

Ex 1: Find the domain of each function:

a/ $f(x) = \sqrt{x+2}$

$$x+2 \geq 0 \Leftrightarrow x \geq -2 \rightarrow D = [-2, \infty)$$

b/ $f(x) = \frac{1}{x^2 - x}$

$$x^2 - x \neq 0 \Leftrightarrow x \neq 0, x \neq 1 \rightarrow D = \mathbb{R} \setminus \{0, 1\}$$

c/ $f(x) = \ln(x^2 - 1) - \frac{x}{\sqrt{x-1}}$

$$\begin{cases} x^2 - 1 > 0 \\ x - 1 > 0 \end{cases} \Leftrightarrow x > 1 \rightarrow D = (1, \infty)$$

Ex 2: Find the **range** of each function:

a/ $f(x) = \sqrt{x-1}$

$$D = [1, \infty) \rightarrow G = [0, \infty)$$

b/ $f(x) = x^2 - 2x$

$$D = \mathbb{R}$$

$$f(x) = x^2 - 2x = x^2 - 2x + 1 - 1 = (x-1)^2 - 1 \geq -1$$

$$G = [-1, \infty)$$

c/ $f(x) = \sin(x)$

$$G = [-1, 1]$$

Ex 3: Determine whether is even, odd, or neither

a/ $f(x) = \frac{x}{x^2 + 1}$ $D = R$

$$f(-x) = \frac{-x}{(-x)^2 + 1} = -\frac{x}{x^2 + 1} = -f(x)$$

b/ $f(x) = \frac{x^2}{x^4 + 1}$ $D = R$

$$f(-x) = \frac{(-x)^2}{(-x)^4 + 1} = \frac{x^2}{x^4 + 1} = f(x)$$

c/ $f(x) = \frac{x}{x+1}$ $D = R \setminus \{-1\}$

$$f(-x) = \frac{-x}{-x+1} \neq f(x), \neq f(-x)$$

Ex 4: Let $f(x) = \frac{x^2 + x + 1}{x}$. Find:

$$\text{a/ } f(2x-1) = \frac{(2x-1)^2 + (2x-1) + 1}{2x-1} = \frac{4x^2 - 2x + 1}{2x-1}$$

$$\text{b/ } f\left(x + \frac{1}{x}\right) = \frac{\left(x + \frac{1}{x}\right)^2 + \left(x + \frac{1}{x}\right) + 1}{x + \frac{1}{x}}$$

$$= \frac{x^4 + x^3 + 3x^2 + x + 1}{x(x^2 + 1)}$$

Ex 5:

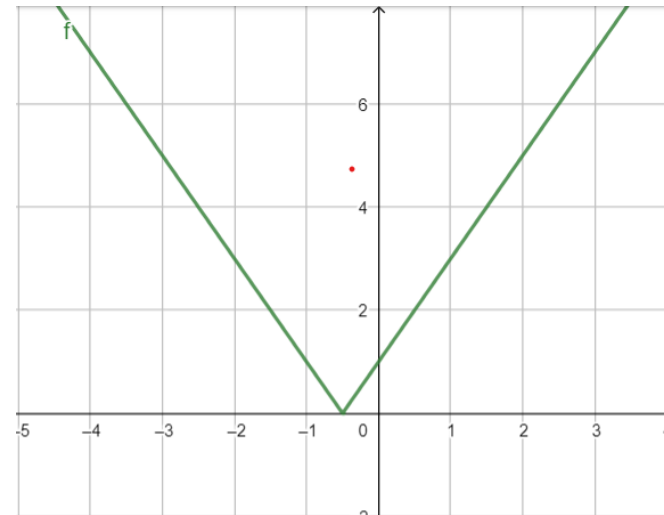
a/ If $f(x) = \frac{x^2 - x}{x - 1}$ and $g(x) = x$ is it true that $f = g$?

