

# Algebra Assignment 03

## Q1:

a/ Compute the rank of each of the following matrices.

$$\text{i/ } A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

$$\text{ii/ } B = \begin{bmatrix} 1 & 1 & -1 & 4 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & 5 & 8 \end{bmatrix}$$

b/ Determine the values of m such that the rank of the matrix is 2

$$\text{i/ } A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 5 \\ 1 & 2 & m \end{bmatrix}$$

$$\text{ii/ } B = \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & 1 & 1 & 5 \\ -3 & 6 & 1 & m \end{bmatrix}$$

## Algebra\_Assignment 03

**Q2:**

Solve the system of linear equation corresponding to the given augmented matrix

$$\text{a/ } A = \begin{bmatrix} 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad \text{b/ } B = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

# Algebra\_Assignment 03

**Q3:**

a/ Solve the system

$$\begin{cases} x + 2y = 12 \\ 3x - y = 8 \\ -x + 5y = 16 \end{cases}$$

b/ Find  $a, b$  and  $c$  so that the system

$$\begin{cases} x + ay + cz = 0 \\ bx + cy - 3z = 1 \\ ax + 2y + bz = 5 \end{cases}$$
 has the solution  $(3, -1, 2)$

# Algebra Assignment 03

## Q4:

Write the augmented matrix for each of the following systems of linear equations and then solve them.

$$a/ \begin{cases} -x + y + 2z = 1 \\ 2x + 3y + z = -2 \\ 5x + 4y + 2z = 4 \end{cases}$$

$$b/ \begin{cases} 2x + 3y + z = 10 \\ 2x - 3y - 3z = 22 \\ 4x - 2y + 3z = -2 \end{cases}$$

$$c/ \begin{cases} x + y + z = 0 \\ 2x - y + 2z = 0 \\ x + z = 0 \end{cases}$$

$$d/ \begin{cases} 2x_1 + 2x_2 - x_3 + x_4 = 0 \\ 2x_1 + 3x_2 - 2x_3 + 3x_4 = 0 \\ x_1 + x_2 - 3x_3 + x_4 = 0 \end{cases}$$

## Algebra Assignment 03

**Q5:**

Consider the matrix  $A = \begin{pmatrix} 2 & -1 & 3 \\ -4 & 2 & k \\ 4 & -2 & 6 \end{pmatrix}$

- a/ If  $A$  is the augmented matrix of a system of linear equations, determine the number of equations and the number of variables.
- b/ If  $A$  is the augmented matrix of a system of linear equations, find the values of  $k$  such that the system is consistent.

# Algebra Assignment 03

## Q6:

Determine the values of  $m$  such that the system of linear equations is inconsistent

a/ 
$$\begin{cases} x - y + 2z = m \\ -x + y - z = 0 \\ x - y + 3z = 1 - m \end{cases}$$

b/ 
$$\begin{cases} x - 2y + 2z = m \\ x + my - z = 0 \\ 2x + y + mz = 2 - m \end{cases}$$

## Algebra\_Assignment 03

**Q7:**

Determine the values of  $a$  and  $b$  such that the system of linear equations is

inconsistent  $\begin{cases} x + y + 3z = 2 \\ x + 2y + 5z = 1 \\ 2x + 2y + az = b \end{cases}$

# Algebra Assignment 03

Q8:

Determine the values of  $m$  such that the system of linear equations has exactly one solution

a/ 
$$\begin{cases} x + y - z = 1 \\ x + my + 2z = m \\ x + 2y + z = 2 \end{cases}$$

b/ 
$$\begin{cases} x + my - mz = m \\ 2x + y - z = 2 \\ x + y + z = 0 \end{cases}$$

c/ 
$$\begin{cases} mx + y + z = 1 \\ x + my + z = m \\ x + y + mz = m^2 \end{cases}$$

d/ 
$$\begin{cases} x - y + 2z = m \\ -x + y - z = 0 \\ -x + my - z = 1 - m \end{cases}$$

# Algebra Assignment 03

**Q9:**

Find all values of  $k$  for which the system has nontrivial solutions and determine all solutions in each case

a/ 
$$\begin{cases} x - y + 2z = 0 \\ -x + y - z = 0 \\ x + ky + z = 0 \end{cases}$$

b/ 
$$\begin{cases} x - 2y + z = 0 \\ x + ky - 3z = 0 \\ x - 6y + 5z = 0 \end{cases}$$

c/ 
$$\begin{cases} x + y + z = 0 \\ x + y - z = 0 \\ x + y + kz = 0 \end{cases}$$

d/ 
$$\begin{cases} x + y - z = 0 \\ kx - z = 0 \\ x + y + kz = 0 \end{cases}$$

# Algebra\_Assignment 03

## Q10:

Find all values of  $k$  so that system of equations has no solution

a/ 
$$\begin{cases} x + y - z = 2 \\ -2y + z = 3 \\ 4y - 2z = k \end{cases}$$

b/ 
$$\begin{cases} x + y - z = 1 \\ 2x + (k+5)y - 2z = 4 \\ x + (k+3)y + (k-1)z = k+3 \end{cases}$$