Chapter 01_Assignment _Answer

Ex 1: Find the domain of each function:

a/ $f(x)=\sqrt{x+2}$

$$x+2\geq 0 \Leftrightarrow x\geq -2 \to D=[-2,\infty)$$

b/ $f(x)=\dfrac{1}{x^2-x}$

$$x^2-x\neq 0 \Leftrightarrow x\neq 0, x\neq 1 \to D=R\setminus\{0,1\}$$

c/ $f(x)=\ln\left(x^2-1\right)-\dfrac{x}{\sqrt{x-1}}$

$$\begin{cases} x^2-1>0 \\ x-1>0 \end{cases} \Leftrightarrow x>1 \to D=(1,\infty)$$

Ex 2: Find the **range** of each function:

a/ $f(x)=\sqrt{x-1}$

$$D=[1,\infty)\to G=[0,\infty)$$

b/ $f(x)=x^2-2x$

$$D=R$$

$$f(x)=x^2-2x=x^2-2x+1-1=(x-1)^2-1\geq -1$$

$$G=[-1,\infty)$$

c/ $f(x)=\sin(x)$

$$G=[-1,1]$$

Chapter 01_Assignment _Answer

Ex 3: Determine whether is even, odd, or neither

a/ $f\left(x\right)=\dfrac{x}{x^2+1}$ $\quad D=R$

$$f\left(-x\right)=\frac{-x}{\left(-x\right)^2+1}=-\frac{x}{x^2+1}=-f\left(x\right)$$

b/ $f\left(x\right)=\dfrac{x^2}{x^4+1}$ $\quad D=R$

$$f\left(-x\right)=\frac{\left(-x\right)^2}{\left(-x\right)^4+1}=\frac{x^2}{x^4+1}=f\left(x\right)$$

c/ $f\left(x\right)=\dfrac{x}{x+1}$ $\quad D=R\setminus\left\{-1\right\}$

$$f\left(-x\right)=\frac{-x}{-x+1}\neq f\left(x\right),\neq -f(x)$$

Chapter 01_Assignment _Answer

Ex 4: Let $f(x)=\dfrac{x^2+x+1}{x}$. Find:

a/ $f(2x-1)=\dfrac{(2x-1)^2+(2x-1)+1}{2x-1}=\dfrac{4x^2-2x+1}{2x-1}$

b/ $f\left(x+\dfrac{1}{x}\right)=\dfrac{\left(x+\dfrac{1}{x}\right)^2+\left(x+\dfrac{1}{x}\right)+1}{x+\dfrac{1}{x}}$

$$=\frac{x^4+x^3+3x^2+x+1}{x\left(x^2+1\right)}$$

Chapter 01_Assignment_Answer

Ex 5:

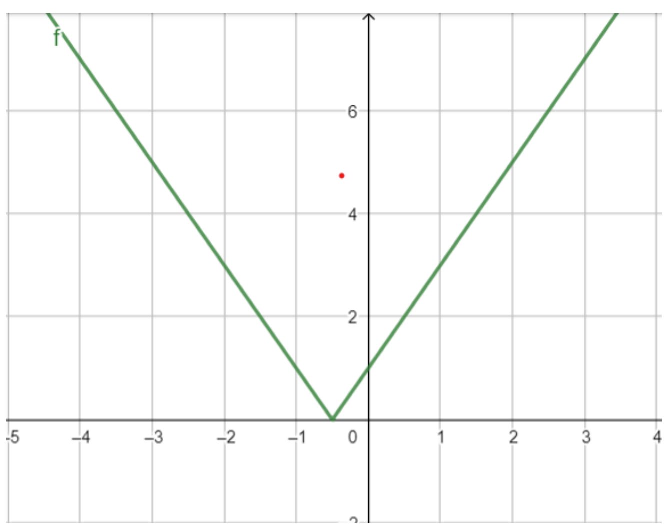
a/ If $f(x)=\dfrac{x^2-x}{x-1}$ and $g(x)=x$ is it true that $f=g$?

