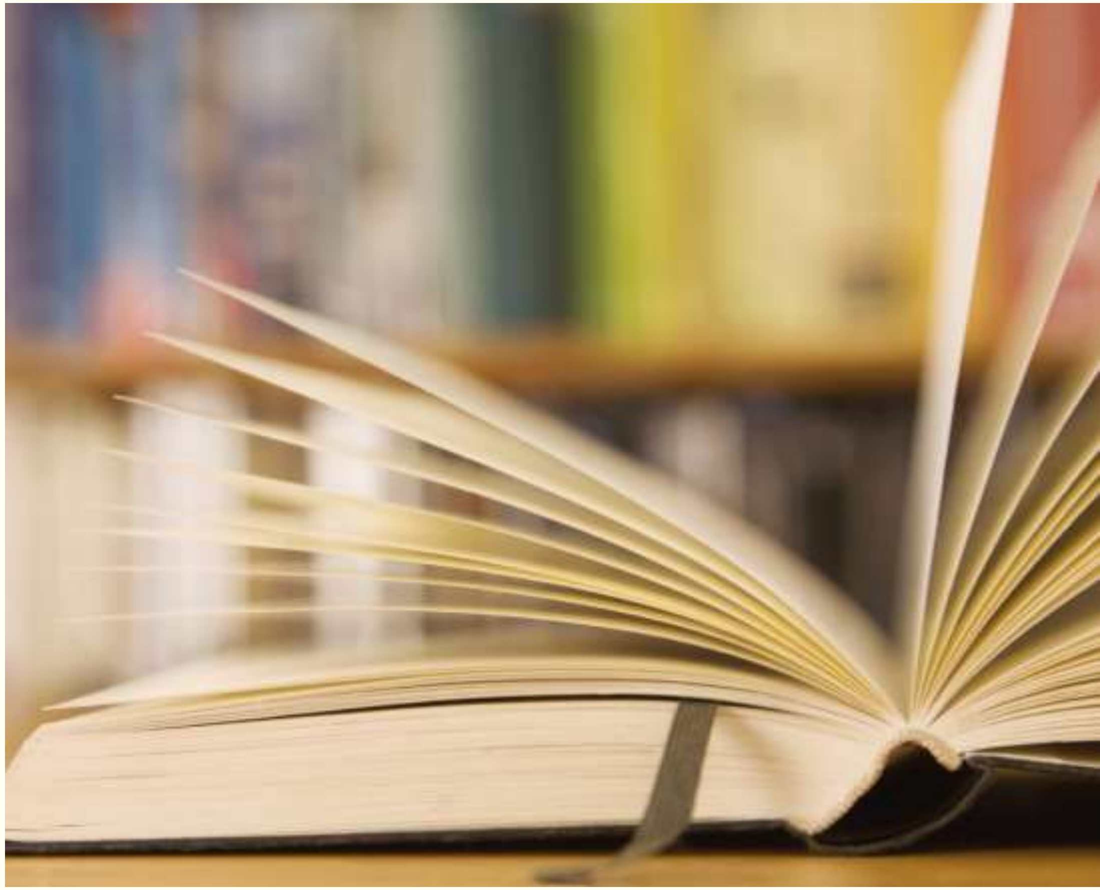
2. NUMBERING SYSTEMS



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- 2.1 Introduction
- 2.2 Positional Number Systems
- 2.3 Conversion

Objectives

After studying this chapter, the student should be able to:

- Understand the concept of number systems.
- Distinguish between non-positional and positional number systems.
- Describe the decimal system (base 10).
- Describe the binary system (base 2).
- Describe the hexadecimal system (base 16).
- Describe the octal system (base 8).
- Convert a number in binary, octal, or hexadecimal to a number in the decimal system.
- Convert a number in the decimal system to a number in binary, octal, or hexadecimal.
- Find the number of digits needed in each system to represent a particular value.

1-INTRODUCTION

1- Introduction

- A **number system** (or numeral system) defines how a number can be represented using distinct symbols.
- A number can be represented differently in different systems.
- For example, the two numbers (2A)16 and (52)8 both refer to the same quantity, (42)10, but their representations are different. This is the same as using.

