Data Structures & Algorithms

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Lecture 6 Deque Array Based

Course Roadmap



Part 1: Linear Data Structures

Lecture 1: Complexity Analysis & Recursion

Lecture 2: Arrays

Lecture 3: Linked List

Lecture 4: Stack

Lecture 5: Queue

Lecture 6: Deque

Lecture 7: STL in C++ (Linear Data Structures)

Lecture Agenda

We will discuss in this lecture the following topics

- 1- Introduction to Deque
- 2- Insertion Operation
- 3- Deletion Operation
- 4- Front & Back Operations
- 5- Traverse Operation
- 6- Time Complexity & Space Complexity

4



Lecture Agenda



Section 1: Introduction to Deque

Section 2: Insertion Operation

Section 3: Deletion Operation

Section 4: Front & Back Operations

Section 5: Traverse Operation

Section 6: Time Complexity & Space Complexity



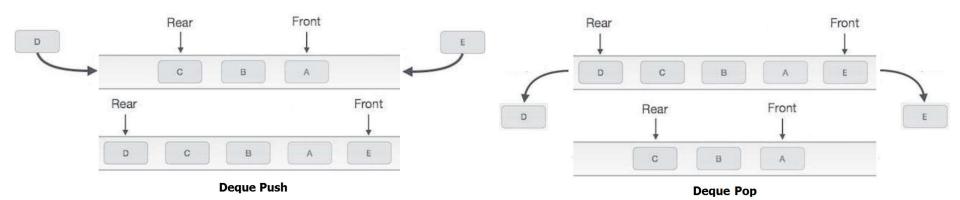
Introduction to Deque

- A double-ended queue (abbreviated to deque) is an abstract data type that generalizes a queue, for which elements can be added to or removed from either the front (head) or back (tail). It is also often called a head-tail linked list.
- A deque, also known as a double-ended queue, is an ordered collection of items similar to the queue. It has two ends, a front and a rear, and the items remain positioned in the collection. What makes a deque different is the unrestrictive nature of adding and removing items. New items can be added at either the front or the rear. Likewise, existing items can be removed from either end. In a sense, this hybrid linear structure provides all the capabilities of stacks and queues in a single data structure.
- A deque, also known as a double-ended queue where by it has two ends, a front and a rear, and the items remain positioned in the collection. New items can be added or removed at either the front or the rear. It does not require the LIFO and FIFO orderings that are enforced by those data structures. It is up to you to make consistent use of the addition and removal operations.



Introduction to Deque

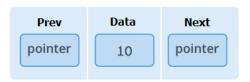
- CODEFORCES AtCoder
- Deque operations may involve initializing or defining the deque, utilizing it, and then completely erasing it from the memory. Here we shall try to understand the basic operations associated with deques
- insert begin() adds a new item to the front of the deque.
- insert end() adds a new item to the rear of the deque.
- delete begin() removes the front item from the deque.
- delete end() removes the rear item from the deque.



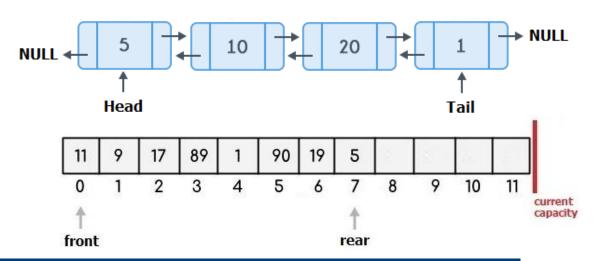
Introduction to Deque

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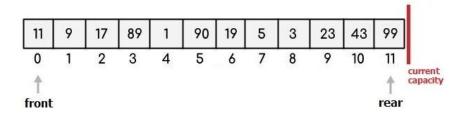
- Following are the basic operations supported by a deque.
- Insert (begin & end) adds a new item to the front & rear of the deque.
- Delete (begin & end) removes the front & rear item from the deque.
- Front & Back: which gets the first element and the last element in the queue.
- Deque Types
- 1. Deque (Linked List Based)



