Given

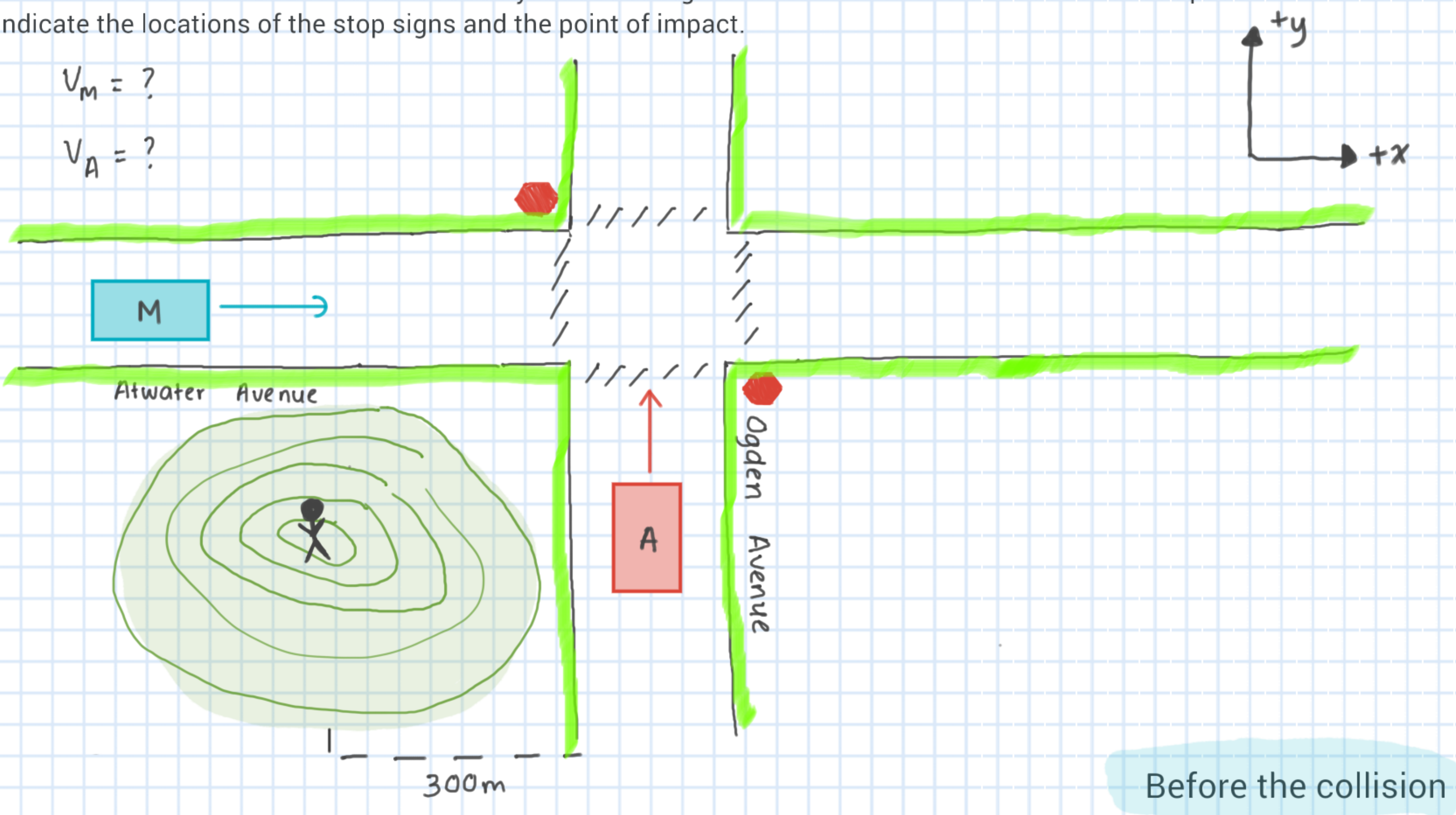
- Crash took place at 10:30 AM
- Speed limit of 40 km/h
- M traveling East, A traveling North prior to crash
- Maximum possible acceleration for A is 1.96 m/s
- A traveled 9.1 m from stop sign to crash
- Hillcrest is 300 m West of the crash site

Statements

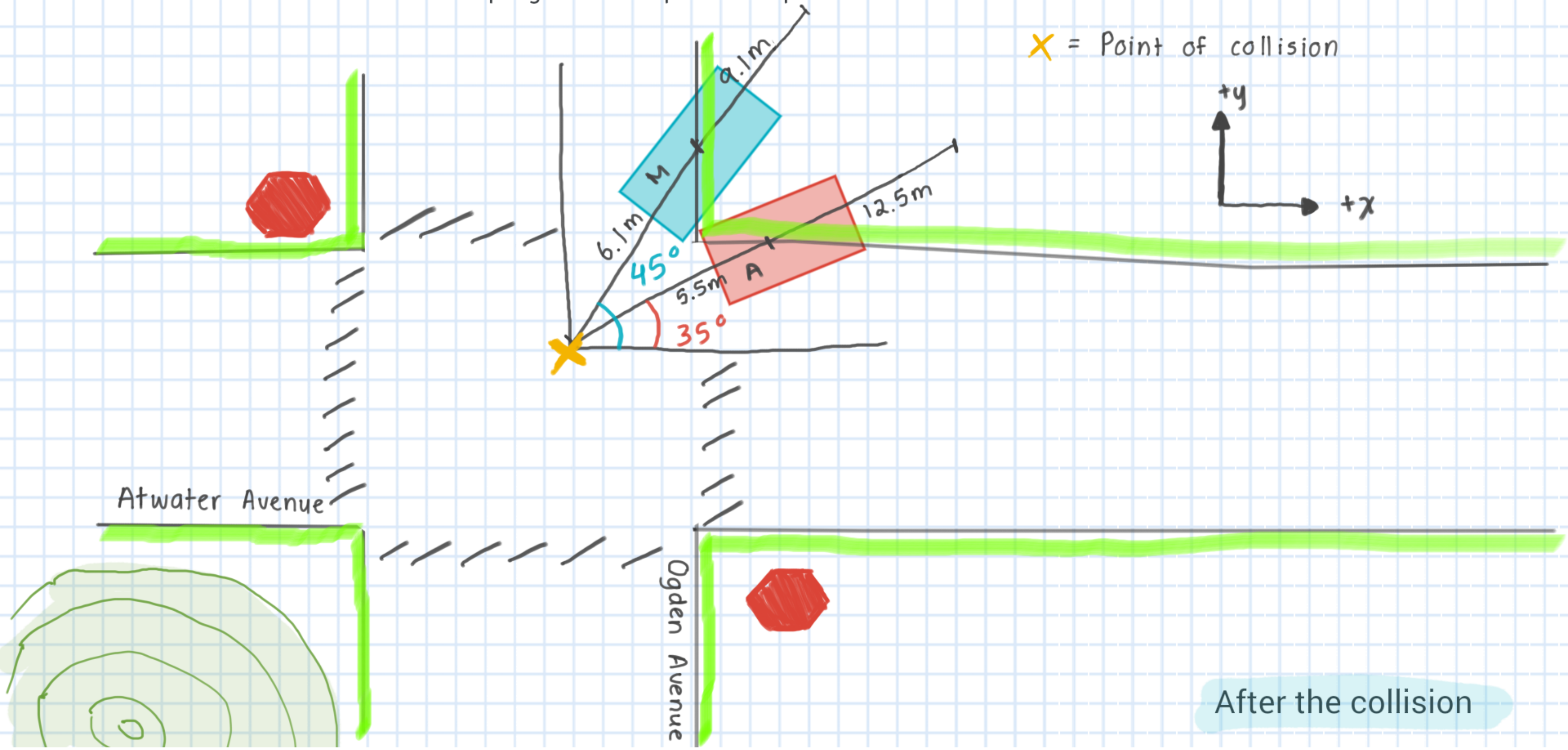
- M claims he's going 40 km/h
- M claims A ran the stop sign
- A claims she stopped at the stop sign
- A claims M was speeding

Item	Vehicle #1	Vehicle #2
Mass including load and occupants	1950 kg	1430 kg
Approach angle	[east]	[north]
Departure angle	E45°N	E35°N
Distance across asphalt	6.1 m	5.5 m
Asphalt coefficient of friction, μ_a	0.72	0.72
Distance across grass	9.1 m	12.5 m
Grass coefficient of friction, μ_g	0.35	0.35

1) Sketch a diagram of the crash scene, showing both vehicles before and after the impact. Label the vehicles, the streets and the different surfaces. Clearly show the angular direction of both vehicles after the impact. Indicate the locations of the stop signs and the point of impact.



