University of Waterloo
Department of Electrical and Computer Engineering

ECE 380, Analog Control Systems
Final Examination
December 17, 2008

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Instructions:

- Time allowed: 2.5 hours.
- No aids allowed other than a nonprogrammable calculator.
- The exam comprises 3 questions with a total value of 100 points; answer all of them.

The following formulas may be of use:

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}\, t + \theta) \; ; \; \theta = \mathrm{Cos}^{-1}\zeta$$

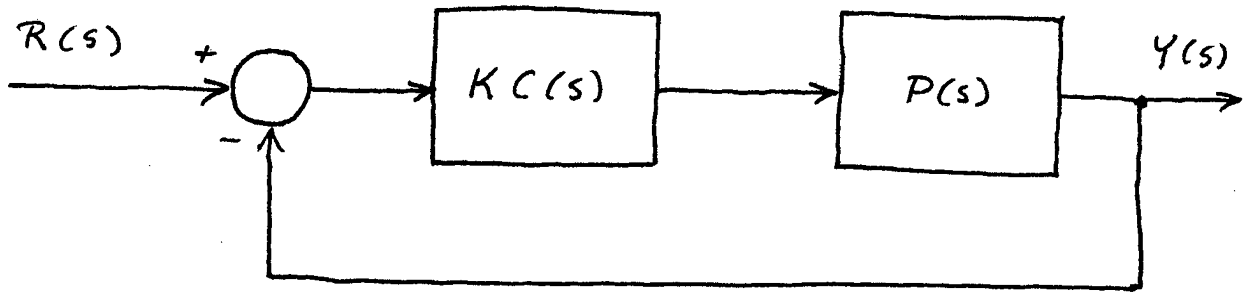
$$y(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}\, t)$$

$$\omega_{\mathrm{max}} = \frac{1}{\sqrt{\alpha}\,\tau}$$

$$\angle C(j\omega_{\mathrm{max}}) = \mathrm{Sin}^{-1}\frac{\alpha - 1}{\alpha + 1}$$

Question No. 1 (35 points)

Consider the following feedback system:



Let $C(s) \equiv 1$ and let $P(s) = \frac{s-z}{(s+15)(s^2+10s+100)}$.

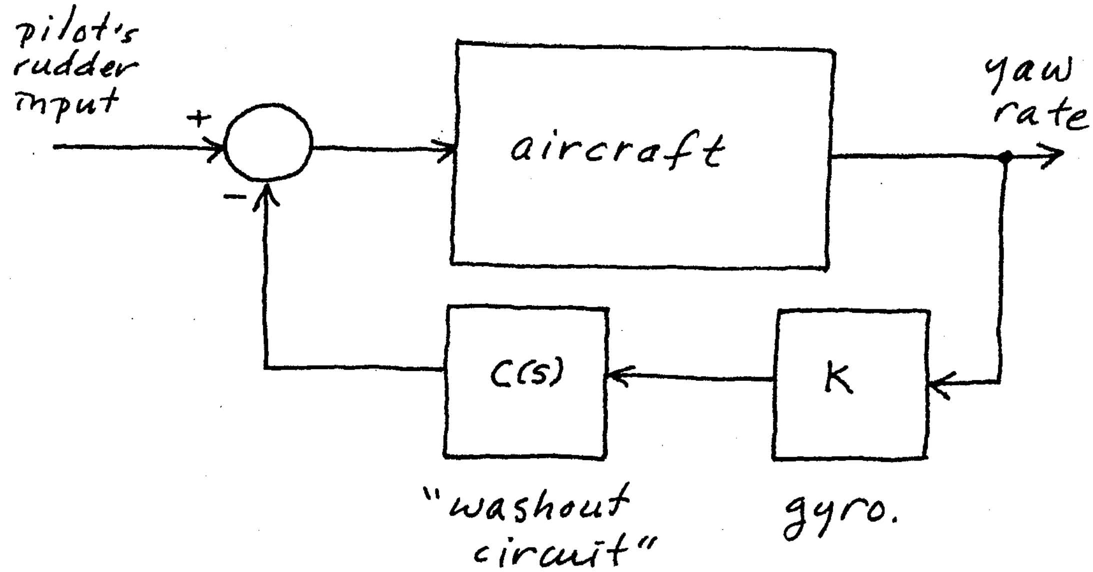
(a) Suppose that $K = 10$. Find the range of values of z for which the feedback system is BIBO stable.