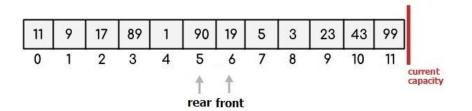
2. Deque (Array Based)



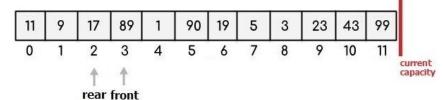




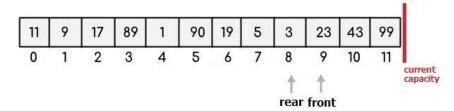
11, 9, 17, 89, 1, 90, 19, 5, 3, 23, 43, 99



19, 5, 3, 23, 43, 99, 11, 9, 17, 89, 1, 90

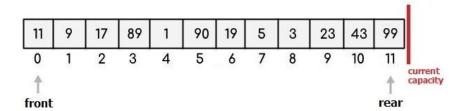


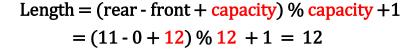
89, 1, 90, 19, 5, 3, 23, 43, 99, 11, 9, 17

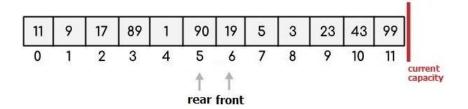


23, 43, 99, 11, 9, 17, 89, 1, 90, 19, 5, 3

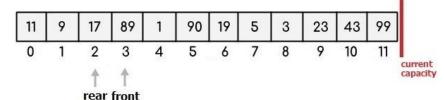




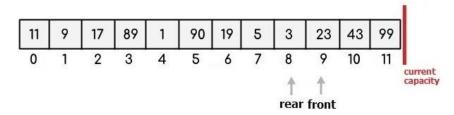




Length = (rear - front + capacity) % capacity +1
=
$$(5 - 6 + 12)$$
 % $12 + 1 = 12$

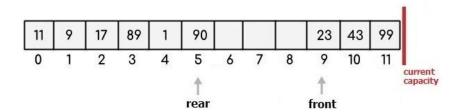


Length = (rear - front + capacity) % capacity +1
=
$$(2 - 3 + 12)$$
 % $12 + 1 = 12$

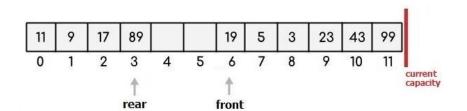


Length = (rear - front + capacity) % capacity +1
=
$$(8 - 9 + 12)$$
 % $12 + 1 = 12$

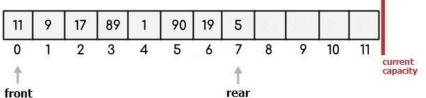




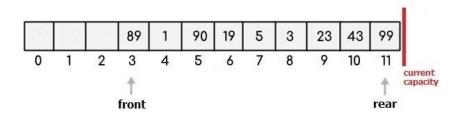
23, 43, 99, 11, 9, 17, 89, 1, 90



19, 5, 3, 23, 43, 99, 11, 9, 17, 89

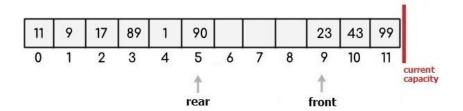


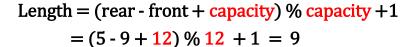
11, 9, 17, 89, 1, 90, 19, 5

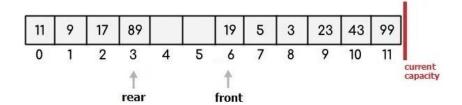


89, 1, 90, 19, 5, 3, 23, 43, 99

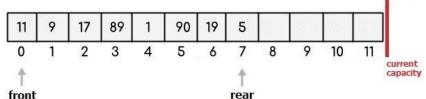




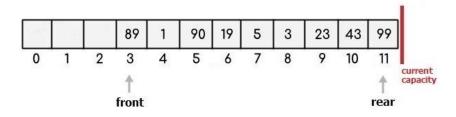




Length = (rear - front + capacity) % capacity +1
=
$$(3 - 6 + 12)$$
 % $12 + 1 = 10$