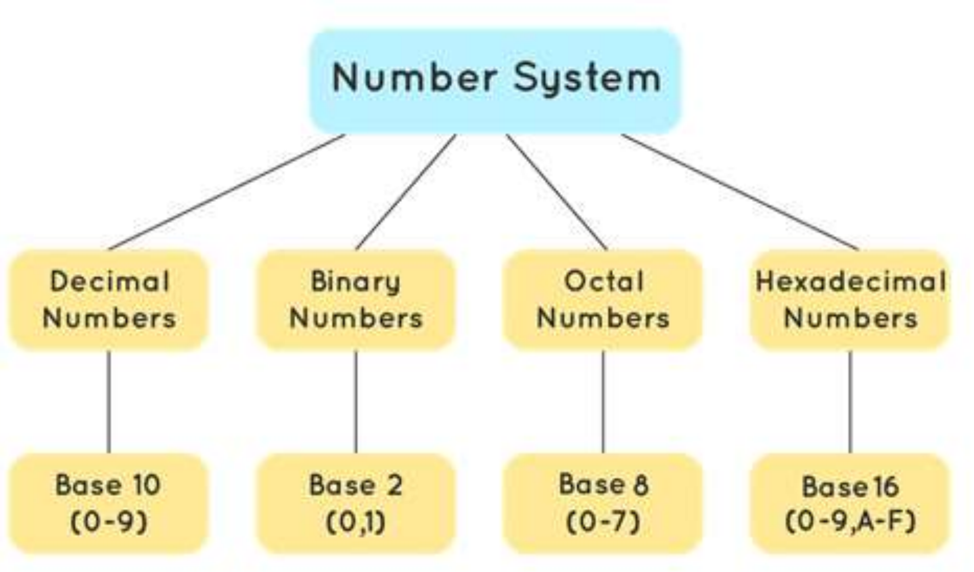


Figure -2.1 Types of number system

2 - POSITIONAL NUMBER SYSTEMS

Introduction

- In a positional number system, the position a symbol occupies in the number determines the value it represents.
- In this system, a number represented as:

$$\boxed{\pm (S_{k-1} \ldots S_2\, S_1\, S_0 .\, S_{-1}\, S_{-2} \ldots S_{-l})_b}$$

- has the value of:

$$\boxed{\begin{aligned} n = \pm\ & S_{K-1} \times b^{K-1} + \ldots + S_1 \times b^1 + S_0 \times b^0 \\ & + \quad S_{-1} \times b^{-1} + S_{-2} \times b^{-2} + \ldots + S_{-L} \times b^{-L} \end{aligned}}$$

- in which S is the set of symbols, b is the base (or radix)

The decimal system (base 10)

- The word *decimal* is derived from the Latin root *decem* (ten). In this system the base b 5 10 and we use ten symbols to represent a number.
- The set of symbols is S 5 {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. As we know, the symbols in this system are often referred to as **decimal digits** or just digits.
- In the decimal system, a number is written as:

$$\boxed{\pm\,(S_{K-1} \ldots S_2 S_1 S_0 . S_{-1} S_{-2} \ldots S_{-L})_{10}}$$

The binary system (base 2)

- The second positional number system we discuss in this chapter is the **binary system**.
- The word **binary** is derived from the Latin root ***bini*** (or two by two). In this system the base b 5 2 and we use only two symbols, S 5 {0, 1}. The symbols in this system are often referred to as **binary digits** or **bits** (**b**inary dig**it.**
- Data and programs are stored in the computer using binary patterns, a string of bits.

		2^2		2^1		2^0		2^{-1}		2^{-2}	Place values
		1		0		1	•	1		1	Number
R	=	1×2^2	+	0×2^1	+	1×2^0	+	1×2^{-1}	+	1×2^{-2}	Values

Example 2.1 the number (101.11)2 in binary = 5.75 in decimal

The hexadecimal system (base 16)

Problems :

- The decimal system does not show what is stored in the computer as binary directly—there is no obvious relationship between the number of bits in binary and the number of decimal digits. Conversion from one to the other is not fast, as we will see shortly.

To **overcome this problem**, two positional systems were devised: hexadecimal and octal.

- We first discuss the **hexadecimal system**, which is more common. The word **hexadecimal** is derived from the Greek root *hex* (six) and the Latin root *decem* (ten).

- To be consistent with decimal and binary, it should really have been called *sexadecimal*, from the Latin roots *sex* and *decem*. In this system the base b 5 16 and we use 16 symbols to represent a number. The set of symbols is S 5 {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}.

