	1	1.25	1.5	1.75	2
r (rate in km/hr)					

Questions to ponder:

1. How are you going to represent the problem through a function?

_____

2. What is/are the given in the problem? What are you going to solve?

_____

3. How can you describe the relationship between the rate r and the time t in the problem? Which do you think is the variable that depends on the value of another variable?

_____

4. What can you say about the graph? Can the values in t increase infinitely?

_____

5. What do you think would be the value of r when t is equal to zero?

_____

From the foregoing activity, you have constructed a function of your speed against your time, and represent a function with a table of values, a graph, and equation.

Congratulation!

What is It

Now, observe that there is a set of values that can be found in x (t, as used in the preceding problem) that corresponds to a unique value in f(x) or in the latter case r(t). In the graph, it can be seen that these x-values represent the points plotted along the x-axis called *abscissa*. On the other hand, those values in the y-axis are called *ordinates*. In a set of ordered pairs (1,6), (2,7), (3,8), (4,9), and (5,10), the points (1,2,3,4,5) are the abscissa since they are on the x-axis while points (6,7,8,9,10) are the ordinates since they are on the y-axis.

Corollary, the x-values are considered the *independent variable (input)* while the y-values are considered the *dependent variables (output)* in forms such as the linear form $y=mx + b$. This can be extended to quadratic, polynomial, and rational functions. In functions, the symbol f*(x)* is used instead of y. So, y = f*(x)* and can be read as "y equals f of x".

In our study of rational function these x-values represent the domain and the y-values represent the range of a rational function. In definition, the **domain** of a function is the set of all values that the variable x can take while the **range** of a function is the set of all values that y or f*(x)* can take. But how do we determine and write the domain and range of a rational function?

If you can recall, we can write the domain and range using different forms:

1. by *roster* format - this method enumerates the lists of all values in the set. Ex. The domain of r(t) are (1, 1.25, 1.5, 1.75, 2).
2. by *set-builder* form or notation - for example, in numbers 10 to 20. you can say {x | x are even numbers from 10 to 20). The | is read as "such that." Assuming that you also include odd numbers in the domain from 10 to 20, then, you can write the domain of the function D(x) as {x | x ϵ R, 10≤x≤20}, read as "x such that x is an element of a real number wherein x is greater than or equal to 10 but less than or equal to 20."
3. by *interval notation* – for example, in a function f*(x)* = $\frac{5}{x-3}$, the domain of this function can be written in the form, (-∞, 3) U (3, ∞). This means that the values of the domain can take all real values of x except 3, otherwise the function is undefined.

In the succeeding activities, you will learn how to find the domain and the range using different methods. But first let us have another activity that will facilitate the understanding of these methods.

Activity 2 – Mobile Plan

Glolibee Telecom would like to offer you the newest smartphone which has 50x zoom in its camera. It is the latest top-of-the-line product. In order to avail this, you only have to pay ₱12,000.00 down-payment while the rest may be paid ₱1.799.00 monthly for 24 months. So that would cost you ₱12000.00 + ₱1799.00(x) in 2 years. Think about it and answer the following questions.

Questions

1. If you are just a student would you ask your parents to buy you this top-of-the-line smartphone? Why or why not?

2. How much would be the total cost of buying this type of smartphone in 24-monthly installment?

3. How much do you think would be the average cost after you have already paid for 12 months? When will be the average monthly cost be less than ₱2,500.00?

Considering the foregoing questions, the answers to questions 2 and 3 above may be mathematically explained using the domain and range of a rational function. While your answer for question number 1, other than having your own personal conviction, may depend on your answers on questions 2 and 3. Why is that so?

First, let us find out the total cost f*(x)* of the new smartphone. This can be expressed in the linear function:

$$f(x) = 12000 + 1799(x),$$

where x is the independent variable (month) while the 12000 is the constant (down-payment). The f(x)=y is the dependent variable which in this case, it is the total cost. Assuming that we have to pay the smartphone in 24 months then, the total cost would be,

$$f(x) = 12000 + 1799(24)$$
$$=55176$$

So, you have to pay ₱55,176.00 in 2 years. This should be your answer in question number 2. Wow, that is a lot of money!

Now to find the average cost, we have to divide the total cost by the number of months you have used the phone service. Thus, we now have a rational function in the form:

$$f(x) = \frac{12000 + 1799(x)}{x}$$

Using a table of values, we can see the average cost in 12 months:

Table 2

Fixed amount	Monthly Payment	Month (x)	Average Cost (y)
₱12,000.00	₱1799.00	1	₱13,799.00
₱12,000.00	₱1799.00	2	₱7,799.00
₱12,000.00	₱1799.00	3	₱5,799.00
₱12,000.00	₱1799.00	4	₱4,799.00
₱12,000.00	₱1799.00	5	₱4,199.00
₱12,000.00	₱1799.00	6	₱3,799.00
₱12,000.00	₱1799.00	7	₱3,513.00
₱12,000.00	₱1799.00	8	₱3,299.00
₱12,000.00	₱1799.00	9	₱3,132.00
₱12,000.00	₱1799.00	10	₱2,999.00
₱12,000.00	₱1799.00	11	₱2,890.00
₱12,000.00	₱1799.00	12	₱2,799.00

In table 2, the x-values (month) is the domain of the function while the y-values (average cost) is the range of the function. The table shows that after 12 months your average cost is ₱2,799.00. But take note you still have 12 more months to pay. And when will be the average monthly cost be less than ₱2,500.00? To answer this question, we will predict situations from this rational function using the inequality:

$$2500 > \frac{12000 + 1799(x)}{x}$$

$$(2500)(x) > 12000 + 1799x$$

$$2500x - 1799x > 120000$$

$$701x > 12000$$

$$x > 17.12 \approx 17 \text{ month}$$

Therefore, starting on the 17th month you will be paying an average cost of less than ₱2,500.00. This real-life situation has shown you the applicability of the domain-range of a rational function which you may apply in your daily life.

This discussion has illustrated how to determine the domain and range of rational function by table of values and by listing elements in the domain and range using what we call the roster method. We also substitute the values in the domain to find the corresponding values in the range. Another method, the set-builder and interval notation may be shown in the following examples:

Example 1:

Find the domain and range of the rational function

$$f(x) = \frac{2x-3}{x^2}$$

first, we equate the denominator $x^2 = 0$, therefore $x = 0$

Domain: $\{x \mid x \in R, x \neq 0\}$ or simply $\{x \in R \mid x \neq 0\}$, that is all values can take the variable x except 0 because when the denominator becomes 0, f(x) will be undefined (undef).

To find the range, we use f(x) = y so that,

$$y = \frac{2x-3}{x^2}$$

$$yx^2 = 2x - 3$$

$$yx^2 - 2x + 3 = 0 \qquad (ax^2 + bx + c)$$

use $b^2 - 4ac \geq 0$ to get real solutions

let $a = y$, $b = -2$, $c = 3$

Therefore, $(-2)^2 - 4(y)(3) \geq 0$

$$4 - 12y \geq 0$$

$$4 \geq 12y$$

$$1/3 \geq y$$

In summary, $D(x) = \{x \in R \mid x \neq 0\}$ and the Range is $\{y \in R \mid y \leq 1/3\}$.

Example 2:

Find the domain and range of the rational function

$$f(x) = \frac{x-2}{x+2}$$

first, we equate the denominator x + 2 = 0, therefore x = -2

Domain: {x | x ϵ R, x ≠ -2}, that is all values can take the variable x except -2 because the denominator becomes 0 and f(x) will be undefined. The interval notation can also be written as D (-∞, -2) U (-2, ∞).

To find the range, we use f(x) = y so that,

$$y = \frac{x-2}{x+2}$$

in solving this, you just multiply y and the denominator x + 2 so that it becomes,

$$xy + 2y = x - 2$$

$$xy - x = -2y - 2$$

$$x(y - 1) = -2(y + 1)$$

$$x = \frac{-2(y+1)}{y-1}$$

Equate y – 1 = 0

y = 1

therefore, y ≠ 1, otherwise the denominator is zero.

Range: {y | y ϵ R, y ≠ 1}, that is all values can take the variable y except 1 because the denominator becomes 0 and x will be undefined.

Example 3:

Find the domain of the rational function

$$f(x) = \frac{3x^2-8x-3}{2x^2+7x-4}$$

first, we equate the denominator $2x^2 + 7x - 4 = 0$,

by factoring we have, (2x - 1) (x + 4) = 0

therefore x = ½, x = -4

Domain: {x ϵ R | x ≠ -4, 1/2}, that is all values can take the variable x except -4 and 1/2 because the denominator becomes 0 and f(x) will be undefined. The interval notation can also be written as D(-∞, -4) U (-4, ½) U (1/2, ∞).

Example 4:

Find the domain and range of the rational function

$$f(x) = \frac{x^2-3x-4}{x+1}$$

first, we equate the denominator x + 1 = 0,

therefore x = -1

Domain: {x ϵ R | x ≠ -1}, that is all values can take the variable x except -1 because the denominator becomes 0 and f(x) will be undefined. The interval notation can also be written as D (-∞, -1) U (-1, ∞).

To find the range, we can factor first the numerator.

$$f(x) = \frac{(x+1)(x-4)}{x+1}$$

You can cancel both (x + 1) of the numerator and denominator so that what remain is f(x) = (x – 4). Then we substitute x = -1 to find y.

y = x – 4

y = -1 – 4

y = -5

Therefore, the Range: {y ϵ R | y ≠ -5}. In interval notation, (-∞, -5) U (-5, ∞).

What's More

Enrichment Activity 1

Find the x-values or the domain of the following:

1. H = {(1,2), (2,3), (3,4), (4,5), (5,6)} D(H) _____

2. B = {(Rizal, 1861), (Bonifacio, 1863), (Mabini, 1864), (Luna. A., 1866), (Del Pilar, G., 1875)} D(B) _____

3. If the ordinates of A are {Quezon, Cavite, Rizal, Batangas, Laguna} and its abscissa are each provinces' corresponding Capitals, what would be the domain of (A)?

_____

4. The table shows: f(x) = 2x + 4. Solve for x.

x					
y	6	8	10	12	14

5. The graph shows:

y = {0, 3, 4, 7, 9}

x = {__, __, __, __, __}

That's it. Good job!

Enrichment Activity 2

Given the domain {-2, -1, 0, 1, 2}, determine the range for each expression. Use a table of values.

1. y = 3x + 2
2. x + y = 8
3. y = 5x – 1
4. y = $3x^2$
5. y = $\frac{2x-1}{2}$
6. x – 2y = 6
7. y = $\frac{(x^2 - 1)}{x}$
8. x = y - 3
9. y = x^2 – 4x - 3
10. y = (x – 1(x +1)

Great job!

Independent Practice

Find the domain and range of the following rational function. Use any notation.

1. f*(x)* = $\frac{2}{x+1}$

2. f*(x)* = $\frac{3x}{x+3}$

3. f*(x)* = $\frac{3-x}{x-7}$

4. f(x) = $\frac{2+x}{x}$

5. f*(x)* = $\frac{(x+1)}{x^2-1}$

Independent Assessment

Find the domain and range of the following rational function. Use any notation.

1. f*(x)* = $\frac{3}{x-1}$

2. f*(x)* = $\frac{2x}{x-4}$

3. f*(x)* = $\frac{x+3}{5x-5}$

4. f*(x)* = $\frac{2+x}{2x}$

5. f*(x)* = $\frac{(x2+4x+3)}{x^2-9}$

What I Have Learned

This module is about the domain and range of a rational function. It laid down the basic concepts of domain and range and showed how to determine them in a rational function. From this module, you learned that a function is a simple rule of correspondence between two variables x and y. The x variable is considered the input which is also called the independent variable while the y variable is the output which is also called the dependent variable. It is a basic notion that for every value of x there corresponds a value in y. This set of values in x is the domain while the set of values in y is the range of a rational function.
Now you try to summarize on your own by filling in the blanks:

To determine the domain and range in rational functions, _____ the denominator to _____ and solve for the variable x. The objective is that it must have _____ denominator. The value that would make it zero is the value that would not be in included in the domain. To find the range, solve the equation for x in terms of ____. Again, it must have non-zero denominator. The value that would make the _____ equal to zero is the value that would not be included in the range.

What I Can Do

Application

There are many ways of applying rational functions in our lives. Examples of these are: average cost, medical dosage, average grade problem, cost of living, and economic production of goods. An example of its application can be seen below.

Field of Application: Medical Dosage

Situation analysis: After a drug is injected into a patient's bloodstream, the concentration C of the drug in the bloodstream t hours after the injection is given by

$$C(t) = \frac{12t}{t^2+5}$$

Use the given formula to find the concentration of the drug after 1-4 hours.

Data manipulation: when t=0, $C(0) = \frac{12(0)}{0^2+5} = 0$

Presentation: Using a table of values, we have:

Table 3

t	1	2	3	4
C(t)	2.00	2.67	2.57	2.29

This is the graph of the function:

Interpretation: The table and the graph show that the drug is most effective after 2 hours where it peaked at 2.67 mg/L. The Domain and Range of the given function are D{x ϵR| x ≥ 0} R{y ϵR| y ≥ 0}

Now, It's Your Turn

1. Create your own or similar real-life situation where rational function is applied.
2. In a bond paper, present the problem from *Field of application* up to *Presentation* as illustrated above.
3. You can use graphing paper, MS Excel, Desmos, or any graphing app to graph the function. You can also use calculators to solve the table of values.
4. Your grade will be according to the criteria below:

Clarity of Presentation	60%
Organization	30%
Applicability to current situation	10%
TOTAL	100%

Assessment

Direction: Read and analyze each item carefully. Shade the entire circle below the letters that corresponds to your answer for each statement.

1. The abscissa of the point (-3, 5) is _____.
   A. 0 C. 5
   B. -3 D. 1

2. Point A is in Quadrant III. The ordinate in this point is _____.
   A. both – and + C. positive (+)
   B. negative (-) D. zero

3. The set of all possible input values (x) which produce a valid output (y) from function is called _____
   A. algebra C. domain
   B. binomial D. range

4. The Range in a rational function is also the _____ variable?
   A. constant C. fixed
   B. dependent D. independent

5. In writing the domain/range of a rational function, the format $(-\infty, 1) \cup (1, \infty)$ is called a/an _____
   A. enumeration C. set-builder notation
   B. roster form D. interval notation

6. In a set of ordered pairs (-5,-4), (-5,1), (-2,3), (2,1), (2,-4), the domain D =_____
   A. {-5,-2,2} C. {-5,-5,-2,2,2}
   B. {-4,1,3,1,-4} D. {1,2,3,4,5}

7. In a set of ordered pairs (-5,-4), (-5,1), (-2,3), (2,1), (2,-4), the range R = _____
   A. {-4,1,3} C. {-4,1,3,1,-4}
   B. {-5,-5,-2,2,2} D. {1,2,3,4,5}

8. Some values for the Domain of the rational function $f(x) = 6x^2 - 5$ are -2, -1, 0, 1, and 2. Find the Range corresponding for each value. _____
   A. {91,19,-5,19,91} C. {19,1,-5,1,19}
   B. {-2,-1,0,1,2} D. {0,1,2,3,4}

9. Some values for the Range of the rational function f(x) = $\frac{3x^2-5}{x}$ are undefined, -2, and 10.75. Find the Domain corresponding to each value. _____

A. {0,-1,-4}
B. {1,2,3}
C. {-2,-1,0}
D. {0, 1, 4}

10. Find the domain and range of the function f(x) = $6x - 4$. D _____ R _____

A. D {x ϵ R| x ≠ -4} R {y ϵ R| y ≠ 6}
B. D {x|x ϵ R} R {y|y ϵ R}
C. D {x ϵ R| x ≠ 0} R {y ϵ R| y ≠ 0}
D. D {x|x ϵ N} R {y|y ϵ N}

11. Find the domain and range of the function f (x) = $x^2 - 8x + 15$. D _____ R _____

A. D {x ϵ R| x ≠ 4} R {y ϵ R| y ≠ -4}
B. D {x|x ϵ N} R {y|y ϵ N}
C. D {x ϵ R| x ≠ -5} R {y ϵ R| y ≠ -3}
D. D {x|x ϵ R} R {y|y ϵ R}

12. Find the domain and range of the rational function f(x) = $\frac{7+x}{x-5}$ D _____ R _____

A. D(-∞, 5) U (5, ∞) R(-∞, 1) U (1, ∞)
B. D(-∞, 7) U (7, ∞) R(-∞, -5) U (-5, ∞)
C. D(-∞, 5) U (5, ∞) R(-∞, -1) U (-1, ∞)
D. D(-∞, -7) U (-7, ∞) R(-∞, 5) U (5, ∞)

13. Find the domain and range of the rational function f(x) = $\frac{x^2-4x+4}{x-2}$ D _____ R _____

A. D(-∞, -1) U (-1, ∞) R(-∞, 4) U (4, ∞)
B. D(-∞, 5) U (5, ∞) R(-∞, -2) U (-2, ∞)
C. D(-∞, 2) U (2, ∞) R(-∞, 0) U (0, ∞)
D. D(-∞, 2) U (2, ∞) R(-∞, 2) U (2, ∞)

14. Find the domain and range of this graph.

A. D(-∞, 0) U (0, ∞) R(-∞, 4) U (4, ∞)
B. D(-∞, 0) U (0, ∞) R(-∞, 1) U (1, ∞)
C. D(-∞, 0) U (0, ∞) R(-∞, -1/4) U (-1/4, ∞)
D. D(-∞, 1/4) U (1/4, ∞) R(-∞, 4) U (4, ∞)

15. The concentration of a drug in the bloodstream can be modeled by the function C(t) = $\frac{30t}{t^2+9}$, 0≤t≤5. Determine when the maximum amount of drug is in the body and the amount at that time.

A. in 2 hrs. with 5 mg/L
B. in 3 hrs. with 5 mg/L
C. in 3 hrs. with 6 mg/L
D. in 4 hrs. with 4.8 mg/L

Additional Activities

Instruction: In doing this activity you may need MS Excel or a mobile app such as Desmos to sketch the graph.

In this module's introduction, you were asked about average grade problem. The rational function for that situation is:

$$f(x) = \frac{22+x}{25+x}$$

Construct a table of values and sketch the graph for this rational function. Find the domain and range.

Answer:

The table of values for this function:

Table 4

x	0	1	2	3	4	5
f(x)						

In answering the questions in this module's introduction, how many correct answers do you need to have a 90% rating, what would be your answer? _____

How about to have an 89% rating? _____ 95%? _____

From this table of values, what do you think would be its domain and range? _____

Sketch the graph of this rational function on a separate sheet.

Answer Key: What's More

Pre-Assessment: 1. A, 2. B, 3. C, 4. D, 5. C, 6. A, 7. B, 8. A, 9. D, 10. B, 11. D, 12. C, 13. D, 14. B, 15. B
Assessment: 1. A, 2. B, 3. C, 4. B, 5. D, 6. A, 7. A, 8. C, 9. D, 10. B, 11. D, 12. A, 13. C, 14. A, 15. B

Independent Assessment

1. D = {x ϵ R | x ≠ 1} or (-∞, 1) U (1, ∞)

   R = {y ϵ R | y ≠ 0} or (-∞, 0) U (0, ∞)

2. D = {x ϵ R | x ≠ 4} or (-∞, 4) U (4, ∞)

   R = {y ϵ R | y ≠ 2} or (-∞, 2) U (2, ∞)

3. D = {x ϵ R | x ≠ 1} or (-∞, 1) U (1, ∞)

   R = {y ϵ R | y ≠ 1/5} or (-∞, 1/5) U (1/5, ∞)

4. D = {x ϵ R | x ≠ 0} or (-∞, 0) U (0, ∞)

   R = {y ϵ R | y ≠ 0} or (-∞, 0) U (0, ∞)

5. D = {x ϵ R | x ≠ -3, 3} or (-∞, -3) U (-3, 3) U (3, ∞)

R = {y ϵ R | y ≠ 1} or (-∞, 1) U (1, ∞)

Independent Practice

1. D = {x ϵ R | x ≠ -1} or (-∞, -1) U (-1, ∞)

   R = {y ϵ R | y ≠ 0} or (-∞, 0) U (0, ∞)

2. D = {x ϵ R | x ≠ -3} or (-∞, -3) U (-3, ∞)

   R = {y ϵ R | y ≠ 3} or (-∞, 3) U (3, ∞)

3. D = {x ϵ R | x ≠ 7} or (-∞, 7) U (7, ∞)

   R = {y ϵ R | y ≠ 1} or (-∞, 1) U (1, ∞)

4. D = {x ϵ R | x ≠ 0} or (-∞, 0) U (0, ∞)

   R = {y ϵ R | y ≠ 1} or (-∞, 1) U (1, ∞)

5. D = {x ϵ R | x ≠ 1} or (-∞, 1) U (1, ∞)

   R = {y ϵ R | y ≠ 0} or (-∞, 0) U (0, ∞)

Enrichment Activity 2

No.	**Domain** *Range*	**-2**	**-1**	**0**	**1**	**2**
#1	*y*	-4	-1	2	5	8
#2	*y*	10	9	8	7	6
#3	*y*	-11	-6	-1	4	9
#4	*y*	12	3	0	3	12
#5	*y*	-2.5	-1.5	-0.5	0.5	1.5
#6	*y*	-4	-3.5	-3	-2.5	-2
#7	*y*	-1.5	0	undef	0	1.5
#8	*y*	-5	-4	-3	-2	-1
#9	*y*	9	2	-3	-6	-7
#10	*y*	3	0	-1	0	3

Enrichment Activity 1

1. D(H) = {1, 2, 3, 4, 5}

2. D(B) = {Rizal, Bonifacio, Mabini, Luna, A. Del Pilar, G.}

3. D(A) = {Lucena City, Imus City, Antipolo City, Batangas City, Santa Cruz}

4. f(x) = 2x + 4

x	1	2	3	4	5
y	6	8	10	12	14

5.

x	-5	-2	-1	2	4
y	0	3	4	7	9

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11 General Mathematics

General Mathematics
Intercepts, Zeroes and Asymptotes of Rational Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Intercepts, Zeroes and Asymptotes of Rational Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics Alternative Delivery Mode (ADM) Module on Determining the Intercepts, Zeroes and Asymptotes of Rational Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Alternative Delivery Mode (ADM) Module on Determining the Intercepts, Zeroes and Asymptotes of Rational Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer What I Know before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written to help you determines the intercepts, zeroes and asymptotes of rational functions. Knowing what a rational function is, you are now ready to learn its other properties. It includes finding the intercepts, zeroes and asymptotes. These will be your guide to easily determine the behavior of a rational function and will prepare you for graphing rational function. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lesson is arranged to follow the standard sequence of the course.

In this module you will determine the intercepts, zeroes and asymptotes of rational functions.

The module consists of one lesson namely: Intercepts, Zeroes, and Asymptotes of Rational Functions.

After going through this module, you are expected to:
1. recall the meaning of intercepts, zeroes and asymptotes;
2. identify the intercepts, zeroes and asymptotes of rational functions;
3. solve for the intercepts, zeroes and asymptotes of rational functions.

What I Know

In this part, let us see how much you know about the lesson by answering the questions in pre-assessment below. If you obtain 100% or a perfect score, skip the module and immediately move to the next module. While in the event you missed a point, please proceed on the module as it will enrich your knowledge in finding the intercepts, zeroes and asymptotes of rational functions. Let's get started!

I. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following is the set of all values that the variable x can take?
   a. Range
   b. Intercept
   c. Domain
   d. Zeroes

2. What is the domain of f(x) $=\frac{x-3}{x+3}$?
   a. All real numbers
   b. All real numbers except – 3
   c. All real numbers except 3
   d. Cannot be determined

3. What value/s of x that will make the function meaningless f(x) = $\frac{x-1}{x}$?
   a. x = -1
   b. x = 0
   c. x = 1
   d. All real numbers

4. Complete the sentence: The real numbers zeroes are also _____ of the graph of the function.
   a. Asymptote
   b. x – intercepts
   c. y – intercepts
   d. Range

5. Which of the following is the set of all values that f(x) can take?
   a. Range
   b. Intercept
   c. Domain
   d. Zeroes

6. What is the range of f(x) $=\frac{1}{x}$?
   a. R $=\{y|y = 1\}$
   b. R $=\{y|y = 0\}$
   c. R = $\{y|y \neq 1\}$
   d. R = $\{y|y \neq 0\}$

7. Which of the following is a true statement?
   a. A rational function is a quotient of functions.
   b. Asymptotes are a common characteristic of rational functions.
   c. An asymptote is a line that a graph approaches, but does not touch.
   d. All of the above.

8. If the degree of the leading coefficient of the numerator is equal to the degree of the leading coefficient of the denominator of a rational function, which of the following statements has to be true?
   a. The graph has no asymptote
   b. The graph of the function has slant asymptote
   c. The graph of the function has a horizontal asymptote
   d. None of the above

9. What is the horizontal asymptote of $f(x) = \frac{x+5}{3x^2}$?
   a. y = 3
   b. y = 0
   c. y = - 2
   d. y = - 3

10. What is the vertical asymptote of $f(x) = \frac{3x+1}{x-5}$?
   a. x = 5
   b. x = 3
   c. x = 1
   d. x = 0

11. What is the oblique asymptote of $f(x) = \frac{x^2-3x}{x+3}$?
   a. y = 3x
   b. y = x – 6
   c. y = x -3
   d. y = 3x + 6

12. Oblique asymptote occurs when there is no horizontal asymptote, the statement is _____.

   a. Always true
   b. Sometimes true
   c. Never true
   d. Cannot be determined

13. How will you describe the horizontal asymptote of $f(x) = \frac{3}{3+x}$?
   a. does not exist
   b. approaching at x = 3
   c. approaching at y = -3
   d. approaching at y = 0

14. If the x – intercept of a rational function is at x = 5, what is the zero of the function?

   a. x = 5
   b. x = 0
   c. x = - 5
   d. cannot be determined

15. What is the y – intercept of $f(x) = \frac{2x^2+x+3}{2x^2+3x+1}$?

   a. 3
   b. 0
   c. – 3
   d. – 6

Lesson 1 Intercepts, Zeroes, and Asymptotes of Rational Functions

In the previous lesson, you learned how to find domain and range of a rational function. In this particular lesson, determining intercepts, zeroes and asymptotes of rational functions will be done. Knowing fully the concept of the different properties of rational function will be your guide to easily determine the behavior of a rational function and it will prepare you for the next topic which is about graphing rational function.

What's In

Let's recall first what you have learned from the previous lesson by answering the following questions:

A. Which of the following is an example of rational function?

1. F(x) = $\frac{3x^2+1}{x-1}$ 2. $\frac{x}{3} = \frac{8}{3}$ 3. $\frac{1}{3x-1} + 3 < 0$

B. Find the domain and range of the functions.

1. F(x) = $\frac{x}{x+3}$ 2. f(x) = $\frac{3}{x-4}$ 3. g(x) = $\frac{x+1}{x^2-1}$

Let us see if you got the correct answer in the activity, if your answer in question A is number 1, you got it right you have a clear understanding of the concept of rational function but if you are incorrect allow me to help you recall what a rational function is, *when two polynomial functions are expressed as a quotient and can be written in the form* $f(x) = \frac{p(x)}{q(x)}$ *and q(x) is a not the zero function it is called a rational function.* Numbers 2 and 3 are not examples of rational function, it is a rational equation and rational inequality, respectively. Number 1 is written as the quotient of two polynomial functions, so it is a rational function.

For activity B, let us review the meaning of domain and range of the function. **Domain** is the set of first coordinates of a relation and it is the value of x that will not make the denominator of the function equal to zero while **Range** is the set of second coordinates. To determine the domain of rational function, simply equate the denominator to zero and then solve for x, this value should be avoided so that the function will not give an undefined or a meaningless function. Example find the domain of F(x) = $\frac{x}{x+3}$, equating the denominator to zero, we have x + 3 = 0, so the value of x = -3, so the domain of the function are all real numbers except -3 remember we will avoid value/s that will make our denominator equal to zero, so if we will substitute -3 to our x in the denominator it will result to 0 and it will give us an undefined function. **In notation, D= (-∞, −3) ∪ (−3, ∞)**

To find the range of the function, change f(x) to y then, solve for x; remember range are real values of y that will make a real value for the function. For example, find the range of F(x) = $\frac{x}{x+3}$;

Changing F(x) to y, the new function is ⟹ y = $\frac{x}{x+3}$

By doing cross multiplication we have ⟹ y(x+3) = x

Distributing y we now have ⟹ xy + 3y = x

Simplifying the equation will give ⟹ xy – x = 3y

Factoring the left side of the equation ⟹ x(y – 1) = 3y

Dividing the equation by (y – 1) ⟹ $\frac{x\cancel{(y-1)}}{\cancel{(y-1)}} = \frac{3y}{(y-1)}$

Removing common factor, the value of x ⟹ $x = \frac{3y}{y-1}$

Since we are looking for the value of y that will give a real value for the function so we need to find value/s for y that will not make the denominator equal to 0.

Equating the denominator to zero ⟹ y – 1 = 0

So, y = 1.

The range of the function F(x) = $\frac{x}{x+3}$ is all real values of y except 1. In notation,

R= (-∞, 1) ∪ (1, ∞).

The following are the answers to Activity B

1. Domain = {x/x≠ −3} or **(-∞, −3) ∪ (−3, ∞)**
   Range = {y/y ≠ 1} or **(-∞, 1) ∪ (1, ∞)**

2. Domain = {x/x≠ 4} or **(-∞, 4) ∪ (4, ∞)**
   Range = {y/y ≠ 0} or **(-∞, 0) ∪ (0, ∞)**

3. Domain = {x/x≠ −1 *or* $x \neq 1$} or **(-∞, −1) ∪ (−1, 1) ∪ (1, ∞)**
   Range = {y/y ≠ 0} or **(-∞, 0) ∪ (0, ∞)**

How is your review of the rational function? I believed you got it all correct. Are you ready to learn new things about rational functions? Let's do the next activity.

Notes to the Teacher

The teacher may say that "the domain refers to the set of possible input values and range is the set of possible output values" is related to the saying "you saw what you reap". Like in our day to day activities if we show good deeds to others, in return we will receive the same treatment.

What's New

Activity

I – Connect Mo!

Connect the given statement/phrase in column A with the answer in column B to complete the statement/phrase in column A. Write the letter of your answer in a separate sheet of paper.

COLUMN A	COLUMN B
1. The intercepts of the graph of a rational function …	M. the x - intercepts
2. To find the x – intercept of a function …	A. let x = 0
3. The zeroes of the function is also …	G. rational function
4. To find the y – intercept of a function …	I. are the points of intersection of its graph and an axis
5. The function of the form $f(x) = \frac{g(x)}{h(x)}$, where g(x) & h(x) are polynomials	C. let y = 0

How was the activity? I believed that you connected it right. So, in this lesson, you will know how to identify intercepts, zeroes and asymptotes of rational function.

What is It

INTERCEPTS AND ZEROES OF RATIONAL FUNCTIONS

The intercepts of the graph of a rational function are the points of intersection of its graph and an axis.

The *y-intercept* of the graph of a rational function r(x) if it exists, occurs at r(0), provided that r(x) is defined at x = 0. To find y-intercept simply evaluate the function at x = 0.

The *x-intercept* of the graph of a rational function r(x), if it exists, occurs at the zeros of the numerator that are not zeros of the denominators. To find x – intercept equate the function to 0.

The *zeroes of a function* are the values of x which make the function zero. The numbered zeroes are also x-intercepts of the graph of the function.

Figure 1. x and y intercepts using GeoGebra

EXAMPLES.

1. Find the x- and y – intercepts, of the following rational functions:

   a. f(x) = $\frac{3-x}{x+1}$ b. f(x) = $\frac{3x}{x+3}$ c. f(x) = $\frac{x^2-3x+2}{x^2-4}$

2. Determine the zeroes of the following rational functions:

a. g(x) = $\frac{x-2}{x+6}$ b. h(x) = $\frac{x-3}{x^2-9}$ c. G(x) = $\frac{x^2+x-2}{x^2-4}$

SOLUTIONS.

1. To find x – intercept equate the function to 0.

$$f(x) = \frac{3-x}{x+1}$$

$0 = \frac{3-x}{x+1}$	Equate the function to 0.
$\frac{3-x}{x+1} = 0$	By Symmetric Property of Equality.
3 – x = 0	Multiply both sides by (x + 1).
$3 + (-3) - x = 0 + (-3)$	By Addition Property of Equality(APE).
$-x = -3$	Simplify.
$(-1)(-x) = (-1)(-3)$	By Multiplication Property of Equality (MPE).
$x = 3$	

So, the x – intercept is (3, 0).

By analyzing the example, we can say that to find the x – intercept simply equate the numerator of the function to 0.

To find the y – intercept, change the x value of the function to 0.

$f(x) = \frac{3-x}{x+1}$	Substitute 0 to x values of the function.
$f(x) = \frac{3-0}{0+1}$	Simplifying the fraction.
$f(x) = \frac{3}{1} = 3$	Value of f(x) or y.

So, the y – intercept is 3 or (0, 3).

2. f(x) = $\frac{3x}{x+3}$

To find the x – intercept, simply equate the numerator to 0,

0 = 3x	Equate the numerator to 0.
3x = 0	By Symmetric Property of Equality.
$\frac{\cancel{3}x}{\cancel{3}} = \frac{0}{3}$	Simplifying the fraction by multiplying both sides by 1/3.
x = 0	

So, the x – intercept is 0 or (0, 0).

To find the y – intercept, change the x value of the function to 0.

$f(x) = \frac{3x}{x+3}$	Substitute 0 to x values of the function.
$f(x) = \frac{3(0)}{0+3}$	Simplifying the fraction.
$f(x) = \frac{0}{3} = 0$	The value of $f(x)$ or y – intercept.

So, the y – intercept is 0 or (0, 0).

3. $f(x) = \frac{x^2-3x+2}{x^2-4}$

$x^2 - 3x + 2 = 0$		Equate the numerator to 0.
(x – 2) (x – 1) = 0		By factoring.
x – 2 = 0	x – 1 = 0	Solve for x, by Zero product property.
x = 2	x = 1	

So, the x – intercepts are x = 2 and x = 1. But by looking at the denominator of the original function if we substitute 2 to the value of x,

$$x^2 - 4 = (2)^2 - 4 = 0,$$

The denominator will become 0, the function becomes meaningless.
So, we will only accept x – intercept at **<u>x = 1 or (1, 0).</u>**

To find the y – intercept:

$f(x) = \frac{x^2-3x+2}{x^2-4}$, change the x value of the function to 0.

$f(x) = \frac{(0)^2-3(0)+2}{(0)^2-4}$ Simplify the fraction.

f(x) = $\frac{2}{-4}$ Reduce the fraction to lowest term.

$f(x) = -\frac{1}{2}$ The value of f(x) or y.

So, the y – intercept is $-\frac{1}{2}$ or (0, $-\frac{1}{2}$).

2. Determine the zeroes of the following rational functions:

a. g(x) = $\frac{x-2}{x+6}$ b. H(x) = $\frac{x-3}{x^2-9}$ c. G(x) = $\frac{x^2+x-2}{x^2-4}$

To find the zeroes of a rational function, equate the function to 0 or solve for the x – intercept of the function by equating the numerator to 0.

a. g(x) = $\frac{x-2}{x+6}$,

x – 2 = 0 Equate the numerator to 0.

x = 2 Solve for x.

Thus, the zero of g(x) is 2.

b. $H(x) = \frac{x-3}{x^2-9}$

$H(x) = \frac{x-3}{x^2-9}$ Simplify by factoring the denominator.

$\frac{\overset{1}{\cancel{x-3}}}{\cancel{(x-3)}(x+3)}$ Remove common factors.

$\frac{1}{x+3} = 0$ Equate the numerator to 0.

1 = 0 False statement.

So, there is **<u>no zero</u>** of the function. Which means that no point on the graph touches the x – axis.

c. G(x) = $\frac{x^2+x-2}{x^2-4}$

$G(x) = \frac{\cancel{(x+2)}(x-1)}{\cancel{(x+2)}(x-2)}$ Simplify by factoring both the numerator and denominator.

$G(x) = \frac{x-1}{x-2}$ Remove common factors.

x -1 = 0 Equate the numerator to 0.

x = 1 Solve for x.

Thus, the zero of G(x) = 1.

ASYMPTOTES

An asymptote is an imaginary line to which a graph gets closer and closer as the x or y increases or decreases its value without limit.

Kinds of Asymptote

- Vertical Asymptote
- Horizontal Asymptote
- Oblique / Slant Asymptote

VERTICAL ASYMPTOTE

The vertical line $x = a$ is a vertical asymptote of a function f if the graph increases or decreases without bound as the x values approach $\boldsymbol{a}$ from the right or left. See illustration below.

Figure 2. Illustration of Vertical Asymptote using geogebra

By looking at the illustration we can see that the graph of the function is approaching at x = - 5 represented by the dotted line and as we can see the graph is getting closer and closer to x = - 5 but it will not pass or intersect the line. So, the vertical asymptote of the graph is at x =- 5. We can find vertical asymptote even without looking at the graph of the function.

Finding Vertical Asymptote

To determine the vertical asymptote of a rational function, first reduce the given function to simplest form then find the zeroes of the denominator that are not zeros of the numerator.

Examples

Determine the vertical asymptote of each rational function.

a. F(x) = $\frac{(x-1)}{(x+5)}$ b. f(x) = $\frac{x+2}{(x+1)(x-4)}$ c. g(x) = $\frac{2x^2-x+1}{x^2-6x+9}$

Solutions

a. The zero of the numerator is 1 and the zero of the denominator is -5. The vertical asymptote for F(x) = $\frac{(x-1)}{(x+5)}$ is x = - 5. The value is zero of the denominator but not of the numerator.

b. The zero of the numerator is -2 and the zeroes of the denominator are -1 and 4. The vertical asymptote for f(x) = $\frac{x+2}{(x+1)(x-4)}$ are x = - 1 and x = 4. These values are zeroes of the denominator but not of the denominator.

c. Since the function is in quadratic form, reduce it to simplest form. The simplest form of g(x) = $\frac{2x^2-x-1}{x^2-5x+6}$ is g(x) = $\frac{(2x+1)(x-1)}{(x-3)(x-2)}$. The zeroes of the numerator are -1/2 and 1. The zeroes of the denominator are 3 and 2. The vertical asymptote for g(x) =$\frac{(2x+1)(x-1)}{(x-3)(x-2)}$ are x = 2 and x = 3. These values are zeroes of the denominator but not of the denominator.

Let us now discuss Horizontal Asymptote

HORIZONTAL ASYMPTOTE

The horizontal line *y=b* is a horizontal asymptote of the function *f* if *f(x)* gets closer to b as x increases or decreases without bound.

Looking at the graph on the next page, we can see that the graph of the function is approaching a line in the y – axis, that line is called the horizontal asymptote. In the graph we can see that it is getting closer and closer at y = 1 but it only approaches but never touches or intersects y = 1. So, the horizontal asymptote of the function is at y = 1. We can determine horizontal asymptote arithmetically by comparing the degree of the leading coefficient of the numerator and denominator of the function.

Figure 3. Illustration of Vertical Asymptote using geogebra

Finding the horizontal asymptote of a rational function.

To determine the horizontal asymptote of a rational function, compare the degree of the numerator n and the degree of the denominator d.

- If n < d, the horizontal asymptote is y= 0
- If n = d, the horizontal asymptote y is the ratio of the leading coefficient of the numerator a, to the leading coefficient of the denominator b. That is $y = \frac{a}{b}$.
- If n > d, there is no horizontal asymptote.

Note: A rational function may or may not cross its horizontal asymptote. If the function does not cross the horizontal asymptote y=b, then b is not part of the range of the rational function.

EXAMPLES

Determine the horizontal asymptote of each rational function.

a. $F(x) = \frac{3x+8}{x^2+1}$ b. $f(x) = \frac{3 + 8x^2}{x^2+1}$ c. $g(x) = \frac{8x^3-1}{1-x^2}$

SOLUTIONS

a. The degree of the numerator 3x + 8 is less than the degree of the denominator $x^2 + 1$. Therefore, the horizontal asymptote is y = 0.

b. The degree of the numerator $3 + 8x^2$ and that of the denominator $x^2 + 1$ are equal. Therefore, the horizontal asymptote y is equal to the ratio of the leading coefficient of the numerator 8 to the leading coefficient of the denominator 1. That is $y = \frac{8}{1} = 8$.

c. The degree of the numerator $8x^3 - 1$ is greater than the degree of the denominator $1 - x^2$. Therefore, there is no horizontal asymptote.

Aside from vertical and horizontal asymptote, a rational function can have another asymptote called oblique or slant. It occurs when there is no horizontal asymptote or when the degree of the numerator is greater than the degree of the denominator.

SLANT / OBLIQUE ASYMPTOTE

An oblique asymptote is a line that is neither vertical nor horizontal. It occurs when the numerator of $f(x)$ has a degree that is one higher than the degree of the denominator.

Figure 4. Illustration of Oblique Asymptote using geogebra

Looking at the graph we can see that there is vertical asymptote and there is no horizontal asymptote. In this case, oblique or slant asymptote occurs. We can determine the oblique / slant asymptote using your knowledge of division of polynomials.

Finding Oblique or Slant Asymptote

To find slant asymptote simply divide the numerator by the denominator by either using long division or synthetic division. The oblique asymptote is the quotient with the remainder ignored and set equal to y.

EXAMPLES

Consider the function $h(x) = \frac{x^2+3}{x-1}$. Determine the asymptotes.

By looking at the function, h(x) is undefined at x = 1, so the vertical asymptote of h(x) is the line at x = 1.

There is no horizontal asymptote because the degree of the numerator is greater than the degree of the denominator.

If the numerator and denominator of h(x) are divided, we get

$$h(x) = \frac{x^2+3}{x-1}$$

$$= x - 1 \overline{\big) x^2 + 0x + 3}$$

$$\begin{array}{r} x + 1 \text{ r. } 4 \\ \hline x - 1 \big) x^2 + 0x + 3 \\ -x^2 \; (+) \; x \\ + \\ \hline x + 3 \\ -x \; (+) \; 1 \\ + \\ \hline 4 \end{array}$$

So, the quotient is $x + 1 + \frac{4}{x-1}$.

Thus, the line y = x + 1 is the oblique asymptote of $h(x) = \frac{x^2+3}{x-1}$.

What's More

Now it's your turn.

Independent Practice 1

Given the rational function f(x) = $\frac{2x+6}{X-3}$, answer the following questions:

1. What are the two functions used to form the rational function?
2. What is the x-intercept of the function? Which function did you use to determine the x-intercept? Why?
3. What is the y – intercept of the function? How did you get the y – intercept?
4. What is the zero of the function?

Remember Me!

- To find the y – intercept, substitute 0 for x and solve for y or f(x).
- To find the x – intercept, substitute 0 for y and solve for x.
- The zero of a rational function is the same as the x – intercept of the function.

Try This!

Independent Assessment 1

Complete the table below by giving the intercepts and zeroes of rational function.

Rational Function	x - intercept	y - intercept	Zeroes of the function
1. $f(x) = \frac{x-9}{x+3}$			
2. $f(x) = \frac{x^2-10x+25}{x+5}$			
3. $f(x) = \frac{x^2+9}{x^2-3}$			

Independent Practice 2

True or False. Tell whether each of the following is **true** or **false.** If the statement is wrong change the underlined word to make it correct. Write your answer on the space provided before each number.

_____ 1. An <u>intercept</u> is a line (or a curve) that the graph of a function gets close to but does not touch.
_____ 2. If <u>n > d</u>, there is no horizontal asymptote.
_____ 3. To determine the vertical asymptote of a rational function, find the zeroes of the <u>numerator</u>.
_____ 4. If n < d, the <u>vertical asymptote</u> is y = 0.
_____ 5. The horizontal asymptote of $f(x) = \frac{x}{x^2-1}$ is <u>y = 1</u>.
_____ 6. The vertical asymptote of $f(x) = \frac{(x-1)(x+3)}{x^2-1}$ are <u>x = 1</u> and <u>x = 2</u>.

Remember Me!

- An asymptote is an imaginary line to which a graph gets closer and closer as the x or y increases or decreases its value without limit.

- To find vertical asymptote of a rational function, first reduce the given function to simplest form then find the zeroes of the denominator that are not zeros of the numerator.

- To determine the horizontal asymptote of a rational function, compare the degree of the numerator n and the degree of the denominator d.

  - *If n < d,* the horizontal asymptote *is y= 0*
  - *If n = d,* the horizontal asymptote y is the ratio of the leading coefficient of the numerator a, to the leading coefficient of the denominator *b. That is* $y = \frac{a}{b}$.
  - *If n > d,* there is no horizontal asymptote.

- An oblique asymptote is a line that is neither vertical nor horizontal. It occurs when the numerator of $f(x)$ has a degree that is one higher than the degree of the denominator. Divide the numerator by the denominator by either using long division or synthetic division. The oblique asymptote is the quotient with the remainder ignored and set equal to y.

Independent Assessment 2

Determine the vertical and horizontal asymptotes of the following rational functions.

	Vertical Asymptote	Horizontal Asymptote
1. $f(x) = \frac{2}{2x+5}$	_____	_____
2. $f(x) = \frac{x+3}{x+7}$	_____	_____
3. $f(x) = \frac{(x+3)(x-2)}{(x+5)(x-4)}$	_____	_____
4. $g(x) = \frac{2+3x}{x^2+3x-4}$	_____	_____
5. $g(x) = \frac{x-3}{2x^2-8}$	_____	_____

What I Have Learned

Let us summarize what you have learned from this module by completing the following statements. Write the correct word/s in a separate sheet of paper.

1. _____ of the graph of a rational function are the points of intersection of its graph and an axis.

2. _____ of a function are the values of x which make the function zero. The numbered zeroes are also _____ of the graph of the function.

3. _____ of the graph of a rational function r(x), if it exists, occurs at the zeros of the numerator that are not zeros of the denominators. To find _____ equate the function to _____.

4. _____of the graph of a rational function r(x) if it exists, occurs at r(0), provided that r(x) is defined at x = 0. To find _____ simply evaluate the function at x = _____.

5. An _____ is an imaginary line to which a graph gets closer and closer as the x or y increases or decreases its value without limit.

6. To find _____of a rational function, first reduce the given function to simplest form then find the zeroes of the denominator that are not zeros of the numerator.

7. To determine the _____ of a rational function, compare the degree of the numerator n and the degree of the denominator d.

   - If n < d, the horizontal asymptote is _____

   - If n = d, the horizontal asymptote y is the ratio of the leading coefficient of the numerator a, to the leading coefficient of the denominator b. That is y = _____.

   - If n > d, there is _____ horizontal asymptote.

8. An oblique asymptote is a line that is _____. To determine oblique asymptote, divide the numerator by the denominator by either using long division or synthetic division. The oblique asymptote is the quotient with the remainder ignored and set equal to y.

What I Can Do

Let's apply what you have learned from the lesson.

The concentration (C) of a given substance in a mixture is the ratio of the amount of substance to the total quantity. In symbols,

$$C = \frac{S}{Q}$$

where C is the concentration, S is the amount of substance, and T is the total quantity. If 8 ounces of punch contains 4 ounces of pure orange juice, the concentration of orange juice in the punch is 4/8 or 50%. The punch is 50% orange juice. Consider the problem where we begin that 8 ounces of punch that is 50% orange juice and want to write a function that gives the orange juice concentration after x ounces of pure orange juice are added.

Questions:

a. How much orange juice do you begin with? Write an expression for the amount of orange juice present after x ounces has been added.

b. Write an expression for the total amount of punch present after x ounces has been added.

c. Using the answers in (a) and (b), write a rational function defining the pineapple juice concentration as a function of x.

d. Give the x and y - intercepts of the rational function.

e. What is the equation of the vertical asymptote and of the horizontal asymptote?

Assessment

Let's Do This!

1. Which of the following is the set of all values that $f(x)$ take?
   a. Range
   b. Intercept
   c. Domain
   d. Zeroes

2. What is the y-intercept of $f(x) = \frac{x-3}{x+3}$?
   a. 0
   b. – 1
   c. – 3
   d. – 5

3. What is the x – intercept of $f(x) = \frac{x-1}{x}$?
   a. x = -1
   b. x = 0
   c. x = 1
   d. All real numbers

4. Complete the sentence: The x- intercept of rational function is also _____ of the graph of the function.
   a. asymptote
   b. range
   c. zero
   d. domain

5. Which of the following are the points of intersection of the graph and the axes?
   a. Range
   b. Intercept
   c. Domain
   d. Zeroes

6. What is the domain of f(x) $= \frac{3}{x}$?
   a. D = $\{x|x = 1\}$
   b. D $=\{x|x = 0\}$
   c. D = $\{x|x \neq 1\}$
   d. D = $\{x|x \neq 0\}$

7. Which of the following is a not a true statement?
   a. A rational function is a quotient of functions.
   b. Asymptotes are a common characteristic of rational functions.
   c. An asymptote is a line that a graph approaches, but does not touch.
   d. Domain and Range of rational functions are always equal

8. If the degree of the leading coefficient of the numerator is less than to the degree of the leading coefficient of the denominator of a rational function, which of the following statements has to be true?
   a. The graph has no asymptote
   b. The graph of the function has slant asymptote
   c. The graph of the function has a horizontal asymptote
   d. None of the above

9. What is the zero of $f(x) = \frac{x+5}{3x^2}$?

a. x = 5
b. x = 0
c. x = - 3
d. x = - 5

10. What is the horizontal asymptote of $f(x) = \frac{3x+1}{x-5}$?

a. y = 5
b. y = 3
c. y = 1
d. y = 0

11. What is the y - intercept of $f(x) = \frac{x^2-3x}{x+3}$?

a. y = 3
b. y = 1
c. y = 0
d. y = - 2

12. When the degree of the leading coefficient of the denominator of a rational function is greater than the degree of the leading coefficient of the numerator, the horizontal asymptote is at $y = \frac{a_n}{a_d}$ the statement is _____.

a. Always true
b. Sometimes true
c. Never true
d. Cannot be determined

13. How will you describe the vertical asymptote of $f(x) = \frac{(x-3)(x-2)(x+5)}{(x-1)(x-3)(x-2)}$?

a. does not exist
b. approaching at x = 1
c. approaching at x = -1
d. approaching at x = 0

14. What is the x – intercept of $f(x) = \frac{x^2-2x-15}{x^2-25}$?

a. x = 5
b. x = 3
c. x = - 3
d. x = - 5

15. What is the horizontal asymptote of $f(x) = \frac{2x^2+x+3}{2x^2+3x+1}$?

a. y = 3
b. y = 2
c. 1
d. 0

Additional Activities

To deepen your knowledge on finding the intercepts, zeroes and asymptotes of rational function you can visit the following websites, https://youtu.be/gDC7XflNbQl and https://youtu.be/GgdGpjiJmkl.

For those who don't have online connections you can answer the following questions to deepen your understanding about the lesson.

Analyze the given function and determine:

a. x – and y- intercepts
b. zeros
c. Asymptotes

1. $f(x) = \frac{x+1}{x-4}$

2. $f(x) = \frac{x^2-4x+5}{x-4}$

Answer Key

What I Know

1. C
2. B
3. B
4. B
5. A
6. D
7. D
8. C
9. B
10. A
11. B
12. A
13. D
14. A
15. A

Activity

1. I
2. C
3. M
4. A
5. G

Independent Practice 1

1. 2x + 6 and x – 3
2. x = - 3 or (-3, 0)
   2x + 6, in getting the x-intercept use the numerator of the function
3. y = -2 or (0, -2)
   substitute 0 to the x value of the function
4. The zero is at
   x = -3

Independent Assessment 1

x-intercept	y-intercept	Zeroes of f(x)
9	-3	9
5 (multiplicity 2)	5	5(multiplicity 2)
none	-3	none

Independent Practice 2

1. Asymptote
2. True
3. Denominator
4. Horizontal asymptote
5. y = 0
6. x = -1

Independent Assessment 2

	Vertical Asymptote	Horizontal Asymptote
1	x = -5/2 or -2.5	y = 0
2	x = - 7	y = 1
3	x = -5 & x = 4	y = 1
4	x = -4 & x = 1	y = 0
5	x = -2 & x = -2	y = 0

Application

a. 4, 4 + x
b. 8 + x
c. $C(x) = \frac{4+x}{8+x}$
d. x = - 4 or (-4, 0)
   y = 0.5 or (0, 0.5)
e. y = 1

Post - Assessment

1. A
2. B
3. C
4. C
5. B
6. D
7. D
8. C
9. D
10. B
11. C
12. C
13. B
14. C
15. C

Additional Activity

1. a. x = -1 and y = -1/4
   b. x = -1
   c. VA at x = 4
      HA at y = 1
      SA = none
2. a. x = none and
      y = -1.25
   b. none
   c. VA at x = 4
      HA none
      SA none

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11 General Mathematics

General Mathematics
Solving Real-Life Problems Involving Rational Functions, Equations, and Inequalities
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Solving Real-Life Problems Involving Rational Functions, Equations, and Inequalities

Introductory Message

For the facilitator:

Welcome to General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Real-Life Problems Involving Rational Functions, Equations and Inequalities.

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Real-Life Problems Involving Rational Functions, Equations and Inequalities.

The hand is one of the most symbolized parts of the human body. It is often used to depict skill, action and purpose. Through our hands, we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module is written to help you solve real-life problems involving rational functions, equations, and inequalities. You will be introduced to different types of word problems and situations. These problems can be transformed into rational functions, equations, and inequalities. Your focus, patience, and determination will play an important role in dealing with these real-life problems. Moreover, you will also realize that rational functions, equations, and inequalities can be applied in real-life strengthening your problem-solving and modeling experience.

In this module, you will learn how to solve real-life problems involving rational functions, equations, and inequalities. Your acquired skills in solving rational equations and inequalities will be of great help in dealing with this module. Furthermore, your knowledge of representation and problem solving will greatly contribute to accomplishing this module.

After going through this module, you are expected to:

1. solve real-life problems involving rational functions, equations, and inequalities;
2. carefully analyze and understand word problems before solving them; and
3. create real-life word problems about rational functions, equations and inequalities.

In this part, let us see how much you know about the lesson by answering the questions in pre-assessment below. If you obtain 100% or a perfect score, skip the module and immediately move to the next module. While in the event you missed a point, please proceed on the module as it will enrich your knowledge in finding the intercepts, zeroes and asymptotes of rational functions. Let's get started!