1) Sketch a diagram of the crash scene, showing both vehicles before and after the impact. Label the vehicles, the streets and the different surfaces. Clearly show the angular direction of both vehicles after the impact. Indicate the locations of the stop signs and the point of impact.



2) Write two expressions: one for the conservation of momentum in the x direction and one for the conservation of momentum in the y direction.

In the x direction:

$$\vec{p}_x = \vec{p}\,'_x$$

$$\vec{p}_{mx} + \vec{p}_{Ax} = \vec{p}\,'_{mx} + \vec{p}\,'_{Ax}$$

$$m_m \vec{V}_m = m_m \vec{V}\,'_{mx} + m_A \vec{V}\,'_{Ax}$$

V(Ax) represents the x component of the speed of vehicle A

In the y direction:

$$\vec{p}_y = \vec{p}\,'_y$$

$$\vec{p}_{my} + \vec{p}_{Ay} = \vec{p}\,'_{my} + \vec{p}\,'_{Ay}$$

$$m_A \vec{V}_A = m_m \vec{V}\,'_{my} + m_A \vec{V}_{Ay}$$

V(Ay) represents the y component of the speed of vehicle A

3) Use the work-energy formula to determine the speed with which vehicle #1 (v 1g) enters the grass. Use the work-energy formula to determine the speed with which vehicle #2 (v 2g) enters the grass.

$$W = \Delta E_K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$-F_f \Delta d = -\frac{1}{2}mv_i^2$$
$$\mu \cancel{m} g \Delta d = \frac{1}{2}\cancel{m}v_i^2$$
$$v_i = \sqrt{2\mu g \Delta d}$$

V(iMg) represents the initial speed of vehicle M on grass

V(iAg) represents the initial speed of vehicle A on grass

Finding speed of M entering the grass:

$$V_{img} = \sqrt{2(0.35)(9.81)(9.1)}$$
$$V_{img} = 7.91 \text{ m/s}$$

Finding speed of A entering the grass:

$$V_{iAg} = \sqrt{2(0.35)(9.81)(12.5)}$$
$$V_{iAg} = 9.26 \text{ m/s}$$

4) Use the work-energy theorem to determine the speed with which vehicle #1 (v1a) enters the asphalt.
Use the work-energy formula to determine the speed with which vehicle #2 (v2a) enters the asphalt.

$$W = \Delta E_k = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$-F_f \Delta d = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$-2 \mu \cancel{m} g \Delta d = \cancel{m} (v_f^2 - v_i^2)$$

$$v_i = \sqrt{2 \mu g \Delta d + v_f^2}$$

V(iMa) represents the initial speed of vehicle M on asphalt

V(iAa) represents the initial speed of vehicle A on asphalt

Finding speed of M entering the asphalt:

$$V_{iMa} = \sqrt{2(0.72)(9.81)(6.1) + (7.91)^2}$$

$$V_{iMa} = 12.2 \, m/s$$

Finding speed of A entering the asphalt:

$$V_{iAa} = \sqrt{2(0.72)(9.81)(5.5) + (9.26)^2}$$

$$V_{iAa} = 12.79 \, m/s$$

5) Determine the x and y components of the velocities with which vehicles #1 and #2 entered the asphalt.

For vehicle M:

$$V_{iMax} = V_{iMa}\cos\theta_M \qquad V_{iMay} = V_{iMa}\sin\theta_M$$

$$V_{iMax} = 12.2\cos(45°) \qquad V_{iMay} = 12.2\sin(45°)$$

$$V_{iMax} = 8.63\ m/s \qquad V_{iMay} = 8.63\ m/s$$

For vehicle A:

$$V_{iAax} = V_{iAa}\cos\theta_A \qquad V_{iAay} = V_{iAa}\sin\theta_A$$

$$V_{iAax} = 12.79\cos(35°) \qquad V_{iAay} = 12.79\sin(35°)$$

$$V_{iAax} = 10.48\ m/s \qquad V_{iAay} = 7.34\ m/s$$

V(iAax) represents the x component of the initial speed of vehicle A on asphalt

V(iAay) represents the y component of the initial speed of vehicle A on asphalt

V(iMax) represents the x component of the initial speed of vehicle M on asphalt

V(iMay) represents the y component of the initial speed of vehicle M on asphalt

6) Use your conservation of momentum expressions from part 2 to determine each vehicle's impact speed.