Because $g(1)=1, f(1)$ **dose not exist, so** $f\neq g$.

b/ Find the domain and sketch the graph of the function

$$y=|2x+1|$$

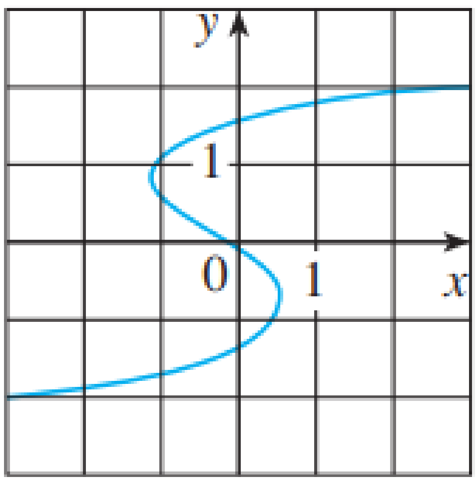
Domain: $D=\mathbb{R}$



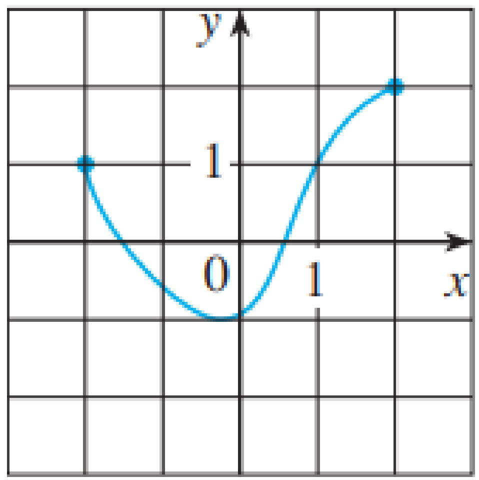
Ex 6:

Determine whether the curve is the graph of a function of x . If it is, state the domain and range of the function.

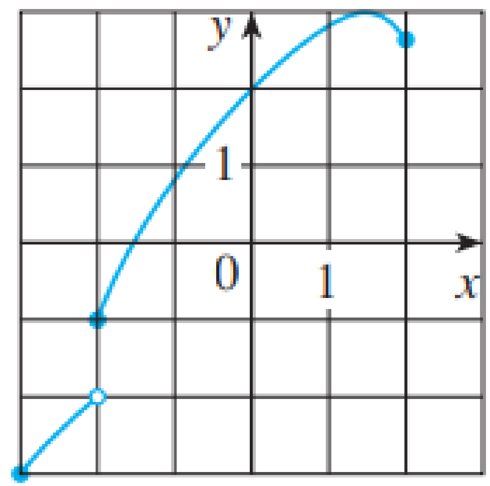
a/



b/



c/



Not a function

A function

$$D=[-2,2]$$

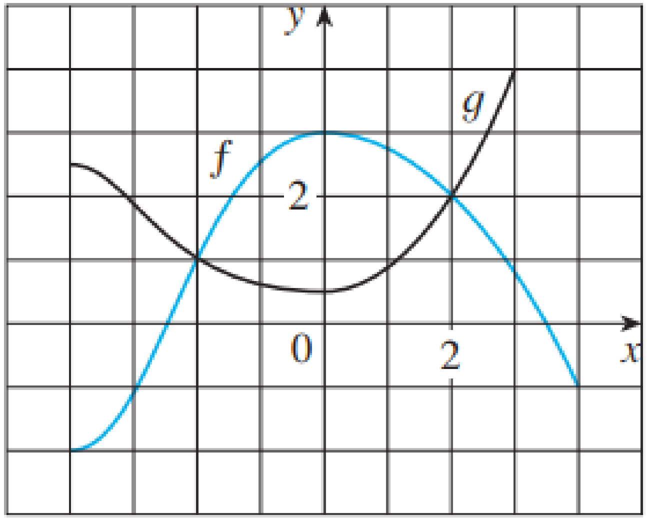
$$G=[-1,2]$$

A function

$$D=[-3,2]$$

$$G=[-3,-2)\cup[-1,3]$$

Ex 7:



The graphs of f and g are given

a/ State the values of $f(-4), g(3)$.

b/ For what values of x is

$f(x) = g(x)$?

c/ Estimate the solution of the equation $f(x) = -1$.