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Mayor Rodriguez received 5000 sacks of rice to be distributed among the families in his municipality during the lockdown. If the municipality has x families, write the function which represents the relationship of the allotted sack of rice per family(y-variable) versus the total number of families.
   a. $y = \frac{x}{5000}$ b. $y = \frac{5000}{x}$ c. $y = \frac{5000x}{x}$ d. $y = \frac{x}{5000+x}$

2. To beat the heat of summer, Mang Berto built a rectangular swimming pool that has a perimeter of 200 meters. Write the function which represents the width(y) of the swimming pool as a function of the length(x).
   a. $y = \frac{200}{x}$ b. $y = \frac{x}{200}$ c. $y = \frac{200}{x+1}$ d. $y = 100 - x$

3. It takes Brad 2 hours to mow his rice field. It takes Kris 3 hours to mow the same rice field. At the same pace, how long would it take them to mow the rice field if they do the job together?
   a. 2 ½ hours b. 1 1/5 hours c. 1 1/6 hours d. 5/6 hours

4. Anne and Maria play tennis almost every weekend. So far, Anne has won **12** out of **20** matches. a. How many matches will Anne have to win in a row to improve her winning percentage to **75**%?
   a. 15 b. 12 c. 9 d. 6

5. In a basket, there are 12 apples and 32 oranges. A buyer requires having a basket of apples and oranges with the ratio greater than or equal to 3:4 respectively. How many apples must be added to the basket to satisfy the buyer's request?
   a. 10 apples b. 15 or more apples c. 12 or more apples d. 8 apples

6. Mario was given 3 hours to practice driving his motorcycle. He plans to travel 100 kilometers at an average speed of 40 kilometers per hour. He wants to maximize his time in driving his motorcycle. How many kilometers more does he need to travel to spend at most 3 hours?
   a. less than or equal 20 kilometers
   b. greater than or equal 20 kilometers
   c. exactly 30 kilometers
   d. less than or equal 30 kilometers

7. Jessie works as a salesman. He earns a daily wage of 250 pesos and an additional 10 pesos for every 3 pieces of cell phone sold. If x represents the number of cell phones sold, write the function for his daily earning (y) as a function of the number of cell phones sold (x).

a. $y = \frac{250}{10x}$ b. $y = 250 + 10\left(\frac{x}{3}\right)$ c. $y = 250 + \frac{3}{x} + 10$ d. $y = \frac{2500}{3x}$

8. Using the problem in number 7, if Jessie sold 48 cell phones in a day, how much money did he earn for that day?

a. 410 pesos b. 250 pesos c. 500 pesos d. 480 pesos

9. Melissa walks **3** miles to the house of a friend and returns home on a bike. She averages **4** miles per hour faster when cycling than when walking, and the total time for both trips is two hours. Find her walking speed.

a. 1 mph b. 2 mph c. 3 mph d. 4 mph

10. You have **10** liters of a juice blend that is **60**% juice. How many liters of pure juice needs to be added to make a blend that is **75**% juice?

a. 10 liters b. 8 liters c. 6 liters d. 4 liters

11. If the sum of a number (x) and 3 is divided by 5, the result is greater than 2. What are the possible values for the given number (x)?

a. x > 5 b. x > 7 c. x < 5 d. x < 7

12. During a pandemic, Brgy. Captain Gerry was given 1,000,000 pesos to support 500 households in his barangay. He plans to give at least 3,000 pesos for every household. How much money does he need to solicit to realize his plan?

a. at least 300,000 b. at least 400,000 c. at least 500,000 d. at least 100,000

13. Coronavirus infection is spreading fast worldwide. The number of people infected by the virus each day is given by the function $P(x) = \frac{100x}{x+3}, 0 \leq x \leq 10$ where x is the number of days, and $P(x)$ is the number of people infected (in thousands). How many people are infected on the first day?

a. 25 b. 25,000 c. 50,000 d. 75,000

14. Sir Paco is thrice as old as his son Javy. 10 years from now, the ratio of their ages will be 2:1 respectively. How old is Javy?

a. 5 b. 15 c. 12 d. 10

15. As part of his exercise routine, Jerson runs 20 kilometers at an average speed of 3 kilometers per hour. If he decided to run at most 2 hours on a specific day, how may kilometers less does he need to run?

a. at least 14 km b. at most 14 km c. exactly 14 km d. less than 14 km

Lesson 1 Problems Involving Rational Functions, Equations, and Inequalities

To be able to solve problems involving rational functions, equations, and inequalities, it is necessary to know the basics of algebra. Solving rational equations and inequalities is very essential in solving word problems. Real-life problems like mixture, work, distance, number, and other related problems might interest you. If you are fond of observing your speed while driving, estimating your time while walking, analyzing your income while selling, determining price increases and decreases, identifying rational relationships and solving real-life problems, then, this module is definitely for you to enjoy!

What's In

Before you proceed to the new lesson, let us recall first what you have learned from the previous lessons so that you will be ready to solve real-life problems involving rational function, rational equation, and rational inequality.

MATCH AND SOLVE!

A. Study the data inside the box and write it in the appropriate column on the next page.

$f(x) = \dfrac{x^2 - 2x + 4}{x}$	$-1 < x < 5$	$\dfrac{4}{x} - \dfrac{x-1}{5} = \dfrac{14}{15}$
$\dfrac{4}{x-2} > 2$	3	$\dfrac{x+1}{x-5} \leq 0$
$\dfrac{4}{x} = \dfrac{x+3}{10}$	5	$y = \dfrac{1000 + x}{20}$
	$-2 < x < 0$	

Rational Equation	Solution to Rational Equations	Rational Functions	Rational Inequalities	Solution to Rational Inequalities

Recall your skill in solving a rational equation and rational inequalities to match the correct data in the appropriate column. This skill is a prerequisite in this module because you cannot solve real-life problems involving rational functions, equation, and rational inequalities if you do not master your previous skill. In that case, let me help you.

On the given, you observed that $\boldsymbol{f(x) = \frac{x^2-2x+4}{x}}$ and $\boldsymbol{y = \frac{1000+x}{20}}$ are written in the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are both polynomial functions, therefore these are examples of rational functionals provided that $q(x)$ is not equal to zero. While $\boldsymbol{\frac{4}{x} = \frac{x+3}{10}}$ and $\boldsymbol{\frac{4}{x} - \frac{x-1}{5} = \frac{14}{15}}$ are both rational equations because they involve rational expressions. Intuitively, you may think that 3 and 5 are the solutions but you need to solve it for you to see the result. On the other hand, $\boldsymbol{\frac{x+1}{x-5} \leq 0}$ and $\boldsymbol{\frac{4}{x-2} > 2}$ are rational inequalities because they are inequalities that involve rational expressions. If you master the skills in solving them, I am sure you got the correct data on the appropriate column.

If you think you are not confident that you are correct, review first your previous lesson before you proceed to take this module, But I am sure, you will do your part because you are willing to learn.

What's New

Speed Me Up!

Read and analyze each situation below and answer the questions that follow.

Mario rides his motorcycle in going to school. He drives at an average speed of 30 kilometers per hour. The distance between his house and the school is 15 kilometers. Every time he sees his best friend Jessica walking on the road, he invites her for a ride and lowers his speed. On the other hand, he increases his speed when he wakes up late for school.

Questions:

a. How long does it take Mario to reach school considering his average speed?

---

b. If x represents the time it takes Mario to drive to school with the given distance of 15 kilometers, how will you represent the relationship of his speed (y) versus the time (x)?

c. Mario's average speed as 30 kilometers per hour. Suppose Mario lowers his speed by 10 kilometers per hour, how long will he reach the school given the same distance?

---

d. Suppose Mario's speed is unknown and represented by (x), he lowers his speed by 10 kilometers per hour at a distance of 15 kilometers and reaches school at $\frac{3}{4}$ hours, how will you write the equation to find his average speed (x)?

e. Mario's average speed was 30 kilometers per hour. He plans to drive for another 30 kilometers from school, how long will it take him to cover the whole distance (house to school to 30 kilometers from school)?

f. If Mario drives another (x) kilometers from his school at an average speed of 30 kilometers per hour and he plans to drive in at most 2 hours, how will you write the inequality to find the additional distance?

What is It

The Speed Me Up Activity is an example of the real-situation involving rational equation and inequality, and to be able to answer the questions given above, it is very important to know the distance-speed-time relationship. The following illustrates these relationships.

$$Distance = Speed\ x\ Time$$

$$Time = \frac{Distance}{Speed}$$

$$Speed = \frac{Distance}{Time}$$

The relationships above, were emphasized when you are in junior high school in the lesson solving distance problem. I am sure that these are familiar to you.

To answer question number 1, you need to consider that the word "how long" pertains to time. Thus, by dividing the distance by the speed, $t = \frac{D}{s}$, you get the time: $Time = \frac{15}{30}\ or\ \frac{1}{2}\ hours$(30 minutes).

Question number 2 requires you to represent the given situation into a functional relationship between speed (y) and time (x) considering a distance of 15 kilometers. Since $s = \frac{D}{t}$, we were able to write $y = \frac{15}{x}, where\ x \neq 0$.

The process of answering question number 3 also considers the distance-speed–time relationship. Since Mario lowers his speed by 10 kilometers per hour, it will take him longer to reach his destination. Thus, we use the formula $t = \frac{D}{s}$.

$$t = \frac{15}{30 - 10}$$

$$t = \frac{15}{20}\ or\ \frac{3}{4}\ hours(45\ minutes)$$

Question number 4 asks you to write the equation in case Mario's speed is unknown or missing. Since x represents Mario's speed, lowering his speed by 10 kilometers per hour will be written as "x – 10". Again, considering the distance-speed-time relationship, we arrive at the equation:

$$t = \frac{D}{s}$$

$$\frac{3}{4} = \frac{15}{x - 10}$$

Solving this equation will give you 30 as his average speed.

To answer question number 5, you need to understand that there is an additional distance of 30 kilometers. The total distance is now 45 kilometers. (Adding 15 and 30). Since the speed remains at 30 kilometers per hour, and we are looking for time, we arrive at the equation:

$$t = \frac{D}{s}$$

$$t = \frac{45}{30} \; or \; 1.5 \; hours(1 \; hour \; and \; 30 \; minutes)$$

Question number 6 requires you to write rational inequality to be able to find the additional distance. Additional distance will be represented by x and the total distance will be "15 + x". Since his speed remains at 30 kilometers per hour and the time that will require him to cover the distance is at most 2 hours (less than or equal to 2), we write the inequality:

$$\frac{D}{s} \leq t$$

$$\frac{15 + x}{30} \leq 2$$

Solving this inequality will give $x \leq 45$. Mario needs to travel an additional distance of not more than 45 kilometers to spend at most 2 hours.

The idea of riding a motorcycle seems very enjoyable. But, always bear in mind that accidents may happen. So, be cautious and consider safe driving by following street rules. Just like analyzing Math problems, little by little, we would arrive at answers if we only know how to follow rules.

Another skill that you will learn in this module is solving real-life problems involving rational function. Consider the examples below:

Example 1

Bamban National High School is preparing for its 25th founding anniversary. The chairperson of the activity allocated ₱90,000.00 from different stakeholders to be divided among various committees of the celebration. Construct a function $C(n)$ which would give the amount of money each of the n numbers of committees would receive. If there are six committees, how much would each committee have?

Solution:

The function $C(n) = \frac{90000}{n}$ would give the amount of money each of the n numbers of committees since the allocated budget is ₱90,000.00 and it will be divided equally to the n number of committees.

If there are six committees, then you need to solve for $C(6)$, thus

$C(6) = \frac{90000}{6} = 15000$

Therefore, each committee will receive ₱15,000.00.

Example 2

Barangay Masaya allocated a budget amounting to ₱100,000.00 to provide relief goods for each family in the barangay due to the Covid-19 pandemic situation. The amount is to be allotted equally among all the families in the barangay. At the same time a philanthropist wants to supplement this budget and he allotted an additional ₱500.00 to be received by each family. Write an equation representing the relationship of the allotted amount per family (y-variable) versus the total number of families (x-variable). How much will be the amount of each relief packs if there are 200 families in the barangay?

Solution:

The amount to be received by each family is equal to the allotted (₱100,000.00), divided by the number of families plus the amount to be given by the philanthropist. Thus the rational function is described as $y = \frac{100000}{x} + 500$. The amount of each relief packs can be computed by finding the value of y when $x = 50$, since there are 50 families in the barangay. Thus,

$$y = \frac{100000}{200} + 500 = 1000$$

Therefore, the amount of each relief packs to be distributed to each family worth ₱1,000.00.

Notes to the Teacher

Remind students that they must: (a) read and analyze the problem carefully, (b) paraphrase and summarize the problem in their own words, (c) find an equation that models the situation, and (d) say how it represents the quantities involved and (e) check to make sure that they understand the problem before they begin trying to solve it.

What's More

Read each problem carefully. Answer them and write your answers on a separate sheet of paper.

Practice Activity 1

A Garden Plot

Vincent is a farmer. He loves to plant vegetables. He found that the area of his rectangular garden is 200 square meters. Let x represent the width of his garden in meters, express the length of the garden L as a function of width x.

Complete the following to solve the problem.

a. The formula in finding the area of a rectangle is _____.
b. Given an area (A) and width (x), the formula in finding length (L) of a rectangle is _____.
c. Using the formula, we may express the length of the garden (L) as a function of width (x) as:

$$L(x) = _____$$

Independent Assessment 1

Triangular Kite

Marco has a triangular kite. The area of the kite is 320 square centimetres. Let x represent the height of the kite in centimetres, express the base of the kite *(B)* as a function of height *x*.

Practice Activity 2

Do it Together

Rodalyn can do a job in 5 days while Apple can do the same job in 3 days. How long will it take them to do the job if they work together?

Complete the following to solve the problem.

a. The part of the job accomplished by Rodalyn on the first day is $\frac{1}{5}$, So, the part of the job accomplished by Apple on the first day is _____.
b. If x represents the time it will take them to do the job together, the part of the job accomplished on the first day of working together is _____.
c. Looking at the relationship, we arrive at the equation:

$$\frac{1}{5}+\frac{1}{3}=\frac{1}{x}$$

d. Solving the rational equation, the value of x is _____.Working together, they can finish the job in ____ day and_____ hours.

Independent Assessment 2

Paint my Wall
Analiza can paint a room in 3 hours. Leoben can do it in 2 hours. Walter can do the painting job in 5 hours. If all of them worked together, how long will it take them to paint the room?

Practice Activity 3

Mix mix mix!
How many liters of pure alcohol must be added to 30 liters of 20 % alcohol solution to make a 25% alcohol solution.

a. Complete table to understand the relationship.

	Original	Added	Result
Concentration	20% = $\frac{20}{100}$	100% = 1	25% = $\frac{25}{100}$
Amount	30 liters	x	30 + x
Multiply	$\frac{20}{100}(30)$	1(x)	?

Note: We use 100% or 1 because pure alcohol was added.

b. Use the relationship to make an equation.

$$\frac{20}{100}(30) + 1(x) = \frac{25}{100}(30 + x)$$

c. Solve the equation by finding the value of x. Multiply the whole equation by LCM which is 100.

$$600 + _____ = _____(30 + x)$$
$$600 + 100x = 750 + 25x$$
$$75x = 150$$
$$x = _____.$$

Independent Assessment 3

Salt solution
Joey has 40 liters of 10% salt solution. How much salt should be added to make it a 20% salt solution?

Practice Activity 4

Volume of a Box
A box with a square base has a volume of 27 cubic inches. If x is the length of its edge and h is the height of the box. What are the possible measurement of its edge if the height should be longer than the edge?

Complete the following to solve the problem.

a. The formula to find the volume of the box is _____.

b. The equation relating to find the value of h is _____.

Since the height is greater than the length of the edge, the inequality can be described as

$$\frac{27}{x^2} - x > 0$$

c. The possible value of x should be _____.

(Hint: Solve for x in the inequality $\frac{27}{x^2} - x > 0$.)

Independent Assessment 4

Who am I?
I am thinking of a number, the sum of twice a number and 8 divided by 12 is greater than or equal to 4. Find the number/numbers.

Independent Assessment 5

Growing Bacteria

Suppose the amount of bacteria growing in a petri dish is represented by the function $B(t) = \frac{100t}{t+2}$ for $0 < t < 15$ where t is in hours and B is in millions. How may bacteria will there be after 10 hours?

What I Have Learned

A. Complete the following statements by writing the correct word or words and formulas.

1. _____is any function which can be defined by a rational fraction, an algebraic fraction such that both the numerator and the denominator are polynomials.

2. An inequality which contains a rational expression is _____.

3. An equation containing at least one fraction whose numerator and denominator are polynomials is called _____.

4. The three formulas which show the relationship among distance, time and speed are:_____, _____ and _____.

B. In your own words, write the different steps to solve real-life problems involving rational functions, equations, and inequalities.

_____

_____

_____

_____

_____

What I Can Do

I Believe I Can Apply!

Observe the surroundings and try to create 3-word problems involving rational functions, equations, and inequalities and show also the solutions to the problems that you created
Here are some of the suggested topics in creating real-life problems:

- Daily sales of your crops
- Number of relief goods in your barangay
- Distance-time-speed relationship in traveling from your place to the town proper
- Rice production
- Areas of your rice fields
- Wage/salary of workers
- Population in your place
- Demand and supply of agricultural products
- Area/Perimeter of a place/structure
- Mixing of brands of rice/sugar

Rubrics for the task:

Categories	**Excellent (4)**	**Satisfactory (3)**	**Developing (2)**	**Beginning (1)**
Representation	Shows a complete understanding of the concept of rational functions, equations, and inequalities.	Shows a partial understanding of the concept of rational functions, equations, and inequalities.	Shows limited understanding of the concept of rational functions, equations, and inequalities.	Not evident
Computation and Solution	Computation is correct and leads to the correct answer	Computation is correct but does not lead to the correct answer	Computation is incorrect and does not relate to the task.	Not evident
Communication	Explained the steps clearly and accurately.	Explained the steps clearly.	Explained the steps,but there some parts which are not clear.	Not evident

Assessment

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Gov. Suarez bought 1000 units of television to be given to disabled persons in Quezon province. If the province has chosen x, disabled persons, write the function which represents the relationship of the allotted unit of television per disabled person(y-variable) versus the total number of disabled persons.

   a. $y = \frac{1000}{x}$ b. $y = \frac{x}{1000}$ c. $y = \frac{1000x}{x}$ d. $y = \frac{x}{1000+x}$

2. Aling Nena cuts a rectangular cloth with a perimeter of 150 meters. Write the function which represents the width(y) of the cloth as a function of the length(x).

   a. $y = \frac{150}{x}$ b. $y = \frac{x}{150}$ c. $y = \frac{150}{x+1}$ d. $y = 75 - x$

3. Maryjoy can bake a cake in 2 hours. Clarissa can do it in 4 hours. How long will it take them to bake a cake if they joined together?

   a. 2 hours b. 1 1/3 hours c. 1 1/6 hours d. 5/3 hours

4. James and Tony play billiard every weekend. So far, James has won 8 out of 14 matches. How many matches will James have to win in a row to improve his winning percentage to 80%?

   a. 16 b. 15 c. 14 d. 12

5. In a jar, there are 10 blue marbles and 15 red marbles. A buyer wants to buy a jar of blue marbles and red marbles with the ratio greater than or equal to 4:5 respectively. How many blue marbles must be added in the jar in order to sell the marbles?

   a. 8 b. 6 c. at least 1 d. at least 2

6. Nerissa was given 2 hours to walk for her morning exercise. She plans to walk 5 kilometers at an average speed of 3 kilometers per hour. How many kilometers more does she need to walk to spend at most 2 hours?

   a. less than or equal 2 kilometers
   b. greater than or equal 2 kilometers
   c. exactly 3 kilometers
   d. less than or equal to 1 kilometer

7. Nimby works as a vendor. He earns a daily wage of 100 pesos and an additional 5 pesos for every 2 pieces of mangoes sold. If x represents the number of mangoes sold, write the function for his daily earning (y) as a function of the number of mangoes sold (x).

a. $y = \frac{100}{x}$ b. $y = 100 + 5\left(\frac{2}{x}\right)$ c. $y = 100 + 5\left(\frac{x}{2}\right)$ d. $y = \frac{1500}{2x}$

8. Using the problem in number 7, if Nimby sold 20 mangoes in a day, how much money did he earn for that day?

a. 120 pesos b. 150 pesos c. 200 pesos d. 130 pesos

9. A boy traveled by train which moved at the speed of 30 mph. He then boarded a bus that moved at the speed of 40 mph and reached his destination. The entire distance covered was 100 miles and the entire duration of the journey was 3 hours. Find the distance he traveled by bus.

a. 50 miles b. 40 miles c. 30 miles d. 20 miles

10. Sterling Silver is 92.5% pure silver. How many grams of Sterling Silver must be mixed to a 90% Silver alloy to obtain a 500g of a 91% Silver alloy?

a. 200 grams b. 400 grams c. 300 grams d. 100 grams

11. Seven divided by the sum of a number and two is equal to half the difference of the number and three. Find all such numbers.

a. -5 and 4 b. 10 and -2 c. 5 and -4 d. -10 and 2

12. Mayor Eleazar solicited 500, 000 pesos to be given to families affected by typhoon Ambo. If he plans to give at least 10,000 pesos for each of the 100 families, how much more money does he need to solicit?

a. at least 500,000 b. at least 400,000 c. at least 300,000 d. at least 200,000

13. A box with a square base and no top is to be constructed so that it has a volume of 1000 cubic centimeters. Let x denote the width of the box, in centimeters. Express the height h in centimeters as a function of the width x.

a. $h(x) = \frac{1000}{x^2}$ b. $h(x) = \frac{1000}{x^3}$ c. $h(x) = \frac{1000}{x}$ d. $h(x) = 1000x$

14. Yen-yen got an average grade of 91 on her 4 subjects. What must be her grade on the fifth subject to get an average of 92?

a. 94 b. 95 c. 96 d. 97

15. The area of a rectangle is $x^2 + 3x - 10$. If it has a side length of $2x - 4$, then the width can be represented by the expression

a. $\frac{x-5}{2}$ b. $\frac{2}{x-5}$ c. $\frac{x+5}{2}$ d. $\frac{2}{x+5}$

Additional Activities

A. Think about this!

A boat that can travel fifteen miles per hour in still water can travel thirty-six miles downstream in the same amount of time that it can travel twenty-four miles upstream. Find the speed of the current in the river.

B. Visit the following links for more lectures and activities about word problems involving rational functions, equations and inequalities.

- https://www.youtube.com/watch?v=09byllGu88Q
- https://www.youtube.com/watch?v=rX8ZBP3nXvI
- https://www.youtube.com/watch?v=gD7A1LA4jO8
- https://www.youtube.com/watch?v=4-a6tkwHZEM
- https://www.youtube.com/watch?v=QLhvLEeS08A

Answer Key

What I Know

1. B
2. D
3. B
4. B
5. C
6. A
7. B
8. A
9. B
10. C
11. B
12. C
13. B
14. D
15. A

What's More

Practice Activity 1

a. The formula in finding the area of a rectangle is A = L x W.
b. Given an area (A) and width (x), the formula in finding length (L) of a rectangle is L = $\frac{A}{x}$.
c. Using the formula, we may express the length of the garden (L) as a function of width (x) as:

$$L(x) = \frac{200}{x}$$

Independent Assessment 1

a. The formula in finding the area of a triangle is A = $\frac{b\,(h)}{2}$.
d. Given an area (A) and height (x), the formula in finding base of a triangle is
$b = \frac{2A}{x}$.
e. Using the formula, we may express the base of the triangle (B) as a function of height (x) as:

$$B(x) = \frac{640}{x}$$

Practice Activity 2

a. The part of the job accomplished by Rodalyn on the first day is $\frac{1}{5.}$
b. The part of the job accomplished by Apple on the first day is $\frac{1}{3}$.
c. If x represents the time it will take them to do the job together, the part of the job accomplished on the first day of working together is $\frac{1}{x}$.
d. Looking at the relationship, we arrive at the equation:

$$\frac{1}{5} + \frac{1}{3} = \frac{1}{x}$$

e. Solving the rational equation, the value of x is $\frac{15}{8}$ or 1 day and 21 hours. Working together, they can finish the job in 1 day and 21 hours.

Independent Assessment 2

a. The part of the job accomplished by Analiza on the first day is $\frac{1}{3.}$
b. The part of the job accomplished by Leoben on the first day is $\frac{1}{2}$.
c. The part of the job accomplished by Walter on the first day is $\frac{1}{5}$.
d. If x represents the time it will take them to do the job together, the part of the job accomplished on the first day of working together is $\frac{1}{x}$.
e. Looking at the relationship, we arrive at the equation:

$$\frac{1}{3} + \frac{1}{2} + \frac{1}{5} = \frac{1}{x}$$

f. Solving the rational equation, the value of x is $\frac{30}{31}$ $or\ 23\ hours$.Working together, they can finish the job 23 hours.

Independent Assessment 4

The amount of bacteria growing in a petri dish is given by the function $B(t) = \frac{100t}{t+2}$ for $0 < t < 15$ where t is in hours and B is in millions.

Substituting t = 10, we have:

$$B(10) = \frac{100(10)}{10+2}$$

$$B(10) = 83.33$$

The amount of bacteria growing in a petri dish after 10 hours is 83,333,333.33.

Assessment

1. A
2. D
3. B
4. A
5. D
6. D
7. C
8. B
9. B
10. A
11. C
12. A
13. A
14. C
15. C

What's More Continuation

Practice Activity 3

a. Complete table to understand the relationship.

	Original	Added	Result
Concentration	20% = $\frac{20}{100}$	100% = 1	25% = $\frac{25}{100}$
Amount	30 liters	x	30 + x
Multiply	$\frac{20}{100}(30)$	1(x)	$\frac{25}{100}(30+x)$

Note: We use 100% or 1 because pure alcohol was added.

b. Use the relationship to make an equation.

$$\frac{20}{100}(30) + 1(x) = \frac{25}{100}(30+x)$$

c. Solve the equation by finding the value of x. Multiply the whole equation by LCM which is 100.

$$600 + 100x = 25(30+x)$$
$$600 + 100x = 750 + 25x$$
$$75x = 150$$
$$x = 2\ liters$$

Independent Assessment 3

a. Complete table to understand the relationship.

	Original	Added	Result
Concentration	10% = $\frac{10}{100}$	100% = 1	20% = $\frac{20}{100}$
Amount	40 liters	x	40 + x
Multiply	$\frac{10}{100}(40)$	1(x)	$\frac{20}{100}(40+x)$

Note: We use 100% or 1 because pure salt was added.

b. Use the relationship to make an equation.

$$\frac{10}{100}(40) + 1(x) = \frac{20}{100}(40+x)$$

c. Solve the equation by finding the value of x. Multiply the whole equation by LCM which is 100.

$$400 + 100x = 20(40+x)$$
$$400 + 100x = 800 + 20x$$
$$80x = 400$$
$$x = 5\ liters$$

Practice Activity 4

Complete the following to solve the problem.

a. The formula to find the volume of the box is V = (x)(x)(h).

b. The equation relating to find the value of h is h=$\frac{V}{x^2}$.

Since the height is greater than the length of the edge, the inequality can be described as

$$\frac{27}{x^2} - x > 0$$

c. The possible value of x should be x < 3.

Independent Activity 4

a. The sum of twice a number (x) and can be written as 2x + 8.

b. The sum of twice a number (x) and 8 divided by 12 can be written as $\frac{2x+8}{12}$.

c. Lastly the whole equation can be written as $\frac{2x+8}{12} \geq 4$.

d. In solving the inequality, we make use of cross multiplication. Then, we solve for the value of x.

e. Finally, the answer is $x \geq 20$

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*DepED Material: General Mathematics Learner's Material

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11 General Mathematics

General Mathematics
One- to-One Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics
One-to-One-Functions

Introductory Message

For the facilitator:

Welcome to the Alternative Delivery Mode (ADM) Module on One-to-One Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the Alternative Delivery Mode (ADM) Module on One-to-One Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you to assess your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical problems. These knowledge and skills will help you understand one-to-one functions. As you go through this lesson, think of this important question: "*How one-to-one functions represents real life situations"?* To find answer, read and perform each activity.

In this module, the learners are expected to demonstrate understanding of key concepts of inverse functions, exponential functions, and logarithmic functions. Learners should also be able to apply concepts of inverse functions, exponential functions, and logarithmic functions to formulate and solve real-life problems with precision and accuracy.

The module

- Lesson 1 – One-to-One Functions

After going through this module, you are expected to:

1. determine if a function is a one-to-one.
2. identify real-life situation using one-to-one function.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. It is a rule which associates each element of set A with at least one element in set B.
   a. Function
   b. Relation
   c. Set
   d. Subset

2. It is a rule which uniquely associates elements of one set A with the elements of another set B; each element in set A maps to a single element in set B.
   a. Function
   b. Relation
   c. Set
   d. Subset

3. It associates two or more values of the independent (input) variable with a single value of the dependent (output) variable.
   a. One-to-one
   b. One-to-many
   c. Many-to-one
   d. Many-to-many

4. It is a single x-value that relates to two different y-values.
   a. One-to-one
   b. One-to-many
   c. Many-to-one
   d. Many-to-many

5. A single x-value relates to only one unique y-values.
   a. One-to-one
   b. One-to-many
   c. Many-to-one
   d. Many-to-many

6. Mabuhay National High School has its own School ID which is 143142 while other school also has their own school ID. Which rule represent the given statement?
   a. One-to-one
   b. One-to-many
   c. Many-to-one
   d. Many-to-many

7. Which of the following does not represent one-to-one function?
   a. My father to its child.
   b. Facebook name to password.
   c. Student's Name to Learner's Reference number (LRN).
   d. Cellphone Number to the owner.

8. Which of the following table of values represent one-to-one function?

a.

Wife	Husband
Raiza	Anthony
Mitchie	Jeff
Sarah	Jordan

c.

x	y
a	b
b	c
d	e

b.

x	y
Klara	Iphone
Kath	Iphone
Loraine	Samsung
Ana	Vivo

d.

x	y
-4	17
-2	5
0	1
2	5
6	17

9. Below is a sample of Venn diagram, what figure doesn't belong to the group?

a.

c.

b.

d.

10. If a _____ can intersect the graph of the function, more than one time then the function is not mapped as one-to-one.

a. Vertical Line Test
b. Horizontal Line Test
c. T-test
d. Z-test

11. Which of the following graph represent a one-to-one function?

a.

c.

b.

d.

12. Functions can be written as _____.
    a. ordered pairs
    b. tables
    c. graphs
    d. all of the choices

13. Let A = {10, 20, 30} and B = {Pandesal, Yema Cake, Mamon, Ensaymada}. Which of the following is a one-to-one function?

    a. {(10,pandesal),(20,Mamon), (30,Pandesal)}
    b. {(10,Yema Cake), (20,Ensaymada), (30,pandesal)}
    c. {(10,pandesal), (20,pandesal), (30,pandesal)}
    d. {(10,Mamon), (20,Yema Cake), (10,Pandesal), (30,Ensaymada)}

14-15. Below are the statements that may represent real life situation using one-to-one function.

14. Which of the following is not included?
    i. One person has one passport.
    ii. A shoe has one place on which you would wear it (your foot).
    iii. Paper has one source.
    iv. A washing machine has two function (to wash)

    a. i only
    b. ii and iii only
    c. iii and iv
    d. iv only

15.Which of the following is an example of one-to-one function?

a. i and ii
b. ii and iv
c. iii and iv
d. i and iv

Lesson 1 Represent Real-Life Situation using One-to-One Functions

Start Lesson 10 of this module by assessing your knowledge of representing real-life situation using one-to-one functions. These knowledge and skill will help you understand easily on how to represent real-life situation using one-to-one functions. Seek the assistance of your teacher if you encounter any difficulty.

What's In

Study the graph below, write the values of y in the table below.

Figure 1

Figure 2

x	y
-2	
-1	
0	
1	
2	

x	y
-2	
-1	
0	
1	
2	

Now you have recall identify the values of x, answer the following questions.

1. What are the values of y in figure 1 and figure 2?
2. What have you noticed on their values?
3. Is the value of x in figure 1 have the same value in y? How about figure 2?
4. Draw horizontal lines each figure. How many times does the horizontal line intersect on figure 1 and figure 2?
5. What function do you call when no two ordered pairs that have the same first component have different second component?

Notes to the Teacher

When working on the coordinate plane, a function is a **one-to-one function** when it will pass the **vertical line test** (to make it a *function*) and also a **horizontal line test** (to make it *one-to-one*).

What's New

Contact five (5) of your classmates to write their Learner's Reference Number (LRN) on the table provided below.

Name of the Member	Learner's Reference Number (LRN)

Questions:

1. What did you observe from the table? Did you notice any repeated LRN?
2. What do you think is the reason why learners have their own LRNs?
3. What kind of function is depicted from the given activity?

What is It

One-to-One Functions

A function f is one-to-one if it never takes the same value twice or $f(x_1) \neq f(x_2)\ whenever\ (x_1) \neq (x_2)$. That is, the same y-value is never paired with two different x-values.

In the Venn diagram below, function f is a one-to-one since not two inputs have a common output.

Figure 1. Venn diagram of a one-to-one function.

In the Venn diagram below, function f is NOT a one-to-one since the inputs -1 and 0 have the same output.

Figure 2. Venn diagram of a function that is not a one-to-one.

On the other hand, the function g(x) = x^2 is not a one-to-one function, because g(−1) = g(1). There are a lot of real-life applications of a one-to-one function. Determine whether the given relation is a function. If it is a function, determine whether it is one-to-one.

Example 1: The relation pairing an SSS member to his or her SSS number.

Solution:
Each SSS member is assigned a unique SSS number. Thus, this relation is a function. Further, two members cannot be assigned the same SSS number, therefore, the function is one-to-one.

Example 2: The relation pairing a citizenship to a person.

Solution:
The relation is a function because each person has a citizenship. However, a person can have two citizenship, (dual citizen) therefore, it is not one-to-one function.

Graph of a One-to-one Function

If f is a one-to-one function then no two points (x_1, y_1) *and* (x_2, y_2) , have the same y-value. Therefore, no horizontal line cuts the graph of the equation y = f(x) more than once. Example. Compare the graphs of the above functions

How to Determine if a Function is One-to-One

Horizontal Line test: A graph passes the Horizontal Line Test if each horizontal line cuts the graph at most once.

A function f is one-to-one if and only if the graph y = f(x) passes the horizontal Line test.

Example. Which of the following functions are one-to-one?

Figure 3 Figure 4

Figure 3. shows that the horizontal line test intersects more than one, while the other horizontal line test intersects no more than one. It means that Figure 4 is an example of one-to-one function.

What's More

Activity 1.1 Understanding One-to-One Functions

Determine whether each of the following situations is a one-to-one function. Elaborate your answer.

1. The relation of a dog to its family members.

   _____
   _____
   _____

2. The relation of a person to his or her passport.

   _____
   _____
   _____

3. A car model to its manufacturer company.

   _____
   _____
   _____

4. A shark to where it lives.

   _____
   _____
   _____

5. True or False questions to answers.

   _____
   _____
   _____

Activity 1.2 True or False

Identify whether the following represents one-to-one functions.

1. Degree Fahrenheit to its equivalent degree Celsius measurement
2. Person to his/her citizenship
3. Fare to the distance travelled
4. Cellphone to its cellphone number
5. Length in meters to its length in inches.
6. Father to his first biological son.
7. ATM Card Number to account name
8. Person to his favorite music.
9. House to telephone number.
10. Brand name to pair of shoes.

What I Have Learned

A. Complete the statements below.

1. _____ is a set of ordered pairs in which no two ordered pairs that have the same first component have different second components.
2. When working on the coordinate plane, a function is a one-to-one function when it will pass the _____ (to make it a *function*) and also a _____ (to make it *one-to-one*).
3. Is the Function f:(m,3), (a,2), (t,9), (h,4) represents one-to-one functions? If yes, why? _____
4. In the diagram below, set A is the _____ of the function and set B is the _____of the function.

5. In a one-to-one function, given any *y* value, there is only one *x* that can be paired with the given *y*. Such functions are also referred to as _____.

B. Which of the following graph shows one-to-one function? State the reason below.

Graph A

Graph B

Graph C

Graph D

What I Can Do

Now that you have deeper understanding of the topic, you are ready to solve the problems below.

Let the students bring several round containers or lids and record the diameter and circumference in a table.

If diameter is the input and circumference is the output, what's the function rule?

As they divide each container's circumference by its diameter to find that rule, they should notice a constant ratio -- a rough approximation of pi.

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. It is a rule that produces a correspondence between the elements of two sets: D (domain) and R (range), such that to each element in D there corresponds one and only one element in R.
   a. Function
   b. Relation
   c. Set
   d. Subset

2. A graph of a function can also be used to determine whether a function is one-to-one using the _____.
   a. vertical line test
   b. horizontal line test
   c. t-test
   d. z-test

3. A method of testing whether a graph represents a function by determining whether a vertical line intersects the graph no more than once.
   a. vertical line test
   b. horizontal line test
   c. t-test
   d. z-test

4. Which of the following is not a one-to-one function?

a.

X	-5	-3	0	3
Y	8	4	0	-4

b.

X	-5	-3	1	3
Y	-5	6	2	-4

c.

X	-5	-3	0	3
Y	-5	-3	-5	-4

d.

X	-5	-3	1	3
Y	8	-3	0	-5

5. Which type of relation wherein every element in the domain is paired with exactly one element in the range?
   a. Function
   b. Asymptote
   c. Inverse
   d. Composite

6. Which of the following relationships DOES NOT indicate a one-to-one function?
   a. A tricycle and its plate number
   b. Chemical symbol to its chemical element
   c. Parents and their children
   d. Husband and Wife

7. Which of the following represents a one-to-one function?
   a. Teacher to students
   b. Student to their LRN
   c. Mother to her children
   d. Students to teacher

8. Consider the graph below. Which of the following line test crosses the graph of a function at no more than one point?

Line Test X

Line Test Y

a. Line Test X only
b. Line Test Y only
c. Both Line Test X and Y
d. None of the following.

9. Which of the following graphs represents a one-to-one function?

a.

c.

b.

d.

10. Which of the following statement represent one-to-one-function?
    a. One person has one passport.
    b. A car model is made by one company.
    c. A house building prototype belongs to one company.
    d. All of the choices

11. Which of the following does not represent one-to-one function?
    a. Gas has one function when put in a car.
    b. A house belongs to one person.
    c. A washing machine has one function (to wash)
    d. An x-ray is associated with the one function of taking internal pictures.

12. The input values make up the _____, and the output values make up the _____.
   a. Domain, horizontal line test
   b. Range, horizontal line test
   c. domain, range
   d. range, domain

13. The coffee shop menu, shown in figure below consists of items and their prices. Does the menu represent one-to-one function?

MENU	
Dunkin Donut	₱25.00
Krispy Kreme	₱45.00
Mister Donut	₱25.00

   a. Yes, because each item on the menu has only one price, so the price is a function of the item.
   b. Yes, because one item on the menu has only one price, so the price is a function of the item.
   c. No, because the two items on the menu have the same price.
   d. No, because one item on the menu have the same price.

14. The table shows the lists of five greatest volleyball players of all time in order of rank. Is the rank a function of the player name? Is the player name a one-to-one function of the rank?

Player	Rank
Tokyo	1
Nairobi	2
Alicia	3
Lisbion	4
Manila	5

   a. Yes
   b. No
   c. Maybe
   d. I don't know.

15. Is the area of a circle a function of its radius? Which of the following statement proves that the area of a circle a function of its radius.
   a. A circle of radius r has a unique area measure given by A= πr2, so for any input r, there is only one output, A.
   b. If the function is one-to-one, the output value, the area, must correspond to a unique input value, the radius.
   c. Any area measure A is given by the formula A=πr2. Because areas and radii are positive numbers, there is exactly one solution: $\sqrt{\frac{A}{\pi}}$.
   d. All of the choices.

Additional Activities

Below are words which can be associated with one-to-one functions. Write a statement below that may prove it is an example of one-to-one function.

Example: Passport ID

Answer: A person has only one passport ID.

1. Citizenship
2. Fare
3. Car
4. Area of a circle.
5. Soap

Answer Key

What I Know

1. B
2. A
3. C
4. B
5. A
6. A
7. A
8. A
9. B
10. B
11. C
12. D
13. B
14. D
15. A

What's More

1. Since the dog is related to all the family members, therefore it is not one-to
   one.
2. A certain passport can only belong to a certain person, therefore it is one-to
3. one.
4. 3. A manufacturer produces thousands of car for a certain model, therefore it is
5. not one-to-one.

Assessment

1. A
2. B
3. A
4. C
5. A
6. C
7. B
8. B
9. A
10. D
11. C
12. C
13. C
14. A
15. D

What I Have Learned

A

1. Function
2. Vertical Line Test, Horizontal Line Test
3. Yes, by the definition of one-to-one function.
4. Domain, Range
5. Injective.

B

Graph A. This cubic function is indeed a "function" as it passes the vertical line test. In addition, this function possesses the property that each *x*-value has one unique *y*-value that is not used by any other *x*-element. This characteristic is referred to as being a 1-1 function.

Notice that this function passes BOTH a vertical line test and a horizontal line test.

Graph B. This absolute value function passes the vertical line test to be a function. In addition, this function has *y*-values that are paired with more than one *x*-value, such as (4, 2) and (0, 2). This function is not one-to-one.

This function passes a vertical line test but not a horizontal line test.

Answer Key

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11 General Mathematics

General Mathematics
The Inverse of One-to-one Function
First Edition, 2020

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Published by the Department of Education
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Undersecretary: Diosdado M. San Antonio

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General Mathematics

The Inverse of One-to-one Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics 11_Alternative Delivery Mode (ADM) Module on The Inverse of One-to-one Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics 11 Alternative Delivery Mode (ADM) Module on The Inverse of One-to-one Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you understand the inverse function. Particularly, this will provide you guide on how to find the inverse of a one-to-one function. Enjoy as you immerse yourself in solving for the inverse function intuitively or using a set of more established steps.

The module is composed of one lesson, namely:

- Lesson 1 – The Inverse of a One-to-one Function

After going through this module, you are expected to:

1. determine the inverse of a one-to-one function.
2. write a letter to a family member or peer about making amends on regretful events which cannot be undone.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which is not related to the word “inverse”?
   a. undo
   b. opposite
   c. delete
   d. interchange

2. What is the inverse of addition?
   a. division
   b. multiplication
   c. subtraction
   d. composition

3. What is the inverse of division?
   a. addition
   b. multiplication
   c. subtraction
   d. composition

4. What is the inverse of multiplication?
    a. addition
    b. division
    c. subtraction
    d. composition

5. What is the inverse of subtraction?
    a. addition
    b. division
    c. multiplication
    d. composition

6. Which function/s has/have inverse function?
    a. one-to-one
    b. many-to-one
    c. both
    d. none

7. What is the mathematical symbol for inverse of $f(x)$?
    a. $\frac{1}{f(x)}$
    b. $f(x)^{-1}$
    c. $'f(x)$
    d. $f^{-1}(x)$

8. What is the inverse of 3x – 4?
    a. 3x + 4
    b. 4x – 3
    c. $\frac{x+3}{4}$
    d. $\frac{x+4}{3}$

9. What is the inverse of $a^3 + 5$?
    a. 3a – 5
    b. $\sqrt[3]{a-5}$
    c. $\sqrt[3]{a} - 5$
    d. $\frac{a}{3} + 5$

10. Which is not a property of an inverse function?
    a. The inverse of $f^{-1}(x)$ is f(x).
    b. $f^{-1}(f(x)) = x$ for all x in the domain of f.
    c. $f^{-1}(f^{-1}(x)) = x$ for all x in the domain of f^{-1}.
    d. $f(f^{-1}(x)) = x$ for all x in the domain f^{-1}.

11. What is the inverse of -2x + 7?
    a. $-\frac{x-7}{2}$
    b. $-\frac{x}{2}+7$
    c. $7x-2$
    d. 2x – 7

12. What is the inverse of b^3+4?
    a. 3b – 4
    b. $\frac{b}{3}+4$
    c. $\sqrt[3]{b}$+4
    d. $\sqrt[3]{b-4}$

13. What is the inverse of g(x) = 2x – 8?
    a. $g'(x)=\frac{x+8}{2}$
    b. $g'(x)=-2x+8$
    c. $g^{-1}(x)=\frac{x+8}{2}$
    d. $g^{-1}(x)=-2x+8$

14. What is the inverse function of $f(c)=\frac{2c+1}{4c-5}$?
    a. $f^{-1}(c)=\frac{4c-1}{2c+5}$
    b. $f^{-1}(c)=\frac{5c+1}{4c-2}$
    c. $f'(c)=\frac{2c-1}{4c+5}$
    d. $f'(c)=\frac{4c-5}{2c+1}$

15. Which is not involved in the process of finding the inverse of a function?
    a. Write the function in the form y = f(x).
    b. Interchange the x and y variables.
    c. Write in the function in the simplest form.
    d. Solve for y in terms of x.

Lesson 1 The Inverse of One-to-One Functions

Among the functions, only a one-to-one function has an inverse which is a function also.

What's In

So far, you have known different faces of functions in the previous lessons. Likewise, you've categorized them already into groups of one-to-one and many-to-one functions. Let's have a quick review. In the first column, identify each of the following as linear function (LF), quadratic function (QF) or rational function (RF). In the second column, decide whether each is one-to-one or many-to-one function.

Function	**LF, QF or RF**	**One-to-one or many-to-one**
1. $f(x) = 2x + 5$		
2. $g(x) = 2x^2 - 4x + 1$		
3. $h(x) = (x-1)^2 - \frac{1}{2}$		
4. $f(x) = \frac{2x-1}{x+5}$		
5. $g(x) = x$		

Do you ever wonder if inverses of these functions are functions as well? That is, both the original equation and its inverse are both functions. In this lesson, you will delve into these functions with function inverses.

Notes to the Teacher

Solutions should be provided for exercises which will not be successfully answered by the learners especially for "Additional Activities" Part.

What's New

I Can See Your Mind

Let's have a mind game. Ready?
Think of a number. Multiply it by 2. Then, subtract 1 from it. Now, add 4 to the difference. Lastly, give me your answer and I'll tell the number you are thinking of.
Can you tell me how I will know the original number you have chosen by giving me the final answer?
The key lies in the command "undo". Familiar with it? Yes, this game follows the same principle as with the "undo" button we click when we are preparing documents using our laptops, cellphones or the likes. When you want to bring back how the document looks like a while ago, you keep clicking this button and the document gradually goes back to its previous layout. It keeps deleting the changes you do to the document one by one from the most recent to the earliest change you made.
Meanwhile, what you did with your chosen number is you multiplied it by 2 and then added 3 to it. Why 3? Because you subtracted 1 and then added 4 to the number which is the same as adding 3 to it. Going back to the principle of "undo", this is how I guessed your original number by telling me your final answer.

Commands	Undo
Step 3. Add 3 to it. (2x + 3)	Step 1. Subtract 3 from your answer y. (y – 3)
Step 2. Multiply it by 2. (2x)	Step 2. Divide it by 2. $\frac{y-3}{2}$
Step 1. Think of a number. (x)	Answer will be the number you are thinking. (x)

By that way, I have seen your mind. Create a new set of commands. It's now your turn to try it with your family members or peer. Experience their oohs and aahs!

What is It

Inverse Function Defined

The inverse of a function is a function with domain B and range A given that the original function has domain A and range B.

This inverse function of function f is denoted by f^{-1}. It is defined by the equation $f^{-1}(y) = x$ if and only if $f(x) = y$ for any y in range B. Since both are functions, then a function has to be one-to-one for its inverse to be a function at the same time. If it is a many-to-one function, its inverse is one-to-many which is not a function.

How to Find the Inverse of One-to-one Function

Intuitively, the inverse of a function may be known by the principle of "undo". That is, by considering the inverse of operations performed, the inverse of a function may be computed easily.

Example 1

Given f(x) = 3x – 8, the inverse of a function may be solved intuitively.

Solution:

Steps	In symbols
Step 1. The last operation performed is subtraction, the inverse operation of which is addition. To x, add 8.	x + 8
Step 2. The second to the last operation performed is multiplication, the inverse operation of which is division. Divide x + 8 by 3.	$\frac{x+8}{3}$
Step 3. Equate it to $f^{-1}(x)$ to denote that it is the inverse function of $f(x) = 3x - 8$.	$f^{-1}(x) = \frac{x+8}{3}$

However, it is not that easy in some case. In later examples, you will understand what I mean by saying that there is a more general method that may be followed.
To find the inverse of a one-to-one function, consider the following:

a. Express the function in the form $y = f(x)$;
b. Interchange the x and y variables in the equation;
c. Solve for y in terms of x.

Example 2

If it exists, solve for the inverse of $g(x) = x^2 - 6x - 7$.

Solution:
Recognize that g(x) is a quadratic function whose graph is a parabola opening upward. It fails the horizontal line test because it has x-values which correspond to the same y-value. And since it is not a one-to-one function, then its inverse is not a function. Simply put, it has no inverse function.

Alternate Solution:

$y = x^2 - 6x - 7$	(change g(x) to y)
$x = y^2 - 6y - 7$	(interchange x and y)
$y^2 - 6y = x + 7$	(solve for y, APE)
$y^2 - 6y + 9 = x + 7 + 9$	(solve for y, by completing the square, by APE)
$(y-3)^2 = x + 16$	(solve for y, by factoring)
$y - 3 = \pm\sqrt{x+16}$	(solve for y, by getting the square root of both sides)
$y = \pm\sqrt{x+16} + 3$	(solve for y, by APE)

Notice that for some values of x, there are two values of y. For instance, if x=1, $y = \sqrt{17} + 3$ and $y = \sqrt{17} + 3$. Therefore, the inverse function of g(x) does not exist.

Example 3

Find the inverse of the rational function $h(x) = \frac{4x+8}{x-3}$.

Solution:

$y = \frac{4x+8}{x-3}$	(change g(x) to y)
$x = \frac{4y+8}{y-3}$	(interchange x and y)
$xy - 3x = 4y + 8$	(solve for y, MPE)
$xy - 4y = 3x + 8$	(solve for y, by APE)
$y(x - 4) = 3x + 8$	(solve for y, by factoring)
$y = \frac{3x+8}{x-4}$	(solve for y, by MPE)
$h^{-1}(x) = \frac{3x+8}{x-4}$	(the inverse function)

What's More

Activity 12.1

Intuitively, give the inverse function of each of the following.

1. $f(x) = x + 2$
2. $g(x) = 12x - 1$
3. $h(x) = -\frac{x}{4}$
4. $f(x) = x$
5. $g(x) = \frac{3x+5}{8}$

Activity 12.2

If it exists, solve for the inverse function of each of the following.

1. $f(x) = 25x - 18$
2. $g(x) = \frac{12x-1}{7}$
3. $h(x) = -\frac{9x}{4} - \frac{1}{3}$
4. $f(x) = x^9$
5. $f(a) = a^3 + 8$
6. $g(a) = a^2 + 8a - 7$
7. $f(b) = (b + 6)(b - 2)$
8. $h(x) = \frac{2x+17}{3x+1}$
9. $h(c) = \sqrt{2c + 2}$
10. $f(x) = \frac{x+10}{9x-1}$

What I Have Learned

Answer the following questions.

1. What is an inverse function?
2. What is the symbol of an inverse function?
3. Do all kinds of functions have inverse function?
4. How do you solve for the inverse of a one-to-one function?

What I Can Do

In real life, can we undo events? Have you experienced any conflict with your family or peer on concerns like showing respect, being honest and trustworthy or being helpful and cooperative? What do you do to make amends? This time try writing a letter to a family member or peer expressing your regret over an event. Pour out your heart and feel light after then.

To make sure you'll put smile on their faces, try scoring your letter using the rubric below:

Criteria	4	3	2	1
Content	Focus on actions to take to resolve the situation; sincere and polite tone; admit one's fault; with follow up	Involves only three of the four characteristics cited at the left	Involves only two of the four characteristics cited	Involves only one of the four characteristics cited
Grammar and mechanics	Sentences are clear; use commas and other punctuations properly; no lengthy narration in every sentence; sentences are arranged properly	Involves only three of the four characteristics cited at the left	Involves only two of the four characteristics cited	Involves only one of the four characteristics cited

If you scored your letter 7 or 8, proceed giving your letter wholeheartedly. If the score you give is 6 or below, consider revising it before giving it to your loved one. This is a rare moment, make it count.

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which relates to "inverse"?
   a. redo
   b. opposite
   c. delete
   d. interchange

2. What is the inverse of $f(x) = x + 36$?
   a. $f(x) = \frac{x}{36}$
   b. $f(x) = 36x$
   c. $f(x) = x - 36$
   d. $f(x) = -x - 36$

3. What is the inverse of $f(x) = \frac{x}{25}$?
   a. $f(x) = x + 25$
   b. $f(x) = 25x$
   c. $f(x) = x - 25$
   d. $f(x) = -25x$

4. What is the inverse of $f(x) = -3x$?
   a. $f(x) = x + 3$
   b. $f(x) = -\frac{x}{3}$
   c. $f(x) = x - 3$
   d. $f(x) = \frac{x}{3}$

5. What is the inverse of $f(x) = x - 10$?
   a. $f(x) = 10 + x$
   b. $f(x) = -\frac{x}{10}$
   c. $f(x) = -10x$
   d. $f(x) = -(x - 10)$

6. Which does not characterize an inverse function?
   a. Given that it is $f^{-1}(x)$, its domain and range are the same as the domain and range of $f(x)$.
   b. It is denoted by $y = f^{-1}(x)$.
   c. Its inverse is one-to-one.
   d. It is one-to-one.

7. What is the mathematical symbol for inverse of $f(x)$?
   a. $\frac{1}{f(x)}$
   b. $f(x)^{-1}$
   c. $'f(x)$
   d. $f^{-1}(x)$

8. What is the inverse of $-2x - 8$?
   a. $8x + 2$
   b. $2x + 8$
   c. $\frac{x+3}{8}$
   d. $\frac{x+8}{-3}$

9. What is the inverse of $b^5 + 2$?
   a. $2b - 5$
   b. $5b + 2$
   c. $\sqrt[5]{b - 2}$
   d. $\sqrt[5]{b} - 2$

10. Which is not a property of an inverse function?
   a. The inverse of $f^{-1}(x)$ is $f(x)$.
   b. $f^{-1}(f(x)) = x$ for all x in the domain of f.
   c. $f^{-1}(f^{-1}(x)) = x$ for all x in the domain of f^{-1}.
   d. $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}.

11. What is the inverse of $-6x - 5$?
   a. $\frac{x+5}{-6}$
   b. $-\frac{x}{6} + 5$
   c. $6x + 5$
   d. $5x + 6$

12. What is the inverse of $(c + 1)^3 - 1$?
   a. $\sqrt[3]{c}$
   b. $\sqrt[3]{c + 1} - 1$
   c. $\sqrt[3]{c - 1} + 1$
   d. $1 - \sqrt[3]{c + 1}$

13. What is the inverse of $g(x) = 9x + 20$?
   a. $g'(x) = \frac{x+20}{-9}$
   b. $g'(x) = -20x + 9$
   c. $g^{-1}(x) = \frac{x-20}{9}$
   d. $g^{-1}(x) = -9x - 20$

14. What is the inverse function of $f(d) = \frac{d-12}{2d+1}$?
   a. $f'(d) = \frac{d+12}{-2d-1}$
   b. $f'(d) = \frac{-d+12}{-2d-1}$
   c. $f'(d) = \frac{d-12}{2d-1}$
   d. $f'(d) = \frac{-d-12}{2d-1}$

15. Which is not involved in the process of finding the inverse of a function?
    a. Write the function in the form $y = f(x)$.
    b. Interchange the x and y variables.
    c. Write in the function in the simplest form.
    d. Solve for y in terms of x.

Additional Activities

Show that $f(x) = |5x|$ has no inverse function.

Answer Key

What I Know

1. C
2. C
3. B
4. B
5. A
6. A
7. D
8. D
9. B
10. C
11. A
12. D
13. C
14. B
15. C

What's More

Activity 12.1

6. $f^{-1}(x) = x - 2$
7. $g^{-1}(x) = \frac{x+1}{12}$
8. $h^{-1}(x) = -4x$
9. $f^{-1}(x) = x$
10. $g^{-1}(x) = \frac{8x-5}{3}$

Activity 12.2

1. $f^{-1}(x) = \frac{x+18}{25}$
2. $g^{-1}(x) = \frac{7x+1}{12}$
3. $h^{-1}(x) = \frac{-4\left(x+\frac{1}{3}\right)}{9}$

   $or\ h^{-1}(x) = -\frac{4x}{9} + \frac{4}{27}$
4. $f^{-1}(x) = \sqrt[9]{x}$
5. $f^{-1}(a) = \sqrt[3]{a-8}$
6. It has no inverse. Initially, the given is not a one-to-one function. Or, by solving for the inverse, $y = \pm\sqrt{x+23} - 4$. There are y-values each of which is paired to two x-values.
7. It has no inverse. Initially, the given is not a one-to-one function. Or, by solving for the inverse, $y = \pm\sqrt{x+16} - 2$. There are ordinates each of which is paired to two abscissas.
8. $h^{-1}(x) = \frac{2x+17}{3x+1}$
9. $h^{-1}(c) = \frac{x^2-2}{2}$
10. $f^{-1}(x) = \frac{x+10}{9x-1}$

Assessment

1. D
2. C
3. B
4. B
5. A
6. A
7. D
8. D
9. C
10. C
11. A
12. B
13. C
14. D
15. C

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11 General Mathematics

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Representations of an Inverse Functions
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Representations of Inverse Functions

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What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you understand the inverse function. Particularly, this will provide you guide on how to find the inverse of a one-to-one function. Enjoy as you immerse yourself in solving for the inverse function intuitively or using a set of more established steps.

The module is composed of one lesson, namely:

- Lesson 1 – Representing an inverse function through table of values, and graph

After going through this module, you are expected to:

1. represent an inverse function through its: (a) table of values and (b) graph.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following graphs do not belong to the group?

a.

c.

b.

d.

