

Ex: a) $f(x) = 5x^4 - 2x^5$, $F_0 = 4$

$$F(x) = 5 \cdot \frac{x^5}{5} - 2 \cdot \frac{x^6}{6} + C$$

$$F(0) = 4 = 0 - \frac{1}{3} \cdot 0 + C \Rightarrow C = 4$$

$$F(x) = -\frac{1}{3}x^6 + x^5 + 4$$

b) $f(x) = 4 - \frac{2x}{x^2+1}$, $F_0 = 1$

$$\int 4 dx = 4x \quad \textcircled{1}$$

$$\frac{2x}{x^2+1} \Rightarrow \ln |2x| \quad \textcircled{2}$$

$$x^2+1 = u \quad du = 2x dx \quad \frac{du}{dx} = 1$$

$$\int \frac{1}{4} dx = \frac{1}{4} \int \frac{1}{u} du$$

$$\textcircled{1} \textcircled{2} \Rightarrow F(x) = 4x - \ln |2x| + C$$

$$F_0 = 1 = 4 \cdot 0 - \ln |2 \cdot 0| + C \Rightarrow C = 1$$

$$F(x) = 4x - \ln |2x| + 1$$

Ex 3: $f'(x) = \sqrt{2x+1}$, $f_0 = 1$

$$(2x)^{\frac{1}{2}} + 1^{\frac{1}{2}} = (2x+1)^{\frac{1}{2}}$$

$$(2x)^{\frac{1}{2}} + 1^{\frac{1}{2}} = 2^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = \sqrt{2} \cdot x^{\frac{1}{2}} = \frac{2\sqrt{2}}{3} x^{\frac{3}{2}}$$

$$t = x$$

$$f(t) = \frac{2\sqrt{2}}{3} t^{\frac{3}{2}} + t + C$$

$$f_0 = 1 = \frac{2\sqrt{2}}{3} 0^{\frac{3}{2}} + 0 + C \Rightarrow C = 1$$

$$f(t) = \frac{2\sqrt{2}}{3} t^{\frac{3}{2}} + t + 1$$

$$b) f(x) = x^2 - 2 \quad x \in [2; 2]$$

$$\Delta x = 0,5$$

left points:

$$f(-1) = -1$$

$$f(-0,5) = -1,75$$

$$f(0) = -2$$

$$f(1) = -1$$

$$f(1,5) = 0,15$$

$$\text{Sumf} = -7,25$$

$$A = 0,5 \cdot -7,25 = -3,625$$

$$c) f(x) = 5 - \frac{1}{2}x$$

$$2 \leq x \leq 14$$

$$\Delta x = 2$$

$$f(2) = 2$$

$$f(8) = -1$$

$$f(4) = 1$$

$$f(10) = -2$$

$$f(6) = 0$$

$$f(12) = -3$$

$$\text{Sumf} = -3$$

$$A = 2 \cdot -3 = -6$$

Ex 6

$$A = \sum_{i=1}^6 f(x_i) \Delta x \quad x_i = a + i \Delta x$$

$$d) f(x) = x + \frac{1}{x} \quad x \in [1, 4]$$

$$\Delta x = 0,5$$

$$+1,5 = 2,166$$

$$f(3) = 3,333$$

$$+2 = 2,5$$

$$f(3,5) = 3,7857$$

$$+2,5 = 2,9$$

$$f(4) = 4,05$$

$$\text{Sum} = 18,9$$

$$A = \frac{1}{2} \cdot 18,9 \approx 9,468$$

$$\rightarrow +\infty = \sqrt{2} - L$$

$$\Delta x = 1$$

$$f(1,5) = \sqrt{1,5} - 2 \approx -0,775$$

$$f(2,5) = -0,419$$

$$f(3,5) \approx -0,134$$

$$f(4,5) \approx 0,121$$

$$f(5,5) \approx 0,545$$

$$A \approx 1 \times (-0,862) = -0,862$$

Ex8:

$$\int_a^b f(x) dx \approx \frac{b-a}{2} [f(a) + f(b)]$$

$$[2, 6] : 40$$

$$[6, 7] : 16$$

$$[7, 10] : 42$$

$$\int_2^{10} f(x) dx \approx 98$$

Ex9:

$$I = \int_{-3}^1 f(x) dx$$

$$f(x) = \begin{cases} -x-1 & -3 \leq x \leq 0 \\ -\sqrt{1-x^2} & 0 \leq x \leq 1 \end{cases}$$

$$I = \int_{-3}^0 (-x-1) dx + \int_0^1 -\sqrt{1-x^2} dx$$

$$\int_{-3}^0 (-x-1) dx = -\frac{x^2}{2} - x \quad \left. \begin{array}{l} I = \frac{3}{2} - \frac{\pi}{4} \\ -3 \rightarrow 0 : \frac{3}{2} \end{array} \right\}$$

$$\int_0^1 -\sqrt{1-x^2} dx = -\frac{\pi}{4}$$

Ex 1:

$$a) f(x) = 6x^2 - 2x + 3$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} (n+1)$$

$$b) \frac{x^3}{3} = 2x^3 - 2 \cdot \frac{x^2}{2} = -x^2$$

$$F(x) = 2x^3 - x^2 + 3x$$

$$b) f(x) = \sqrt[4]{x} + \frac{1}{x^2}$$

$$x^{1/4} = \frac{x^{5/4}}{x^5} = \frac{1}{5} x^{5/4}$$

$$\frac{1}{x^2} = 1 \cdot \ln|x^2| \quad \frac{1}{x^2} = x^{-2} = \frac{1}{x^2} = -\frac{1}{x^2}$$

$$\cancel{A} \quad x^{-2} = \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1} = -\frac{1}{x}$$

$$F = \frac{1}{5} x^{5/4} - \frac{1}{x} + C$$

$$c) f(x) = \frac{x^2 + x + 2}{x}$$

$$\frac{x^2}{x} = x = \frac{x^2}{2} + 1 \quad 1 = x \quad 1 \cdot \frac{2}{x} = 2 \cdot \frac{1}{x} = 2 \cdot \ln|x|$$

$$F(x) = \frac{x^2}{2} + x + 2 \ln|x| + C$$

$$d) f(u) = 2x(x^2 + 1)$$

$$2x^3 + 2x$$

$$2x^3 = 2 \cdot \frac{x^4}{4} = \frac{1}{2} x^4 \quad 2x = 2 \cdot \frac{x^2}{2} = x^2$$

$$F(x) = \frac{1}{2} x^4 + x^2 + C$$



b) $f(x) = x^2 - L$ $[-1; 2]$
 $\Delta x = \frac{2 - (-1)}{6} = 0,5$

$$\begin{array}{ll} f_{-1} = -1,75 & f_1 = -1 \\ f_0 = L & f_{1,5} = 0,25 \\ f_{1,25} = -1,75 & f_2 = 2 \\ A = 0,5 \cdot -4,125(-4,125) = -2,125 \end{array}$$

c) $f(x) = x^2 - Lx$ $[0, 3]$
 $\Delta x = 0,5$
 $A = 0,5 \cdot 1,75 = 0,875$

Ex 7

a) $f(x) = x + \frac{1}{x}$ $[1, 4]$ $n=6$

$$\begin{array}{ll} f_{1,25} = 2,05 & f_{2,75} = 0,3711 \\ f_{1,75} = 2,314 & f_{3,25} = 3,155 \\ f_{2,25} = 2,6944 & f_{3,75} = 4,01 \\ A \approx 0,5 \times 17,7538 = 8,877 \end{array}$$

b) $f(x) = x^2 - L$ $[-1, 2]$ $n=6$
 $\Delta x = 0,5$

$\circ [-1; 0,5] [0,5; 1] [1; 1,5] [1,5; 2]$
 \bullet Tâm: $-1,75, -1,25, -0,75, -0,25, 0,25, 0,75, 1,25, 1,75$.
 $A = -5,0625$



$$S' = V$$

$$V' = a$$

Thứ ngày

$$\text{Ex4: } a) V(t) = \sin t - \cos t, S_0 = 0$$

$$\sin t = -\cos t \quad -\cos t = -\sin t$$

$$\int V dt = -\cos t + \sin t + C$$

$$C_0 + \sin t + C = 0$$

$$C = 0$$

$$S(t) = \sin t - \cos t$$

$$b) V(t) = 10 \sin t + 3 \cos t, S(0) = 0$$

$$S(t) = -10 \cos t + 3 \sin t + C$$

$$-10 \cdot 6.667 + 3 \sin 0 + C = 0$$

$$10 + 3 \cdot 0 + C =$$

$$C = -10$$

$$S(t) = -10 \cos t + 3 \sin t - 10$$

$$c) V(t) = 10 + 3t - 3t^2, S(2) = 10$$

$$S(t) = 10t + \frac{3}{2}t^2 - \frac{3}{3}t^3 + C$$

$$S(2) = 10 = 10 + \frac{3}{2} \cdot 2^2 - \frac{3}{3} \cdot 2^3 + C$$

$$10 = 10 + 6 - 8 + C \rightarrow C = 2$$

$$S(t) = 10 + \frac{3}{2}t^2 - \frac{3}{3}t^3 + 2$$

Ex5

$$a) f(x) = 71 + \frac{1}{x} \quad x \in [1; 4]$$

$$Dx = \frac{a-b}{6} = 0,5$$

$$t_1 = 2$$

$$f(2) = 71,65 = 2,9$$

$$t_{1,5} = 2,166$$

$$+ 3 = 3,45$$

$$t_2 = 2,5$$

$$Dx = 16,686 + 3,5 = 3,7857$$

$$\rightarrow A = 0,5 \cdot 16,686 \approx 8,343$$

