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ENGR 2300 Applied Linear Algebra

Homework CH 3.1, 3.2

(1) Determine which property of determinants the equation illustrates.

$$\begin{vmatrix} 3 & 4 & 5 \\ 3 & -5 & 3 \\ 4 & 6 & 7 \end{vmatrix} = -\begin{vmatrix} 3 & 4 & 5 \\ -3 & 5 & -3 \\ 4 & 6 & 7 \end{vmatrix}$$

If a row is multiplied by a scalar c, then the determinant is multiplied by c. $c = -1$

d. If a row of a matrix is multiplied by a scalar, then the determinant of the matrix is multiplied by that scalar.

(2) Use expansion by cofactors to find the determinant of the matrix

$$\begin{bmatrix} 2 & 3 & -4 \\ 0 & 2 & 3 \\ -2 & 4 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} = (2\cdot 1) - (4\cdot 3) = 2 - 12 = -10 \Rightarrow 2\cdot \mathrm{Cof}(-10)$$

$+ \quad - \quad +$

$|A| = 2\cdot \mathrm{Cof}(1) + 3\cdot \mathrm{cof}(2) + -4\cdot \mathrm{cof}(3)$

$$\begin{bmatrix} 2 & 3 & -4 \\ 0 & 2 & 3 \\ -2 & 4 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 3 \\ -2 & 1 \end{bmatrix} = -((0\cdot 1) - (3\cdot -2)) = -(0 - (-6)) = -(6) \Rightarrow -(3\cdot \mathrm{Cof}(-6))$$

$$\begin{bmatrix} 2 & 3 & -4 \\ 0 & 2 & 3 \\ -2 & 4 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 2 \\ -2 & 4 \end{bmatrix} = (0\cdot 4) - (2\cdot -2) = (0) + 4 = 4 \Rightarrow -4\cdot \mathrm{Cof}(4)$$

$|A| = 2\cdot \mathrm{Cof}(-10) - (3\cdot \mathrm{Cof}(-6)) + (-4)\cdot \mathrm{Cof}(4)$

$|A| = 2(-10) - 3(6) - 4(4)$

$|A| = -20 - 18 - 16 = \boxed{-54}$

checkerboard signs

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

(3) T or F?

a) To find the determinant of a triangular matrix, add the entries on the main diagonal. False, the determinant of a triangular matrix is equal to the product of the entries in the main diagonal, which may not be equal to the sum of the entries in the main diagonal.

b) To find the determinant of a matrix, expand by cofactors in any row or column. True, to find the determinant of a matrix, expand by cofactors in any row or column.

c) When expanding by cofactors, you need not evaluate the cofactors of zero entries. True, in a cofactor expansion each cofactor gets multiplied by the corresponding entry. If this entry is 0, the product will be zero.

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$$A = \begin{vmatrix} 1 & 3 & 2 \\ 5 & 1 & 4 \\ 2 & 4 & 2 \end{vmatrix} \qquad \det(A) = 1\begin{vmatrix} 1 & 4 \\ 4 & 2 \end{vmatrix} - 3\begin{vmatrix} 5 & 4 \\ 2 & 2 \end{vmatrix} + 2\begin{vmatrix} 5 & 1 \\ 2 & 4 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 3 & 2 \\ 5 & 1 & 4 \\ 2 & 4 & 2 \end{vmatrix} \qquad \begin{vmatrix} 1 & 3 & 2 \\ 5 & 1 & 4 \\ 2 & 4 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 4 \\ 4 & 2 \end{vmatrix} = (2\cdot 1) - (4\cdot 4) = -14$$

$$\begin{vmatrix} 5 & 4 \\ 2 & 2 \end{vmatrix} = (5\cdot 2) - (2\cdot 4) = 2$$

$$\begin{vmatrix} 5 & 1 \\ 2 & 4 \end{vmatrix} = (5\cdot 4) - (2\cdot 1) = 18$$

$$\det(A) = (1\cdot -14) - (3\cdot 2) + (2\cdot 18) = -14 - 6 + 36 = \boxed{16 = \det(A)}$$

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$$A = \begin{vmatrix} -3 & 6 \\ \frac{1}{2} & 8 \end{vmatrix} \qquad \det(A) = (a\cdot d) - (b\cdot c)$$

$$= (-3\cdot 8) - (6\cdot \tfrac{1}{2}) = -24 - 3 = \boxed{-27 = \det(A)}$$

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$$A = \begin{vmatrix} -3 & 0 & 0 \\ 9 & 6 & 0 \\ -3 & 9 & -3 \end{vmatrix} \qquad \det(A) = -3\begin{vmatrix} 6 & 0 \\ 9 & -3 \end{vmatrix} - 0 + 0$$