2. The graph of a one-to-one function and its inverse function is _____

a. Hyperbolic b. Parabola c. Parallel d. Symmetric

3. Which of the following terms deals with inverse?

a. alternate b. delete c. eliminate d. interchange

4. Which of the following equations is used to test if the graph of a one-to-one function and its inverse function?

a. f(x) = y b. y = x c. f(x) = x d. f(x) = x+y

5. Which of the following ordered pairs of an inverse function has its one-to-one function indicated in the table below?

f(x)	3	4	5	6	7
y	-2	-1	0	1	2

a. $f^{-1}(x)$ = {(-3,-2), (-4,-1), (0,5), (-1,6), (-2, 7)}

b. $f^{-1}(x)$ = {(3,2), (4,1), (0,-5), (1,-6), (2,-7)}

c. $f^{-1}(x)$ = {(3,-2), (4,-1), (0,5), (1,6), (2, 7)}

d. $f^{-1}(x)$ = {(-3,-3), (-4,-1), (-5,0), (-6,1), (-7,2)}

6. What test is used to determine if a function is one-to-one function?

a. Diagonal line test

b. Horizontal line test

c. Straight line test

d. Vertical line test

7. What is the missing value in the table below with the equation, f(x) = x -3?

f(x)	3	5	7	9	11
y	0	2	____	6	8

a. 3 b. 4 c. 5 d. 6

8. The illustration shows the one-to-one function and its inverse, which pair has the correct functions?

a. $f(x) = 2x - 3$ and $f^{-1}(x) = \frac{x+3}{2}$

b. $f(x) = 3x - 2$ and $f^{-1}(x) = \frac{x+2}{3}$

c. $f(x) = x - 3$ and $f^{-1}(x) = \frac{x+1}{3}$

d. $f(x) = 2x + 3$ and $f^{-1}(x) = \frac{x-3}{2}$

9. Which term was used to represent the x-values of the function are the y-values of its inverse, and the y-values of the function are the x-values of its inverse?

a. Coordinating values
b. Intersecting values
c. Table of values
d. True values

10. Which is true about f(x) = 2x - 1 and $y = \frac{x+1}{2}$?

a. They are inverse functions.
b. They are not one-to-one functions.
c. The graphs are parallel to each other.
d. The graphs intersect at two or more points.

11. Which of the following graphs best described f(x) = 2x - 1 and $y = \frac{x+1}{2}$?

a.

c.

b.

d.

12. If the function is one-to-one and has x and y-values of (2, -6), what is its inverse values?

a. (-2, 6) b. (6,-2) c. (-6,2) d. (6, 2)

13. Which term best complete the sentence: To graph the inverse all you need to do in the coordinates of each ordered pair is to _____.

a. delete b. investigatec. switch d. replace

14. All of the following are used in the representation of inverse function, EXCEPT.

a. $f^{-1}(x)$ b. graph c. range d. table of values

15. Which of the following table of values represent the correct inverse function of $f(x) = 2x + 3$?

a.

$f^{-1}(x)$	-2	-1	0	1	2
y	-1	1	3	5	7

b.

$f^{-1}(x)$	-1	1	3	5	7
y	-2	-1	0	1	2

c.

$f^{-1}(x)$	1	-1	-3	-5	-7
y	-2	-1	0	1	2

d.

$f^{-1}(x)$	-1	1	3	5	7
y	2	1	0	-1	-2

Lesson 1 Representing an inverse function through table of values, and graph

Among the functions, only a one-to-one function has an inverse which is a function also which can be represented in table of values and graphs.

What's In

You have learned different types of functions in the previous lessons. Do you ever wonder if inverses of functions are functions as well? That is, both the original equation and its inverse are both functions. In this lesson, you will delve into these functions with function inverses.

Notes to the Teacher

Enable learners to perform each task or activity in this module. Solutions should be provided for exercises which will not be successfully answered by the learners especially for "Additional Activities" Part.

What's New

You Complete Me!

Below is a table with function f(x) and its inverse $f^{-1}(x)$; and possible values. To enjoy this activity, you need to review the concept of finding the inverse of a one-to-one function and investigate on the possible values of the function and its inverse by completing the statements below the table as many as you can.

Functions			Possible Values		
$f(x)=2x+3$	$f^{-1}(x)=\frac{x+3}{5}$	$f(x)=3x-1$	$f(0)=-3$	$f^{-1}(-1)=-2$	$f^{-1}(1)=\frac{2}{3}$
$f(x)=3x-2$	$f^{-1}(x)=\frac{x-1}{3}$	$f^{-1}(x)=\frac{x-3}{2}$	$f(1)=5$	$f(-1)=-4$	$f(5)=13$
$f(x)=5x-3$	$f^{-1}(x)=\frac{x+2}{3}$	$f^{-1}(x)=\frac{x+1}{3}$	$f^{-1}(1)=1$	$f^{-1}(0)=-\frac{3}{2}$	$f^{-1}(0)=\frac{3}{5}$

1. The function _____ has an inverse function of _____ with possible values of the function as _____ and _____.

2. The function _____ has an inverse function of _____ with possible values of the function as _____ and _____.

3. The function _____ has an inverse function of _____ with possible values of the function as _____ and _____.

4. The function _____ has an inverse function of _____ with possible values of the function as _____ and _____.

What is It

Inverse Function Defined with table of values and graph

The inverse of a function is a function with domain B and range A given that the original function has domain A and range B.

This inverse function of function f is denoted by f^{-1}. It is defined by the equation $f^{-1}(y)=x$ if and only if $f(x) = y$ for any y in range B. Since both are functions, then a function has to be one-to-one for its inverse to be a function at the same time. If it is a many-to-one function, its inverse is one-to-many which is not a function.

In using **table of values** of the functions, first we need to ascertain that the given function is a one-to-one function wherein no x-values are repeated. It is represented as the x-values of the function resulted as the y-values of its inverse, and the y-values of the function are the x-values of its inverse. Also, the graph should correspond to a one to one function by applying the **Horizontal Line test**. If it passes the test, the corresponding function is one-to-one. Given the graph of a one-to-one function, the **graph of its inverse** can be obtained by reflecting the graph about the line **y = x.**

Example 1

In the given function f(x) = 2x + 3, with an inverse function of $f^{-1}(x) = \frac{x-3}{2}$ as discussed in the previous activity. Let us use the x-values to complete the table of values in y-values for the f(x) = 2x + 3.

f(x)	-2	-1	0	1	2
y					

In order to complete the y-values, let us substitute each x-value from the function, f(x) = 2x + 3.

If x = -2, f(-2) = 2(-2) + 3, by solving it, f(-2) = -4 + 3, then, f(-2) = -1.

If x = -1, f(-1) = 2(-1) + 3, by solving it, f(-1) = -2 + 3, then, f(-1) = 1.

If x = 0, f(0) = 2(0) + 3, by solving it, f(0) = 0 + 3, then, f(0) = 3.

If x = 1, f(1) = 2(1) + 3, by solving it, f(1) = 2 + 3, then, f(1) = 5.

If x = 2, f(2) = 2(2) + 3, by solving it, f(2) = 4 + 3, then, f(2) = 7.

Thus, the table of values for f(x) = 2x + 3 is presented below with its corresponding graph.

f(x)	-2	-1	0	1	2
y	-1	1	3	5	7

At this point, let us investigate on the inverse function $f^{-1}(x)=\frac{x-3}{2}$ by using the y-values from the original function as x-values of the inverse function. Observe the same process in completing the table of values by substituting the x-values to the given inverse function. Now the table of values will be as follows:

$f^{-1}(x)$	-1	1	3	5	7
y					

In order to complete the y-values, let us substitute each x-value from the given inverse function, $f^{-1}(x)=\frac{x-3}{2}$.

If $f^{-1}(-1)$, $f^{-1}(-1)=\frac{(-1)-3}{2}$, by solving it, $f^{-1}(-1)=\frac{-4}{2}$, then $f^{-1}(-1)=-2$.

If $f^{-1}(1)$, $f^{-1}(1)=\frac{(1)-3}{2}$, by solving it, $f^{-1}(1)=\frac{-2}{2}$, then $f^{-1}(1)=-1$.

If $f^{-1}(3)$, $f^{-1}(3)=\frac{(3)-3}{2}$, by solving it, $f^{-1}(3)=\frac{0}{2}$, then $f^{-1}(3)=0$.

If $f^{-1}(5)$, $f^{-1}(5)=\frac{(5)-3}{2}$, by solving it, $f^{-1}(5)=\frac{2}{2}$, then $f^{-1}(5)=1$.

If $f^{-1}(7)$, $f^{-1}(7)=\frac{(7)-3}{2}$, by solving it, $f^{-1}(7)=\frac{4}{2}$, then $f^{-1}(7)=2$.

Thus, the table of values for $f^{-1}(x)=\frac{x-3}{2}$ is presented below.

$f^{-1}(x)$	-1	1	3	5	7
y	-2	-1	0	1	2

This is the graph of the inverse function, $f^{-1}(x) = \frac{x-3}{2}$. In the next page, the graphs of the two functions will be presented to you. Let us see how it looks like!

As you observed, there is an diagonal line (represented as broken line) across the origin to the point of intersection of the line f(x) = 2x + 3 and $f^{-1}(x) = \frac{x-3}{2}$.

However, for easy steps, if you're asked to graph a function and its inverse, all you have to do is graph the function and then switch all x and y values in each point to graph the inverse. Just look at all those values switching places from the f(x) function to its inverse $f^{-1}(x)$ (and back again). Furthermore, the two graphs will be **symmetric** about the line y = x.

What's More

Activity 13.1

Intuitively, give the table of values of each of the following functions. (Use -2 to 2).

1. $f(x) = x + 2$
2. $g(x) = 12x - 1$
3. $h(x) = -\frac{x}{4}$
4. $f(x) = x$
5. $g(x) = \frac{3x+5}{8}$

Activity 13.2

Illustrate the graph of the given one-to-one function and its inverse.

1. $f(x) = 3x - 4$
2. $f(x) = 5x + 3$
3. $f(x) = 7x - 5$
4. $f(x) = \dfrac{x-2}{3}$
5. $f(x) = \dfrac{x+3}{2}$
6. $f(x) = \dfrac{x+5}{4}$

What I Have Learned

Now, answer the following questions.

1. Describe the graph of a one-to-one function and its inverse?
2. What will you do to graph the inverse function?
3. How important it is to present table of values of a function?

What I Can Do

In real life, can we undo events? Have you experienced any conflict with yourself as a student like showing respect, being honest and trustworthy or being helpful and cooperative? What do you do to make amends? This time, try representing a model or a function or an equation that shows negative to positive outlook in life that can be reflected through a simple illustration of your model. Feel free to write something your heart desires to write or a brief explanation of your unique output.

To make sure you'll get proud of yourself, try scoring your output using the rubric below:

Criteria	5	4	3	2	1
Visual Appearance and Ideas	The illustration is well done depicting the progress of a student from one aspect of life to a very progressive one. The illustration is very unique and promotes self-worth and progress	The illustration is well done but there are missing ideas in the output which slightly promote self-worth and progress as a student.	The illustration is limited to point out ideas and students' self-word and progress was not observed.	The illustration, ideas, and progress was not observed but rare thoughts on self-worth and progress as a student was written.	The illustration and ideas was not observed.

If you scored your output 4 to 5, proceed publishing your output in any social media platforms, use it as your profile picture for at least a week representing your personalities as a student. If the score you give 3 or less, consider reflecting on your actions that will somehow improve the ideas in the illustration that may best represent yourself as a student. This is a rare moment, make it count.

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following graphs do not belong to the group?

a.

c.

b.

d

2. The following are the behaviors of a graph of a one-to-one function and its inverse function, EXCEPT.

   a. intersecting b. Parabola c. Parallel d. Symmetric

3. Which of the following terms do not deals with inverse?

   a. alternate b. interchange c. switch d. reverse

4. Which of the following equations is used to test if the graph of a one-to-one function and its inverse function?

   a. f(x) = y b. f(x) = x c. f(x) = x+y d. y = x

5. Which of the following ordered pairs of an inverse function has its one-to-one function indicated in the table below?

f(x)	-2	-1	0	1	2
y	0	1	2	3	4

c. $f^{-1}(x)$ = {(-2,0), (-1,1), (0,2), (1,3), (2, 4)}

d. $f^{-1}(x)$ = {(-2,0), (-1,-1), (0,-2), (1,-3), (2,-4)}

c. $f^{-1}(x)$ = {(2,0), (1,1), (0,2), (-1,3), (-2, 4)}

d. $f^{-1}(x)$ = {(0,-2), (1,-1), (2,0), (3,1), (4,2)}

6. What is the missing ordered pair in the table below with a f(x) = 2x +9?

f(x)	3	5	___	9	11
y	15	19	____	27	31

a. (6,20) b. (7,22) c. (7,23) d. (8,25)

7. In item number 8, which is the missing inverse ordered pair?

a. (20, 6) b. (22,7) c. (23,7) d. (25,8)

8. The illustration shows the one-to-one function and its inverse, which pair has the correct functions?

a.. $f(x) = 3x - 4$ and $f^{-1}(x) = \frac{x+4}{3}$

b. $f(x) = 3x - 2$ and $f^{-1}(x) = \frac{x+2}{3}$

c. $f(x) = 4x - 3$ and $f^{-1}(x) = \frac{x+3}{4}$

d. $f(x) = 2x + 3$ and $f^{-1}(x) = \frac{x-3}{2}$

9. Which term was used to represent the x and y-values of the functions.

a. Coordinating values

b. Intersecting values

c. Table of values

d. True values

10. Which is true about f(x) = 5x - 3 and $y = \frac{x+5}{3}$?

a. They are inverse functions.

b. They are not inverse functions.

c. The graphs are parallel to each other.

d. The graphs intersect at two or more points.

11. Which of the following graphs best descirbed $f(x) = 5x + 3$ and $y = \frac{x-3}{5}$?

a.

c.

b.

d.

12. If the function is one-to-one and has x and y-values of (-2, 7), what is its inverse values?

c. (-2, 7) b. (7,-2) c. (-7,2) d. (7, 2)

13. If the function is one-to-one and has x and y-values of (5, 0), what is its inverse values?

d. (5, 0) b. (0,-5) c. (0, 5) d. (-5, 0)

14. All of the following are used in the representation of inverse function, EXCEPT.

a. domain b. $f^{-1}(x)$ c. graph d. table of values

15. Which of the following table of values best represents the correct inverse function of f(x) = 3x + 7?

a.

$f^{-1}(x)$	1	-1	-3	-5	-7
y	10	4	-2	-8	-14

b.

$f^{-1}(x)$	-2	-1	0	1	2
y	1	4	7	10	13

c.

$f^{-1}(x)$	1	4	7	10	13
y	-2	-1	0	1	2

d.

$f^{-1}(x)$	10	4	-2	-8	-14
y	1	4	7	10	13

Additional Activities

Show the inverse function and construct a table of values for each of the following function.

1. $f(x) = \dfrac{x+7}{3}$

2. 2. $f(x) = \dfrac{x-2}{5}$

Additional Activities

1. $f(x)=\frac{x+7}{3}$;

$f(x)$	-4	-1	2	5	8
y	1	2	3	4	5

$f^{-1}(x)=3x-7$

$f^{-1}(x)$	1	2	3	4	5
y	-4	-1	2	5	8

The graph of $f(x)=\frac{x+7}{3}$ and

$f^{-1}(x)=3x-7$

2. $f(x)=\frac{x-2}{5}$

$f(x)$	-3	2	7	12	17
y	-1	0	1	2	3

$f^{-1}(x)=5x+2$

$f^{-1}(x)$	-1	0	1	2	3
y	-3	2	7	12	17

The graph of $f(x)=\frac{x-2}{5}$ and

$f^{-1}(x)=5x+2$

What's More

Activity 13.1

1. f(x)=x+2

$f(x)$	-2	-1	0	1	2
y	0	1	2	3	4

2. f(x)=12x - 1

$f(x)$	-2	-1	0	1	2
y	-25	-13	-1	11	23

3. h(x)= - x/4

$f(x)$	-2	-1	0	1	2
y	1/2	1/4	0	1/4	1/2

4. f(x) = x

$f(x)$	-2	-1	0	1	2
y	-2	-1	0	1	2

5. g(x) = (3x+5)/8

$f(x)$	-2	-1	0	1	2
y	-1/8	1/4	5/8	1	11/8

What I Know

What's New (In any order)

1. f(x)=2x+3, $f^{-1}(x)=\frac{x-3}{2}$,

f(1)=5, $f^{-1}(0)=-\frac{3}{2}$

2. f(x)=3x-1, $f^{-1}(x)=\frac{x+1}{3}$,

$f(-1)=-4$, $f^{-1}(1)=\frac{2}{3}$

3. f(x)=3x-2, $f^{-1}(x)=\frac{x+2}{3}$,

f(5)=13, $f^{-1}(1)=1$

4. f(x)=5x-3, $f^{-1}(x)=\frac{x+3}{5}$,

f(0)=-3, $f^{-1}(0)=\frac{3}{5}$

Answer Key

Assessment

1. c	6. c	11.a
2. b	7. c	12.b
3. a	8. a	13.c
4. d	9. c	14.a
5. d	10.b	15.b

What's More

Activity 13.2

1.

2.

3

4.

5

6

References

Dimasuay, Lynie, et. al. 2016. *General Mathematics*. Philippines: C & E Publishing, Inc.

Verzosa, Debbie Marie, et.al. 2016. *General Mathematics: Learner's Material, First Edition.* Philippines: Lexicon Press Inc.

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11 General Mathematics

General Mathematics
Domain and Range of Inverse Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics
Domain and Range of Inverse Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics Alternative Delivery Mode (ADM) Module on Domain and Range of Inverse Functions!

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Alternative Delivery Mode (ADM) Module on Finding the Domain and Range of an Inverse Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!
This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
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6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

In this learning module, you will know more about the domain and range, and how to determine the domain and range of an inverse function. This module was designed and written with you in mind. It is here to help you easily master the procedure in finding the domain and range of an inverse function.

After going through this module, you are expected to:

1. Define domain and range.
2. Find the domain and range of a given inverse function.
3. Represent the domain and range using set builder notation.

What I Know

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What do you call to the set of all allowable values of the independent variable?
   a. Range
   b. Domain
   c. Real Numbers
   d. Inverse Function

2. Which of the following is not allowed as the value of the independent variable if the function is a fraction?
   a. zero
   b. negative number
   c. decimal number
   d. irrational number

3. What is the domain and range of the function $(x) = x - 5$?
   a. The domain is all real numbers except -5 and the range is all real numbers except 0.
   b. The domain is all real numbers and the range is all real numbers except 0.
   c. The domain is all real numbers except -5 and the range is all real numbers.
   d. The domain and range are all real numbers.

4. What is the inverse of $(x) = 3x + 6$?

   a.. $f^{-1}(x) = \frac{x+6}{3}$
   b. $f^{-1}(x) = \frac{3}{x-6}$
   c. $f^{-1}(x) = \frac{x-6}{3}$
   d. $f^{-1}(x) = \frac{3}{x+6}$

5. Which of the following pairs of functions is NOT the inverse of each other?

a. $f(x) = 2x + 5 \; and \; g(x) = 2x - 5$

b. $f(x) = 3x \; and \; g(x) = \frac{x}{3}$

c. $f(x) = x - 3 \; and \; g(x) = x + 3$

d. $f(x) = x + 2 \; and \; g(x) = x - 2$

For numbers 6-10, consider the function $(x) = \frac{3}{x+1}$.

6. What is the domain of the function?

a. $\{x \in R1\}$
b. $\{x \neq 0\}$
c. $\{x \neq -1\}$
d. $\{x \neq 1\}$

7. What is the Range of the function?

a. $\{y \neq 0\}$
b. $\{y \neq 1\}$
c. $\{y \neq 3\}$
d. $\{y \neq -1\}$

8. What is the inverse of the function?

a. $f^{-1}(x) = \frac{3+x}{x}$
b. $f^{-1}(x) = \frac{3-x}{x}$
c. $f^{-1}(x) = \frac{x}{3+x}$
d. $f^{-1}(x) = \frac{x}{3-x}$

9. What is the domain of f^{-1}?

a. $\{x \neq 0\}$
b. $\{x \neq 3\}$
c. $\{x \neq -3\}$
d. $\{x \neq -1\}$

10. What is the range of f^{-1}?

a. $\{y \neq 0\}$
b. $\{y \neq y\}$
c. $\{y \neq 1\}$
d. $\{y \neq -1\}$

For numbers 11-15, consider the function $f(x) = x^2 + 2$.

11. What is the domain of the function?

a. $\{x > 2\}$
b. $\{x \in \neq 2\}$
c. $\{x > 0\}$
d. $\{x \in R\}$

12. What is the range of the function?

a. $\{y \geq 2\}$
b. $\{y > 2\}$
c. $\{y < 2\}$
d. $\{y > 0\}$

13. What is the inverse of the function?

a. $f^{-1}(x) = x^2 - 2$

b. $f^{-1}(x) = 2 + x^2$

c. $f^{-1}(x) = \sqrt{x - 2}$

d. $f^{-1}(x) = \sqrt{x + 2}$

14. What is the domain of f^{-1}?

a. $\{x \geq -2\}$

b. $\{x \geq 2\}$

c. $\{x < -2\}$

d. $\{x < 2\}$

15. What is the range of f^{-1}?

a. $\{y \in R\}$

b. $\{y \in > 2\}$

c. $\{y < 2\}$

d. $\{y > -2\}$

Lesson 1 Finds the Domain and Range of an Inverse Function

Start Lesson 1 of this module by assessing your knowledge of the basic skills in finding the inverse of a function. This knowledge and skill will help you understand easily on how to find the domain and range of an inverse function. Seek the assistance of your teacher if you encounter any difficulty. This topic is about finding the domain and range of an inverse function.

What's In

Recall that a function has an inverse if and only if it is one-to-one and every one-to-one function has a unique inverse function.

Below are the steps in solving for the inverse of a function:

a. Write the function in the form y=f(x);
b. Interchange the x and y variables;
c. Solve for y in terms of x;
d. Replace y by $f^{-1}(x)$;
e. Verify if f and f-1 are inverse functions.

Example 1: Find the inverse of $f(x) = 3x - 8$.

Solution: The equation of a function is $y = 3x - 8$. Interchanging the $\boldsymbol{x}$ and $\boldsymbol{y}$ variables, we get $x = 3y - 8$.

Solving y for x: $3y = x + 8$

$$y = \frac{x+8}{3}$$

Therefore, the inverse of $f(x) = 3x - 8$ is $f^{-1}(x) = \frac{x+8}{3}$

To verify if f and f^{-1} are inverse functions:

$$f[f^{-1}(x)] = 3\left(\frac{x+8}{3}\right) - 8$$
$$= x+8$$
$$= x$$

$$f^{-1}[f(x)] = \frac{3x-8+8}{3}$$
$$= \frac{3x}{3}$$
$$= x$$

Therefore, f^{-1} is the inverse of f.

Example 2: Find the inverse of $f(x) = \sqrt{2x+1}$.

Solution: The equation of a function is $y = \sqrt{2x+1}$. Interchanging the ***x*** and ***y*** variables, we get $x = \sqrt{2y+1}$.

Solving y for x: $2y = x^2 - 1$

$y = \frac{x^2-1}{2}$

Therefore, the inverse of $f(x) = \sqrt{2x+1}$ is $f^{-1}(x) = \frac{x^2-1}{2}$

To verify if f and f^{-1} are inverse functions:

$f[f^{-1}(x)] = \sqrt{2\left(\frac{x^2-1}{2}\right)+1}$

$= \sqrt{x^2-1+1}$

$= x$

$f^{-1}[f(x)] = \frac{\left(\sqrt{2x+1}\right)^2-1}{2}$

$= \frac{2x+1-1}{2}$

$= x$

Therefore, f^{-1} is the inverse of *f*.

Example 3: Find the inverse of $f(x) = x^2 + 4$.

Solution: The equation of a function is $y = x^2 + 4$. Interchanging the ***x*** and ***y*** variables, we get $x = y^2 + 4$.

Solving y for x: $y^2 = x - 4$

$y = \sqrt{x-4}$

Therefore, the inverse of $f(x) = x^2 + 4$ is $f^{-1}(x) = \sqrt{x-4}$

To verify if f and f^{-1} are inverse functions:

$f[f^{-1}(x)] = (\sqrt{x-4})^2 + 4$

$= x - 4 + 4$

$= x$

$f^{-1}[f(x)] = \sqrt{x^2+4-4}$

$= \sqrt{x^2}$

$= x$

Therefore, f^{-1} is the inverse of *f*.

Notes to the Teacher

The notation f^{-1} is used to represent the inverse of a function *f*.

To verify that the *f* and f^{-1} are inverse functions:

and

What's New

Let's Find Out!

A. Complete the table of each given function.

1. $f(x) = 5x + 20$

x	-2	0	2	4	6
f(x)					

2. $f(x) = 4 + \frac{x}{5}$

x	-30	-20	-10	0	10
f(x)					

B. Graph the functions in one Cartesian Plane

C. Answer the following questions:

1. What can you say about the two given functions?

_____
_____
_____
_____
_____

2. Based from the table of values, describe the domain and range of the first function with respect to the domain and range of the other function.

_____
_____
_____
_____
_____

3. What can you say about the graphs of the two functions?

_____
_____
_____
_____
_____

4. Drawing a diagonal line (y=x), what can you say about the graphs with respect to line y=x?

_____
_____
_____
_____
_____

5. Can you give any other observation/s?

_____
_____
_____
_____
_____

What is It

In the activity that you have done, were you able to determine the relationship of the domain and range of the function and its inverse? Have you seen their graphs? You will find out the easy way and understand it clearly as you go through the next session of this module.

From the previous lesson, you already learned that the domain of a function is the set of input values that are used for the independent variable and the range of a function is the set of output values for the dependent variable. But, from this lesson, how will you determine the domain and range of an inverse function?

A relation reversing the process performed by any function f(x) is called inverse of f(x).

To determine the domain and range of an inverse function:

The outputs of the function f are the inputs to f^{-1}, so the range of f is also the domain of f^{-1}. Likewise, because the inputs to f are the outputs of f^{-1}, the domain of f is the range of f^{-1}. We can visualize the situation.

This means that the domain of the inverse is the range of the original function and that the range of the inverse is the domain of the original function.

Original Function

x	2	3	5	10
y	6	8	12	21

Inverse Relation

x	6	8	12	21
y	2	3	5	10

The domain of the original function is (2,3,5,10) and the range is (6,8,12,21). Therefore the domain of the inverse relation will be (6,8,12,21) and the range is (2,3,5,10).

Properties of an Inverse Function

If the f^{-1} inverse function exists,

1. f^{-1} is a one to one function, f is also one-to-one.
2. Domain of f^{-1}= Range
3. Range of f^{-1} = Domain of f.

Example 1. Find the domain and range of the inverse function $\boldsymbol{f^{-1}(x) = \frac{x+2}{3}}$

Solution:

To find the domain and range of an inverse function, go back to the original function and then interchange the domain and range of the original function.

The original function is f(x) = 3x-2. The original function's domain is the set of real numbers and the range is also the set of real numbers. Thus, the domain and range of $\boldsymbol{f^{-1}(x) = \frac{x+2}{3}}$ is the set of all real numbers.

Example 2. Find the domain and range of $\boldsymbol{f(x) = 3x + 12}$ and its inverse.

Solution:

Let $\boldsymbol{y = 3x + 12}$

Interchange x and y: $\boldsymbol{x = 3y + 12}$

Solve for y.

$$\boldsymbol{3y = x - 12}$$
$$\boldsymbol{y = (x - 12)/3}$$
$$\boldsymbol{f^{-1}(x) = (x - 12)/3}$$

Determine the domain and range of *f* and f^{-1}.

You have $\boldsymbol{f(x) = 3x + 12}$ a and $\boldsymbol{f^{-1}(x) = (x - 12)/3}$

Domain of (f) ={$\boldsymbol{x}$€$\boldsymbol{R}$}Range of (f)= {$\boldsymbol{y}$€$\boldsymbol{R}$}

Domain of (f^{-1}) = {$\boldsymbol{x}$€$\boldsymbol{R}$} Range of (f^{-1}) ={$\boldsymbol{y}$€$\boldsymbol{R}$}

To verify if f and f^{-1} are inverse functions:

$f[f^{-1}(x)] = 3(x - 12/3) + 12$
$= x-12+12$
$= x$

$f^{-1}[f(x)] = (3x - 12 - 12)/3$
$= 3x/3$
$= x$

Therefore, f^{-1} is the inverse of f.

Example 3. Find the domain and range of $\boldsymbol{f(x) = \sqrt{x+2}}$ and its inverse.

Solution:

Let $\boldsymbol{y = \sqrt{x+2}}$

Interchange x and y: $\boldsymbol{x = \sqrt{y+2}}$

Solve for y.

$\boldsymbol{x^2 = y + 2}$
$\boldsymbol{y = x^2 - 2}$
$\boldsymbol{f^{-1}(x) = x^2 - 2}$

Determine the domain and range of f and f^{-1}.

You have $\boldsymbol{f(x) = \sqrt{x+2}}$ a and $\boldsymbol{f^{-1}(x) = x^2 - 2}$

Domain of (f) $=\{\boldsymbol{x \geq -2}\}$ Range of (f)= $\{\boldsymbol{y \geq 0}\}$

Domain of (f^{-1}) = $\{\boldsymbol{x \geq 0}\}$ Range of (f^{-1}) $=\{\geq \boldsymbol{-2}\}$

To verify if f and f^{-1} are inverse functions:

$f[f^{-1}(x)] = \sqrt{x^2 - 2 + 2}$
$= \sqrt{x^2}$
$= x$

$f^{-1}[f(x)] = \left(\sqrt{x+2}\right)^2 - 2$
$= x+2-2$
$= x$

Therefore, f^{-1} is the inverse of f.

Example 4. Consider $\mathrm{f}(\boldsymbol{x}) = \boldsymbol{x^2 - 5}$. Find the inverse and its domain and range.

Solution:

Let $\boldsymbol{y = x^2 - 5}$

Interchange x and y: $\boldsymbol{x = y^2 - 5}$

Solve for y.

$\boldsymbol{y^2 = x + 5}$
$\boldsymbol{y = \sqrt{x+5}}$
$\boldsymbol{f^{-1}(x) = \sqrt{x+5}}$

Determine the domain and range of f and f^{-1}.

You have $\boldsymbol{f(x) = x^2 - 5}$ and $\boldsymbol{f^{-1}(x) = \sqrt{x+5}}$

Domain of (f) $=\{\boldsymbol{x \in R}\}$ Range of (f)= $\{\boldsymbol{y > -5}\}$

Domain of (f^{-1}) = $\{\boldsymbol{x > -5}\}$ Range of (f^{-1}) $=\{\boldsymbol{y \in R}\}$

To verify if f and f^{-1} are inverse functions:

$f^{-1}[f(x)] = \left(\sqrt{x+5}\right)^2 - 5$

$= x+5-5$

$= x$

$f[f^{-1}(x)] = \sqrt{x^2 - 5 + 5}$

$= \sqrt{x^2}$

$= x$

Therefore, f^{-1} is the inverse of f.

What's More

Practice Activity

A. Find the inverse of f. Determine the domain and range of each resulting inverse functions. Write your answer inside the box provided.

1. $f(x) = 2x - 1$

$f^{-1} =$	Solution:
Domain	
Range	

2. $f(x) = 5x + 2$

$f^{-1} =$	Solution:
Domain	
Range	

3. $f\ (x) = \frac{x+2}{5}$

$f^{-1} =$	Solution:
Domain	
Range	

4. $f(x) = x^2 + 2$

$f^{-1} =$	Solution:
Domain	
Range	

5. $f(x) = \sqrt{1+x}$

$f^{-1} =$	Solution:
Domain	
Range	

What I Have Learned

Think It Over And Complete Me!

A. Complete The Paragraph

Remember that an inverse function is a _____ function. Whereas, the _____ of the inverse function is the range of the one-to-one function and the _____ of the inverse function is the domain of the one-to-one function.

To find the domain and range of an inverse function, go back to the _____ function and then _____ the domain and range of the original function.

B. How is the skill in operating fractions and radicals relevant in determining the domain and range of the inverse function? Explain.

_____
_____
_____
_____
_____
_____
_____

C. You have understood that inverse function is a function that reverses another function. In life, if it so happens that you have done some mistakes, you can only correct it and not reverse it. But if you would be given a chance to reverse one thing in your life, what would it be and why?

_____
_____
_____
_____
_____
_____
_____

What I Can Do

EXPLORE DEEPER AND THINK WISELY

Now that you have deeper understanding of the topic, you are ready to solve the problems below.

1. Temperatures are normally expressed in degrees Celcius or degress Fahrenheit. A temperature reading expressed in degrees Celsius can be converted to degrees Fahrenheit, and vice versa.
   a. Determine a function F that expresses a given temperature in degrees Fahrenheit to degrees Celsius.
      Solution:

   b. Determine a function C that expresses a given temperature in degrees Celsius to degrees Fahrenheit.

      Solution:

   c. Verify if the functions F and C are inverse Functions.
      Solution:

   d. Determine the domain and range of the functions and its inverse.

2. The formula S= (n-2) 180 gives the sum of the measures of the angles of n-sided polygon where n is the input and S is the output.
   a. Solve the formula for n so that S becomes the input and n becomes the output.
      Solution:

   b. Write the formula in (a) as the inverse function of f(x) = (x-2) 180.
      Solution:

   c. Verify if the two functions are inverse functions.

      Solution:

   d. Determine the domain and range of the function and its inverse.

Rubrics:

Score	For letters a and b	For letter c	For letter d
4	The function or formula was determined or formulated with properly shown procedures.	The functions were verified as inverse functions with completely shown procedures.	The domain and range of the function and its inverse were correctly determined and properly written.
3	The function or formula was determined or formulated with partially shown procedures.	The functions were verified as inverse functions with partially shown procedures.	The domain and range of the function and its inverse were correctly determined but it was improperly written.
2	The function or formula was not determined or formulated and other alternative procedures was shown.	The functions were not verified as inverse functions and other alternative procedures was shown.	The domain was correct but the range is incorrect or vice versa.
1	The function or formula was not determined or formulated without any procedure or solution.	The functions were not verified as inverse functions without any procedure.	The domain and range was not correctly determined and improperly written.

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What do you call to the resulting y-values after we have substituted in the possible x - values?

   a. Range
   b.Domain
   c. Real Numbers
   d. Inverse Function

3. Which of the following is not allowed as the value of the independent variable under the square root sign?

   a. zero
   b. negative number
   c. decimal number
   d. fraction

3. What is the domain and range of the function $(x) = 5x + 2$?

   a. The domain is all real numbers except 2 and the range is all real numbers except 0.
   b. The domain is all real numbers and the range is all real numbers except 0.
   c. The domain is all real numbers except 2 and the range is all real numbers.
   d. The domain and range are all real numbers.

4. What is the inverse of $(x) = 9x + 5$?

   a. $f^{-1}(x) = \frac{x-5}{9}$
   b. $f^{-1}(x) = \frac{9}{x-5}$
   c. $f^{-1}(x) = \frac{x+5}{3}$
   d. $f^{-1}(x) = \frac{9}{x+5}$

5. Which of the following pair of functions is NOT the inverse of each other?

   a. $f(x) = 5x \text{ and } g(x) = \frac{x}{5}$

   b.$f(x) = 2 - 3x \text{ and } g(x) = \frac{2-x}{3}$

   c. $f(x) = \frac{1}{x} \text{ and } g(x) = \frac{1}{x}$

   d.$f(x) = x^2 \text{ and } g(x) = \sqrt{x}$

For numbers 6-10, consider the function $f(x)\frac{3}{x-2}$.

6. What is the domain of the function?

a. $\{x \neq 3\}$
b. $\{x \neq 0\}$
c. $\{x \neq 2\}$
d. $\{x \neq -2\}$

7. What is the Range of the function?

a. $\{y \neq -2\}$
$b.\{y > 0\}$
c.$\{y \neq 3\}$
d. $\{y \neq 0\}$

8. What is the inverse of the function?

$a. f^{-1}(x) = \frac{2x-3}{x}$
b. $f^{-1}(x) = \frac{2x+3}{x}$
c. $f^{-1}(x) = \frac{x}{2x-3}$
d. $f^{-1}(x) = \frac{x}{2x+3}$

9. What is the domain of f^{-1}?

$a.\{x \neq 0\}$
$b.\{x \neq 2\}$
c. $\{x \neq -2\}$
d. $\{x \neq 3\}$

10. What is the range of f^{-1}?

a. $\{y \neq 3\}$
b. $\{y \neq -3\}$
c. $\{y \neq -2\}$
d. $\{y \neq 2\}$

For numbers 11-15, consider the function $(x) = \sqrt{x-1}$.

11. What is the domain of the function?

a. $\{x \geq -1\}$
b. $\{x \geq 1\}$
c. $\{x \geq 0\}$
d. $\{x < 1\}$

12. What is the range of the function?

a. $\{y < 1\}$
b. $\{y > 0\}$
c. $\{y \in R\}$
d. $\{y > 1\}$

13. What is the inverse of the function?

a. $f^{-1}(x) = x^2 - 1$
b. $f^{-1}(x) = x^2 + 1$
c. $f^{-1}(x) = \frac{1}{x^2}$
d. $f^{-1}(x) = x^2$

14. What is the domain of f^{-1} ?

a. $\{x > 1\}$
b. $\{x \in R\}$
c. $\{x < 1R\}$
d. $\{x > 0\}$

15. What is the range of f^{-1}?

a. $\{\geq 1\}$
b. $\{\geq -1\}$
c. $\{y < 1\}$
d. $\{y \geq 1\}$.

Additional Activities

Give Me More Companions

In this section, you are going to think deeper and test further your understanding of domain and range of inverse function. Ask someone who can help you to find the correct solutions and answer.

Tom and Jerry are school mates and they are playing a number- guessing game. Tom asks Jerry to think of a positive number, triple the number, square the results and then add 7. If Jerry's answer is 43, what was the original number? Use the concept of the inverse function and its domain and range in your solution.

Answer Key

What I Know

A
D
D
A
B
B
D
A
C
D
B
C
B
B
A

What's More

1.

f^{-1}	$f^{-1} = \frac{x+1}{2}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \in \mathbb{R}\}$

2.

f^{-1}	$f^{-1} = \frac{x-2}{5}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \in \mathbb{R}\}$

3.

f^{-1}	$f^{-1} = 3x - 2$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \in \mathbb{R}\}$

4.

f^{-1}	$f^{-1} = \sqrt{x-2}$
Domain	$\{x \mid x > 2\}$
Range	$\{y \mid y \in \mathbb{R}\}$

5.

f^{-1}	$f^{-1} = x^2 - 1$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -1\}$

Assessment

B
A
D
C
A
C
A
B
A
D
D
A
C
B
A

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*DepED Material: General Mathematics Learner's Material

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11 General Mathematics

General Mathematics
Solving Real-Life Problems Involving Inverse Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Real-life Problems Involving Inverse Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Real-life Problems Involving Inverse Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Real-life Problems Involving Inverse Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be able to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was intended and written to guide and help you apply inverse functions to real-life situations such as finding the original number, conversion of currency, converting units of temperature from degree Celsius to degree Farenheit and a lot more.

Likewise, you will learn how to evaluate inverse functions and interpret results. The knowledge and skills you have learned from the previous lessons are significant for you to solve real-life problems involving inverse functions.

After going through this module, you are expected to:

1. recall how to finding the inverse of the functions;
2. solve problems involving inverse functions; and
3. evaluate inverse functions and interpret results.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following is the inverse $f(x) = 3x^2 + 5$?

   a. $f^{-1}(x) = \sqrt{\frac{x-5}{3}}$ b. $f^{-1}(x) = \sqrt{\frac{x+5}{3}}$ c. $f^{-1}(x) = \sqrt{\frac{x-3}{5}}$ d. $f^{-1}(x) = \sqrt{\frac{x+3}{5}}$

2. Which of the following is the inverse $f(x) = 5x - 2$?

   a. $f^{-1}(x) = \frac{x+5}{2}$ b. $f^{-1}(x) = \frac{x+5}{2}$ c. $f^{-1}(x) = \frac{x-2}{5}$ d. $f^{-1}(x) = \frac{x+2}{5}$

3. A study found that the relationship between the students' exam scores (x) and the number of hours they spent in studying $f(x)$ is given by the equation of function $f(x) = \frac{x-55}{10}$. Using this information, what will be the estimated number of hours that the student spent studying if he scored 85 on the test?

   a. 4 hours b. 3 hours c. 2 hours d. 1 hour

4. The relationship between temperatures in degree Fahrenheit (°F) and in degree Celsius (°C) is given by $°F = \frac{9}{5}°C + 32$. What is the corresponding value in degree Celsius of $100°F$?

   a. 37.78 $°C$ b. 42.50$°C$ c. 65.28$°C$ d. 89.92$°C$

For items number 5-7, refer to the following:

Audrey and her mother are planning for a debut party. Audrey suggested that she wants to celebrate her birthday at Jardin De Emilia Hall. The reception hall costs a flat rate of ₱2000.00 and an additional rental fee of ₱50.00 per guest. If their budget for hall expenses is limited at ₱10,00.00

5. Which of the following is the cost as a function of the number of guests?
   a. y = 2000 + 50x b. y = 2000 – 50x c. y = 50 + 2000x d. y = 50 – 2000x

6. Which of the following is the inverse of cost as a function of the number of guests?
   a. $f^{-1}(x) = \frac{x-50}{2000}$ c. $f^{-1}(x) = \frac{x-2000}{50}$
   b. $f^{-1}(x) = \frac{x+50}{2000}$ d. $f^{-1}(x) = \frac{x+2000}{50}$

7. What is the domain and range of the inverse?
   a. D = {x ϵ N | 0 ≤ x ≤ 260}
      R = {y ϵ R | 0 ≤ y ≤ 10,000}
   b. D = {x ϵ N | 0 ≤ x ≤ 2000}
      R = {y ϵ R | 0 ≤ y ≤ 10,000}
   c. D = {x ϵ N | 0 ≤ x ≤ 160}
      R = {y ϵ R | 0 ≤ y ≤ 10,000}
   d. D = {x ϵ N | 0 ≤ x ≤ 10,000}
      R = {y ϵ R | 0 ≤ y ≤ 2000}

8. Suppose I am travelling at 50 miles per hour, and I want to know how I have gone in x hours. Then, it can be represented by the function $f(x) = 50x$. Find the inverse of the function.
   a. $f^{-1}(x) = \frac{x}{25}$ b. $f^{-1}(x) = \frac{x}{50}$ c. $f^{-1}(x) = \frac{x}{75}$ d. $f^{-1}(x) = \frac{x}{100}$

For items number 9-10, refer to the following:

Luis is standing on the ground to take a series of photographs of a kite rising vertically. The distance between Luis at (B) and the launching point of the kite (A) is 500 meters. Luis must keep the kite on sight and therefore its angle of elevation must change with height x of the kite.

9. Find the angle t as a function of the height x.
   a. $t = \tan^{-1}\left(\frac{x}{500x}\right)$ c. $t = \tan^{-1}\left(\frac{x}{500}\right)$
   b $t = \tan^{-1}\left(\frac{500x}{300}\right)$ d. $t = \tan^{-1}(500)$

10. Find the angle t in degrees when x is equal to 150 meters.
    a. 25.6 b. 26.6 c. 27 d. 28

11. Find the angle t in degrees when x is equal to 300 meters.
    a. 48 b. 47 c. 46 d. 45

For items number 12-13, refer to the following:

The function defined by $g(x) = 5.3x$ converts a volume of x gallons into *g(x)* liters.

12. Which of the following is the inverse of g(x)?
    a. $g^{-1}(x) = \frac{x}{5.3}$
    b. $g^{-1}(x) = \frac{x}{5x+3}$
    c. $g^{-1}(x) = \frac{5.3x}{5.3+x}$
    d. $g^{-1}(x) = \frac{3x}{5.3}$

13. Find the equivalent volume in gallons of a 40 – liter cooking oil.
    a. 9.5 b. 8.5 c. 7.5 d. 6.5

For items number 14-15, refer to the following:

Joshua resides in a certain city, but he starts a new job in the neighbor city. Every Monday, he drives his new car 90 kilometers from his residence to the office and spends the week in a company apartment. He drives back home every Friday. After 4 weeks of this routinary activity, his car's odometer shows that he has travelled 870 kilometers since he bought the car.

14. If the mathematical model that gives the distance y covered by the car as a function of x number of weeks is y = 180x + 150. Find its inverse.
    a. $f^{-1}(x) = = \frac{x-150}{180}$
    b. $f^{-1}(x) = = \frac{x-180}{90}$
    c. $f^{-1}(x) = = \frac{x+510}{90}$
    d. $f^{-1}(x) = = \frac{x+90}{150}$

15. If he travelled 1590 kilometers how many weeks he drives his car?
    a. 10 b. 8 c. 6 d. 4

Lesson 1 Solving Real-life Problems Involving Inverse Functions

You have learned from your previous modules the representations inverse functions through its table of values, graphs, and equations. You also learned how to find its domain and range which are important in the study of solving real-life problems involving inverse functions. This module will help you solve real-life problems involving inverse functions.

What's In

Let us start your journey by recalling the previous lessons you already learned about inverse functions. Here is the list of functions and its inverse, match column B to column A by finding the inverse of the items in column A. Write the letter of the answer in the box below that will reveal a "word" or the name of the "building" that you are looking for.

The United Arab Emirates was given the title of "The Tallest Building in the World" on January 4, 2010. What is the name of the building?

Column A	Column B
1. $g(x) = x^5 - 3$	R. $y = \frac{5x+1}{9x-4}$
2. $f(x) = 7x + 10$	A. $y = \frac{t}{3}$
3. $h(x) = \frac{4x+1}{9x-5}$	A. $y = \pm\sqrt{\frac{x^2}{5}}$
4. $k = \frac{5}{9}(t - 32) + 273.15$	J. $y = \frac{9}{5}(k - 273.15) + 32$
5. $w(x) = 2x + 9$	L. $y = \sqrt[3]{\frac{x+7}{2}}$
6. $t(x) = \frac{x+2}{3x-5}$	K. $y = \frac{x-9}{2}$
7. $r(x) = \lvert 5x \rvert$	B. $y = \sqrt[5]{x+3}$
8. $s(x) = 2x^3 - 7$	H. $y = \frac{5x+2}{3x-1}$
9. $q(x) = 3x - 5$	F. $y = \frac{x-11}{5}$
10. $n(x) = 5x + 11$	I. $y = \frac{x+5}{3}$
11. $z(x) = 3t$	R. $y = \sqrt[3]{\frac{x-2}{7}}$
	U. $y = \frac{x-10}{7}$

1	2	3	4

5	6	7	8	9	10	11

Notes to the Teacher

To be able to arrive in an accurate and similar answer, the teacher must advise the learners to recall the steps in finding the inverse of the function and the properties of an inverse function.

What's New

Now, that you already know how to find the inverse of the function, and how to evaluate inverse functions, as well as finding the domain and range. I am confident that you are now ready for the new lesson.

Exchange Rate!

Anna's mother works in South Carolina USA as a domestic helper for a living. She sends off money in the Philippines each month. Recently the exchange was $ 1.00 to ₱50.85.

(a) Complete the table by converting U.S. dollar to peso

$	1	5	25	50	100
₱	50.85				

(b) Describe how did you convert US dollar to peso.

_____

_____

(c) Write an equation that converts dollar into peso.

_____

_____

(d) Write an equation that converts peso into dollar using the equation in (c).

_____

_____

(e) Sketch and describe the graphs of the original function and its inverse. Write your answer on a separate sheet of paper.

(f) If Anna needs to buy a new laptop for her online classes, how much dollars should her mother give her if it costs ₱17,000.00?

_____

_____

(g) How important the conversion of currency in real-life situations?

_____

_____

In the previous activity, first you need to write a model that would represent the situation. To represent the equation of converting Philippine Peso to US dollar,

What is It

you need to think about the value of the US dollar as the input and the equivalent amount in peso as the output. Since the exchange rate is ₱50.85 per US dollar, then the function can be described as ₱ = 50.85$ and its inverse as $\$ = \frac{₱}{50.85}$ where ₱ and $ as are the amount in peso and dollar respectively. For you to complete the table of values, you need to evaluate the function. After completing the table of values you can now sketch the graph of the function and its inverse. Remember, that the domain of the original function is the range of the inverse function and the range of the original function is the domain of its inverse. Take note, that the graph of the inverse function is the reflection of the graph of the original function about the line $y = x$.

Going back to the situation, if Anna needs to buy a new laptop for her studies, how much dollars should her mother give her if it costs ₱17,000.00? Anna's mother should consider giving her 334.32$.

How important the knowledge of conversion of currency in real-life situations? If you are aware of the exchange rate, it is an advantage for you to choose the right institution or establishment for your money. You can calculate the amount you will receive as the less or high value after the currency is converted depending on the current exchange rates.

The inverse function is a function that switches the input and the ouput. But, not all functions have inverse functions. The reverse process performed by any function *f(x)* is called inverse of *f(x).* It means that the domain of the original function is the range of the inverse function and that the range of the original function is the domain of the inverse function.

The graph of the inverse is the reflection of the graph of the original function. The axis of symmetry is the line y = x.

Steps in finding the inverse of a function is given below.

To find $f^{-1}(x)$:

1. Replace $f(x)$ with y.
2. Interchange x and y.
3. Solve for the new y from the equation in Step 2.
4. Replace the new y with $f^{-1}(x)$ if the inverse is a function

For better understanding, study the examples below and reflect on the different steps to solve real-life problems involving inverse function.

Example 1

Andreau and his friend are playing a number - guessing game. Andreau asks his friend to think a positive number, then add four to the number. Next, square the resulting number, and multiply the result by 3. Finally, divide the result by 2. If you are his friend and you get a result of 50, (a) write an inverse function that will give you the original number and (b) determine the original number.

Solutions:

To find the inverse, you need first to represent a model for the situation

Let x be the number that you think of

$x + 4$ represents the statement "add four to the number"

$(x + 4)^2$ represents the statement "square the resulting number"

$3(x + 4)^2$ represents the statement "multiply the result by 3"

$\frac{3(x+4)^2}{2}$ represents the statement "divide the result by 2"

Therefore, the model for the situation is $f(x) = \frac{3(x+4)^2}{2}$

To find the inverse.

$y = \frac{3(x+4)^2}{2}$ Write $f(x)$ as y

$x = \frac{3(y+4)^2}{2}$ Interchange x and y

$2x = 4(y + 3)^2$ Multiply both sides by 2

$\frac{2x}{4} = (y + 3)^2$ Multiply both sides by $\frac{1}{4}$

$\sqrt{\frac{2x}{4}} = \sqrt{(y+3)^2}$ Get the square root of both sides

$\sqrt{\frac{2x}{4}} = y + 3$

$\sqrt{\frac{x}{2}} - 3 = y$ Apply the addition property of equality

Therefore, the inverse of the function is $f^{-1}(x) = \sqrt{\frac{x}{2}} - 3$.

(b) To find the original number, use the inverse of the function $f^{-1}(x) = \sqrt{\frac{x}{2}} - 3$, and evaluate $f^{-1}(50)$.

$f^{-1}(50) = \sqrt{\frac{50}{2}} - 3$

$f^{-1}(50) = \sqrt{25} - 3$

$f^{-1}(50) = 5 - 3$

$f^{-1}(50) = 2$

Therefore, the original number is 2.

Example 2

To convert from degrees Fahrenheit to Kelvin, the function is $k(t) = \frac{5}{9}(t - 32) + 273.15$, where t is the temperature in Fahrenheit (Kelvin is the SI unit of temperature). Find the inverse function converting the temperature in Kelvin to degrees Fahrenheit

Solution:

The equation of the function is: $k = \frac{5}{9}(t - 32) + 273.15$

We do not interchange the variables k and t because it refers to the temperatures in Kelvin and Fahrenheit respectively.

Solve for t in terms of k:

Use the given formula

$k = \frac{5}{9}(t - 32) + 273.15$

$k - 273.15 = \frac{5}{9}(t - 32)$ Apply the addition property of equality

$\left(\frac{9}{5}\right)k - 273.15 = \frac{5}{9}(t - 32)\left(\frac{9}{5}\right)$ Multiply both sides by $\frac{9}{5}$

$\left(\frac{9}{5}\right)k - 273.15 = (t - 32)$

$\frac{9}{5}(k - 273.15) + 32 = t$ Apply the addition property of equality

Therefore, the inverse function is $t(k) = \frac{9}{5}(k - 273.15) + 32$ where k is the temperature in Kelvin

Example 3

The SSG officers of Camohaguin National High School are planning for a JS Prom. The allocated budget for decorations, sounds, and other miscellaneous expenses is ₱10,000.00 and an additional ₱150.00 for meal expenses for each guest. The organization received an amount of ₱40,000.00 from its external stakeholders.

a. Write the total allocated budget as a function of the number of guests.
b. Find the inverse of the function.
c. State the domain and range for this situation.
d. Find the possible number of guest for a budget of ₱40,000.00

Solutions:

(a) Let x be number of guest

$f(x)$ be the allocated budget as a function of the number of guests.

Thus, $f(x) = 10000 + 150x$

(b) To find the inverse

y = 10,000 + 150x	Write f(x) in terms of y
x = 10,000 + 150y	Interchange x and y and solve for y
x – 10,000 = 150y	Apply the addition property of equality
$\frac{x-10,000}{150} = \frac{150y}{150}$	Divide both sides by 150
$\frac{x-10,000}{150} = y$	

Therefore, the inverse of the function is $f^{-1}(x) = \frac{x-10,000}{150}$

(c) Use the inverse of the function to find the domain and range of the situation.

$$f^{-1}(x) = \frac{x-10,000}{150}$$
$$f^{-1}(40,000) = \frac{40,000-10,000}{150}$$
$$f^{-1}(40,000) = 200$$

Domain : $\{\boldsymbol{x} \in \boldsymbol{N} \mid \boldsymbol{0} \leq x \leq 200\}$

Range : $\{\boldsymbol{y} \in \boldsymbol{R} \mid \boldsymbol{0} \leq y \leq 40,000\}$

(d) Therefore, for a budget of ₱40,000.00, two hundred (200) students may attend the JS prom. If there are more than 200 students, the organization needs to think of other means to raise additional funds for the prom.

Example 4

A Google Play Music allows member to download songs for ₱203.40 pesos each after paying a monthly service charge of ₱762.75. The total monthly cost C(x) of the service in peso is C(x) = 762.75 + 203.40x, where x is the number of songs downloaded.

(a) Find the inverse function
(b) What do x and $C^{-1}(x)$ represent in the context of the inverse function?
(c) How many songs were downloaded if a member's monthly bill is ₱3813.75?

Solutions:

(a) Use the given equation to find the inverse of the function.

$C(x)$ = 762.75 + 203.40x

y = 762.75 + 203.40x	Write $C(x)$ in terms of y
x = 762.75 + 203.40y	Interchange x and y and solve for y
x – 762.75 = 203.40y	Apply the addition property of equality
$\frac{x-762.75}{203.40} = \frac{203.40y}{203.40}$	Divide both sides by 49.50
$\frac{x-762.75}{203.40} = y$	

Therefore, the inverse of the function is $C^{-1}(x) = \frac{x-762.75}{203.40}$

(b) x is the total monthly cost of the service, and $C^{-1}(x)$ is the number of songs downloaded.

(c) 15 songs downloaded if a member's monthly bill is ₱3,813.75

Example 5

Maria wants to buy a particular breed of *bangus*. And she is aware that the weight W (in kilograms) of a particular breed of *bangus* is related to its length L (in centimeter). Given this function $W = (5.32\ x\ 10^{-3})L^2$, find its inverse and determine the approximate length of a *bangus* that weighs 0.769 kilogram

Solutions:

(a) To find the inverse

$W = (5.32\ x\ 10^{-3})L^2$

$\frac{W}{5.32\ x\ 10^{-3}} = \frac{(5.32\ x\ 10^{-3})}{5.32\ x\ 10^{-3}} L^2$ Divide both sides by $(5.32\ x\ 10^{-3})$.

$\frac{W}{5.32\ x\ 10^{-3}} = L^2$

$\sqrt{\frac{W}{5.32\ x\ 10^{-3}}} = \sqrt{L^2}$ Get the square root of both sides.