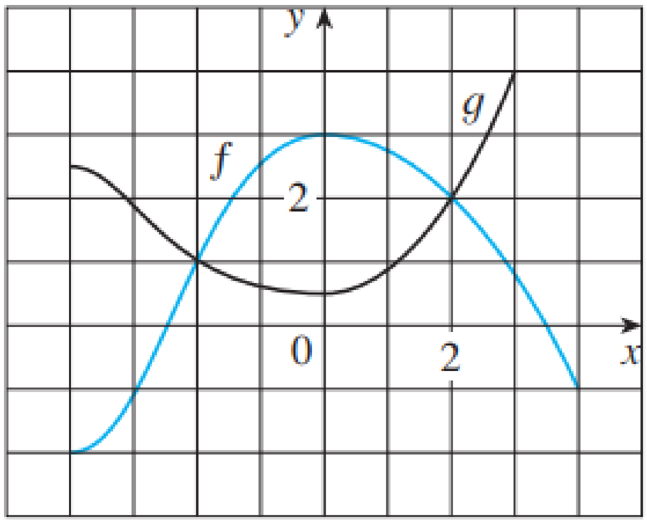
d/ State the domain and range of f.

e/ State the domain and range of g.

Chapter 01_Assignment _Answer

Ex 7:

a/ $f(-4) = -2, g(3) = 4.$



b/ $x = \pm 2$

c/ $x = 4 \text{ or } x = -3$

d/ Domain $D = [-4, 4]$ and range $G = [-2, 3]$

e/ Domain $D = [-4, 3]$ and range $G = \left[\frac{1}{2}, 4\right]$

Ex 8:

Explain how the following graphs are obtained from the graph of $f\left(x\right)$:

a/ $f\left(x-4\right)$

Shift the graph 4 units to the right.

b/ $f\left(x\right)+3$

Shift the graph 3 units up.

Chapter 01_Assignment_Answer

Ex 8:

Explain how the following graphs are obtained from the graph of $f(x)$:

c/ $f(x-2)-3$

Shift the graph 2 units to the right and 3 units down.

d/ $f(x+5)-4$

Shift the graph 5 units to the left and 4 units down.

Chapter 01_Assignment_Answer

Ex 9:

Suppose that the graph of $f\left(x\right)=\sqrt{x}$ is given. Describe how the graph of the function $g\left(x\right)=\sqrt{x-1}+2$ can be obtained from the graph of $f\left(x\right)$.

We have $g\left(x\right)=\sqrt{x-1}+2=f\left(x-1\right)+2$

Shift the graph 1 units to the right and 2 units up.

Chapter 01_Assignment_Answer

Ex 10: Let $f(x)=\sqrt{x}$ and $g(x)=\sqrt{2-x}$. Find:

a/ $f\circ g=f\left(g\left(x\right)\right)=f\left(\sqrt{2-x}\right)=\sqrt{\sqrt{2-x}}$

b/ $g\circ f=g\left(f\left(x\right)\right)=g\left(\sqrt{x}\right)=\sqrt{2-\sqrt{x}}$

Chapter 01_Assignment_Answer

Ex 10: Let $f(x)=\sqrt{x}$ and $g(x)=\sqrt{2-x}$. Find:

c/ $f\circ f = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$

d/ $g\circ g = g(g(x)) = g(\sqrt{2-x}) = \sqrt{2-\sqrt{2-x}}$

Chapter 01_Assignment_Answer

Ex 11: Use the table to evaluate each expression

x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	2	3

a/ $f\left(g\left(1\right)\right) = f\left(6\right) = 5$

b/ $g\left(f\left(1\right)\right) = g\left(3\right) = 2$

c/ $f\left(f\left(1\right)\right) = f\left(3\right) = 4$

Chapter 01_Assignment_Answer

Ex 11: Use the table to evaluate each expression

x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	2	3

d/ $g\left(g(1)\right) = g(6) = 3$

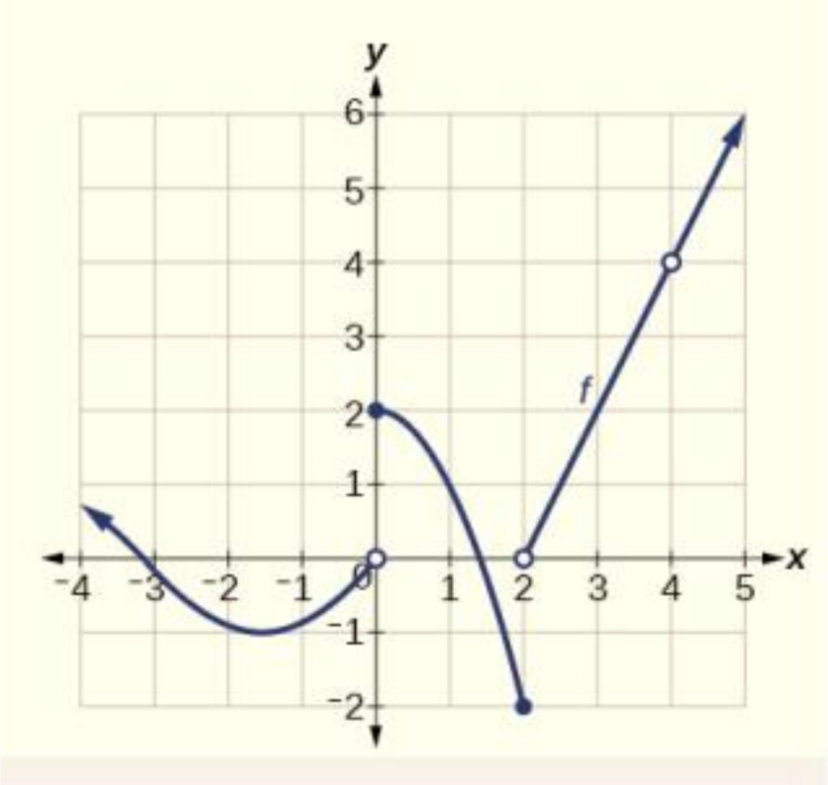
e/ $(g \circ f)(3) = g\left(f(3)\right) = g(4) = 1$

g/ $(g \circ f)(6) = g\left(f(6)\right) = g(5) = 2$

Chapter 01_Assignment_Answer

Ex 12: The graph of f is given.

a/ Find each limit, or explain why it does not exist



i/ $\lim_{x\to 0^+} f(x) = 2$

$\lim_{x\to 0^-} f(x) = 0$

$\lim_{x\to 0} f(x)$

does't exist

ii/ $\lim_{x\to 1} f(x) = 1 = f(1)$

iii/ $\lim_{x\to 4} f(x) = 4 \neq f(4)$

Ex 13: Find the limit, if it exists. If the limit does not exist, explain why.

a/ $\lim\limits_{x\to -2}\dfrac{2-|x|}{2+x}\left(\dfrac{0}{0}\right)=\lim\limits_{x\to -2}\dfrac{2+x}{2+x}=1$

b/ $\lim\limits_{x\to 0.5}\dfrac{2x-1}{|2x^3-x^2|}\left(\dfrac{0}{0}\right)$

$\lim\limits_{x\to 0.5^+}\dfrac{2x-1}{|2x^3-x^2|}\left(\dfrac{0}{0}\right)=4;\ \lim\limits_{x\to 0.5^-}\dfrac{2x-1}{|2x^3-x^2|}\left(\dfrac{0}{0}\right)=-4$

Because $\lim\limits_{x\to 0.5^+}\dfrac{2x-1}{|2x^3-x^2|}\neq\lim\limits_{x\to 0.5^-}\dfrac{2x-1}{|2x^3-x^2|}$**,so** $\lim\limits_{x\to 0.5}\dfrac{2x-1}{|2x^3-x^2|}$

Dose not exist.

Chapter 01_Assignment_Answer

Ex 14: Given that $\lim\limits_{x\to 2} f(x)=4;\ \lim\limits_{x\to 2} g(x)=-2;\ \lim\limits_{x\to 2} h(x)=0$ find the limits that exist. If the limit does not exist, explain why.

a/ $\lim\limits_{x\to 2}\left(f(x)+5g(x)\right)=\lim\limits_{x\to 2} f(x)+\lim\limits_{x\to 2} 5g(x)$

$=\lim\limits_{x\to 2} f(x)+5\lim\limits_{x\to 2} g(x)=4+5(-2)=-6$

b/ $\lim\limits_{x\to 2}\left[g(x)\right]^3=\left[\lim\limits_{x\to 2} g(x)\right]^3=(-2)^3=-8$

c/ $\lim\limits_{x\to 2}\sqrt{f(x)}=\sqrt{\lim\limits_{x\to 2} f(x)}=\sqrt{4}=2$

Chapter 01_Assignment_Answer

Ex 14: Given that $\lim_{x\to2} f(x)=4;\ \lim_{x\to2} g(x)=-2;\ \lim_{x\to2} h(x)=0$ find the limits that exist. If the limit does not exist, explain why.

