To Do

Read Section 4.7

Do End-of-Chapter Problems 1-30.

Read Section 5.1

Today's Lecture

(1) Confidence Interval for Gaussian Standard Deviation σ when mean μ is unknown

(2) Introduction to Tests of Hypothesis

Gaussian data with unknown mean μ and unknown standard deviation σ

Suppose $Y_1, Y_2, \ldots, Y_n$ is a random sample from a G(μ,σ) distribution where E(Y_i) = μ is unknown and sd(Y_i) = σ is also unknown.

A point estimator for σ^2 is

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y})^2$$

Theorem

Suppose $Y_1, Y_2, \ldots, Y_n$ is a random sample from a G(μ,σ) distribution.

Then

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

Pivotal Quantity

The random variable $$\frac{(n-1)S^2}{\sigma^2}$$

is a function of the data $Y_1, Y_2, \ldots, Y_n$ and the unknown parameter σ.

Since $$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

the distribution of $\frac{(n-1)S^2}{\sigma^2}$ **is completely known.**

We use this pivotal quantity to construct confidence intervals for σ and σ^2.

100p% Confidence Interval for σ^2, when μ is unknown

We need values *a* and b such that
P($a \leq W \leq b$) = p where $W \sim \chi^2(n-1)$

Since the Chi-squared distribution is not symmetric about its mean we find *a* and *b* such that

P($W \leq a$) = $(1 - p)/2$

and

P($W > b$) = $(1 - p)/2$ or equivalently
P($W \leq b$) = $(1 + p)/2$

100p% Confidence Interval for σ^2, when μ is unknown

Since P($a \le W \le b$) = p where $W \sim \chi^2$(n-1)

and

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

therefore

$$P\left(a \le \frac{(n-1)S^2}{\sigma^2} \le b \right) = p$$

or

$$P\left(\frac{(n-1)S^2}{b} \le \sigma^2 \le \frac{(n-1)S^2}{a} \right) = p$$

100*p*% Confidence Interval for σ^2, when μ is unknown

Since

$$P\left(\frac{(n-1)S^2}{b} \le \sigma^2 \le \frac{(n-1)S^2}{a}\right) = p$$

a 100*p*% confidence interval for σ^2 is

$$\left[\frac{(n-1)s^2}{b}, \frac{(n-1)s^2}{a}\right]$$

100p% Confidence Interval for σ^2, when μ is unknown

The 100p% confidence interval for σ^2

$$\left[\frac{(n-1)s^2}{b}, \frac{(n-1)s^2}{a}\right]$$

is NOT symmetric about s^2 the point estimate of σ^2.

100p% Confidence Interval for σ^2, when μ is unknown

Choosing *a* and *b* such that
$P(W \leq a) = P(W > b) = ((1- p)/2)$ where
$W \sim \chi^2(n-1)$ does NOT give the narrowest interval.

A numerical solution is required to obtain the narrowest interval and for $n > 30$ the intervals are nearly identical.

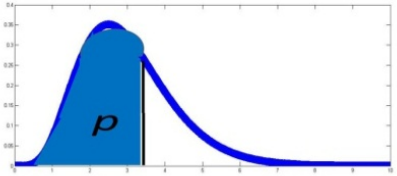
Useful Result

If $p = 0.9$ then $(1 - p)/2 = 0.05$ and $(1 + p)/2 = 0.95$ so use the columns labelled 0.05 and 0.95 for a 90% confidence interval.

If $p = 0.95$ then $(1 - p)/2 = 0.025$ and $(1 + p)/2 = 0.975$ so use the columns labelled 0.025 and 0.975 for a 95% confidence interval.

If $p = 0.99$ then $(1 - p)/2 = 0.005$ and $(1 + p)/2 = 0.995$ so use the columns labelled 0.005 and 0.995 for a 99% confidence interval.

Chi-squared table



99%

95%

90%

CHI-SQUARED DISTRIBUTION QUANTILES

df\p	0.005	0.01	0.025	0.05	0.1	0.2	0.8	0.9	0.95	0.975	0.99	0.995
1	0.000	0.000	0.001	0.004	0.016	0.064	1.642	2.706	3.842	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	0.446	3.219	4.605	5.992	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	1.005	4.642	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	1.649	5.989	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.146	1.610	2.343	7.289	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	3.070	8.558	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	3.822	9.803	12.017	14.067	16.013	18.475	20.278
8	1.344	1.647	2.180	2.733	3.490	4.594	11.030	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	5.380	12.242	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	6.179	13.442	15.987	18.307	20.483	23.209	25.188
11	2.603	3.054	3.816	4.575	5.578	6.989	14.631	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	7.807	15.812	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	8.634	16.985	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	9.467	18.151	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	10.307	19.311	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	11.152	20.465	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	12.002	21.615	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.391	10.865	12.857	22.760	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	13.716	23.900	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	14.578	25.038	28.412	31.410	34.170	37.566	39.997
25	10.520	11.524	13.120	14.611	16.473	18.940	30.675	34.382	37.652	40.646	44.314	46.928
30	13.787	14.953	16.791	18.493	20.599	23.364	36.250	40.256	43.773	46.979	50.892	53.672

100p% Confidence Interval for σ

Since $$P\left(\frac{(n-1)S^2}{b} \le \sigma^2 \le \frac{(n-1)S^2}{a}\right) = p$$

implies $$P\left(\sqrt{\frac{(n-1)S^2}{b}} \le \sigma \le \sqrt{\frac{(n-1)S^2}{a}}\right) = p$$

therefore $$\left[\sqrt{\frac{(n-1)s^2}{b}}, \sqrt{\frac{(n-1)s^2}{a}}\right]$$

a 100*p*% confidence interval for *σ* is

Confidence Interval for Handspan Variance for Females

A point estimate for σ^2 is $s^2 = 2.0550$.

An interval estimate is given by a 95% confidence interval.

Using R, we obtain:

$P(W \le 42.9503) = (1- 0.95)/2 = 0.025$

$P(W \le 86.8296) = (1+ 0.95)/2 = 0.975$ for $W \sim \chi^2(63)$

A 95% confidence interval for σ^2 is

$$\left[\frac{63(2.0550)}{86.8296}, \frac{63(2.0550)}{42.9503}\right] = [1.491, 3.014]$$

Confidence Interval for Handspan Standard Deviation for Females

A point estimate for *σ* is *s* = 1.4335.

A 95% confidence interval for *σ* is

$$\left[\sqrt{\frac{63(2.0550)}{86.8296}},\sqrt{\frac{63(2.0550)}{42.9503}}\right]=[1.221,1.736]$$

Exercise

Find 95% confidence intervals for σ and σ^2 for male handspans.

Tests of Hypothesis (Chapter 5): Introduction

Student claims they have ESP. You think this is rather doubtful.

We decide to conduct an experiment to determine how plausible their claim is.

The are two possibilities: “Student has ESP" or “Student does not have ESP".

Before we collect the data do you think both statements are equally plausible? Why or why not?

North America Criminal Court System

In the North American criminal court system the two possibilities “the defendant is innocent” and “the defendant is guilty” are not treated symmetrically.

A criminal trial begins by assuming that the defendant is innocent.

The prosecution then attempts to find sufficient evidence (data) to show that the hypothesis (statement) of innocence is not plausible.

North America Criminal Court System

It is not necessary for the defendant to be proved innocent.

By the end of the trial there may still be insufficient evidence to prove that the defendant is guilty "beyond a shadow of a doubt".

North America Criminal Court System

There are two types of mistakes that can be made in such a system:

(1) convict an innocent defendant

(2) fail to convict a guilty defendant.

These two mistakes have very different consequences.