$\sqrt{\frac{W}{5.32\ x\ 10^{-3}}} = L$

Therefore, the inverse of the function is $\boldsymbol{L} = \sqrt{\frac{W}{5.32\ x\ 10^{-3}}}$

(b) To determine the approximate length of a *bangus* that weighs 0.769 kilogram , evaluate the inverse $f^{-1}(L) = \sqrt{\frac{W}{5.32\ x\ 10^{-3}}}$ when W=0.769 kilograms

$L = \sqrt{\frac{W}{5.32\ x\ 10^{-3}}}$

$L = \sqrt{\frac{0.769}{5.32\ x\ 10^{-3}}}$

$L \approx 12.02$

Therefore, the length of a particular breed of bangus is approximately equal to 12.02 cm.

Example 6

The balloon is rising vertically and Dennis wants to take a series of photographs. The distance between Dennis at (B) and the launching point of the balloon (A) is 250 meters. The angle of elevation must change with the height x of the balloon.

(a) Find the angle t as a function of the height x

(b) Find the angle t in degrees when x is equal to 125, 250, 500 and 1000 meters (approximate your answer to 1 decimal place)

(c) Graph t as a function of x.

Solutions:

(a) The opposite and adjacent sides to angle t are x and 250 meters.

$$\tan(t) = \frac{x}{250}$$

Use the property of the tangent function and it's inverse.

$$\tan^{-1}(\tan(t)) = x$$

Rewrite the equation $\tan(t) = \frac{x}{250}$

$$\tan^{-1}(\tan(t)) = \tan^{-1}\left(\frac{x}{250}\right)$$

Simplify the left side of the equation to obtain $t = \tan^{-1}\left(\frac{x}{250}\right)$

$$\tan^{-1}(\tan(t)) = \tan^{-1}\left(\frac{x}{250}\right)$$

$$\frac{1}{\tan}(\tan(t)) = \tan^{-1}\left(\frac{x}{250}\right)$$

$$\frac{\tan\tan(t)}{\tan} = \tan^{-1}\left(\frac{x}{250}\right)$$

$$\mathbf{t = \tan^{-1}\left(\frac{x}{250}\right)}$$

Therefore, the angle t as a function of the height x is $\boldsymbol{t = \tan^{-1}\left(\frac{x}{250}\right)}$

(b) Use your calculator to find the values of 125, 250, 500, and 1000.

$t(125) = \tan^{-1}\left(\frac{125}{250}\right)$ $t(250) = \tan^{-1}\left(\frac{250}{250}\right)$ $t(500) = \tan^{-1}\left(\frac{500}{250}\right)$ $t(1000) = \tan^{-1}\left(\frac{1000}{250}\right)$

t(125) = 26.6° **t(250) = 45°** **t(500) = 63.4°** **t(1000) = 76°**

Table of values

x	0	125	250	500	1000
t	0	26.6°	45°	63.4°	76°

(c)

The graph of *t* as a function of x

What's More

Read each situation carefully to solve each problem. Write your answer on a separate sheet of paper.

Activity 1.1

The ABS CBN News reports foreign exchange rate are closed on March 13, 2020 at ₱51.25. Therefore the formula that gives Philippine Peso in terms of US dollars on that day is:

$$P = 51.25D$$

Where D represents US dollar and P represents Philippine Peso.

(a) Complete the table by converting U.S. dollar to Peso

$	1	25	50	100	200
₱					

(b) Describe how did you convert US dollars to Peso.

_____

_____

(c) Find the inverse of the function to determine the value of a United States dollar in terms of Philippine Peso on March 13, 2020.

_____

_____

(d) Interpret and evaluate P (1000) and $P^{-1}(1000)$.

_____

_____

Activity 1.2

The cost of producing laptops by a JOB Company is given by C(x) = 1300x + 5500 (in pesos) where x is the number of produced laptops.

(a) Find the inverse of the function.

_____

_____

(b) How many laptops will produce if the cost is ₱12,000.00?

_____

_____

Activity 1.3

The formula for converting Celsius to Fahrenheit is given by $F = \frac{9}{5}C + 32$ where C is the temperature in degree Celsius and F is the temperature in degree Fahrenheit.

(a) Write the inverse of the function which converts temperature from degree Celsius to degree Fahrenheit.

_____

_____

(b) Find the equivalent temperatures in degree Fahrenheit of the following $20°C$, $10°C$, $5°C$, and $0°C$.

_____

_____

(c) Graph the inverse function.

Activity 1.4

Juan is making a collage, and he planned to form a circle by putting together various pieces of construction paper. Given the formula of the area of the circle $A = \pi r^2$.

(a) Find the inverse of the area in terms of radius.

_____

_____

(b) Use the inverse to find the radius of a circle with an area of 48 cm^2.

_____

_____

Activity 1.5

Engineers have determined that the maximum force t in tons that a particular bridge can carry is related the distance d in meters between its supports by the following function: $t(d) = \left(\frac{12.5}{d}\right)^3$

a. How far should the supports be if the bridge is to support 6.5 tons?

_____

_____

b. Construct an inverse function to determine the result.

_____

_____

What I Have Learned

A. Fill in the blanks with the correct term or phrase to complete the sentence.

1. The domain of the original function is the _____of the inverse functions.
2. The range of the original function is the _____of the inverse functions.
3. The graph of the inverse is the _____ of the graph of the original function about the line $y = x$?

B. In your own words, how important is your knowledge of solving real-life problems involving inverse functions?

_____
_____
_____
_____

What I Can Do

Read and understand the situation below, and make a complete plan to solve Mang Jose's problem.

Paint My House!

Mang Jose wants to paint the exterior of his house. He needs to know how many gallons of paint he would need. So, he asks his friend Juan to help him, according to Juan one (1) gallon of paint can cover an area of 250 square feet. Help Mang Jose prepare a budget for his project if his house exterior is 2700 square feet. Do a research on the different prices of one (1) gallon of paint depending on its brand name and choice of colors. Make a proposal budget for three (3) different colors of paint with its corresponding brand name.

Your output will be graded using this rubric.

CRITERIA	EXCELLENT 4 points	SATISFACTORY 3 points	DEVELOPING 2 points	BEGINNING 1point
Accuracy of the Solution	Shows accurate solution and estimation of the possible expenses.	Shows solution and estimation of the possible expenses with minimal errors.	Shows solution and estimation of the possible expenses with plenty of errors.	The solution and estimation of the possible expenses are all erroneous.
Mathematical Concept	Shows excellent understanding of the concept of solving real-life problems involving inverse functions and other concepts related to the problem.	Shows clear understanding of the concept of solving real-life problems involving inverse functions.	Shows limited understanding of the concept of solving real-life problems involving inverse functions.	Did not apply the concept of solving real-life problems involving inverse functions.

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following is the inverse $f(x) = \sqrt{\frac{x+3}{7}}$?

   a. $f^{-1}(x) = 7x^2 + 3$ b. $f^{-1}(x) = 7x^2 - 3$ c. $f^{-1}(x) = 3x^2 + 7$ d. $f^{-1}(x) = 3x^2 - 7$

2. Which of the following is the inverse $f(x) = 6x + 5$?

   a. $f^{-1}(x) = \frac{x-5}{6}$ b. $f^{-1}(x) = \frac{x+5}{6}$ c. $f^{-1}(x) = \frac{x-2}{5}$ d. $f^{-1}(x) = \frac{x+2}{5}$

3. A study found that the relationship between the number of hours (x) and the student's exam scores $f(x)$ is given by the equation of function $f(x) = 10x + 55$ Using this information, what will be the estimated number of scores of the student if he spent 4 hours in studying?

   a. 95 b. 85 c. 75 d. 65

4. The relationship between temperatures in degree Celsius (°C) and in degree Fahrenheit (°F) is given by $°C = \frac{5}{9}(°F - 32)$. What is the corresponding value in degree Fahrenheit of $37.78°C$?

   a. $80°F$ b. $90°F$ c. $100°F$ d. $110°F$

For items number 5-7, refer to the following:

Cath and Arvin are planning for their wedding. Cath suggested that she wants Casa de Aurora to cater their reception. The reception hall rental fee starts at a flat rate of ₱3,500.00 and an additional rental fee of ₱60.00 per guest. If their budgetis limited at ₱20,000.00.

5. Which of the following represents the total rental fee as a function of the number of guests?

a. y = 3500 + 60x
b. y = 3500 – 60x
c. y = 60 + 3500x
d. y = 60 – 3500x

6. Which of the following is the inverse function in item 5?

a. $f^{-1}(x) = \frac{x-60}{2000}$
b. $f^{-1}(x) = \frac{x+50}{3500}$
c. $f^{-1}(x) = \frac{x+35000}{50}$
d. $f^{-1}(x) = \frac{x-3500}{60}$

7. What is the domain and range of the inverse?

a. D = {x ϵ N | 0 ≤ x ≤ 275}
R = {y ϵ R | 0 ≤ y ≤ 20,000}
b. D = {x ϵ N | 0 ≤ x ≤ 2000}
R = {y ϵ R | 0 ≤ y ≤ 10,000}
c. D = {x ϵ N | 0 ≤ x ≤ 160}
R = {y ϵ R | 0 ≤ y ≤ 20,000}
d. D = {x ϵ N | 0 ≤ x ≤ 10,000}
R = {y ϵ R | 0 ≤ y ≤ 2000}

8. Suppose I am travelling at 30 miles per hour, and I want to know how I have gone in x hours. Then, this could be represented by the function $f(x) = 30x$. Find the inverse of the function.

a. $f^{-1}(x) = \frac{x}{10}$
b. $f^{-1}(x) = \frac{x}{20}$
c. $f^{-1}(x) = \frac{x}{30}$
d. $f^{-1}(x) = \frac{x}{100}$

For items number 9-11, refer to the following:
Marx is standing on the ground to take a series of photographs of a kite rising vertically. The distance between Luis at (B) and the launching point of the kite (A) is 800 meters. Luis must keep the kite on sight and therefore its angle of elevation must change with height x of the kite.

9. Find the angle *t* as a function of the height x.

a. $t = \tan^{-1}\left(\frac{x}{800}\right)$
b. $t = \tan^{-1}\left(\frac{500x}{300}\right)$
c. $t = \tan^{-1}\left(\frac{800x}{500}\right)$
d. $t = \tan^{-1}(800)$

10. Find the angle *t* in degrees when x is equal to 150 meters

a. 31.6
b. 21.6
c. 11.6
d. 10.6

11. Find the angle *t* in degrees when x is equal to 300 meters.

a. 20.6
b. 21.6
c. 22.6
d. 23.6

For items number 12-13, refer to the following:
The function defined by $g(x) = \frac{x}{5.3}$ converts a volume of x liters into *g(x)* gallons.

12. Which of the following is the inverse of $g(x)$?

a. $g^{-1}(x) = 5.3x$
b. $g^{-1}(x) = \frac{x}{5x+3}$
c. $g^{-1}(x) = \frac{5.3x}{5.3+x}$
d. $g^{-1}(x) = \frac{3x}{5.3}$

13. Find the equivalent volume in liters of a 7.5 – gallon cooking oil.
    a. 43 b. 42 c. 40 d. 50

For items number 14-15, refer to the following:
Mark resides in a Quezon City, but he starts a new job in the neighbor city. Every Monday, he drives his new car 80 kilometers from his residence to the office and spends the week in a company apartment. He drives back home every Friday. After 5 weeks of this routinary activity, his car's odometer shows that he has travelled 1000 kilometers since he bought the car. (Note: He only use his car for his job.)

14. If the mathematical model that gives the distance y covered by the car as a function of x number of weeks is y = 160x + 200. Find its inverse.
    a. $f^{-1}(x) = = \frac{x+150}{80}$
    b. $f^{-1}(x) = = \frac{x-180}{90}$
    c. $f^{-1}(x) = = \frac{x-510}{90}$
    d. $f^{-1}(x) = = \frac{x-200}{160}$

15. If he travelled 1640 kilometers how many weeks he drives his car?
    a. 10 b. 9 c. 6 d. 4

Additional Activities

Now, that you have gained skills in representing and solving real–life situations involving inverse functions, try to sharpen your skills by working on the task below.

John pays an amount ₱12.00 per hour for using the internet at Cyber Cafe. During Saturdays and Sundays, he enjoys and spends most of his time playing online games with his friends. The maximum number of hours he spend at Cyber Café ever weekend is 4 hours.
(a) How much will John pay for using the internet for 1 hour? 2 hours? 3 hours? 4 hours?
(b) Make a table of values. (c) Write a function that relates that amount spend and the time consumed. (d) Find the inverse of the function in item 2.
(e) If John has decided not to play the game in the internet café this weekend, what is the maximum amount that he would have saved?

Answer Key

What I Know	What 's In	What's More	Assessment
1. A 2. D 3. B 4. A 5. A 6. C 7. C 8. B 9. C 10. B 11. D 12. A 13. C 14. A 15. A	1. B 2. U 3. R 4. J 5. K 5. H 6. A 7. L 8. I 9. F 10. A	1. 1 (a) answers may vary ***(b)*** Multiply the US dollars to the given exchange rate at 51.25 to obtain the amount in peso. ***(c)*** $ = ***(d)*** P (1000) = 51,250 $P^{-1}(1000)$= 19.51 1.2. (a) $f^{-1}(x)$ (b) 5 1.3. (a) F = + 32 (b) 68, 50, 41, 32 © 1.4. (a) $r = \sqrt{\frac{A}{\pi}}$ (b) 3.91 cm 1.5 (a) 6.70 meters (b) $d(t) = \frac{12.5}{\sqrt[3]{t}}$	1. B 2. A 3. A 4. C 5. B 6. D 7. A 8. C 9. A 10. D 11. A 12. A 13. C 14. D 15. B

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*DepED Material:
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11 General Mathematics

General Mathematics
Representing Real-life Situations Using Exponential Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics
Representing Real-life Situations Using Exponential Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Representing Real-life Situations Using Exponential Functions.

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Representing Real-life Situations Using Exponential Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

Population growth is believed to be continuous overtime and there is an increase in growth rate over time. This scenario illustrates the exponential function. Population growth of organisms, growth of money in the bank, as well as decay of a substance, are some of the occurrences where exponential functions are used. Exponential function belongs to the so-called transcendental functions because they cannot be expressed by a finite number of algebraic operations.

In this learning module, you will know more about exponential function, and how the concept of an exponential function is utilized in our daily life. This module was designed and written with you in mind. It is here to help you master representing and solving real-life situations using exponential functions.

After going through this module, you are expected to:

1. define exponential functions;
2. show illustrations of exponential functions that represent real-life situations;
3. represent real-life situations using the exponential functions; and
4. solve problems involving real-life situations using the exponential functions.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. In the exponential function $f(x) = b^x$, x is the ____.
   a. base
   b. dependent variable
   c. exponent
   d. independent variable

2. Which of the following is an exponential function?
   a. $f(x) = x^2 + 3x - 4$
   b. $f(x) = 2^x - 3x + 4$
   c. $f(x) = 2^{3x-4}$
   d. $f(x) = x^2 + 3^x - 4$

3. Which of the given situations illustrates an exponential function?
   a. The distance travelled varies directly as the speed.
   b. The area of a square is s^2 where s is the length of the side of a square.
   c. Radioactive material has a half-life of 1500 years.
   d. As x increases, the value of y increases.

4. Solve $f(x) = 2^x$, if $x = -4$.
   a. $1/16$
   b. $1/8$
   c. $1/4$
   d. ½

5. Which of the following depicts the increase in number or size at a constantly growing rate?
   a. Half-life
   b. Exponential growth
   c. Exponential decay
   d. Compound interest

6. What is the rate of change in the formula $y = y_0(2)^{\frac{t}{T}}$ every T units of time?
   a. doubles
   b. half
   c. triples
   d. multiples

7. In the formula $A = P(1 + r)^t$A; what is P?
   a. principal compounds
   b. principal invested
   c. principal time
   d. principal year

8. Which of the following statements modeled an exponential growth?
   a. The cost of pencils as a function of the number of pencils.
   b. The distance when a stone is dropped as a function of time.
   c. The distance of a swinging pendulum bob from the center as a function of time.
   d. The compound interest of the principal amount as a function of time.

For nos. 9-10. Suppose a culture of 300 bacteria is put in a petri dish and the culture doubles every hour.

9. What is the exponential model on the given situation?
   a. $y = 2(300)^{\frac{1}{t}}$
   b. $y = 300(2)^{\frac{1}{t}}$
   c. $y = 2(300)^t$
   d. $y = 300(2)^t$

10. How many bacteria will be there after 9 hours?
   a. 93,660
   b. 153,600
   c. 653,100
   d. 393,660

For nos. 11-12. The half-life of a substance is 400 years. Initially there are 200 grams.

11. What is the exponential model for the given situation?
   a. $y = 200(\frac{1}{2})^{\frac{t}{400}}$
   b. $y = 400(\frac{1}{2})^{\frac{t}{200}}$
   c. $y = 200(\frac{1}{2})^{\frac{400}{t}}$
   d. $y = 400(\frac{1}{2})^{\frac{200}{t}}$

12. How much will remain after 800 years?
   a. 100 g
   b. 25 g
   c. 50 g
   d. 12.5g

For nos. 13-14. Lino invested ₱5,000.00 into an account which increases annually at the rate of 5.5%.

13. What equation best describes this investment after t years?
    a. $A = 5000(0.055)^t$
    b. $A = 5000(1.055)^t$
    c. $A = 5000(1.55)^t$
    d. $A = 5000\text{A}$

14. How much is his investment after 5 years?
    a. ₱6,534.80
    b. ₱7,843.20
    c. ₱25,204.50
    d. ₱45,354.80

15. A large slab of meat is taken from the refrigerator and placed in a pre-heated oven. The temperature T of the slab t minutes after being placed in the oven is given by $T = 170 - 165e^{-0.006t}$. What is the temperature rounded to the nearest integer after 30 minutes?
    a. 32°C
    b. 42°C
    c. 52°C
    d. 64°C

Lesson 1 Representing Real-Life Situations Using Exponential Functions

The beauty of Mathematics can be found everywhere. Sometimes, you are not aware that in front of you are situations which can be written as a Mathematics model. Some conditions in life increase and decrease tremendously such as the growth of bacteria, interest of an investment or an amount loaned, depreciation or appreciation of the market value of a certain product, and even the decay of microorganism. These real-life situations exhibit exponential patterns.

This lesson is about modeling real-life situations using exponential functions like population growth, population decay, growth of an epidemic, interest in banks and investments.

What's In

Before you proceed to the new lesson, study the following, and recall what you have learned from the previous lesson so that you will be ready for your next journey.

Definition

An exponential function with the base b is a function of the form
or, where
Some examples are:, , , and

The following will help you to recall, how to evaluate functions.

Example 1. If $f(x) = 4^x$, evaluate $f(2)$, $f(-2)$, $f(1/2)$, and $f(\pi)$.
Solution:

$f(2) = 4^2 = 16$

$f(-2) = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

$f(1/2) = 4^{1/2} = \sqrt{4} = 2$

$f(\pi) = 4^{\pi}$

Example 2. Complete the table of values for x = -3, -2, -1, 0, 1, 2, and 3 for the exponential functions $f(x) = 3^x$ and $f(x) = (1/3)^x$.

x	-3	-2	-1	0	1	2	3
$f(x) = 3^x$	1/27	1/9	1/3	1	3	9	27
$f(x) = (1/3)^x$	27	9	3	1	1/3	1/9	1/27

Let b a positive number not equal to 1. A transformation of an exponential function with base b is a function of the form

$$f(x) = a * b^{x-c} + d \text{ where } a, c, \text{and } d \text{ are real numbers}$$

Notes to the Teacher

Since is irrational, the rules for rational exponents are not applicable. We define using rational numbers: can be approximated by $4^{3.14}$. A better approximation is $4^{3.14159}$. Intuitively, one can obtain any level of accuracy for 4π by considering sufficiently more decimal places of . Mathematically, it can be proved that these approximations approach a unique value, which we define to be .

What's New

Helping Hands!

Read and analyze the problem carefully to complete the table and to answer the questions that follow.

Ms. Love Reyes, a Mathematics teacher introduces a new project to teach her students the values of helpfulness and sharing through peer tutoring while learning Math. She believes that her students will be more comfortable and open when interacting with a peer. To teach a short cut technique in solving rational equations and inequalities, she demonstrates the strategy to one of her students and requires this student to do the same to two of his classmates, with a condition that each student who undergoes the peer tutorial will repeat the process until everyone in the class will be able to learn the short cut technique. Also, each student is required to submit a reflection paper of their experienced while doing the peer tutoring and learning with classmates, for her to assess if she is successful to attain her objectives.

a. Based on the given situation, complete the table below.

Tutorial Stage	0	1	2	3	4	5	6
Number of Students who undergo the tutorial	1						

(Hint: In 0 stage, only one student undergoes the tutorial, he is the first student chose by Ms. Reyes, stage 1 is the stage where the first students share his learning to his classmate and continue up to stage 6)

b. What pattern can be observed from the data?

_____
_____
_____
_____
_____
_____

c. Write a formula to determine the number of students who are already involved with the tutorial project in a particular stage?

_____
_____
_____
_____
_____

d. If the project will be extended to other students within the school, in what stage will it reach 512 students?

_____
_____
_____
_____
_____

e. Illustrate the situation above using a tree diagram.

f. What kind of teacher is Mrs. Reyes?

_____
_____
_____
_____
_____

g. Given a chance, will you join the project? Why?

_____
_____
_____
_____
_____

What is It

The problem in the previous activity is an example of real-life situations using exponential functions. Hence, exponential functions occur in various real-world situations. Exponential functions are used to model and illustrate real-life situations such as population growth, radioactive decay and carbon dating, growth of an epidemic, loan interest and investments.

In the previous activity, you need to complete the table for you to see the pattern. Have you seen the pattern? The pattern represents the exponential functions. You may observe that as the stage increases, the number of students involved also increases in the pattern which is equal to $f(x) = 2^x$. If you got it correctly, congratulations! You already representing the exponential function to a real-life situation and I am sure you can now answer the question, if the project will be extended to other students within the school, in what stage will it reach 512 students? So, the answer is stage 9.

Going back to the project, what can you say to Mrs. Reyes? What kind of teacher is she? Well, it's up to you to answer the question to yourself. What I believe is that, you will be lucky if you will be a student of Mrs. Reyes because she is not only teaching Mathematics but she is also infusing good values to her students. You may now reflect on the question, if given the chance, will you join the project? Why or why not?

Exponential Function

An exponential function with the base b is a function of the form $f(x) = b^x$ or $y = b^x$, where $(b > 0, b \neq 1)$.

Some of the most common applications in real-life of exponential functions and their transformations are population growth, exponential decay, and compound interest.

The following are examples of representing an exponential function in real-life situations.

Example 1

Suppose a culture of 300 bacteria at MJD Farm is put into a Petri dish and the culture doubles every 10 hours. Give an exponential model for the situation. How many bacteria will there be after 90 hours?

Solution:

a. Let y = number of bacteria
   At $t = 0, y = 300$

   $t = 10, \quad y = 300(2) = 600$
   $t = 20, \quad y = 300(2)^2 = 1200$
   $t = 30, \quad y = 300(2)^3 = 2400$
   $t = 40, \quad y = 300(2)^4 = 4800$

   An exponential model for this situation is $y = 300(2)^{t/10}$

b. If $t = 90$, then $y = 300(2)^{90/10}$, $y = 300(2)^9$, $y = 153{,}600$. There will be 153,600 bacteria after 90 hours.

Exponential Models and Population Growth

Suppose a quantity y doubles every T units of time. If is the initial amount, then the quantity after t units is given by

Example 2

A certain radioactive substance decays half of itself every 5 days. Initially, there are 50 grams. Determine the amount of substance left after 30 days, and give an exponential model for the amount of remaining substance.

Solution:

a. Let t= time in days
   At t= 0 Amount of Substance = 50g
   t= 5 Amount of Substance = 50 (1/2) = 25 g
   t = 10 Amount of Substance = 50 $(1/2)^2$ = 12.5 g
   t = 15 Amount of Substance = 50 $(1/2)^3$ = 6.25 g

   An exponential model for this situation is $y = 50\,(1/2)^{t/5}$

b. $y = 50(1/2)^{30/5} = 50(1/2)^6 = 0.78125$ g

Exponential Decay

The half-life of a radioactive substance is the time it takes for half of the substance to decay. The exponential decay formula is $y = y_o\,(1/2)^{t/T}$.

Example 3

Aling Dionisia deposits ₱10,000.00 in BDO that pays 3% compound interest annually. Define an exponential model for this situation. How much money will she have after 11 years without withdrawal?

Solution: Compound Interest means the interest earned at the end of the period is added to the principal and this new amount will earn interest in the nesting period.

a. At $t = 0$ ₱10,000
$t = 1$ ₱10,000+ ₱10,000(0.03) = ₱10,300.00
$t = 2$ ₱10,300+ ₱10,300(0.03) = ₱10,609.00
$t = 3$ ₱10,609 + ₱10,609(0.03) = ₱10,927.27

From the above, the principal amount together with the interest earned as computed is as follows:

At $t = 0$ ₱10,000
$t = 1$ ₱10, 000(1+0.03) = ₱10,000(1.03) = ₱10,300.00
$t = 2$ ₱10,000$(1+0.03)^2$ = ₱10,000$(1.03)^2$ = ₱10,609.00
$t = 3$ ₱10, 000$(1+0.03)^3$= ₱10,000$(1.03)^3$ =₱10,927.27

An exponential model for this situation is $A = 10{,}000(1.03)^t$

b. A = ₱10,000$(1.03)^{11}$

= ₱13,842.34

After 11 years without withdrawal there will be ₱13,842.34 in bank.

Compound Interest

If a principal P (*initial amount of money*) is invested at an annual rate of r; compounded annually, then the amount after t years is given by

$A = P(1+r)^t$.

Example 4

The Natural Exponential Function

While an exponential function may have various bases, a frequently used base is the irrational number ***e***, whose value is approximately 2.71828. Because ***e*** is a commonly used base, the natural exponential function is defined as having ***e*** as the base.
The predicted population of a certain city is given by $P=200{,}000\ e^{(0.03y)}$ where y is the number of years after the year 2020. Predict the population for the year 2030.

Solution:

The number of years from 2020 to 2030 is 10, so y= 10.

$P = (200{,}000)(2.71828)^{(0.03)(10)}$

P = 269, 971.70

The predicted population for the year 2030 is 269, 971.

The **natural exponential function** is the function $f(x) = e^x$.

Notes to the Teacher

Remind the students about the units of the final answer. Explain to them when to round off the result. If the problem involve money two decimal place is ok, but if it is about population it should be a whole number.

What's More

Activity 1.1

Solve the following:

1. A culture of 100 bacteria in a petri dish doubles every hour.
   a. Complete the table.

t	0	1	2	3	4
No. of bacteria					

   b. Write the exponential model for the number of bacteria inside the box.

   c. How many bacteria will there be after 6 hours?
   Solution:

   Answer: _____

2. The half-life of a radioactive substance is 12 hours and there are 100 grams initially.

   a. Complete the table.

t	0	12	24	36	48
Amount					

   b. Write the exponential model for the amount of substance inside the box.

   c. Determine the amount of substance left after 3 days.
   Solution:

   Answer: _____

3. Kim deposited ₱10,000.00 in a bank that pays a 3% compound interest annually.
   a. Identify the given: P = _____
      r = _____

   b. Write the exponential model for the amount of substance inside the box.

   c. How much money will he have after 2 years?
      Solution:

      Answer: _____

Activity 1.2

1. Suppose the half-life of a certain radioactive substance is 20 days and there are 10g initially. Determine the exponential model and the amount of substance remaining after 75 days.
   Solution:

2. Danzel deposited an amount of ₱10,000.00 in a bank that pays 4% annual interes compounded annually. How much money will he have in the bank after 2 years.
   Solution:

3. The population of a certain country can be approximated by the function P(x) = 20,000,000 $e^{0.0251x}$ where x is the number of years. Use this model to get the approximate number of the population after 30 years.
   Solution:

What I Have Learned

A. Fill in the blanks with the correct term or phrase to complete the sentence.

1. A function of the form $f(x) = b^x$ or $y = b^x$, where $b > 0$ and $b \neq 1$ is called _____.
2. Suppose a quantity y doubles every T units of time. If y_o is the initial amount, then the quantity y after t units is given by the formula _____.
3. The time it takes for half of the substance to decay is called _____.
4. The exponential decay formula is _____.
5. If a principal P is invested at an annual rate of r; compounded annually, then the amount after t years is given by the formula _____.

B. In your own words, what are the steps to represent exponential function to real-life situation?

_____
_____
_____
_____
_____
_____
_____
_____
_____

C. Our population today increases exponentially which results to some economic problems. If you will become the president of the Philippines, what programs will you suggest to solve the problems? Explain.

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

What I Can Do

Read and understand the situation below, then answer the questions or perform the tasks that follow.

Wise Decision and Friendship Goal

You have a best friend, and she is also an 18-year old senior high school student and asking for your advice as to which between the two "25th birthday gift options" posted by her parents she should choose for her 25th birthday.

Option A: Her parents will give her ₱3,000.00 each year starting from her 19th birthday until her 25th birthday.

Option B: Her parents will give her ₱400.00 on her 19th birthday, ₱800.00 on her 20th birthday, ₱1,600.00 on her next birthday, and the amount will be doubled each year until she reaches 25.

Task:

You need to prepare a written report highlighting the amount of money (y) your best friend gets each year (x) starting from her 19th birthday using options A and B in tabular form. Write equations that represent the two options with a complete set of solutions. At the end of your report, write a conclusion stating the option you will choose and the explanation of your decision.

Written Report:

Conclusion:

_____
_____
_____
_____
_____
_____
_____

Rubrics for rating the output:

Score	Descriptors
20	The situation is correctly modeled with an exponential function, appropriate mathematical concepts are fully used in the solution and the correct final answer is obtained.
15	The situation is correctly modeled with an exponential function, appropriate mathematical concepts are partially used in the solution and the correct final answer is obtained.
10	The situation is not modeled with an exponential function, other alternative mathematical concepts are used in the solution and the correct final answer is obtained.
5	The situation does not model an exponential function, a solution is presented but has an incorrect final answer.

The additional 5 points will be determined from the conclusions or justifications made.

5-States a conclusion with complete and appropriate justification based on a reasonable interpretation of the data.

4-States a conclusion with enough justification, based on a reasonable interpretation of the data.

3-States a conclusion with some justification, based on a reasonable interpretation of the data.

2-States a conclusion on a reasonable interpretation of the data.

1-The conclusion is based on an unreasonable interpretation of the data.

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. In the exponential function $f(x) = b^x$, b is called as the _____.
   a. base
   b. dependent variable
   c. exponent
   d. independent variable

2. Which of the following defines an exponential function?
   a. $f(x) = 2x^2$
   b. $f(x) = 2x-1$
   c. $f(x) = 3^x$
   d. $f(x) = x^2+1$

3. Solve $f(x) = 2^{x+1}$, if $x = 2$.
   a. 2
   b. 4
   c. 8
   d. 16

4. Which of the given situations illustrate an exponential change?
   a. A store has 100 regular customers and each month 5 new customers come
   b. The number of organisms in a culture doubles every 5 hours
   c. The monthly wage of a laborer increase by 75 every year
   d. As x increases the value of y increases

5. What do you call a quantity that decreases at a rate proportional to its current value?
   a. Population growth
   b. Exponential growth
   c. Exponential decay
   d. Compound interest

6. Which of the following situations describe an exponential decay?
   a. The number of rabbits doubles every month.
   b. The population decreases every year by 100.
   c. The atmospheric pressure decreases as you go higher.
   d. The amount of money increases every year.

7. What is the approximate value of e in the equation $y = e^x$?
   a. 3.1416
   b. 31.416
   c. 2.71828
   d. 27.1828

8. Half-life is the time required for a quantity to reduce to half its initial value. Which of the following represents exponential function involving half-life?
   a. $y= y_o(2)^{t/T}$
   b. $y=y_o (1/2)^{t/T}$
   c. $A = P(1+r)^t$
   d. $y= e^x$.

For nos. 9-10. What if the 200 bacteria in a certain culture doubles every 3 hours?

9. What is the exponential model for the given situation?

a. $y= 2(200)^{t/3}$
b. $y= 200(2)^{t/3}$
c. $y= 2(200)^{3/t}$
d. $y= 200(2)^{3/t}$

10. How many bacteria are there after 9 hours?

a. 1600
b. 1800
c. 2000
d. 2100

For nos. 11-12. The half-life of a radioactive substance is 10 days and there are 10 grams initially.

11. What is the exponential model for the given situation?

a. $y= (1/2)(10)^{10/t}$
b. $y= 10(1/2)^{t/10}$
c. $y= 10(1/2)^{10/t}$
d. $y= (1/2)(10)^{t/10}$

12. What is the amount of substance left after 20 days?

a. 5 g
b. 2.5 g
c. 0.025 g
d. 1.25 g

For nos. 13-14. Alex deposited ₱1,000.00 in a bank at Lucena City that pays 5% compound interest annually.

13. What equation best describes this investment after t years?

a. $A= 1{,}000\ (1.5)^t$
b. $A= 1{,}000\ (1.05)^t$
c. $A= 1{,}000\ (15)^t$
d. $A= 1{,}000\ (1.005)^t$

14. How much money will he have after 2 years?

a. ₱1,100.50
b. ₱1,102.50
c. ₱1,201.50
d. ₱1,220.50

15. The predicted population of a certain city is given by $P=5{,}000e^{(0.15y)}$ where y is the number of years after 2020. What is the population in the year 2028?

a. 6,600
b. 16,600
c. 17,600
d. 18,000

Additional Activities

Now that you have gained skills in representing and solving real-life situations using exponential functions, try to sharpen your skill by working on the task below:

Your task is to study the exponential function of the Corona Virus. Look for the different exponential model for the virus.

Answer Key

What I Know	What's More	Assessment
1. C 2. C 3. C 4. A 5. B 6. A 7. B 8. D 9. D 10. B 11. A 12. C 13. B 14. A 15. A	1.a. b. $y= 10(2)^t$; c. $y= 10(2)^6$; 6400 bacteria 2. a. $y= 5(1/2)^{t/5}$; $y= 5(1/2)^{72/5;}$ 0.625g a. P=P10,000; r= 3% $A= 10,000(1.03)^t$; $A= 10,000(1.03)^t$; A= P10,609 a. $P(x)= 20,000(e)^{(0.0251)(30)}$ b. 42,467	1. A 2. C 3. C 4. B 5. C 6. B 7. C 8. B 9. D 10. A 11. B 12. C 13. B 14. B 15. B

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*DepED Material: General Mathematics Learner's Material

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11 General Mathematics

General Mathematics
Exponential Functions, Equations, and Inequalities
First Edition, 2020

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Published by the Department of Education
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Undersecretary: Diosdado M. San Antonio

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General Mathematics

Exponential Functions, Equations, and Inequalities

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Exponential Functions, Equations, and Inequalities!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Exponential Functions, Equations and Inequalities!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be able to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you master Exponential Function, Exponential Equation and Exponential Inequality. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

1. distinguish logarithmic function, logarithmic equation, and logarithmic inequality; and
2. formulate own examples of exponential functions, equations, and inequalities.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What do you call an expression that is of the form $a \cdot b^{x-c}d$, where a, b, c, and d are real numbers and x is a variable?
   a. Rational Algebraic Expression
   b. Mathematical Expression
   c. Exponential Expression
   d. Logarithmic Expression

2. Which of the following expresses the relationship between two variables?
   a. Exponential Equation
   b. Exponential Inequality
   c. Exponential Function
   d. Exponential Expression

3. Which of the following is commonly expressed as an independent variable?
   a. y
   b. x
   c. $f(x)$
   d. $g(x)$

4. In which of the following is $16 = 4^{x-2}$ classified?
   a. Exponential Expression
   b. Exponential Equation
   c. Exponential Function
   d. Exponential Inequality

5. In which of the following is 2^{3x-4} classified?
   a. Exponential Expression
   b. Exponential Equation
   c. Exponential Function
   d. Exponential Inequality

6. In which of the following is $f(x) = 7x^3$ classified?
   a. Exponential Expression
   b. Exponential Equation
   c. Exponential Function
   d. Exponential Inequality

7. In which of the following is $\left(\frac{1}{2}\right)^{3x} \leq 16$ classified?
   a. Exponential Expression
   b. Exponential Equation
   c. Exponential Function
   d. Exponential Inequality

8. What is true about the exponential function $y = 10^{x-2}$?
   a. The possible values for x can be solved based on y-values.
   b. This can also be considered as an exponential equation.
   c. This shows the relationship between two variables.
   d. The possible values for y can be assigned beforehand.

9. Which of the following is an exponential function?
   a. $y = 9x^2$
   b. $h(x) = 4^x$
   c. $2^{x+1} = 4$
   d. $3x^2 = 81$

10. Which of the following is an exponential inequality?
   a. $9^x < 27^{2x}$
   b. $3^{4x} = y$
   c. $2x = 64x^2$
   d. $f(x) = 6^x$

11. Which of the following is an exponential equation?
   a. $10^{x-2} \geq 100^x$
   b. $g(x) = 8^{3x}$
   c. $12 = 144^x$
   d. $(0.25)^{x+4} > (0.5)^{5x}$

12. In which of the following is $2^2(400^{x+1}) = 80$ classified?
    a. Exponential Model
    b. Exponential Function
    c. Exponential Inequality
    d. Exponential Equation

13. In which of the following is $64 \geq 4^{x+1}$ classified?
    a. Exponential Model
    b. Exponential Function
    c. Exponential Inequality
    d. Exponential Equation

14. Which of the following is an exponential equation?
    a. $36 = x^2$
    b. $3^{4x} > 27$
    c. $f(x) = 10^{2x}$
    d. $81 = 9^x$

15. Which of the following is an exponential inequality?
    a. $(0.64) \leq (0.8)^x$
    b. $3 > 27x^3$
    c. $10^2 \geq 1000x$
    d. $49 < 7^3$

Lesson 1 Exponential Functions, Equations and Inequalities

What is the pride of your city or province? Are you aware of the natural sources of income your city or province have? Have you ever thought of the pattern in the production of some natural sources of income in your hometown? Are they increasing or declining quickly? As a Senior High School learner and a concern citizen, that is one good thing you need to be aware of.

What's In

Anything that increases or decreases rapidly is said to be exponential. You have learned in the previous module that there are a lot of real-life situations involving such conditions. For instance, population growth, exponential decay, and compound interest. And these situations depict the so-called exponential functions.

Recall that an exponential function with base b is of the form $f(x) = b^x$ or $y = b^x$, where $b > 0$, but $b \neq 1$. Have you noticed anything with the exponent? How does it differ from the exponents of some other functions? Good! The exponential function has exponents that are variables. In the past lesson, you have learned that this exponent determines how fast a function increases or decreases.

Notes to the Teacher

Lead learners to the concept that f(x) and y are the same in dealing with functions, as they both refer to dependent variable. Guide learners as well in understanding that the symbols "$b > 0,\ b \neq 1$" mean all positive numbers, but one.

What's New

Spot the Similarity and Difference!

Below are three expressions. Observe them and spot their similarities and differences.

(a) $3^{x-2} = 81$ (b) $y = 4^x$ (c) $2^x \geq 32$

Questions:

1. What is the similarity of (a), (b), and (c)?
2. What is the similarity of (a) and (c)?
3. How do (a) and (c) differ from (b)?
4. How are (a) and (b) similar to each other?
5. How do (a) and (b) differ from (c)?
6. Can you recall which among them is an exponential function?
7. Which among the three is/are exponential expression/s?

As you have noticed, all the three givens are expressions involving variable as the exponent (a) and (c) both involve one variable only, while (b) involves the relationship between two variables. Nevertheless, (a) and (b) both use an equal sign. (b) however, it contains an inequality symbol.

Recall that (b) is an exponential function since it shows the relation between dependent variable y and independent variable x. Nevertheless, all three are exponential expressions, including (a) and (c). It is so since all are expressions with a variable used as an exponent.

What is It

From the previous activity, you have learned that there are other exponential expressions aside from exponential function. Based on what you have observed in the activity, they are those exponential expressions that do not involve a dependent variable y or *f(x).* But taking a deeper look, you have noticed that these expressions can be further classified into two – exponential equation and inequality. Let us now differentiate the three.

An **exponential function** is a function involving exponential expression showing a relationship between the independent variable x and dependent variable y or *f(x).* Examples of which are $f(x) = 2^{x+3}$ and $y = 10^{2x}$.

On the other hand, an **exponential equation** is an equation involving exponential expression that can be solved for all x values satisfying the equation. For instance, $121 = 11^x$ and $3^x = 9^{x-2}$.

Lastly, an **exponential inequality** is an inequality involving exponential expression that can be solved for all x values satisfying the inequality. For example, $64^{1/3} > 2^x$ and $(0.9)^x > 0.81$.

After learning the differences among the three exponential expressions, can you give your own examples for each? What are they?

What's More

Activity 17.1 Where Do They Belong?

Below is a list of exponential expressions. Classify each as to whether it is an exponential function, equation, inequality, or does not belong to any of these three.

$32^{x-4} \leq 16^{x+2}$	$36^x = 6$	$6 > \left(\frac{1}{36}\right)^x$	$x^7 + 1 < 10x^8$	$y = x^5$	$64 = 2^{x+2}$
$100 > 10^{2x}$	$f(x) = x^3$	$g(x) = 4^{5x}$	$y = 5^{x-1}$	$\left(\frac{1}{2}\right)^{x+2} = \left(\frac{1}{8}\right)^x$	$f(x) = 2^x$
$7 = 49^x$	$27 < 3^x$	$f(x) = 5^{x+2}$			

Exponential Function	**Exponential Equation**	**Exponential Inequality**	**None of these**

Activity 17.2 Classify and Justify!

On the blank provided before each number, classify each exponential expression into an exponential function, equation, inequality, or none of these three. Justify your decision in one sentence for every item. Write the justification on the blanks provided below each given expression.

_____ 1. $3^{2x} = 81$

_____

_____

_____ 2. $x^5 < 15x^3$

_____

_____

_____ 3. $5 > \left(\frac{1}{25}\right)^x$

_____

_____

_____ 4. $h(x) = 6^{3x}$

_____

_____

_____ 5. $3^2 = 2^{x+1}$

_____

_____

What I Have Learned

Fill in the blanks of the following statements with the correct missing words or phrases.

1. An expression involving a rapid increase or decrease is said to be _____ _____.
2. The exponent of an exponential expression is a _____ _____ showing how fast the function increase or decrease.
3. When an exponential expression depicts a relationship between two variables, it involves _____.
4. An _____ can be solved for all x values of those involving equations.
5. An exponential inequality is a/an _____ that can also be solved for all values of x.

What I Can Do

Coordinate with authorities regarding the production rate of the natural sources of income in your areas such as rice, coconut, or fish production. Construct an exponential function corresponding to the production rate that you will be given. Based on the exponential expression you formulated, how would you be able to contribute in their promotion or restoration as a responsible citizen? How would you promote your own city's pride to others?

Try scoring your essay using the rubric below.

Criteria	**4**	**3**	**2**	**1**
Formulated Exponential Expression	Correct exponential expression	Exponential expression with mistakes	Mathematical expression that is not exponential	No mathematical expression presented
Content	Very relative to the topic and well-organized	Somewhat relative to the topic and organized	Less relative to the topic and poorly organized	Not relative to the topic and very poorly organized
Spelling, Grammar, and Punctuations	No spelling, punctuation or grammatical errors	Very few spelling, punctuation, and grammatical errors	Several spelling, punctuation, and grammatical errors	Many spelling, punctuation, and grammatical errors

When you scored 9-12, send your work as a private message to some of your friends residing in the same area where you are. Let them feel your appreciation of your local officials' service. At the same time, encourage them to support your local officials. But when you scored 8 and below, try revising it first. Then, see to it that right after, you will also motivate your friends to have the same concern as yours.

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What is true about exponential expression?
   a. It has an exponent.
   b. Its exponent is a variable.
   c. Its exponent is constant.
   d. Its exponent is an expression.
2. Which of the following is an exponential function?
   a. $f(x) = (4 + x)^2$
   b. $f(x) = 4x^2$
   c. $f(x) = 4x$
   d. $f(x) = 4^x$
3. In which of the following is $36^x = 6^{x+4}$ classified?
   a. Exponential Model
   b. Exponential Function
   c. Exponential Inequality
   d. Exponential Equation
4. Which of the following is commonly used as a dependent variable in an exponential function?
   a. $f(x)$
   b. x
   c. $2x$
   d. 2^x

5. In which of the following is $169 \geq 13^{2x}$ classified?
   a. Exponential Inequality
   b. Exponential Equation
   c. Exponential Function
   d. Exponential Model

6. In which of the following is $3^{5x-1} = 27$ classified?
   a. Exponential Expression
   b. Exponential Equation
   c. Exponential Function
   d. Exponential Inequality

7. Which of the following exponential function?
   a. $5^{x+3} < 25^x$
   b. $12 = 144^x$
   c. $y = 12^{x-1}$
   d. $(0.64)^x > (0.8)^{x+4}$

8. Which of the following is an exponential inequality?
    a. $(0.04) \leq 0.2x$
    b. $3 > 27^x$
    c. $30^2 \geq 900x$
    d. $64x < 2^6$
9. Which of the following is an exponential function?
    a. $y = 11x^2$
    b. $f(x) = 11x$
    c. $j(x) = 11^x$
    d. $11x^2 = x^{11}$
10. Which of the following is an exponential inequality?
    a. $4^{x-1} > 16^{2x}$
    b. $4x - 1 < 16x^2$
    c. $y = 16x^2$
    d. $f(x) = (16 + x)^2$
11. Which of the following is an exponential equation?
    a. $x^2 = 144$
    b. $15^{x+2} = 225$
    c. $2x + 3 = 29$
    d. $f(x) = 8^x$
12. In which of the following is $y = 2^{3x+1}$ classified?
    a. Exponential Expression
    b. Exponential Equation
    c. Exponential Function
    d. Exponential Inequality
13. What is true about an exponential function?
    a. It contains the inequality symbol.
    b. It has a numerical exponent only.
    c. It is written in the form $f(x) = b^x$ where $b > 0, b \neq 1$.
    d. It involves radical expression.
14. In which of the following is $g(x) = 4^{x-1}$ classified?
    a. Exponential Function
    b. Exponential Equation
    c. Exponential Inequality
    d. None of these
15. What is true about $1000 = 100^x$?
    a. It can be considered as an exponential function.
    b. It is an exponential equation whose x-value can be solved.
    c. It shows the relationship between the independent and dependent variables.
    d. It can also be expressed as an exponential inequality with one variable

Additional Activities

Formulate your own 5 examples for each of the following:

- Exponential function
- Exponential equation
- Exponential inequality

Answer Key

What I Know

1. c
2. c
3. b
4. b
5. a
6. c
7. d
8. c
9. b
10. a
11. c
12. d
13. c
14. d
15. a

What's More

Activity 1.1

Exponential Function

$$g(x) = 4^{5x}$$
$$y = 5^{x-1}$$
$$f(x) = 2^{x}$$
$$f(x) = 5^{x+2}$$

Exponential Equation

$$36^{x} = 6$$
$$64 = 2^{x+2}$$
$$\left(\frac{1}{2}\right)^{x+2} = \left(\frac{1}{8}\right)^{x}$$
$$7 = 49^{x}$$

Exponential Inequality

$$32^{x-4} \leq 16^{x+2}$$
$$6 > \left(\frac{1}{36}\right)^{x}$$
$$100 > 10^{2x}$$
$$27 < 3^{x}$$

None of these

$$x^{7} + 1 < 10x^{8}$$
$$y = x^{5}$$
$$f(x) = x^{3}$$

Activity 1.2

1.Exponential Equation
It is an exponential expression involving equation with only 1 variable.
2.None of these
It is not an exponential expression.
3.Exponential Inequality
It is an exponential expression involving equation with only 1 variable.
4.Exponential Function
It is an exponential expression involving relationship between 2 variables.
5. Exponential Equation
It is an exponential expression involving equation with only 1 variable.

Assessment

1. b
2. d
3. d
4. a
5. a
6. b
7. c
8. b
9. c
10. a
11. b
12. c
13. c
14. a
15. b

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*DepED Material: General Mathematics Learner's Material

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11 General Mathematics

General Mathematics
Solving Exponential Equations and Inequalities
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Solving Exponential Equations and Inequalities

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Exponential Equations and Inequalities!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Exponential Equations and Inequalities!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you master how to solve exponential equation and inequality. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

1. identify the properties used in solving exponential equations and inequalities; and
2. solve exponential equations and inequalities.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What should be considered in solving an exponential equation?
   a. Bases on both sides must be the same.
   b. Bases on both sides must be simplified.
   c. Exponents on both sides must be the same.
   d. Exponents on both sides must be simplified.

2. Which of the following describe the statement: If $x_1 \neq x_2$, then $b^{x_1} \neq b^{x_2}$. Conversely, if $x_1 = x_2$, then $b^{x_1} = b^{x_2}$?
   a. Addition Property of Equality
   b. Multiplication Property of Equality
   c. Distributive Property of Equality
   d. One-to-one Property of Exponential Function

3. In solving for the value of the unknown variable in $4^{x+1} = 16$, what is the best thing to do first?
   a. Divide 16 by 4.
   b. Multiply 4 by x+1.
   c. Write x+1 as the exponent for both 4 and 16.
   d. Express 16 as 4^2.

4. Which of the following best leads to the value of the unknown in $4^{x+1} = 16$?
   a. $4^{x+1} = 16^x$
   b. $(4)^{2(x+1)} = 16$
   c. $4^{x+1} = 4^2$
   d. $4^{x+1} = 2^4$

5. What is the value of x in $4^{x+1} = 16$?
   a. 0
   b. 1
   c. 2
   d. 4

6. Which of the following best leads to the value of the unknown in $27^x = 9$?
   a. $9^{3x} = 9$
   b. $9^{3x} = 3^2$
   c. $3^{9x} = 9$
   d. $3^{3x} = 3^2$

7. What is the value of x in $27^x = 9$?
   a. 2/3
   b. 3/2
   c. 1/3
   d. 3

8. What is the first step in solving for x in the exponential inequality $2(5)^x > 10$?
   a. Multiply 2 by 5.
   b. Divide both sides by 2.
   c. Divide both sides by 10.
   d. Divide both sides by 5.

9. What is the value of x in the given exponential inequality in item 8?
   a. $x > 5$
   b. $x < 2$
   c. $x > 1$
   d. $x < 0$

10. Which of the following is equivalent to $10^{x-5} > 100^{x-10}$?
   a. $10(10)^{x-5} > 100^{x-10}$
   b. $10^{x-5} > 10^{2x-20}$
   c. $1^{x-5} > 10^{2x-20}$
   d. $1^{x-5} < 10^{2x-20}$

11. Which best leads to the correct answer in solving for the unknown variable in the given exponential inequality in number 10?
    a. $x - 5 < x - 10$
    b. $x - 5 > x - 10$
    c. $x - 5 < 2x - 20$
    d. $x - 5 > 2x - 20$

12. What is the value of x for the given in number 10?
    a. $x < 10$
    b. $x > 10$
    c. $x < 15$
    d. $x > 15$

13. Which best describes base b of $\left(\frac{1}{3}\right)^{x+4} \geq \left(\frac{1}{9}\right)^{x}$?
    a. $0 < b < 1$
    b. $b < 1$
    c. $b > 1$
    d. $b > 0$

14. Which of the following best leads to the value of x in $\left(\frac{1}{3}\right)^{x+4} \geq \left(\frac{1}{9}\right)^{x}$?
    a. $\left(\frac{1}{3}\right)^{x+4} \leq \left(\frac{1}{3}\right)^{2x}$
    b. $\left(\frac{1}{3}\right)^{x+4} \geq \left(\frac{1}{3}\right)^{x}$
    c. $\left(\frac{1}{3}\right)^{x+4} \geq \left(\frac{1}{3}\right)^{-2x}$
    d. $\left(\frac{1}{3}\right)^{x+4} \leq \left(\frac{1}{3}\right)^{-2x}$

15. What is the solution to $\left(\frac{1}{3}\right)^{x+4} \geq \left(\frac{1}{9}\right)^{x}$?
    a. $(4, +\infty)$
    b. $[4, +\infty)$
    c. $(-\infty, 4)$
    d. $(-\infty, 4]$

Lesson 1 Solving Equations and Inequalities

We are living in a diverse world. Differences among men exist. Things vary from each other. Various decisions lead to several differing results. These differences, nevertheless, are always present. In the same manner, in the previous module, we have learned that exponential expressions may take various forms. And we have understood that despite their differences, what is important is to know how to classify each accordingly and how to deal with them.

What's In

Listed below are exponential expressions. Which are exponential functions? exponential equations? exponential inequalities?

(a) $4^x = 2^{x+1}$	(d) $81 = 9^{3x}$	(g) $y = e^x$	(j) $g(x) = 6x^3$
(b) $100 > 10^{x-2}$	(e) $f(x) = 5^{x-4}$	(h) $(0.81)^{2x} \geq 0.9$	
(c) $16x = x^2$	(f) $27 < 3x$	(i) $25^{x-2} = 5^{3x}$	

Recall that an exponential expression can either be a function, an equation, or an inequality. An exponential function is not intended to be solved as it simply shows relationship between two variables. In the above list, (e) and (g) are both exponential functions. Why is (j) not considered as one? Though it has two variables, yet it is not an exponential expression since the exponent is not a variable.

Both exponential equation and inequality, on the other hand, are the ones whose x values satisfying the given expressions, are meant to be solved. They both involve only one variable. Among the expressions above, (a), (d), and (i) are exponential equations. They are all exponential expressions with equations and are consisted of only one variable. How about (c)? It is not even an exponential expression since its exponent is a constant.

(b) and (h) are the only exponential inequalities in the list. They are both exponential expressions consisting of one variable only and with inequality symbols. (f) has inequality symbol, but why can't you consider it as exponential inequality? It is since its exponent is not a variable, hence it is not an exponential expression.

Notes to the Teacher

Stress once again that not all functions, equations, or inequalities with exponents, are exponential expressions. Instead, only those involving variable exponents are considered as so.

What's New

Raise It!

This time let us see how fast you can compute mentally and how smart your reasoning power is. Just answer as fast as you can without looking at the solutions below. Ready? Let us begin!

1. How many times do you have to multiply 4 by itself to obtain 64?
2. How will you write it in symbols?
3. What kind of mathematical expression is it?
4. How many times do you have to multiply 4 by itself so that the result will be greater than 64?
5. How will you write the fourth question in symbols?
6. What kind of mathematical expression is being depicted by it?

Solutions:

1. Sounds like you find it easy. Correct, three times!
2. Expressing it in symbols, it is $4^x = 64$. Then, it becomes $4^3 = 64$, based on your first answer.
3. In the previous module, you have learned that $4^x = 64$ is an exponential equation since it is an exponential expression with only one variable and involving an equation.
4. You might answer 5, 6, 7, 8, and so on. They are all correct. But actually, even non-integers may be solutions as long as they are greater than 4. Hence the solution is $x > 4$.
5. Writing it in symbols, it is $4^x > 64$.
6. It is an exponential inequality since it involves an exponential expression with only one variable and an inequality.

What is It

In the previous activity you have been able to solve for the values of the unknown in both exponential equation and inequality. Most probably you have successfully solved them mentally. How do your solutions really work? Let us try to understand it further.

Solving Exponential Equation

One-to-one Property of Exponential Functions states that in $f(x) = b^x$, if $x_1 \neq x_2$, then $b^{x_1} \neq b^{x_2}$. Conversely, if $b^{x_1} = b^{x_2}$, then $x_1 = x_2$. This property paves the way in understanding how to solve exponential equation.

Example 1: Solve for the value of x in $4^{x+1} = 64$.

Solution:

$4^{x+1} = 4^3$	Express 64 as 4^3, in order for both sides of the equation to have same bases.
$x + 1 = 3$	One-to-one Property of Exponential Functions states that if $b^{x_1} = b^{x_2}$, then $x_1 = x_2$.
$x + 1 - 1 = 3 - 1$	Use Addition Property of Equality in order to solve for the value of x.
$x = 2$	Combine like terms.