Length = (rear - front + capacity) % capacity +1
=
$$(7 - 0 + 12)$$
 % 12 + 1 = 8



Length = (rear - front + capacity) % capacity +1
=
$$(11 - 3 + 12)$$
 % $12 + 1 = 9$

Reserve Method - Deque (Array Based)

```
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```
// This function updates the capacity of the deque
void reserve(int new capacity) {
    // Initialize a new deque with the new capacity
    int* temp = new int[new capacity];
    // copy the elements in the current deque to the new deque
    for (int i = 0; i < n; i++)
        temp[i] = arr[(front idx + i) % capacity];
    // delete the old deque
    delete[] arr;
                                                                                rear front
    // set the temp deque with new capacity to be the deque
    arr = temp;
                                                                       17
    // reset front_idx and rear_idx
    front idx = 0;
    rear idx = n - 1;
                                                       front
                                                                      rear
    // set the current capacity of the deque to be the new capacity
    capacity = new capacity;
```

Lecture Agenda





✓ Section 1: Introduction to Deque

Section 2: Insertion Operation

Section 3: Deletion Operation

Section 4: Front & Back Operations

Section 5: Traverse Operation

Section 6: Time Complexity & Space Complexity







 Insert Operation is to add (store) an item to the deque.

• Insertion Algorithm:

If the array is full increase the capacity

1.
$$if n == 0 then front = rear = 0$$

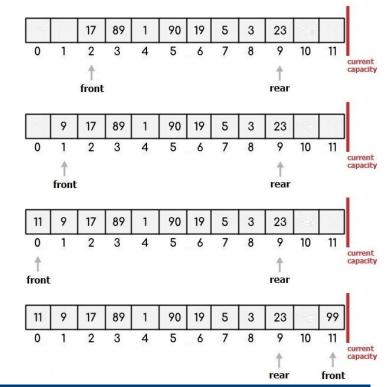
2. otherwise front

$$= (front - 1 + capacity) \% capacity$$

3. arr[front] = data

4.
$$n = n + 1$$

Insert at begin 9, 11, 99



```
// This function inserts an element at the begin of the deque
void insert begin(int new data) {
    // check if we need to update the capacity of the deque
    if (n == capacity)
        reserve(2 * capacity + 1);
    // check if the deque is empty
    if (n == 0)
        front idx = rear idx = 0;
    // update the front idx of the deque
    else
        front idx = (front idx - 1 + capacity) % capacity;
    // insert current element into Deque
    arr[front idx] = new data;
    // update the size of the deque
    n = n + 1;
```







 Insert Operation is to add (store) an item to the deque.

• Insert at end 43, 99, 11



• Insertion Algorithm:

If the array is full increase the capacity

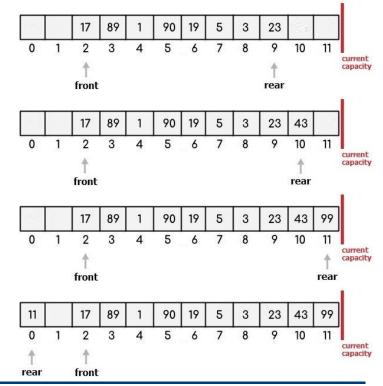
1.
$$if n == 0 then front = rear = 0$$

2. otherwise rear

$$= (rear + 1) \% capacity$$

3.
$$arr[rear] = data$$

4.
$$n = n + 1$$



```
This function inserts an element at the end of the deque
void insert end(int new data) {
    // check if we need to update the capacity of the deque
    if (n == capacity)
        reserve(2 * capacity + 1);
    // check if the degue is empty
    if (n == 0)
        front idx = rear idx = 0;
    // update the rear idx of the deque
    else
        rear idx = (rear idx + 1) % capacity;
    // insert the new element
    arr[rear idx] = new data;
    // update the size of the deque
   n = n + 1;
```





Lecture Agenda



- ✓ Section 1: Introduction to Deque
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 Delete Operation remove (access) an item from the deque.

