

$$-2x^3 + x^2$$

$$\frac{2x-1}{-2x^3 + x^2}$$

$$\frac{2x-1}{-x^2(2x-1)}$$

$$\frac{1}{-x^2}$$

~~Set~~ $x = \frac{1}{2}$ out this situation: $\frac{1}{-(\frac{1}{2})^2} = -4$.

$$Af: S(1) \underset{x \rightarrow \frac{1}{2}}{\lim} f(x) = u \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{No Limits}$$

$$S(2) \underset{x \rightarrow \frac{1}{2}}{\lim} f(x) = -4$$

$$Fx 1d: \underset{x \rightarrow 2}{\lim} f(x) = u; \underset{x \rightarrow 2}{\lim} g(x) = -2; \underset{x \rightarrow 2}{\lim} h(x) = 0$$

$$a) \underset{x \rightarrow 2}{\lim} (f(x) + 5g(x)) = \underset{x \rightarrow 2}{\lim} (u + 5(-2)) = -6$$

$$b) \underset{x \rightarrow 2}{\lim} [g(x)]^3 = (-1)^3 = 0$$

$$c) \underset{x \rightarrow 2}{\lim} \sqrt{f(x)} = \sqrt{u} = 2$$

$$d) \underset{x \rightarrow 2}{\lim} \frac{3f(x) - 2g(x)}{h(x)} = \frac{3 \cdot u - 2 \cdot (-2)}{0} = -6$$

$$e) \underset{x \rightarrow 2}{\lim} \frac{g(x)}{h(x)} = \frac{-2}{0} = 0$$

$$f) \underset{x \rightarrow 2}{\lim} \frac{g(x) \cdot h(x)}{f(x)} = \frac{-2 \cdot 0}{4} = 0$$

Thứ ngày



Ex 16
a) $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x - 3}$

$$\frac{(x-3)(x+4)}{(x-3)}$$

$$\lim_{x \rightarrow 3} \frac{x+4}{(x-3)} = 7$$

b) $\lim_{x \rightarrow -1} \frac{x^2 + 2x + 1}{x^4 - 1}$

$$\frac{(x+1)(x+1)}{[(x-1)(x+1)]^2}$$

$$\frac{1}{(x-1)^2}$$

$$\frac{1}{x^2 - 2x + 1}$$

$$\lim_{x \rightarrow -1} \frac{1}{x^2 + 2 + 1} = \frac{1}{4}$$

c) $\lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h}$

$$\frac{(3+h-3)(3+h+3)}{h}$$

$$\lim_{h \rightarrow 0} \frac{6+h}{h} = 6$$

d) $\lim_{t \rightarrow 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$

$$\frac{(t^2 + 9 - 9)(t^2 + 9 + 9)}{t^4}$$
$$\frac{(t^2)(t^2 + 18)}{t^4}$$
$$\frac{(t^2 + 18)}{t^2}$$

EX 12:

i) $\lim_{x \rightarrow 0^+} f(x) = 2$

$\lim_{x \rightarrow 0^-} f(x)$ not exist.

$\lim_{x \rightarrow 0} f(x)$ not exist

ii) $\lim_{x \rightarrow 1} f(x)$

this function not exists at a given point
 \rightarrow no given point

function continuous, not have limit.

iii) $\lim_{x \rightarrow 4} f(x)$

This function have a given point but not exists.

This function is a continuous function, because does not have any ~~shy~~ split point in curve line.

EX 13:

a) $\lim_{x \rightarrow -2} \frac{2 - |x|}{2 + x}$

• $x \rightarrow -2 \rightarrow |x| = -x$.

• $\lim_{x \rightarrow -2} \frac{2 - (-x)}{2 + x}$

$\lim_{x \rightarrow -2} \frac{2 + x}{2 + x}$

- Does not split 2 situation because -2^+ L.O.
 $\Rightarrow -2^-$ L.O.