Example 2: Solve for the value of x in $3^{4x} = 9^{x+1}$.

Solution:

$3^{4x} = (3)^{2(x+1)}$	Express 9 as 3^2, in order for both sides of the equation to have same bases.
$4x = 2x + 2$	One-to-one Property of Exponential Functions states that if $b^{x_1} = b^{x_2}$, then $x_1 = x_2$.
$4x - 2x = 2x + 2 - 2x$	Use Addition Property of Equality in order to solve for the value of x.
$2x = 2$	Combine like terms.
$x = 1$	Use Multiplication Property of Equality by multiplying both sides of the equation by ½.

Solving Exponential Inequality

Recall that in an exponential function $f(x) = b^x$, $b > 0$ but $b \neq 1$. Now, the key to solving exponential inequality is the fact that if $b > 1$ and $x_1 > x_2$, then $b^{x_1} > b^{x_2}$. Otherwise, if $0 < b < 1$, then $b^{x_1} < b^{x_2}$. Let us further make this clearer by considering the next examples.

Example 3: Solve for the values of x in $5^x > 125^{x+8}$

Solution:

$5^x > (5)^{3(x+8)}$	Express 125 as 5^3, for both sides of the inequality to have same bases.
$x > 3x + 24$	$b = 5$. It is a fact that if $b > 1$ and $b^{x_1} > b^{x_2}$, then $x_1 > x_2$.
$x - x - 24 > 3x + 24 - x - 24$	Use Addition Property of Equality in order to solve for the value of x.
$-24 > 2x$	Combine like terms.
$x < -12$	Use Multiplication Property of Equality by multiplying both sides of the equation by ½.

Hence, the solution to the exponential inequality $5^x > 125^{x+8}$ is the set of all real numbers less than -12. In symbols, that is, $x < -12$ or $(-\infty, -12)$.

Example 4: Solve for the values of x in $\left(\frac{1}{7}\right)^{2x+9} \leq \left(\frac{1}{343}\right)^{x-5}$.

Solution:

$\left(\frac{1}{7}\right)^{2x+9} \leq \left(\frac{1}{7}\right)^{3(x-5)}$	Express $\frac{1}{343}$ as $\left(\frac{1}{7}\right)^3$, in order for both sides of the inequality to have same bases.
$2x + 9 \geq 3x - 15$	$b = \frac{1}{7}$. It is a fact that if $0 < b < 1$ and $b^{x_1} < b^{x_2}$, then $x_1 > x_2$.
$2x + 9 - 2x + 15 \geq 3x - 15 - 2x + 15$	Use Addition Property of Equality in order to solve for the value of x.
$24 \geq x$ or $x \leq 24$	Combine like terms.

Thus, the solution to the exponential inequality $\left(\frac{1}{7}\right)^{2x+9} \leq \left(\frac{1}{343}\right)^{x-5}$ is the set of all real numbers less than or equal to 24. In symbols, that is, $x \leq 24$ or $(-\infty, 24]$.

What's More

Activity 18.1 Who Has a Point?

Observe each of the following pairs of solutions. Decide whether anyone of them got the correct answer. Answer the sets of guide questions.

Becca

$$16^{x^2} = 4^{x+3}$$
$$(2)^{4x^2} = (2)^{2(x+3)}$$
$$4x^2 = 2x + 6$$
$$\left(\frac{1}{2}\right)(4x^2 - 2x - 6 = 0)$$
$$2x^2 - x - 3 = 0$$
$$(2x - 3)(x + 1) = 0$$
$$2x - 3 = 0 \text{ and } x + 1 = 0$$
$$x = \frac{3}{2},\ x = -1$$

Celia

$$16^{x^2} = 4^{x+3}$$
$$(4)^{2x^2} = 4^{x+3}$$
$$2x^2 = x + 3$$
$$2x^2 - x - 3 = 0$$
$$(2x - 3)(x + 1) = 0$$
$$2x - 3 = 0 \text{ and } x + 1 = 0$$
$$x = \frac{3}{2},\ x = -1$$

1. How did you check whether 3/2 and -1 are really solutions of the given exponential equation?
2. Were Becca and Celia both correct or both wrong?
3. What is the difference between their solutions?
4. Did the difference affect the solutions? Why?
5. Did Becca and Celia both use the One-to-one Property of Exponential Function? How?

Hector	**Dindo**
$(0.5)^{x-1} > (0.25)^{-x-2}$	$(0.5)^{x-1} > (0.25)^{-x-2}$
$(0.5)^{x-1} > (0.5)^{2(-x-2)}$	$(0.5)^{x-1} > (0.5)^{2(-x-2)}$
$x - 1 > -2x - 4$	$x - 1 < -2x - 4$
$3x > -3$	$3x < -3$
$x > -1 \; or \; (-1, +\infty)$	$x < -1 \; or \; (-\infty, -1)$

6. Have you noticed any difference in the solutions? What is it?
7. Who used the property for exponential inequality? How did he use it?
8. Taking Hector's solution, can 0 be a value of x?
9. Considering Dindo's solution, will -2 make the inequality correct?
10. Were Hector and Dindo both correct? If not, whose work is right?

Activity 18.2 Find the Missing x!
Solve for the values of x for each of the following exponential equations and inequalities.

1. $8^{2-x} = 2$
2. $\left(\frac{1}{2}\right)^x < \frac{1}{8}$
3. $5^x = 25^{x-2}$
4. $3^{x+2} \geq 27$
5. $4^{3x} = 8^{x-1}$

What I Have Learned

Fill in the blanks of the following statements with the correct missing words or phrases.

1. _____ states that in $f(x) = b^x$, if $b^{x_1} = b^{x_2}$, then $x_1 = x_2$.
2. This property, as stated in the previous statement, applies in solving _____.
3. Given an exponential equation, first thing we see to it is that the bases of both sides of the equation are _____.
4. In solving an exponential inequality, if base b is greater than 1 and $b^{x_1} > b^{x_2}$, then _____.
5. Given that _____, and $b^{x_1} > b^{x_2}$, then $x_1 < x_2$.

What I Can Do

When solving exponential equation and inequality, you look for values of the unknown variable that would accurately fit in the exponential expression.

Similarly, you have a group of people where you perfectly fit. And you become the best version of you when you are with those whom you originally belong. They are no more than your own family. With them, you can show the true you, because you know they accept you no matter what. From your family is where you first learn desirable traits towards your fellows. What others see from you reflects how you were raised by your family. Write an essay of your own experience showing your characters towards your peers that are instilled in you by your family. Include also your own constructed and solved exponential equation or inequality that would represent your shared experience. Try scoring your essay using the rubric below.

	4	**3**	**2**	**1**
Content	Very relative to the topic and well-organized	Somewhat relative to the topic and organized	Less relative to the topic and poorly organized	Not relative to the topic and very poorly organized
Spelling, Grammar, and Punctuations	No spelling, punctuation or grammatical errors	Very few spelling, punctuation, and grammatical errors	Several spelling, punctuation, and grammatical errors	Many spelling, punctuation, and grammatical errors
Constructed and Solved Exponential Equation or Inequality	Constructed exponential equation or inequality and solved correctly	Constructed exponential equation or inequality but with incorrect solution	Constructed exponential equation or inequality but without solution	Did not construct an exponential equation nor inequality

When you scored 9-12, share and read it aloud to your family and make them feel appreciated. But when you scored 8 and below, try revising it first. Then see to it that right after, you will also make your family proud after sharing it with them.

Assessment

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What should be considered in solving an exponential equation with base b and exponents x_1 and x_2?
   a. If $b^{x_1} = b^{x_2}$, then $x_1 \neq x_2$.
   b. If $b^{x_1} \neq b^{x_2}$, then $x_1 = x_2$.
   c. If $b^{x_1} = b^{x_2}$, then $x_1 = x_2$.
   d. If $b^{x_1} = b^{x_2}$, then $x_1 > x_2$.

2. Which of the following is true in solving an exponential inequality with base b and exponents x_1 and x_2?
   a. If $b > 1$ and $b^{x_1} > b^{x_2}$, then $x_1 > x_2$.
   b. If $b > 1$ and $b^{x_1} > b^{x_2}$, then $x_1 < x_2$.
   c. If $0 < b < 1$ and $b^{x_1} > b^{x_2}$, then $x_1 > x_2$.
   d. If $0 < b < 1$ and $b^{x_1} < b^{x_2}$, then $x_1 < x_2$.

3. In solving for the value of the unknown variable in $2^{5x} = 64$, what is the best thing to do first?
   a. Simplify 64 into 8^2.
   b. Divide 64 by 2.
   c. Express 64 as 2^6.
   d. Multiply 5x by 2.

4. Which of the following best leads to the value of the unknown in $2^{5x} = 64$?
   a. $2^{5x} = 64/2$
   b. $2^{5x} = 2^6$
   c. $2^{5x} = 8^2$
   d. $2^{5x} = 64/2^{5x}$

5. What is the value of x in $2^{5x} = 64$?
   a. 6/5
   b. 6
   c. 8
   d. 32

6. Which of the following best leads to the value of the unknown in $8^{x^2} = 2^{2x+1}$?
   a. $8^{3x^2} = 2^{2x+1}$
   b. $8^{x^2} = 8^{2x+1}$
   c. $(2)^3 = 2^{2x+1}$
   d. $(2)^{3x^2} = 2^{2x+1}$

7. What are the values of x in $8^{x^2} = 2^{2x+1}$?
   a. $-\frac{1}{3}, -1$
   b. $-\frac{1}{3},\ 1$
   c. $\frac{1}{3}, -1$
   d. $\frac{1}{3}, 1$

8. What is an important step in solving for x in any exponential inequality?
   a. Consider if $b > 1$ or if $0 < b < 1$.
   b. Assume that $b < 0$.
   c. See to it that the exponents are equal.
   d. Always divide both sides by the common exponent.
9. Which among the following is a significant observation when solving for x value of $25^x < 125^{x-3}$?
   a. The exponents are almost the same.
   b. The exponents both use x variable.
   c. The bases are greater than 1.
   d. The bases are both multiples of 5.
10. Which of the following best leads to the solution for the given in item 9?
   a. $25^x < (100 + 25)^{x-3}$
   b. $25^x > (100 + 25)^{x-3}$
   c. $(5)^{2x} < (5)^{3(x-3)}$
   d. $(5)^{2x} > (5)^{3(x-3)}$
11. What is the value of x in the given exponential inequality in item 9?
   a. $x < 6$
   b. $x > 7$
   c. $x < 8$
   d. $x > 9$
12. Which among the following is a significant observation when solving for x value of $0.49^x > 0.7^{x+1}$?
   a. The exponents both used variable x.
   b. The exponent on the right side is 1 greater than the other.
   c. The bases are multiples of 0.7.
   d. The bases are greater than 0 but less than 1.
13. Which best leads to the correct answer in solving for the unknown variable in the given exponential inequality in number 12?
   a. $0.49^x < (0.7)^{2(x+1)}$
   b. $(0.7)^{2x} < 0.7^{x+1}$
   c. $0.49^x > (0.7)^{2(x+1)}$
   d. $(0.7)^{2x} > (0.7)^{2(x+1)}$
14. Which of the following is a correct part of the solution for the given in item 12?
   a. $x > x + 1$
   b. $x < x + 1$
   c. $2x > x + 1$
   d. $2x < x + 1$

15. hat is the value of x for the given in number 12?
    a. $(-\infty, 1)$
    b. $(-\infty, 1]$
    c. $(1, +\infty)$
    d. $[1, +\infty)$

Additional Activities

Solve for the values of x in $32^{3x^2} = 4^{11x+24}$ and in $\left(\frac{1}{16}\right)^{2x+5} = \left(\frac{1}{64}\right)^{x-1}$.

Answer Key

What I Know

1. a
2. d
3. d
4. c
5. b
6. d
7. a
8. b
9. c
10. b
11. d
12. c
13. a
14. a
15. b

What's More

Activity 1.1

By substituting each to x of the given.
They are both correct.
Becca used b=2, while Celia used b=4.
It did not, since both were correctly used as exponential form of the given.
Yes, they both did. They both used same bases for both sides of the equation, before equating their respective exponents.
Dindo interchanged the inequality symbol, while Hector used the same symbol all throughout.
Dindo used the property. Since 0<b(0.5)<1, he interchanged the inequality symbol.
0 cannot be.
-2 can be.
Only Dindo got it right.

Activity 1.2

1. $x = \frac{4}{3}$
2. $x > 3$
3. $x = 4$
4. $x \geq 1$
5. $x = -1$

Assessment

1. c
2. a
3. c
4. b
5. a
6. d
7. b
8. a
9. c
10. c
11. d
12. d
13. b
14. d
15. a

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*DepED Material: General Mathematics Learner's Material

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11 General Mathematics

General Mathematics
Representations of Exponential Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Representations of Exponential Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Representations of Exponential Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Representations of Exponential Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you master the representations of exponential functions through the table of values, graphs, and equations. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

1. define the exponential function; and
2. demonstrate how to represent exponential function using table of values, graphs, and equations.

What I Know

Directions: Read and analyze each item carefully. Circle the letter that corresponds to your answer for each statement.

1. In the function, $f(x) = b^x$, x is called _____.
   a. base c. intercept
   b. exponent d. coefficient
2. Which of the following is **NOT** an application of exponential function?
   a. population decline c. compound interest
   b. bacterial growth d. monthly salary
3. Which of the following is an example of exponential function?
   a. $f(x) = x^2 + 1$ c. $f(x) = 4^{x+3}$
   b. $f(x) = \frac{2}{x}$ d. $f(x) = 3x^2$
4. The following are examples of exponential function **except one**.
   a. $f((x) = 5^{-2x}$ c. $f(x) = 3^{3x}$
   b. $f(x) = 10^{x+3}$ d. $f(x) = (x^{-2})^{-2}$
5. In exponential function, the function has the value of 1 when the exponent is equal to _____.
   a. 0 c. -1
   b. 1 d. none of the choices
6. The function $f(x) = 2^x$ is an increasing function. The function $f(x) = 2^{-x}$ is _____ function.
   a. increasing c. either increasing or decreasing
   b. decreasing d. no conclusion can be made from the given
7. In the function $f(x) = 2^x$, $x < 0$, the x-axis becomes the _____ of the graph.
   a. intercept c. base
   b. asymptote d. slope

8. The domain of the function $f(x) = 2^x$ is the set of _____.
   a. negative real numbers c. zero
   b. positive real numbers d. set of real numbers
9. The range of the function $f(x) = 2^x$ is the set of _____.
   a. negative real numbers c. zero
   b. positive real numbers d. set of real numbers
10. The function $f(x) = 4^{-x}$ is the same as the function _____.
   a. $f((x) = \left(\frac{1}{4}\right)^x$ c. $f(x) = -4x$
   b. $f(x) = \left(\frac{1}{2}\right)^x$ d. $f(x) = \left(\frac{1}{4}\right)^{-x}$
11. In the function $f(x) = 2(1.25)^x$, find f(x) if x = 0, 1, 2.
   a. f(x) = 1.6, 2, 2.5
   b. f(x) = 1.28, 1.6, 2
   c. f(x) = 2, 2.5, 3.125
   d. f(x) = 2.5, 3.125, 3.906

12. In order to determine if the function $f(x) = 5(1.5)^x$ is exponential growth or decay you have to look at _____.
    a. x c. 5
    b. 1.5 d. 5^x
13. The exponential function in question no.12 is an example of _____.
    a. decay c. growth
    b. progress d. stagnant
14. The exponential function $f(x) = 0.65^x$ is an example of _____.
    a. decay c. growth
    b. progress d. stagnant
15. The annual sales at a company are ₱400,000.00 in the year 2019 and increasing at the rate of 5% per year. What will be the total amount after 20 years?
    a. ₱1,010,780.08 c. ₱1,114,385.04
    b. ₱962,647.69 d. ₱1,061319.08

Lesson 1 Representations of Exponential Functions

In your previous lesson, you learned about exponential equations and inequalities. In this module, you will extend your knowledge of exponents to growth and decays. Specifically, this module is about the exponential function. Several examples of exponential growth and decay can be observed in some real-life situations in the field of business and economics, health, demography and sciences. But why is it called the exponential function? What are its characteristics that differentiate it from other functions?

The lessons and activities in this module will explain what you have to know about exponential functions.

After going through this module, you are expected to:

1. define the exponential function; and
2. represent an exponential function through its table of values, graphs and equations.

What's In

In your previous lesson, you learned how to solve exponential equations and inequalities. As a review, ready yourself in doing this first drill.
Let us have an equation $32 = 2^x$.
To find the value of x, first transform 32 into its exponential form. So, 32 becomes 2^5. Then equate it with 2^x, this becomes $2^5 = 2^x$. Since they both have the same base, therefore x=5. Now, find the value of x on the following equations:

a) $3^{x+1} = 243$
b) $2^{4x+1} = 512$
c) $9^{2x} = 27$

What's New

Recently, a new corona virus has caused a world pandemic. In the Philippines, it spreads from 5 cases in March up to ten thousand by May. The contamination of cases is likened to what we call exponential spread.

For your **Activity 1**, construct a table of values that would represent the covid19 confirmed cases in the Philippines on the first 15 days of March starting on March 5. You may do your research online, then plot the values that you get on a Cartesian plane. You may use paper and pencil or any applicable graphing apps such as MS Excel, GeoGebra, or Desmos. You may use the table below as your reference.

Table 1

t, Days	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
x, cases															

Questions to ponder:

1. What can you observe about the table and graph? Is it linear? _____
2. Does it curve slowly or rapidly? _____
3. In order to slow down the growth what can you do to the curve? In real-life, what must be done to flatten the curve? _____

From the foregoing activity, you have constructed an exponential function that depicts actual cases and are represented in a form of table of values, a graph and equation.

What is It

Now, observe that there is a set of values that can be found in x (t, as used in the preceding problem) which corresponds to a certain value in f(x). In the case of the exponential function, the values of f(x) curves rapidly on a given value of x. This is the characteristic of an exponential function that differentiates it from other functions. Exponential function can be described using the form $f(x) = b^x$, where b is a constant called the base while x is a variable power or simply the exponent.

Let us study the different behaviors of the graphs of exponential functions relative to its independent and dependent variables. Take a look at the function $f(x) = 2^x$. Here x is the input and f(x) is the output. Consider the values in the following table. Substitute the values of x to the function to get f(x).

Table 2

x	0	1	2	3	4	5
f(x)	1	2	4	8	16	32

Looking at Table 2, you can observe that as x increases by 1 unit, f(x) doubles its value from its previous value. To show how rapidly f(x) changes a graph of the function is shown in fig.1 below:

Fig.1. You can see how steep the curve moves upward from its initial value. This nature of the function has x values that are real numbers, i.e., $x \in R$. Here, the y-intercept is 1 (x=0). At $x < 0$, the x-axis becomes the *asymptote* of the graph. On the other hand, y-values or f(x) contains only positive integers. Moreover, you can observe that the constant 2 is greater than 1 and is not equal to zero. If this is the case where b is greater than and not equal to 1, you will have what we call exponential growth. It is an increasing function.

Fig.2. Now take a look at the function $f(x) = (½)^x$, here $0<b<1$ where $b=1/2$. You may notice that this function is also the same as $f(x) = 2^{-x}$. Observe the graph of the function on the left. The curve moves steeply downward going to the right but not touching x-axis. Still, the x inputs are real numbers. If this is the case, where $0<b<1$, you will have what we call exponential decay. It is a decreasing function.

Variations of graphs may be tried relative to functions $y=2^x$ and $y=2^{-x}$.

But first let us have an activity that will facilitate more understanding of these methods.

Activity 2 – Move Me

Materials Needed: pen/pencil, graphing paper/bond paper, MS Excel, Desmos,

Mechanics:

1. In a bond paper/graphing paper plot the following functions.
   a. $f(x) = 3^x$ c. $f(x) = 2^x + 1$
   b. $f(x) = -2^x$
2. Create a table of values for question no.1 (altogether in one table) using the following x-values: -2, -1, 0, 1, 2.
3. Identify which among the functions are growth and decay.

Questions

1. Relative to $f(x) = 2^x$, what can you say about the movement of the curves?
2. Were you able to correctly identify which is an exponential growth and decay? Explain how you did that.

In the foregoing discussion, you were able to learn about the graph of the exponential function of $y=2^x$ relative to other functions. To cite some behaviors of these functions relative to $y=2^x$, let us briefly summarize the phenomena: Fig.3. In $y=3^x$, both functions have y=1 when x=0. The difference is that $y=3^x$ has move 1 unit up at x=1. By looking at the graph $y=3^x$ moves more rapidly and steeply towards the y-axis. In $y=-2^x$ each y of $y=2^x$ is multiplied by -1 so that they become opposites. $y=-2^x$ is the reflection of $y=2^x$ about the x-axis. In $y=2^x+1$ the graph shifts 1 unit up relative to $y=2^x$. Let us try some real-life examples:

Example 1:

Jose is planning to buy a gift worth ₱500 for his mom's birthday. So, he planned to save money from what remains on his daily allowance. On the first day he was able to save ₱5.00. Each day he decided to double his previous savings. At what day can he be able to buy the gift?

Table 3 shows the pattern how Jose saves his money:

Table 3

day	savings
1	5
2	10
3	20
4	40
5	80
6	160
7	320
8	640

Fig.4

Conclusion: Jose can buy the gift for her mother's birthday on the 8th day.

Example 2:
Mang Leonardo bought his son a motorcycle worth ₱125,000.00. But father and son planned to sell the same motorcycle after 3 years. The value of the motorcycle depreciates at 5% per annum. How much would be the value of the motorcycle after 3 years?
To solve this problem, use the function $A(t) = P(1 - r)^t$, take note that instead of addition we use subtraction in (1-r). Again, P is the initial amount, r is the rate of interest, and t is the time. Substituting values, we get:

$$A(t) = 125000(1 - .05)^3$$

The table of values for 3 years is shown below:

Table 4

t, time	0	1	2	3
A(t)	125,000	118,750	112,812.5	107171.88

Fig.5

Conclusion: The motorcycle has depreciated to an amount of ₱107,171.88.

What's More

Activity 1.1
Find the values of the exponential function f(x) when x=1, 2, 3, 4, 5. Identify whether it is growth or decay.

1. $f(x) = \left(\frac{1}{3}\right)^{x-1}$
2. $f(x) = -\left(\frac{1}{2}\right)^{x} + 1$
3. $f(x) = 5(2)^x$
4. $f(x) = 2(2)^{\left(\frac{1}{2}\right)x}$
5. $f(x) = \frac{2}{5}(2)^x - 1$

Activity 1.2
Use MS Excel or any graphing apps to plot the graph of the following function. Identify if it is exponential growth or decay

1. $f(x) = -2^{x-2}$
2. $f(x) = 3^{-x}$
3. $f(x) = 4^x$
4. $f(x) = -\left(\frac{1}{2}\right)^{-2x}$
5. $f(x) = -4^{-x}$

Activity 1.3
Analyze and solve.

A new car costs ₱450.000.00. This value subsides by 10% each year.

1. Write an exponential model that represents this situation after t years.
   Exponential function f(x) = _____
2. How much will the car be worth after 5 years? _____
   Given: A_0 = _____ r = _____ t = _____
3. Create a table of values for t = 1-4 years

Activity 1.4

Analyze and solve.

Mr. Morales invests ₱10.000.00 in a company stock. This stock value depreciates by 1.5% each year.

1. Write an exponential model that represents this situation after t years.
2. How much will be the value of the stock after 5 years? _____
3. Create a table of values for t = 1-4 years

What I Have Learned

Now, try to summarize the behavior of this function at b>1 and 0<b<1 by filling in the blanks with correct word or words.

The function $f(x) = 2^x$ curves upward from its initial value. This nature of the function has x values that are _____ numbers. Here, the y-intercept is 1 (x=0). At x < 0, the x-axis becomes the _____ of the graph. On the other hand, y-values or f(x) contain only _____ integers. Moreover, you can observe that the constant 2 is greater than 1 and is not equal to zero. If this is the case where b is greater than and not equal to 1, you will have what we call an exponential _____. It is an increasing function. In the case, where 0<b<1, you will have what we call exponential _____. It is a decreasing function.

What I Can Do

There are many ways of applying exponential functions in our lives. Examples of these are population growth, bacterial growth, radioactive decay, medical dosage and compound interest. An example of its application can be seen below.

Field of Application: Compound Interest

Situation analysis: Ms. Gomez decided to have an initial deposit to CDO Bank worth ₱5,000.00. This decision came after an agent of the bank told her that the bank is offering a 5% interest compounded annually. How much would be Ms. Morales' total money in the bank after 5 years?

$$A(t) = P(1+r)^t$$

Use the given formula to find the total amount of money after 5 years.

Data manipulation: when t=0, $A(0) = P(1+.05)^0 = 5000$

Presentation: Using a table of values, we have:

Table 3

t	1	2	3	4	5
A(t)	5250.00	5512.50	5788.13	6077.53	6381.41

This is the graph of the function:

Interpretation: The table and the graph show that it has an increasing function. Thus, the values increase at certain period of time. It is an exponential growth. Ms. Gomez' money earned ₱1,181.14 after 5 years in the bank.

Now, It's Your Turn

1. Create your own or similar real-life situation where exponential function is applied.
2. In a bond paper, present the problem from **Field of application** up to **Presentation** as illustrated above.
3. You can use graphing paper, MS Excel, Desmos or any graphing app to graph the function. You can also use calculators to solve the table of values.
4. Your grade will be according to these criteria: Clarity of Presentation 60%, Organization and Accuracy, 30%, and Applicability to current situations, 10% with a total of 100%.

Assessment

Directions: Read and analyze each item carefully. Circle the letter that correspond to your answer for each statement.

1. In the function, $f(x) = 2^x$, 2 is called _____.
   a. constant
   b. variable power
   c. coefficient
   d. base

2. Which of the following is an application of exponential function?
   a. monthly salary
   b. shooting a cannon
   c. radioactive decay
   d. distance travelled

3. The following are examples of exponential function ***except one.***
   a. $f(x) = 2^{-x}$
   b. $f(x) = 2^{x-3}$
   c. $f(x) = 5^{2x}$
   d. $f(x) = \left(\frac{3}{x-1}\right)^2$

4. One of the properties of the base in the exponential function is that it cannot be ___.
   a. equal to1
   b. greater than 0
   c. greater than 0 but less than 1
   d. greater than 1

5. In exponential function, when the exponent is equal to zero the function has a value equal to _____.
   a. 0
   b. 1
   c. -1
   d. imaginary number

6. The function $f(x) = 3^{-x}$ is a decreasing function. The function $f(x) = 3^{x}$ is _____ function.

a. increasing
b. decreasing
c. either increasing or decreasing
d. slanting

7. In the function $f(x) = 4^{x}$, the x-axis becomes the asymptote of the graph at _____.

a. $x = 0$
b. $x > 0$
c. $x < 0$
d. $x = 1$

8. The domain of the function $f(x) = 2^{x-1}$ is the set of _____.

a. negative real numbers
b. positive real numbers
c. zero
d. real numbers

9. The range of the function $f(x) = \left(\frac{1}{2}\right)^{x}$ is the set of _____.

a. positive numbers
b. zero
c. negative numbers
d. all of the choices

10. The exponential function $f(x) = \left(\frac{1}{20}\right)^{x}$ is an example of _____.

a. decay
b. progress
c. growth
d. stagnant

11. The exponential function $f(x) = 2(10)^{x}$ is an example of _____.

a. decay
b. progress
c. growth
d. stagnant

12. In the function $f(x) = \frac{1}{2}(1.5)^{x}$, find f(x) if x = -1, 0, 1

a. f(x) = 0.22, 0.33, 0.5
b. f(x) = 0.5, 0.75, 1.125
c. f(x) = 0.22, 0.5, 1.125
d. f(x) = 0.33, 0.5, 0.75

13. The annual sales at a company are ₱100,000.00 in the year 2020 and increasing at the rate of 4% per year. What is its total amount after 10 years?

a. ₱136,856.91
b. ₱153,945.41
c. ₱148,024.43
d. ₱142,331.18

14. A new car costs ₱150.000.00. This value subsides by 5% each year. How much will the car be worth after 6 years?

a. ₱116,067.14
b. ₱110,263.78
c. ₱122,175.94
d. ₱104,750.59

15. At the start of the experiment in the laboratory, there are 1000 bacteria in a petri dish. The relationship between time t, in minutes, and the number of bacteria, N(t), can be represented by the function $N(t) = 15(2)^{\frac{t}{10}}$. How many bacteria will there be after 150 minutes?

a. 491,520
b. 15,360`
c. 61,440
d. 983,040

Additional Activities

Instructions: In doing this activity you may need MS Excel or a mobile app such as Desmos to sketch the graph.

One of the regions in the Philippines is Region IV-A more popularly called CALABARZON region. In the last Philippine census, CALABARZON has around 14 million Filipinos living in the region with 2.58% population growth rate. Now using the formula: $N(t) = N_0 b^t$

where N(t) is the number of the population, N_o is the initial count of the number of population, b is the growth factor, and t is the time period. Construct a table of values and sketch the graph of its population for five years from 2021 – 2025.

Answer:
The following are given on this problem:

N_o = _____ b = (1+.0258) = _____

t = _____

1. The table of values for this function: (round off your answers to three decimal digits)

Table 4

t	0	1	2	3	4	5
N(t) (in million)	14					

2. The graph of this function: (use the space provided)

Questions:

1, What is the value of N(t) after 5 years? _____
2. After 5 years, how many people have been added from the initial number of the population? _____
3. Is this an increasing or decreasing function? _____

Answer Key

What's More

Activity 1.1

1. f(x) = 1, 0.33, 0.11 0.4, 0.01
2. f(x) = 0.5, 0.75, 0.88, 0.94, 0.97
3. f(x) = 10, 20, 30, 40, 80, 160
4. f(x) = 2.83, 4, 5.66, 8, 11.31
5. f(x) = -0.2, 0.6, 2.2, 5.4, 11.8

Activity 1.2

1. growth
2. decay
3. growth
4. decay
5. decay

Activity 1.3

a. $A(t) = A_0(1-r)^t$

b. Given: $A_o = 450000$

$r = 10\%$

$t = 5$

$A(t) = 450000(1-0.10)^5$

$= 450000(0.90)^5$

$= 265,720.50$

c.

t	1	2	3	4
A(t)	405000	364500	328050	295245

Activity 1.4

a. $I(t) = I_0(1-r)^t$

b. $I(t) = 10000(1-.015)^5$

$= 10000(0.985)^5$

$= 9272.17$

c.

t	1	2	3	4
I(t)	9850	9702.25	9556.72	9413.37

What I know	Assessment
1. B	1. D
2. D	2. C
3. C	3. D
4. D	4. A
5. A	5. B
6. C	6. A
7. B	7. C
8. D	8. D
9. B	9. A
10. A	10. A
11. C	11. C
12. B	12. D
13. C	13. C
14. A	14. B
15. D	15. A

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*DepED Material: General Mathematics Learner's Material

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11 General Mathematics

General Mathematics
Domain and Range of Exponential Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Domain and Range of Exponential Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Domain and Range of an Exponential Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Domain and Range of Exponential Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

In this learning module, you will know more about the domain and range. These two concepts are always included in the study of functions. Identifying the domain of the function will help you to determine if the function will exist in a particular value. The domain of a function is always associated with the possible inputs for a function to exist, while the range is associated with outputs after substituting the possible inputs to the unknown variable.

This module was designed and written with you in mind. It is hoped to answer the questions, "Why domain and range are important?" and "How can I determine the domain and range of an exponential function?". It is here to help you master finding the domain and range of an exponential function.

After going through this module, you are expected to:

1. define domain and range;
2. find the domain and range of a given function; and
3. represent the domain and range using the set builder and interval notation.

What I Know

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. The set of all allowable values of the independent variable is called _____.
   a. Range
   b. Domain
   c. Real Numbers
   d. Exponential Function

2. Which of the following does not exist?
   a. Zero value on the denominator
   b. Positive values under the square root sign
   c. Negative values on the denominator
   d. Zero value under the square root sign

3. What is the domain and range of the function: $(x) = \frac{3}{x+7}$?
   a. The domain is all real numbers except -7 and the range is all real numbers except 0.
   b. The domain is all real numbers and the range is all real numbers except 0.
   c. The domain is all real numbers except -3 and the range is all real numbers.
   d. The domain and range are all real numbers.

4. Which of the following statement is always true?
   a. For any given x-value, the y-value of $y = 2^x$ is negative.
   b. For any given x-value, the y-value of $y = 2^x$ is all real numbers.
   c. The domain of exponential function $y = 2^x$ is positive numbers.
   d. The domain of exponential function $y = 2^x$ is all real numbers.

5. What is the domain of the function: $(x) = 3^x$?
   a. All negative numbers
   b. All positive numbers
   c. All real numbers except 0
   d. All real numbers

For item numbers 6-7, refer to the graph below:

6. What is the domain of the exponential function?
   a. $\{x|x \in \mathbb{R}, x < 0\}$
   b. $\{x|x \in \mathbb{R}, x \geq 0\}$
   c. $\{x|x \in \mathbb{R}, x \leq 0\}$
   d. $\{x|x \in \mathbb{R}\}$

7. What is the range of the exponential function?
   a. $\{y|y \in \mathbb{R}\}$
   b. $\{y|y \in \mathbb{R}, y < 0\}$
   c. $\{y|y \in \mathbb{R}, y > 0\}$
   d. $\{y|y \in \mathbb{R}, y \geq 0\}$

For item numbers 8-9, refer to the given function: $(x) = 4^x$.

8. What is the domain of the exponential function?
   a. $\{x|x \in \mathbb{R}\}$
   b. $\{x|x \in \mathbb{R}, x \geq 0\}$
   c. $\{x|x \in \mathbb{R}, x \leq 0\}$
   d. $\{x|x \in \mathbb{R}, x > 0\}$

9. What is the range of the exponential function?
   a. $\{y|y \in \mathbb{R}\}$
   b. $\{y|y \in \mathbb{R}, y \geq 0\}$
   c. $\{y|y \in \mathbb{R}, y \leq 0\}$
   d. $\{y|y \in \mathbb{R}, y > 0\}$

For item numbers 10-11, refer to the given the function: $f(x) = \left(\frac{1}{4}\right)^x$.

10. What is the domain of the exponential function?
    a. $\{x|x \in \mathbb{R}\}$
    b. $\{x|x \in \mathbb{R}, x > 1/4\}$
    c. $\{x|x \in \mathbb{R}, x > 0\}$
    d. $\{x|x \in \mathbb{R}, x < 1/4\}$

11. What is the range of the exponential function?
    a. $\{y|y \in \mathbb{R}\}$
    b. $\{y|y \in \mathbb{R}, y > 1/4\}$
    c. $\{y|y \in \mathbb{R}, y > 0\}$
    d. $\{y|y \in \mathbb{R}, y < 0\}$

For item numbers 12-13, refer to the given the function: $f(x) = 3^x + 2$.

12. What is the domain of the exponential function?
    a. $\{x|x \in \mathbb{R}\}$
    b. $\{x|x \in \mathbb{R}, x > 2\}$
    c. $\{x|x \in \mathbb{R}, x > 0\}$
    d. $\{x|x \in \mathbb{R}, x < 3\}$

13. What is the range of the exponential function?
    a. $\{y|y \in \mathbb{R}\}$
    b. $\{y|y \in \mathbb{R}, y > 2\}$
    c. $\{y|y \in \mathbb{R}, y > 0\}$
    d. $\{y|y \in \mathbb{R}, y < 3\}$

For item numbers 14-15, refer to the given the function: $f(x) = -3 \cdot 2^x + 4$.

14. What is the domain of the exponential function?
    a. $\{x|x \in \mathbb{R}\}$
    b. $\{x|x \in \mathbb{R}, x > 4\}$
    c. $\{x|x \in \mathbb{R}, x < 3\}$
    d. $\{x|x \in \mathbb{R}, x < 2\}$

15. What is the range of the exponential function?
    a. $\{y|y \in \mathbb{R}\}$
    b. $\{y|y \in \mathbb{R}, y > -3\}$
    c. $\{y|y \in \mathbb{R}, y > 2\}$
    d. $\{y|y \in \mathbb{R}, y < 4\}$

Lesson 1 Domain and Range of Exponential Functions

For you to begin this module, you need to assess your knowledge of the basic skills in finding the domain and range of a function. From your previous lessons in Math, you already encountered the domain and range of functions. Your acquired knowledge and skill will help you understand easily how to find the domain and range of an exponential function. Seek the assistance of your teacher if you encounter any difficulty.

What's In

Recall that the **domain of a function** is the set of all allowable values of x, commonly known as the independent variable or possible inputs of the function.

The **range of a function** is the set of output values commonly known as the dependent variable when all x-values in the domain are evaluated into the function.

This means that you need to find the domain first to describe the range.
The following will help you to recall how to find the domain and range of a function.

Find the domain and range of the following:

1. $y = 2x + 3$

Solution:

In a linear function, any real number can be substituted to x to get an output. Therefore the domain and range are all real numbers. You can express your answer in set notation or interval notation.

	Domain	**Range**
Set Notation	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y \in \mathbb{R}\}$
Interval Notation	$(-\infty, +\infty)$	$(-\infty, +\infty)$

2. $y = x^2 + 2$

Solution:
This is a quadratic function, like the linear function any real number can be substituted to x to get an output. You also learned that the graph of a quadratic

function is a parabola that opens upward or downward. Therefore, it has a minimum or a maximum point called the vertex of the parabola.

The given $y = x^2 + 2$ is a parabola opening upward with vertex at (0, 2), and so lowest possible value of the function is 2. That means that the range of the function is all real numbers greater than 2.

	Domain	**Range**
Set Notation	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y \in \mathbb{R}, y \geq 2\}$
Interval Notation	$(-\infty, +\infty)$	$[2, +\infty)$

3. $y = \frac{2}{x+3}$

Solution:

This is a rational function. The domain of a rational function is restricted at the value of its denominator. The denominator of a rational function should not be equal to zero for the value of the function to exist. In this case, the denominator is $x + 3$, and when $x = -3$ you will get a value of zero. Therefore, the domain of the function is all real numbers except -3.

Because the function $y = \frac{2}{x+3}$ will never be zero, you need to exclude 0 from the range. That means the range is all real numbers except 0.

	Domain	**Range**
Set Notation	$\{x \mid x \in \mathbb{R}, x \neq -3\}$	$\{y \mid y \in \mathbb{R}, y \neq 0\}$
Interval Notation	$(-\infty, -3) \cup (-3, +\infty)$	$(-\infty, 0) \cup (0, +\infty)$

3. $y = \sqrt{x + 5}$

Solution:
The given is a radical function also known as the square root function. The domain of a radical function is any x value for which the radicand (the value inside the radical symbol) is not negative. If the radicand has a negative value the roots or the solution are imaginary roots or no real roots. Thus, it is not allowed in the domain of the function to have a negative value inside the radical sign. Since, inside the radical symbol is $x + 5$ the domain of the function is a set of all possible values which are greater than or equal to -5.

Likewise, the value of the function at its domain is a all real numbers which means the range is all real numbers.

	Domain	**Range**
Set Notation	$\{x \mid x \in \mathbb{R}, x \geq -5\}$	$\{y \mid y \in \mathbb{R}\}$
Interval Notation	$[-5, +\infty)$	$(-\infty, +\infty)$

Notes to the Teacher

Remind the students that to find the domain of the functions they need to avoid zero (0) value in the denominator of a fraction, or negative values inside the square root sign. Moreover, do mention to them that the domain of the polynomial functions which includes linear, quadratic or any polynomial function of degree n is always the set of real numbers.

Tell them also that the range of a function is the spread of possible y-values (minimum y-value to maximum y-value) and it can be computed by substituting different x-values into the expression for y to see what is happening. (Ask yourself: Is y always positive? Always negative? Or maybe not equal to certain values?). Finally, make sure you look for minimum and maximum values of y because it is very important to find the range of the function.

What's New

Can You Show Me the Way?

Complete the table of values of the exponential function. Then, draw its graph in the given coordinate plane.

x	-3	-2	-1	0	1	2	3
$f(x) = 3^x$							

1. How will you describe the graph?

_____

_____

_____

2. In which quadrant/s do the graph occupies?

_____

_____

3. What are the other possible values of x? Describe the range of the possible values of x?

_____

_____

_____

4.What do you think now is the range of the function?

_____

_____

5.What other observations can you give?

_____

_____

_____

What is It

In the activity that you have done, were you able to determine the possible values of x? How about the range of the given function? You will find whether your answers are correct as you go through the next session of this module.

The Domain and Range

The domain of a function is the set of input values that are used for the independent variable. The range of a function is the set of output values for the dependent variable. For any exponential function, $f(x) = b^x$ the domain is the set of all real numbers. The range, however, is bounded by the horizontal asymptote of the graph of $f(x)$.

Example 1: Find the domain and range of the function $y = 3^{x+2}$

Solution:
Look at the graph of the function.

The function is defined for all real numbers. So, the domain of the function is a set of real numbers. As x extends to approach positive infinity $(+\infty)$, the value of the function also extends to $+\infty$, and as x extends to approach negative infinity $(-\infty)$, the function approaches the x-axis but never touches it. Therefore, the range of the function is a set of real positive numbers greater than 0 or $\{y|y \in \mathbb{R}, y > 0\}$.

Thus, the domain and range of the given function is given below and can be written as:

	Domain	**Range**
Set Notation	$\{x\|x \in \mathbb{R}\}$	$\{y\|y \in \mathbb{R}, y > 0\}$
Interval Notation	$(-\infty, +\infty)$	$(0, +\infty)$

Example 2: Find the domain and range of the function $y = \left(\frac{1}{4}\right)^{2x}$

Solution:
Look at the graph on the next page in the coordinate plane.

The function is defined for all real numbers. So, the domain of the function is the set of real numbers.

As x tends to approach $+\infty$, the value of the function tends to approach zero, and the graph approaches the x-axis but never touches it. As x tends to approach $-\infty$, the function also tends to $-\infty$. Therefore, the range of the function is the set of real positive numbers greater than 0 or $\{y|y \in \mathbb{R}, y > 0\}$.

	Domain	**Range**
Set Notation	$\{x|x \in \mathbb{R}\}$	$\{y|y \in \mathbb{R}, y > 0\}$
Interval Notation	$(-\infty, +\infty)$	$(0, +\infty)$

Domain and Range of Exponential Functions

Let f(x) = $a \cdot b^{P(x)} + h$ be an exponential function where $P(x)$ is linear. Then

Domain of the function is $\mathbb{R}$

Range of the function = $\begin{cases} (h, +\infty), & if\ a > 0 \\ (-\infty, h), & if a < 0 \end{cases}$

In cases of exponential functions where $P(x)$ is linear, in which case, $b^{p(x)}$ will always be defined for any value of $\boldsymbol{x}$. Thus, the domain of an exponential function is the set of real numbers or $\mathbb{R}$. For the range, note that $b^{p(x)} > 0$ for any values of $\boldsymbol{x}$. Hence, the range of an exponential function will depend on $\boldsymbol{a}$ and $\boldsymbol{h}$.

Example 3. Let $f(x) = 3^x$ Find the domain and range.

Solution:
The domain of the function is the set of real numbers since $f(x) = 3^x$ is defined for any real number x. It means that any value of x from the set of real numbers can be substituted to variable x. Note that any power of 3 is always positive. Hence, the range is $(0, +\infty)$.

	Domain	**Range**
Set Notation	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y \in \mathbb{R}, y > 0\}$
Interval Notation	$(-\infty, +\infty)$	$(0, +\infty)$

Example 4. Let $f(x) = 4^{x+1} - 2$. Find the domain and range.

Solution:
The domain of the function is the set of real numbers, because $P(x) = x + 1$ and it is linear. Also, in the given function you may observe that $a > 0$ ($a = 1\ and\ b = 2$) and $h = -2$, hence the range of the function is equal to $(h, +\infty)$.

	Domain	**Range**
Set Notation	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y \in \mathbb{R}, y > -2\}$
Interval Notation	$(-\infty, +\infty)$	$(-2, +\infty)$

Example 5. Let$f(x) = -2^{x-1} + 3$. Find the domain and range.

Solution:
The domain of the function is the set of real numbers because $P(x) = x - 1$ and it is linear. Also, in the given function you may observe that $a < 0$ $(a = -2 \; and \; b = \frac{1}{2})$ and $h = 3$, hence the range of the function is equal to $(-\infty, h)$.

	Domain	**Range**
Set Notation	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y \in \mathbb{R}, y < 3\}$
Interval Notation	$(-\infty, +\infty)$	$(-\infty, 3)$

What's More

Activity 1.1
Answer the guide questions to complete the table of domain and range of the following exponential functions.

a. $f(x) = 5^x$
Is $f(x) = 5^x$ defined at any values of x? _____.
What is the minimum value of $f(x)$? _____.
Can you determine the the maximum value of $f(x)$? _____.

	Domain	**Range**
Set Notation		
Interval Notation		

b. $f(x) = \left(\frac{1}{5}\right)^{3x}$
Is $f(x) = \left(\frac{1}{5}\right)^{3x}$ defined at any values of x? _____.
What is the minimum value of $f(x)$? _____.
Can you determine the maximum value of $f(x)$? _____.

	Domain	**Range**
Set Notation		
Interval Notation		

c. $f(x) = \frac{1}{3}(2)^x + 3$

Which is $P(x)$? _____Is $P(x)$ linear? _____
Is $a > 0$ or $a < 0$? _____
What is the value of h? _____

	Domain	**Range**
Set Notation		
Interval Notation		

d. $g(x) = -(4)^{2x-3} + 1$
Which is $P(x)$? _____Is $P(x)$ linear? _____
Is $a > 0$ or $a < 0$? _____
What is the value of h? _____

	Domain	**Range**
Set Notation		
Interval Notation		

e. $h(x) = 2(3)^{3x-1} + 2$
Which is $P(x)$? _____Is $P(x)$ linear? _____
Is $a > 0$ or $a < 0$? _____
What is the value of h? _____

	Domain	**Range**
Set Notation		
Interval Notation		

Activity 1.2
Find the domain and range of the following exponential functions:

1. $f(x) = \left(\frac{1}{2}\right)^{x+1}$
2. $f(x) = 3^{x+2}$
3. $f(x) = -(5)^x$
4. $(x) = 4^{-x}$
5. $(x) = \left(\frac{1}{3}\right)^{-x+1}$
6. $f(x) = 2^{x-1} + 1$
7. $f(x) = -5^x - 2$
8. $f(x) = \left(\frac{1}{4}\right)^{2x+1} + 3$
9. $f(x) = 5^{3x} - 4$
10. $f(x) = -\left(\frac{2}{3}\right)^x - 3$

What I Have Learned

Warm that Mind Up!

A. Fill in the blanks with the correct word or phrase to complete the sentence.

1. The set of values of the independent variable (usually x) is called _____.
2. The resulting y-values after we have substituted in the possible x-values is called _____.
3. In the function f(x) = $a \cdot b^{P(x)} + h$ where P(x) is linear, the domain of the function is _____.
4. In the function f(x) = $a \cdot b^{P(x)} + h$ where P(x) is linear, if $a > 0$ the range of the function is _____.
5. In the function f(x) = $a \cdot b^{P(x)} + h$ where P(x) is linear, if $a < 0$ the range of the function is _____.

B. In your own words, what technique or strategy can you think of to facilitate your way of finding the domain and range of an exponential function?

_____

_____

_____

_____

C. This lesson helps you understand the procedure in finding the domain and range of an exponential function. You also discover your own technique or strategy on how to easily find ways in determining the domain and range of an exponential function. If you have given the chance to give an advise to Jethro who is a Senior High School learner that is working as a part time at the canteen near his house at the same time studying, what strategy or technique can you suggest to finish his study. Explain

_____

_____

_____

_____

What I Can Do

Now, that you have a deeper understanding of the topic, I believe that you are ready to solve the problems below.

Mission Possible

1. The volume V of air remaining in an inflated balloon can be modeled by the function $V = 1{,}000(0.85)^x$ where x represents the number of days that have passed since inflating the balloon. What is the reasonable domain for the situation? Explain.

   _____
   _____
   _____
   _____
   _____
   _____
   _____

2. The function $f(x) = 65{,}000(1.5)^x$can be modeled the population of a city for x, the number of years that have passed since 2010. What inequality represents the reasonable range of the function based on the situation? Explain.

   _____
   _____
   _____
   _____

Rubrics:
5- Shows in-depth comprehension of the pertinent concepts and /or processes, and provides explanations wherever appropriate.
4- Shows in-depth comprehension of the pertinent concepts and/or processes.
3- Shows in-depth comprehension of major concepts although neglects or misinterprets less significant ideas or details.
2- Shows gaps in theoretical comprehension.
1- Demonstrate minor comprehension not being able to develop an approach.

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. The resulting y-values after we have substituted in the possible x-values is called _____.
   a. Range
   b. Domain
   c. Real Numbers
   d. Exponential Function

2. Which of the following does not exist?
   a. zero on the numerator
   b. zero values under the square root sign
   c. negative values on the denominator
   d. negative values under the square root sign

3. What is the domain and range of the function: $f(x) = \sqrt{x+7}$?
   a. Both the domain and the range is a set of all real numbers.
   b. The domain is the set of all real numbers while the range is a set of real numbers greater than or equal to 0.
   c. The domain is a set of real numbers greater than or equal to -7 while the range is a set of real numbers all real numbers.
   d. The domain is a set of real numbers greater than or equal to -7 while the range is a set of real numbers greater than or equal to 0.

4. Which of the following statement is never true?
   a. For any given x-value, the y-value of $y = 5^x$ is negative.
   b. For any given x-value, the y-value of $y = 5^x$ is positive.
   c. The domain of an exponential function $y = 5^x$ is positive numbers.
   d. The domain of an exponential function $y = 5^x$ is all real numbers.

5. Which of the following statements is true about the function $y = 3^x$?
   a. The range of the function is the set of all real numbers.
   b. The domain of the function is the set of real numbers less than 3.
   c. The range of the function is the set of all real numbers less than 3.
   d. The domain of the function is the set of all real numbers.

Use the graph below for nos. 6-7.

6. What is the domain of the exponential function $f(x) = b^x$?
   a. $\{x|x \in \mathbb{R}, x \geq 0\}$
   b. $\{x|x \in \mathbb{R}, x < 0\}$
   c. $\{x|x \in \mathbb{R}\}$
   d. $\{x|x \in \mathbb{R}, x \leq 0\}$

7. What is the range of the exponential function $f(x) = b^x$?
   a. $\{y|y \in \mathbb{R}\}$
   b. $\{y|y \in \mathbb{R}, y \geq 0\}$
   c. $\{y|y \in \mathbb{R}, y < 0\}$
   d. $\{y|y \in \mathbb{R}, y > 0\}$

For item numbers 8-9, refer to the given function $f(x) = 3^{x+1}$

8. What is the domain of the exponential function?
   a. $\{x|x \in \mathbb{R}\}$
   b. $\{x|x \in \mathbb{R}, x \geq 0\}$
   c. $\{x|x \in \mathbb{R}, x \leq 0\}$
   d. $\{x|x \in \mathbb{R}, x > 0\}$

9. What is the range of the exponential function?
   a. $\{y|y \in \mathbb{R}\}$
   b. $\{y|y \in \mathbb{R}, y \geq 0\}$
   c. $\{y|y \leq 0\}$
   d. $\{y|y \in \mathbb{R}, y > 0\}$

For item numbers 10-11, refer to the given fudnction $f(x) = \left(\frac{1}{3}\right)^x$.

10. What is the domain of the exponential function?
   a. $\{x|x \in \mathbb{R}\}$
   b. $\left\{x \middle| x \in \mathbb{R}, x > \frac{1}{3}\right\}$
   c. $\{x|x \in \mathbb{R}, x > 0\}$
   d. $\left\{x \middle| x \in \mathbb{R}, x < \frac{1}{3}\right\}$

11. What is the range of the exponential function?
    a. $\{y|y \in \mathbb{R}\}$
    b. $\left\{y \middle| y \in \mathbb{R}, y > \frac{1}{3}\right\}$
    c. $\{y|y \in \mathbb{R}, y > 0\}$
    d. $\{y|y \in \mathbb{R}, y < 0\}$

For item numbers 12-13, refer to the given function $f(x) = 5^x + 2$

12. What is the domain of the exponential function?
    a. $\{x|x \in \mathbb{R}\}$
    b. $\{x|x \in \mathbb{R}, x > 2\}$
    c. $\{x|x \in \mathbb{R}, x > 0\}$
    d. $\{x|x \in \mathbb{R}, x < 5\}$

13. What is the range of the exponential function?
    a. $\{y|y \in \mathbb{R}\}$
    b. $\{y|y \in \mathbb{R}, y > 2\}$
    c. $\{y|y \in \mathbb{R}, y > 0\}$
    d. $\{y|y \in \mathbb{R}, y < 2\}$

For item numbers 14-15, refer to the given function $f(x) = -4(3)^x - 5$.

14. What is the domain of the exponential function?
    a. $\{x|x \in \mathbb{R}\}$
    b. $\{x|x \in \mathbb{R}, x > -4\}$
    c. $\{x|x \in \mathbb{R}, x < 3\}$
    d. $\{x|x \in \mathbb{R}, x < -5\}$

15. What is the range of the exponential function?
    a. $\{y|y \in \mathbb{R}\}$
    b. $\{y|y \in \mathbb{R}, y > -4\}$
    c. $\{y|y \in \mathbb{R}, y > 3\}$
    d. $\{y|y \in \mathbb{R}, y < -5\}$

Additional Activities

Let me Formulate!

In this section, you are going to think deeper and test further your understanding of the domain and range of the exponential function.

Give five examples of exponential functions in the form $f(x) = b^x$ and $f(x) = a \cdot b^{P(x)} + h$ with its domain and range.

Answer Key

What I Know

1. B
2. A
3. A
4. D
5. D
6. D
7. C
8. A
9. D
10. A
11. C
12. A
13. B
14. A
15. D

What's More

Activity 1.1

1.

	Domain	Range
Set Notation	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y > 0\}$
Interval Notation	$(-\infty, +\infty)$	$(0, +\infty)$

2.

	Domain	Range
Set Notation	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y > 0\}$
Interval Notation	$(-\infty, +\infty)$	$(0, +\infty)$

3.

	Domain	Range
Set Notation	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y > 3\}$
Interval Notation	$(-\infty, +\infty)$	$(3, +\infty)$

4.

	Domain	Range
Set Notation	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y < 1\}$
Interval Notation	$(-\infty, +\infty)$	$(-\infty, 1)$

5.

	Domain	Range
Set Notation	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y > 2\}$
Interval Notation	$(-\infty, +\infty)$	$(2, +\infty)$

Activity 1.2

1. D:$\mathbb{R}$ R: $\{y \mid y > 0\}$
2. D:$\mathbb{R}$ R: $\{y \mid y > 0\}$
3. D:$\mathbb{R}$ R: $\{y \mid y < 0\}$
4. D:$\mathbb{R}$ R: $\{y \mid y > 0\}$
5. D:$\mathbb{R}$ R: $\{y \mid y > 0\}$
6. D:$\mathbb{R}$ R: $(1, +\infty)$
7. D:$\mathbb{R}$ R: $(-\infty, -2)$
8. D:$\mathbb{R}$ R: $(3, +\infty)$
9. D:$\mathbb{R}$ R: $(-4, +\infty)$
10. D:$\mathbb{R}$ R: $(-\infty, -3)$

Assessment

1. A
2. D
3. D
4. A
5. D
6. C
7. D
8. A
9. D
10. A
11. C
12. A
13. B
14. A
15. D

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*DepED Material: General Mathematics Learner's Material

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11 General Mathematics

General Mathematics
Intercepts, Zeroes and Asymptotes of Exponential Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Intercepts, Zeroes and Asymptotes of Exponential Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Intercepts, Zeroes, and Asymptotes of Exponential Functions.

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Determining Intercepts, Zeroes and Asymptotes of an Exponential Function.

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you master the different ways to determine the zeroes, intercepts, and asymptotes of exponential functions. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

1. determine zeroes of an exponential function; and
2. determine intercepts and asymptotes of an exponential function given the graph of an exponential function.