Because $g(1) = 1$, $f(1)$ dose not exist, so $f \neq g$.

b/ Find the domain and sketch the graph of the function

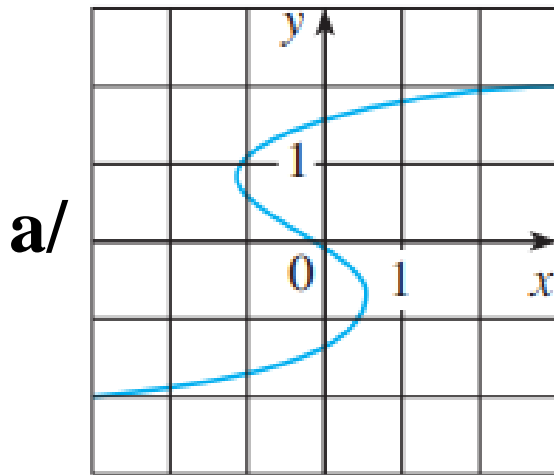
$$y = |2x + 1|$$

Domain: $D = \mathbb{R}$

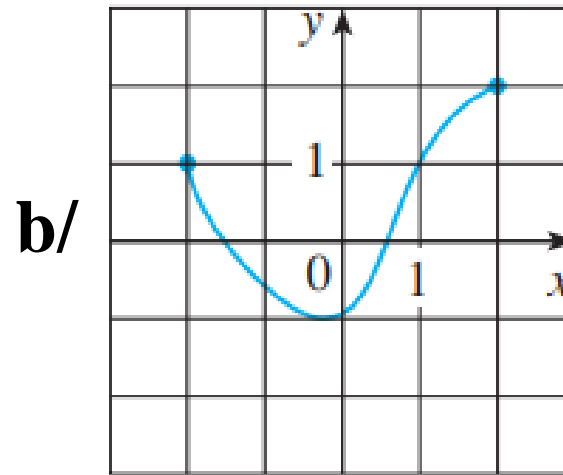


Ex 6:

Determine whether the curve is the graph of a function of x .
If it is, state the domain and range of the function.



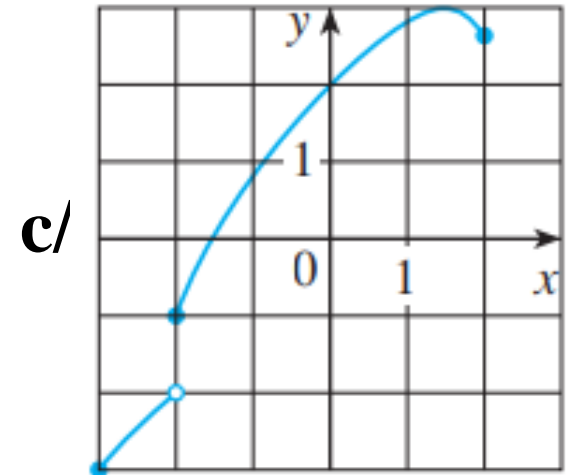
Not a function



A function

$$D = [-2, 2]$$

$$G = [-1, 2]$$



A function

$$D = [-3, 2]$$

$$G = [-3, -2) \cup [-1, 3]$$

Ex 7:

The graphs of f and g are given

a/ State the values of $f(-4)$, $g(3)$.

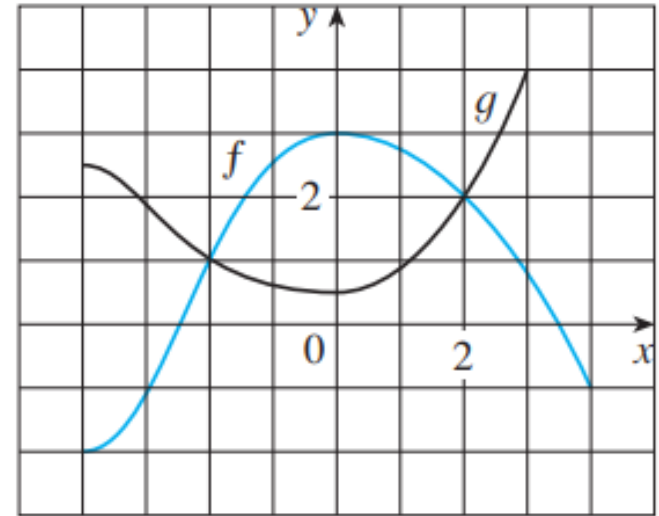
b/ For what values of x is

$$f(x) = g(x) ?$$

c/ Estimate the solution of the equation $f(x) = -1$.

d/ State the domain and range of f .

e/ State the domain and range of g .

