UNIVERSITY OF WATERLOO

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

E&CE 380: Analog Control Systems

FINAL EXAM

December 11, 2001

Professor D.E. Davidson

Instructions

- Answer all four questions. The questions do not have equal value, so be wise in how you use your time. The exam is out of 80 marks in total.
- You have exactly three hours.
- Only non-programmable calculators are allowed.
- Please clearly indicate all final answers.
- If something is unclear to you, make and state assumptions, and continue.
- A Bode plot is provided for Question 4. You may hand the plot in with your solutions. Please make sure that your name and student number are on everything you hand.

Potentially useful mathematical expressions

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}, \quad \frac{1}{2\xi\sqrt{1-\xi^2}}, \quad \omega_n\sqrt{1-2\xi^2}, \quad 100\exp\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right), \quad \frac{\pi}{\omega_n\sqrt{1-\xi^2}}, \quad \frac{4}{\xi\omega_n}$$

$$\frac{2.16\xi + 0.6}{\omega_n}, \quad (-1.196\xi + 1.85)\omega_n, \quad E = mc^2$$

QUESTION 1 [Worth 20 marks]

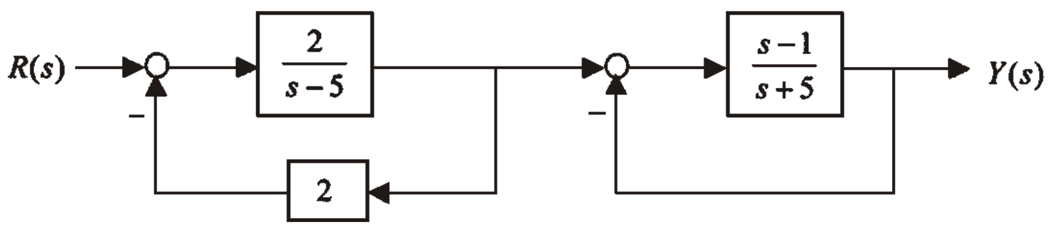
Answer all four of the following totally unrelated short questions:

---

[5] (a) Consider the polynomial $Q(s) = s^5 + 2s^4 - 2s^3 + 16s^2 - 24s + 32$. Determine how many roots of $Q(s)$ are in the open left-half plane, how many are in the open right-half plane, and how many are on the imaginary axis.

---

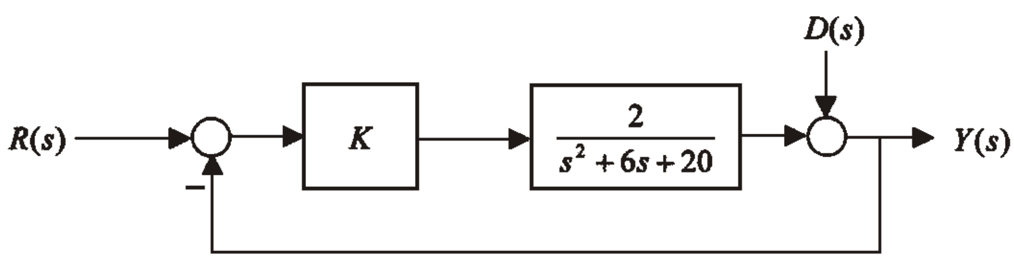
[4] (b) Consider the following control system:



Determine whether or not the closed-loop system is stable. You must show work to get any credit.

---

[5] (c) Consider the following control system:



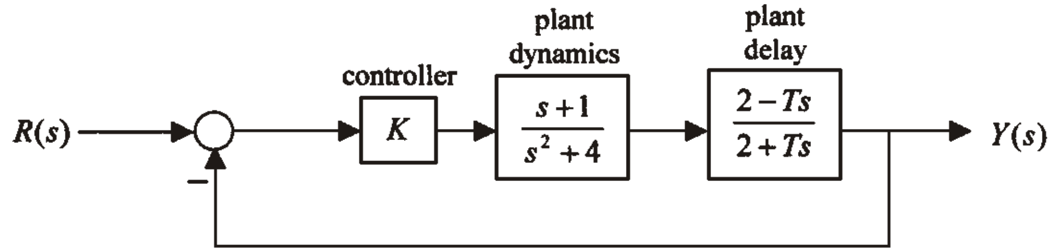
Assume that $r(t)$ is a step of height +2 and $d(t)$ is a step of height -20. Find the value of the controller gain, K, that will make the steady-state value of $y(t)$ equal to zero.