What I Know

Test yourself on the topics to be discussed in this module.
Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Simplify the following expression: $3^{(x+3)}$.
   a. $3^{(x+3)} = 3^{2x} \bullet 3^3 = 27(3^{3x})$
   b. $3^{(x+3)} = 3^{x+3} \bullet 3^3 = 27(3^{x+3})$
   c. $3^{(x+3)} = 3^{x} \bullet 3^3 = 27(3^{x})$
   d. $3^{(x+3)} = 3^{x/3} \bullet 3^3 = 27(3^{x/2})$

2. Where should the y-intercept of the graph of the function $b^x a^x$ be?
   a. The y-intercept is at (0, b)
   b. The y-intercept is at (0,0)
   c. The y-intercept is at (b,0)
   d. The y-intercept is at (b, b)

3. For what values of x is the function $f(x) = 3^x 3^x$ less than 1?
   a. $f(x) < 1$ for all $x < 1$
   b. $f(x) > 1$ for all $x < 1$
   c. $f(x) > 1$ for all $x < -1$
   d. $f(x) < 1$ for all $x < -1$

4. Where do the graphs of $y = a^x$ and $y = a^{-x}$ intersect?
   a. They intersect at the point (0,0)
   b. They intersect at the point (1,0)
   c. They intersect at the point (1,1)
   d. They intersect at the point (0,1)

5. Does the function $f(x) = (\frac{1}{3})^x$ increase or decrease?
   How about the function $f(x) = 3^x$?
   a. $f(x) = (\frac{1}{3})^x$ decreases as x decreases and increases as x increases.
      $f(x) = (3)^x$ increases as x decreases and decreases as x increases.
   b. $f(x) = (\frac{1}{3})^x$ increases as x decreases and decreases as x increases.
      $f(x) = (3)^x$ decreases as x decreases and increases as x increases.
   c. $f(x) = (\frac{1}{3})^x$ does not increase as x decreases and does not decrease as x increases.
      $f(x) = (3)^x$ increases as x decreases and decreases as x increases.
   d. $f(x) = (\frac{1}{3})^x$ increases as x decreases and decreases as x increases.
      $f(x) = (3)^x$ does not decrease as x increases and decreases as x increases.

6. If $3^x = 3^8$, what is x?
   a. -4
   b. -2
   c. 6
   d. 8

7. Find x if $2^{x-1} = 8$.
   a. 4 b. 3 c. 2 d. 1

8. Find the zeroes of $h(x) = 2^{x-3}$.
   a. 3 b. 5 c. 7 d. 9

9. What value of x will make the function $y = 2^{3x} - 1$ equal to 0?
   a. 2 b. 1 c. 0 d. -1

10. Determine the zeroes of the exponential function $f(x) = 2^x$.
    a. (0, -1) b. (0, -2) c. no zero d. (0, 2)

11. The graph of a function of the form $y = a^x$ passes through which of the following points?
    a. (-1, 0)
    b. (1, 0)
    c. (0, 1)
    d. (0, -1)

12. Which of the statements is best modeled by exponential growth?
    a. The cost of pencils as a function of the number of pencils.
    b. The distance when a stone is dropped as a function of time
    c. The distance of a swinging pendulum bob from the center as a function of time
    d. The compound interest of an amount as a function of time.

For numbers 13 to 15, please refer to the given function y = $\left(\frac{1}{3}\right)^{x} - 2$.

13. Which of the following is the y-intercept?

    a. -1 b. -2 c. 1 d. 2

14. What can you say about the trend of the graph?

    a. increasing
    b. decreasing
    c. either increasing or decreasing
    d. no conclusion can be made

15. Which of the following is the horizontal asymptote?

    a. $y = -1$ b. $y = -2$ c. $y = 1$ d. $y = 2$

Lesson 1 Intercepts, Zeroes, and Asymptotes of Exponential Functions

In the previous lessons, you learned how to determine domain and range of an exponential function. You were to only consider cases of exponential functions where *P(x)* is linear, in which case, $b^{P(x)}$ will always be defined for any value of *x*. Thus, the domain of an exponential function is the set of real numbers or R. For the range, note that $b^{P(x)} > 0$ for any value of *x*. hence, the range of an exponential function will depend on a and h.

What's In

To fully understand the topic, you must make a recall on the laws of exponents. You should be able to pay more attention to those properties. Know its application and be able to distinguish one after the other. Be patient enough to practice more in enhancing your skills. Keep in mind that an exponential function is different from other functions as its exponent is a variable.

Let us review the laws of exponents and the properties of equality for exponential equation.

Laws of Exponents

For any real numbers, a and b and any positive real numbers m and n,

a. $a^m a^n = a^{m+n}$
b. $(a^m)^n = a^{mn}$
c. $(ab)^n = a^n b^n$
d. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
e. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
f. $a^0 = 1$

Simplify each expression and express the answers with positive exponents.

1. $x^3 x^5 = x^{3+5}$

   $= x^8$

2. $\left(x^{-2}\right)^{3} = x^{(-2)(3)}$

$= x^{-6}$

$= \frac{1}{x^{6}}$

3. $\left(2x^{-5}\right)^{-3} = 2^{-3}x^{15}$

$= \frac{x^{15}}{2^{3}}$

$= \frac{x^{15}}{8}$

4. $\frac{16x^{5}}{12x^{7}} = \frac{16}{12} \bullet \frac{x^{5}}{x^{7}}$

$= \frac{4}{3} \bullet x^{5-7}$

$= \frac{4}{3} \bullet x^{-2}$

$= \frac{4}{3} \bullet \frac{1}{x^{2}}$

$= \frac{4}{3x^{2}}$

5. $\left[\frac{64x^{\frac{1}{3}}}{125x^{-\frac{2}{3}}}\right]^{\frac{2}{3}}$

$\left[\frac{64x^{\frac{1}{3}}}{125x^{-\frac{2}{3}}}\right]^{\frac{2}{3}} = \left[\frac{64}{125}\right]^{\frac{2}{3}} \bullet \left[\frac{x^{\frac{1}{3}}}{x^{-\frac{2}{3}}}\right]^{\frac{2}{3}}$

$= \left[\sqrt[3]{\frac{64}{125}}\right]^{2} \bullet \left[x^{\frac{1}{3}\bullet\frac{2}{3}}\right]^{\frac{2}{3}}$ Express rational exponents in radical form and simplify

$= \left[\frac{4}{5}\right]^{2} \bullet \left[x^{\frac{3}{3}}\right]^{\frac{2}{3}}$

$= \left[\frac{16}{25}\right] \bullet \left[x^{1}\right]^{\frac{2}{3}}$

$= \frac{16x^{\frac{2}{3}}}{25}$

Suppose you were asked to solve for the value of the variable that would make the equation true, how are you going to begin the task? So, to help you with this matter, let us recall what you have learned previously.

The Property of Equality for Exponential Equation

An exponential equation in one variable is an equation where the variable is an exponent.

In solving exponential equations, the property of equality for exponential equation also known as equating-exponents property implies that, if the bases are equal, the exponents must also be equal.

This could also be stated as follows,

"If a, b and c are real numbers and a ≠ 0, then $a^{b} = a^{c}$ if and only if b = c."

Examples: Solve for the value of the variable that would make the equation true.

1. $2^x = 2^4$ Since the bases are equal,
   $x = 4$ the exponents must be equal too.
   Since the $x = 4$, then $2^x = 2^4$.

2. $3^{4y} = 3^{16}$ Since the bases are equal,
   $4y = 16$ the exponents must be equal too.
   $y = 4$
   Since $y = 4$, then $3^{4y} = 3^{16} \rightarrow 3^{4(4)} = 3^{16} \rightarrow 3^{16} = 3^{16}$

3. $5^6 = 5^{x-2}$ The bases are equal,
   $6 = x - 2$ the exponents must be equal too.
   $x = 8$
   Since $x = 8$, then $5^6 = 5^{x-2} \rightarrow 5^6 = 5^{8-2} \rightarrow 5^6 = 5^6$

Use laws of exponents to solve to make the bases equal. Then apply the Equating-Exponents Property.

Solve the equation $2^{x-1} = 8$.
Solution: Write both sides with 2 as the base.

$2^{x-1} = 8$
$2^{x-1} = 2^3$
$x - 1 = 3$ By the additive inverse property
$x = 4$

Finding the Roots of Exponential Equation

1. Solve the exponential equation $2^{4x-1} = 8^{x-2}$.
Solution: Use laws of exponents to make the bases equal. Then apply the Equating-Exponents.

$2^{4x-1} = 8^{x-2}$
$2^{4x-1} = 2^{3(x-2)}$
$4x - 1 = 3(x - 2)$
$4x - 1 = 3x - 6$
$x = -5$

2. Solve the exponential equation $2^{x^2-5x} = \frac{1}{16}$.

$2^{x^2-5x} = \frac{1}{16}$
$2^{x^2-5x} = 16^{-1}$
$2^{x^2-5x} = (2^4)^{-1}$
$2^{x^2-5x} = 2^{-4}$
$x^2 - 5x = -4$
$x^2 - 5x + 4 = 0$
$(x - 1)(x - 4) = 0$
$x = 1$ or $x = 4$

Notes to the Teacher

Provide solutions for problems which will not be correctly solved by the learners. Students should have a mastery of the simpfilfying exponential expressions, laws of exponents, properties of equality and finding the roots of exponential equztions.

What's New

Who Says Who?

Maria Corazon C. Tolentino

What could go wrong if my mind explodes?
The absence of my "x" that left my side,
to completely heal my heart.
There could have been us but if not "asymptote" decides,
numbers and variables collide
and my mind might collapse.

Who says who?
Exponents could be bossy too.
While base awaits, raise to power too.
It's just that my heart wants to subside,
from this pool of miseries of confusion.
Even inspiration is a piece of cake,
to cater to my mind's undecided state.

Who says who?
Nothing is yet to decide.
My "*x*" or your "*y*", who could be it now?
Absolute affection is indeed my direction,
To value the things in my perception.

Hey, you, who brings my heart,
be positive enough to my delight.
Can I conquer my fear without you at my sight?

Be brave, be brave, My heart!

Take note. not everyone shares the same idea as you have. We all have different ways of dealing with our problems. What might be easy for me might be difficult for you? You might be afraid to try learning this topic. Be not afraid. Little by little, things that do not matter to us are the most essential for all you know. This poem reaches out to your inner self. The same with this module. It is talking to you as if it is your friend. Try to test your limit and appreciate that no matter what happens, you can always go back to basic. As there are new lessons that will be introduced to you in this module, try to think and learned this poem by heart, for it will lead you to the right path.

What is It

Today, we will unlock the concepts on the properties of the exponential function. If you could notice, the exponential function has a great connection in Algebra, in Trigonometry, in Calculus, in all Sciences and Mathematics, and so on. There may be things that are still unclear to you, but the idea is for you to stay focus on what are the properties of an exponential function. What you should know about before taking this module? How will you be able to find the intercepts, zeroes, and asymptote of an exponential function? How should you apply knowledge of these topics in the real-life situation? How should you react to each of the examples given and can you discuss what you have learned to a partner?

The focus of this module is on determining intercepts, zeroes and asymptotes of an exponential function.

Determining the Zeroes of Exponential Equation

The zero of an exponential function refers to the value of the independent variable x that makes the function 0. Graphically it is the abscissa of the point of intersection of the graph of the exponential function and the x-axis. To find the zero of an exponential function $f(x)$, equate $f(x)$ to 0 and solve for x.

Examples:

Determine the zero of the given exponential function.

1. $f(x) = 3^x$

Solution:

To find the zero of the function, equate it to 0 and solve for x.

$f(x) = 3^x = 0$

$3^x = 0$

The resulting equation suggests that $f(x)$ has no zero since no real value of x will make $3^x = 0$ a true statement.

2. $g(x) = 5^{3x-12} - 1$

Solution:

To find the zero of the function, equate it to 0 and solve for x.

$$g(x) = 5^{3x-12} - 1 = 0$$
$$5^{3x-12} - 1 = 0$$
$$5^{3x-12} = 1$$
$$5^{3x-12} = 5^0$$
$$3x - 12 = 0$$
$$3x = 12$$
$$x = 4$$

the zero of $g(x)$ is 4.

3. $h(x) = \left(\frac{1}{2}\right)^{3x+5} - 8$

Solution:

To find the zero of the function, equate it to 0 and solve for x.

$$h(x) = \left(\frac{1}{2}\right)^{3x+5} - 8 = 0$$
$$\left(\frac{1}{2}\right)^{3x+5} - 8 = 0$$
$$\left(\frac{1}{2}\right)^{3x+5} = 8$$
$$\left(\frac{1}{2}\right)^{3x+5} = 2^3$$
$$\left(2^{-1}\right)^{3x+5} = 2^3$$
$$-3x - 5 = 3$$
$$-3x = 8$$
$$x = -\frac{8}{3}$$

The zero of $h(x)$ is $-\frac{8}{3}$.

4. $y = 4^{3x+2} - \left(\frac{1}{256}\right)^{2x-1}$

Solution:

To find the zero of the function, equate it to 0 and solve for x.

$$y = 4^{3x+2} - \left(\frac{1}{256}\right)^{2x-1} = 0$$

$$4^{3x+2} - \left(\frac{1}{256}\right)^{2x-1} = 0$$

$$(2^2)^{3x+2} = \left(256^{-1}\right)^{2x-1}$$

$$2^{6x+4} = \left[(2^8)^{-1}\right]^{2x-1}$$

$$2^{6x+4} = (2^{-8})^{2x-1}$$

$$2^{6x+4} = (2)^{-16x+8}$$

$$6x + 4 = -16x + 8$$

$$22x = 4$$

$$x = \frac{2}{11}$$

The zero of y is $\frac{2}{11}$.

Intercepts of an Exponential Function

The y-intercept is a point at which the graph crosses the y-axis. The x-value is always at zero. When you want to find the intercepts from an equation, let the y-value equal to zero, then solve for x.

Examples:

1. Find the x-intercept and y-intercept of $y = 4^{x+1} - 2$.

Solution:

To find the y-intercept, let $x = 0$, then by substitution, we have

$$y = 4^{x+1} - 2$$

$$y = 4^{0+1} - 2$$

$$y = 4^1 - 2$$

$$y = 2.$$

Then, the y-intercept is at (0, 2).

To find the x-intercept, let $y = 0$, then by substitution, we have

$$y = 4^{x+1} - 2$$

$$0 = 4^{x+1} - 2$$

$$2 = 4^{x+1}$$

$$2^1 = \left(2^2\right)^{x+1}$$

$$2^1 = (2)^{2(x+1)}$$

$$1 = 2(x + 1)$$

$$1 = 2x + 2$$

$1 - 2 = 2x$

$-1 = 2x$

$\frac{-1}{2} = x$

Thus, the x-intercept is at $(\frac{-1}{2}, 0)$.

2. Find the x-intercept and y-intercept of $y = 2^x - 64$.

Solution:

To find the y-intercept, let $x = 0$, then by substitution, we have

$y = 2^x - 64$

$y = 2^0 - 64$

$y = 1 - 64$

$y = -63.$

Then, the y-intercept is at (0, -63).

To find the x-intercept, let $y = 0$, then by substitution, we have

$y = 2^x - 64$

$0 = 2^x - 64$

$64 = 2^x$

$2^6 = 2^x$

$6 = x$

Thus, the x-intercept is at (6, 0).

3. Find the x-intercept and y-intercept of $y = 3.2^x + 8$.

Solution:

To find the y-intercept, let $x = 0$, then by substitution, we have

$y = (3.2)^x + 8$

$y = (3.2)^0 + 8$

$y = 1 + 8$

$y = 9$

Then, the y-intercept is at (0, 9).

To find the x-intercept, let $y = 0$, then by substitution, we have

$y = 3.2^x + 8$

$0 = 3.2^x + 8$

$-8 = 3.2^x$

$-2^3 = 3.2^x$

Since no way could make their bases equal, there is no x-intercept.

4. Find the x-intercept and y-intercept of $f(x) = -2(0.3^{2x+1}) + 4$.

Solution:

To find the y-intercept, let $x = 0$, then by substitution, we have

$$f(x) = -2(0.3^{2x+1}) + 4$$
$$f(0) = -2(0.3^{2(0)+1}) + 4$$
$$f(0) = -2[\left(\tfrac{3}{10})^{-1}\right)^{1}] + 4$$
$$f(0) = -2\left(\tfrac{3}{10}\right)^{-1} + 4$$
$$f(0) = -2\left(\tfrac{10}{3}\right) + 4$$
$$f(0) = \left(\tfrac{-20}{3}\right) + 4$$
$$f(0) = -\tfrac{8}{3}$$

Then, the y-intercept is at $(0, -\frac{8}{3})$.

To find the x-intercept, let $y = 0$, then by substitution, we have

$$f(x) = -2(0.3^{2x+1}) + 4$$
$$y = -2(0.3^{2x+1}) + 4$$
$$0 = -2(0.3^{2x+1}) + 4$$
$$4 = -2(0.3^{2x+1})$$
$$2^2 = -2(0.3^{2x+1})$$

Since no way could make their bases equal, there is no x-intercept.

Asymptotes of an Exponential Function Given by a Graph

A line that a curve approaches arbitrarily closely is an asymptote. An asymptote may be vertical, oblique or horizontal. As for this topic, horizontal asymptotes correspond to the value the curve approaches as x gets very large or very small.

With the help of a table of values and a graph you can determine the asymptote of an exponential function. Let us first take a look at the properties of the function $f(x) = 2^x$. In this case, $a = 1$, $P(x) = x$, and $h = 0$.

Assign integer values to x and find the corresponding values of $f(x)$.

For $x \geq 0$:

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	1	2	4	8	16	32	64	128	256	512

Please take note, that as x increases, the value of $f(x)$ keeps on increasing rapidly.

For x < 0:

x	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
$f(x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{256}$	$\frac{1}{512}$	$\frac{1}{1{,}024}$

Observe that as the value of x decreases, the value of $f(x)$ decreases as well. Notice that when x is negative and decreasing, the value of the function approaches zero. Thus, the graph has $y = 0$ as a horizontal asymptote. (Note: You will learn more about graphing an exponential function on another module.)

One property of the graph is that it passes the point (0, 1) or the graph has its y-intercept = 1.

Let us take this next example. Suppose $f(x) = 2^x + 2$. Our table of values in this case is as shown below,

x	-3	-2	-1	0	1	2	3
$f(x)$	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{1}{2}$	3	4	6	10

Comparing the two, we can see that the graph of $f(x) = 2^x + 2$ is shifted up by two units that of the $f(x) = 2^x$ on the graph.

Figure 1. Asymptotes of $f(x) = 2^x$ and $f(x) = 2^x + 2$

From the graph, you can see that the horizontal asymptote of $f(x) = 2^x$ is $y = 0$, while the horizontal asymptote of $f(x) = 2^x + 2$ is $y = 2$

To help you understand more on this topic, here are some more samples for you to try. (Hint: Observe the value of d in the exponential function $f(x) = a \cdot b^x + d$)

Determine the asymptote of the following:

a. $f(x) = 5^x$

Solution: The asymptote is at $y = 0$.

b. $f(x) = 3^x + 2$.

Solution: The asymptote is at $y = 2$.

c. $y = 3^{x+2} - 5$.

Solution: The asymptote is at $y = -5$.

d. $y = -2 + 3^x$.

Solution: The asymptote is at $y = -2$.

e. $f(x) = 4^{x-3}$.

Solution: The asymptote is at $y = 0$.

Did you get the technique? Now, if there are still confusing processes to you, do not hesitate to go back once again and verify the answers. Don't be hesitant to ask for help from your teacher. Have a happy attitude to get you where you want to be.

What's More

Activity 1.1

If there is any, solve for the zero of each exponential function below.

1. $f(x) = 14^x - 1$
2. $g(a) = -3^a + 27$
3. $h(x) = \left(\frac{1}{2}\right)^x - \frac{1}{8}$
4. $f(x) = 4^x$
5. $h(b) = -2\left(2^{b+3}\right) + 8$

Activity 1.2

Solve for the y-intercept of each exponential function below.

1. $f(c) = 3^c$
2. $g(x) = -\left(\frac{1}{3}\right)^x$
3. $h(x) = 5(2^x)$
4. $f(d) = 7^{d+2} - 1$
5. $g(x) = -6(2^{2x+3}) + 4$

Activity 1.3

Solve for the asymptote of each exponential function below.

1. $g(x) = -7^x$
2. $h(x) = \left(\frac{3}{4}\right)^x$
3. $f(x) = 2^x + 5$
4. $g(z) = -4^{z-1} + 1$
5. $h(k) = \left(\frac{1}{2}\right)^k - 3$

Activity 1.4

Complete the table below.

exponential function	y-intercept	zero	asymptote
1. $f(x) = 3^x$			
2. $g(x) = -3^x$			
3. $g(x) = \left(\frac{1}{3}\right)^x - 1$			
4. $h(x) = 2(3^x) - 18$			
5. $h(x) = 81 - 3^{x+1}$			
6. $g(x) = 3^x + 1$			
7. $f(x) = -2\left(\frac{1^x}{3}\right)$			
8. $g(x) = 2\left(\frac{1^x}{3}\right) + 1$			
9. $f(x) = \left(\frac{1}{4}\right)^{x+2} + 3$			
10. $f(x) = -3^{2x+1} + 2$			

What I Have Learned

This time, complete the statements below.

1. To solve for the y-intercept of $f(x) = a(b^{x-c}) + d$, replace _____ with 0, and solve for _____.
2. To solve for the zero of $f(x) = a(b^{x-c}) + d$, replace _____ with 0, and solve for _____.
3. If the range of $f(x) = a(b^{x-c}) + d$ is $(d, +\infty)$ or $y > d$, the equation of the asymptote is y=_____.

4. If the range of $f(x) = a(b^{x-c}) + d$ is $(-\infty, d)$ or $y < d$, the equation of the asymptote is y=_____.
5. Regardless of the value of _____ in $f(x) = b^x$, there is _____ zero of a function. Meanwhile, the y-intercept is _____ and the asymptote's equation is _____.

What I Can Do

COVID-19 has caused damages to the world even up to taking lives of many. Death due to this pandemic follows an exponential pattern among nations across the globe. In times like this, what can you do to help hinder the spread of a deadly virus? Express your answer through an essay.

Rubrics shown below will be used in scoring your essay.

FEATURES	**4**	**3**	**2**	**1**
Quality of writing	Piece was written in an extraordinary style and voice; very informative and well organized	Piece was written in an interesting style and voice; somewhat informative and organized	Piece has little style and voice; gives some new information but poorly organized	Piece has no style and voice; gives no new information and very poorly organized
Grammar, usage and mechanics	Virtually no spelling, punctuation or grammatical errors	Few spelling or punctuation errors, minor grammatical errors	A number of spelling, punctuation or grammatical errors	So many spelling, punctuation or grammatical errors that it interferes with the meaning of

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What will you find if zero is substituted to x-variable of an exponential function?
   a. asymptote
   b. y-intercept
   c. zero
   d. domain

2. What will you find if zero is substituted to y-variable of an exponential function?
   a. asymptote
   b. y-intercept
   c. zero
   d. domain

3. Which determines the equation of the asymptote in $f(x) = a(b^{x-c}) + d$?
   a. a
   b. b
   c. c
   d. d

4. Of the following, which is not true about $f(x) = -5^x$?
   a. There is no zero.
   b. The x-intercept is zero.
   c. The y-intercept is 1.
   d. The asymptote is $y = 0$.
5. What is the zero of $f(x) = 2^x - 8$?
   a. 0
   b. 1
   c. 2
   d. 3
6. What is the y-intercept of $f(x) = -\left(\frac{5}{12}\right)^x$?
   a. -1
   b. 0
   c. $\frac{5}{12}$
   d. 1

7. What is the y-intercept of $g(x) = -6^{x+1} + 1$?
   a. -5
   b. -1
   c. 1
   d. 7

8. What is the asymptote of $g(x) = 2^x + 7$?
   a. 2
   b. 7
   c. $y = 2$
   d. $y = 7$

9. Which of the following is true about $h(x) = 3^x - 9$?
   a. Its zero is -2.
   b. Its y-intercept is -8.
   c. It is a decreasing function.
   d. Its asymptote is $y = 9$.

10. Which is/are similar among $f(x) = 2^x, g(x) = 4^x \; and \; h(x) = 7^x$?
   a. asymptotes
   b. y-intercepts
   c. both a and b
   d. none

11. Which characteristic is not the same for all the following functions:
$f(x) = \; 2^x, g(x) = -2^x, h(x) = \left(\frac{1}{2}\right)^x \; and \; j(x) = -\left(\frac{1}{2}\right)^x$?
   a. asymptotes
   b. range
   c. y-intercepts
   d. zeroes

12. Which is not the same for all the functions: $f(x) = 5^x, g(x) = 5^x + 1, h(x) = 5^x - 2$?
   a. asymptotes
   b. x-intercepts
   c. y-intercepts
   d. zeroes

13. Which is not true for $f(x) = 7^{x+1} \; to \; g(x) = -7^{x+1} \; and \; h(x) = 2(7^{x+1})$?
   a. Each exponential function has no zero.
   b. The y-intercepts are 7, -7 and 14, respectively.
   c. The asymptote of each exponential function is $y = 0$.
   d. The range of each exponential function is $y > 7$.

14. Which is true about $f(c) = 4(2^c) - 8$?
    a. The asymptote is the same as the asymptote of $g(c) = 4(2^c)$.
    b. The y-intercept is the same as the y-intercept of $h(c) = (2^c) - 8$.
    c. The zero is the same as the zeroes of $g(c) = 4(2^c)$.
    d. The asymptote is the same as the asymptote of $h(c) = (2^c) - 8$

15. Which is not true about $f(x) = -4(2^{x+3}) - 16$
    a. The y-intercept is 48.
    b. The zero of the exponential function is -1.
    c. Its asymptote is $y = -16$.
    d. Its domain is the set of real numbers.

Supply each set of exponential functions in the table below with correct data. Write also your observations about the similarities and differences in the features of each set of exponential functions, if there is any.

Additional Activities

	y-intercept	**zero**	**asymptote**	**observations**
Set A				
$f(x) = 2^x$				
$g(x) = -2^x$				
Set B				
$f(x) = 3^x$				
$g(x) = \left(\frac{1}{3}\right)^x$				
Set C				
$f(x) = 4^x$				
$g(x) = 4^x + 1$				
$h(x) = 4^x - 1$				
Set D				
$f(x) = 2^x$				
$g(x) = 2^{x+1}$				
$h(x) = 2^{x-1}$				

Answer Key

What I Know

1. C
2. A
3. D
4. D
5. B
6. D
7. A
8. B
9. C
10. C
11. C
12. D
13. A
14. A
15. B

What's More

Activity 20.1

1. 0
2. 3
3. 3
4. None
5. -1

Activity 20.2

1. 1
2. -1
3. 5
4. 48
5. -44

Activity 20.3

1. y=0
2. y=0
3. y=5
4. y=1
5. y=-3

Activity 20.4

	y-inter-cept	Zero	asymptote
1	1	None	y=0
2	-1	None	y=0
3	2	0	y=-1
4	1	2	y=-18
5	-5	3	y=81

Assessment

1. B
2. B
3. D
4. B
5. D
6. A
7. A
8. D
9. B
10. C
11. C
12. A
13. D
14. D
15. A

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*DepED Material: General Mathematics Learner's Material

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11 General Mathematics

General Mathematics
Solving Real-life Problems Involving, Equations and Inequalities
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Solving Real-life Problems Involving Exponential Functions, Equations and Inequalities

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Real-life Problems Involving Exponential Functions, Equations and Inequalities!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Real-life Problems Involving Exponential Functions, Equations and Inequalities!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you solve real-life problems involving exponential functions, equations and inequalities. Most of the time, students like you ask why you need to study Mathematics. Even though you know the answer, still you keep on asking this question because perhaps you did not realize how important it is to real-life situations. This module hopes to help you make a wise decision in the future because it involves money matter problems. After going through this module, you are expected to solve real-life problems involving exponential functions, equations, and inequalities.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following situations can you apply solving exponential functions?
   a. finding the age of your father if he is 15 more than thrice your age
   b. calculating the area of rectangular field given the dimensions
   c. finding the number of bacteria with a growth rate of 25% after a certain period given the initial number
   d. getting the probability of a discrete random variable

2. If ₱1,000.00 is invested at the rate of 5% compounded quarterly, at the end of the year it is equal to _____.
   a. ₱1,000.00+0.05x 4
   b. $(₱1000(1.05)^4$
   c. $(₱1000 + 0.05)^4$
   d. $\left(₱1000(1 + \frac{0.05}{4}\right)^4$

3. Which of the following formulas can be used to find the exponential growth of the population?
   a. $A = P(1 + \frac{r}{n})^{nt}$
   b. $A = P(1 + r)^t$
   c. $A = Pe^{rt}$
   d. $A = \pi r^2$

4. A bank offers you a time deposit with 7% interest compounded annually; give an exponential model for the offer if you wanted to invest ₱100,000.00 under this investment.
   a. A = ₱100,000$(1.07)^t$
   b. A = ₱100,000(1.07)t
   c. A = ₱100,000$(.07)^t$
   d. A = ₱100,000(.07)t

5. Using item no. 4 how much would your money be after 15 years?
   a. ₱250,000.00 c. ₱275,903.15
   b. ₱275,000.00 d. ₱375,903.15

For numbers 6 – 7, refer to the problem below:

Andy deposited an amount of ₱1,000.00 in a bank which offers 3% interest compounded annually and forgot about it due to his busy life. After 5 years, he remembered that he has money on the bank and check his balance.

6. Which of the following is the formula to determine the total amount on his passbook after 5 years?
   a. $A = P(1+r)^n$ c. $A = (1-r)^n$
   b. $A = P(1-r)^n$ d. $A = n(1+r)^P$

7. How much money did Andy have on his account after 5 years?
   a. ₱1,010.96 b. ₱1,129.24 c. ₱1,159.27 d. ₱1,231.05

For numbers 8 – 9, refer to the problem below:
The half-life of a radioactive substance is 3,000 years, with an initial amount of substance of 500 grams.
8. Give an exponential model of the amount remaining after t years.
   a. $y = 500(1/2)^{t/3000}$ c. $y = 500(1/4)^{t/3000}$
   b. $y = 5{,}000(1/2)^{t/300}$ d. $y = 5{,}000(1/4)^{t/300}$

9. What amount of substance remains after 2,000 years
   a. 198.43 g b. 314.98 g c. 200 g d. 320 g

10. In 2010, Barangay Santolan has a population of 3,200. Its rate increases 1.05% every year. What is the population of the barangay after 3 years? (Use $P = P_0 e^{rt}$)
   a. 3,860 b. 3,680 c. 3,423 d. 3,303

11. A car bought for ₱1,500,000.00 depreciates by 20% per year. After how many years can one buy the car at about half of its original price?
   a. 5 years b. 6 years c. 7 years d. 8 years

12. The half-life of a radioactive substance is 20 days and there are 5 grams initially. Determine the amount of substance left after 80 days?
   a. 5 g b. 3 g c. 1.25 g d. 0.3125 g

For numbers 13 – 14, refer to the problem below.
Gerson Joseph opened a savings account and deposited ₱15,000.00. Each year the account increases by 5%.

13. Which of the following equations best represents the situation?

a. $A_t = 15000(1 + 0.05)^t$ c. $A_t = 15000(1 + 0.05)^{\frac{n}{t}}$

b. $A_t = 15000(1 - 0.5)^t$ d. $A_t = 15000(1 - 0.05)$

14. How many years will it take the account reach to ₱20,101.43?

a. 5 b. 6 c. 7 d. 8

15. The growth of a culture of bacteria is defined by the formula $y = 5000e^{0.03t}$, where t is the time (in days). How many bacteria will there be after two weeks?

a. 8000 b. 7805 c. 7610 d. 6705

Lesson 1 Solving Real-life Problems Involving Exponential Functions, Equations and Inequalities

Exponential growth and decay are the common applications of the exponential functions. The population growth is modeled by an exponential function, which includes the growth of investment under a compound interest, the increase in the number of bacteria as time passes by and a lot more. In the previous module, you already learned how to represent the exponential functions to real-life situations. This module will help you to gain a deeper understanding of the application of exponential functions, equations and inequalities.

What's In

Before we proceed in solving real-life problems involving exponential functions, equations, and inequalities. Let us first recall how to solve exponential equations and inequalities.

To solve exponential equations and inequalities, you should be familiar with the one-to-one property of exponential equations which state that if $x_1 \neq x_2$, then $b^{x_1} \neq b^{x_2}$. Conversely, if $b^{x_1} = b^{x_2}$ then $x_1 = x_2$, Also, you should know the property of exponential equalities:

If $b > 1$, then the exponential function $y = b^x$ is increasing for all x, which means that $b^x < b^y$ if and only if $x < y$.

If $0 < b < 1$, then the exponential function $y = b^x$ is decreasing for all x, which means that $b^x > b^y$ if and only if $x < y$.

Example 1

Solve the equation $2^{4x+2} = 64$.

Solution:

$$2^{4x+2} = 2^6$$
$$4x + 2 = 6$$
$$4x = 6 - 2$$
$$4x = 4$$
$$x = 1$$

Example 2

Solve the equation $\left(\frac{1}{10}\right)^{3x-1} = 1000$.

Solution:

$$\left(\frac{1}{10}\right)^{3x-1} = 10^3$$
$$\left(\frac{1}{10}\right)^{3x-1} = \left(\frac{1}{10}\right)^{-3}$$
$$3x - 1 = -3$$
$$3x = -3 + 1$$
$$3x = -2$$
$$x = -\frac{2}{3}$$

Example 3

Solve the inequality $5^x < 5^{2(x+1)}$.

Solution:

$$\boldsymbol{x < 2(x + 1)}$$
$$\boldsymbol{x < 2x + 2}$$
$$\boldsymbol{-2 < 2x - x}$$
$$\boldsymbol{-2 < x}$$
$$\boldsymbol{x > -2}$$

Example 4

Solve the inequality $\left(\frac{343}{125}\right)^{x+2} \geq \frac{25}{49}$.

Solution:

$$\boldsymbol{\left(\frac{7}{5}\right)^{3(x+2)} \geq \left(\frac{5}{7}\right)^{2}}$$
$$\boldsymbol{3(x + 2) \geq -2}$$
$$\boldsymbol{3x + 6 \geq -2}$$
$$\boldsymbol{3x \geq -8}$$
$$\boldsymbol{x > -\frac{8}{3}}$$

For the four examples given, I do hope you remember what you have learned in your previous module.

What's New

Read and analyze the problem below to answer the questions that follow.

What a Surprise!

Today is Alexa's birthday. Her parents want to give her a surprise, it is a savings account passbook with her name as the account holder. Her parents deposited an amount of ₱20,000.00 on the account at the time she was born. They think that it is about time for Alexa to manage her account. If you were Alexa, what would be your reaction if the passbook will be given to you as a birthday gift? Now that she is 18 years old, how much money will be in her savings account, if the money was invested with an interest of 3% compounded quarterly since the time it was deposited.

What is It

The previous activity is an example of real-life situations involving exponential function. If you doubt your answer, that's okay, or if you don't know what are you going to do to answer the problem above. We'll you may read first the examples here, and you may go back to the activity after you fully understand how to solve involving compound interest. The following are the applications of exponential functions, equations and inequalities to real-life problems.

Real-Life Problems Involving Exponential Function
Compound Interest

Example 1:
Danielle deposited ₱5,000.00 in an account that offers 6% interest compounded semi-annually. How much money is in his account at the end of three years?
The formula for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$
where $A =$ final amount, $P =$ principal or the initial amount, $r =$ interest rate, $n =$ number of times interest is compounded in one year, $t =$ number of years

Solution:
Given: $P = 5000 \quad r = 6\% \; or \; 0.06 \qquad n = 2 \; (semi - annually) \qquad t = 3$
Find A.

$$A = 5000\left(1 + \frac{0.06}{2}\right)^{2(3)}$$
$$A = 5000(1 + 0.03)^6$$
$$A = 5000(1.03)^6 = 5000(1.194) = 5970.26$$

Therefore, after three years the amount of money in Danielle's account is ₱5,970.26

Note: If interest is compounded annually $n = 1$.
If interest is compounded semi-annually $n = 2$.
If interest is compounded quarterly $n = 4$.
If interest is compounded monthly $n = 12$.

Looking at this example, I believe that you are now ready to check your answer on the What's a Surprise Problem. Do you think you got it right? I believe you are.

Population Growth and Decay

In the module entitled Representing Real-Life Situations Using Exponential Functions, you encounter problems like population growth and decay. This time, you will encounter the population once again but with the concept of the natural exponential function. The natural exponential function is the function $f(x) = e^x$. (If you want to know about this number, you can read the book "e: The Story of a Number", by Eli Maor.)

Example 2:
A certain bacteria, given favorable growth conditions, grow continuously at a rate of 5.4% a day. Find the bacterial population after twenty-four hours, if the initial population was 500 bacteria.
When you read a problem that suggests growth continuously, you should be thinking "continuously-compounded growth formula". For this situation, the formula is $A = P_0 e^{rt}$

where A = population after a certain period
P_0 = initial population
r = rate of change (growth rate but sometimes it is called decay rate)
t = time (growth/decay rates in contexts might be measured in minutes, hours, days, etc.)

Solution:
Given: $P = 500$ $r = 5.4\%\ or\ 0.054$ $t = 1$ day
Find A.
Note: 24 hours is converted to 1 day because the growth rate was expressed in terms of a given percentage *per day*. Thus,
$A = 500e^{0.054(1)}$
$A = 527.74$
Therefore, there will be about 528 bacteria after twenty-four hours.

Real-Problems involving Exponential Equation and Inequalities

Exponential equations and inequalities are equations and inequalities in which one (or both) sides involve a variable exponent. They are useful in situations involving repeated multiplication, especially when being compared to a constant value, such as in the case of interest. For instance, exponential inequalities can be used to determine how long it will take to double one's money based on a certain rate of interest.

Example 3:
Suppose that a population of a colony of bacteria increases exponentially. At the start of the experiment, there are 1000 bacteria. And one hour later, the population has increased to 1200 bacteria. How long will it take for the population to reach 5000 bacteria? Round your answer to the nearest hour.
Solution.
Given: $A = 6000$ $P = 1000$ $r = \frac{200}{1000} = 0.2$
Find t.

$6000 = 1000e^{(0.2)t} \rightarrow$ (This is an exponential equation)
$\frac{6000}{1000} = \frac{1000e^{(0.2)t}}{1000}$ Multiplication Property of Equality
$6 = e^{(0.2)t}$
$\ln 6 = ln\, e^{(0.2)t}$ Changing exponential to logarithm
$\ln 6 = 0.2t$ Property of logarithm
$t = \frac{\ln 6}{0.2}$
$t = 8.96$
Therefore, it will take 8.96 hours to reach 5000 bacteria.

Example 4:

Michael owns ₱15,000.00 and he wants to invest his money into an account that will double his money. He is thinking of a financial institution that can make his dream come true. He is considering to invest his money in a lending company which offers a 15% interest compounded quarterly. For how long, will he invest his money in that company to earn at least twice as much as he has now?

Given: $A \geq 2(15000)$ (to earn at least twice as much as he has now)

$P = 15000$

$r = 15\%\ or\ 0.15$

$n = 4$

Find t. Why?

$2(15000) \geq 15000(1 + \frac{0.15}{4})^{4t} \rightarrow$ (This is an exponential inequality)

$3000015000 \geq (1 + 0.0375)^{4t}$ Simplify

$\frac{30000}{15000} \geq \frac{15000(1.0375)^{4t}}{15000}$ Multiplication Property of Equality

$2 \geq (1.0375)^{4t}$

$4t \geq \log_{1.0375} 2$ Changing exponential to the logarithm

$\frac{\log 2}{\log 1.0375} = 18.83$ Change-of-base formula $log_b x = \frac{\log x}{\log b}$

$4t \geq 18.83$ Substitution

$t \geq 4.71$ Multiplication Property of Equality

Therefore, after at least 4.71 years Michael's money will be ₱30,000.0

What's More

Analyze the given problem and then answer the questions that follow:

Activity 1.1

"I – Predict Mo"

In 2015, a certain municipality in Quezon Province has a population of 45, 300. Each year, the population increases at a rate of about 5%.

a. What is the growth factor of the municipality?
b. Determine an equation to represent the problem?
c. What is the population of the municipality in 2020, use the equation in letter (b)?
d. If the population continues to increase at the same rate, what is the population in 2025?

Activity 1.2

"Let's Invest!"

1. Jeanelle has ₱5,000.00 in a bank which is below her required maintaining balance. As a penalty, her money decreases at the rate of 5% every month. How much money will she have after 1 year?

2. If ₱20,000.00 is invested at 10% compounded quarterly, when will the amount of investment be tripled?
3. Mr. Jolo deposited an amount of ₱20,000.00 in a bank that gives 3% annual interest compounded monthly. How much money will he have in the bank after 4 years?

What I Have Learned

Reflect and answer the following:
1. What are the common application of exponential functions, equations and inequalities to real-life situations?
2. In solving real-life problems involving exponential functions, equations and inequalities, what do you think are the important skills that you should have to solve the problems?
3. Enumerate the different steps that you should consider solving real-life problems involving exponential functions, equations and inequalities.

What I Can Do

Exponential Decay

Some things "decay" (get smaller) exponentially.

Example: Atmospheric pressure (the pressure of air around you) decreases as you go higher. It decreases about 12% for every 1000 m: an **exponential decay**. The pressure at sea level is about 1013 hPa (depending on weather).
Note: hPa stands for hectopascal (100 x 1 pascal) pressure
If the model that represent the situation is $y(t) = ae^{rh}$
where: a =(the pressure at sea level =1013 hPa)
h = is in meters (distance, not time, but the formula still works)
$y(t) = y(1000)$ (It is a 12% reduction on 1013 hPa = 891 hPa)

a. Find the pressure on the roof of Grand Hyatt Hotel Manila (one of the tallest buildings in the Philippines if its height is 1,043 feet.
b. Find the pressure on the top of Mount Pulag which is 2,922 meters tall.

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following depicts the increase in number or size at a constantly growing rate?
   a. Half-life c. Exponential decay
   b. Exponential growth d. Time elapsed
2. Which of the following statements is best modeled by exponential growth?
   a. The cost of pencils as a function of the number of pencils.
   b. The distance when a stone is dropped as a function of time.
   c. The distance of a swinging pendulum bob from the center as a function of time.
   d. The compound interest of an amount as a function of time.
3. Lino invested ₱5,000.00 into an account that has a 5.5% annual increasing rate. What equation best describes this investment after ***t*** years?
   a. $A = 5000\ (0.055)^t$ c. $A = 5000\ (1.55)^t$
   b. $A = 5000\ (1.055)^t$ d. $A = 5000\ (5.5)^t$
4. If the population in 1995 of Barangay Manggahan is 1,500, and is increasing at a rate of 2.3% every 5 years, what is the projected population of the town in 2025?
   a. 2,967 b. 1,681 c. 1,722 d. 1,759

For numbers 5-6, refer to the following:
Ms. Juana Care plans to invest her ₱1,000,000.00 in a company that offers 8% interest compounded annually.

5. Define an exponential model for this situation.
   a. $A = 1000000(1.08)^t$ c. $A = 1000000(1.08)^{t+1}$
   b. $A = 1,000,000(1.08)(t)$ d. $A = 1000000(1.08)(t)+1$
6. How much is the investment after 5 years.
   a. ₱5,400,000 b. ₱5,400,001 c.₱1,586,874.32 d. ₱1,469,328.08
7. The half-life of Zn-71 is 4.25 minutes. At t = 0, there were y_0 grams of Zn-71, but only 1/64 of this amount remains after some time. How much time has passed?
   a. t = 20.5 b. t = 19.5 c. t = 21. d. t = 25.5

8. Which of the following situations ***does not*** describes an exponential decay?
   a. The number of rabbits doubles every month.
   b. The amount of substance decreases every 10 minutes.
   c. The atmospheric pressure decreases as you go higher.
   d. The value of a car depreciates every year.

9. A photocopier is purchased for ₱15,200.00 and depreciates in value by 15% per year. Which equation best describes the value of the photocopier in ***x*** years?
   a. $y= 15200\ (0.15)^x$
   b. $y= 15200\ (0.85)^x$
   c. $y= 15200\ (1.15)^x$
   d. $y= 15200\ (1.85)^x$

10. Suppose ₱4000.00 is invested at 6% interest compounded annually. How much money will there be in the bank at the end of 5 years?
   a. ₱5,352.90
   b. ₱5,325.90
   c. ₱5,253.90
   d. ₱5,235.90

11. In 2012 the population of schoolchildren in a city was 90,000. This population increases at a rate of 5% each year. What will be the population of school children in year 2022?
   a. 148,385 school children.
   b. 150, 625 school children
   c. 165, 373 school children
   d. 190, 428 school children

For items 12-13, refer to the following:
The population of Lucena City is estimated to increase by 1.49% per year. According to 2015 census, the population of the city is 266, 248.

12. Which of the following best modelled the situation?
   a. $A = 266248e^{(0.0149)(t)}$
   b. $A = 266248(1.0149)^t$
   c. $A = 266248(\frac{1.0149}{n})^t$
   d. $A = 266248e^{(1.0149)(t)}$

13. What will be the population ten years from now?
   a. 309,027 b. 332,929 c. 350,456 d. 402, 123

14. Joana earned ₱1500 last summer. If she deposited the money in a bank account that earns 5% interest compounded yearly, how much money will he have after five years?
   a. ₱2,015.35 b. ₱1,914.42 c. ₱1,846.48 d. ₱3,560.15

15. If the population of a town doubles in 30 years, when will it be quadruple?
   a. in 45 years b. in 60 years c. in 90 years d. in 100 years

Additional Activities

Solve the following problems.

1. How much money will you have after 5 years, if you invest ₱2,000.00 at the rate of 2% compounded monthly?

2. At the start of the experiment, there are 400 bacteria. If the bacteria follow an exponential growth pattern with $r = 0.03$, what will be the population after 6 hours? How long will it take for the population to double?

3. Consider a population of bacteria that grows according to the function $f(t) = 500e^{0.05t}$, where t is measured in minutes. How many bacteria are present in the population after 4 hours?

Answer Key

What I Know

1. C
2. D
3. C
4. A
5. C
6. A
7. C
8. A
9. B
10. A
11. D
12. D
13. A
14. B
15. C

What's More

ACT 1.1: a. 5%

b. $P = 45300e^{0.05t}$

c. 58,167

d. 74,688

ACT 1.2: a. times 3

b. $y = y_0(3)^{\frac{t}{3}}$

c. 31,749

Activity 1.3

1. ₱2701.80

2. The amount will be tripled in 12 years

3. The money will be ₱22,546.56 in 4 years.

Assessment

1. B
2. D
3. C
4. C
5. A
6. D
7. D
8. A
9. B
10. A
11. A
12. A
13. B
14. B
15. A

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11 General Mathematics

General Mathematics
Representing Real-Life Situations Using Logarithmic Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Representing Real-Life Situations Using Logarithmic Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Representing Real-Life Situations Using Logarithmic Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Representing Real-Life Situations Using Logarithmic Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you represent logarithmic functions using real-life situations. Likewise, this module will give you the idea of how the exponential function and logarithmic function are related to each other. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

1. define logarithmic functions;
2. transform exponential function to logarithmic function or vice versa;
3. evaluate logarithmic expression; and
4. represent real-life situations using logarithmic functions.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What do you call to a function of the form $f(x) = log_b x$ where $b > 0$ and $b \neq 1$?
   a. inverse function
   b. rational function
   c. exponential function
   d. logarithmic function
2. In the expression $log_a x = y$, which is the base?
   a. log
   b. a
   c. x
   d. y
3. What is the logarithmic form of $5^3 = 125$?
   a. $\log_5 3 = 125$
   b. $\log_3 5 = 123$
   c. $\log_5 125 = 3$
   d. $\log_3 125 = 5$

4. Which of the following is the exponential form of $\log_{0.1} 10000 = -4$?
   a. $(0.1)^{-4} = 10000$
   b. $(-4)^{0.1} = 10000$
   c. $(10000)^{-4} = 0.1$
   d. $(0.1)^{10000} = -4$

5. What is the value of $\log_2 128$?
   a. 4
   b. 5
   c. 6
   d. 7

6. Which of the following is the logarithmic form of $a^b = c$?
   a. $\log_a b = c$
   b. $\log_b c = a$
   c. $\log_a c = b$
   d. $\log_c b = a$

7. Find the exponential form of $\log_m (x + y) = n$
   a. $m^n = x + y$
   b. $n^m = x + y$
   c. $(x + y)^m = n$
   d. $m^{(x+y)} = n$

8. Which of the following is equal to $\log_{100} 10$?
   a. 1/2
   b. 1/10
   c. 1
   d. 2

9. Find the value of $\log_5 \left(\frac{1}{25}\right)$?
   a. -5
   b. -3
   c. 3
   d. 5

10. Evaluate: $\log_2 16 + \log_7 49$
   a. 2
   b. 4
   c. 6
   d. 8

11. Find the exact value of$\log_4 64 + \log_2 32 - \log_3 27$.
   a. 3
   b. 4
   c. 5
   d. 6

12. What is the value of $2\log_3 81$?
    a. 8
    b. 12
    c. 16
    d. 20

13. What is the magnitude in the Richter Scale of an earthquake that released 10^{10} joules of energy?
    a. 2.4
    b. 3.7
    c. 4.5
    d. 5.3

14. A solution contains hydrogen ion concentration of $1x10^{-7}$ moles. Calculate its pH value.
    a. 10
    b. 9
    c. 8
    d. 7

15. The intensity of sound in a certain forest is 10^{-8} watts/m^2. What is the corresponding sound intensity in decibels?
    a. 4
    b. 5
    c. 6
    d. 7

Lesson 1 Representing Real-life Situations Using Logarithmic Functions

You have learned in previous modules that polynomial function, piece-wise function, rational and exponential functions can be used to model real-life situations. The logarithmic function is just one of them. Since the logarithmic function is the inverse of the exponential function, most of the real-life problems involving exponential functions can also be solved by logarithmic functions.

This module will help you to represent logarithmic function to real-life situations like finding the magnitude of an earthquake in a Richter scale, the intensity of a sound in decibel, the acidity or the alkalinity of a solution, and a lot more.

What's In

For you to begin, let us recall some important concepts and skills from the previous lessons which are needed to understand the logarithmic function. In the previous module, you learned that exponential equations are equations involving exponential expressions like $4^{x-2} = 32$, $49^x = 7$, and $5^{2x} = \frac{1}{25}$. You also learned that exponential inequalities are inequalities involving exponential expressions like $3^x \leq 27, 10^{2x+1} = \frac{1}{1000}$, and $32^{x-1} = 128$. Moreover, you learned that exponential functions are functions of the form $f(x) = b^x$ where $b > 0$ and $b \neq 1$ and it can be used to model exponential growth and decay, the half-life of a substance, and the compound interests.

The example below shows how exponential function is use to represent a real-life situation.

Example

Laboratory findings show that the SARS-causing corona virus, upon reaching maturity, divides itself into two after two hours. How many cells of the virus will be present after 1 day if it started with just one cell?

Solution:

Let t = number of hours elapsed
f(t) = number of corona virus present after t hour elapsed

t	0	1	2	3	4	5	6
f(t)	$1=2^0$	$2=2^1$	$4=2^2$	$8=2^3$	$16=2^4$	$32=2^5$	$64=2^6$

The table shows a pattern: as t increases by 1, f(t) increases rapidly by 2^t. In symbols, $f(t) = 2^t$.

Hence, if t = 24 hours (1 day), $f(24) = 2^{24}$ = **16, 777, 216.**

In a matter of 1 day, a virus that started as a single cell can increase to millions of cells each of which has the same ability to reproduce exponentially.

Notes to the Teacher

Advise the students that they may use calculator in solving for bigger numbers because it saves time. But at first, it is important to grasp the concepts and understand the computation on a manual basis. Use of calculator is just an aid to make their work easier.

Similarly, teach the students on how to read mathematical symbols in a proper way.

What's New

Investigate!

Investigate and discover the transformation of exponential equations to logarithmic equations.

Exponential Equation	Logarithmic Equation
$2^2 = 4$ $\longrightarrow$	$\log_2 4 = 2$
$3^4 = 81$ $\longrightarrow$	$\log_3 81 = 4$
$9^{\frac{1}{2}} = 3$ $\longrightarrow$	$\log_9 3 = \frac{1}{2}$
$n^3 = 64$ $\longrightarrow$	$\log_n 64 = 3$
$4^a = 2$ $\longrightarrow$	$\log_4 2 = \frac{1}{2}$
$a^x = y$ $\longrightarrow$	$\log_a y = x$

1. What have you noticed in the transformation from exponential equation to logarithmic equation?

   _____

   _____

   _____

2. What happened to the exponent in the exponential form upon changing it to logarithmic form?

   _____

   _____

   _____

3. Give three examples exponential equations and its equivalent logarithmic form.

   _____

   _____

   _____

What is It

There are exponential equations that are not easy to solve. For instance, the equation

$$2^x = 3$$

cannot be easily solved but for sure, it has a solution. Since $2^1 < 3 < 2^2$, therefore, $1 < x < 2$. The solution to $2^x = 3$ can be written as $x = \log_2 3$. This is read as "x is equal to the logarithm of 3 to the base 2." This suggests that $2^x = 3$ is equivalent to $x = \log_2 3$.

From the activity earlier, you noticed the transformation of exponential equations to logarithmic equations. The said activity leads to the description of the logarithm as follows.

The Logarithm of a Number

> Let a, b and c be positive real numbers such that $b > 0$ and $b \neq 1$. The logarithm of a with base b is denoted by $\log_b a$, and is defined as $c = log_b a$ if and only if $a = b^c$.

Note:

1. Logarithmic functions and exponential functions are inverses.
2. In logarithmic form $\log_b a$, b cannot be negative.
3. The value of $\log_b a$, can be negative.

Examples 1. Rewrite the following exponential equations in logarithmic form whenever possible.

a. $7^2 = 49$

b. $27^{\left(\frac{1}{3}\right)} = 3$

c. $(m-2)^3 = x$

d. $e^x = 3$

e. $\left(\frac{1}{2}\right)^{-2} = 4$

f. $\left(\sqrt{7}\right)^2 = 7$

g. $\left(\frac{5}{2}\right)^{-2} = \frac{4}{25}$

h. $81^{\frac{1}{2}} = 9$

Solutions:

a. $7^2 = 49 \Longrightarrow \log_7 49 = 2$

b. $27^{\left(\frac{1}{3}\right)} = 3 \Longrightarrow \log_{27} 3 = \frac{1}{3}$

c. $(m-2)^3 = x \Longrightarrow \log_{(m-2)} x = 3$

d. $e^x = 3 \Longrightarrow \ln 3 = x$

e. $\left(\frac{1}{2}\right)^{-2} = 4 \Longrightarrow \log_{\frac{1}{2}} 4 = -2$

f. $\left(\sqrt{7}\right)^2 = 7 \Longrightarrow \log_{\sqrt{7}} 7 = 2$

g. $\left(\frac{5}{2}\right)^{-2} = \frac{4}{25} \Longrightarrow \log_{\frac{5}{2}} \frac{4}{25} = -2$

h. $81^{\frac{1}{2}} = 9 \Longrightarrow \log_{81} 9 = \frac{1}{2}$

Examples 2. Rewrite the following logarithmic equations in exponential forms whenever possible.

a. $\log_3 81 = 4$

b. $\log_x mn = p$

c. $\log 5 = m$

d. $\log_2 \frac{1}{16} = -4$

e. $\log 0.00001 = -5$

f. $\ln 7 = a$

g. $\log_{169} 13 = \frac{1}{2}$

h. $\log_3 3 = 1$

Solutions:

a. $\log_3 81 = 4 \Longrightarrow 3^4 = 81$

b. $\log_x mn = p \Longrightarrow x^p = mn$

c. $\log 5 = m \Longrightarrow 10^m = 5$

d. $\log_2 \frac{1}{16} = -4 \Longrightarrow 2^{-4} = \frac{1}{16}$

e. $\log 0.00001 \Longrightarrow 10^{-5} = 0.00001$

f. $\ln 7 = a \Longrightarrow e^a = 7$

g. $\log_{169} 13 = \frac{1}{2} \Longrightarrow 169^{\frac{1}{2}} = 13$

h. $\log_3 3 = 1 \Longrightarrow 3^1 = 3$

Note: If the base is not written, it is understood to be in the base 10

The next examples illustrate how to evaluate logarithms.