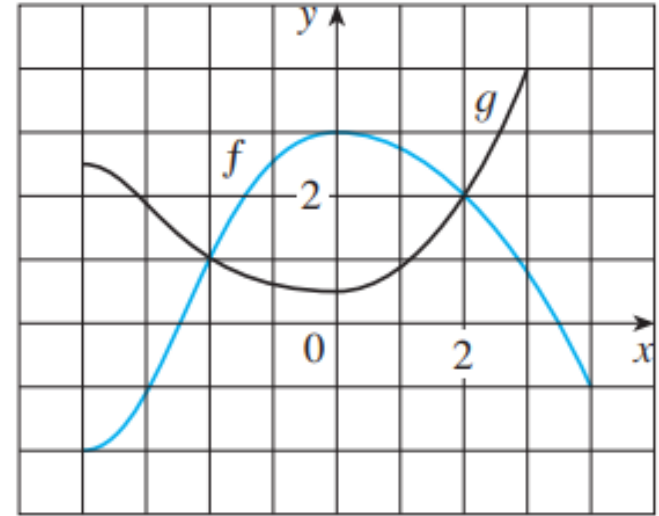


Ex 7:

a/ $f(-4) = -2, g(3) = 4.$

b/ $x = \pm 2$

c/ $x = 4$ or $x = -3$



d/ Domain $D = [-4, 4]$ and range $G = [-2, 3]$

e/ Domain $D = [-4, 3]$ and range $G = \left[\frac{1}{2}, 4\right]$

Ex 8:

Explain how the following graphs are obtained from the graph of $f(x)$:

a/ $f(x - 4)$

Shift the graph 4 units to the right.

b/ $f(x) + 3$

Shift the graph 3 units up.

Ex 8:

Explain how the following graphs are obtained from the graph of $f(x)$:

c/ $f(x - 2) - 3$

Shift the graph 2 units to the right and 3 units down.

d/ $f(x + 5) - 4$

Shift the graph 5 units to the left and 4 units down.

Ex 9:

Suppose that the graph of $f(x) = \sqrt{x}$ is given. Describe how the graph of the function $g(x) = \sqrt{x-1} + 2$ can be obtained from the graph of $f(x)$.

We have $g(x) = \sqrt{x-1} + 2 = f(x-1) + 2$

Shift the graph 1 units to the right and 2 units up.

Ex 10: Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$. Find:

$$\mathbf{a/} \quad f \circ g = f(g(x)) = f(\sqrt{2-x}) = \sqrt{\sqrt{2-x}}$$

$$\mathbf{b/} \quad g \circ f = g(f(x)) = g(\sqrt{x}) = \sqrt{2-\sqrt{x}}$$

Ex 10: Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$. Find:

c/ $f \circ f = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$

d/ $g \circ g = g(g(x)) = g(\sqrt{2-x}) = \sqrt{2-\sqrt{2-x}}$

Chapter 01_Assignment_Answer

Ex 11: Use the table to evaluate each expression

x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	2	3

a/ $f(g(1)) = f(6) = 5$

b/ $g(f(1)) = g(3) = 2$

c/ $f(f(1)) = f(3) = 4$

Ex 11: Use the table to evaluate each expression

x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	2	3

d/ $g(g(1)) = g(6) = 3$

e/ $(g \circ f)(3) = g(f(3)) = g(4) = 1$

g/ $(g \circ f)(6) = g(f(6)) = g(5) = 2$

Ex 12: The graph of f is given.