d/ $\lim_{x\to2}\dfrac{3f(x)}{g(x)}=\dfrac{3\lim_{x\to2} f(x)}{\lim_{x\to2} g(x)}=\dfrac{3.4}{-2}=-6$

e/ $\lim_{x\to2}\dfrac{g(x)}{h(x)}$ dose not exist.

Because , if $x\to2^+ : h(x)\to0^+ : \lim_{x\to2^+}\dfrac{g(x)}{h(x)}=\dfrac{\lim_{x\to2^+} g(x)}{\lim_{x\to2^+} h(x)}=-\infty$

$x\to2^- : h(x)\to0^- : \lim_{x\to2^-}\dfrac{g(x)}{h(x)}=\dfrac{\lim_{x\to2^-} g(x)}{\lim_{x\to2^-} h(x)}=+\infty$

Chapter 01_Assignment_Answer

Ex 14: Given that $\lim_{x\to 2} f(x) = 4; \lim_{x\to 2} g(x) = -2; \lim_{x\to 2} h(x) = 0$ find the limits that exist. If the limit does not exist, explain why.

Then $\lim_{x\to 2^+} \dfrac{g(x)}{h(x)} \neq \lim_{x\to 2^-} \dfrac{g(x)}{h(x)}$

f/ $\lim_{x\to 2} \dfrac{g(x)h(x)}{f(x)} = \dfrac{\lim_{x\to 2} g(x)h(x)}{\lim_{x\to 2} f(x)} = \dfrac{\lim_{x\to 2} g(x) \lim_{x\to 2} h(x)}{\lim_{x\to 2} f(x)} = 0$

Ex 15:

- $\lim\limits_{x \to 1^-} g(x) = \lim\limits_{x \to 1^-} x = 1$

- $\lim\limits_{x \to 1^+} g(x) = \lim\limits_{x \to 1^+} \left(2 - x^2\right) = 1$

- $\lim\limits_{x \to 1} g(x) = 1$ • $g(1) = 3$

- $\lim\limits_{x \to 2^-} g(x) = \lim\limits_{x \to 2^-} \left(2 - x^2\right) = -2$

- $\lim\limits_{x \to 2^+} g(x) = \lim\limits_{x \to 2^+} (x - 3) = -1$

- $\lim\limits_{x \to 2} g(x)$ does't exist because $\lim\limits_{x \to 2^-} g(x) \neq \lim\limits_{x \to 2^+} g(x)$

Ex 16: Evaluate the following limits

a/ $\lim_{x \to 3} \dfrac{x^2 + x - 12}{x - 3} = 7$

b/ $\lim_{x \to -1} \dfrac{x^2 + 2x + 1}{x^4 - 1} = 0$

Chapter 01_Assignment_Answer

Ex 16: Evaluate the following limits

c/ $\lim_{h \to 0} \frac{(3+h)^2 - 9}{h} = 6$

d/ $\lim_{t \to 0} \frac{\sqrt{t^2 + 9} - 3}{t^2} = \frac{1}{6}$

Chapter 01_Assignment_Answer

Ex 16: Evaluate the following limits

e/ $\lim_{x\to -1} \dfrac{2x^2+3x+1}{x^2-2x-3} = \dfrac{1}{4}$

f/ $\lim_{x\to 1} \dfrac{x^2-1}{|x-1|}$ **does't exists.**

Ex 17: Evaluate the following limits

a/ $\lim_{x\to 3}\frac{x^6-1}{x^{10}-1}=\frac{3^6-1}{3^{10}-1}=\frac{91}{7381}$

b/ $\lim_{x\to -4}\frac{\frac{1}{4}+\frac{1}{x}}{x+4}\left(\frac{0}{0}\right)=\lim_{x\to -4}\frac{\frac{x+4}{4x}}{x+4}=\lim_{x\to -4}\frac{1}{4x}=-\frac{1}{16}$

