15-110 Principles of Computing – F21

Lecture 14:
Functions

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Functions: callable, named subprograms (procedures)

- **Function:** informally, a *subprogram*
  - we write a sequence of statements and give that sequence a name
  - the instructions can then be executed at any point in a program by referring to the function name

```
def function_name(parameters):
    function_body
    return something
```

User function definition

```
def sum_two(x, y):
    z = x + y
    return z
```

```
a, b = 3, 4
s = sum_two(a, b)

print("Sum is: ", s)
```

Returned value

Input arguments

Calling (invoking) the user-defined function

Built-in function (python standard library)

All functions *return* something!

```
def function_name(parameters):
    function_body
```

is <u>implemented</u> as:

```
def function_name(parameters):
    function_body
    return None
```

↓

All function calls return something, `None` when left unspecified in the function body

Built-in functions (Python standard library)

abs()	delattr()	hash()	memoryview()	set()
all()	dict()	help()	min()	setattr()
any()	dir()	hex()	next()	slice()
ascii()	divmod()	id()	object()	sorted()
bin()	enumerate()	input()	oct()	staticmethod()
bool()	eval()	int()	open()	str()
breakpoint()	exec()	isinstance()	ord()	sum()
bytearray()	filter()	issubclass()	pow()	super()
bytes()	float()	iter()	print()	tuple()
callable()	format()	len()	property()	type()
chr()	frozenset()	list()	range()	vars()
classmethod()	getattr()	locals()	repr()	zip()
compile()	globals()	map()	reversed()	__import__()
complex()	hasattr()	max()	round()	

https://docs.python.org/3/library/functions.html

Functions: organizing the code, putting aside functionalities

- Functions are a fundamental way to *organize* the code into **procedural elements** that can be *reused*
- Functions provide **structure and organization,** that facilitate:

Decomposition

Abstraction

Reusability

Fundamental ingredients in the design of computational solutions

Decomposition – Abstraction – Reusability: an example

❖ A natural number n with D digits is named **armstrong** if the sum of D-th power of digits equals to n itself

Given $n = d_1 d_2 d_3 \cdots d_D$ If $d_1^D + d_2^D + d_3^D + \cdots + d_D^D = n$ $\rightarrow n$ is **armstrong**

- $n = 1450$ has $D = 4$ digits: $1^4 + 4^4 + 5^4 + 0^4 = 882$ $\rightarrow$ 1450 is NOT armstrong
- $n = 153$ has $D = 3$ digits: $1^3 + 5^3 + 3^3 = 153$ $\rightarrow$ 153 is armstrong!
- $n = 3$ has $D = 1$ digit: $3^1 = 3$ $\rightarrow$ 3 is armstrong!

Write the function `is_special_number(n)` that takes as input a natural number `n` and returns `True` if the number is **either an armstrong or a prime number**, `False` otherwise.
The function also prints out if the number is prime or armstrong.

Decomposition – Abstraction - Reusability: an example

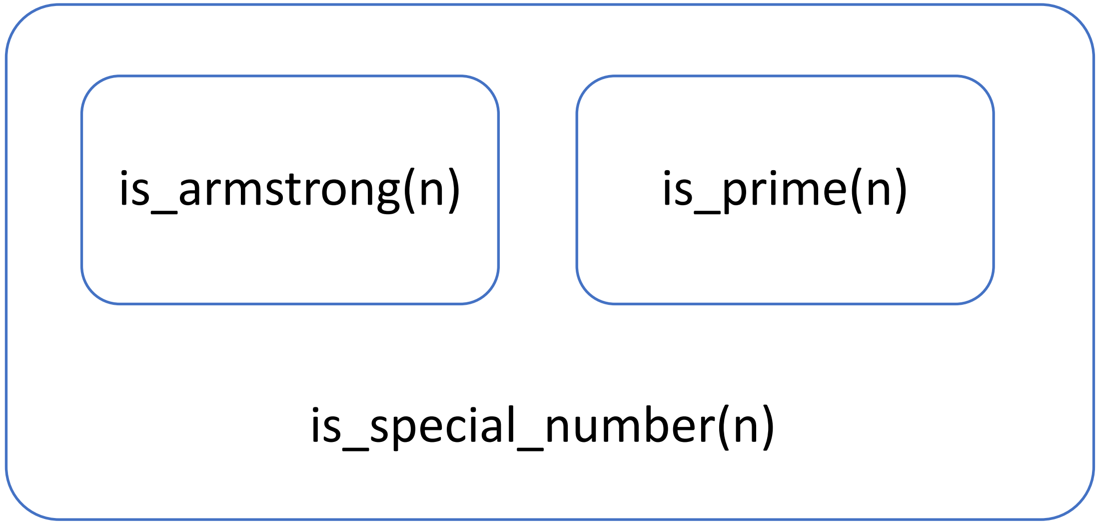
How do we design the function?

- **Decompose** the problem into sub-problems

1. Check if it is armstrong or not
   → Write the function `is_armstrong(n)`

2. Check if it is prime or not
   → Write the function `is_prime(n)`

3. Combine the results of the checks and return the value



Decomposition – Abstraction - Reusability: an example

```python
def is_prime(n):
    if n < 2:
        return False
    for i in range(2,n):
        if n % i == 0:
            return False
    return True
```

is_armstrong(n)

is_prime(n)

is_special_number(n)

- What about the function `is_armstrong(n)`?
  - A natural number n with D digits is named **armstrong** if the sum of D-th power of digits equals to n

➢ What do we need to build the function?

1. Get the **number of digits** of n
2. Get **each digit**, one by one
3. Perform the **square and sum** operations

Decomposition – Abstraction - Reusability: an example

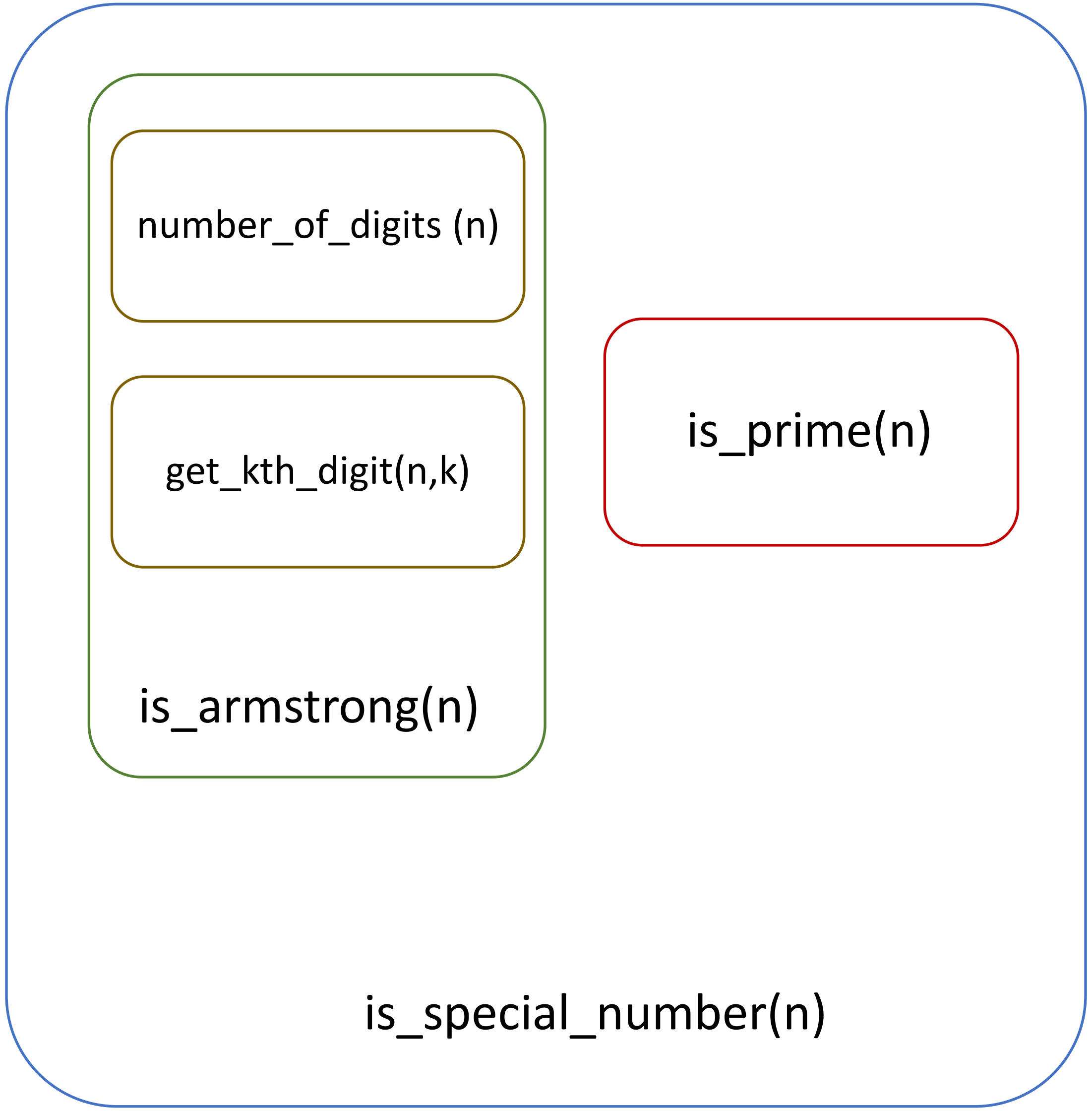
1. Get the **number of digits** of n

   → Write the function `number_of_digits(n)`

2. Get **each digit**, one by one

   → Write the function `get_kth_digit(n,k)`

3. Perform the **square and sum** operations



Decomposition – Abstraction - Reusability: an example

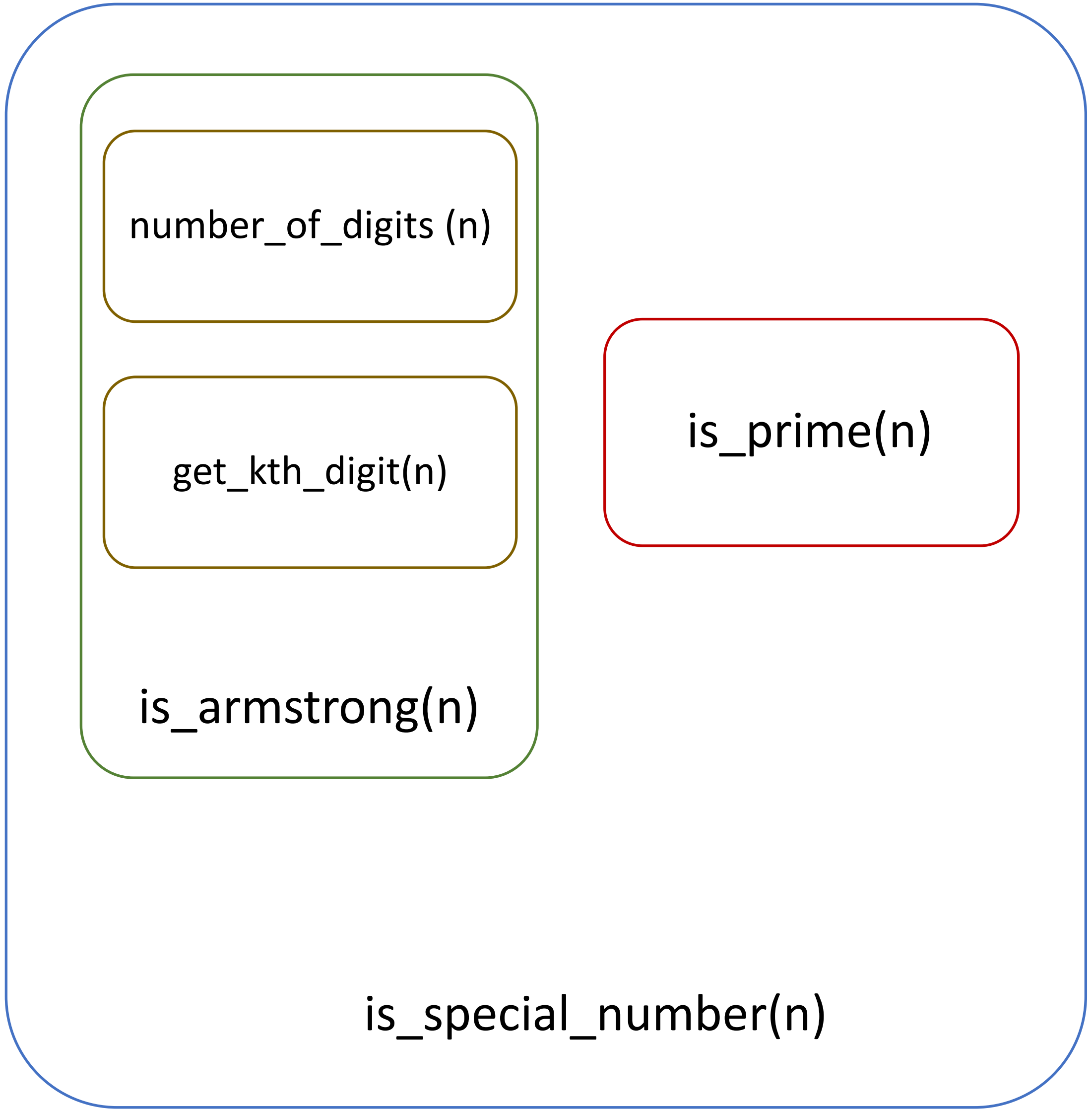
```python
def number_of_digits(n):
    count = 1
    digit = n
    while True:
        digit //= 10
        if digit == 0:
            return count
        else:
            count += 1
```

```python
def is_armstrong(n):
    n_digits = number_of_digits(n)
    power_sum = 0
    for k in range(n_digits):
        digit = get_kth_digit(n, k)
        power_sum += digit ** n_digits
    if power_sum == n:
        return True
    else:
        return False
```

```python
def get_kth_digit(n, k):
    n = abs(n)
    n = n // 10 ** k
    return n % 10
```

Decomposition – Abstraction - Reusability: an example

```python
def is_special_number(n):

    prime, armstrong = False, False
    if is_prime(n):
        print(n, 'is prime!')
        prime = True

    if is_armstrong(n):
        print(n, 'is armstrong!')
        armstrong = True

    if prime or armstrong:
        return True
    else:
        return False
```

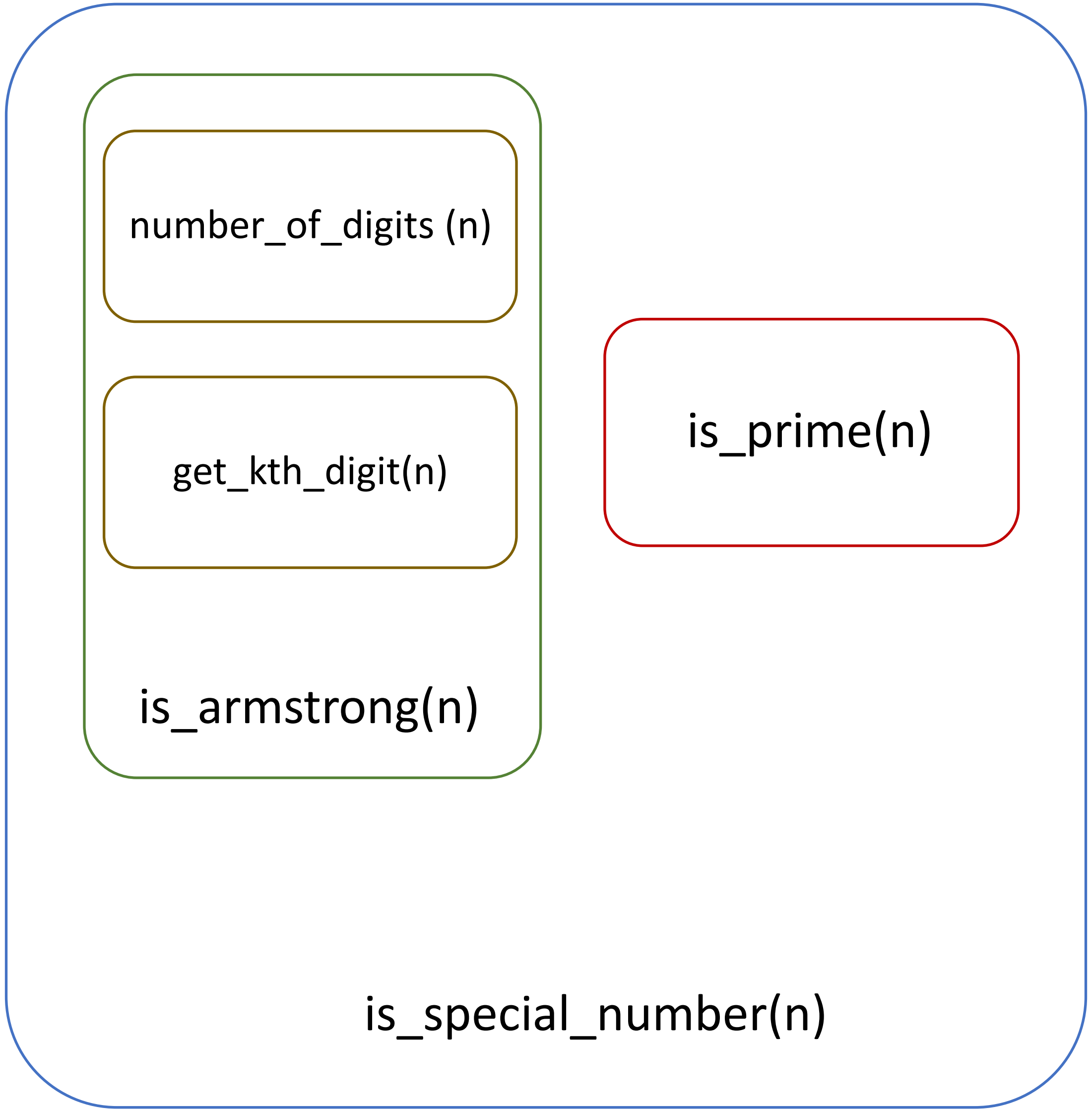


Decomposition

- Instead of writing ONE function, we wrote FIVE functions!

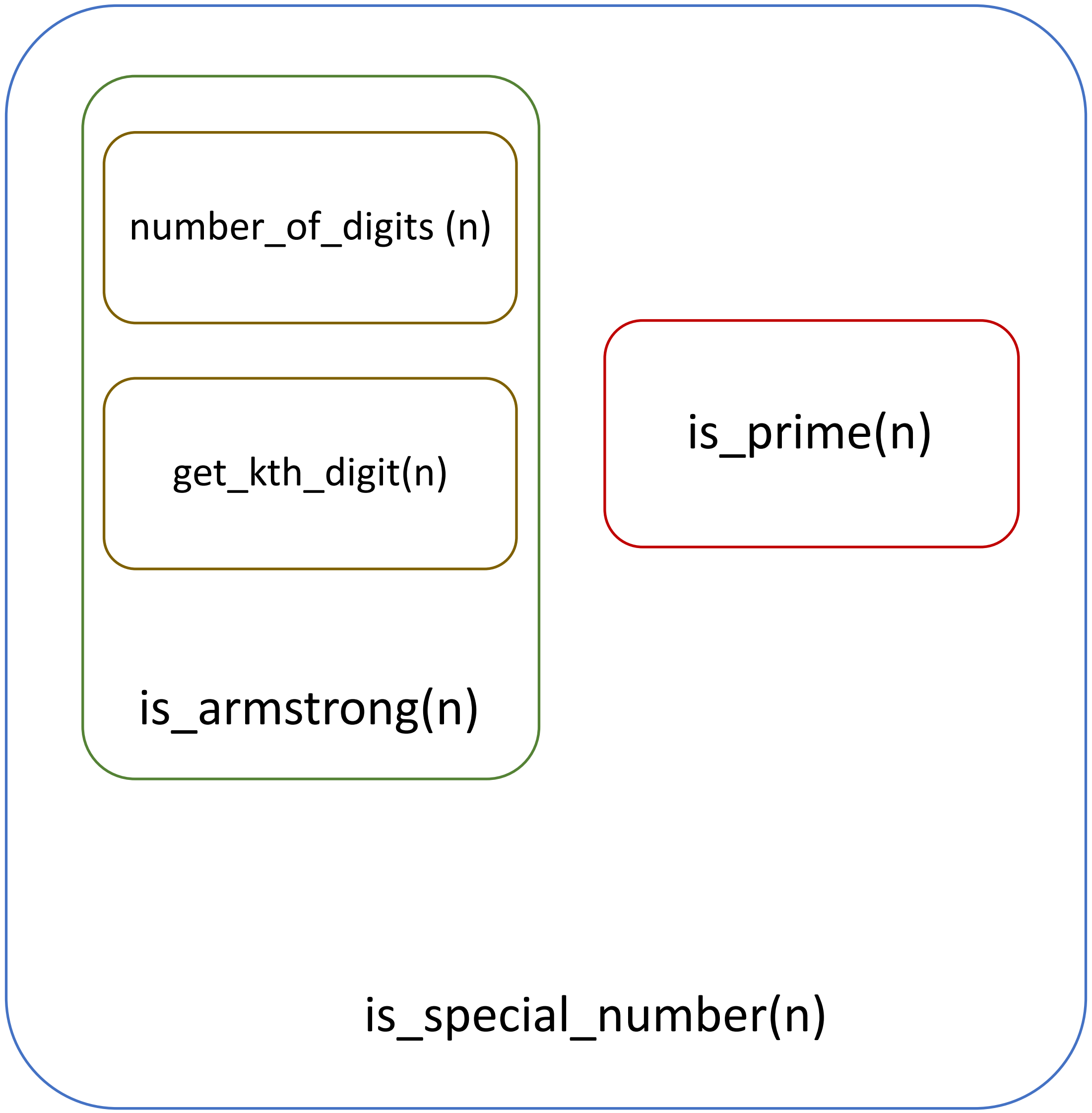
- **Problem decomposition**: we solved four sub-problems and combined them to tackle the target problem

  - ✓ Each sub-problem was easier to tackle than the whole, original problem
  - ✓ Each sub-problem is relatively easy to debug and test for correctness and efficiency standalone
  - ✓ We gained in readability of the whole solution: we can actually *read* the code of `is_special_number(n)`
  - ✓ We can split the job in parallel among multiple programmers, each doing a function! ;-)



Reusability

- **Reusability**: we have now five functions (working, well debugged!) each doing a different thing (i.e., providing a different service)
  - ✓ We can reuse the individual functions in different, new problems!
    - E.g., find all the happy prime numbers up to n
  - ✓ We can reuse existing functions to define new functions with improved / extended functionalities
    - E.g., find the number of digits that are even/odd



Reusability

❖ **Reusability**:

✓ We can reuse the same function as many times as we want in our code, without having to repeat the code, but juts invoking the function by its name

```
def is_armstrong(n):
    n_digits = number_of_digits(n)
    power_sum = 0
    for k in range(n_digits):
        digit = get_kth_digit(n, k)
        power_sum += digit ** n_digits
    if power_sum == n:
        return True
    else:
        return False
```

We call the `get_kth_digit()` function `n_digits` times!

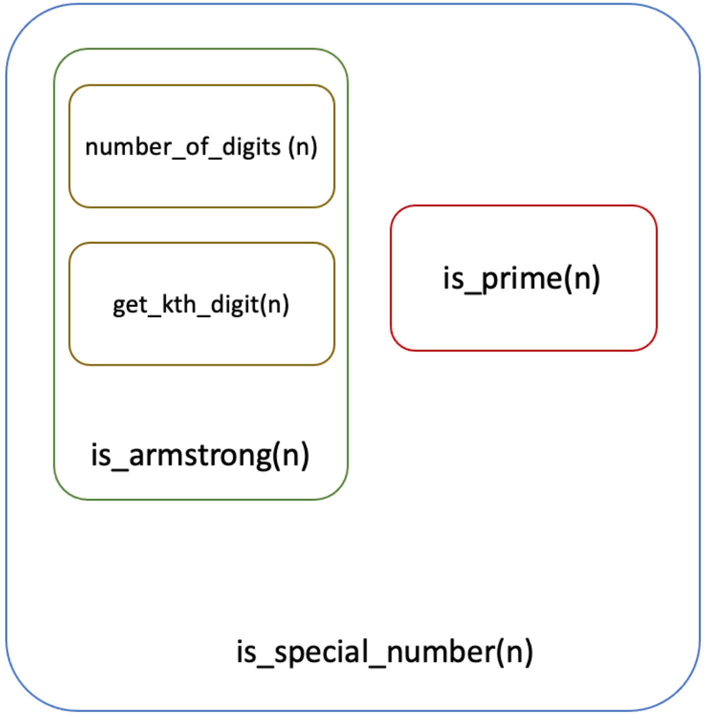
```
def happy():
    print("Happy birthday to you!")
```

```
def sing(person):
    happy()
    happy()
    print "Happy Birthday, dear", person)
    happy()
```

Abstraction

❖ **Abstraction**: functions provides a service hiding the details about *how* the service is being provided. We can use a function by **invoking its name** and using its **results!**

- ✓ Solution details are abstracted away (to the user)
- ✓ A function is not limited to a specific input, instead it describes a computation that applies to *any* admissible input (any natural number n in this case) since the input is parametric



Abstraction

```python
def number_of_digits(n):
    count = 1
    digit = n
    while True:
        digit //= 10
        if digit == 0:
            return count
        else:
            count += 1
```

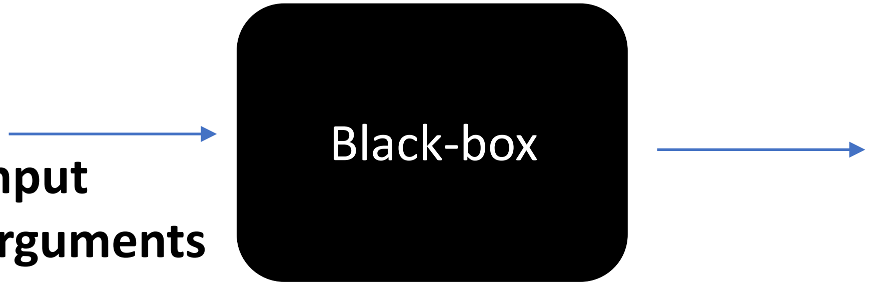
Three different ways of implementing the same functionality (i.e., returning the same values for the same inputs)

```python
def number_of_digits(n):
    return len(str(n))
```

```python
import math
def number_of_digits(n):
    '''
        n: integer > 0
        Observe: log10(1) is 0, log10(10) is 1, log10(100) is 2,
        log10(1000) is 3 ...
    '''
    return math.floor( math.log10(n)) + 1
```

Abstraction using functions

Input arguments



Function

Outputs

- returned values
- modifications that propagates outside the function
- side effects on the program environment

- The only information relevant to use a function :
  - **input parameters**: types and admitted values
  - **returned objects**: types, range of values, adopted conventions
  - **description of what the function does**, including possible side effects

The precise details about *how* processing is performed are hidden by the abstraction

Local and global scope of variables

❖ **Local Scoping**: where do function variables *live*?

```
def average(L):
    n_elements = len(L)
    avg = sum(L) / n_elements
    return avg
```

```
numbers = [1, 3, 6, 2]
x = average(numbers)
print(x)

print(avg)  ?

    NameError: name 'avg' is not defined

print(n_elements) ?

print(L)   ?
```

➢ Variables `avg, n_elements, L` are local to the function

➢ Their scope is limited to the function and to the duration of the function call

❖ Variable defined inside a function **cannot be referenced outside the function**!

Local and global scope of variables

❖ **Global Scoping**: where do variables *live*?

```
def average_global():
    n_elements = len(L)
    avg = sum(L) / n_elements
    return avg
```

✓ This works, variable L is defined outside of a function and as such can be accessed globally

```
L = [1, 3, 6, 2]
x = average_global()
print(x)
```

➢ `L` is a variable that can be referred to anywhere
→ Variable with a *global* scope

```
numbers = [1, 3, 6, 2]
x = average_global()
```

```
NameError: name 'L' is not defined
```

- This doesn't work, `L` isn't defined anywhere

Local and global scope of variables

❖ **Global Scoping**: *clean* way to define global variables

```
def average_global():
    global L
    n_elements = len(L)
    avg = sum(L) / n_elements
    return avg
```

```
L = [1, 3, 6, 2]
x = average_global()
print(x)
```

➢ In the function `L` is explicitly *defined* as a global variable: we assume that it has been *declared* somewhere

Local and global scope of variables

- **Global Scoping**: changes are propagated outside of the function!

```
def median():
    global L
    L.sort()
    return L[ len(L)//2 ]
```

```
L = [1, 3, 6, 2, 0]
m = median()

print(L)
```

`L` has changed now: [0, 1, 2, 3,6]

> Be careful: **Avoid** as much as possible to use global variables!

Functions without input parameters

```
def function_name():
    function_body
    return something
```

The function does something and doesn't require any input specifications for doing it!

```
def happy():
    print("Happy birthday to you!")
```

```
def product():
    global L
    prod = 1
    for v in L:
        prod = prod * v
    return prod
```

```
def square():
    sizeh = 22
    sizev = 10
    for i in range(sizeh):
        print('-', end ='')
    print()
    for i in range(sizev):
        print('|', end = '')
        for j in range(sizeh-2):
            print(' ', end = '')
        print('|', end = '')
        print()
    for i in range(sizeh):
        print('-', end ='')
    print('\n')
```

Functions with input parameters

```
def function_name(parameters):
    function_body
    return something
```

The function does something and does require input data / specifications for doing it!

- ✓ If we only need to pass one **single argument**, there's no much to say
- ▪ The situation is different when passing **multiple parameters** …

```
def quadratic_roots(a, b, c):
    d = 1 / (2 * a)
    sqr = math.sqrt(b**2 - (4 * a *c))
    return (-b + sqr) * d, (-b - sqr) * d
```

$$ax^2 + bx + c = 0$$

$$x_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

`quadratic_roots(31, 93, 62)` is different than: `quadratic_roots(93, 31, 62)`

Order matters!

Functions with input parameters: use of *named* arguments

```
def quadratic_roots(a, b, c):
    d = 1 / (2 * a)
    sqr = math.sqrt(b**2 - (4 * a *c))
    return (-b + sqr) * d, (-b - sqr) * d
```

✓ Passing arguments as **positional arguments**

```
quadratic_roots(31, 93, 62)
```

is different than:

```
quadratic_roots(93, 31, 62)
```

Order matters!

✓ Passing arguments as **keyword arguments**

```
quadratic_roots(a=31, b=93, c=62)
```

is the same as:

```
quadratic_roots(b=93, a=31, c=62)
```

Order doesn’t matter!

Keyword arguments and the `help()` function

- Passing arguments as **keyword arguments** works because python **knows the name function arguments**, and therefore it can perform automatic matching without errors

→ We can ask python **help** on function's parameters using the `help(function_name)`:

`help(quadratic_roots)` gives as answer: `quadratic_roots(a,b,c)`

✓ Use of keyword arguments also increases the clarity of a program!

```
random_password(upper=1, lower=1, digits=1, length=8)
```

vs.

```
random_password(1, 1, 1, 8)
```

Default values for the parameters, equivalent function calls

- When defining a function, a **default value can be defined for each argument**
  - If a value is passed for a parameter with a default value when the function is called, then the parameters takes the value of the provided argument
  - Otherwise, the parameter takes its default value

```
def quadratic_roots(a = 1, b = -3, c = 2):
    d = 1 / (2 * a)
    sqr = math.sqrt(b**2 - (4 * a *c))
    return (-b + sqr) * d, (-b - sqr) * d
```

Default equation:

$$x^2 - 3x + 2 = 0$$

```
x1, x2 = quadratic_roots()
```

x1 $\rightarrow 2.0$
x2 $\rightarrow 1.0$

```
x1, x2 = quadratic_roots(a=2, b=3, c=1)
```

x1 $\rightarrow -0.5$
x2 $\rightarrow -1.0$

Default values for the parameters, equivalent function calls

```
def quadratic_roots(a = 1, b = -3, c = 2):
    d = 1 / (2 * a)
    sqr = math.sqrt(b**2 - (4 * a *c))
    return (-b + sqr) * d, (-b - sqr) * d
```

```
x1, x2 = quadratic_roots(b=2, c=-3)
```

x1 $\rightarrow 1.0$
x2 $\rightarrow -3.0$

Equivalent to call the function as:

```
quadratic_roots(a=1, b=2, c=-3)
```

```
x1, x2 = quadratic_roots(b=4)
```

x1 $\rightarrow -0.58$
x2 $\rightarrow -3.41$

Equivalent to call the function as:

```
quadratic_roots(a=1, b=4, c=2)
```

Be careful mixing up positional and named parameters!

```
def quadratic_roots(a = 1, b = -3, c = 2):
    d = 1 / (2 * a)
    sqr = math.sqrt(b**2 - (4 * a *c))
    return (-b + sqr) * d, (-b - sqr) * d
```

```
x1, x2 = quadratic_roots(2, c=-3)
```

x1 $\rightarrow 2.18$
x2 $\rightarrow -0.68$

Equivalent to call the function as:

```
quadratic_roots(a=2, b=-3, c=-3)
```

```
x1, x2 = quadratic_roots(2, a=3)
```

```
TypeError: quadratic_roots() got multiple values for argument 'a'
```

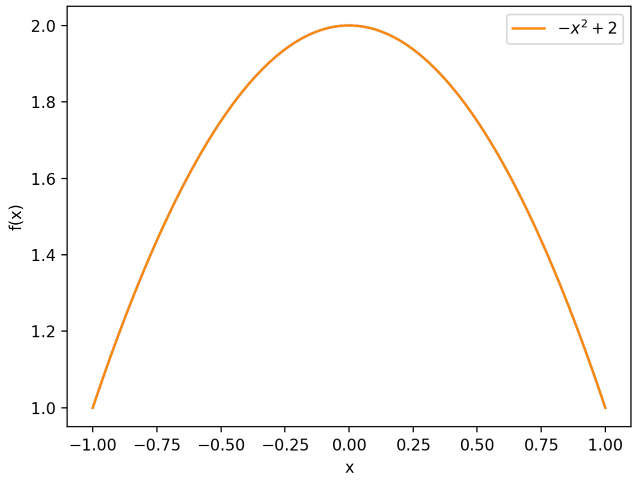
```
x1, x2 = quadratic_roots(b=2, 3, 4)
```

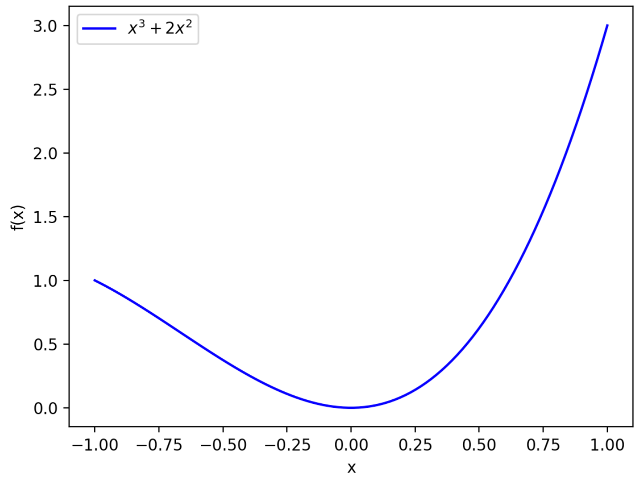
```
SyntaxError: positional argument follows keyword argument
```

Passing *functions* as arguments!

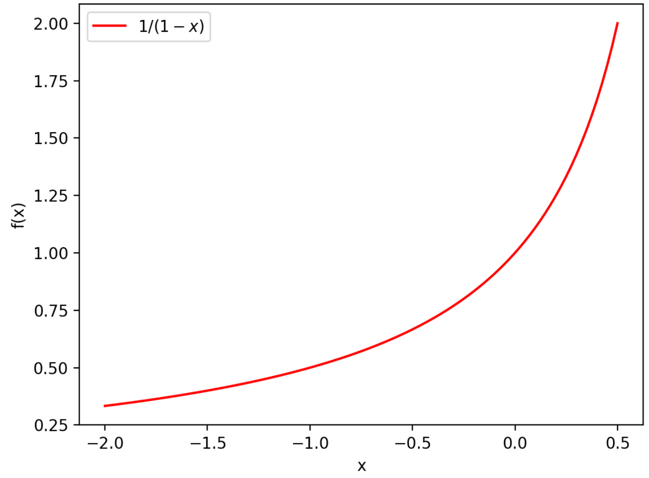
- Function parameters can also include a **function**, **that can be then invoked inside the function itself**

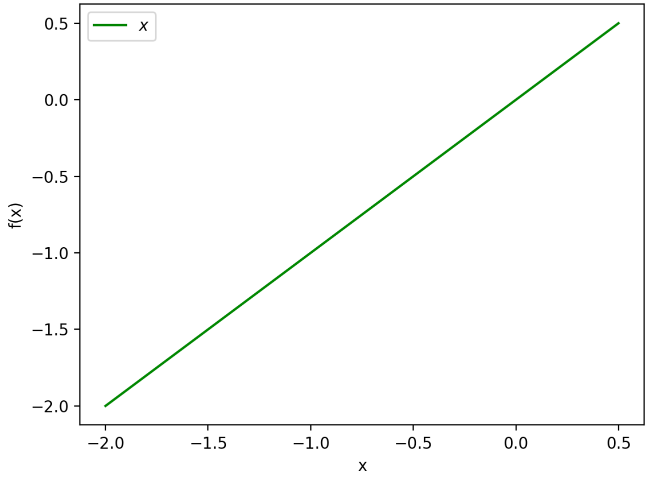
```
def parabola(x):
    return -x**2 + 2

def cubic(x):
    return x**3 + 2*x**2
```






```
def geometric(x):
    return 1 / (1-x)

def line(x):
    return x
```





Passing functions as arguments!

- Function parameters can also include a **function**, **that can be then invoked inside the function itself**

```
def estimate_max_in_interval(f,
                             xmin, xmax,
                             samples):
    x = xmin
    step = (xmax - xmin) / samples
    max_val = f(xmin)
    for i in range(samples):
        y = f(x)
        if y > max_val:
            max_val = y
        x += step
    return max_val
```

```
def parabola(x):
    return -x*x + 2

def cubic(x):
    return x*x*x + 2*x*x

def geometric(x):
    return 1 / (1-x)

def line(x):
    return x
```

```
print(estimate_max_in_interval(parabola, -1, 1, 100))
```

Function specifications, docstrings, help()

- The only information relevant to use a function :
  - ✓ **input parameters**: types and admitted values
  - ✓ **returned objects**: types, range of values, adopted conventions
  - ✓ **description of what the function does**, including possible side effects

Where do we need to write/store this information?

```python
def average(L):
    '''
    Parameters
    ----------
    L : List of numbers

    Returns
    -------
    avg : float, the average of the input list
    '''
    n_elements = len(L)
    avg = sum(L) / n_elements
    return avg
```

➢ If the first line after the name of the function is a string delimited by triple quotes → **docstring**

A docstring shall contain the description of the I/O of the function and what the function does

Function specifications, docstrings, help()

```python
def average(L):
    '''
    Parameters
    ----------
    L : List of numbers

    Returns
    -------
    avg : float, the average of the input list
    '''
    n_elements = len(L)
    avg = sum(L) / n_elements
    return avg
```

- ✓ If you don't remember about a function or want to know about it: ask python!

- ➢ The docstring is returned by the `help()` function:

```
help(average)

Help on function average in module __main__:

average(L)
    Parameters
    ----------
    L : List of numbers

    Returns
    -------
    avg : float, the average of the input list
```

Function specifications, docstrings, help()

- ✓ Can ask help about any function!

```
help(sum)

Help on built-in function sum in module builtins:

sum(iterable, start=0, /)
    Return the sum of a 'start' value (default: 0) plus an iterable of numbers

    When the iterable is empty, return the start value.
    This function is intended specifically for use with numeric values and may
    reject non-numeric types.
```

Specifications

A specification **describes the abstraction**, the function and its elements in our case, in order to properly use it and reuse it!

- **Specification:** defines a *contract* between the provider/implementer of an abstraction (a function) and those who will be using the abstraction (the function), the users
- **User ↔ Client** for the services provided by the abstraction (the function)

Specifications: example

Assumptions: what the client must do to use the function

Guarantees: what the provider guarantees about outputs and effects of the function

Numeric examples: show function usage and provide basic test cases

```
def find_root(f, a, b, N):
    '''Approximate solution of f(x)=0 on interval [a,b] by the bisection method.

    Parameters
    ----------
    f : function
        The function for which we are trying to approximate a solution f(x)=0.
    a,b : numbers
        The interval in which to search for a solution. The function returns
        None if f(a)*f(b) >= 0 since a solution is not guaranteed.
    N : (positive) integer
        The number of iterations to implement.

    Returns
    -------
    x_N : number
        The midpoint of the Nth interval computed by the bisection method. The
        initial interval [a_0,b_0] is given by [a,b]. If f(m_n) == 0 for some
        midpoint m_n = (a_n + b_n)/2, then the function returns this solution.
        If all signs of values f(a_n), f(b_n) and f(m_n) are the same at any
        iteration, the bisection method fails and return None.

    Examples
    --------
    >>> f = lambda x: x**2 - x - 1
    >>> bisection(f,1,2,25)
    1.618033990263939
    >>> f = lambda x: (2*x - 1)*(x - 3)
    >>> bisection(f,0,1,10)
    0.5
    '''
```

… the program code would follow