Matlab Confirm ✓

$$\begin{vmatrix} 6 & 0 \\ 9 & -3 \end{vmatrix} = (6\cdot -3) - (0\cdot 9) = -18 - 0 = -18$$

$$\det(A) = -3(-18) = \boxed{54}$$

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$$\begin{vmatrix} 5 & 1 & 0 & 1 \\ 1 & 0 & -1 & -1 \\ 4 & 0 & 1 & 4 \\ -1 & 0 & 2 & 1 \end{vmatrix}$$

TI-84 calculator Matrix > Math

$$\boxed{\det([A]) = 8}$$

Python confirm ✓

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$$A = \begin{vmatrix} 3 & -1 & -2 \\ 2 & 1 & 3 \\ -6 & 2 & 4 \end{vmatrix} \qquad \boxed{\det(A) = 0}$$

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a) Adding a multiple of one column of a square matrix to another column changes only the sign of the determinant.

False. Adding a multiple of one column to another does not change the value of the determinant.

b) Two matrices are column equivalent when one matrix can be obtained by performing elementary column operations on the other.

True. Column-equivalent matrices are matrices that can be obtained from each other by performing elementary column operations on the other.

c) If one row of a square matrix is a multiple of another row, then the determinant is 0.

True. You can achieve a row of all zeros by adding a multiple of one row to another which does not change the determinant of the matrix.

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$$A = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$$

$$\det(A) = (\cos\theta \cdot \cos\theta) - (-\sin\theta \cdot \sin\theta)$$
$$= \cos^2\theta + \sin^2\theta$$
$$\boxed{\det(A) = 1}$$

$$B = \begin{vmatrix} \sin\theta & -1 \\ -1 & \sin\theta \end{vmatrix}$$

$$\det(B) = (\sin\theta \cdot \sin\theta) - (-1 \cdot -1)$$
$$= \sin^2\theta - 1 = -(\cos^2\theta)$$
$$\boxed{\det(B) = -\cos^2\theta}$$

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$$A = \begin{vmatrix} \lambda & 9 & 0 \\ 0 & \lambda+2 & 3 \\ 0 & 1 & \lambda \end{vmatrix}$$

Solved w/ MATLAB

$$\boxed{\lambda = -3, 0, 1}$$

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$$A = \begin{vmatrix} -3 & 2 & 1 \\ 5 & 4 & 6 \\ 2 & -1 & 3 \end{vmatrix}$$

Solved w/MATLAB

Minors: $\begin{vmatrix} 18, & 3, & -13 \\ 7, & -11, & -1 \\ 8, & -23, & -22 \end{vmatrix}$

Cofactors: $\begin{vmatrix} 18 & -3 & -13 \\ -7 & -11 & 1 \\ 8 & 23 & -22 \end{vmatrix}$

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Solve for x

$$\begin{vmatrix} x-10 & 11 \\ 2 & x-1 \end{vmatrix} = 0$$

$$\det(A) = (a \cdot d) - (b \cdot c)$$
$$(x-10)(x-1) - (11 \cdot 2) = 0$$
$$x^2 - x - 10x + 10 - 22 = 0$$
$$x^2 - 11x - 12 = 0$$

Quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(1)(-12)}}{2(1)}$$
$$x = \frac{11 \pm \sqrt{121 + 48}}{2}$$
$$x = \frac{11 \pm \sqrt{169}}{2} = \frac{11 \pm 13}{2}$$
$$x = \frac{11+13}{2} = \frac{24}{2} \qquad x = \frac{11-13}{2} = \frac{-2}{2}$$
$$\boxed{x = 12, -1}$$

Confirm w/ Python ✓

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Use elementary row or column operations to find the determinant.

$$A = \begin{vmatrix} 1 & 6 & -3 \\ 1 & 3 & 1 \\ 3 & 6 & 1 \end{vmatrix}$$

$$\det(A) = 1 \cdot \begin{vmatrix} 3 & 1 \\ 6 & 1 \end{vmatrix} - 6 \cdot \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + (-3) \cdot \begin{vmatrix} 1 & 3 \\ 3 & 6 \end{vmatrix}$$

$(3 \cdot 1) - (1 \cdot 6) = -3$

$(1 \cdot 1) - (1 \cdot 3) = -2$

$(1 \cdot 6) - (3 \cdot 3) = -3$

$$\det(A) = (1 \cdot -3) - (6 \cdot -2) + (-3 \cdot -3) = -3 + 12 + 9 = \boxed{18 = \det(A)}$$

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Use expansion by cofactors

$$A = \begin{array}{cccc} w & x & y & z \\ \hline 15 & -21 & 18 & 30 \\ -10 & 24 & -18 & 32 \\ -22 & 40 & 32 & -35 \end{array}$$

Solved w/ MATLAB

$$91074w + 46362x - 5826y - 9588z$$