Ex 10: Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$. Find:

a/ $f \circ g$ **b/** $g \circ f$

c/ $f \circ f$ **d/** $g \circ g$

Ex 11: Use the table to evaluate each expression

x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	2	3

a/ $f\big(g(1)\big)$

b/ $g\big(f(1)\big)$

c/ $f\big(f(1)\big)$

d/ $g\big(g(1)\big)$

e/ $(g \circ f)(3)$

g/ $(g \circ f)(6)$

Ex 12: The graph of f is given.
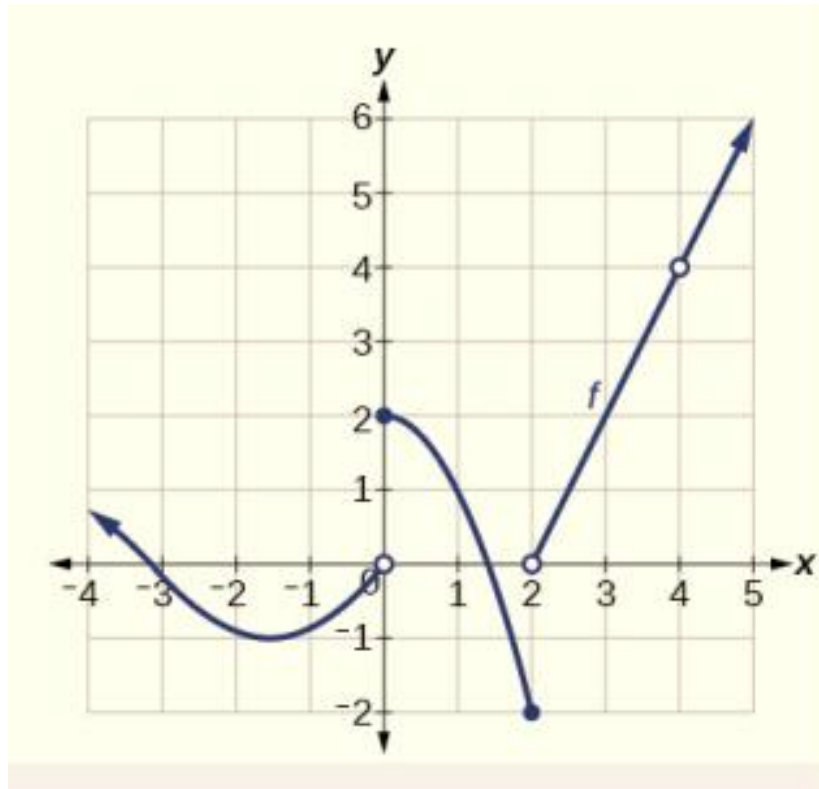
a/ Find each limit, or explain why it does not exist

i/ $\lim\limits_{x \to 0^+} f(x)$

$\lim\limits_{x \to 0^-} f(x)$

$\lim\limits_{x \to 0} f(x)$

ii/ $\lim\limits_{x \to 1} f(x)$

iii/ $\lim\limits_{x \to 4} f(x)$

Ex 13: Find the limit, if it exists. If the limit does not exist, explain why.

a/ $\displaystyle\lim_{x\to-2}\frac{2-|x|}{2+x}$

b/ $\displaystyle\lim_{x\to0.5}\frac{2x-1}{|2x^3-x^2|}$

Ex 14: Given that

$$\lim_{x \to 2} f(x) = 4; \lim_{x \to 2} g(x) = -2; \lim_{x \to 2} h(x) = 0$$

find the limits that exist. If the limit does not exist, explain why:

a/ $\lim_{x \to 2}\left(f(x) + 5g(x)\right)$

e/ $\lim_{x \to 2} \dfrac{g(x)}{h(x)}$

b/ $\lim_{x \to 2}\left[g(x)\right]^3$

f/ $\lim_{x \to 2} \dfrac{g(x)h(x)}{f(x)}$

c/ $\lim_{x \to 2} \sqrt{f(x)}$

d/ $\lim_{x \to 2} \dfrac{3f(x)}{g(x)}$

Ex 15: Let

$$g(x) = \begin{cases} x, x < 1 \\ 3, x = 1 \\ 2 - x^2, 1 < x \leq 2 \\ x - 3, x > 2 \end{cases}$$

a/ Evaluate each of the following limits, if it exists

- $\lim\limits_{x \to 1^-} g(x)$
- $\lim\limits_{x \to 1} g(x)$
- $g(1)$

- $\lim\limits_{x \to 2^-} g(x)$
- $\lim\limits_{x \to 2^+} g(x)$
- $\lim\limits_{x \to 2} g(x)$

b/ Sketch the graph of g.

<u>Ex 16:</u> Evaluate the following limits

a/ $\displaystyle\lim_{x\to 3}\frac{x^2+x-12}{x-3}$

b/ $\displaystyle\lim_{x\to -1}\frac{x^2+2x+1}{x^4-1}$

c/ $\displaystyle\lim_{h\to 0}\frac{(3+h)^2-9}{h}$

d/ $\displaystyle\lim_{t\to 0}\frac{\sqrt{t^2+9}-3}{t^2}$

e/ $\displaystyle\lim_{x\to -1}\frac{2x^2+3x+1}{x^2-2x-3}$

f/ $\displaystyle\lim_{x\to 1}\frac{x^2-1}{|x-1|}$

<u>Ex 17:</u> Evaluate the following limits

a/ $\lim\limits_{x\to 3}\dfrac{x^6-1}{x^{10}-1}$

b/ $\lim\limits_{x\to -4}\dfrac{\frac{1}{4}+\frac{1}{x}}{x+4}$

c/ $\lim\limits_{x\to 0}\dfrac{x}{\sqrt{1+3x}-1}$

d/ $\lim\limits_{x\to 0}\dfrac{\sqrt{3+x}-\sqrt{3}}{x}$

e/ $\lim\limits_{t\to 0}\left(\dfrac{1}{t}-\dfrac{1}{t^2+t}\right)$

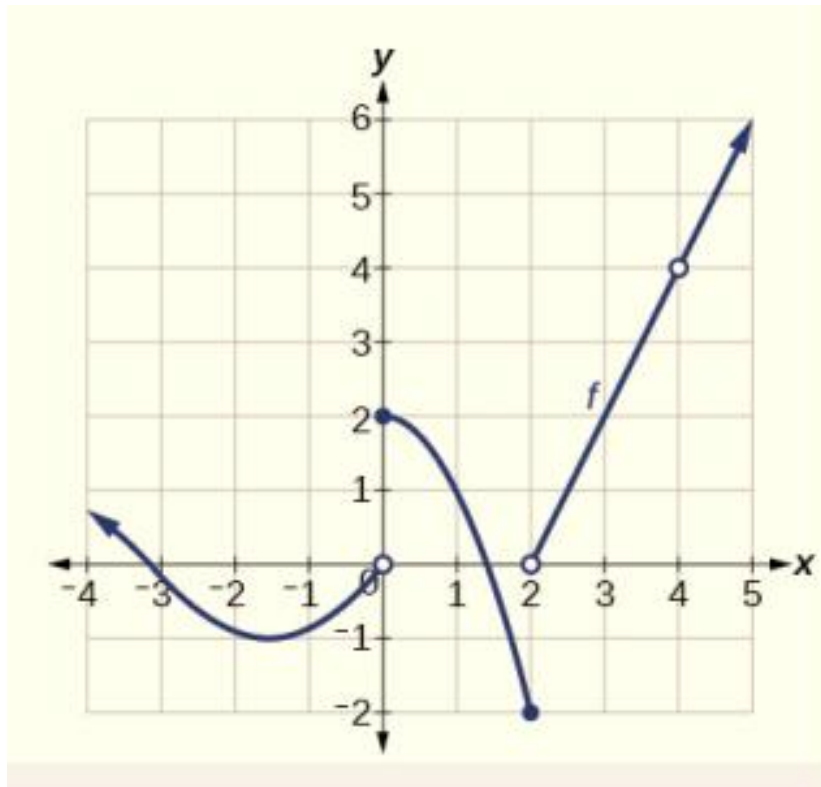**f/** $\lim\limits_{h\to 0}\dfrac{(x+h)^3-x^3}{h}$

Ex 18:

a/ If $4x - 9 \le f(x) \le x^2 - 4x + 7$, find $\lim\limits_{x \to 4} f(x)$

b/ Find the limit $\lim\limits_{x \to 0} \sqrt{x^3 + x^2} \sin\left(\dfrac{\pi}{x}\right)$

Ex 19: The graph of f is given. At what numbers is f discontinuous ?

Ex 20: Determine where the function $f(x)$ is continuous

a/ $f(x) = \dfrac{2x^2 + x - 1}{x - 2}$

b/ $f(x) = \dfrac{x - 9}{\sqrt{4x^2 + 4x + 1}}$

c/ $f(x) = \ln(2x + 5)$

Ex 21: Find the constant m that makes f continuous on R

a/ $f(x) = \begin{cases} x^2 - m^2, & x < 4 \\ mx + 20, & x \geq 4 \end{cases}$

b/ $f(x) = \begin{cases} mx^2 + 2x, & x < 2 \\ x^3 - mx, & x \geq 2 \end{cases}$

c/ $f(x) = \begin{cases} \dfrac{x^2 - 1}{\sqrt{x} - 1}, & x > 1 \\ mx + 1, & x \leq 1 \end{cases}$

Ex 22: Is there a number m such that

$$\lim_{x \to -2} \frac{3x^2 + mx + m + 3}{x^2 + x - 2}$$

exists ? If so, find the value of m and the value of the limit.

Ex 23:

If $\quad \displaystyle\lim_{x \to 1} \frac{f(x) - 8}{x - 1} = 10$, find $\quad \displaystyle\lim_{x \to 1} f(x)$

Ex 24:

Find the horizontal and vertical asymptotes of each curve.

a/ $y = \dfrac{2x^2 + x - 1}{x^2 + x - 2}$

b/ $y = \dfrac{x - 9}{\sqrt{4x^2 + 3x + 2}}$

<u>Ex 25:</u> Evaluate the limits:

a/ $\displaystyle \lim_{x \to 0} \frac{\sin 3x}{5x^3 - 4x}$

b/ $\displaystyle \lim_{\theta \to 0} \frac{1 - \cos \theta}{\sin \theta}$

c/ $\displaystyle \lim_{x \to 0} \frac{x \sin 2x}{e^{9x^2} - 1}$