---

[6] (d) Sketch the Nyquist plot of $L(s) = \dfrac{1}{s^2(s+1)}$.

QUESTION 2 [Worth 16 marks]

Consider the following control system:



The plant has a delay of T seconds, approximated by a first-order transfer function, as shown in the block diagram.

In any root locus plots you draw for this question, you do *not* have to compute breakaway points or angles of departure/arrival or imaginary-axis crossing, unless stated otherwise or if you find that such computations are needed to answer the quantitative questions.

[3] (a) Fix $T = 0$ (i.e., suppose there is no time delay). Sketch how the closed-loop poles vary with $K \geq 0$. Compute any breakaway points.

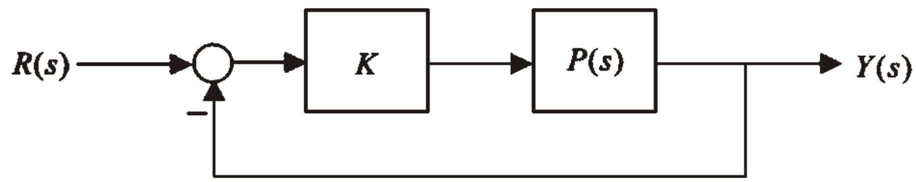
[6] (b) Now fix $K = 1$. Sketch a root locus plot showing how the closed-loop poles vary with $T \geq 0$. What happens to the closed-loop stability as $T \to \infty$? Is this what you would expect, given that T is (or, more accurately, is an approximation to) a time delay?

[5] (c) Again fix $K = 1$. Find which values of $T \geq 0$ result in a table closed-loop system.

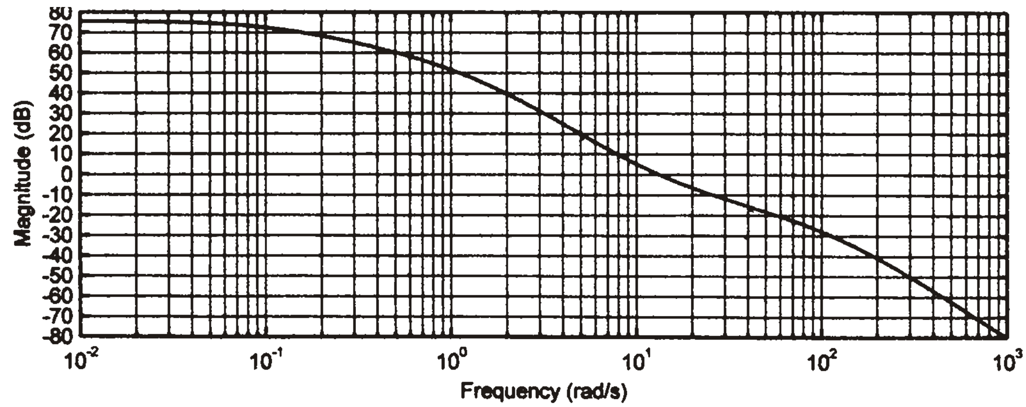
[2] (d) Again fix $K = 1$. Find which values of $T \geq 0$ result in perfect "steady-state" tracking for the reference signal $r(t) = 3\sin(2t)$. [Hint: Think of the Bode plot of the plant.]