		16^2		16^1		16^0	Place values
		2		A		E	Number
N	=	2×16^2	+	10×16^1	+	14×16^0	Values

Example 2.2 the number $(2AE)_{16}$ in hexadecimal = 686 in decimal

The octal system (base 8)

- The second system that was devised to show the equivalent of the binary system outside the computer is the **octal system**.
- The word *octal* is derived from the Latin root *octo* (eight). In this system the base b 5 8 and we use eight symbols to represent a number.
- The set of symbols is S 5 {0, 1, 2, 3, 4, 5, 6, 7}. The symbols in this system are often referred to as **octal digits**

		8^3		8^2		8^1		8^0	Place values
		1		2		5		6	Number
N	=	1×8^3	+	2×8^2	+	5×8^1	+	6×8^0	Values

Example 2.3 the number (1256)8 in octal = 686 in decimal:

Summary of the four positional systems

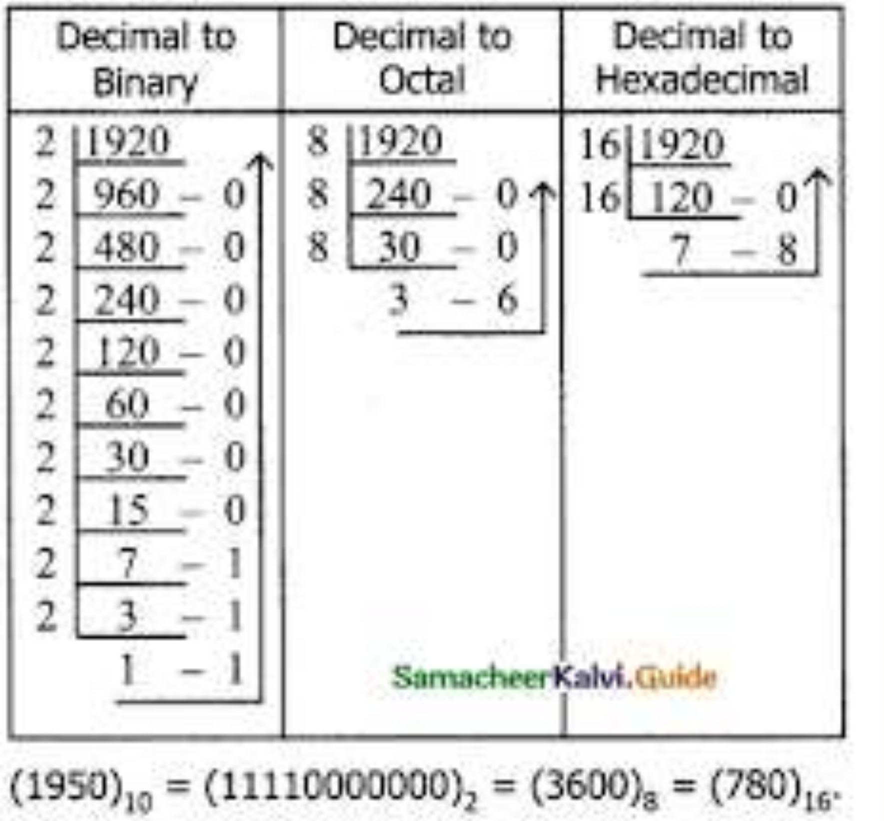
System	*Base*	*Symbols*	*Examples*
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	2345.56
Binary	2	0, 1	$(1001.11)_2$
Octal	8	0, 1, 2, 3, 4, 5, 6, 7	$(156.23)_8$
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	$(A2C.A1)_{16}$

Table 2.1 Summary of the four positional number systems

3 - CONVERSION

Introduction

- We need to know how to convert a number in one system to the equivalent number in another system. Since the decimal system is more familiar than the other systems,.
- **First** show how to covert from any base to decimal.
- **Then** show how to convert from decimal to any base.
- **Finally**, show how we can easily convert from binary to hexadecimal or octal and vice versa.



