

Ex 1:

Compute $u \cdot v$ where:

a/ $u = (2, -1, 3)$, $v = (-1, 1, 1)$; b/ $u = (-2, 1, 4)$, $v = (-1, 5, 1)$

Ex 2:

Compute $\|v\|$ if v equals

a/ $(2, -1, 2)$ b/ $2(1, 1, -1)$ c/ $-3(1, 1, 2)$ d/ $(1, 2, 3) - (4, 1, 2)$

Ex 3:

a/ Find a **unit vector** in the direction from $(3, -1, 4)$ to $(1, 3, 5)$.

b/ Let $u = (1, -3, -2)$, $v = (-1, 1, 0)$ and $w = (1, 2, -3)$. Compute $\|u - v + w\|$

Ex 4:

a/ Find $\|v - 3w\|$ when $\|v\| = 2$, $\|w\| = 1$, and $v \cdot w = 2$.

b/ Let $u, v \in \mathbb{R}^3$ such that $\|u\| = 3$, $\|v\| = 4$ and $u \cdot v = -2$.

Evaluate $\|2u + 3v\|$ and $\|2u - v\|$.

Ex 5:

Compute the angle between u and v :

a/ $u = (-1, 1, 2)$, $v = (-1, 2, 1)$

b/ $u = (1, -1, 4)$, $v = (5, 2, -1)$

c/ $u = (2, 1, 5)$, $v = (0, 3, 1)$

Ex 6:

Find all real numbers x such that:

a/ $(3, -1, 2)$ and $(3, -2, x)$ are orthogonal.

b/ $(2, -1, 1)$ and $(1, x, 2)$ are at an angle of $\pi/3$.

Ex 7:

Find the three internal angles of the triangle with vertices:

a/ $A(3, 1, -2)$, $B(3, 0, -1)$, and $C(5, 2, -1)$

b/ $A(3, 1, -2)$, $B(5, 2, -1)$, and $C(4, 3, -3)$

Ex 8:

In each case, compute the projection of u on v

$$\text{a/ } u = \begin{bmatrix} 5 \\ 7 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$\text{b/ } u = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$$\text{c/ } u = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, v = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

$$\text{d/ } u = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}, v = \begin{bmatrix} -6 \\ 4 \\ 2 \end{bmatrix}$$

Ex 9:

Find the projection of $u = (2, -3, 1)$ on $d = (-1, 1, 3)$ and express $u = u_1 + u_2$ where u_1 is parallel to d and u_2 is orthogonal to d .

Ex 10:

a/ Find the equations of the line through the points $P_0(2,0,1)$ and $P_1(4,-1,1)$

b/ Find the equations of the line through $P_0(3,-1,2)$ parallel to the line with

$$\text{equations: } \begin{cases} x = -1 + 2t \\ y = 1 + t \\ z = -3 + 4t \end{cases}$$

c/ Determine whether the following lines intersect and, if so, find the point of

$$\text{intersection } \begin{cases} x = 1 - 3t \\ y = 2 + 5t \\ z = 1 + t \end{cases}, \begin{cases} x = -1 + s \\ y = 3 - 4s \\ z = 1 - s \end{cases}$$

Ex 11:

- a/ Find an equation of the plane through $P_0 (1, -1, 3)$ with $n = (-3, -1, 2)$ as normal
- b/ Find an equation of the plane through $P_0(3, -1, 2)$ that is parallel to the plane with equation $2x - 3y - z = 6$.
- c/ Find the equation of the plane through $P(1, 3, -2)$, $Q(1, 1, 5)$, and $R(2, -2, 3)$.

Ex 12:

a/ Show that the points $P(3, -1, 1)$, $Q(4, 1, 4)$, and $R(6, 0, 4)$ are the vertices of a right triangle.

b/ Find the area of the triangle.

Ex 13:

- a/ Find the area of the triangle with vertices $P(2, 1, 0)$, $Q(3, -1, 1)$, and $R(1, 0, 1)$
- b/ Find the volume of the parallelepiped determined by the vectors
 $u = (1, 2, -1)$, $v = (3, 4, 5)$ and $w = (-1, 2, 4)$.

Ex 14:

Find the shortest distance from the point $P(2, -1, -3)$ to the plane with equation $3x - y + 4z = 1$. Also find the point Q on this plane closest to P .

Ex 15:

Find the equations of the line of intersection of the following planes.

a/ $2x - 3y + 2z = 5$ and $x + 2y - z = 4$.

b/ $3x + y - 2z = 1$ and $x + y + z = 5$.

Ex 16:

Find the shortest distance between the nonparallel lines

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + s \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$