• Deletion Algorithm:

1. if front == rear then
$$front = rear = -1$$

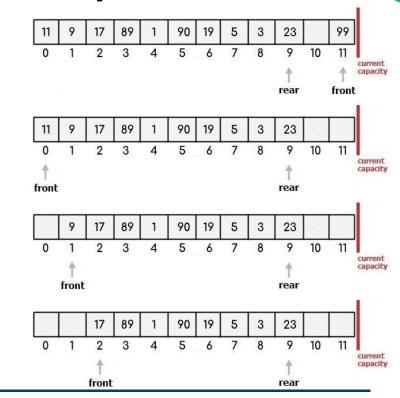
2. otherwise front

$$= (front + 1) \% capacity$$

3.
$$n = n - 1$$

If the array is half full decrease the capacity

Delete at begin 99, 11, 9



```
This function deletes the first element in the deque
void delete begin() {
    // check if the deque is empty
    if (n == 0)
        return:
    // check if the deque has only one element
    if (front idx == rear idx)
        front idx = rear idx = -1;
    // otherwise update the front idx of the queue
    else
        front idx = (front idx + 1) % capacity;
    // update the size of the deque
    n = n - 1;
    // check if we need to update the capacity of the deque
    if (n < capacity / 2)</pre>
        reserve(capacity / 2);
```







• Delete Operation remove (access) an item from the deque.

• Deletion Algorithm:

1. if front == rear then
$$front = rear = -1$$

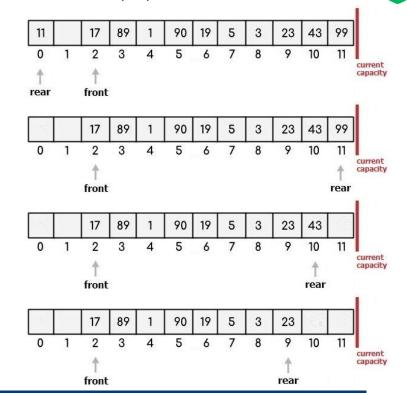
2. otherwise rear

$$= (rear - 1 + capacity) \% capacity$$

3.
$$n = n - 1$$

If the array is half full decrease the capacity

Delete at end 11, 99, 43



```
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```

```
This function deletes the last element in the deque
void delete end() {
    // check if the deque is empty
    if (n == 0)
        return:
    // check if the degue has only one element
    if (front idx == rear idx)
        front idx = rear idx = -1;
    // update the rear idx of the deque
    else
        rear idx = (rear idx - 1 + capacity) % capacity;
    // update the size of the deque
    n = n - 1;
    // check if we need to update the capacity of the deque
    if (n < capacity / 2)</pre>
       reserve(capacity / 2);
```



Lecture Agenda



- ✓ Section 1: Introduction to Deque
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Section 6: Time Complexity & Space Complexity

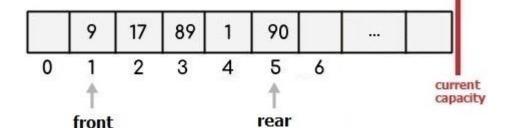


Front & Back Operations - Deque (Array Based)

- CODEFORCES AtCoder
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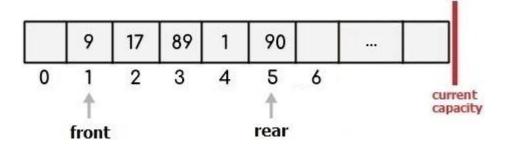
- Front Operation gets the first item in the queue.
- Front 9

- Front Algorithm:
- 1. return arr[front]



- Back Operation gets the last item in the queue.
- Back 90

- Back Algorithm:
- 1. return arr[rear]



Front & Back Operations - Deque (Array Based)