Covert from any base to decimal

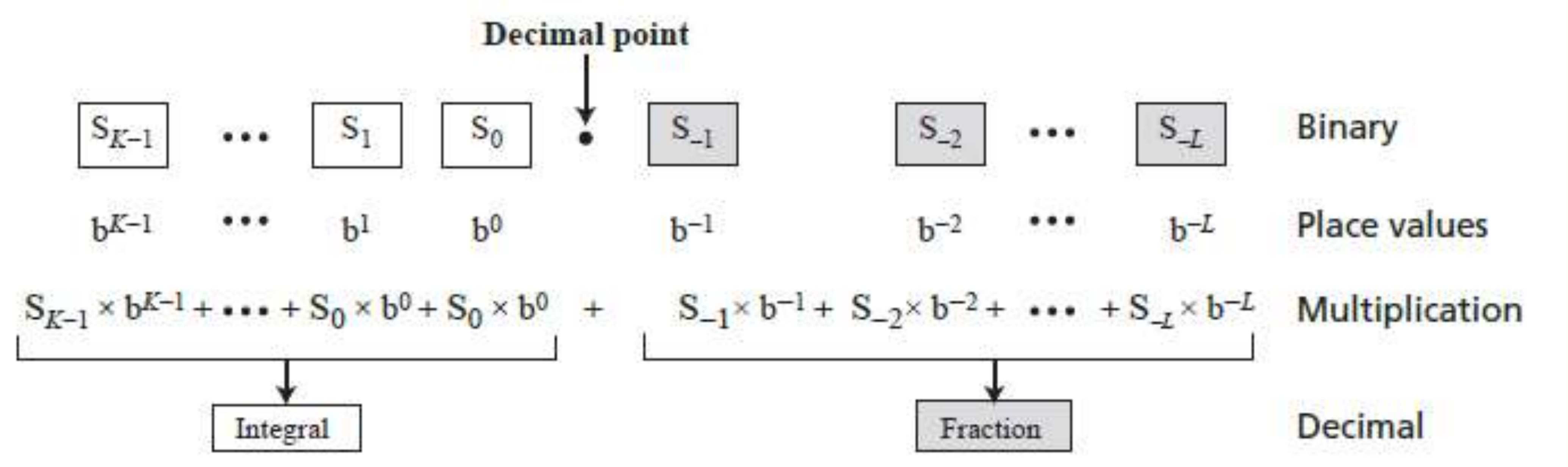


Figure 2.2 *Converting other bases to decimal*

Covert from any base to decimal (examples)

Binary	1		1		0	•	1		1
Place values	2^2		2^1		2^0		2^{-1}		2^{-2}
Partial results	4	+	2	+	0	+	0.5	+	0.25
Decimal: 6.75									

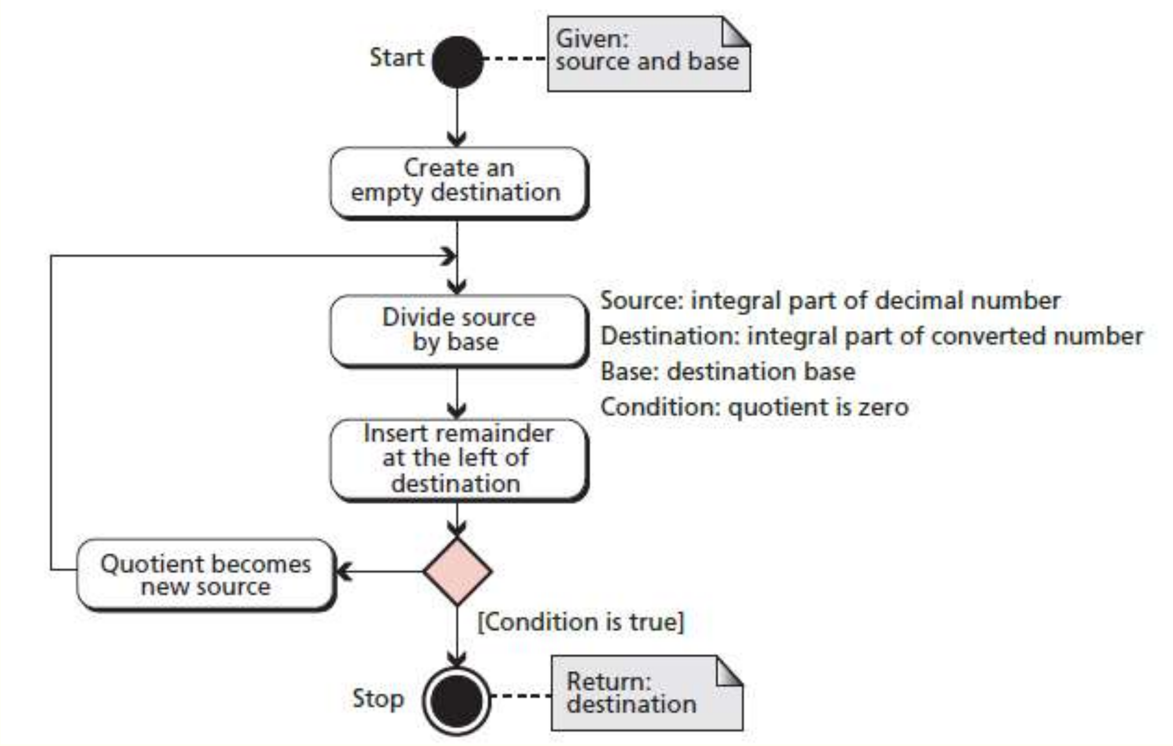
Example 3.1 Convert the binary number $(110.11)_2$ to decimal: **$(110.11)_2 = 6.75$**