a/ Find each limit, or explain why it does not exist

i/ $\lim_{x \rightarrow 0^+} f(x) = 2$

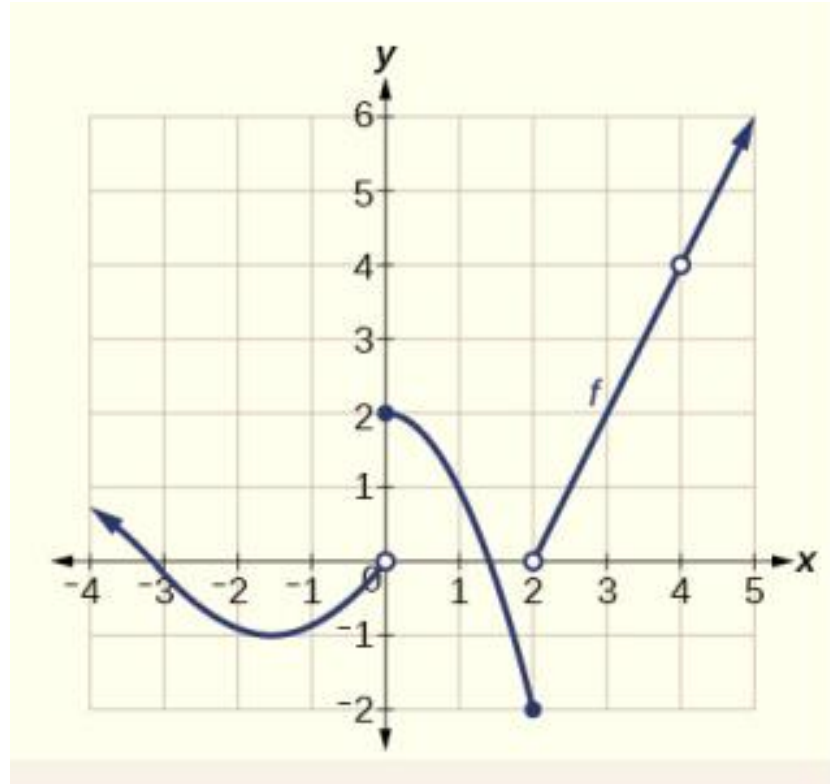
$\lim_{x \rightarrow 0^-} f(x) = 0$

$\lim_{x \rightarrow 0} f(x)$

doesn't exist

ii/ $\lim_{x \rightarrow 1} f(x) = 1 = f(1)$

iii/ $\lim_{x \rightarrow 4} f(x) = 4 \neq f(4)$



Ex 13: Find the limit, if it exists. If the limit does not exist, explain why.

$$\text{a/ } \lim_{x \rightarrow -2} \frac{2 - |x|}{2 + x} \left(\frac{0}{0} \right) = \lim_{x \rightarrow -2} \frac{2 + x}{2 + x} = 1$$

$$\text{b/ } \lim_{x \rightarrow 0.5} \frac{2x - 1}{|2x^3 - x^2|} \left(\frac{0}{0} \right)$$

$$\lim_{x \rightarrow 0.5^+} \frac{2x - 1}{|2x^3 - x^2|} \left(\frac{0}{0} \right) = 4; \quad \lim_{x \rightarrow 0.5^-} \frac{2x - 1}{|2x^3 - x^2|} \left(\frac{0}{0} \right) = -4$$

$$\text{Because } \lim_{x \rightarrow 0.5^+} \frac{2x - 1}{|2x^3 - x^2|} \neq \lim_{x \rightarrow 0.5^-} \frac{2x - 1}{|2x^3 - x^2|}, \text{ so } \lim_{x \rightarrow 0.5} \frac{2x - 1}{|2x^3 - x^2|}$$

Dose not exist.

Ex 14: Given that $\lim_{x \rightarrow 2} f(x) = 4$; $\lim_{x \rightarrow 2} g(x) = -2$; $\lim_{x \rightarrow 2} h(x) = 0$ find the limits that exist. If the limit does not exist, explain why.

$$\begin{aligned} \text{a/ } \lim_{x \rightarrow 2} (f(x) + 5g(x)) &= \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} 5g(x) \\ &= \lim_{x \rightarrow 2} f(x) + 5 \lim_{x \rightarrow 2} g(x) = 4 + 5(-2) = -6 \end{aligned}$$

$$\text{b/ } \lim_{x \rightarrow 2} [g(x)]^3 = \left[\lim_{x \rightarrow 2} g(x) \right]^3 = (-2)^3 = -8$$

$$\text{c/ } \lim_{x \rightarrow 2} \sqrt{f(x)} = \sqrt{\lim_{x \rightarrow 2} f(x)} = \sqrt{4} = 2$$

Chapter 01_Assignment_Answer

Ex 14: Given that $\lim_{x \rightarrow 2} f(x) = 4$; $\lim_{x \rightarrow 2} g(x) = -2$; $\lim_{x \rightarrow 2} h(x) = 0$
find the limits that exist. If the limit does not exist, explain why.

$$\text{d/ } \lim_{x \rightarrow 2} \frac{3f(x)}{g(x)} = \frac{3 \lim_{x \rightarrow 2} f(x)}{\lim_{x \rightarrow 2} g(x)} = \frac{3 \cdot 4}{-2} = -6$$

e/ $\lim_{x \rightarrow 2} \frac{g(x)}{h(x)}$ does not exist.