```
CODEFORCES At Coder
```

```
This function returns the value of the first element in the deque
int front() {
   // check if the deque is empty
   // to return the biggest integer value as an invalid value
   if (n == 0)
        return INT MAX;
   // otherwise return the real value
   else
        return arr[front idx];
   This function returns the value of the last element in the deque
int back() {
    // check if the degue is empty
    // to return the biggest integer value as an invalid value
    if (n == 0)
        return INT MAX;
    // otherwise return the real value
    else
        return arr[rear idx];
```



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Traverse Operation - Deque (Array Based)

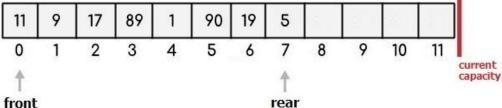


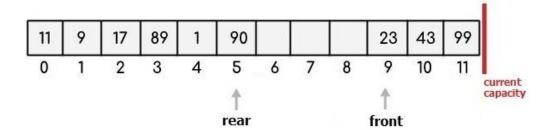
Traverse operation is to traverse through the elements of an array.

Traverse this deque



- Traverse Algorithm:
- 1. i = 0
- 2. print arr[(front + i) % capacity]
- 3. i = i + 1
- 4. Repeat Step 3 if i < n





Traverse Operation - Deque (Array Based)

```
CODEFORCES AtCoder
```

```
// This function prints the contents of the deque
void print deque() {
    // loop to print the elements in the deque
    for (int i = 0; i < n; i++)
        cout << arr[(front_idx + i) % capacity] << ' ';</pre>
  This function prints the contents of the deque
void print deque reverse() {
    // loop to print the elements in the deque
    for (int i = n-1; i \rightarrow 0; i-1) i = n-1; i \rightarrow 0;
        cout << arr[(front idx + i) % capacity] << ' ';</pre>
```





> Initialize a global struct

```
#include <bits/stdc++.h>
using namespace std;

// Initialize an deque with dynamic length
int front_idx = -1, rear_idx = -1;
int n;
int capacity;
int* arr;
```

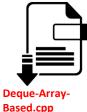


cout << "the above elements have been added to the deque\n";</pre>



➤ In the Main function:

```
cout << "Deque front: " << front() << " and Deque back: " << back() << '\n';</pre>
cout << "Deque items forward: ";</pre>
print deque();
                                           Deque front: 2147483647 and Deque back: 2147483647
cout << '\n';
                                           Deque items forward:
cout << "Deque items backward: ";</pre>
                                           Deque items backward:
print deque reverse();
cout << '\n';
cout << "adding the following elements 10 20 30 40 50\n";
insert end(10);
                                           adding the following elements 10 20 30 40 50
insert end(20);
                                           the above elements have been added to the deque
insert end(30);
insert end(40);
insert end(50);
```





➤ In the Main function:

cout << '\n';

```
cout << "Deque front: " << front() << " and Deque back: " << back() << '\n';</pre>
cout << "Deque items forward: ";</pre>
print deque();
                                           Degue front: 10 and Degue back: 50
cout << '\n';
                                           Deque items forward: 10 20 30 40 50
cout << "Deque items backward: ";
                                           Degue items backward: 50 40 30 20 10
print deque reverse();
cout << '\n';
cout << "add element 60 at the end of the deque\n";</pre>
insert end(60);
                                           add element 60 at the end of the deque
cout << "Deque front: " << front() << " and Deque back: " << back() << '\n';</pre>
cout << "Deque items forward: ";</pre>
                                           Deque front: 10 and Deque back: 60
print deque();
                                           Deque items forward: 10 20 30 40 50 60
cout << '\n';
                                           Deque items backward: 60 50 40 30 20 10
cout << "Deque items backward: ";</pre>
print deque reverse();
```





➤ In the Main function:





➤ In the Main function:





► In the Main function:

```
Expected Output:
```





► In the Main function:

```
Expected Output:
```





In the Main function:

```
cout << "delete the first element \n":
delete begin();
cout << "delete the last element \n";</pre>
delete end();
cout << "Deque items forward: ";</pre>
print deque();
cout << '\n';
cout << "Deque items backward: ";</pre>
print deque reverse();
cout << '\n';
```

```
delete the first element
                                          delete the last element
cout << "Deque front: " << front() << " and Deque back: " << back() << '\n';</pre>
                                          Deque front: 10 and Deque back: 40
                                          Degue items forward: 10 20 30 40
                                          Deque items backward: 40 30 20 10
```





➤ In the Main function:

cout << '\n';