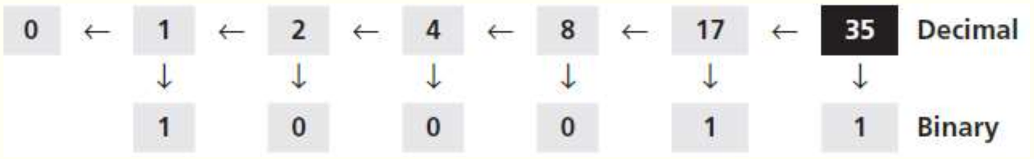
Hexadecimal	1		A	•	2		3
Place values	16^1		16^0		16^{-1}		16^{-2}
Partial result	16	+	10	+	0.125	+	0.012
Decimal: 26.137							

Example 2.3 Convert the hexadecimal number $(1A.23)_{16}$ to decimal : **1A.23 = 26.137**

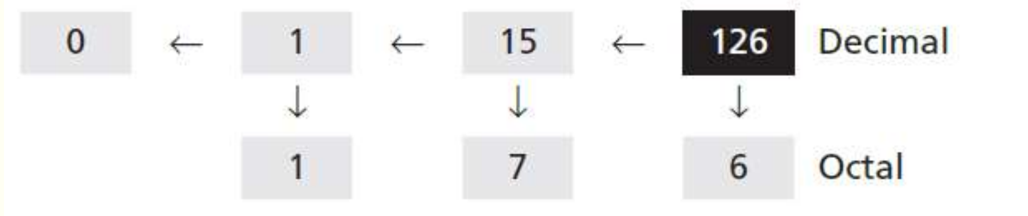
Convert from decimal to any base.

We can convert a decimal number to its equivalent in any base. We need two procedures, one for **the integral part** and one for **the fractional part.**