$$m_m \vec{V}_m = m_m \vec{V}'_{mx} + m_A \vec{V}'_{Ax}$$

$$\vec{V}_m = \vec{V}'_{mx} + \frac{m_A \vec{V}'_{Ax}}{m_m}$$

$$\Downarrow$$

$$V_m = V_{iMax} + \frac{m_A V_{iAax}}{m_m}$$

$$V_m = 8.63 + \frac{(1430)(10.48)}{1950}$$

$$V_m = 16.32 \text{ m/s}$$

$$V_m = 58.75 \text{ km/h}$$

$$m_A \vec{V}_A = m_m \vec{V}'_{my} + m_A \vec{V}'_{Ay}$$

$$\vec{V}_A = \frac{m_m \vec{V}'_{my}}{m_A} + \vec{V}'_{Ay}$$

$$\Downarrow$$

$$V_A = \frac{m_m V_{iMay}}{m_A} + V_{iAay}$$

$$V_A = \frac{(1950)(8.63)}{1430} + 7.34$$

$$V_A = 19.11 \text{ m/s}$$

$$V_A = 68.8 \text{ km/h}$$

Therefore, M's impact speed was 58.75 km/h

Therefore, A's impact speed was 68.8 km/h

V(iAax) represents the x component of the initial speed of vehicle A on asphalt

V(iAay) represents the y component of the initial speed of vehicle A on asphalt

V(iMax) represents the x component of the initial speed of vehicle M on asphalt

V(iMay) represents the y component of the initial speed of vehicle M on asphalt

6) Given vehicle #2's maximum acceleration, calculate its minimum speed at the stop sign, given that the crash site was 9.1 m north of the stop sign. Calculate the time it took for vehicle #2 to accelerate from this speed to the impact speed.

$$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a} \cdot \Delta\vec{d}$$

$$\vec{v}_1 = \sqrt{\vec{v}_2^{\,2} - 2\vec{a} \cdot \Delta\vec{d}}$$

$$\vec{v}_1 = \sqrt{(19.11)^2 - 2(1.96)(9.1)}$$

$$\vec{v}_1 = 18.15 \text{ m/s}$$

$$\vec{v}_1 = 65.34 \text{ km/h}$$

Therefore, at maximum acceleration, A must have been traveling at a speed of 65.34 km/h when crossing the stop sign.

$$\Delta t = \frac{2\Delta d}{\vec{v}_1 + \vec{v}_2}$$

$$\Delta t = \frac{2(9.1)}{(18.15 + 19.11)}$$

$$\Delta t = 0.49 \text{ s}$$

Therefore, A traveled from the stop sign to the crash in 0.49 seconds

7) Calculate how far vehicle #1 would go during this time if it traveled at a constant speed equal to its impact speed.

$$\Delta\vec{d} = \vec{v}\Delta t$$

$$\Delta\vec{d} = (16.32)(0.49)$$

$$\Delta\vec{d} = 8.00\,m$$

Therefore, M would have traveled 8.00 m at a constant velocity

Assess the claims of Michael James and Lady Agatha. Whose story is confirmed by the crash analysis?

- Vehicle 1, M, was speeding, going at 59 km/h in a 40 km/h zone
- Vehicle 2, A, was also speeding, going at 69 km/h in a 40 km/h zone
- Given a maximum acceleration for vehicle 2 (A) of 1.96 m/s^2, vehicle 2 could NOT have accelerated to 69 km/h from rest
- Given the 300 m distance from the Hillcrest, Lady Agatha should have been able to see Michael before she entered the intersection, as he was less than 300 m away when she was at the stop sign

Michael was speeding, however, Lady Agatha ran the stop sign at a 69 km/h, meaning she was also speeding - and, she should have been able to see him prior to crossing the intersection. Thus, NEITHER of their stories are true according to the crash analysis.

Write a paragraph summarizing the crash analysis, including any recommendations for charges to be laid.

Both Michael and Lady Agatha should be charged with speeding fines, but Michael should have a smaller fine, as he only went over the limit by 19 km/h, whereas and Lady Agatha went over by 29 km/h. Lady Agatha should also be charged for running a stop sign, as well as driving negligently, as she clearly did not check for cars before entering the intersection.