Because , if $x \rightarrow 2^+ : h(x) \rightarrow 0^+ : \lim_{x \rightarrow 2^+} \frac{g(x)}{h(x)} = \frac{\lim_{x \rightarrow 2^+} g(x)}{\lim_{x \rightarrow 2^+} h(x)} = -\infty$

$x \rightarrow 2^- : h(x) \rightarrow 0^- : \lim_{x \rightarrow 2^-} \frac{g(x)}{h(x)} = \frac{\lim_{x \rightarrow 2^-} g(x)}{\lim_{x \rightarrow 2^-} h(x)} = +\infty$

Ex 14: Given that $\lim_{x \rightarrow 2} f(x) = 4$; $\lim_{x \rightarrow 2} g(x) = -2$; $\lim_{x \rightarrow 2} h(x) = 0$ find the limits that exist. If the limit does not exist, explain why.

Then $\lim_{x \rightarrow 2^+} \frac{g(x)}{h(x)} \neq \lim_{x \rightarrow 2^-} \frac{g(x)}{h(x)}$

$$\lim_{x \rightarrow 2} \frac{g(x)h(x)}{f(x)} = \frac{\lim_{x \rightarrow 2} g(x)h(x)}{\lim_{x \rightarrow 2} f(x)} = \frac{\lim_{x \rightarrow 2} g(x) \lim_{x \rightarrow 2} h(x)}{\lim_{x \rightarrow 2} f(x)} = 0$$

Ex 15:

- $\lim_{x \rightarrow 1^-} g(x) = \lim_{x \rightarrow 1^-} x = 1$
- $\lim_{x \rightarrow 1^+} g(x) = \lim_{x \rightarrow 1^+} (2 - x^2) = 1$
- $\lim_{x \rightarrow 1} g(x) = 1$ • $g(1) = 3$
- $\lim_{x \rightarrow 2^-} g(x) = \lim_{x \rightarrow 2^-} (2 - x^2) = -2$
- $\lim_{x \rightarrow 2^+} g(x) = \lim_{x \rightarrow 2^+} (x - 3) = -1$
- $\lim_{x \rightarrow 2} g(x)$ does't exist because $\lim_{x \rightarrow 2^-} g(x) \neq \lim_{x \rightarrow 2^+} g(x)$

Ex 16: Evaluate the following limits

$$\text{a/ } \lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x - 3} = 7$$

$$\text{b/ } \lim_{x \rightarrow -1} \frac{x^2 + 2x + 1}{x^4 - 1} = 0$$

Ex 16: Evaluate the following limits

$$\text{c/ } \lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h} = 6$$

$$\text{d/ } \lim_{t \rightarrow 0} \frac{\sqrt{t^2 + 9} - 3}{t^2} = \frac{1}{6}$$

Ex 16: Evaluate the following limits

e/ $\lim_{x \rightarrow -1} \frac{2x^2 + 3x + 1}{x^2 - 2x - 3} = \frac{1}{4}$

f/ $\lim_{x \rightarrow 1} \frac{x^2 - 1}{|x - 1|}$ **doesn't exist.**

Ex 17: Evaluate the following limits

$$\mathbf{a/} \lim_{x \rightarrow 3} \frac{x^6 - 1}{x^{10} - 1} = \frac{3^6 - 1}{3^{10} - 1} = \frac{91}{7381}$$

$$\mathbf{b/} \lim_{x \rightarrow -4} \frac{\frac{1}{4} + \frac{1}{x}}{x + 4} \left(\frac{0}{0} \right) = \lim_{x \rightarrow -4} \frac{\frac{x + 4}{4x}}{x + 4} = \lim_{x \rightarrow -4} \frac{1}{4x} = -\frac{1}{16}$$

Ex 17: Evaluate the following limits

c/ $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x}-1} \left(\frac{0}{0} \right) = \frac{2}{3}$

d/ $\lim_{x \rightarrow 0} \frac{\sqrt{3+x}-\sqrt{3}}{x} \left(\frac{0}{0} \right) = \frac{1}{2\sqrt{3}}$