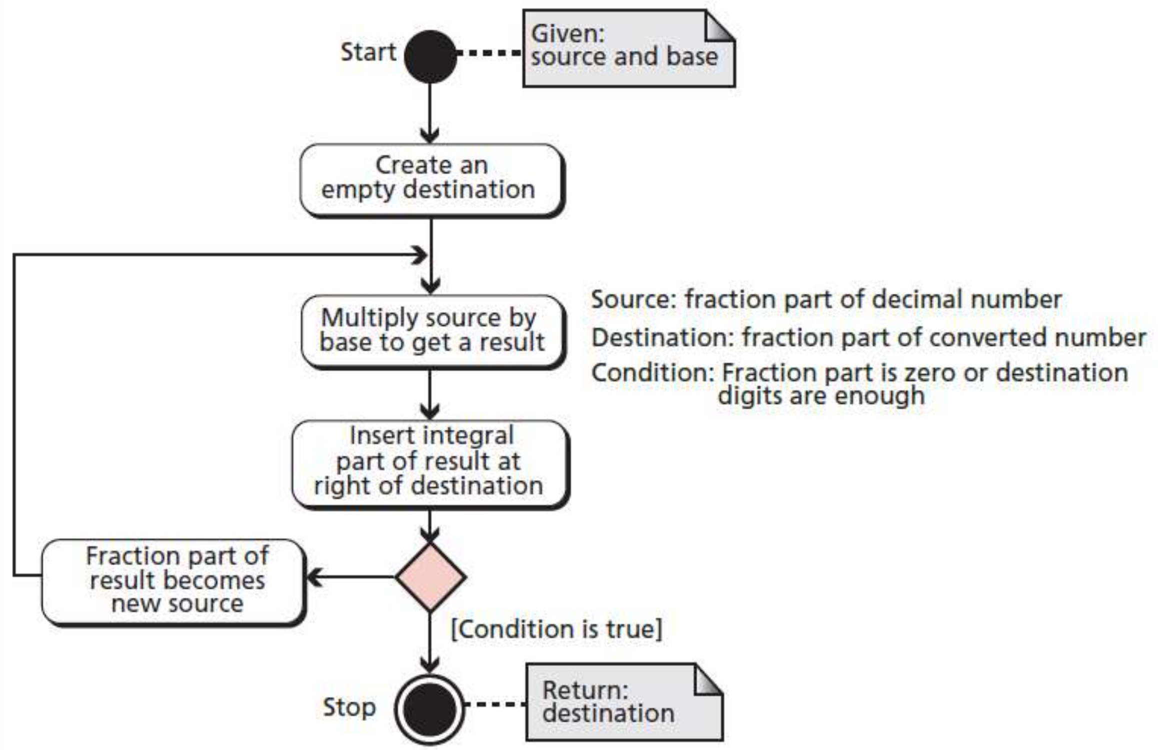
Convert 35 in decimal to binary.



Convert 126 in decimal to Octal.

Figure 2.3 *Algorithm to convert* ***the integral part***

Convert from decimal to any base (cont).



Decimal		0.625	→	0.25	→	0.50	→	0.00
		↓		↓		↓		
Binary	•	1		0		1		

Convert decimal number 0.625 to binary

Decimal		0.634	→	0.072	→	0.576	→	0.608	→	0.864
		↓		↓		↓		↓		
Octal	•	5		0		4		4		

Convert 0.634 to octal using a maximum of four digits

Figure 2.4 *Algorithm to convert* ***the fractional part***

Binary–hexadecimal conversion

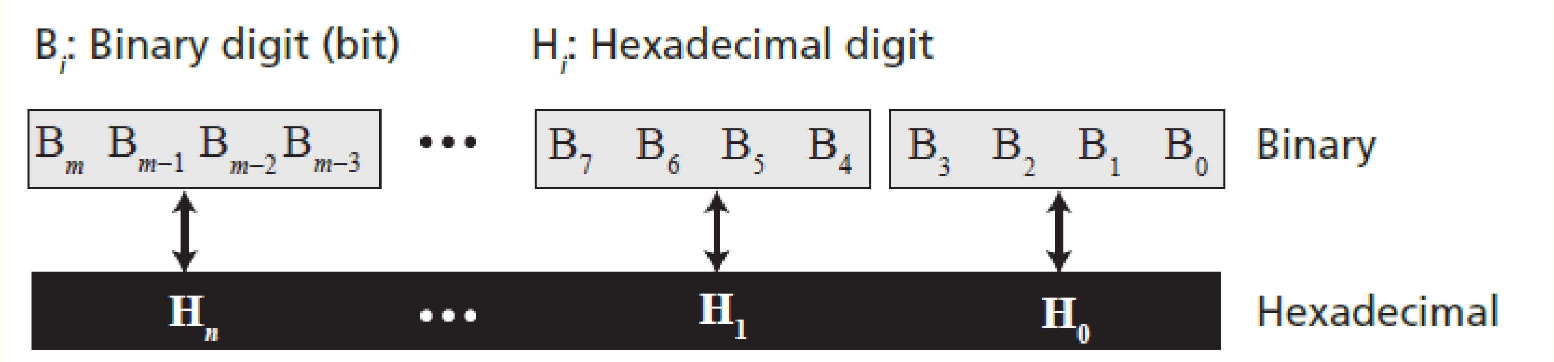


Figure 2.5 *Binary to hexadecimal and hexadecimal to binary*

Example 2.4

What is the binary equivalent of (24C)16?

Solution

Each hexadecimal digit is converted to 4-bit patterns: 2 → 0010, 4 → 0100, and C → 1100.

The result is (001001001100)2.

