$-2x^3+x^2$

$$\frac{2x-1}{-2x^3+x^2}$$

$$\frac{\cancel{2x-1}}{-x^2(2x-1)}$$

$$\frac{1}{-x^2}$$

~~x~~ Set $x=\frac{1}{2}$ ~~at~~ this situation: $\frac{1}{-(\frac{1}{2})^2}=-4$.

At: $S(1)$ $\lim\limits_{x\to\frac{1}{2}} f(x)=4$
$S(2)$ $\lim\limits_{x\to\frac{1}{2}} f(x)=-4$ } No Limits

Ex 14: $\lim\limits_{x\to 2} f(x)=4$; $\lim\limits_{x\to 2} g(x)=-2$; $\lim\limits_{x\to 2} h(x)=0$

a) $\lim\limits_{x\to 2} (f(x)+5g(x)) = \lim\limits_{x\to 2}(4+5.(-2))=-6$

b) $\lim\limits_{x\to 2} [g(x)]^3 = 0^3=0$

c) $\lim\limits_{x\to 2} \sqrt{f(x)} = \sqrt{4}=2$

d) $\lim\limits_{x\to 2} \frac{3f(x)}{g(x)} = \frac{3.4}{-2}=-6$

e) $\lim\limits_{x\to 2} \frac{g(x)}{h(x)} = \frac{-2}{0}=0$

f) $\lim\limits_{x\to 2} \frac{g(x).h(x)}{f^x} = \frac{-2.0}{4}=0$

Thứ ngày

Ex16:

a) $\lim_{x \to 3} \dfrac{x^2 + x - 12}{x - 3}$

$\dfrac{(x-3)(x+4)}{(x-3)}$

$\lim_{x \to 3} x + 4 = 7$

b) $\lim_{x \to -1} \dfrac{x^2 + 2x + 1}{x^4 - 1}$

$\dfrac{(x+1)(x+1)}{[(x-1)(x+1)]^2}$

$\dfrac{1}{(x-1)^2}$

$\dfrac{1}{x^2 - 2x + 1}$

$\cancel{\lim}_{x \to -1} \dfrac{1}{1^2 + 2 + 1} = \dfrac{1}{4}$

c) $\lim_{h \to 0} \dfrac{(3+h)^2 - 9}{h}$

$\dfrac{(3+h-3)(3+h+3)}{h}$

$\lim_{h \to 0} 6 + h = 6$

d) $\lim_{t \to 0} \dfrac{\sqrt{t^2 + 9} - 3}{t^2}$ $\qquad \left(\dfrac{1+2}{3}\right)^2 \qquad \dfrac{(1+2)^2}{3^2}$

$\dfrac{(t^2 + 9 - 9)(t^2 + 9 + 9)}{t^4}$

Thứ ngày

Ex 12:

i> $\lim_{x \to 0^+} f(x) = 2$

$\lim_{x \to 0^-} f(x)$ not exist.

-> $\lim_{x \to 0} f(x)$ not exist

ii> $\lim_{x \to 1} f(x)$

this function not exists at a given point

-> no given point

~~function~~ continious, ~~not~~ have limit.

iii> $\lim_{x \to 4} f(x)$

This function have a given point but not exists.

This function is a continious function, because does not have any ~~sht~~ slipt point in carve line.

Ex 13:

a> $\lim_{x \to -2} \frac{2-|x|}{2+x}$

. $x \to -2 \to |x| = -x$.

. $\lim_{x \to -2} \frac{2-(-x)}{2+x}$

$\lim_{x \to -2} \frac{2+x}{2+x}$

- Does not slipt 2 situation because $-2^+ < 0$.

Close -> $-2^- < 0$.