Chapter 01_Assignment_Answer

Ex 17: Evaluate the following limits

c/ $\lim_{x\to 0} \dfrac{x}{\sqrt{1+3x}-1} \left(\dfrac{0}{0}\right) = \dfrac{2}{3}$

d/ $\lim_{x\to 0} \dfrac{\sqrt{3+x}-\sqrt{3}}{x} \left(\dfrac{0}{0}\right) = \dfrac{1}{2\sqrt{3}}$

Chapter 01_Assignment_Answer

Ex 17: Evaluate the following limits

e/ $\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right)(\infty-\infty)=\lim_{t\to 0}\frac{1}{t+1}=1$

f/ $\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}\left(\frac{0}{0}\right)=3x^2$

Chapter 01_Assignment_Answer

Ex 18:

a/ If $4x-9\le f(x)\le x^2-4x+7$, find $\lim\limits_{x\to 4} f(x)$

Beacause $\lim\limits_{x\to 4}(4x-9)=\lim\limits_{x\to 4}\left(x^2-4x+7\right)=7$

So, $\lim\limits_{x\to 4} f(x)=7.$

Ex 18:

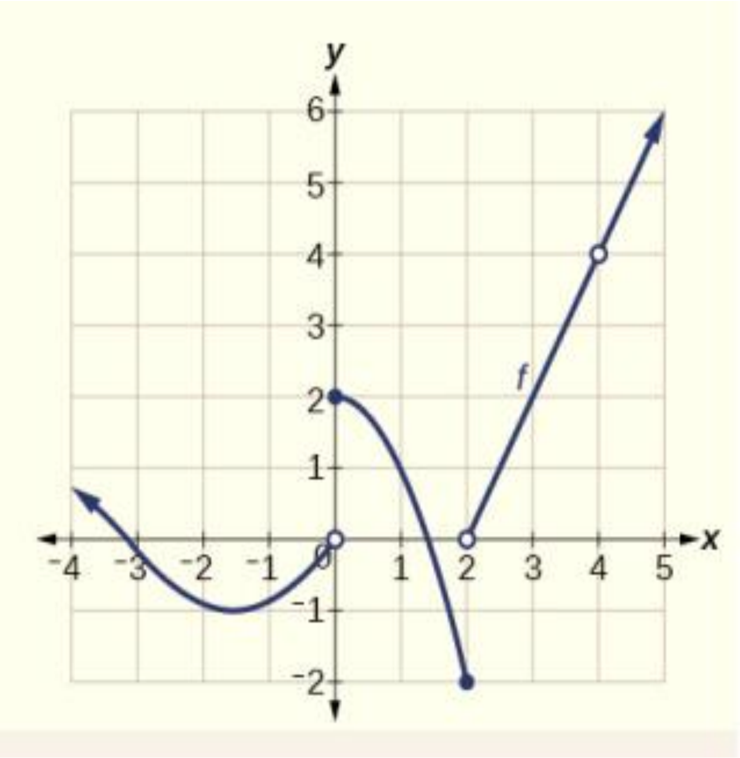
b/ Find the limit $\lim\limits_{x\to 0}\sqrt{x^3+x^2}\sin\left(\frac{\pi}{x}\right)$

Hint:

$$-\sqrt{x^3+x^2} \le \sqrt{x^3+x^2}\sin\left(\frac{\pi}{x}\right) \le \sqrt{x^3+x^2}$$

Chapter 01_Assignment_Answer

Ex 19: The graph of f is given. At what numbers is discontinuous ?



- f **is discontinuous at** $x = 4$

because $D = \mathbb{R} \setminus \{4\}$

- f **is discontinuous at** $x = 0$

because

$$\lim_{x \to 0^+} f(x) = 2 \neq \lim_{x \to 0^-} f(x) = 0$$

- f **is discontinuous at** $x = 2$

because $\lim_{x \to 2^+} f(x) = 0 \neq \lim_{x \to 2^-} f(x) = -2$

Chapter 01_Assignment_Answer

Ex 20: Determine where the function $f(x)$ is continuous

a/ $f(x)=\dfrac{2x^2+x-1}{x-2}$ $\quad D=R\setminus\{2\}$

b/ $f(x)=\dfrac{x-9}{\sqrt{4x^2+4x+1}}$ $\quad D=R\setminus\left\{-\dfrac{1}{2}\right\}$