Binary–octal conversion

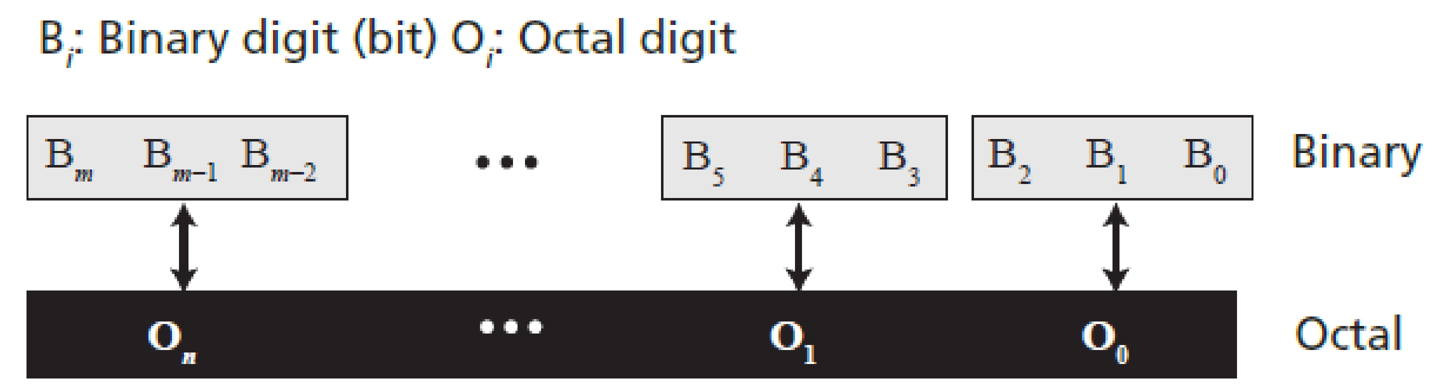


Figure 2.6 Binary to octal conversion

Example 2.5

What is the binary equivalent of for $(24)_8$?

Solution

Write each octal digit as its equivalent bit pattern to get $(010100)_2$.

Octal–hexadecimal conversion

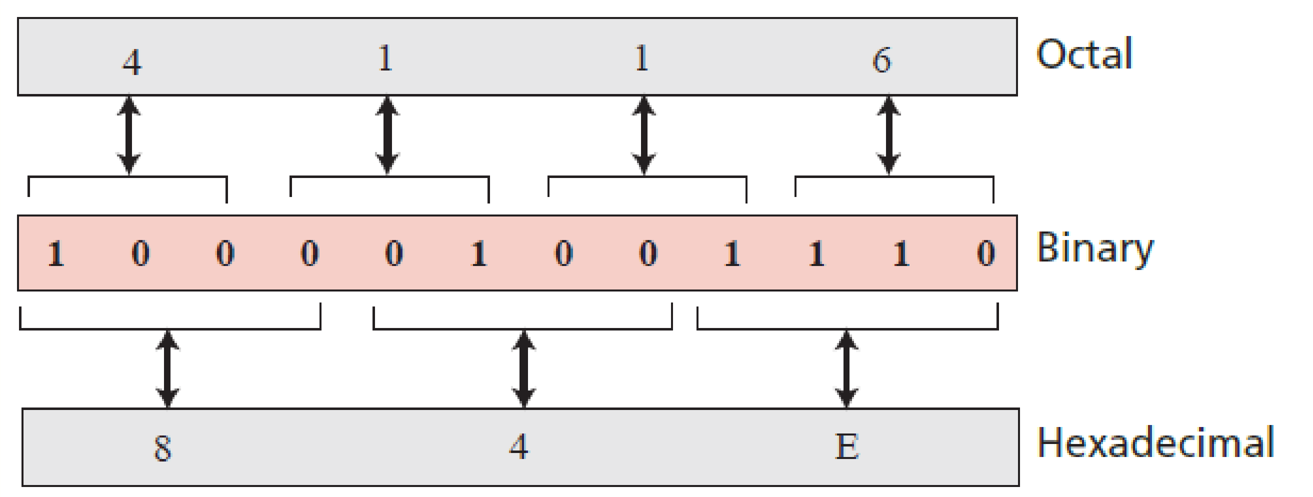


Figure 2.7 *Octal to hexadecimal and hexadecimal to octal conversion*