Thứ ngày



$$Ex_{10}: f(x) = \sqrt{x} \quad g(x) = \sqrt{2-x}$$

$$a) f \circ g$$

$$f(\sqrt{2-x}) = \sqrt[4]{2-x}$$

$$b) g \circ f$$

$$g(\sqrt{x}) = \sqrt{2-\sqrt{x}}$$

$$c) f \circ f$$

$$f(\sqrt{x}) = \sqrt[3]{x}$$

$$d) g \circ g$$

$$g(\sqrt{2-x}) = \sqrt{2-\sqrt{2-x}}$$

$$Ex_{11}$$

$$a) f(g(1)) \\ = f(6) = 5$$

$$b) g(f(1)) \\ = g(3) = 2$$

$$c) f(f(1)) \\ = f(3) = 4$$

$$d) g(g(1)) \\ = g(6) = 3$$

$$e) (g \circ f)(3) \\ = g(f(3)) = g(4) = 1$$

$$f) (g \circ f)(6) \\ = g(f(6)) = g(5) = 2$$

5

10

15

20

$$\lim_{x \rightarrow -2} \frac{x^2 - 4}{x - 1} = \frac{-4 - 4}{-2 - 1} = -\frac{8}{3} = -\frac{2}{3}$$

Ex 2.1

$$\lim_{x \rightarrow 4^-} f(x) = \begin{cases} x^2 - m^2 & x < 4 \\ mx + 20 & x \geq 4 \end{cases}$$

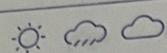
$$\lim_{x \rightarrow 4^-} x^2 - m^2 = \lim_{x \rightarrow 4^+} mx + 20 = f(4)$$

$$16 - m^2 = 4m + 20$$

$$m^2 + 4m + 4 = 0$$

$$m = -2$$

Thứ ngày



Ex 17:

$$\text{a)} \lim_{x \rightarrow 3} \frac{x^6 - 1}{x^{10} - 1}$$

$$\lim_{x \rightarrow 3} \frac{1}{x^4 - 1} = \frac{1}{80}$$

$$\text{b)} \lim_{x \rightarrow -4} \frac{\frac{1}{4} + \frac{1}{x}}{x+4}$$

~~$x \neq -4$~~

$$\lim_{x \rightarrow -4} \frac{x+4}{4x(x+4)} = \frac{1}{4x} = \frac{1}{-16}$$

$$\text{c)} \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x} - 1}$$

$$\frac{x(\sqrt{1+3x} + 1)}{1+3x - 1}$$

~~$\sqrt{1+3x} + 1$~~

$$\frac{x(\sqrt{1+3x} + 1)}{3x}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+3x} + 1}{3} = \frac{2}{3}$$

$$\frac{1-1}{1-1} = m+1$$
$$-2m = -1$$

Ex 19

at $x = 0^-$

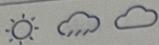
$$x \rightarrow 0^- = 0$$
$$x \rightarrow 0^+ = 2$$

x not defined. $\rightarrow x = 0$ this function
not continuous

AHOMA

HONGHA

Thứ ngày



Ex 18:

$$4x - 9 \leq f(x) \leq x^2 - 4x + 7$$

up Range of $f(x)$: $x^2 - 4x + 7$

$$x \geq 4 \quad x^2 - 4x + 7$$

$$4^2 - 4 \cdot 4 + 7 = 7 \quad \textcircled{1}$$

$$\begin{aligned} &\text{down range of } f(x): 4x - 9 \\ &x \geq 4 \quad 4x - 9 = 16 - 9 = 7 \quad \textcircled{2} \end{aligned}$$

① ② \Rightarrow $\begin{cases} x^2 - 4x + 7 \\ 4x - 9 \end{cases}$ is 2 sides of $f(x)$ limits

$\lim_{x \rightarrow 4} f(x) = 7$.