Ex 10 : $f(x) = \sqrt{x}$ $g(x) = \sqrt{2-x}$

a> $f \circ g$

$f(\sqrt{2-x}) = \sqrt[4]{2-x}$

b> $g \circ f$

$g(\sqrt{x}) = \sqrt{2-\sqrt{x}}$

c> $f \circ f$

$f(\sqrt{x}) = \sqrt[4]{x}$

d> $g \circ g$

$g(\sqrt{2-x}) = \sqrt{2-\sqrt{2-x}}$

Ex 11

a> $f(g(1))$

$= f(6) = 5$

b> $g(f(1))$

$= g(3) = 2$

c> $f(f(1))$

$= f(3) = 4$

d> $g(g(1))$

$= g(6) = 3$

e> $(g \circ f)(3)$

$= g(f(3)) = g(4) = 1$

g $(g \circ f)(6)$

$= g(f(6)) = g(5) = 2$

$x \to -2 \qquad \dfrac{\ldots}{x-1} = \dfrac{\ldots}{-3} = -1$

Ex 21

a) $f(x) = \begin{cases} x^2 - m^2, & x < 4 \\ mx + 20, & x \geq 4 \end{cases}$

$$\lim_{x \to 4^-} x^2 - m^2 = \lim_{x \to 4^+} mx + 20 = f(4)$$

$$16 - m^2 = 4m + 20$$

$$m^2 + 4m + 4 = 0$$

$$m = -2$$

Thứ ngày

Ex 17:

a) $\lim_{x \to 3} \frac{x^6 - 1}{x^{10} - 1}$

$\lim_{x \to 3} \frac{1}{x^4 - 1} = \frac{1}{80}$

b) $\lim_{x \to -4} \frac{\frac{1}{4} + \frac{1}{x}}{x+4}$

$\cancel{x^2 - 16}$

$\lim_{x \to -4} \frac{x + 4}{4x(x+4)} = \frac{1}{4x} = \frac{1}{-16}$

c) $\lim_{x \to 0} \frac{x}{\sqrt{1+3x} - 1}$

$\frac{x(\sqrt{1+3x} + 1)}{1 + 3x - 1}$

$\cancel{\frac{\sqrt{1+3x} + 1}{\frac{2x}{1+2x-1}}}$

$\frac{x(\sqrt{1+3x} + 1)}{3x}$

$\lim_{x \to 0} \frac{\sqrt{1+3x} + 1}{3} = \frac{2}{3}$

$$\frac{1-1}{1-1} = m+1$$

$$\Rightarrow m = -1$$

Ex 19

at $x = 0$

$x \to 0^- = 0$, $x \to 0^+ = 2$

x not defined $\Rightarrow x = 0$ this functions not continuous

HONGHA

Thứ ngày

Ex18:

a> $4x-9 \le f(x) \le x^2-4x+7$

up ~~range~~ range of $f(x)$: x^2-4x+7

x^2-4x+7

$x \to 4$ $4^2-4.4+7=7$ (1)

down range of $f(x)$: $4x-9$

$x \to 4$ $4x-9 = 16-9=7$ (2)

(1) (2) -> $\begin{cases} x^2-4x+7 \\ 4x-9 \end{cases}$ is 2 sides of f(x) limits