$$4x^2+4x+1>0\Leftrightarrow(2x+1)^2>0\Leftrightarrow 2x+1\neq 0\Leftrightarrow x\neq-\frac{1}{2}$$

c/ $f(x)=\ln(2x+5)$ $\quad D=\left(-\dfrac{5}{2},\infty\right)$

Chapter 01_Assignment_Answer

Ex 21: Find the constant m that makes f continuous on R

a/ $$f(x)=\begin{cases} x^2-m^2, x<4 \\ mx+20, x\geq 4 \end{cases}$$

For all $x\neq 4$, $f(x)$ is polynomial function, so $f(x)$ is continuous.

At $x=4$: $\lim\limits_{x\to 4^+} f(x)=\lim\limits_{x\to 4^+}(mx+20)=4m+20$

$$\lim_{x\to 4^-} f(x)=\lim_{x\to 4^-}\left(x^2-m^2\right)=16-m^2$$

$$f(4)=4m+20$$

$$\lim_{x\to 4^+} f(x)=\lim_{x\to 4^-} f(x)=f(4)\Leftrightarrow 16-m^2=4m+20\Leftrightarrow m=-2$$

Chapter 01_Assignment_Answer

Ex 21: Find the constant m that makes f continuous on R

b/ $f\left(x\right)=\begin{cases} mx^2+2x, x<2 \\ x^3-mx, \quad x\geq 2 \end{cases}$ **Ans:** $m=\dfrac{2}{3}$

c/ $f\left(x\right)=\begin{cases} \dfrac{x^2-1}{\sqrt{x}-1}, x>1 \\ mx+1, \quad x\leq 1 \end{cases}$ **Ans:** $m=3$

Chapter 01_Assignment_Answer

Ex 22: Is there a number m such that

$$\lim_{x\to -2}\frac{3x^2+mx+m+3}{x^2+x-2}$$

exists ? If so, find the value of m and the value of the limit.

$$m=15$$

$$\lim_{x\to -2}\frac{3x^2+15x+18}{x^2+x-2}\left(\frac{0}{0}\right)=\lim_{x\to -2}\frac{(x+2)(3x+9)}{(x+2)(x-1)}$$

$$=\lim_{x\to -2}\frac{3x+9}{x-1}=-1$$

Ex 23:

If $\lim\limits_{x\to1}\dfrac{f(x)-8}{x-1}=10$, find $\lim\limits_{x\to1} f(x)$

If $\lim\limits_{x\to1} f(x)\neq 8$ then $\lim\limits_{x\to1}\dfrac{f(x)-8}{x-1}$ does't exist.

So, $\lim\limits_{x\to1} f(x)=8$

Chapter 01_Assignment_Answer

Ex 24:

Find the horizontal and vertical asymptotes of each curve.

a/ $y = \dfrac{2x^2 + x - 1}{x^2 + x - 2}$

$x = 1, x = -2$

$y = 2$

b/ $y = \dfrac{x - 9}{\sqrt{4x^2 + 3x + 2}}$

$y = \dfrac{1}{2}; y = -\dfrac{1}{2}$

Ex 25: Evaluate the limits:

$$\text{a/ } \lim_{x\to 0}\frac{\sin 3x}{5x^3-4x}\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\sin 3x}{5x^3-4x}=\lim_{x\to 0}\frac{3x}{5x^3-4x}$$

$$=\lim_{x\to 0}\frac{3}{5x^2-4}=-\frac{3}{4}$$

Ex 25: Evaluate the limits:

b/ $$\lim_{\theta\to 0}\frac{1-\cos\theta}{\sin\theta}\left(\frac{0}{0}\right)=\lim_{\theta\to 0}\frac{(1-\cos\theta)(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$=\lim_{\theta\to 0}\frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}=\lim_{\theta\to 0}\frac{\sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$=\lim_{\theta\to 0}\frac{\sin\theta}{1+\cos\theta}=0$$

Chapter 01_Assignment_Answer

Ex 25: Evaluate the limits:

c/ $$\lim_{x\to 0}\frac{x\sin 2x}{e^{9x^2}-1}\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{x\times 2x}{9x^2}=\lim_{x\to 0}\frac{2}{9}=\frac{2}{9}$$