

Ex 1:

Find the most general anti-derivative of the function

a/ $f(x) = 6x^2 - 2x + 3$

b/ $f(x) = \sqrt[4]{x} + \frac{1}{x^2}$

c/ $f(x) = \frac{x^2 + x + 2}{x}$

d/ $f(x) = 2x(x^2 + 1)$

Ex 2:

Find the anti-derivative of that satisfies the given condition

a/ $f(x) = 5x^4 - 2x^5, F(0) = 4$

b/ $f(x) = 4 - \frac{2x}{x^2 + 1}, F(0) = 1$

Ex 3:

Find $f(x)$ for $f'(x) = \sqrt{2x+1}$ and $f(0) = 1$

Ex 4:

A particle is moving with the given data. Find the position of the particle

a/ $v(t) = \sin t - \cos t, s(0) = 0$

b/ $v(t) = 10\sin t + 3\cos t, s(\pi) = 0$

c/ $v(t) = 10 + 3t - 3t^2, s(2) = 10$

Ex 5:

Estimate the area under the graph of $y = f(x)$ using 6 rectangles and **left endpoints**.

a/ $f(x) = x + \frac{1}{x}, x \in [1, 4]$

b/ $f(x) = x^2 - 2, x \in [-1, 2]$

c/ $f(x) = 3 - \frac{1}{2}x, 2 \leq x \leq 14$

Ex 6:

Estimate the area under the graph of $y = f(x)$ using 6 rectangles and **right endpoints**.

a/ $f(x) = x + \frac{1}{x}, x \in [1, 4]$

b/ $f(x) = x^2 - 2, x \in [-1, 2]$

c/ $f(x) = x^2 - 2x, 0 \leq x \leq 3$

Ex 7:

Estimate the area under the graph of $y = f(x)$ using **midpoints**.

a/ $f(x) = x + \frac{1}{x}, x \in [1, 4], n = 6$

b/ $f(x) = x^2 - 2, x \in [-1, 2], n = 6$

c/ $f(x) = \sqrt{x} - 2, 1 \leq x \leq 6, n = 5$

Ex 8:

The function f is continuous on the closed interval $[2,10]$ and has values given in the table below.

x	2	6	7	10
$f(x)$	5	15	17	11

Using the subintervals $[2,6]$, $[6,7]$, and $[7,10]$, what is the

trapezoidal approximation of $\int_2^{10} f(x) dx$?

Ex 9:

Use the **Trapezoidal Rule** to approximate the given integral with the specified value of n

$$\mathbf{a/} \int_0^3 \sqrt{x} dx, n = 4$$

$$\mathbf{b/} \int_1^3 \frac{\sin x}{x} dx, n = 6$$

Ex 10:

Use the **Simpson's Rule** to approximate the given integral with the specified value of n

$$\mathbf{a/} \int_0^3 \sqrt{x} dx, n = 4$$

$$\mathbf{b/} \int_1^3 \frac{\sin x}{x} dx, n = 6$$

Ex 11:

Use the Midpoint Rule the given value of n to approximation the integral. Round the answer to four decimal places.

$$1. \int_0^8 \sin \sqrt{x} dx, \quad n = 4$$

$$2. \int_0^2 \frac{x}{x+1} dx, \quad n = 5$$

$$3. \int_0^{\pi/2} \cos^4 x dx, \quad n = 4$$

$$4. \int_1^4 \sqrt{x^3 + 1} dx, \quad n = 6$$

Ex 12:

Let $f(x) = \begin{cases} -x-1, & -3 \leq x \leq 0 \\ -\sqrt{1-x^2}, & 0 \leq x \leq 1 \end{cases}$. Evaluate $\int_{-3}^1 f(x) dx$

Ex 13:

Find the derivative of the function

$$\mathbf{a/} \quad g(x) = \int_0^x \sqrt{1+t^2} dt$$

$$\mathbf{b/} \quad g(x) = \int_1^{x^4} \frac{1}{\cos t} dt$$

$$\mathbf{c/} \quad g(x) = \int_1^{\sqrt{x}} \frac{\sin u}{u} du$$

$$\mathbf{d/} \quad g(x) = \int_{2x}^{x^2+x+2} \frac{e^t}{t} dt$$

$$\mathbf{e/} \quad g(x) = \int_{\sin x}^{\cos x} (1+v^2)^{10} dv$$

Ex 14:

Find $g'(0)$ if

a/ $g(x) = \int_0^{x^2} e^{2t+1} dt$

b/ $g(x) = \int_{2x-1}^{x^3} u\sqrt{u+1} du$

Ex 15:

Suppose the acceleration function and initial velocity are

$$a(t) = t + 3 \left(m / s^2 \right), v(0) = 5 \left(m / s \right).$$

Find the velocity at time t and the distance traveled when
 $0 \leq t \leq 5$

Ex 16:

A particle moves along a line with velocity function $v(t) = t^2 - t$, where t is measured in meters per second. Find the displacement and the distance traveled by the particle during the time interval $t \in [0, 2]$.

Ex 17:

Find the average value of the function on the given interval

a/ $f(x) = x^2, [-1, 1]$

b/ $f(x) = \frac{1}{x}, [1, 5]$

c/ $f(x) = x\sqrt{x}, [1, 4]$

d/ $f(x) = x \ln x, [1, e^2]$