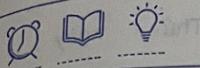
$$\lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin\left(\frac{\pi}{x}\right)$$

$$\sqrt{\pi^2(x+1)} \sin\left(\frac{\pi}{x}\right)$$

$$0 \quad \pi^2 \cdot 10; \quad |x| \sqrt{\pi+1} \cdot \sin \frac{\pi}{x}$$

$$\lim_{x \rightarrow 0} |x| \sqrt{\pi+1} \cdot \sin \frac{\pi}{x}$$

= 0



Thứ 5 ngày

$$d) \lim_{t \rightarrow 0} \frac{\sqrt{t^2 + 9} - 3}{t}$$

$$\begin{aligned} & \frac{(t^2 + 9 - 9)}{t^2} \cdot \frac{(t^2 + 9 + 9)}{(t^2 + 9 - 9)} \\ & \frac{t^2(t^2 + 18)}{t^2(\sqrt{t^2 + 9} - 3)} \end{aligned}$$

$$\lim_{t \rightarrow 0} \frac{t^2 + 18}{\sqrt{t^2 + 9} - 3} = \frac{18}{3+3} = 3$$

$$e) \lim_{x \rightarrow -1} \frac{2x^2 + 3x - 11}{x^2 - 2x - 3}$$

$$\frac{2(x + \frac{1}{2})(x - 1)}{(x - 3)(x + 1)}$$

$$\lim_{x \rightarrow -1} \frac{2x + 1}{x - 3} = \frac{-1}{+4}$$

$$f) \lim_{x \rightarrow 1} \frac{x^2 - 1}{|x - 1|}$$

$$S_1 : \frac{(x+1)(x-1)}{x-1}$$

$$\lim_{x \rightarrow 1} \frac{x+1}{x-1} = 2$$

$$S_2 : \frac{(x+1)(x-1)}{x+1}$$

$$\lim_{x \rightarrow 1} \frac{x-1}{x+1} = 0$$



Thứ ngày

at $x = 2$

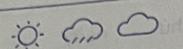
$x \rightarrow 0^- = -2$ $x \rightarrow 0^+ = 2$

x not defined at $x=2$.
→ this function not continuous.

at $x = 4$

$x \rightarrow 4^- = 4$ $x \rightarrow 4^+ = 4$

But x not defined at $x=4$ when $x=4$
→ This function not continuous.



Domain. $x \neq -2$

$$\cancel{x \neq -2} \quad \frac{2+x}{2+x} = 1$$

So if $x \neq -2$ $\lim f(x) = 1$

$$\Rightarrow \lim_{x \rightarrow 0.5} \frac{2x-1}{|2x^3-x^2|}$$

$|2x^3-x^2|$ of $\lim_{x \rightarrow 0.5}$, 2 situation:

$$1) 2x^3-x^2 / D: x \neq \frac{1}{2}; x \neq 0$$

$$2) -2x^3+x^2 / D: x \neq -\frac{1}{2}; x \neq 0$$

$$\frac{2x-1}{|2x^3-x^2|} = \frac{2x-1}{(x^2)(|2x-1|)}$$

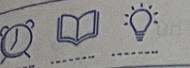
$$= \frac{2x-1}{x^2 |2x-1|}$$

$$|2x-1| \neq 0 \text{ at } 2x-1$$

$x > \frac{1}{2}$ and x must $\neq 0$

If $x > \frac{1}{2}$ + So this function change to: $\frac{1}{x^2}$

$$\lim_{x \rightarrow \frac{1}{2}^+} \frac{1}{(\frac{1}{x})^2} = \frac{1}{\frac{1}{\frac{1}{2}}} = 4$$



$$d) \lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3}}{x} = \frac{1}{2\sqrt{3}}$$