```
cout << "deleting the following elements 10 20 30 40\n";
delete end();
                                          deleting the following elements 10 20 30 40
delete end();
                                          the above elements have been deleted from the deque
delete end();
delete end();
cout << "the above elements have been deleted from the deque\n";
cout << "Deque front: " << front() << " and Deque back: " << back() << '\n';</pre>
cout << "Deque items forward: ";</pre>
                                           Degue front: 2147483647 and Degue back: 2147483647
print deque();
                                           Deque items forward:
cout << '\n';
                                           Deque items backward:
cout << "Deque items backward: ";</pre>
print deque reverse();
```





In the Main function:

```
cout << "adding the following elements 40 30 20 10\n";
insert begin (10);
                                               adding the following elements 40 30 20 10
insert begin (20);
                                               the above elements have been added to the deque
insert begin (30);
insert begin (40);
cout << "the above elements have been added to the deque\n";</pre>
cout << "Deque front: " << front() << " and Deque back: " << back() << '\n';</pre>
cout << "Deque items forward: ";</pre>
                                           Degue front: 40 and Degue back: 10
print deque();
                                           Deque items forward: 40 30 20 10
cout << '\n';
                                           Deque items backward: 10 20 30 40
cout << "Deque items backward: ";</pre>
print deque reverse();
cout << '\n';
```





➤ In the Main function:

cout << '\n';

```
cout << "deleting the following elements 40 30 20 10\n";
delete begin();
                                          deleting the following elements 40 30 20 10
delete begin();
                                          the above elements have been deleted from the deque
delete begin();
delete begin();
cout << "the above elements have been deleted from the deque\n";
cout << "Deque front: " << front() << " and Deque back: " << back() << '\n';</pre>
cout << "Deque items forward: ";</pre>
                                           Deque front: 2147483647 and Deque back: 2147483647
print deque();
                                           Deque items forward:
cout << '\n';
                                           Deque items backward:
cout << "Deque items backward: ";</pre>
print deque reverse();
```



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Section 6: Time Complexity & Space Complexity



Time Complexity & Space Complexity

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> Time Analysis

	Worst Case	Average Case
 Insert at begin 	⊙ (n)	θ(1)
 Insert at end 	$\mathbf{\Theta}(n)$	$\Theta(1)$
 Delete at begin 	$\mathbf{\Theta}(n)$	$\Theta(1)$
• Delete at end	$\mathbf{\Theta}(n)$	$\Theta(1)$
 Front 	$\Theta(1)$	$\Theta(1)$
• Back	$\Theta(1)$	$\Theta(1)$
 Traverse 	$\mathbf{\Theta}(n)$	$\Theta(n)$

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CODEFORCES

- 1- Reverse string using stack
- 2- Check string is palindrome or not
- 3- Convert Infix Expression to Postfix Expression
- 4- Convert Infix Expression to Prefix Expression
- 5- Convert Postfix Expression to Infix Expression
- 6- Convert Prefix Expression to Infix Expression
- 7- Convert Postfix Expression to Prefix Expression
- 8- Convert Prefix Expression to Postfix Expression
- 9- Evaluation of Postfix Expression
- 10- Reverse a stack using recursion
- 11- Check for balanced parentheses in an expression
- 12- Length of the longest valid substring
- 13- Minimum number of bracket reversals needed to make an expression balanced
- 14- Next Greater Element
- 15- Delete middle element of a stack

CODEFORCES

- 16- Reverse individual words
- 17- Largest Rectangular Area in a Histogram
- 18- Find maximum depth of nested parenthesis in a string
- 19- Expression contains redundant bracket or not
- 20- Check if two expressions with brackets are same
- 21- Delete consecutive same words in a sequence
- 22- Remove brackets from an algebraic string
- 23- Range Queries for Longest Correct Bracket Subsequence
- 24- Check if stack elements are pairwise consecutive
- 25- Reverse a number using stack
- 26- Tracking current Maximum Element in a Stack
- 27- Decode a string recursively encoded as count followed by substring
- 28- Find maximum difference between nearest left and right smaller elements
- 29- Find if an expression has duplicate parenthesis or not
- 30- Find index of closing bracket for a given opening bracket in an expression

CODEFORCES AtCoder

- 31- Sliding Window Maximum summation of all sub-arrays of size k
- 32- Check string is palindrome or not
- 33- Generate Binary Numbers from 1 to n
- 34- Reversing a queue using recursion
- 35- Reversing the first K elements of a Queue
- 36- Find the largest multiple of 3
- 37- Smallest multiple of a given number made of digits 0 and 9 only
- 38- Delete all elements in the deque
- 39- Sum of minimum and maximum elements of all subarrays of size k
- 40- First negative integer in every window of size k

Assignment



- Deque (usually pronounced like "deck") is an irregular acronym of double-ended queue. Double-ended queues are sequence containers with dynamic sizes that can be expanded or contracted on both ends (either its front or its back).
- Specific libraries may implement deques in different ways, generally as some form of dynamic array. But in any case, they allow for the individual elements to be accessed directly through random access iterators, with storage handled automatically by expanding and contracting the container as needed.
- Therefore, they provide a functionality similar to vectors, but with efficient insertion and deletion of elements also at the beginning of the sequence, and not only at its end. But, unlike vectors, deques are not guaranteed to store all its elements in contiguous storage locations: accessing elements in a deque by offsetting a pointer to another element causes undefined behavior.
- Double ended queues are sequence containers with the feature of expansion and contraction on both the ends. They are similar to vectors, but are more efficient in case of insertion and deletion of elements. Unlike vectors, contiguous storage allocation may not be guaranteed.



- Both vectors and deques provide a very similar interface and can be used for similar purposes, but internally both work in quite different ways: While vectors use a single array that needs to be occasionally reallocated for growth, the elements of a deque can be scattered in different chunks of storage, with the container keeping the necessary information internally to provide direct access to any of its elements in constant time and with a uniform sequential interface (through iterators). Therefore, deques are a little more complex internally than vectors, but this allows them to grow more efficiently under certain circumstances, especially with very long sequences, where reallocations become more expensive.
- Double Ended Queues are basically an implementation of the data structure double ended queue. A queue data structure allows insertion only at the end and deletion from the front. This is like a queue in real life, wherein people are removed from the front and added at the back. Double ended queues are a special case of queues where insertion and deletion operations are possible at both the ends. The functions for deque are same as vector, with an addition of push and pop operations for both front and back.

More Info: cplusplus.com/reference/deque/deque/deque/
More Info: en.cppreference.com/w/cpp/container/deque

More Info: geeksforgeeks.org/deque-cpp-stl/



CODEFORCES

Member functions: (constructor) Construct vector (public member function)

(destructor) Vector destructor (public member function)

(operator=) Assign content (public member function)

Iterators: (begin) Return iterator to beginning (public member function)

(end) Return iterator to end (public member function)

(rbegin) Return reverse iterator to reverse beginning (public member function)

(rend) Return reverse iterator to reverse end (public member function)

(cbegin) Return const<u>i</u>terator to beginning (public member function)

(cend) Return const_iterator to end (public member function)

(crbegin) Return const_reverse_iterator to reverse beginning (public member function)

(crend) Return const_reverse_iterator to reverse end (public member function)

CODEFORCES

Capacity: (size) Return size (public member function)

(max_size) Return maximum size (public member function)

(resize) Change size (public member function)

(empty) Test whether vector is empty (public member function)

(shrink to fit (public member function)

• Element access: (operator[]) Access element (public member function)

(at) Access element (public member function)

(front) Access first element (public member function)

(back) Access last element (public member function)

CODEFORCES

Modifiers:

(assign) Assign vector content (public member function)
(push_back) Add element at the end (public member function)
(push_front) Insert element at beginning (public member function)
(pop_back) Delete last element (public member function)
(pop_front) Delete first element (public member function)
(insert) Insert elements (public member function)
(erase) Erase elements (public member function)
(swap) Swap content (public member function)
(clear) Clear content (public member function)

More Info: cplusplus.com/reference/deque/d