Examples 3. Find the value of each logarithm.

a. $\log_2 64$
b. $\log_4 256$
c. $\log_{(\frac{1}{2})} 32$
d. $\log_{\frac{1}{9}} 3$
e. $\log_3 81$
f. $\log 1000$
g. $\log \frac{1}{1000}$
h. $\log_{0.5} 16$

Solution:

a. $\log_2 64$ What should be the exponent of 2 to get 64? Since $2^6 = 64$, then, $\log_2 64 = 6$.

b. $\log_4 256$ What should be the exponent of 4 to get 256? Since $4^4 = 256$, then, $\log_4 256 = 4$.

c. $\log_{(\frac{1}{2})} 32$ What should be the exponent of ½ to get 32? Since $\left(\frac{1}{2}\right)^{-5} = 32$ then, $\log_{(\frac{1}{2})} 32 = -5$.

d. $\log_{\frac{1}{9}} 3$ What should be the exponent of $\frac{1}{9}$ to get 3?" Since $\frac{1}{9}^{-\frac{1}{2}} = 3$, then, $\log_{\frac{1}{9}} 3 = -\frac{1}{2}$.

e. $\log_3 81$ What should be the exponent of 3 to get 81? Since $3^4 = 81$, then, $\log_3 81 = 4$.

f. $\log 1000$ What should be the exponent of 10 to get 1000? Since $10^3 = 1000$, then, $log\, 1000 = 3$.

g. $\log \frac{1}{1000}$ What should be the exponent of 10 to get $\frac{1}{1000}$? Since $10^{-3} = \frac{1}{1000}$, then, $\log \frac{1}{1000} = -3$.

h. $\log_{0.5} 16$ What should be the exponent of 0.5 to get 16? Since $0.5^{-4} = 16$, then, $\log_{0.5} 16 = -4$.

From the brief discussion of finding the value of each logarithm, I think you are now ready to represent logarithmic functions to real-life situations.

Here are some of the real-life applications of logarithms.

Richter Scale

The Richter magnitude scale was developed in 1935 by Charles F. Richter of the California Institute of Technology as a mathematical device to compare the size of earthquakes. The magnitude of an earthquake is determined from the logarithm of the amplitude of waves recorded by seismographs.

The magnitude R of an earthquake is given by

$$\boldsymbol{R = \frac{2}{3} log \frac{E}{10^{4.40}}}$$

where E (in joules is the energy released by the earthquake (the quantity $10^{4.40}$ joules is the energy released by a very small reference earthquake).

The formula indicates that the magnitude of an earthquake is based on the logarithm of the ration between the energy it releases and the energy released by a reference earthquake.

Example 1

Suppose that an earthquake released approximately 10^8 joules of energy. (a) What is the magnitude on a Richter scale? (b) How much more energy does this earthquake release than the reference earthquake?

Solution:

(a) Since $E = 10^8, R = \frac{2}{3} log \frac{10^8}{10^{4.40}}$

$$R = \frac{2}{3} log\ 10^{3.6}$$

By, definition $\boldsymbol{log\ 10^{3.6}}$ is the exponent by which 10 must be raised to obtain $10^{3.6}$, so $log\ 10^{3.6} = 3.6$.

Thus $R = \frac{2}{3} log\ 10^{3.6} \approx 2.4$

(b) This earthquake releases $\frac{10^8}{10^{4.40}} = 10^{3.6} \approx 3981$ times more energy than the reference earthquake.

Sound Intensity in Decibel

The loudness of a sound is expressed as a ratio comparing the sound to the least audible sound. The range of energy from the lowest sound that can be heard to a sound so loud that is produces pain rather than the sensation of hearing is so large that an exponential scale is used. The lowest possible sound that can be heard is called the threshold of hearing.

In acoustics, the decibel (dB) level of a sound is

$$\boldsymbol{D = 10 \log \frac{l}{10^{-12}}}$$

where l is the sound intensity in watts$/m^2$ (the quantity m^{-12} watts$/m^2$ is the least audible sound a human can hear)

Example 2

The intensity of sound of a lawn mower is 10^{-3} watts$/m^2$. (a) What is the corresponding sound intensity in decibels? (b) How much more intense is this sound than the least audible sound a human can hear?

Solution:

(a) Since $l = 10^{-3}$ *then* $D = 10\log\frac{10^{-3}}{10^{-12}}$

$D = 10\log 10^9$

By, definition $\boldsymbol{log\ 10^9}$ is the exponent by which 10 must be raised to obtain 10^9, so $log\ 10^9 = 9$

$D = 10(9)$

$D = 90\ decibels$

(b) This sound is $\frac{10^{-3}}{10^{-12}} = 10^9 = 1{,}000{,}000{,}000$ times more intense than the least audible sound a human can hear

pH Scale

Acidic and basic are two extremes that describe a chemical property. Mixing acids and bases can cancel out or neutralize their extreme effects. A substance that is neither acidic nor basic is neutral.

The pH scale measures how acidic or basic a substance is. The pH scale ranges from 0 to 14. A pH of 7 is neutral. A pH less than 7 is acidic. A pH greater than 7 is basic. The pH level of a water-based solution is defined as

$$\boldsymbol{pH = -log[H^+]}$$

where $[H^+]$ is the concentration of hydrogen ions in moles per liter.

Example 3

A 1-liter solution contains 0.01 moles of hydrogen ions. Determine and describe its pH level.

Solution:

Since there are 0.01 moles of hydroegen ions in 1 liter, then the concentration of hydrogen ions is 10^{-2} moles per liter. The pH level is $-\log 10^{-2}$. By, definition $\boldsymbol{log\ 10^{-2}}$ is the exponent by which 10 must be raised to obtain 10^{-2}, so $log\ 10^{-2} = -2$,

So, $pH = -(-2) = 2$, therefore, the pH level is 2

Since the pH level is 2, then it is acidic.

The application of logarithmic function will further discuss on the lesson solving real-life problems involving logarithmic functions, equations and inequalities.

What's More

Activity 1.1

Transform the following logarithmic expression to exponential form or vice versa.

1. $9^2 = 81$ _____
2. $125^{1/3} = 5$ _____
3. $m^n = p$ _____
4. $(x - 1)^2 = 12$ _____
5. $m^{-3} = 1/27$ _____
6. $\log_4 16 = 2$ _____
7. $\log_m x = 10$ _____
8. $\log_5 (a - b) = 0$ _____
9. $\log x = 2$ _____
10. $\log 1 = 0$ _____

Activity 1.2.

Evaluate the following.

1. $\log_8 64$ _____
2. $\log_5 625$ _____
3. $\log_3 81$ _____
4. $\log 0.0001$ _____
5. $\log_9 \frac{1}{729}$ _____
6. $\log_{\frac{1}{2}} \frac{1}{32}$ _____
7. $\ln 5$ _____
8. $\log_3 1$ _____
9. $\log_5 \frac{1}{25}$ _____
10. $\log_{\frac{3}{2}} \frac{27}{8}$ _____

Activity 1.3.

Solve the following problems.

1. A 1-liter solution contains 0.0000001 moles of hydrogen ions. Find its pH level.

2. Suppose that an earthquake released approximately 10^6 joules of energy, what is the magnitude on a Richter scale? How much more energy does this earthquake release than the reference earthquake?

3. The intensity of sound of a background noise at a restaurant is 10^{-6} watts/m2. What is the corresponding sound intensity in decibels? (b) How much more intense is this sound than the least audible sound a human can hear?

4. The July 16, 1990 earthquake in Baguio City killed more than 2000 people. What is the magnitude in the Richter Scale if it releases approximately 10^{16} joules of energy? How much more energy does this earthquake release than the reference earthquake?

What I Have Learned

Complete the following statements by writing the correct word or words and formulas.

1. Logarithm is the inverse of _____.
2. The logarithm of a with base b is denoted by _____ , and is defined as $c = \log_b a$ if and only if_____.
3. In logarithmic form $\log_b a$,the value of b cannot be _____.
4. The value of $\log_b a$ can be _____.
5. Let a, b, and c be _____ real numbers such that b ≠ 1. The logarithm of a with base b is denoted by _____, and is defined as _____ if and only if $a = b^c$
6. Logarithmic functions and exponential functions are _____.
7. In logarithmic form $\log_b a$, b cannot be _____.
8. The base in the given logarithmic expression $\log_3 5$ is _____.
9. If the base is not written in the logarithmic expression, then it is understood to be _____.
10. From the given $\log_7 343$, it is the same as asking "What will be the exponent of _____ to get _____? Since 7— = 343Therefore, $\log_7 343$ = _____, then, $\log_7 343 =$ ___.

What I Can Do

What to do before, during and after the earthquake?

A brochure is an informative paper document (often used for advertising) that can be folded into pamphlet, or leaflet. Brochures are promotional documents, primarily used to introduce a company, organization, products, or services and inform potential customers or members of the public of the benefits.

Task:

Make a brochure informing the public regarding different tips on what to do before, during and after the earthquake. You may use the following guidelines in creating your brochure.

1. Determine your purpose. Go straight to the point.
2. Know your brochure folds.
3. Be creative. Be unique.
4. Limit your font choices into just three. Avoid big words.
5. Use high-quality paper. Choose the right colors.
6. Add appropriate images.
7. Make the brochure worth keeping.
8. The content must focus on what to do before, during, and after an earthquake.

Please include the description of earthquake effects in different magnitude levels in the Richter Scale.

Name of the Project:

_____

Brief Description:

_____

_____

_____

_____

_____

_____

Rubrics for the task:

	4	3	2	1
Organization	The brochure has excellent formatting and very well-organized information	The brochure has appropriate formatting and well-organized information	The brochure has some organized information with random formatting	The brochure's format and organization of material are confusing to the reader.
Ideas	The brochure communicates relevant information appropriately and effectively to the intended audience.	The brochure communicates relevant information appropriately to the intended audience.	The brochure communicates irrelevant information or communicates inappropriately to the intended audience.	The brochure communicates irrelevant information, and communicates inappropriately to the intended audience.
Conventions	All of the writing is done in a complete sentence. Capitalization and punctuation are correct throughout the brochure.	Most of the writing is done in a complete sentence. Most of the capitalization and punctuation are correct throughout the brochure.	Some of the writing is done in a complete sentence. Some of the capitalization and punctuation are correct throughout the brochure.	Most of the writing is not done in a complete sentence. Most of the capitalization and punctuation are not correct throughout the brochure.
Graphics	The graphics go well with the text and there is a good mix of text and graphics.	The graphics go well with the text, but there are so many that they distract from the text.	The graphs go well with the text, but there are too few	The graphics do not go with the accompanying text appears to be randomly chosen.

Assessment

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. It is a function defined as $y = \log_b a$ if and only if $b^y = a$ where a, b, and c are positive real numbers and b is not equal to 1.
   a. exponential function
   b. inverse function
   c. logarithmic function
   d. rational function

2. Which is the base of the given logarithm $\log_p m = n$?
   a. log
   b. m
   c. n
   d. p

3. What is the logarithmic form of $10^3 = 1000$?
   a. $\log 3 = 1000$
   b. $\log 1000 = 3$
   c. $\log_3 1000 = 10$
   d. $\log_3 100 = 1000$

4. Which of the following is the exponential form of $\log_{0.5} 4 = -2$?
   a. $(0.5)^{-2} = 4$
   b. $(-2)^{0.5} = 4$
   c. $(4)^{-2} = 0.5$
   d. $(0.5)^4 = -2$

5. Find the value of $\log_4 \frac{1}{16}$.
   a. -2
   b. -1/2
   c. 1/2
   d. 2

6. Which of the following is the logarithmic form of $x^a = y$?
   a. $\log_x a = y$
   b. $\log_y x = a$
   c. $\log_x y = a$
   d. $\log_a x = y$

7. Find the exponential form of $\log_a d = b + c$.
   a. $(b+c)^a = d$
   b. $a^{b+c} = d$
   c. $(b+c)^d = a$
   d. $d^a = b+c$

8. Which of the following is the correct value of $\log_9 27$?
   a. 3/2
   b. 2/3
   c. -2/3
   d. -3/2

9. Find the value of $\log_3 \frac{1}{243}$.
   a. 7
   b. 5
   c. -5
   d. -7

10. Evaluate: $\log_3 27 + \log_9 729$.

a. 2
b. 4
c. 6
d. 8

11. Find the exact value of $\log_2 64 + \log_3 9 - \log_5 625$.

a. 3
b. 4
c. 5
d. 6

12. What is the value of $3\log_8 512$?

a. 7
b. 8
c. 9
d. 10

13. What is the magnitude in the Richter Scale of an earthquake that released 10^{14} joules of energy?

a. 3.6
b. 5.1
c. 6.4
d. 7.2

14. The decibel level of sound in a certain forest is 10^{-7} watts/m^2. What is the corresponding sound intensity in decibels?

a. 4
b. 5
c. 6
d. 7

15. A solution contains hydrogen ion concentration of 1×10^{-11} moles per liter. Calculate its pH value.

a. 11
b. 10
c. 9
d. 8

Additional Activities

If you want to try more, these activities are for you. It will help you to practice your skill in solving real-life problems involving logarithmic expression. Study and analyze each situation to solve the problem.

1. The magnitude M of an earthquake is a function of energy E measured in ergs. Richter and Gutenberg developed the so-called Richter scale and the formula for the magnitude is given earlier in this module. In 2013, a 7.2 magnitude earthquake hit Central Visayas and killed more than 150 people, destroyed century-old churches, and affected more than 3 million families. What is the amount of energy released by this earthquake?

2. To measure the brightness of a star from earth, the brightness of the star Vega is used as a reference, and is assigned a relative intensity $l_0 = 1$. The magnitude m of any given star is defined by $m = 2.5 \log l$, where l is the relative intensity of that star. (a) What is the magnitude of Vega? (b) Suppose that light arriving from another star has a relative intensity of 2.4. What is the magnitude of this star?

Answer Key

What I Know

1. C
2. D
3. B
4. A
5. A
6. C
7. B
8. A
9. D
10. C
11. B
12. C
13. B
14. B
15. A

What's More

Activity 1.1

1. $\log_9 81 = 2$
2. $\log_{125} 5 = 1/3$
3. $\log_m p = n$
4. $\log_{x-1} 12 = 2$
5. $\log_m 1/27 = -3$
6. $4^2 = 16$
7. $m^{10} = x$
8. $5^0 = a - b$
9. $10^2 = x$
10. $10^0 = 1$

Activity 1.2

1. 2
2. 4
3. 4
4. -4
5. -3
6. 5
7. 1.609
8. 0
9. -2
3

Activity 1.3

1. 7, neutral
2. 1.07 magnitude
3. 60 decibels
4. 7.7 magnitude

Assessment

1. D
2. B
3. C
4. A
5. D
6. A
7. A
8. A
9. B
10. C
11. C
12. A
13. C
14. D
15. A

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pH Scale Introduction and definition. http://chemistry.elmhurst.edu/vchembook/184ph.html

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11 General Mathematics

General Mathematics
Solving Logarithmic Equations, and Inequalities
First Edition, 2020

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Published by the Department of Education
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General Mathematics

Solving Logarithmic Equations, and Inequalities

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Logarithmic Equations, and Inequalities!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Logarithmic Equations, and Inequalities!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you distinguish logarithmic function, logarithmic equation, and logarithmic inequality. Furthermore, it is made for you to master solving logarithmic equations and logarithmic inequalities. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains two lessons:

- Lesson 1 – Logarithmic Functions, Equations and Inequalities
- Lesson 2 – Solving Logarithmic Equations and Inequalities

After going through this module, you are expected to:

1. distinguishes logarithmic function, logarithmic equation, and logarithmic inequality;
2. apply basic properties of logarithms and laws of logarithms; and
3. solves logarithmic equations and inequalities.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. It is an equation involving logarithms which can be solved for any values that satisfy the mathematical sentence.
   a. logarithmic equation c. logarithmic inequality
   b. logarithmic function d. none of these
2. It is a function of the form $f(x) = b^x$ where $b > 0$ *and* $b \neq 1$. It can be represented by a table of values, equations and graph.
   a. logarithmic equation c. logarithmic inequality
   b. logarithmic function d. logarithmic expression
3. The following symbols can be expressed in the logarithmic inequality EXCEPT:
   a. < c. >
   b. ≥ d. =
4. Which of the following is an example of a logarithmic inequality?
   a. $\log_2 128 = \log_2 x$ c. $\log_3 x < 4$
   b. $y = \log_2 2x$ d. $(0.1)1000 = -4$
5. The following is an example of logarithmic equation EXCEPT:
   a. $\log_2 4 = \log_2 x$
   b. $\log_2 128$
   c. $\log_3 x + 3 = 2$
   d. $\log_5 x = \log_5 2(x-4) + 2$
6. Which of the following is an example of a logarithmic function?
   a. $\log_3 27 = \log_3 x$
   b. $\log_2 x = \log_2 2$
   c. $\log_3 x < 10$
   d. $y = \log_2 2x - 1$
7. What is the value of $5^{\log_5 10}$?
   a. 5 b. 10 c. 15 d. 50
8. Give the value of $\log_7 7 + \log_5 1$.
   a. 0 b. 1 c. 9 d. 10

9. What is the expanded form of $\log_2 3xy^2$?

a. $\log_2 3+\log_2 x+2\log_2 y$

b. $\log_2 3x+2\log_2 y$

c. $\log_2 3+\log_2 x+\log_2 y^2$

d. $\log_2 3x + \log_2 y^2 \log_2 3x+\log_2 y^2$

10. What is the expanded form of $\log_4 \frac{y}{z}$?

a. $\log_4 y+\log_4 z$

b. $\log_4 y-\log_4 z$

c. $\log_4 z+\log_4 y$

d. $\log_4 z-\log_4 y$

11. Express $3\log_5 x-\log_5 y$ as a single logarithm.

a. $\log_5 \frac{y^3}{z}$

b. $\log_5 \frac{3x}{y}$

c. $\log_5 x^3 y$

d. $\log_5 \frac{x^3}{y}$

12. What is the value of n is the logarithmic equation: $\log_2 n=\log_2 2n+6$?

a. 6 b. -6 c. 3 d. -3

13. Find x if $\log_8 x-3=\log_8 2$.

a. 2 b. 3 c. 4 d. 5

14. Which of the following satisfies the inequality: $\log_5 25<n$?

a. n ≥ 1 b. n ≤ 3 c. n > 2 d. n < 2

15. Solve for n: $\log_{n-3} 36>2$.

a. x > 9

b. x > 6

c. x > 3

d. x > -3

Lesson 1 Logarithmic Functions, Equations and Inequalities

Functions, equations and inequalities are some of the most important terms in Algebra. These separate a single concept into three different ideas. Such that we cannot say that equation is the same as a function or a function the same as inequalities and so on. We will now focus on the three different ideas of the logarithm. If you are excited to uncover it, then this lesson will be perfect for you!

What's In

For you to begin let us recall the definition of a logarithm. The logarithm of a with base b is denoted by $\log_b a$, and is defined as $c = \log_b a$ if and only if $a = b^c$. Previously, you learned that exponential and logarithm are the inverses of each other. So, you can transform exponential form to logarithmic form.

Now, look at the following examples. How many can you identify as logarithmic functions? Name all the functions according to their types.

a. $f(x) = 3x - 5$

b. $g(x) = \log_3 \frac{3x}{4}$

c. $h(x) = \log_3 2x - 4$

d. $i(x) = 2x^2 - 3x + 5$

e. $j(x) = \frac{2x-1}{3x+1}$

f. $k(x) = 4^x$

g. $l(x) = \begin{cases} 2x+1, x > 1 \\ x, -1 < x < 1 \\ x-1, x < -1 \end{cases}$

h. $y = \log_x 3$

i. $n(x) = \log_{1/2} 5 - 4x$

j. $\log_x 3 + \log_x 5 = y$

From the different functions above, did you see logarithmic functions? How many are they? If your answer is 5, definitely you are correct. Letters b, c, h, i, and j are all examples of logarithmic functions. Letter a is an example of linear function while letter d is a quadratic function. Letters e and f are rational function and exponential function, respectively. Lastly, letter g is an example of a piece-wise function.

Since you are already familiar with the logarithmic function. Let us identify or distinguish the difference among logarithmic functions, logarithmic equations, and logarithmic inequalities.

What's New

Classify Me!

Identify whether the following as logarithmic (A) equation, logarithmic (B) inequality or logarithmic (C) function. Write the letter of the correct answer on the space provided for.

1. $\log_4 3x = 5$ _____
2. $f(x) = \log_2 4x + 1$ _____
3. $8 \le \log_2 3x - 3$ _____
4. $y = \log_5 2x$ _____
5. $\log_7 3x + \log_7 2 = 1$ _____
6. $\log_6 3x > 2$ _____

1. Based on your answers, how will you define logarithmic equation?

_____

_____

2. How can you distinguish a logarithmic equation from a logarithmic inequality?

_____

_____

3. When do you say that the given is a logarithmic function?

_____

_____

4. Can you identify the symbols that will help you determine the difference among logarithmic equation, inequality and function?

_____

_____

What is It

From the first activity, you classify the given expression as to logarithmic equation, function or inequality. The following will give you the definition and examples of the logarithmic function, equation and inequality.

	Definition	**Examples**
Logarithmic Function	It is a function of the form $f(x) = log_b x$, such that $b > 0$ and $b \neq 1$.	$f(x) = \log_3 x - 4$ $y = \log_2(2x + 3)$

Logarithmic Equation	It is an equation involving logarithms.	$\log_2 4 = \log_2 x$ $\log_3 x + 3 = 2$
Logarithmic Inequality	It is an inequality involving logarithms.	$\log_3 x < 4$ $\log_2 x - 5 \geq 4$

Classify the following into logarithmic function, logarithmic equation, logarithmic inequality or neither of the three, then justify your answer.

1. $\log_4 n + 3 = 2$

2. $\log_4 x > 3/2$

3. $f(x) = \log_2 2x - 5$

4. $f(x) = 2^{3x-2}$

Solution*:*

1. $\log_4 n + 3 = 2$ ⟶ This is a logarithmic equation because it is an equation involving logarithms.
2. $\log_4 x > 3/2$ ⟶ This is a logarithmic inequality because it is an inequality involving logarithms.
3. $f(x) = \log_2 2x - 5$ ⟶ This is a logarithmic function because It is a function of the form $f(x) = log_b x$, with $b = 2$.
5. 4. $f(x) = 2^{3x-2}$ ⟶ This is neither a logarithmic function, logarithmic equation, nor logarithmic inequality because it is an exponential function.

What's More

Activity 1.1

Determine whether the given is a logarithmic function, logarithmic equation, logarithmic inequality or neither of the three options.

_____ 1. $f(x) = log_5 2x$
_____ 2. $2log_3 x = log_3 5x - 4$
_____ 3. $y = log_9 2(x - 5)$
_____ 4. $x + y = 7$
_____ 5. $log_4 x \geq 0$
_____ 6. $2log_8(x - 2) < 2$

_____ 7. $log_4 x = log_4 4$
_____ 8. $x^2 - 3x + 6 = 9$
_____ 9. $g(x) = log_8 4x - 1$
_____ 10. $2x - y = 90$

Activity 1.2.

Justify or explain your answer in Activity 1.1.

1. _____.
2. _____.
3. _____.
4. _____.
5. _____.
6. _____.
7. _____.
8. _____.
9. _____.
10. _____

What I Have Learned

A. Complete the following statements by writing the correct word or words and formulas.

1. _____ is an equation involving logarithms.
2. _____ is an inequality involving logarithms.
3. Logarithmic function is a function of the form_____, such that $b > 0$ and $b \neq 1$.
4. The expression $\log_3(x-1) > 2$ is an example of _____.
5. $\log_4(2x+4) = 1$ is an example of _____.
6. The expression $\log_7 \frac{3x}{2} = y$ is a _____.

What I Can Do

Journal Writing

Write a mathematical journal connected to what you have learned about logarithmic functions, equations and inequalities. Try to relate your writing or reflections on our lesson.

> Many things in our life operate like an equation or inequality where certain inputs result outputs. By investing in friendships, we reap happiness. By working hard, we reap success. Can you cite examples in life that appear to unequally related?

Rubrics for the task:

	4	**3**	**2**	**1**
Conventions of Journal Writing (includes the date, references to text or data, and personal thoughts and opinions)	The writer follows the conventions.	The writer follows most of the conventions.	The writer follows some of the conventions.	The writer does not follow any conventions.
Capitalization and Punctuation	The writer makes no mistakes.	The writer makes 1-2 mistakes.	The writer makes 3-4 mistakes.	The writer makes more than 4 mistakes.
Effective Written Communication	The writer communicates thoughts in a clear and organized manner.	The writer understandably communicates thoughts, but the organization could have been better.	The writer communicates in a somewhat organized manner, but ideas were not very clear.	The writer communicates showed no organization or consideration.
Reflection and thoughts	The writer demonstrates a deep understanding of the topic.	The writer demonstrates some understanding of the topic.	The writer demonstrates minimal understanding of the topic.	The writer demonstrates no understanding of the topic.

Additional Activities

If you want to try more, these activities are for you.
An acrostic is a series of lines or verses in which the first, last, or other particular letters when taken in order spell out a word, phrase, etc. Try to create acrostics from the word equation, function, and inequality.

1. E -
Q -
U -
A -
L –

2. F -
U -
N -
C -
T -
I -
O -
N -

3. U -
N -
E -
Q -
U -
A -
L -

Lesson 2 Solving Logarithmic Equations and Inequalities

Equations and inequalities may be true or false. Depending on the context, solving equations and inequalities may consist of finding either any solution, all solutions, or a solution that satisfies further properties. When the task is to find the solution, generally, it determines the possible value or values of the unknown variable. It can be a single value or a set of values. Have you experienced finding the value/s of the unknown from the previous lessons? How about trying to solve logarithmic equations and inequalities? If you are excited to learn about it, then this lesson is for you!

What's In

For you to begin, let us recall once again the previous lessons in this module. Logarithm comes in three forms: the logarithmic function, equation and inequality.

Example 1

Classify the following as logarithmic function, logarithmic equation or logarithmic inequality and explain why.

1. $\log_5 7 < 4x$
2. $f(x) = \log_p(5x+7)$
3. $\log_8(2m-7) = 1$

Solution:

1. $\log_5 7 < 4x$ $\longrightarrow$ This is a logarithmic inequality because it is an inequality involving logarithms.

2. $f(x) = \log_p(5x+7)$ $\longrightarrow$ This is a logarithmic function because it is a function of the form $f(x) = log_b x$ where $b > 0, b \neq 1$.

3. $\log_8(2m-7) = 1$ $\longrightarrow$ This is a logarithmic equation because it is an equation involving logarithms.

Another thing that you need to be successful in lesson 2 are your skills in rewriting the logarithmic form to exponential form, applying the logarithmic properties and laws, and identifying the domain of the equations and inequalities.
To check if you are ready, try to do the next activity.

What's New

Investigate!

Study the table below and provide the information needed to complete the table. Use the calculator to check your answer and explain your answer.

Examples	**Why?**
$\log_3 3 = 1$ $\quad$ $\log_m m = ?$ $\log_{11} 11 = 1$	
$\log_3 1 = 0$ $\quad$ $\log_m 1 = ?$ $\log_9 1 = 0$	
$5^{\log_5 10} = 10$ $\quad$ $m^{\log_m x} = ?$ $2^{\log_2 5} = 5$	
$\log_3 12 = 2.26$ $\log_3 (4)(3) = \log_3 4 + \log_3 3 = 2.26$ $\log_m xy = ?$	
$\log_2 \frac{16}{2} = \log_2 16 - \log_2 2 = 3$ $\log_m \frac{x}{y} = ?$	
$\log_2 2^4 = 4\log_2 2$ $\log_7 7^2 = 2\log_7 7$ $\log_m m^x = ?$	

1. Based on your answers in the second column, what can you say about the activity?
2. Describe the property or rule applied in each item on the table.

What is It

From your What's New activity, you try to apply the properties and laws of logarithms. Now, let us check if we have the same idea.

The next table shows the basic properties and laws of logarithms.

Basic Properties and Laws of Logarithm

Let b, x and y be real numbers such that b > 0 and b ≠ 1, the basic properties and laws of logarithms are as follows:

PROPERTIES	**EXAMPLES**
I. $\log_b b = 1$	$\log_3 3 = 1$ $\log_{11} 11 = 1$
II. $\log_b 1 = 0$	$\log_3 1 = 0$ $\log_9 1 = 0$
III. $b^{\log_b x} = x$	$5^{\log_5 10} = 10$ $2^{\log_2 5} = 5$
IV. $\log_b b^x = x$	$\log_2 2^4 = 4$ $\log_7 7^2 = 2$
LAWS	**EXAMPLES**
Product Law I. $\log_b MN = \log_b M + \log_b N$	$\log_3 6m = \log_3 6 + \log_3 m$ $\log_6 mn = \log_6 m + \log_6 n$
Quotient Law II. $\log_b \frac{M}{N} = \log_b M - \log_b N$	$\log_3 \frac{m}{n} = \log_3 m - \log_3 n$ $\log_2 16 - \log_2 2 = 3$
Power Law III. $\log_b M^n = n \log_b M$	$\log_4 m^3 = 3 \log_4 m$ $\log_2 n^x = x \log_2 n$

Example 2

Now, let us use properties of the logarithms to evaluate the following:

1. $\log_4 4$ = _____
2. $\log_5 5^7$ = _____
3. $\log_8 1$ = _____
4. $17^{\log_7 10}$ = _____
5. $3\log_9 9^3$ = _____

Solution:

1. $\log_4 4$ = 1 (Property I)
2. $\log_5 5^7$ = 7 (Property IV)
3. $\log_8 1$ = 0 (Property II)
4. $7^{\log_7 10}$ = 10 (Property III)
5. $3\log_9 9^3$ = 9 (Property IV)

Example 3

This time, let use the law of logarithms to expand the following expressions

1. $\log_9 ab^2$
2. $\log_b \frac{x^3}{y^2}$

Solution:

1. $\log_9 ab^2 = \log_9 a + \log_9 b^2 = \log_9 a + 2\log_9 b$ *(Product and Power Laws)*
2. $\log_b \frac{x^3}{y^2} = \log_b x^3 - \log_b y^2 = 3\log_b x - 2\log_b y$ *(Quotient and Power Laws)*

Now, that you are familiar with the basic properties of logarithms and laws of logarithms, read and analyze the following concepts and examples for you to learn how to solve logarithmic equations and inequalities.

Properties of Logarithmic Equations

If b > 0, then the logarithmic function $f(x) = \log_b x$ is increasing for all x.

If 0 < b < 1, then the logarithmic function $f(x) = \log_b x$ is decreasing for all x.

This means that $\log_b u = \log_b v$ if and only if $u = v$.

Here are some techniques or strategies in solving the logarithmic equation.

1. Rewriting to exponential form.
2. Using logarithmic properties.
3. Applying the one–to–one property of logarithmic functions.
4. The Zero Factor Property: If ab = 0, then a = 0 or b = 0.
5. Take into consideration the domain of logarithmic expression.

Example 4

Find the value of x in the following.

1. $\log_5(x+3) = \log_5 22$
2. $\log_3(9x) - \log_3(x-8) = 4$

Solution:

1. $\log_5(x+3) = \log_5 22$	Given
$x+3 = 22$	One-to-one Property
$x = 22-3$	Addition Property of Equality
$x = 19$	Simplify

2. $\log_3(9x) - \log_3(x-8) = 4$	Given
$\log_3 \frac{9x}{x-8} = 4$	Quotient Law of Logarithm
$\frac{9x}{x-8} = 3^4$	Change into exponential form
$9x = 81(x-8)$	Multiplication Property of Equality
$9x = 81x - 648$	Distributive Property
$-72x = -648$	Addition Property of Equality
x = 9	Multiplication Property of Equality

Solving Logarithmic Inequalities

Remember: If b > 0, then the logarithmic function y = $\log_b$ x is increasing for all x. If 0 < b < 1, then the logarithmic function y = $\log_b$ x is decreasing for all x. This means that $\log_n$ a > $\log_n$ b implies a > b. Moreover, bear in mind that the domain of the logarithmic function is the set of all positive real numbers.
The techniques or strategies in solving logarithmic inequality are the same in solving logarithmic equations.

Example 5
Find all values of x that will satisfy the inequality.
1. $\log_2(2x+1) < 3$ 2. $\log_4 9 > 2\log_4 x$

Solution:

1. $\log_2(2x+1) < 3$	Given
$2^3 < 2x+1$	Changing into exponential form
$8 < 2x+1$	Simplify
$7 < 2x$	Addition Property of Equality
7/2 < x	Multiplication Property of Equality

Since the domain of logarithmic function is the set of all positive real numbers, the given $\log_2(2x+1)$ will be defined if x > -1/2 ($2x + 1 > 0 \Rightarrow x > -1/2$). Therefore, the solution set of the inequality is still **x > 7/2.**

2. $\log_4 9 > 2\log_4 x$	Given
$\log_4 9 > \log_4 x^2$	Laws of Logarithm
$9 > x^2$	One-to-one Property
x < 3 or x < -3	Taking square root on both sides.

Since the domain of the logarithmic function is the set of all positive real numbers, the given $2\log_4 x$ will be defined if x > 0. Therefore, the solution set of the inequality is **0 < x < 3.**

What's More

Activity 2.1. Use properties of logarithms to find the value of the following.

1. $\log_2 2 =$ _____
2. $9^{\log_9 6} =$ _____
3. $\log_9 1 =$ _____
4. $4\log_8 8 =$ _____
5. $7^{\log_7 10} =$ _____
6. $\log_m 1 =$ _____
7. $\log_c c - 2^{\log_2 19} =$ _____
8. $\log_{13} 1 + \log_5 5^2 =$ _____
9. $7^{\log_7 10} + 10\log_5 5 =$ _____
10. $\log_5 5 - 2^{\log_2 9} =$ _____

Activity 2.2. Use the law of logarithm to expand the following.

1. $\log_9 4np^3$
2. $\log_b \frac{4y^5}{x^4 z}$

Activity 2.3. Find the value/s of x in the following equations/inequalities

1. $\log_5 25 = 3x - 3$
2. $\log_4(x+3) = 3/2$
3. $\log_{1/2}(3-x) = -3$
4. $\log_3 2x - \log_3(x+5) = 0$
5. $\log_x(\log_2 256) = 3$
6. $\log_3(3x-2) < 2$
7. $\log_3(x-1)^2 > 2$
8. $\log_3 x + \log_3 6 \geq 2$
9. $\log_3 x - 3\log_3 2 \leq 1$
10. $\log_2 x + \log_2(x+4) < 5$

What I Have Learned

A. Complete the following statements by writing the correct word or words and formulas. Write your answer on another sheet of paper.

1. A logarithm of a number with the same number as its base is always equal to _____.
2. The logarithm of 1 at any base will always be equal to _____.
3. The logarithm of a number to a power x with the same number as its base is equal to _____.
4. The logarithm of the product of a positive real number is equal to the _____ of the logarithm of the factors to the given base.
5. The logarithm of the quotient of two positive real numbers is equal to the logarithm of the dividend _____the logarithm of the divisor.
6. The logarithm of the power of positive real numbers is equal to the _____ times the logarithm of the number to the given base.
7. In logarithm, $y = \log_b x$ if and only if $x =$ _____
8. The one-to-one property of equality of logarithmic equation states that if $\log_a x = \log_a y$, then _____.
9. To solve for the unknown of the logarithmic equations and inequalities, you need to rewrite it into its equivalent _____.
10. The domain of logarithmic functions is _____.

B. Enumerate the five (5) strategies or techniques that may help you to solve logarithmic equations and inequalities.

1.
2.
3.
4.
5.

What I Can Do

Journal Writing

Write a mathematical journal connected to what you have learned about solving logarithmic equations and inequalities. Try to relate your writing or reflections on our lesson.

.

Law is a system of rules and procedures that are created and enforced through social or governmental institutions to regulate behavior. There is a saying that "In every rule, there is an exemption". Can you cite an example wherein this saying is applied?

*The same rubric in lesson 1 will be used to rate you out.

Assessment

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. This expresses a relationship between the input and the output and can be represented through a table of values, graph, and equation,
   a. logarithmic equation
   b. logarithmic function
   c. logarithmic inequality
   d. none of these

2. Which of the following is an example of a logarithmic function?
   a. $\log_3 27 = \log_3 x$
   b. $y = \log_2(2x-1)$
   c. $\log_2 x < 10$
   d. $\log_2 x = \log_2 2$

3. It is an inequality involving logarithms.
   a. logarithmic equation
   b. logarithmic function
   c. logarithmic inequality
   d. logarithmic expression

4. The following is an example of a logarithmic equation EXCEPT:

a. $\log_2 4 = \log_2 x$

b. $\log_2 128$

c. $\log_3(x+3) = 2$

d. $\log_5 x = \log_5 2(x-4)+2$

5. What is the value of $8^{\log_8 9}$?

a. 0 b. 3 c. 6 d. 9

6. Give the value of $\log_9 9 + \log_5 1$.

a. 10 b. 9 c. 1 d. 0

7. Find the value of $4\log_3 3^{11}$.

a. 44 b. 15 c. 7 d. 0

8. What is the expanded form of $\log_7 7mn^4$?

a. $\log_7 7 + \log_7 m + \log_7 n^4$

b. $\log_7 7m + 4\log_7 n$

c. $\log_7 7 + \log_7 m + 4\log_7 n$

d. $1 + \log_7 m + \log_7 n$

9. What is the expanded form of $\log \frac{x^2}{\sqrt{y}}$?

a. $\log x + \log y$

b. $2\log x - 1/2\log y$

c. $2\log x + \log y^{\sqrt{2}}$

d. $\log x^2 - \log y^{1/2}$

10. Express $2\log_5 a + 4\log_5 b$ as a single logarithm.

a. $\log_5 a^2 b^4$

b. $\log_5 2a4b$

c. $\log_5 a^2 + b^4$

d. $\log_5 8ab$

11. What is the value of x in the logarithmic equation: $\log_7 x = \log_7 2x + 4$?

a. -4 b. -2 c. 2 d. 4

12. Find b if $\log_3 (b + 2) = \log_3 8$.

a. 0 b. 4 c. 6 d. 12

13. Which of the following satisfies the inequality: $\log_3 27 < m$?

a. $m \geq 3$

b. $m > 3$

c. $m \leq 3$

d. $m < 3$

14. Solve for n: $\log_{x-3} 36 > 2$.
    a. $x > 9$
    b. $x > 6$
    c. $x > 3$
    d. $x > -3$

15. Solve for the unknown in the logarithmic equation: $\log_4 x + \log_4 (x - 3) = 1$.
    a. $x = 1$
    b. $x = 4$ and $x = -1$
    c. $x = 4$
    d. $x = 1$ and $x = -4$

Additional Activities

If you want to try more, these activities are for you. It will help you to practice your skill in solving logarithmic equations and inequalities.

Study and analyze each situation to solve the problem.

1. If $\log_b 3 = 0.48$, $\log_b 4 = 0.60$, and $\log_b 5 = 0.70$, find the following logarithms. First item is done for you.
    a. $\log_b 15 = \log_b (3)(5) = \log_b 3 + \log_b 5 = 0.48 + 0.70 =$ **1.18**
    b. $\log_b 30$
    c. $\log_b 45$
    d. $\log_b 120$
    e. $\log_b 2.5$
2. Natural Logarithm

    Base e logarithms are called natural logarithms. The irrational number e, which is approximately equal to 2.718281 is described in calculus. The following are the properties of natural logarithms.

    a. $e^{\ln x} = x$ **b.** $\ln e^x = x$

    Evaluate the following:
    a. $e^{\ln 5} + \ln e^7 =$
    b. $\ln e^{12} - e^{\ln 9} =$
    c. $5\ln e^2 + 2e^{\ln 5} =$
    d. $(e^{\ln 5})^2 =$
    e. $e^{\ln 81 - \ln 9} =$

Answer Key

What I Know

1. A
2. B
3. D
4. C
5. B
6. D
7. B
8. C
9. A
10. B
11. D
12. B
13. D
14. C
15. C

What's More

LESSON 1

Activity 1.1

1. Log. Function
2. Log. Equation
3. Log. Function
4. Neither
5. Log. Inequality
6. Log. Inequality
7. Log. Equation
8. Neither
9. Log. Function
10. Neither

Activity 1.2

1.	1	6.	0
2.	6	7.	-18
3.	0	8.	2
4.	4	9.	20
5.	10	10.	-8

Activity 1.3

1. $\log_9 4 + \log_9 n + 3\log_9 p$
2. $\log_b 4 + 5\log_b y - 4\log_b x - \log_b z$

LESSON 2

Activity 2.1

1. $x = 5/3$
2. $x = 5$
3. $x = -5$
4. $x = 5$
5. $x = 2$

Activity 2.2

1. $2/3 < x \leq 11/3$
2. $-2 > x > 4$
3. $x \geq 3/2$
4. $0 < x \leq 6$
5. $0 < x \leq 4$

Assessment

1. B
2. B
3. C
4. B
5. D
6. C
7. A
8. D
9. B
10. A
11. A
12. C
13. B
14. C
15. C

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*DepED Material: General Mathematics Learner's Material

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11 General Mathematics

General Mathematics
Representations of Logarithmic Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics
Representations of Logarithmic Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Representations of Logarithmic Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Alternative Delivery Mode (ADM) Module on Representations of Logarithmic Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module focuses on logarithmic functions represented through the table of values, graphs and its transformation, and equation. Logarithmic function as the inverse of the exponential function have properties that are somewhat related to exponential functions. Using the representations of logarithmic functions will give the ideas of how are these two functions related to each other.

After going through this module, you are expected to:

1. convert logarithmic equation to exponential equation;
2. represent logarithmic function through its table of values, graph and equation; and
3. sketch the transformation of the graph of logarithmic function.

What I Know

Before studying this module, take this test to determine what you already know about the topic covered.
Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. The graph of the logarithmic function $f(x)=\log_b x$ is _____ if $b > 1$.
   a. decreasing b. increasing c. shifted down d. shifted up
2. The graph of the logarithmic function $f(x)=\log_b(x)+d$ is shifted _____ if $d < 0$.
   a. downward b. left c. right d. upward

3. What is the inverse of the function $x = b^y$?
   a. $b=\log_x y$ b. $x=\log_b y$ c. $y=\log_b x$ d. $y=\log_x b$

4. The graph of the logarithmic function $f(x)=\log_b(x+c)$ is shifted _____ if $c > 0$.
   a. downward b. left c. right d. upward

5. The graph of the logarithmic function $f(x)=a \log_b(x)$ is stretches if _____.
   a. $a>1$ b. $a<1$ c. $0 < a < 1$ d. $1 < a < 0$

6. Which of the following is the graph of $y = \log_2 x$?

a.

c.

b.

d.

7. The graph of the logarithmic function $f(x)=\log_b x$ is _____ if $0 < b < 1$.
   a. Decreasing b. increasing c. shifted down d. shifted up

8. What is the inverse of the logarithmic function?
   a. Exponential b. linear c. polynomial d. quadratic

9. Which of the following is the inverse of $y = \log_2 x$?
   a. $y=x^2$ b. $y=2^x$ c. $2^y=x$ d. $x=y^2$

10. The graph of the logarithmic function $f(x)=\log_b(x)+d$ is shifted _____ if $d > 0$.
    a. down b. left c. right d. up

11. Which of the following is the graph of $y = \log_{\frac{1}{2}} x$?

a.

c.

b.

d.

12. The graph of the logarithmic function $f(x)=a \log_b(x)$ is compresses if _____.
    a. $a>1$ b. $a<1$ c. $0 < a < 1$ d. $1 < a < 0$

13. The graph of the logarithmic function $f(x)=\log_b(x+c)$ is shifted _____ if $c< 0$.
    a. down b. left c. right d. up

14. Which of the following is the graph of y=2logx?

a.

c.

b.

d.

15. Which of the following is the graph of y=log (x-2)?

a.

c.

b.

d.

Lesson 1 Representations of Logarithmic Functions

This topic will explore the graphs of the logarithmic function through the table of values and its transformation. The transformations of the graph of logarithmic function are either shifting to the left or the right, shifting upward or downward, stretching, or compressing. Also, it will help you understand other concepts related to logarithmic functions.

What's In

In representing logarithmic function through the table of values and graph, it is important to recall your knowledge on how to transform the logarithmic equation to an exponential equation. In doing so, let us start with the following examples on how to change from logarithmic equation to exponential equation which will help you as you go along with this module.

a. $\log_4 64 = 3$
b. $\log_{36} 6 = \frac{1}{2}$
c. $\log_2 \left(\frac{1}{4}\right) = -2$

In writing logarithmic equation to exponential function, it can be recalled that the logarithmic function $y=\log_b x$ is the inverse of the exponential function $y=b^x$ and you just need to remember that you are answering the question "To what power must ***b*** be raised to obtained the number x?".

base power(exponent) base power(exponent)

$\log_b x = y$ means $b^y = x$

logarithmic form exponential form

Let us now try the examples given above.

a. $\log_4 64 = 3$

$\downarrow$

$4^3 = 64$

In this example, b=4, y=3 and x=64

b. $\log_{36} 6 = \left(\frac{1}{2}\right)$

$\downarrow$

$36^{\frac{1}{2}} = 6$

In this example, b=36, y= $\frac{1}{2}$ and x=6

c. $\log_2\left(\frac{1}{4}\right) = -2$

$\downarrow$

$2^{-2} = \frac{1}{4}$

In this example,
b=2, y= -2 and
$x=\frac{1}{4}$

Now that you know how to change the logarithmic equation to an exponential function, it is now time to have a glimpse of the graph of a logarithmic function by doing the activity prepared for you.

What's New

Visualizing the Graphs

Match the function with its graph by writing the letter of your answer on a sheet of paper. (Note: You may use the available graphing tools online like Desmos, Geogebra, etc.)

____ 1. f(x) = log (x+2)

____ 2. f(x) = $\log_3$ x

____ 3. f(x) = log(x-3) + 2

____ 4. f(x) = $\log_{\frac{1}{4}} x$

____ 5. f(x) = 2log

____ 6. f(x) = log (x-2)

What is It

In the previous activity, you are going to match the given graphs to their respective equations or functions and try to identify which are the graphs of a logarithmic function.

After doing the activity, it is time for you to reflect on the following:

1. Do you find difficulty in determining the graph of a function?
2. What can you say about the graph of a logarithmic function?
3. How about the properties logarithmic function $y=\log_b x$?

Graph of a Logarithmic Function

Let us begin with the parent function $f(x) = log_b x$. Because every logarithmic function of this form is the inverse of the exponential function with the form $f(x) = b^x$, their graphs are reflections of each other across the line y = x, as shown below.

For any real number x and constant ***b*** > 0, ***b*** ≠ 1, we can see the following characteristics in the graph ***f(x)=log$_b$x***: one-to-one function; vertical asymptote x = 0; domain: (0, ∞) and range: (-∞, ∞); x-intercept: (1,0) and key point (b, 1), y-intercept: none; increasing if b > 1 and decreasing if 0 < b < 1

Example 1. Sketch the graph of $y = \log_2 x$.

Solution:

Step 1: Construct a table of values of ordered pairs for the given function by assigning values for y then solve for x. A table of values for $y=\log_2 x$ is as follows:

x	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
y	-4	-3	-2	-1	0	1	2	3

Now let us solve x given $y = log_2 x$ where y=-4 from the table.

$y = \log_2 x$	Convert into exponential equation
$\log_2 x = -4 \rightarrow b^y = x$	You need to rewrite $\log_2 x = y$ in the form $b^y = x$. Here the base is 2 and the exponent is -4.
$b^y = x \rightarrow 2^{-4} = x$	Substitute for b, y and x in the exponential equation, $b^Y = x$.
$x = \frac{1}{2^4} = \frac{1}{16}$	Solve for x.

Now continue with the remaining values of y to complete the table.

Step 2. Plot the points found in the table and connect them using a smooth curve.

Example 2. Sketch the graph of $= \log_{\frac{1}{2}} x$.

Solution:

Step 1: Construct a table of values of ordered pairs for the given function by assigning values for y then solve for x. A table of values for $y = \log_{\frac{1}{2}} x$ is as follows:

x	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
y	-4	-3	-2	-1	0	1	2	3	4

Now let us solve x given $y = \log_{\frac{1}{2}} x$ where y=-4 from the table.

$\log_{\frac{1}{2}} x = y$	Convert into exponential equation
$\log_{\frac{1}{2}} x = -4 \rightarrow b^y = x$	You need to rewrite $\log_{\frac{1}{2}} x = y$ in the form $b^y = x$. Here the base is ½ and the exponent is -4.
$b^y = x \rightarrow \left(\frac{1}{2}\right)^{-4} = x$	Substitute for b, y and x in the exponential equation, $b^Y = x$.

$x = \frac{1}{\left(\frac{1}{2}\right)^4} = \frac{1}{\left(\frac{1}{16}\right)} = 16$ Solve for x.

Now continue with the remaining values of y to complete the table.

Step 2. Plot the points found in the table and connect them using a smooth curve.

Example 3. Sketch the graph of $y = \log_4 x$.

Solution:

Step 1: Construct a table of values of ordered pairs for the given function by assigning values for y then solve for x. A table of values for $y=\log_4 x$ is as follows:

x	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16
y	-2	-1	0	1	2

Now let us solve x given $y = \log_4 x$ where y=-2 from the table.

$y = \log_4 x$	Convert into exponential equation
$\log_4 x = -2 \rightarrow b^y = x$	You need to rewrite $\log_4 x=y$ in the form $b^y=x$. Here the base is 4 and the exponent is -2.
$b^y = x \rightarrow 4^{-2} = x$	Substitute for b, y and x in the exponential equation, $b^Y=x$.
$x = \frac{1}{4^2} = \frac{1}{16}$	Solve for x.

Now continue with the remaining values of y to complete the table.

Step 2. Plot the points found in the table and connect them using a smooth curve.

Plotting of points for y=log$_3$x

Graph of y=log$_4$x

Example 4. Sketch the graph of $= \log_{\frac{1}{4}} x$.

Solution:

Step 1: Construct a table of values of ordered pairs for the given function by assigning values for y then solve for x. A table of values for $y = \log_{\frac{1}{4}} x$ is as follows:

x	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$
y	-2	-1	0	1	2

Now let us solve x given $y = \log_{\frac{1}{4}} x$ where y=-2 from the table.

$\log_{\frac{1}{4}} x = y$	Convert into exponential equation
$\log_{\frac{1}{4}} x = -2 \rightarrow b^y = x$	You need to rewrite $\log_{\frac{1}{4}} x = y$ in the form b^y=x. Here the base is $\frac{1}{4}$ and the exponent is -2.
$b^y = x \rightarrow \left(\frac{1}{4}\right)^{-2} = x$	Substitute for b, y and x in the exponential equation, b^Y=x.
$x = \frac{1}{\left(\frac{1}{4}\right)^2} = \frac{1}{\left(\frac{1}{16}\right)} = 16$	Solve for x.

Now continue with the remaining values of y to complete the table.

Step 2. Plot the points found in the table and connect them using a smooth curve.

Plotting of points for $y = \log_{\frac{1}{4}} x$

Graph of $y = \log_{\frac{1}{4}} x$

In addition to the graphs of the logarithmic function, let us also take a look at how the graph of the parent function ***y=log$_b$x*** transform either by shifting to the left or right, shifting up or down, stretches and compresses.

Graphing Transformations of Logarithmic Functions

Transformation of logarithmic graphs behave similarly to those of other parent functions. We can shift, stretch, and compress the parent function $y = \log_b(x)$ without loss of shape.

A. Horizontal Shifts of the Parent Function $y = \log_b (x)$

For any constant ***c***, the function $f(x) = \log_b (x+c)$

- shifts the parent function $y = \log_b (x)$ left ***c*** units if ***c*** > 0.
- shifts the parent function $y = \log_b (x)$ right ***c*** units if ***c*** < 0.
- has the vertical asymptote x = **- *c***.
- has domain (-c, ∞).
- has range (-∞, ∞).

Example 1. Sketch the graph of $f(x) = \log_3(x-2)$ alongside its parent function.

Solution:

Since the function $f(x) = \log_3(x-2)$, we notice $x + (-2) = x - 2$.
Thus, $c = -2$, so $c < 0$. This means we will shift the function $f(x) = \log_3(x)$ right 2 units. The vertical asymptote is $x = -(-2)$ or $x = 2$.

- Consider three key points from the parent function such as $(\frac{1}{3}, -1)$, $(1,0)$, and $(3,1)$ then add 2 to the x coordinates for the new coordinates $(\frac{7}{3}, -1)$, $(3,0)$, and $(5,1)$.

Graph:

B. Vertical Shifts of the Parent Function $y = \log_b(x)$

For any constant d, the function $f(x) = \log_b(x) + d$.

- shifts the parent function $y = \log_b(x)$ up d units if $d > 0$.
- shifts the parent function $y = \log_b(x)$ down d units if $d < 0$.
- has the vertical asymptote $x = 0$.

Example 2. Sketch the graph of $f(x) = \log_3(x) - 2$ alongside its parent function.

Solution:

Since the function is f(x) = $\log_3(x)$ -2, we will notice ***d*** = -2. Thus, ***d*** < 0. This means we will shift the function f(x) = $\log_3(x)$ down 2 units. The vertical asymptote is x = 0.

- Consider three key points from the parent function such as ($\frac{1}{3}$,-1), (1,0), and (3,1) then subtract 2 from the y coordinates for the new coordinates ($\frac{1}{3}$, -3), (1,-2), and (3,-1).

Graph:

C. Vertical Stretches and Compressions of the Parent Function y=$\log_b(x)$

For any constant b > 1, the function f(x) = a $\log_b(x)$

- stretches the parent function y = $\log_b(x)$ vertically by a factor of ***a*** if ***a***>1.
- compresses the parent function y = $\log_b(x)$ vertically by a factor of ***a*** if 0 < ***a*** < 1.
- has the vertical asymptote x = 0.
- has the x-intercept (1,0).

Example 3. Sketch the graph of $f(x) = 2\log_4(x)$ alongside its parent function.

Solution:

Since the function $f(x) = 2\log_4(x)$, we will notice $a = 2$.
This means we will stretch the function $f(x) = \log_4(x)$ by a factor of 2. The vertical asymptote is $x = 0$.

- Consider three key points from the parent function such as $(\frac{1}{4}, -1)$, $(1,0)$, and $(4,1)$ then multiply the y-coordinate by 2 for the new coordinates $(\frac{1}{4},-2)$, $(1,0)$, and $(4,2)$.

Graph:

What's More

Activity 1.1

Match It!

Match the following logarithmic functions to its corresponding graph by drawing a line to connect them.

1.) $f(x) = \log(x-1)+2$

2.) $f(x) = \log_2 x$

3.) $f(x) = \log x^2$

4.) $f(x) = \log(x+3)$

5.) $f(x) = 3 - \frac{1}{2}\log x$

a.)

b.)

c.)

d.)

e.)

Activity 1.2

Let's be "Pair"

Complete the table of values and find the inverse of the given function. Sketch the graphs of the function and its inverse on a separate sheet of paper.

1.

X	$\frac{1}{1000}$			1			1000
y = log x		-2	-1		1	2	

2.

X	-1	-2	-4	-8	-16	-32
y= $\log_4(-x)$						

Activity 1.3

Graph the following using table of values:

1. $f(x) = \log_2 (x+3)$
2. $f(x) = \log_3 (x) -$
3. $f(x) = \frac{1}{2} \log x$`
4. $f(x) = 4\log x$
5. $f(x) = \log_3 (x - 2)$

What I Have Learned

Fill in the blanks with the correct answer.

- The graph of a logarithmic function has a vertical asymptote at _____.
- The graph of the logarithmic function $f(x)=\log_b(x)$ is increasing if _____ and decreasing if _____.
- The graph of the function $f(x)=\log_b (x+c)$ shifted the parent function $f(x)=\log_b(x)$ to the _____ if $c > 0$ and to the _____ if $c < 0$.
- The graph of the function $f(x)=\log_b(x) + d$ shifted the parent function $f(x)=\log_b(x)$ upward if _____ and downward if ____.
- The graph of the function $f(x)=a \log_b x$ _____ the parent function $f(x)=\log_b x$ if _____ and compresses if _____.

