Q1:

a> $U = \{(1, s, t) \mid s \text{ and } t \text{ in } \mathbb{R}\}$

$1 \neq \vec{0}$ -> not subspace

b> $U = \{(0, s, t) \mid s \text{ and } t \text{ in } \mathbb{R}\}$

-> has $\vec{0}$, $s, t \in \mathbb{R}$ => subspace.

c> $U = \{(r, s, t) \mid r, s \text{ and } t \text{ in } \mathbb{R}, -r + 3s + 2t = 0\}$

$ax \neq 0$ subspace

~~$x + y \in U$~~ .

d> $\{(r, 3s, r-2) \mid r, s \text{ in } \mathbb{R}\}$

$x + y \notin U$

-> not subspace

e> $U = \{(r, 0, s) \mid r^2 + s^2 = 0, r, s \in \mathbb{R}\}$

$\vec{0}$

{ $x + x \in U$ → sub space

ax thoả mãn số thực.

f> $U = \{(2r, -s^2, t) \mid r, s \text{ and } t \text{ in } \mathbb{R}\}$

$-(2, 1, -(1)^2, 1)$ → $r = 1, t = 1, s = 1$

{ $-1 \neq ?$ not - numbers

→ no sub space.

g> $U = \{(2+a, b-a, b) \mid a, b \in \mathbb{R}\}$

{ (000) → $a = -2$ $b = 0$ $b = 0 \Rightarrow t : 2 \neq 0$.

thay vào pt → $2 \neq 0$

→ not subspace.

h> $U = \{(a+b, a, b)\}$

$a+b, a, b = a(1\,1\,0) + b(1\,0\,1)$

$v_1 = (1\,1\,0)$

$v_2 = (1\,0\,1)$

- $a = 0, b = 0$ → $0, 0, 0$ OK
- $u = a_1 + b_1, b_1, 1$

→ sub space

i> $U = \{(2a+b, 0, b) \mid a, b \in \mathbb{R}\}$

$(2a+b, 0, b) = a(2, 0, 0) + b(1\,0\,1)$

- $a = 0, b = 0$ → OK
- $u = (2a_1 + b_1, 0, b_1)$ $v = (2a_2 + b_2, 0, b_2)$

$u + v = (2a_1 + 2a_2 + b_1 + b_2 \dots)$

- $u = (2a+b, 0, b)$

$Au = A(2a+b, 0, b)$

$= A(2a+b), A0, Ab$

$= (2Aa + Ab, 0, Ab)$

→ sub space

Thứ ngày

Q2 :

$x = (-1, -2, -2)$ $u = (0, 1, 4)$ $v = (-1, 2, 2)$

$w = (3, 1, 2)$

$$\begin{matrix} 0 & -1 & 3 & -1 \\ 1 & 1 & 1 & -2 \\ 4 & 2 & 2 & -2 \end{matrix} \qquad \begin{matrix} 1 & 1 & 1 & -2 \\ 0 & -1 & 3 & -1 \\ 4 & 2 & 2 & -2 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 1 & -2 \\ 0 & -2 & -2 & 6 \\ 0 & -1 & 3 & -1 \end{matrix} \qquad \begin{matrix} 1 & 1 & 1 & -2 \\ 0 & -2 & -2 & 6 \\ 0 & 0 & -8 & 8 \end{matrix}$$

$$\rightarrow \begin{cases} a = 1 \\ b = -2 \\ c = -1 \end{cases}$$

Q3 : $u = 1, 2$ $w = 1, -1$

a> $v = 0, 1$

$$\begin{matrix} 1 & 1 & 0 \\ 2 & -1 & 1 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & 0 \\ 0 & -3 & 1 \end{matrix} \rightarrow V = \frac{1}{3}u - \frac{1}{3}w$$

$$= \frac{1}{3}(1,2) - \frac{1}{3}(1,-1) = \left(\frac{1}{3} - \frac{1}{3}, \frac{2}{3} + \frac{1}{3}\right) = (0, 1)$$

b> $(2, 3)$

$$\begin{matrix} 1 & 1 & 2 \\ 2 & -1 & 3 \end{matrix} \Rightarrow \begin{matrix} 1 & 1 & 2 \\ 0 & -1 & -1 \end{matrix} \rightarrow V = u + w$$

$1, 2 + 1, -1 = 2, 3$

c> $(1, 4)$

$$\begin{matrix} 1 & 1 & 1 \\ 2 & -1 & 4 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & 1 \\ 0 & -1 & 2 \end{matrix} \rightarrow V = 3u - 2w$$

$3(1,2) - 2(1,-1) = 3 - 2, 6 + 2 = (1, 4)$

d> $v = (-5, 1)$

$$\begin{matrix} 1 & 1 & -5 \\ 2 & -1 & 1 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & -5 \\ 0 & -1 & 11 \end{matrix} \rightarrow V = 6u - 11w$$

$6(1,2) - 11(1,-1) = (6 - 11, 12 + 11) = (-5, 1)$

Thứ ngày

Q4: $VA = r(A|B)$

a) $x = (-3, 2, m)$ $S = \{(-1,-1,1), (2,-3,-4)\}$

$\begin{matrix} -1 & 2 & -3 \\ -1 & -3 & 2 \\ 1 & -4 & m \end{matrix} \begin{matrix} -1 & 2 & -3 \\ 0 & -5 & 5 \\ 0 & -2 & m-3 \end{matrix} \begin{matrix} -1 & 2 & -3 \\ 0 & -5 & 5 \\ 0 & 0 & +5m-25 \end{matrix}$

$+5m-25 = 0$

$\rightarrow m = +5.$

b) $x = (4, 5, m)$ $S = \{(1,-1,1), (2,-3,4)\}$

$\begin{matrix} 1 & 2 & 4 \\ -1 & -3 & 5 \\ 1 & 4 & m \end{matrix} \rightarrow \begin{matrix} 1 & 2 & 4 \\ 0 & -1 & 9 \\ 0 & 2 & m-4 \end{matrix} \rightarrow \begin{matrix} 1 & 2 & 4 \\ 0 & -1 & 9 \\ 0 & 0 & 14+m \end{matrix}$

$2m + 14 = 0$

$\rightarrow m = -14$

c) $x = (m+1, 5, m)$ $S = \{(1,1,1)(2,3,1)(3,4,2)\}$

$\begin{matrix} 1 & 2 & 3 & m+1 \\ 1 & 3 & 4 & 5 \\ 1 & 1 & 2 & m \end{matrix} \begin{matrix} 1 & 2 & 3 & m+1 \\ 0 & 1 & 1 & 4-m \\ 0 & -1 & -1 & m-5 \end{matrix} \begin{matrix} 1 & 2 & 3 & m+1 \\ 0 & 1 & 1 & 4-m \\ 0 & 0 & 0 & 5-m \end{matrix}$

$-2(4-m)$

$\rightarrow m = 5$

$2m - m - 3 = 0$

$m = 3.$

d) $(3, 5, 7, m)$ $S = \{(1,1,1,-1)(1,2,3,1)(2,3,4,0)\}$

$\begin{matrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 5 \\ 1 & 3 & 4 & 7 \\ -1 & 1 & 0 & m \end{matrix} \begin{matrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & -2 & -2 & 4 \\ 0 & 2 & 2 & m+3 \end{matrix} \begin{matrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & m+7 \end{matrix}$

$m = -7.$

rA < n -> phụ thuộc tuyến tính

rA = n -> độc lập tuyến tính

Thứ ngày

Q5:

a) S = {(-1,2) (3,1) (2,1)}

b) S = {(-1,2,3) (3,1,1) (1,3,5)}

c) S = {(1,-2,2) (2,3,5) (3,1,7)}

d) S = {(-1,2,1) (2,4,0) (3,1,1)}

e) S = {(1,-2,2,1) (1,2,3,5) (-1,3,1,7)}

a) $\begin{matrix} -1 & 3 & 2 \\ 2 & 1 & 1 \end{matrix} \rightarrow \begin{matrix} -1 & 3 & 2 \\ 0 & 7 & 5 \end{matrix}$

{k=3 > n=2 -> depend. $7x = 5$ -> phụ thuộc

b) $\begin{matrix} -1 & 3 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 5 \end{matrix}$ det = -8 ≠ 0 -> phụ thuộc dependence

c) S = {(1,-2,2) (2,3,5) (3,1,7)}

$\begin{matrix} 1 & 2 & 3 \\ -2 & 3 & 1 \\ 2 & 5 & 7 \end{matrix}$ det C = 0, independence.

d) S = {(-1,2,1) (2,4,0) (3,1,1)}

$\begin{matrix} -1 & 2 & 3 \\ 2 & 4 & 1 \\ 1 & 0 & 1 \end{matrix}$ det = -18 ≠ 0 -> dependence

e) S = {(1,-2,2,1) (1,2,3,5) (-1,3,1,7)}

$\begin{matrix} 1 & 1 & -1 \\ -2 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & 5 & 7 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & -1 \\ 0 & 4 & 1 \\ 0 & 1 & 3 \\ 0 & 4 & 8 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & -1 \\ 0 & 1 & 3 \\ 0 & 4 & 1 \\ 0 & 4 & 8 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & -11 \\ 0 & 0 & 7 \end{matrix}$

$\begin{matrix} 1 & 1 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & -11 \\ 0 & 0 & 0 \end{matrix}$ det = -11 ≠ 0 -> dependence.

Thứ ngày

Q6.

a> $S = \{(-1,2,1)\ (K,4,0)\ (3,1,1)\}$

$\begin{matrix} -1 & K & 3 \\ 2 & 4 & 1 \\ 1 & 0 & 1 \end{matrix}$ $\begin{matrix} 1 & 0 & 1 \\ -1 & K & 3 \\ 2 & 4 & 1 \end{matrix}$ $\begin{matrix} 1 & 0 & 1 \\ 0 & K & 4 \\ 0 & 4 & -1 \end{matrix}$ $\begin{matrix} 1 & 0 & 1 \\ 0 & K & 4 \\ 0 & 0 & K+16 \end{matrix}$

For make it independence $K(K+16) = 0 \rightarrow K^2 = 0$ or $K = -16$

b> $S = \{(K,1,1)\ (1,K,1)\ (1,1,K)\}$

$\begin{matrix} K & 1 & 1 \\ 1 & K & 1 \\ 1 & 1 & K \end{matrix}$ $\begin{matrix} 1 & 1 & K \\ 1 & K & 1 \\ K & 1 & 1 \end{matrix}$ $\begin{matrix} 1 & 1 & K \\ 0 & K-1 & 1-K \\ K-1 & 0 & 1-K \end{matrix}$ $\begin{matrix} 1 & 1 & K \\ 0 & K-1 & 1-K \\ 0 & -K+1 & K-1 \end{matrix}$

$\begin{matrix} 1 & 1 & K \\ 0 & K-1 & 1-K \\ 0 & 0 & 0 \end{matrix}$

For independence $\rightarrow K = 1$

c> $S = \{(1,2,1,0)\ (-2,1,1,-1)\ (-1,3,2,K)\}$

$\begin{matrix} 1 & -2 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \\ 0 & -1 & K \end{matrix}$ $\begin{matrix} 1 & -2 & -1 \\ 0 & 5 & 5 \\ 0 & +3 & 3 \\ 0 & -1 & K \end{matrix}$ $\begin{matrix} 1 & -2 & -1 \\ 0 & -1 & K \\ 0 & 5 & 5 \\ 0 & 3 & 3 \end{matrix}$ $\begin{matrix} 1 & -2 & -1 \\ 0 & -1 & K \\ 0 & 5 & 5 \end{matrix}$

$\begin{matrix} 1 & -2 & -1 \\ 0 & -1 & K \\ 0 & 0 & 5+5K \end{matrix}$

For make it independence : $-(5+5K) = 0$

$-5 \Rightarrow 5K$

$\rightarrow -1 = K$