Ex 17: Evaluate the following limits

$$\mathbf{e/} \lim_{t \rightarrow 0} \left(\frac{1}{t} - \frac{1}{t^2 + t} \right) (\infty - \infty) = \lim_{t \rightarrow 0} \frac{1}{t + 1} = 1$$

$$\mathbf{f/} \lim_{h \rightarrow 0} \frac{(x + h)^3 - x^3}{h} \left(\frac{0}{0} \right) = 3x^2$$

Ex 18:

a/ If $4x - 9 \leq f(x) \leq x^2 - 4x + 7$, find $\lim_{x \rightarrow 4} f(x)$

Beacause $\lim_{x \rightarrow 4} (4x - 9) = \lim_{x \rightarrow 4} (x^2 - 4x + 7) = 7$

So, $\lim_{x \rightarrow 4} f(x) = 7$.

Ex 18:

b/ Find the limit $\lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin\left(\frac{\pi}{x}\right)$

Hint:

$$-\sqrt{x^3 + x^2} \leq \sqrt{x^3 + x^2} \sin\left(\frac{\pi}{x}\right) \leq \sqrt{x^3 + x^2}$$

Ex 19: The graph of f is given. At what numbers is discontinuous ?

• f is discontinuous at $x = 4$

because $D = \mathbb{R} \setminus \{4\}$

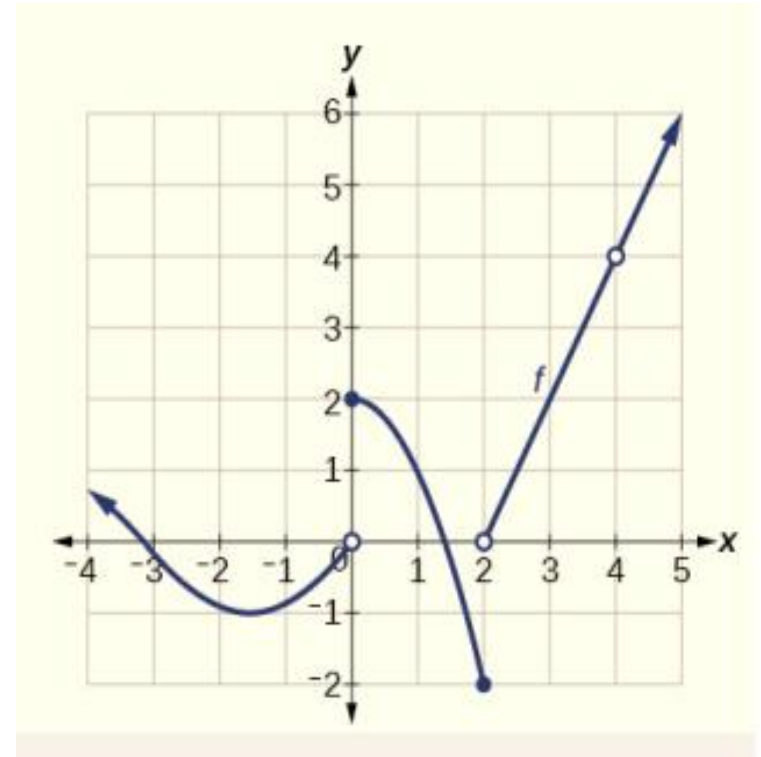
• f is discontinuous at $x = 0$

because

$$\lim_{x \rightarrow 0^+} f(x) = 2 \neq \lim_{x \rightarrow 0^-} f(x) = 0$$

• f is discontinuous at $x = 2$

because $\lim_{x \rightarrow 2^+} f(x) = 0 \neq \lim_{x \rightarrow 2^-} f(x) = -2$



Ex 20: Determine where the function $f(x)$ is continuous

$$\text{a/ } f(x) = \frac{2x^2 + x - 1}{x - 2} \quad D = \mathbb{R} \setminus \{2\}$$

$$\text{b/ } f(x) = \frac{x - 9}{\sqrt{4x^2 + 4x + 1}} \quad D = \mathbb{R} \setminus \left\{-\frac{1}{2}\right\}$$

$$4x^2 + 4x + 1 > 0 \Leftrightarrow (2x + 1)^2 > 0 \Leftrightarrow 2x + 1 \neq 0 \Leftrightarrow x \neq -\frac{1}{2}$$

$$\text{c/ } f(x) = \ln(2x + 5) \quad D = \left(-\frac{5}{2}, \infty\right)$$

Ex 21: Find the constant m that makes f continuous on \mathbb{R}

$$\mathbf{a/} \quad f(x) = \begin{cases} x^2 - m^2, & x < 4 \\ mx + 20, & x \geq 4 \end{cases}$$