$\lim_{x \to 4} f(x) = 7.$

b> $\lim_{x \to 0} \sqrt{x^3+x^2}\, \sin\left(\frac{\pi}{x}\right)$

$\sqrt{x^2(x+1)}\, \sin\left(\frac{\pi}{x}\right)$

∘ $x^2 \ge 0$: $|x|\sqrt{x+1} . \sin\frac{\pi}{x}$

$\lim_{x \to 0} |x|\sqrt{x+1} . \sin\frac{\pi}{x}$

$= 0$

Thứ ngày

d) $\lim_{t \to 0} \dfrac{\sqrt{t^2+9}-3}{t^2}$

$\dfrac{(t^2+9-9)(t^2+9+9)}{t^2(\sqrt{t^2+9}-3)}$

$\dfrac{t^2(t^2+18)}{t^2(\sqrt{t^2+9}-3)}$

$\lim_{t \to 0} \dfrac{t^2+18}{\sqrt{t^2+9}-3} = \dfrac{18}{3+3} = 3$

e) $\lim_{x \to -1} \dfrac{2x^2+3x+1}{x^2-2x-3}$

$\dfrac{2(x+\frac{1}{2})(x+1)}{(x-3)(x+1)}$

$\lim_{x \to -1} \dfrac{2x+1}{x-3} = \dfrac{+1}{+4}$

f) $\lim_{x \to 1} \dfrac{x^2-1}{|x-1|}$

C_1 : $\dfrac{(x+1)(x-1)}{x-1}$

$\lim_{x \to 1} x+1 = 2$

C_2 : $\dfrac{(x+1)(x-1)}{x+1}$ =

$\lim_{x \to 1} x-1 = 0$

Thứ ngày

at $x = 2$

$x \to 2^- = -2$, $x \to 2^+ = 2$

x not defined at $x = 2$.

$\to$ this function not continouses

at $x = 4$

$x \to 4^- = 4$, $x \to 4^+ = 4$

but x not defined at $\cancel{0}x$ when $x = 4$

$\to$ This function not continouses.

Thứ ngày

Domain $x \neq -2$

$\cancel{\lim_{x \to 2}}$ $\frac{2+x}{2+x} = 1$

So, if $x \neq -2$ $\lim f(x) = 1$

b) $\lim_{x \to 0,5} \frac{2x-1}{|2x^3 - x^2|}$

∘ $|2x^3 - x^2|$ of $\lim_{x \to 0,5}$, 2 situation:

1, $2x^3 - x^2 / 0$: $x \neq \frac{1}{2}$; $x \neq 0$

2, $-2x^3 + x^2 / 0$: $x \neq -\frac{1}{2}$; $x \neq 0$

$$\frac{2x-1}{|2x^3 - x^2|} = \frac{2x-1}{|x^2| \cdot |2x-1|}$$

$$= \frac{2x-1}{x^2 \, |2x-1|}$$

$|2x-1| \neq 0$ at $2x-1$

$x \neq \frac{1}{2}$

and x must $\neq 0$ } $x \neq \frac{1}{2}$

If $x \to \frac{1}{2}^+$ So this function change to: $\frac{1}{x^2}$

$\cancel{\lim}$ $\frac{1}{\left(\frac{1}{2}\right)^2} = \frac{1}{\frac{1}{4}} = 4$

Thứ ngày

d) $\lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$

$\frac{3+x-3}{x(\sqrt{3+x}+\sqrt{3})}$

$\lim_{x \to 0} \frac{1}{\sqrt{3+x}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$

e) $\lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right)$

$\frac{t^2+t-t}{t(t^2+t)}$

$\frac{t}{t^2+t}$

$\lim_{t \to 0} \frac{1}{t+1} = 1$

f) $\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

$(x+h)^3 = x^3 + \cancel{3x^2h} + 3x^2h + 3h^2x + h^3$

$\frac{x^3 - x^3 + 3x^2h + 3h^2x + h^3}{h}$

$\lim_{h \to 0} 3x^2 + 3hx + h^3 = 3x^2$

Thứ ngày

b> $f(x) = \begin{cases} mx^2 + 2x, & x < 2 \\ x^3 - mx, & x \geq 2 \end{cases}$

for function continuous:

$$\lim_{x \to 2^-} mx^2 + 2x = \lim_{x \to 2^+} x^3 - mx = f(2)$$

$\cancel{2x^2}$

$$m4 + 4 = 8 - 2m$$

$$6m = 4$$

$$\to m = \frac{2}{3}$$

c> $f(x) \begin{cases} \dfrac{x^2 - 1}{\sqrt{x} - 1}, & x > 1 \\ mx + 1, & x \leq 1 \end{cases}$

for fuction continuous:

$$\lim_{x \to 1^-} \frac{x^2 - 1}{\sqrt{x} - 1} = \lim_{x \to 1^+} mx + 1 = f(1)$$

$$\frac{1 - 1}{1 - 1} = m + 1$$

$$\to m = -1$$

Ex 19

at $x = 0$

$x \to 0^- = 0$, $x \to 0^+ = 2$

x not defined. $\to$ $x = 0$ this functions not continuous

Thứ ngày

Ex 22

$$\lim_{x \to -2} \frac{3x^2 + mx + m + 3}{x^2 + x - 2}$$

$x \to -2$ $\quad 3(-2)^2 + (-2)m + m + 3$

$12 - 2m + m + 3 \Leftrightarrow m = 15$

$$\frac{3x^2 + 15x + 15 + 3}{x^2 + x - 2}$$

$$\frac{3x^2 + 15x + 18}{x^2 + x - 2}$$

$$\frac{3(x+2)(x+3)}{(x-1)(x+2)}$$

$$\lim_{x \to -2} \frac{3x + 9}{x - 1} = \frac{3}{-3} = -1$$

Ex 21

a) $f(x) = \begin{cases} x^2 - m^2, & x < 4 \\ mx + 20, & x \geq 4 \end{cases}$

$mx + 20 = f(4)$

Thứ ngày

Ex23 $\lim_{x \to 1} \frac{f(x) - 8}{x - 1} = 10$

$\lim_{x \to 1} f(x) = ?$

$\frac{f(x) - 8}{x - 1} = 10$

$f(x) - 8 = 10x - 10$

$f(x) = 10x - 2$

$\lim_{x \to 1} f(x) = 8$.

$(x+1)(x+2)$

$\frac{a}{2}x = 0$

Ex 24:

a) $y = \frac{2x^2 + x - 1}{x^2 + x - 2}$

- horizontal asymptotes:

$\lim_{x \to \infty} y = 2$

$\lim_{x \to -\infty} y = 2$

HONG HA

~~Ngang $\lim_{x \to \infty} = L$ 1) ; d~~

~~Đứng $\lim \to {}^{+}L^{-} ; {}^{-}L^{+}$~~

o vertical asymptotes:

$\lim_{x \to 1^+} y = -\infty$ $\quad$ $\lim_{x \to 1^-} y = +\infty$

$\lim_{x \to -2^+} y = -\infty$ $\quad$ $\lim_{x \to 2^-} y = +\infty$

This function have horizontal line at $x = 2$

vertical line at $\left[\begin{array}{l} x = -1 \\ x = 2 \end{array}\right.$

$x \to 4^- = 4$, $x \to 4^+ = 4$

but x not defined at ~~θx~~ when $x = 4$

→ This function not continouses.

Ex 20:

a) $\dfrac{2x^2 + x - 1}{x - 2}$

$D = x \neq 2$.

This function is a ~~inter~~ rational fraction, continious ~~R~~ = expect $\{2\}$

b) $f(x) = \dfrac{x - 9}{\sqrt{4x^2 + 4x + 1}}$

D: $4x^2 + 4x + 1 > 0$

$\to D: x \neq -\dfrac{1}{2}$

This fuction is a rational fraction, continious expect $\{-\dfrac{1}{2}\}$

c) $f(x) = \ln(2x + 5)$

D: $x > -\dfrac{5}{2}$

This fuction is continious ~~I~~f $x > -\dfrac{5}{2}$

Thứ ngày

b) $y = \dfrac{x-9}{\sqrt{4x^2+3x+2}}$

- horizontal line:

$\lim\limits_{x \to +\infty} y = \dfrac{1}{2}$ $\lim\limits_{x \to -\infty} y = -\dfrac{1}{2}$

- vertical line:

~~$\lim\limits_{x \to}$~~ $4x^2 + 3x + 2$

-> This function do not have x_1, x_2

Because:

$\left.\begin{array}{l} 4 > 3 > 2 \\ x^2 > x > nums \end{array}\right\} \to x > 0$

This's function have ~~roots~~.

So, this function does not have vertical line.

Ex 23: