Task 1

Formulate the Sudoku puzzle as a constraint satisfaction problem (CSP).

A detailed description of all variables, domains and constrains that are sufficient to model Sudoku as a CSP.

A Sudoku board consists of a grid of 81 *cells,* along 9 rows and 9 columns, partitioned into 3-cell by 3-cell *sections,* with each cell containing a number in the closed range [1-9]. Each cell can be represented with a variable, C_{11} through C_{99}, using a matrix-like two-index notation. For the sake of brevity, these 3x3 sections will be referred to as S_{11} through S_{33}, using the same matrix-like notation.

C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	C_{19}
C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}	C_{27}	C_{28}	C_{29}
C_{31}	C_{32}	C_{33}	C_{34}	C_{35}	C_{36}	C_{37}	C_{38}	C_{39}
C_{41}	C_{42}	C_{43}	C_{44}	C_{45}	C_{46}	C_{47}	C_{48}	C_{49}
C_{51}	C_{52}	C_{53}	C_{54}	C_{55}	C_{56}	C_{57}	C_{58}	C_{59}
C_{61}	C_{62}	C_{63}	C_{64}	C_{65}	C_{66}	C_{67}	C_{68}	C_{69}
C_{71}	C_{72}	C_{73}	C_{74}	C_{75}	C_{76}	C_{77}	C_{78}	C_{79}
C_{81}	C_{82}	C_{83}	C_{84}	C_{85}	C_{86}	C_{87}	C_{88}	C_{89}
C_{91}	C_{92}	C_{93}	C_{94}	C_{95}	C_{96}	C_{97}	C_{98}	C_{99}

The domains of each of these variables is identical. $D(C_n) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

Informally, no number may appear multiple times in the same row, column, or section. Formally, this defines 8 constraints per row, 8 constraints per column, and 8 constraints per section. All 216 constraints could be enumerated explicitly, however, that would make this document far too long. These constrains are defined generally

In the following definitions, let:

$$\mathbb{D} = 1, 2, 3, 4, 5, 6, 7, 8, 9$$

$$a, b, c, d, i, j, k \in \mathbb{D}$$

such that:

$$C_{ij} \neq C_{kj}$$

$$C_{ij} \neq C_{ik}$$

$$\text{if } C_{ab} \in S_{ij} \wedge C_{cd} \in S_{ij}, \text{ then } C_{ab} \neq C_{cd}$$