What I Can Do

Answer the problem given below.

Loudness of Sound

The loudness ***L***, in bels (after Alexander Graham Bell), of a sound of intensity ***I*** is defined to be

$$L = log \frac{I}{I_o}$$

where $\boldsymbol{I_o}$ is the minimum intensity detectable by the human ear (such as the tick of a watch at 20 ft under quiet conditions). If a sound is 10 times as intense as another, its loudness is 1 bel greater than that of the other. If a sound is 100 times as intense as another, its loudness is 2 bels greater, and so on. The bel is a large unit, so a subunit, the decibel, is generally used. For ***L***, in decibels, the formula is

$$L = 10\ log \frac{I}{I_o}$$

Find the loudness, in decibels, of each sound with the given intensity.

SOUND	INTENSITY
a. Jet engine at 100 ft.	$10^{14} \bullet \boldsymbol{I_o}$
b. Loud rock concert	$10^{11.5} \bullet \boldsymbol{I_o}$
c. Bird calls	$10^{4} \bullet \boldsymbol{I_o}$
d. Normal conversation	$10^{6.5} \bullet \boldsymbol{I_o}$
e. Thunder	$10^{12} \bullet \boldsymbol{I_o}$
f. Loudest sound possible	$10^{19.4} \bullet \boldsymbol{I_o}$

Rubrics for rating this activity.

Score	Descriptors
20	The situation is correctly modeled with an exponential and logarithmic function, appropriate mathematical concepts are fully used in the solution and the correct final answer is obtained.
15	The situation is correctly modeled with an exponential and logarithmic function, appropriate mathematical concepts are partially used in the solution and the correct final answer is obtained.
10	The situation is not modeled with an exponential and logarithmic function, other alternative mathematical concepts are used in the solution and the correct final answer is obtained.
5	The situation does not model an exponential and logarithmic function, a solution is presented but has an incorrect final answer.

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. The graph of the logarithmic function $f(x)=\log_b(x)+d$ is shifted upward if _____.
   a. d=0 b. d < 0 c. d > 0 d. o < d < 1

2. Which of the following is the graph of $= \log_{\frac{1}{2}} x$?
   a.
   c.
   b.
   d.

3. The graph of the logarithmic function $f(x)=a \log_b(x)$ is _____ if $0 < a < 1$.
   a. compresses c. shifted to the right
   b. shifted downward d. stretches

4. The graph of the logarithmic function $f(x)=\log_b(x+c)$ is shifted to the right if _____.
   a. c > 0 b. c < 0 c. c = 0 d. 0 < c < 1

5. Which of the following is the graph of y=log (x+2)?
   a.
   c.
   b.
   d.

6. Which of the following is the exponential form of $-4 = \log_{\frac{1}{2}} x$?

a. $x = -4^{\frac{1}{2}}$ b. $-4 = x^{\frac{1}{2}}$ c. $x = \left(\frac{1}{2}\right)^{-4}$ d. $\frac{1}{2} = x^{-4}$

b.

7. The graph of the logarithmic function $f(x)=\log_b x$ is decreasing if_____.

a. b < 1 b. b = 0 c. 0 < b < 1 d. b > 1

8. The graph of the logarithmic function $f(x)=\log_b(x)+d$ is shifted downward if _____.

a. d=0 b. d < 0 c. d > 0 d. o < d < 1

9. The graph of the logarithmic function $f(x)=\log_b(x+c)$ is shifted to the left if _____.

a. c > 0 b. b. c < 0 c. c = 0 d. 0 < c < 1

10. The graph of the logarithmic function $f(x)=a \log_b(x)$ is _____ if 0 a > 1.

a. compresses c. shifted to the right

b. shifted downward d. stretches

11. The graph of the logarithmic function $f(x)=\log_b x$ is increasing if_____.

a. b < 1 b. b = 0 c. 0 < b < 1 d. b > 1

12. Which of the following is the graph of y=-2logx?

a.

c.

b.

d.

13. How many units do the graph of the logarithmic function f(x)=log(x+3) is shifted to the left?

a. 2 b. 3 c. 4 d. 5

14. What is the inverse of $y = \log_3 x$?

a. $y=x^3$ b. $y=3^x$ c. $3^y=x$ d. $x=y^3$

15. What is the inverse of the exponential function?

a. logarithmic c. polynomial

b. linear d. quadratic

Additional Activities

Sketch the graph of the following logarithmic functions. Show your solution, table of values then describes the graph of the function against the function ***y=log$_b$x***. Write your answer on a separate sheet of paper.

1. $y = \log_2(x+5)$		
Table of values:	Solution:	Graph:

2. $y = \log_{\frac{1}{3}} x$		
Table of values:	Solution:	Graph:

Answer Key

What I Know

1. D
2. B
3. B
4. D
5. D
6. C
7. C
8. B
9. C
10. A
11. C
12. B
13. A
14. C
15. B

What's More

Activity 1.1

1. d
2. e
3. a
4. b
5. c

Activity 1.2

x	$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000
y	-3	-2	-1	0	1	2	3

x	-1	-2	-4	-8	-16	-32
y	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$

Assessment

1. c
2. b
3. a
4. d
5. b
6. c
7. a
8. b
9. a
10. d
11. d
12. c
13. b
14. c
15. a

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*DepED Material: General Mathematics Learner's Material

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11 General Mathematics

General Mathematics
Domain and Range of a Logarithmic Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics
Domain and Range of Logarithmic Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Domain and Range of Logarithmic Functions!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Domain and Range of Logarithmic Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was written for students to understand the concept of domain and range logarithmic function. The topic to be discussed in this module includes finding the domain and range of a logarithmic function algebraically. The language used in this module is appropriate to the diverse communication and language ability of the learners.

After going through this module, you are expected to:

1. define domain and range;
2. understand the properties of logarithmic function; and
3. determine the domain and range of the logarithmic function.

What I Know

Directions: Choose the letter of the best answer. Write your chosen letter on a sheet of paper.

1. What is the inverse of the function $x = b^y$?
   a. $b = y$ c. $y = x$
   b. $x = y$ d. $y = b$

2. What is the domain of the logarithmic function $f(x) = x$?
   a. (0, ∞) c. (-∞,∞)
   b. (0, -∞) d. (∞, -∞)

3. What is known as the possible values of the independent variable x?
   a. domain c. outputs
   b. inputs d. range

4. What is the range of the function $f(x) = x$ where $b < 1$?
   a. (0, ∞) c. (∞, -∞)
   b. (-∞, 0) d. (-∞,∞)

5. What is the domain of the function $f(x) = (3x - 6)$?
   a. (2, ∞) c. (-∞, 2)
   b. (-2, ∞) d. (-∞, 2)

6. The range of the function are the corresponding values of the independent variable y which are often called _____.

a. domain
b. inputs
c. outputs
d. range

7. What is a set of all y values?

a. domain
b. inputs
c. outputs
d. range

8. What is the domain of the function $f(x) = (x - 1)$?

a. (-1, ∞)
b. (1, ∞)
c. (-∞,1)
d. (-∞. -1)

9. What is a set of all x values?

a. domain
b. inputs
c. outputs
d. range

10. What is the domain of the function $f(x) = log\ log\ (5 - 3x)$?

a. $(-\infty, \frac{3}{5})$
b. (-∞, 5)
c. $(-\infty, \frac{5}{3})$
d. (-∞, 3)

11. What is the inverse of the logarithmic function?

a. exponential
b. linear
c. polynomial
d. quadratic

12. What is the range of the function $f(x) = (x - 2)$?

a. (0, ∞)
b. (-∞, 0)
c. (∞, -∞)
d. (-∞,∞)

13. Which of the following is NOT an exponential function?

a. $y = 1^x$
b. $y = 2^x$
c. $y = 3^x$
d. $y = 4^x$

14. What is the inverse of $y = x$?

a. $y = x^2$
b. $y = 2^x$
c. $2^y = x$
d. $x = y^2$

15. What is the domain of the function $y = (x - 5) + 2$?

a. (-5, ∞)
b. (5, ∞)
c. (2, ∞)
d. (-2, ∞)

Lesson 1 Domain and Range of Logarithmic Functions

The domain of a function is the set of possible values of the independent variable. The range is the set of the resulting values that the dependent variable can have as x varies throughout the domain. This module focuses on the domain and range of a logarithmic function.

What's In

To fully understand this topic, let us recall some concepts from the previous lessons on exponential function and its relationship with its inverse function, the logarithmic function. So that it will be easier for you to grasp the next lesson.

Let us start with the definition of an exponential function which is a function of the form

$$f(x) = b^x \text{ , where } \boldsymbol{b > 0} \text{ and } \boldsymbol{b \neq 1}.$$

Examples are as follows: f(x) = 3^X, $f(x) = \left(\frac{1}{2}\right)^x$, g(x) =$2^{-X}$, and $y = \left(\frac{1}{5}\right)^{-2x}$. Its domain is a set of real numbers while its range is a set of all positive real numbers.

Let us also recall that if a function is a one-to-one function, then an inverse function exists denoted by f^{-1} having the following properties:

- f^{-1} is a one-to-one function
- domain of f^{-1} is the range of f
- range of f^{-1} is the domain of f

If the position of x and y in y = b^x are interchanged and then y is solved for the resulting equation, the rule of correspondence of the inverse of the exponential function is obtained. This rule is denoted by the symbol:

$$y = x$$

Since the exponential function is one-to-one, its inverse must also be a function. Just like in the exponential function, where $b > 0$, and $b \neq 1$.

Let us also recall lessons regarding domain and range of a function by answering the following:

a. $f(x) = 2x + 5$
b. $f(x) = \sqrt{x - 3}$
c. $f(x) = \frac{x+3}{x-2}$

Solution:

a. $f(x) = 2x + 5$

Since the linear function - *2x+5* is a polynomial function, and based on its graph, its domain is {x|x is a real number} and generally its range is {y|y is a real number}.

b. $f(x) = \sqrt{x - 3}$

Solving for the domain and range:

x-3 ≥ 0
x ≥ 3

therefore, the domain is {x|x≥3} and the range is {y|y≥0} as you can see from the graph.

Take note that the number under a square root sign must be positive,

c. $f(x) = \frac{x+3}{x-2}$

In a rational function, the denominator of a fraction cannot be zero, therefore, the domain is $\{x \mid x \neq 2\}$.

For the range, interchange the variable of the given function, then solve for y.

$x(y-2)=y+3$
$xy-2x = y+3$
$xy - y = 2x+3$
$y(x-1) = 2x+3$

Therefore the range is $\{y \mid y \neq 1\}$

What's New

Let us help Mang Kulas to find his lost carabao by going through the maze. Find the domain of the given logarithmic functions to get to the carabao.

What is It

In the previous activity, you need to help Mang Kulas to find his lost carabao by going through the maze. In doing so, you need to answer by finding the domain of the logarithmic functions given.

After you finish the activity, reflect to the following questions:

1. Do you find difficulty in finding the domain of a function?
2. How do you find the domain of the logarithmic function?
3. How can you define domain and range?

If you think that the activity is difficult, that is okay because after you read more about domain and range of logarithmic function, you can go back to the activity and help Mang Kulas to find his carabao. The discussion below will help you to understand more the domain and range of the logarithmic function.

Domain and Range of Logarithmic Function

The domain of a function is the set of all possible values of the independent variable x. The possible values of the independent variable x are often called ***inputs***. The range of the function are the corresponding values of the dependent variable y. The corresponding values of the dependent variable y are often called ***outputs***.

In the case of a logarithmic function, its domain is defined as a set of all positive real numbers while its range is a set of real numbers.

, where *b>1*

Domain: (0, ∞)
Range: (-∞, ∞)

, where *b<1*

Domain: (0, ∞)
Range: (-∞, ∞)

Transformation of the parent function $f(x) = x$ either by shift, stretch, compression, or reflection changes the domain of the parent function. When finding the domain of a logarithmic function, therefore, it is important to remember that the domain consists *only of positive real numbers*. That is, the argument of the logarithmic function must be greater than zero.

Example 1. Find the domain and range of

Solution

$2x - 4 > 0$	set up an inequality showing an argument greater than zero
$2x > 4$	solve for x
or 2	
Domain: (2, ∞)	write the domain in interval notation
Range: (-∞, ∞)	

Graph

From the graph of the function , it can be seen that the curve is asymptotic at x = 2. Therefore the domain and range are as follows:

Domain: (2, ∞)
Range: (-∞, ∞)

Example 2. Find the domain and range of .

Solution:

$x-3 > 0$
$x-3+3 > 0+3$
$x > 3$

Domain: (3, ∞)
Range: (-∞, ∞)

Graph:

Example 3. Find the domain and range of .

Graph

Solution:

3-2x > 0
-3+3-2x > 0-3
-2x > -3

Domain: (-∞,)
Range: (-∞, ∞)

What's More

Activity 1.1

Directions: Arrange the small triangles to fit into the larger triangle accordingly. Make sure that the given function corresponds to its right domain. Write the number of your answer inside the triangle to form a larger triangle

Activity 1.2

MATCH IT: Match column A with column B by drawing a line to connect.

1.	$y = (x - 1)$	a.	Domain: (-2,∞), Range: (-∞,∞)
2.	$y = x - 1$	b.	Domain: (3,∞), Range: (-∞, ∞)
3.	$y = (x + 2) - 5$	c.	Domain: (-5,∞), Range: (-∞, ∞)
4.	$y = (x - 3)$	d.	Domain: (1,∞), Range: (-∞,∞)
5.	$y = (x + 5) - 3$	e.	Domain: (0,∞), Range: (-∞,∞)

Activity 1.3

Determine the domain and range of the following:

1. $f(x) = (3 - 2x)$
2. $f(x) = log\ log\ (x) - 5$
3. $f(x) = (x + 2) + 4$
4. $f(x) = (x + 1) - 2$
5. $f(x) = (4 - 7x)$

What I Have Learned

1. The _____ of a function is the set of all possible values of the independent variable ____. These possible values are often known as _____.
2. The _____ of a function is the set of all possible values of the dependent variable ____. These possible values are often known as _____.
3. The _____ $f(x) = x$ is the inverse of _____ $f(x) = b^x$.
4. The domain of the function $f(x) = x$ is _____ while its range is _____.

What I Can Do

Answer the problem given below.

Loudness is measured in decibels. The formula for the loudness of a sound is given by "dB = $10\frac{loglog\ I}{I_0}$" where I_0 is the intensity of "threshold sound", or sound that can barely be perceived. Other sounds are defined in terms of how many times more intense they are than threshold sound. For instance, a cat's purr is about 316 times as intense as threshold sound, for a decibel rating of:

$$\begin{aligned} \text{dB} &= 10\frac{loglog\ I}{I_0} \\ &= 10\frac{loglog\ (316\ I_0)}{I_0} \\ &= 10log[\ 316\ ] \\ &= 24.9968708262\ldots,\ldots \text{or } 25 \text{ decibels.} \end{aligned}$$

Considering that prolonged exposure to sounds above 85 decibels can cause hearing damage or loss, and considering that a gunshot from a .22 rimfire rifle has an intensity of about $I = (2.5 \times 10^{13})I_0$, should you follow the rules and wear ear protection when relaxing at the rifle range?

Rubrics for rating this activity

20	The problem is correctly answered applying the concept and properties of logarithmic functions. The model is used appropriately.
15	The problem is correctly answered applying the concept and properties of logarithmic function. The model is used with some misrepresentation.
10	The problem is partially answered by applying a different solution. The model is not used at all.
5	The problem is not correctly answered applying the concept and properties of logarithmic functions. The model is not used at all.

Assessment

Multiple Choice: Choose the letter of the best answer. Write your answer in your notebook.

1. Range is a set of all _____ values.
   a. w c. y
   b. x d. z

2. What is the range of the exponential function $y = b^x$?
   a. $(0, \infty)$ c. $(-\infty,\infty)$
   b. $(-\infty, 0)$ d. $(\infty, -\infty)$

3. What is the domain of the function $f(x) = (3x - 2)$?
   a. $(\frac{3}{2}, \infty)$ c. $(-\infty, \frac{2}{3})$
   b. $\left(-\infty, \frac{3}{2}\right)$ d. $(\frac{2}{3}, \infty)$

4. What is known as the possible values of the independent variable x?
   a. domain c. outputs
   b. inputs d. range

5. What is the inverse of the function $f(x) = x$?
   a. $b = y^x$ c. $x = b^y$
   b. $b = x^y$ d. $y = b^x$

6. What is a set of all x values?
   a. domain c. outputs
   b. inputs d. range

7. What is the domain of the function $f(x) = log\ log\ (7 - 4x)$?
   a. $(-\infty, \frac{4}{7})$ c. $(\frac{7}{4}, \infty)$
   b. $(\frac{4}{7}, \infty)$ d. $(-\infty, \frac{7}{4})$

8. Outputs are the possible values of the _____ variable y.
   a. constant
   b. dependent
   c. independent
   d. real

9. What is the domain of the function $f(x) = (x + 1)$?
   a. (-∞, 1)
   b. (-∞, -1)
   c. (-1, ∞)
   d. (1, ∞)

10. What is the range of the function $f(x) = x$?
   a. (-∞,∞)
   b. (-∞, 0)
   c. (0, ∞)
   d. (1, ∞)

11. What is the inverse of an exponential function?
   a. linear
   b. logarithmic
   c. polynomial
   d. quadratic

12. What is the range of the function $f(x) = (x + 2)$?
   a. (-∞, 2)
   b. (-∞, -2)
   c. (2, ∞)
   d. (-2, ∞)

13. Which of the following is the correct logarithmic form of $16^{\frac{1}{4}} = \frac{1}{8}$?
   a. $\frac{1}{8} = \frac{3}{4}$
   b. $\frac{3}{4} = \frac{1}{8}$
   c. $16 = \frac{3}{4}$
   d. $16 = \frac{1}{8}$

14. Which of the following is an exponential function?
   a. $y = 2x^4$
   b. $y = x^4$
   c. $y = 4^x$
   d. $y = x^{-4}$

15. What is the domain of the function $f(x) = (x + 6) - 2$?
   a. (-∞, 6)
   b. (-∞, -6)
   c. (6, ∞)
   d. (-6, ∞)

Additional Activities

Graph the following logarithmic functions using an online graphing calculator then find its domain and range.

1. $y = (x - 2)$
2. $y = (x + 3)$
3. $y = (x - 1)$
4. $y = (x + 2)$
5. $y = (x + 1)$
6. $y = x - 2$
7. $y = x + 3$
8. $y = x - 1$
9. $y = x + 2$
10. $y = x + 1$

Answer Key

What I Know

1. c
2. a
3. b
4. d
5. a
6. c
7. d
8. b
9. a
10. c
11. a
12. d
13. a
14. c
15. b

What's More

Activity 1.1

Activity 1.2
1. D
2. E
3. A
4. B
5. C

Activity 1.2
1.D= R=
2. D= R=
3. D= R=
4. D= R=
5. D= R=

Assessment

1. c
2. a
3. d
4. b
5. c
6. a
7. d
8. b
9. c
10. a
11. b
12. d
13. a
14. c
15. d

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11 General Mathematics

General Mathematics
Intercepts, Zeroes, and Asymptotes of Logarithmic Functions
First Edition, 2020

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Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

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General Mathematics

Intercepts, Zeroes, and Asymptotes of Logarithmic Functions

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Intercepts, Zeroes and Asymptotes of Logarithmic Function!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Intercepts, Zeroes and Asymptotes of Logarithmic Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module will help you determine the intercepts and zeroes of logarithmic functions using the algebraic solution and its asymptotes through its domain which are essentials in the next chapter. The topics to be discussed in this module will able you to prepare to solve real-life applications of logarithmic functions. The language used in this module is appropriate to a diverse communication and language ability of the learners.

After going through this module, you are expected to:

1. find the intercepts of logarithmic functions;
2. solve for the zeroes of logarithmic functions; and
3. determine the asymptotes of logarithmic functions.

What I Know

Directions: Choose the letter of the best answer. Write your chosen letter on a sheet of paper.

1. What is a line that the curve approaches, as it heads toward infinity?
   a. asymptote c. intercept
   b. domain d. range

2. It is where a function crosses the x or y-axis?
   a. asymptote c. intercept
   b. domain d. range

3. What is the x-intercept of $f(x) = (x - 4)$?
   a. 4 c. -5
   b. -4 d. 5

4. Logarithmic function is not defined for _____ numbers and zero.
   a. negative c. real
   b. positive d. whole

5. The graph of the function $f(x) = x$ has a vertical asymptote at _____.
   a. x =1 c. x = 0
   b. x = -1 d. x = 2

6. What is the inverse of the exponential function?
   a. logarithmic
   b. linear
   c. polynomial
   d. rational

7. What is known as the x-value that makes the function equal to 0?
   a. asymptote
   b. intercept
   c. range
   d. zeroes

8. What is a function of the form $f(x) = b^x$?
   a. exponential
   b. logarithmic
   c. linear
   d. polynomial

9. It is where the functions cross the x-axis and where the height of the function is zero.
   a. asymptote
   b. x-intercept
   c. y-intercept
   d. zeroes

10. What is the x-intercept of the function $f(x) = log\ log\ (2x + 3)$?
   a. (1,0)
   b. (0,1)
   c. (0, -1)
   d. (-1,0)

11. What are the zeroes of the function $f(x) = x^2$?
   a. x=0 and x=1
   b. x=1 and x=-1
   c. x=0 and x=-1
   d. x=2 and x=-2

12. The graph of the function $f(x) = (3x - 2)$ has a vertical asymptote at _____.
   a. $x = \frac{2}{3}$
   b. $x = \frac{3}{2}$
   c. x=2
   d. x=3

13. What is the x-intercept of the function $f(x) = ln\ ln\ (x - 3)$?
   a. (0,4)
   b. (0,-4)
   c. (-4,0)
   d. (4,0)

14. The graph of the function $f(x) = log\ log\ x - 2$ has a vertical asymptote at _____.
   a. x=1
   b. x=0
   c. x=-1
   d. x=2

15. What is the inverse of $y = x$?
   a. $y = x^2$
   b. $y = 2^x$
   c. $2^y = x$
   d. $x = y^2$

Lesson 1 Intercepts, Zeroes and Asymptotes of Logarithmic Functions

This topic focuses on how to determine the intercept, zeroes, and asymptote of a logarithmic function. It is also about the concept of finding the intercept and zeroes of a logarithmic function applying the transformation of logarithmic function to exponential form and determining the asymptote of a logarithmic function using the idea of its domain.

What's In

Let us start our discussion by recalling some important topics that will guide you as you go along with this module.

It can be remembered that the logarithmic function $f(x) = x$ is the inverse of the exponential function $f(x) = b^x$ and since the logarithmic function is the inverse of the exponential function, the domain of the logarithmic function is the range of exponential function, and vice versa.

In general, the function $f(x) = x$ where $b, x > 0$ and $b \neq 1$ is a continuous and one-to-one function. Note that the logarithmic function is ***not defined*** for negative numbers or zero. The graph of the function approaches the y-axis as x tends to ∞, but never touches it. The function rises from $-\infty$ to ∞ as x increases if $b > 1$ and falls from ∞ to $-\infty$ as x increases if $0 < b < 1$.

Therefore, the domain of the logarithmic function $y = x$ is the set of positive real numbers and the range is the set of real numbers.

What's New

Decode It: Solve for the zero and asymptote of the given logarithmic functions. Blacken the circle that corresponds to your answer and write the letter in the appropriate box to decode the word.

1.) $y = (x + 2)$

- (E) x=-1, VA: x=2
- (T) x=-2, VA: x=-1
- (H) x=-1, VA: x=-2

3.) $y = (x - 1)$

- (D) x=-1, VA: x=2
- (R) x=-2, VA: x=1
- (E) x=2, VA: x=-1

5.) $y = (2x - 6)$

- (R) x=7/2, VA: x=3
- (A) x=2/7, VA: x=3
- (P) x=7/2, VA: x=-3

2.) $y = x - 1$

- (I) x=-3, VA: x=-0
- (B) x=3, VA: x=-0
- (D) x=0, VA: x=--3

4.) $y = (3x - 5)$

- (S) x=2, VA: x=-3/5
- (C) x=2, VA: x=-5/3
- (N) x=3/5, VA: x=2

6.) $y = (4x + 5)$

- (A) x=-2, VA: x=-1
- (R) x=1, VA: x=-2
- (P) x=-1, VA: x=-2

The number 0 is originally called

4	2	6	1	3	5

What is It

In order to decode the activity above, you are going to solve the zero of the function and find its vertical asymptote. Then, you are going to blacken the circle that corresponds to your answer and from the letters of the word will be revealed to decode the answer.

After you go through the activity, reflect on the following questions:

1.) How do you find the activity?
2.) Did you decode the answer? What is your answer?
3.) What did you do to find the zero of the given logarithmic function? How about finding the vertical asymptote?

Since you are now ready to learn the lesson with the idea that you gained from the previous activity. Let us now start our lesson.

Intercepts and Zeroes of Logarithmic Functions

An intercept in Mathematics is where a function crosses the x or y-axis. x-intercepts are where functions cross the x-axis. They are also called roots, solutions, and zeroes of a function. They are found algebraically by setting $y=0$ and solving for x. The zero of a function is the x-value that makes the function equal to 0, that is, $f(x) = 0$. In this section, our discussion will focus only on the x-intercept of a given logarithmic function.

Example 1. Find the intercept and zeroes of $f(x) = log\ log\ (2x + 3)$.

To find the intercept, we let y = 0 then solve for x.

$f(x) = log\ log\ (2x + 3)$

$0 = log\ log\ (2x + 3)$

$10^0 = 2x + 3$ → change from logarithmic to exponential function

$1 = 2x+3$ → since $10^0 = 1$

$2x = 1-3$

$2x = -2$ → dividing both sides by 2

$x = -1$

Therefore, the x-intercept is at (-1,0) and the zero of the function is -1.

Example 2. Find the intercept and zeroes of $f(x) = x^2$.

To find the intercept, we let y = 0 then solve for x.

$f(x) = x^2$

$0 = x^2$

$2^0 = x^2$ → change from logarithmic to exponential function

$1 = x^2$ → since $2^0 = 1$

$x = \pm\sqrt{1}$

$x = \pm 1$

Therefore, the x-intercepts are at (1,0) and (-1,0) and the zeroes of the function are 1 and -1.

Example 3. Find the intercept and zeros of $f(x) = ln\ ln\ (x - 3)$

To find the intercept, we let y = 0 then solve for x.

$f(x) = ln\ ln\ (x - 3)$

$0 = ln\ ln\ (x - 3)$

$x - 3 = e^0$ → change from logarithmic to exponential function

$x-3 = 1$ → since $e^0 = 1$

$x = 1 + 3$

$x = 4$

Therefore, the x-intercept is at (4,0) and the zero of the function is 4.

Vertical Asymptote of Logarithmic Function

An asymptote is a line that a curve approaches, as it heads towards infinity. It is a vertical asymptote when as x approaches some constant value c (either from the left or from the right) then the curve goes towards ∞ or -∞.

In dealing with the vertical asymptote of a logarithmic function, it is a must to remember that logarithmic function is not defined for negative numbers or zero, and the domain of a logarithmic function $f(x) = x$ x is a set of positive real numbers. A logarithmic function will have a vertical asymptote precisely where its argument (i.e. the quantity inside the parentheses) is equal to zero.

Example 1. Find the vertical asymptote of the graph of $f(x) = log\ log\ x - 2$.

Since the domain of the logarithmic function is *(0, ∞)*, thus the graph has a vertical asymptote at x = 0.

Example 2. Find the vertical asymptote of the graph of $f(x) = (3x - 2)$.

Set the argument *(3x-2)* equal to zero then solve for x, that is,

$3x - 2 = 0$

$3x = 2$ dividing both sides by 3

$x = \frac{2}{3}$

Since the logarithmic function is defined for $x > \frac{2}{3}$, thus, the graph has a vertical asymptote at $x = \frac{2}{3}$.

Example 3. Find the vertical asymptote of the graph of $f(x) = (x + 3) + 2$.

Set the argument *(x+3)* equal to zero then solve for x, that is,

$x + 3 = 0$

$x = -3$

Since the logarithmic function is defined for $x > -3$, thus, the graph has a vertical asymptote at $x = -3$.

What's More

Activity 1.1

Match It: Match column A with column B by drawing a line to connect.

Column A	Column B
1. $y = 2x$	a. VA: x=-2, int.: (-1,0) zero: -1
2. $y = x - 1$	b. VA: x=0, int.: (0.125,0) zero: 0.125
3. $y = (x + 2)$	c. VA: x=0, int.: (1,0) zero: 1
4. $y = (x - 3)$	d. VA: x=3, int.: (4,0) zero: 4
5. $y = (x) - 3$	e. VA: x=0, int.: (3,0) zero:3

Activity 1.2

Directions: Unscramble the letters to find the correct answer then write your answers in the boxes provided before each number.

(tysatomep)

1. A line that the curve approaches but never touches it.

(narge)

2. A set of all y-values.

(atmlocgrihi)

3. The inverse of exponential function.

(oseerz)

4. The x-value that makes the function equal to 0.

(ncprteite)

5. It is where a function crosses the x or y-axis.

(moadni)

6. The set of all x-values.

(oxetlapenni)

7. A function of the form $f(x)=b^x$.

(atvneegi)

8. Logarithmic function is not defined for _____ numbers and zero.

(ifev)

9. The x-intercept of $f(x)=\log_2(x-4)$.

(lriectva)

10. The graph of the function $f(x)=\log_b x$ has a _____ asymptote at $x = 0$.

Activity 1.3

Determine the x-intercepts, zeroes and vertical asymptotes of the following:

1. $f(x) = x$
2. $f(x) = log\ log\ (x) - 3$
3. $f(x) = (x - 2) + 4$
4. $f(x) = (x + 1) - 2$
5. $f(x) = (x\ ) + 2$

What I Have Learned

Complete the following statement with correct word/s.

1. The logarithmic function _____ is the inverse of $f(x) = b^x$.
2. An _____ is where the functions cross the x or y-axis and _____ is where the curve cross the x-xis.
3. An _____ is a line that a curve approaches as it approaches_____.
4. The _____ of a function is the x-value that makes the function equal to _____.
5. A logarithmic function is _____ on negative numbers and_____.

What I Can Do

Answer the problem given below.

pH Level In chemistry, the pH of a substance is defined as $pH = -\ log\ log\ [H^+]$ where H+ is the hydrogen ion concentration, in moles per liter. Find the pH level of each substance.

SUBSTANCE	HYDROGEN ION CONCENTRATION
a.) Pineapple juice	1.6×10^{-4}
b.) Hair conditioner	0.0013
c.) Mouthwash	6.3×10^{-7}
d.) Eggs	1.6×10^{-8}
e.) Tomatoes	6.3×10^{-5}

Rubrics for rating this activity:

20	All questions are answered correctly using the model given in the problem.
15	4 questions are answered correctly using the model given in the problem.
10	2-3 questions are answered correctly using the model given in the problem.
5	0-1 questions are answered correctly using the model given in the problem.

Assessment

Multiple Choice: Choose the letter of the best answer. Write your answer in your notebook.

1. Intercept is where a function crosses the _____.
   a. x-axis
   b. x and y-axis
   c. y-axis
   d. y and z-axis

2. Logarithmic function is not defined for negative numbers and _____.
   a. one
   b. three
   c. two
   d. zero

3. What is the x-intercept of the function $f(x) = (3x - 2)$?
   a. x=1
   b. x=-1
   c. x=3
   d. x=2

4. The graph of $f(x) = x$ has a _____ at x=0.
   a. horizontal asymptote
   b. vertical asymptote
   c. x-intercept
   d. y-intercept

5. What is the zero of $f(x) = (x - 4)$?
   a. -4
   b. 4
   c. 5
   d. -5

6. Asymptote is a line that the curve approaches as it approaches _____,
   a. curve
   b. infinity
   c. one
   d. zero

7. What is the inverse of the function $y=b^x$?
   a. $y = b$
   b. $x = b$
   c. $y = x$
   d. $b = x$

8. What is the x-intercept of the function $f(x) = (2x + 5)$?
   a. (-2,0)
   b. (2,0)
   c. (1,0)
   d. (-1,0)

9. What is the zero of the function $f(x) = (x + 1)$?
   a. 2
   b. -1
   c. 0
   d. 1

10. The x-intercept is where the function crosses the x-axis and where the height of the function is _____.
   a. maximum
   b. negative
   c. one
   d. zero

11. What is the inverse of a logarithmic function?
   a. exponential
   b. linear
   c. polynomial
   d. quadratic

12. What is the intercept of the function $f(x) = (x + 2)$?
   a. x=2
   b. x=-1
   c. x=-2
   d. x=1

13. What is the zero of the function $f(x) = ln\ ln\ (x - 3)$?
   a. 4
   b. -4
   c. 2
   d. -2

14. The graph of the function $f(x) = log\ log\ (2x - 3)$ has a vertical asymptote at _____.
   a. x=2
   b. x=3
   c. x = $\frac{2}{3}$
   d. $x = \frac{3}{2}$

15. What is the intercept of the function $f(x) = (x + 6)$?
   a. x=5
   b. x=-5
   c. x=6
   d. x=-6

Additional Activities

Determine the intercept, zero and vertical asymptote of the following logarithmic functions. Write your answer in a sheet of paper.

1. $y = (x + 3)$
2. $y = x + 1$
3. $y = (x - 1)$
4. $y = (x + 1)$
5. $y = x + 2$
6. $y = x - 2$
7. $y = (x - 2)$
8. $y = x + 3$
9. $y = x - 1$
10. $y = (x + 2)$

Answer Key

What I Know

1. a
2. c
3. d
4. a
5. c
6. a
7. d
8. a
9. b
10. d
11. b
12. a
13. d
14. b
15. c

What's More

Activity 1.1

1. c
2. e
3. a
4. d
5. b

Activity 1.2

1. asymptote
2. range
3. logarithmic
4. zeroes
5. intercept
6. domain
7. exponential
8. negative
9. five
10. vertical

Activity 1.3

1. VA: , Int. (, 0) Zero: 1
2. VA: , Int. (,0)
Zero: 1000
3. VA: , Int. (
Zero:
4. VA: , Int. (, 0) Zero: 3
5. VA: , Int. (, 0) Zero: 2

Assessment

1. b
2. d
3. a
4. b
5. c
6. b
7. c
8. a
9. c
10. d
11. a
12. b
13. a
14. d
15. b

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**General Mathematics Learner's Material*. First Edition. 2016. pp. 124-133

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11 General Mathematics

General Mathematics
Solving Real-Life Problems Involving Logarithmic Functions, Equations and Inequalities
First Edition, 2020

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Published by the Department of Education
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Undersecretary: Diosdado M. San Antonio

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General Mathematics

Solving Real-life Problems Involving Logarithmic Functions, Equations, and Inequalities

Introductory Message

For the facilitator:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Real-life Problems Involving Logarithmic Functions, Equations and Inequalities!

This module was collaboratively designed, developed and reviewed by educators from public institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the General Mathematics Grade 11 Alternative Delivery Mode (ADM) Module on Solving Real-life Problems Involving Logarithmic Functions, Equations and Inequalities!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

Previously, you learned how to simplify and solve logarithmic functions, equations, and inequalities. Also, you already have the background of the properties, techniques, and steps in solving problems using logarithmic functions. You are now aware of the use of the Richter Scale to find the magnitude of an earthquake, determining for the acidity and pH level of a solution concentration, computing the population, and solving compound interest.

Can you still remember the formulas to solve those real-life applications of logarithmic functions? It is not enough that you know the formulas, what matters most is you know how to apply it in real-life situations. In this module, you will gain a deeper understanding of the application of a logarithmic function, equation, and inequalities to real-life situations.

You will realize that aside from the mentioned real-life problem above there are still other real-life situations that you could use logarithm like computing for the decay rate, how bacteria and viruses multiply, how to get the age of a decomposed bone by knowing the carbon-14 content. You might also find it interesting to solve for your future savings account or how you could possibly get a higher amount if you will save earlier.

And now, are you ready for the new lesson? Fasten your seatbelt and focus on the world of solving numerous ways of using logarithm is a real-life situation.

After going through this module, you are expected to:

1. recall how to solve logarithmic equations and inequalities; and
2. solve problems involving logarithmic functions, equations, and inequalities.

What I Know

Let's find out how far you might already know about this topic! Please take this challenge! Have Fun!

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following situations show the application of the logarithmic function to the real-life situation?
   a. Getting the number of teachers in one division
   b. Looking for the missing value of a variable
   c. Computing the age of Maria given her sibling true age
   d. Getting the pH level of water from an unknown water tunnel
2. The following situation shows the application of the logarithmic functions to real-life situation **EXCEPT**:
   a. Determining time your money may double in amount
   b. Measuring the size of human statistics
   c. Determining the vital statistics of a person
   d. Getting the total number of population in one particular region in a certain time frame
3. An earthquake is measured with a wave amplitude of 1012 times. What is the magnitude of this earthquake using the Richter scale to the nearest tenth? *(Hint:* $R = \frac{2}{3} log \frac{E}{10^{4.40}}$*)*
   a. 5.07
   b. 6.07
   c. 7.57
   d. 7.87
4. A particular running experiment is initially 100 bacteria cells. She expects that the number of cells is given by the function $c(t) = 100(2)^{\frac{t}{15}}$, where time t is the number of hours since the experiment started. After how many hours would the scientist expect to have 300 bacteria cells? Give your answer to the nearest hour.
   a. 2 hours
   b. 24 hours
   c. 104 hours
   d. 1, 048 hours
5. Which of the following logarithmic inequalities is correct? Round off your answer to 2 decimal places.
   a. log(x-1) + log(x+1) < 2logx if x = 2
   b. log(x-1) + log(x+1) < 2logx if x = 100
   c. log(x+1) > 2log(x) if x = 2
   d. log(x+5) > 5log(-x) if x = -2

6. Simplify $log_5 x \geq 3$.

a. x ≥ 125 c. x ≥ 15

b. x ≥ 85 d. x ≥ 225

7. The formula in the risk of having an increasing car accident as the concentration of alcohol in blood increases is $A = 6e^{12.75x}$ where x is the blood alcohol concentration and A is the given percentage of car accident risk. What blood alcohol concentration corresponds to a 50% risk of a car accident?

a. 0.20 c. 0.17

b. 0.25 d. 0.19

8. Evaluate the logarithmic form $\log_6 8$.

a. 1.16 c. 2.16

b. 2.25 d. 1.25

9. Determine the depreciated value of a teacher's table that has discounted 50% of its original value of ₱5000.00 using a decay factor.

a. ₱5000.00 c. ₱3000.00

b. ₱2500.00 d. ₱4500.00

10. Find the inverse of $f(x) = b^x$.

a. $f^{-1}(x) = \log_x b$ c. $f(x) = \log_b x$

b. $f^{-1}(x) = \log_b x$ d. $f^{-1}(b) = \log_b x$

11. The magnitude of an earthquake in Matanao, Davao Del Sur on December 15, 2019, is 6.8. And it is predicted that there will be another earthquake that will strike somewhere in the Philippines that is 4 times stronger than the mentioned earthquake. What could be the possible magnitude of the predicted earthquake? (Earthquake Magnitude on a Richter scale $R = \frac{2}{3} log \frac{E}{10^{4.40}}$)

a. 7 c. 8.40

b. 8 d. 7.20

12. Suppose that you are observing the behavior of bacteria duplication in a laboratory. You observe that the bacteria triple every hour. Write an equation with base 3 to determine the population of bacteria after one day.

a. 3.02×10^{11} c. 2.90×10^{11}

b. 3.20 x 1011 d. 2.82 x 1011

13. Using item number 12, determine how long it would take the population of bacteria to reach 300,000 bacteria.

a. 11.48 days c. 12.5 days

b. 13 days d. 14 days

For item numbers 14-15, refer to the following:

A Senior High School student plans to invest in a bank since he knew that his family struggles financially. He thought that if he will not prepare for the future it will be hard for him to continue to study at the university. This decision is very wise for a student like him. It suggests that even as early as Grade 7 students should have the urge and initiative to save for the future. His initial amount for his savings is ₱5,500.00. Help him to decide to save his money with the formula $A = P(1 + r)^n$ and by answering the questions that follow:

14. A bank offers 12% compounded annually, predict the balance after 5 years.

a. ₱9,500.00 c. ₱9,692.88

b. ₱10,692.88 d. ₱10,500.00

15. If he would like to have ₱20,000 in the future how long will it take him to save with the same amount of initial investment and the same interest rate?

a. 8 years
b. 10 years
c. 12 years
d. 13 years

Lesson 1 Solving Problems involving Logarithmic Functions, Equations and Inequalities

Learning new things like discovering the importance of learning logarithm and its significance in real-life situations is fun. You will notice that some of the problems here are somewhat the same with the problems you already solved involving exponential function. Yes! You already know about solving some problems here, but this time you will solve them using logarithmic functions, equations, and inequalities.

What's In

As the saying goes, "A person who does not remember where he came from will never reach his destination". Because of that here are some exercises to refresh your mind.

Activity 1

Determine whether each of the given below is a logarithmic function, a logarithmic equation, a logarithmic inequality, or neither of the three. Enjoy working while recalling your previous lessons regarding logarithm. Have fun!

1. $g(x) = 2\log x$
2. $y = \log 4(2x-1)$
3. $x\log 8(2x) = -\log(3x-5)$
4. $\log(4x - 1) > 0$
5. $g(x) = 2x-7$

How did you distinguish logarithmic functions, logarithmic equations and logarithmic inequalities from each other?

Activity 2

Pick, Pair and Solve

Complete the table below by selecting your answer inside the box and putting them in the column where they belong. In the columns, logarithmic equations and logarithmic inequalities, make sure you will **pick** and **pair it** with the correct solutions. Have fun!

$\log_4(x-2)+\log_4(x+2)=16$ | **5ln2x = 15** | **(2/3,+∞)**

$\log_4(x+2) < \log_8 4x$ | $-10 < \log x < 10$

$h(x)=\log_9(x-64)$ | $p(x)=\log_2(x)$ | **6.70** | **-4 and 4**

$\left(\frac{1}{10{,}000{,}000{,}000} \middle| 10{,}000{,}000{,}000\right)$

Logarithmic Equations	Solutions to Logarithmic Equations	Logarithmic Functions	Logarithmic Inequalities	Solutions to Logarithmic Inequalities

What's New

Why oh Why?

In a far-flung area somewhere in Quezon Province, the school principal observed that the number of graduating students decreases every year. In the year 2018, the number of graduating students is 200, but in the year 2020, it becomes 150 only. Use the formula $A = Pe^{rt}$ and the information given to answer the following questions:

Questions:

1. What is the decay rate of the number of graduating students?
2. Using the decay rate that you get in item 1, about how many years will there are less than 100 graduating students?
3. Do you think the way of living in a remote area affects the decreasing population of learners per year?
4. What could be the other reasons for the decreasing population of graduating learners per year?
5. Were you able to solve the problem with the given formula? Justify your answer.

What is It

You have noticed that you were given a formula on the problem above under ***What's New*** to solve for the decay rate. Sometimes, this formula is also used for problems involving exponential growth. Let us now try to solve the problem above. Using the formula $A = Pe^{rt}$ we can substitute the given value for the first question which is you were asked to look for the decay rate.

Given: A = 150 P = 200 t = 2 years r = ?

Using substitution in the formula $A = Pe^{rt}$, we have

$$150 = 200e^{r(2)}$$

To simplify: divide both sides by 200 that becomes

$$0.75 = e^{r(2)}$$

$$\ln 0.75 = 2r \ln e$$

from this equation divide both sides by 2 that makes the equation

$$0.1438 = r \ln e$$

Since ln e is equal to one then the final answer is r = 0.1438 or 14.38% decay rate.

To answer question number 2, do it with the same process but this time look for the time instead of rate and use the 0.1438 for the value of r. This will become inequality since we are looking for the time that a population decayed to less than 100 graduating students. Thus, $100 < 200e^{0.1438t}$

Using the same process this will give us the answer 4.82 years < t or t > 4.82 years. Therefore, if the number of graduating students will be continued to decrease following the decay rate of 14.38%, intuitively, in five years there will be less than 100 graduating students. This information will provide the school administration and teachers to look for a solution regarding the declining number of graduating students. This is the role of mathematics to real-life problems, it gives us the information we need to make wise decisions.

Word problems involving logarithmic functions, equations, and inequalities generally involve solving and evaluating exponential form. Exponential and logarithm cannot be separated from each other. If the given problem is in logarithmic form, it is necessary to transform them to exponential and solve for the unknown value which will satisfy the original equation or function.

This is just one of the applications of logarithmic inequality, function, and equation. Aside from this, you will be given other examples of the logarithm that will be applied in real life.

Example 1

COVID-19 pandemic according to news is spreading rapidly, transferring from human to human. It is a kind of virus that affects the human respiratory system and it is commonly associated with cough, pneumonia, SARS (Severe Acute Respiratory Syndrome), and other respiratory-related infections.

Let us assume that the virus has an initial population of 10,000 and grows to 25,000 after 50 minutes. Assume that its growth follows an exponential model $f(t) = Ae^{kt}$ representing the number of viruses after **t** minutes. The e is used in the model because the virus continuously grows over time.

a. Find **A and k.**

b. Use the model to determine the number of viruses after 6 hours.

Solution:

(a) Given: $f(0) = 10{,}000$

$f(50) = 25{,}000$

Thus, $f(0) = Ae^{k(0)}$

$A = 10{,}000$

$F(50) = 25{,}000e^{k(50)}$

$= 25{,}000$

$e^{50k} = 5/2$

$\ln e^{50k} = \ln 5/2$

Take the ln of both sides

$50k = \ln 5/2$

$= 0.01832$

Therefore, A = 10,000 and k=0.01832.

Also, the exponential model is $f(t) = 10{,}000e^{0.01832t}$

(b) 6 hours = 360 minutes;

$f(360) = 10{,}000e^{.01832(360)}$

$= 7{,}315{,}752$

Therefore, the number of viruses after 6 hours is 7,315,752.

Example 2

Under certain circumstances, a virus spreads according to the equation $p(t) = \frac{1}{1+15e^{-0.3t}}$ where p(t) is the proportion of the population of the virus spread at time t days. How long will it take the virus to spread at 75% of the population?

Solution:

$$0.75 = \frac{1}{1+15e^{-0.3t}}$$

$$0.75 + 11.25e^{-0.3t} = 1$$

$$11.25e^{-0.3t} = 0.25$$

$$e^{-0.3t} = 0.25/11.25$$

$$-0.3t \ln e = \ln 0.25/11.25$$

$$t = 12.69$$

Therefore, it will take approximately 13 days for the virus to spread to 75% of the population.

Example 3

When an organism dies, the amount of carbon-14 in its system starts to decrease. The Carbon-14 is about 7,200 years. An archaeologist found a bone in Mountain Province of Cordillera Region that contains ¼ of the carbon-14 it originally had, how long ago did the human die?

Solution:

The mathematical model of the situation is $y = (\frac{1}{2})^{t/7,200}$ where y is the amount of carbon-14 in the organism after t years and y_0 initial amount of carbon-14.

Since the bone is only ¼ of the carbon-14 it originally had, we have

$$\frac{1}{4} y_o = y_o (1/2)^{t/7,200}$$

Taking the ln of both sides, $\ln\frac{1}{4} = (t/7,200) \ln(1/2)$

$$\ln\frac{1}{4} \div \ln\frac{1}{2} = t/7,200$$

$$t = 14,400$$

Therefore, the human died 14,400 years ago and this must be a big contribution to our history.

Example 4

Mr. Boy a fisherman from Mulanay Quezon Province initially invested ₱500,000.00 in a local cooperative and wanted a double amount form its initial investment. Using the formula from the previous lesson on exponential function $A = P(1+r)^n$ where: A is the future value; P is the present value; r is the interest rate and n is the number of years, how many years will it take an investment to triple if the annual interest rate is 6%?

Solution:

Triple of the initial investment means that three (3) times ₱500,000.00 which is equal to ₱1,500,000.00

Given: A = ₱1,500,000.00, P = ₱500,000.00, r = 6% or .06, n = ?

$A = P(1+r)^n$

₱1,500,000.00 = ₱500,000.00$(1+.06)^n$

$3 = (1.06)^n$

$\log 3 = \log(1.06)^n$

$\log 3 = n\log(1.06)$

$n = \frac{\log 3}{\log 1.06}$

n = 18.85 years

therefore the money will triple approximately after 19 years.

What's More

Read each problem carefully and answer each question to solve the problem. Have Fun!

Activity 1.1

One of the remote areas in Manila which happens to be the capital of the Philippines has recorded an increasing case of diarrhea. It is found out that a certain bacteria has been discovered which causes this disease. This culture starts at 5,000 bacteria, and doubles every 100 minutes. How long will it take a number of bacteria to reach 20,000.

1. What could be the mathematical model for this situation? _____
2. Identify the given. _____
3. Substitute the given to the mathematical model _____
4. How long will it take the number of bacteria to reach 20,000?

Activity 1.2

1. Using the world population formula P = 6.9(1.011)t, where t is the number of years after 2010 and P is the world population in billions of people, estimate: a) the population in the year 2030 to the nearest hundred million, and b) by what year will the population be double from 2010?

2. An earthquake during October 2019 at Tulunan Cotabato was recorded to have a magnitude of 6.3. Another earthquake somewhere in Davao was recorded to have a 7.1 magnitude in December 2018. How much more energy was released by the 2018 earthquake compared to that of 2019 recorded earthquake? You can refer to the discussion in the introduction to logarithm for computation.

3. How much money should be invested at 5% compounded annually for 30 years so that you have ₱25,000.00 at the end of 30 years? Round your answer to the nearest two decimal places.

What I Have Learned

A. Please read the sentences carefully and fill in the missing word/s by writing your answer on the line/s provided.

1. Logarithmic equation is a _____.
2. Logarithmic inequality is a _____.
3. Logarithmic function is a _____.
4. Logarithmic function is the _____ of exponential function.

B. Give at least three examples of real-life situations which can be modelled by a logarithmic functions, equations or inequalities.

What I Can Do

Read and analyze the situation below then answer the question given.

Exponential function cannot be separated in solving problems involving logarithmic function. Most of the time, professionals like chemists, engineers, and scientists encounter problems that require the application of exponential and logarithmic functions.

Chemists define the acidity or alkalinity of a substance according to the formula "pH = –log[H+]" where [H+] is the hydrogen ion concentration, measured in moles per liter. Solutions with a pH value of less than 7 are considered acidic while solutions with a pH value of greater than 7 are basic. On the other hand, solutions with a pH of 7 (such as pure water) are neutral. Suppose that you test apple juice and find that the hydrogen ion concentration is [H+] = 0.0003. Find the pH value and determine whether the juice is basic or acidic.

Here are the steps to solve the problem and the rubric that will guide you in giving the correct solution to the problem.

Steps in Problem Solving	Possible Highest Points	Your Score
1. Give the Appropriate model or equation to find the pH Level.	2 points	
2. Identify the given	2 points	
3. Substitute the given and show the solution	3 points	
4. Give the final answer	3 points	
Total	10 points	

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following situations show the application of the logarithmic function?
   a. Determining the level of acid in a solution
   b. Determining time your money may double in amount
   c. Measuring the size of human statistics
   d. Getting the ion component of a chemical
2. Compute for the value of x in a given logarithmic inequalities
   log2(x+1) > log4(x2).
   a. x > ½ c. x > ½ x ≠ 0
   b. x > ¾ d. x > ¾ x ≠ 1
3. An earthquake is measured with wave amplitude of 1015 times. What is the magnitude of this earthquake using the Richter scale R = 2/3 log $(E/10^{4.40})$ to the nearest tenth?
   a. 6.07 c. 7.57
   b. 7.07 d. 8.00
4. A particular bacterial colony doubles its population every 15 hours. A scientist running an experiment is starting with 100 bacteria cells. She expects the number of cells to be given by the function $c(t) = 100(2)^{\frac{t}{15}}$, where t is the number of hours since the experiment started. After how many hours would the scientist expect to have 500 bacteria cells? Give your answer to the nearest hour.
   a. 5 hours c. 25 hours
   b. 15 hours d. 35 hours

5. If log 0.3 (x-1) < log 0.09 (x-1), then x lies in the interval _____.
   a. 2 < x < ∞ c. – 2 < x < -1
   b. – ∞ < x < 2 d. 1 < x < 2
6. What is the depreciated value of a smartphone discounted 35% of its original price of ₱36,000.00?
   a. ₱23,400.00 c. ₱12,000.00
   b. ₱12,600.00 d. ₱23,000.00
7. Solve the logarithmic inequality $\log_2 x \leq 4$.
   a. 0 ≥ x ≤16 c. x ≤16
   b. 0 ≤ x ≤ 8 d. 0 ≤ x ≤16
8. The formula in the risk of having an increasing car accident as the concentration of alcohol in blood increases is $A = 6e^{12.75x}$ where x is the blood alcohol concentration and A is the given percentage of car accident risk. What blood alcohol concentration corresponds to a 75% risk of a car accident?
   a. 0.20 c. 0.17
   b. 0.25 d. 0.19

9. You observed that the behavior of bacteria laboratory tripled every minute. Write an equation with base 3 to determine the population of bacteria after one hour.
   a. 3.23×10^{28}
   b. 4.23×10^{28}
   c. 2.23×10^{28}
   d. 1.23×10^{28}
10. Using item number 9, determine how long it would take the population of bacteria to reach 1,000,000 bacteria.
   a. 12 days
   b. 12.58 days
   c. 13.58 days
   d. 14.68 days
11. Find the value of x in the equation $\log_4(2x - 1) = 2$
   a. 8.5
   b. 8
   c. 9.5
   d. 9
12. The magnitude of an earthquake in San Luis Aurora Province in May 2020 is 5.4. And it is predicted that there will be another earthquake that will strike somewhere in the Philippines that is 5 times stronger than the mentioned earthquake. What could be the possible magnitude of the predicted earthquake? (Use Earthquake Magnitude on a Ritcher scale $R = \frac{2}{3} log \frac{E}{10^{4.40}}$)
   a. 7
   b. 8
   c. 6.13
   d. 7.10

For item numbers 13-15, refer to the following:

Mr. Juan Bayan thought of investing or saving some of his money after all the leisures that he enjoyed. He believes in the saying "early comer is better than hard worker". With ₱10,000.00 remaining cash on hand he plans to save it in a bank, but he is still in doubt where to invest the money. Using the formula $A = P(1 + r)^n$ help him to solve his problem by answering the questions that follow.

13. A bank offers him a time deposit of 36% compounded annually, how much will his money be after 10 years?
   a. ₱216,000.00
   b. ₱116,000.00
   c. ₱116,465.70
   d. ₱216,465.70
14. If he would like to have ₱500,000 in the future, how long will it take him to save with the same amount of initial investment and the same interest rate?
   a. 19 years
   b. 20 years
   c. 25 years
   d. 30 years
15. He's been thinking that if only he save at an early age he could have gotten a lot bigger. Based on question no. 14 if he starts to invest at the age of 24 how old is he to get the ₱500,000.00?
   a. 44 years old
   b. 32 years old
   c. 52 years old
   d. 60 years old

Additional Activities

Solve the following:

1. You find a skull in a nearby tribe ancient burial site and with the help of a spectrometer, you discovered that the skull contains 9% of the C-14 found in a modern skull. Assuming that the half-life of C-14 (radiocarbon) is 5,730 years, how old is the skull?

2. Suppose that the population of a colony of bacteria increases exponentially. At the start of an experiment, there are 10,000 bacteria, and one hour later, the population has increased to 10,500. How long will it take for the population to reach 25,000? Round off your answer to the nearest hour.

Answer Key

What I Know

1. D
2. B
3. A
4. B
5. A
6. A
7. C
8. A
9. B
10. B
11. D
12. D
13. A
14. C
15. C

What's More

Activity 1.1

$Y = 5,000(2)^{t/100}$

$20,000 = 5,000(2)^{t/100}$

$t = 200$

Activity 1.2

1. a. 8.6 billon people
   b. 2074
2. The earthquake recorded during 2018 of December released 15.85 times more energy than that released on October 2019.

3. The initial amount should be ₱5, 784.44.

Assessment

1. C
2. C
3. B
4. D
5. C
6. A
7. D
8. A
9. B
10. B
11. A
12. C
13. D
14. B
15. A

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