(b) Let $z = 10$. Draw the root locus that shows how the roots of the characteristic polynomial vary as K varies from 0 to ∞.

(c) Let $z = 10$. Draw the system's Nyquist plot.

Question No. 2 (35 points)

A "yaw damper" autopilot for an aircraft has the following block diagram:



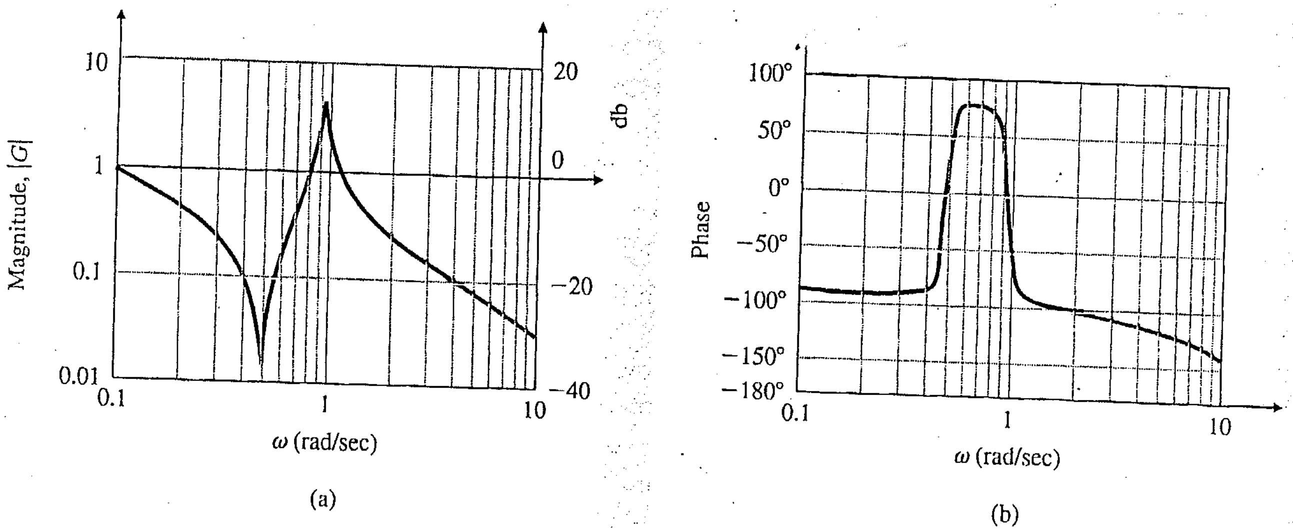
The purpose of this control system is not to track a reference signal but to assist the pilot by damping out "yawing" motion – rotational oscillation of the aircraft about a vertical axis. The yaw rate is measured by a gyroscope and fed back, with the controller located in the feedback path.

The controller has the form of a "washout circuit":

$$C(s) = \frac{s}{s-p}$$

Note that this controller is similar to a lead compensator, except that its zero lies at the origin of the s-plane. (Because the zero is at the origin, the controller has a dc gain of zero; hence it filters out (or "washes out") the dc component of the yaw rate. Intuitively, this feature allows the washout circuit to damp out transient yawing motion without interfering with the constant, nonzero steady-state yaw rate necessary for controlled turns.)

The frequency response of the "uncompensated system" $KP(s)$ is shown below (reproduced from Franklin, Powell and Emami-Naeini, *Feedback Control of Dynamic Systems*, 5th ed., Prentice-Hall):



(a) Sketch the Bode plot of the frequency response of the washout circuit. (Use a piecewise-linear approximation.)

(b) Choose the pole p so as to increase the phase margin of the feedback system by approximately 22.5°. (Strictly speaking, the phase margin may be undefined, owing to unstable pole-zero cancellations – ignore this technicality for the moment.)

(c) Assuming that $K = 1$ and $p = 1$ rad/s, what is the steady-state response of the closed-loop system to a unit-impulse input from the pilot?

Question No. 3 (30 points)

A plant has the transfer function

$$P(s) = \frac{7/16}{s^2 + s + 7/16}.$$

Design a feedback controller such that the closed-loop system has a pair of poles at $s = -1 \pm j\sqrt{3}/4$. What is the steady-state error in response to a unit-step input? Propose a modification to your controller that reduces this steady-state error by a factor of 5.