For all $x \neq 4$, $f(x)$ is polynomial function, so $f(x)$ is continuous.

$$\text{At } x = 4: \lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^+} (mx + 20) = 4m + 20$$

$$\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^-} (x^2 - m^2) = 16 - m^2$$

$$f(4) = 4m + 20$$

$$\lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^-} f(x) = f(4) \Leftrightarrow 16 - m^2 = 4m + 20 \Leftrightarrow m = -2$$

Ex 21: Find the constant m that makes f continuous on R

b/ $f(x) = \begin{cases} mx^2 + 2x, & x < 2 \\ x^3 - mx, & x \geq 2 \end{cases}$

Ans: $m = \frac{2}{3}$

c/ $f(x) = \begin{cases} \frac{x^2 - 1}{\sqrt{x} - 1}, & x > 1 \\ mx + 1, & x \leq 1 \end{cases}$

Ans: $m = 3$

Ex 22: Is there a number m such that

$$\lim_{x \rightarrow -2} \frac{3x^2 + mx + m + 3}{x^2 + x - 2}$$

exists ? If so, find the value of m and the value of the limit.

$$m = 15$$

$$\lim_{x \rightarrow -2} \frac{3x^2 + 15x + 18}{x^2 + x - 2} \left(\frac{0}{0} \right) = \lim_{x \rightarrow -2} \frac{(x+2)(3x+9)}{(x+2)(x-1)}$$

$$= \lim_{x \rightarrow -2} \frac{3x+9}{x-1} = -1$$

Ex 23:

If $\lim_{x \rightarrow 1} \frac{f(x) - 8}{x - 1} = 10$, find $\lim_{x \rightarrow 1} f(x)$

If $\lim_{x \rightarrow 1} f(x) \neq 8$ then $\lim_{x \rightarrow 1} \frac{f(x) - 8}{x - 1}$ doesn't exist.

So, $\lim_{x \rightarrow 1} f(x) = 8$

Ex 24:

Find the horizontal and vertical asymptotes of each curve.

a/ $y = \frac{2x^2 + x - 1}{x^2 + x - 2}$

$$x = 1, x = -2$$

$$y = 2$$

b/ $y = \frac{x - 9}{\sqrt{4x^2 + 3x + 2}}$

$$y = \frac{1}{2}; y = -\frac{1}{2}$$

Ex 25: Evaluate the limits:

$$\begin{aligned} \text{a/ } \lim_{x \rightarrow 0} \frac{\sin 3x}{5x^3 - 4x} \left(\frac{0}{0} \right) &= \lim_{x \rightarrow 0} \frac{\sin 3x}{5x^3 - 4x} = \lim_{x \rightarrow 0} \frac{3x}{5x^3 - 4x} \\ &= \lim_{x \rightarrow 0} \frac{3}{5x^2 - 4} = -\frac{3}{4} \end{aligned}$$

Ex 25: Evaluate the limits:

$$\text{b/ } \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin \theta} \left(\frac{0}{0} \right) = \lim_{\theta \rightarrow 0} \frac{(1 - \cos \theta)(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)}$$

$$= \lim_{\theta \rightarrow 0} \frac{1 - \cos^2 \theta}{\sin \theta (1 + \cos \theta)} = \lim_{\theta \rightarrow 0} \frac{\sin^2 \theta}{\sin \theta (1 + \cos \theta)}$$

$$= \lim_{\theta \rightarrow 0} \frac{\sin \theta}{1 + \cos \theta} = 0$$

Ex 25: Evaluate the limits:

$$\text{c/ } \lim_{x \rightarrow 0} \frac{x \sin 2x}{e^{9x^2} - 1} \left(\frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{x \times 2x}{9x^2} = \lim_{x \rightarrow 0} \frac{2}{9} = \frac{2}{9}$$