$$e) \lim_{t \rightarrow 0} \left(\frac{1}{t+1} - \frac{1}{t^2+t} \right)$$

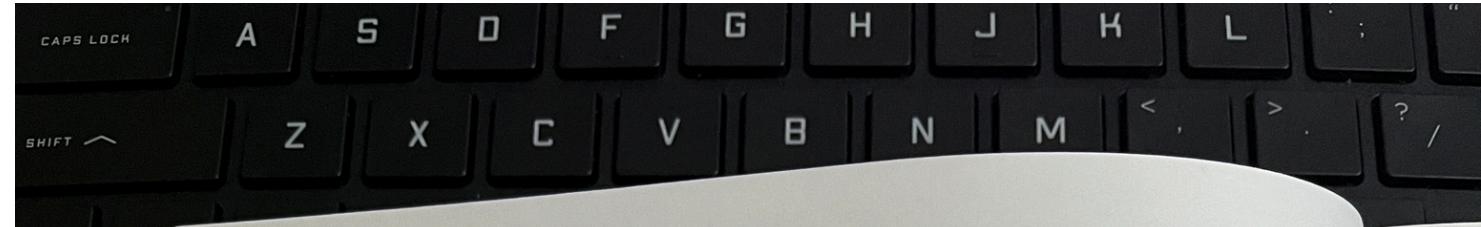
$$\frac{t^2+t - t}{t(t^2+t)}$$

$$\lim_{t \rightarrow 0} \frac{1}{t+1} = 1$$

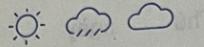
$$f) \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$$

$$\begin{aligned} (x+h)^3 &= x^3 + 3x^2h + 3xh^2 + h^3 \\ &\quad - x^3 + 3x^2h + 3xh^2 + h^3 \\ &= h \end{aligned}$$

$$\lim_{h \rightarrow 0} \frac{3x^2 + 3hx + h^2}{h} = 3x^2$$



Thứ ngày



IC

$$b) f(x) = \begin{cases} mx^2 + 2x, & x < 2 \\ x^3 - mx, & x \geq 2 \end{cases}$$

for function continuous:

$$\lim_{x \rightarrow 2^-} mx^2 + 2x = \lim_{x \rightarrow 2^+} x^3 - mx = f(2)$$

~~2²~~

$$m \cdot 4 + 4 = 8 - 2m$$

$$6m = 4$$

$$\rightarrow m = \frac{2}{3}$$

$$c) f(x) = \begin{cases} \frac{x^2 - 1}{\sqrt{x} - 1}, & x > 1 \\ mx + 1, & x \leq 1 \end{cases}$$

for function continuous:

$$\lim_{x \rightarrow 1^-} \frac{x^2 - 1}{\sqrt{x} - 1} = \lim_{x \rightarrow 1^+} mx + 1 = f(1)$$

$$\frac{1-1}{1-1} = m+1$$

$$\rightarrow m = -1$$

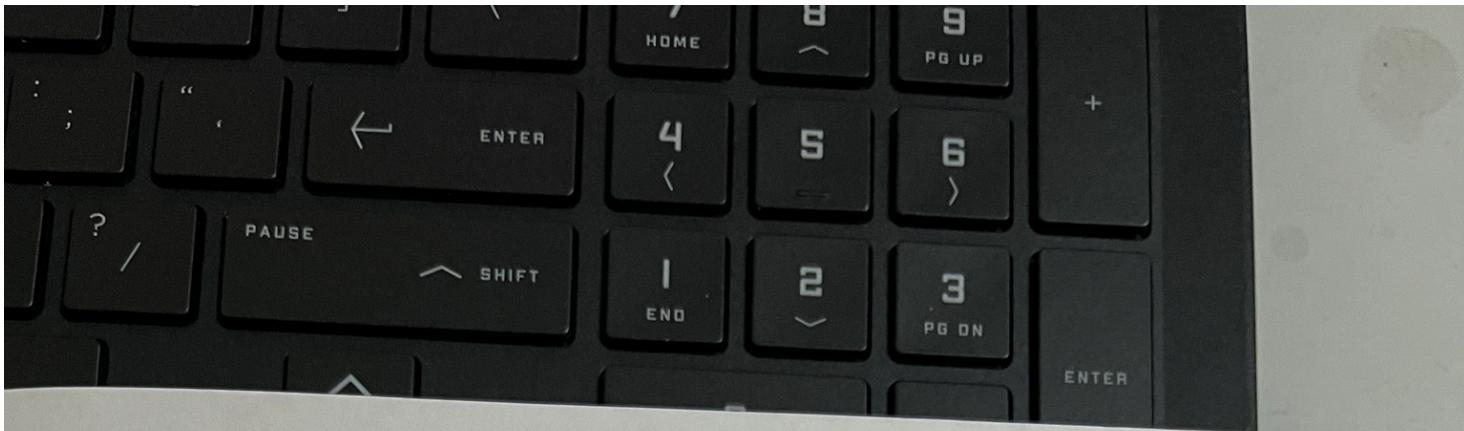
Ex 19

at $x = 0$

$$x \rightarrow 0^- = 0$$

$$x \rightarrow 0^+ = 2$$

x not defined. $\rightarrow x = 0$ this functions
not continuous



Thứ ngày

Ex 22.

