HYPOTHESIS TESTING

There is a claim made about the population.

We want to test this claim, using a sample, as to whether the claim is supported by the evidence.

H_0 : null hypothesis

: Conventional wisdom / current belief

Challenger → H_1 : Alternate hypothesis

Applications :

Jeopardy problem :

$\{y_1, \ldots, y_n\}$ y_i = # of shows the i^{th} Canadian participated in.

θ = probability that a Canadian wins Jeopardy

$$\left.\begin{array}{l} H_0 : \theta = 1/3 \\ H_1 : \theta > 1/3 \end{array}\right\} \Rightarrow \text{ONE SIDED HYPOTHESIS}$$

(2).

The claim UW Brochure

Average starting salary of UW math grads

Null hypothesis = \$75,000/year.

$$H_0 : \mu = 75,000$$

(3). Will Hillary win the Election

tomorrow:

$$H_0 : A \geq 270$$

$$H_1 . \quad A < 270$$

A = # of electoral votes

Other examples.

(iv) Are men & women wages equal?

$$\left.\begin{array}{l} H_0 : \mu_1 = \mu_2 \\ \text{or } H_1 : \mu_1 \neq \mu_2 \end{array}\right\} \text{ EQUALITY OF MEANS}$$

(v) EQUALITY OF PROPORTIONS

- Geese example.
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How do we test hypotheses?

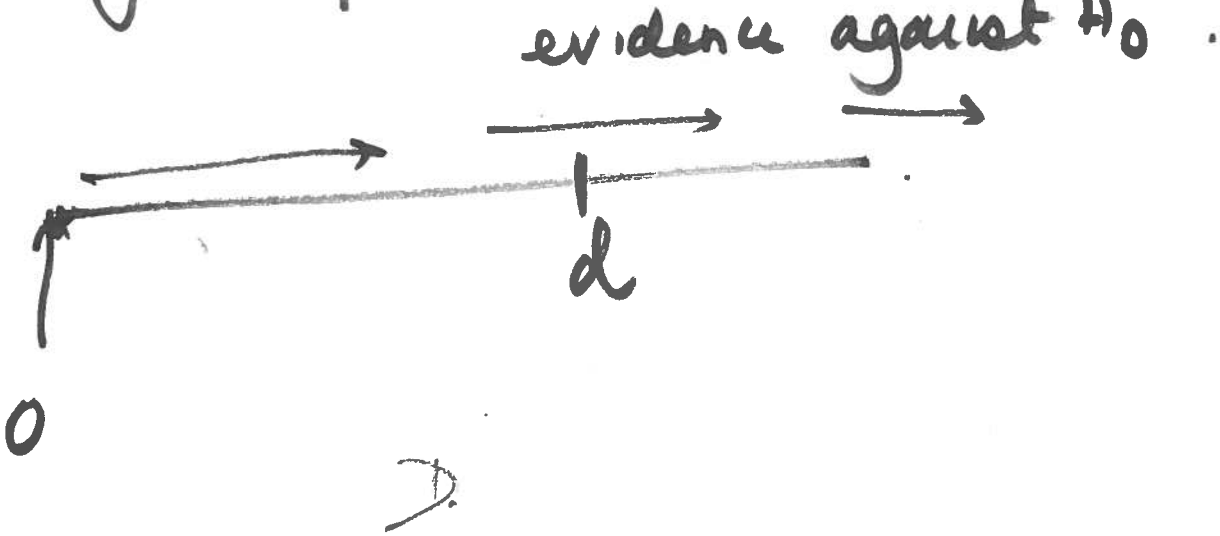
DISCREPANCY MEASURE : D :

a r.v. that measures the level of disagreement of the data from the Null hypothesis

Typically, we try to make sure that D satisfies the following properties

- $D \geq 0$
- $D = 0 \Rightarrow$ best possible outcome for the null hypothesis
- The larger the value of D, the more evidence against

TEST STATISTIC: d : Value of D from my sample.



p-value = $P(D \geq d;\ H_0 \text{ is true})$.

= Probability of observing your sample (or worse) given that the null hypothesis is true.

p-value measures how unusual your sample is, given H_0 is true.

p value = 0.3.

(If the experiment was done a lot of times, 30% of those outcomes would be as unusual than the one we observed, given that H_0 is true)

Does not Mean: ~~The probability of H_0 is true = 0.3~~

Convention

$\text{p-value} > 0.1 \Rightarrow$ No evidence against H_0

$0.05 < \text{p-value} \leq 0.1 \Rightarrow$ Weak evidence against H_0.

$0.01 < \text{p-value} \leq 0.05 \Rightarrow$ Evidence against H_0.

$\text{p-value} \leq 0.01 \Rightarrow$ Strong evidence against H_0

"STATISTICALLY SIGNIFICANT"

$\text{p-value} < 0.05$

Example: We are wondering whether a coin is fair

Toss the coin 10 times

y = # of heads.

$H_0 : \theta = 1/2 \qquad \theta = P(H)$

$H_1 : \theta \neq \frac{1}{2}$ (TWO-SIDED HYPOTHESIS)

Y = # of heads

$$D = |Y - 5|$$

can be used as a discrepancy measure

We look at our sample:

$y = 9$

d = Observed value of the discrepancy measure.

$= |9-5| = 4 \quad d.$

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p-value $= P(D \geq 4)$

$= P(|Y-5| \geq 4)$

$= P(Y=0) + P(Y=1) + P(Y=9) + P(Y=10)$

$^{10}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{10} + \ldots$

Hillary

$$\left.\begin{array}{l} H_0: \theta = 1/2 \\ H_1: \theta \neq \frac{1}{2} \end{array}\right\}$$

$n = 1000$. Y = # of successes.

$$D = |Y - 500|$$

Clicker

Are you left/right handed?

$L \to 1$. $R \to 2$

(i) R over L or L over R

$\| \; 44\%$ $\qquad \| \; 56\%$.

R over L $\rightarrow$ 1

L over R $\rightarrow$ 2

183

16% $\rightarrow$ L.

84% $\rightarrow$ R