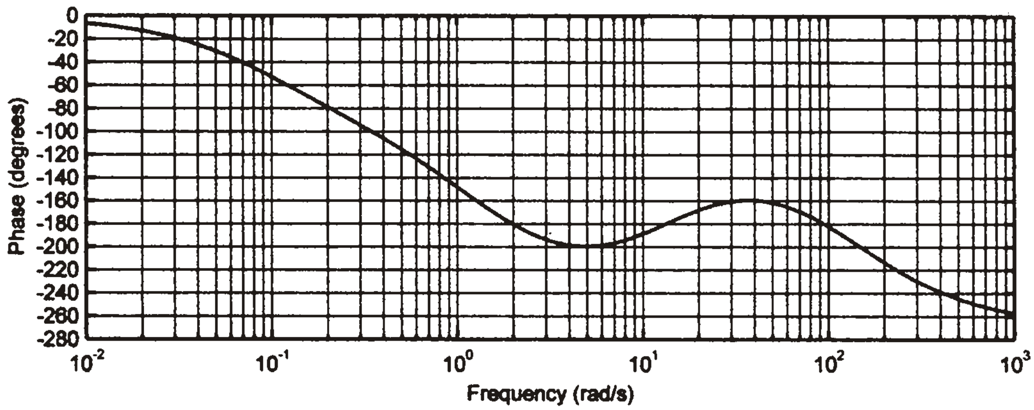
QUESTION 3 [Worth 19 marks]

Consider the following feedback control system. Assume $P(s)$ is (open-loop) stable.



A Bode plot of the plant, $P(s)$, was experimentally found to be as follows:





[5] (a) Sketch the Nyquist plot of $P(s)$. The plot does not have to be drawn to scale.

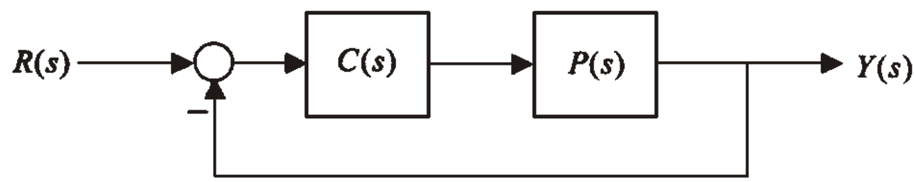
[5] (b) Estimate the values of the controller gain $K \geq 0$ that stabilize the closed-loop system.

[5] (c) For $K = 10$, estimate the gain margin, phase margin, and delay margin.

[4] (d) Find the controller gain that maximizes the phase margin, subject to the constraint that tracking for a step input must have no more than 1% steady-state error.

QUESTION 4 **[Worth 25 marks]**

Consider the following feedback control system:



A Bode plot of the plant, which is open-loop stable, is included on the next page. An extra copy of the plot is provided in case you need it.

The design specifications are:

- The phase margin must be (to within ±3 degrees) 50 degrees.

  and

- The tracking error for a step input must be no more than 5%.

Answer the following questions:

[4] (a) Design, if possible, a proportional compensator that satisfies the specifications. If a proportional compensator cannot be designed to satisfy the specifications, then provide a brief explanation stating why.

[6] (b) Repeat part (a) using a lead compensator.

[6] (c) Repeat part (a) using a lag compensator.

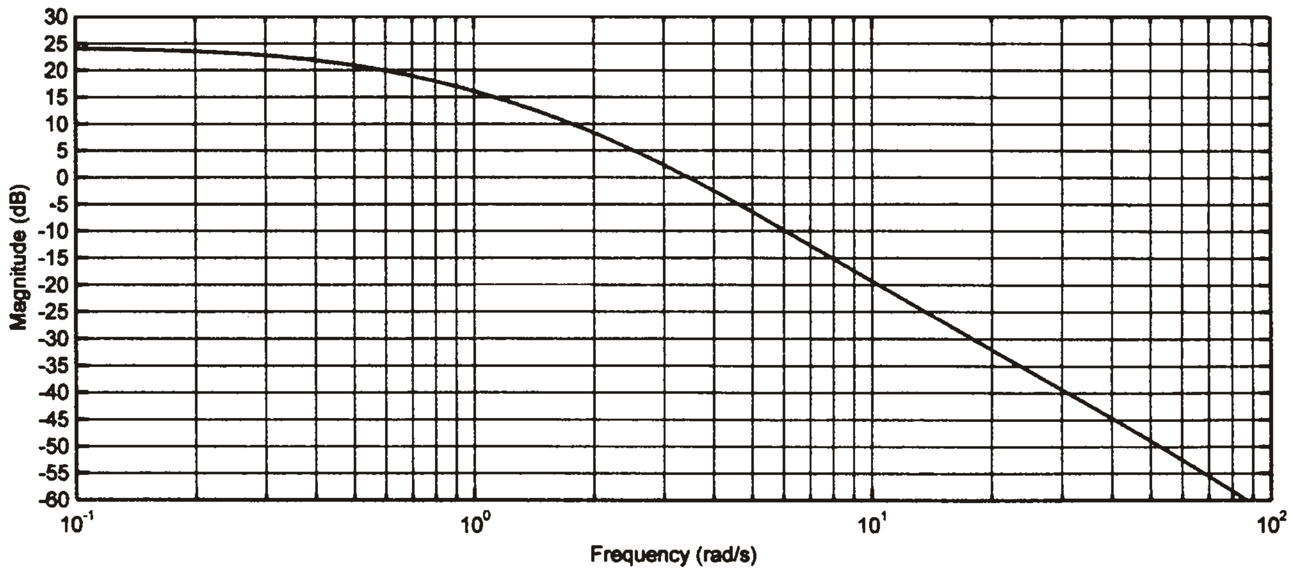
[6] (d) Repeat part (a) using an ideal PD compensator.

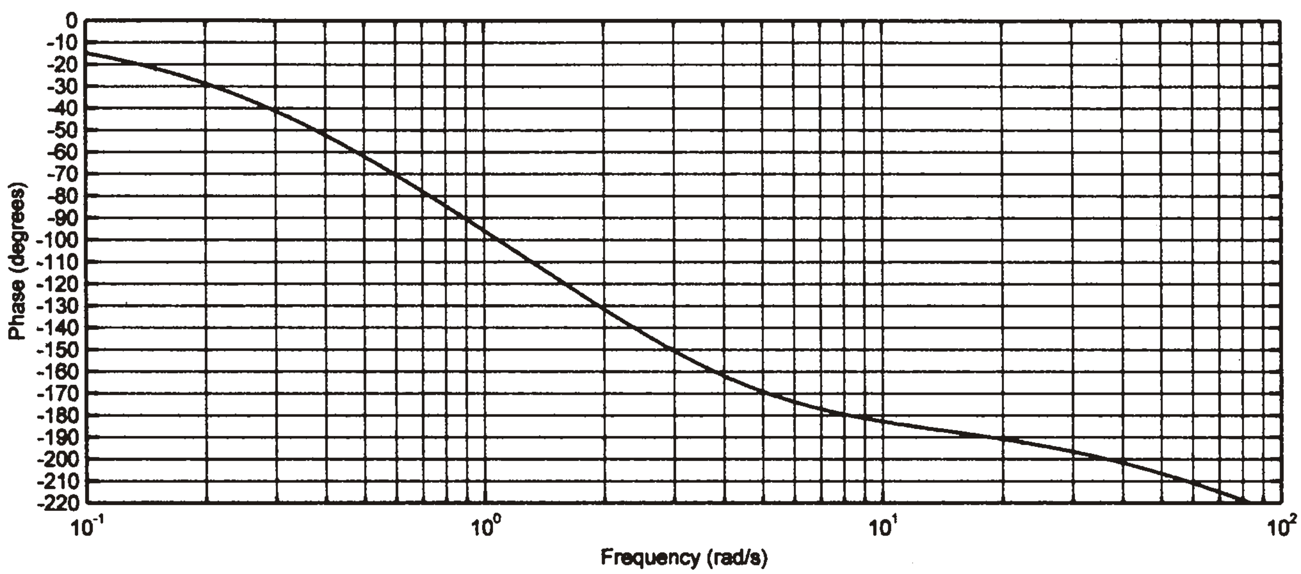
[3] (e) Of all the controllers that you designed above, predict which one results in the fastest system response and which one results in the slowest system response (where "fastest" and "slowest" are interpreted in any reasonable way). Briefly explain your reasoning.

BODE PLOT FOR QUESTION 4

Name: ____

Student number: ____________________________________





BODE PLOT FOR QUESTION 4

Name: ____

Student number: ____________________________________