Giảm
 $x \rightarrow -2$

$$\frac{3x^2 + mx + m + 3}{x^2 + x - 2}$$

$$x \rightarrow -2 \quad 3(-2)^2 + (-2)m + m + 3 \\ 12 - 2m + m + 3 = 15$$

$$\frac{3x^2 + 15x + 15 + 3}{x^2 + x - 2}$$

$$\frac{3x^2 + 15x + 18}{x^2 + x - 2}$$

$$\frac{3(x+2)(x+3)}{(x-1)(x+2)}$$

Giảm
 $x \rightarrow -2$

$$\frac{3x + 9}{x-1} = \frac{3}{-3} = -1$$

Ex 21

$$a > f(x) = \begin{cases} x^2 - m^2, & x < 4 \\ mx + 20, & x \geq 4 \end{cases}$$

$$mx + 20 = f(4)$$



Thứ ngày

$$\lim_{n \rightarrow 1} f(n) = ?$$

$$\lim_{n \rightarrow 1} \frac{f(n) - 8}{n - 1} = 10$$

$$\frac{f(n) - 8}{n - 1} = 10$$

$$f(n) - 8 = 10n - 10$$

$$f(n) = 10n - 2$$

$$\lim_{n \rightarrow 1} f(n) = 8$$

$$f(n) = n^2 + 19(n+2)$$

$$\frac{dy}{dx} = 0$$

Ex29.

$$y = \frac{2x^2 + x - 1}{x^2 + x - 2}$$

horizontal asymptotes:

$$\lim_{x \rightarrow \infty} y = 2$$

$$\lim_{x \rightarrow -\infty} y = 2$$



$$\begin{aligned} \lim_{n \rightarrow \infty} y &= -1 \\ \lim_{n \rightarrow -\infty} y &= 1 \end{aligned}$$

o Vertical asymptotes:

$$\lim_{x \rightarrow 1^+} y = -\infty \quad \lim_{x \rightarrow 1^-} y = +\infty$$

$$\lim_{x \rightarrow -2^+} y = -\infty \quad \lim_{x \rightarrow -2^-} y = +\infty$$

This function have horizontal line at $y = 2$
vertical line at $x = -1$

$$x - 1 \neq 0, x - 2 \neq 0$$

but DC not defined at $0 < x < 2$ when $x = 1$

\rightarrow This function not continuous.

Ex 20:

$$\text{D: } \frac{2x^2 + x - 1}{x - 2}$$

$$\text{D: } x \neq 2$$

This function is a rational fraction
, continuous except $x = 2$

$$\text{b) } f(x) = \frac{x - 9}{\sqrt{4x^2 + 4x + 1}}$$

$$\text{D: } 4x^2 + 4x + 1 \neq 0 \\ \rightarrow \text{D: } x \neq -\frac{1}{2}$$

This function is a rational fraction, continuous except $x = -\frac{1}{2}$

$$\Rightarrow f(x) = \ln(2x + 5)$$

$$\text{D: } x > -\frac{5}{2}$$

This function is continuous if $x > -\frac{5}{2}$.

$$\lim_{x \rightarrow +\infty} y = \frac{2x - 9}{4x^2 + 3x + 2}$$

◦ horizontal line :

$$\lim_{x \rightarrow +\infty} y = \frac{1}{2} \quad \lim_{x \rightarrow -\infty} y = -\frac{1}{2}$$

◦ vertical line :

$$\lim_{x \rightarrow 2} y = 4x^2 + 3x + 2$$

→ This function do not have x_1, x_2
because.

$$y > 3 > 2$$

$x > 0$

So, this function does not have vertical line.

Ex 23:

