

## Task 1

Formulate the Sudoku puzzle as a constraint satisfaction problem (CSP).

A detailed description of all variables, domains and constraints that are sufficient to model Sudoku as a CSP.

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A Sudoku board consists of a grid of 81 *cells*, along 9 rows and 9 columns, partitioned into 3-cell by 3-cell *sections*, with each cell containing a number in the closed range [1-9]. Each cell can be represented with a variable,  $C_{11}$  through  $C_{99}$ , using a matrix-like two-index notation. For the sake of brevity, these 3x3 sections will be referred to as  $S_{11}$  through  $S_{33}$ , using the same matrix-like notation.

$C_{11}$	$C_{12}$	$C_{13}$	$C_{14}$	$C_{15}$	$C_{16}$	$C_{17}$	$C_{18}$	$C_{19}$
$C_{21}$	$C_{22}$	$C_{23}$	$C_{24}$	$C_{25}$	$C_{26}$	$C_{27}$	$C_{28}$	$C_{29}$
$C_{31}$	$C_{32}$	$C_{33}$	$C_{34}$	$C_{35}$	$C_{36}$	$C_{37}$	$C_{38}$	$C_{39}$
$C_{41}$	$C_{42}$	$C_{43}$	$C_{44}$	$C_{45}$	$C_{46}$	$C_{47}$	$C_{48}$	$C_{49}$
$C_{51}$	$C_{52}$	$C_{53}$	$C_{54}$	$C_{55}$	$C_{56}$	$C_{57}$	$C_{58}$	$C_{59}$
$C_{61}$	$C_{62}$	$C_{63}$	$C_{64}$	$C_{65}$	$C_{66}$	$C_{67}$	$C_{68}$	$C_{69}$
$C_{71}$	$C_{72}$	$C_{73}$	$C_{74}$	$C_{75}$	$C_{76}$	$C_{77}$	$C_{78}$	$C_{79}$
$C_{81}$	$C_{82}$	$C_{83}$	$C_{84}$	$C_{85}$	$C_{86}$	$C_{87}$	$C_{88}$	$C_{89}$
$C_{91}$	$C_{92}$	$C_{93}$	$C_{94}$	$C_{95}$	$C_{96}$	$C_{97}$	$C_{98}$	$C_{99}$

The domains of each of these variables is identical.  $D(C_n) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ .

Informally, no number may appear multiple times in the same row, column, or section. Formally, this defines 8 constraints per row, 8 constraints per column, and 8 constraints per section. All 216 constraints could be enumerated explicitly, however, that would make this document far too long. These constraints are defined generally

In the following definitions, let:

$$\mathbb{D} = 1, 2, 3, 4, 5, 6, 7, 8, 9$$

$$a, b, c, d, i, j, k \in \mathbb{D}$$

such that:

$$C_{ij} \neq C_{kj}$$

$$C_{ij} \neq C_{ik}$$

if  $C_{ab} \in S_{ij} \wedge C_{cd} \in S_{ij}$ , then  $C_{ab} \neq C_{cd}$

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