STAT 231

October 31, 2016

Video Review for TQ 2 →

Posted on Learn.

(Weekly materials – Lectures)

Tutorial from last week ⟶ Tutorial Section

Interval Estimation

- T-pivot
- χ^2-pivot
- Relationship between the Likelihood Interval and the Confidence Interval.

Interval Estimation

Case I Gaussian estimation of μ when σ is known.

Case II: Binomial model. Estimation of θ, n is large.

Case III Gaussian estimation of μ when σ is unknown.

Case IV: Gaussian estimation of σ.

Case V : Other Non-Gaussian models with large sample sizes

For each of these cases

(1) Pivotal Quantity ✓

(2) Pivotal Distribution ✓

(3) Coverage Interval ✓

(4) Confidence " ✓

<u>Case I</u> : Gaussian problem with known σ.

P.Q : a function of $Y_1, \ldots, Y_n$ and θ whose prob. can be calculated without knowing θ

PIVOTAL QUANTITY

$$\frac{\bar{Y} - \mu}{\frac{\sigma}{\sqrt{n}}} = \text{PIVOTAL QUANTI}$$

Z : PIVOTAL DISTRIB

COVERAGE INTERVAL

$$\boxed{\bar{Y} \pm z^{*} \frac{\sigma}{\sqrt{n}}}$$

where z^{α} is calculated using the confidence level.

e.g. 95% C.I $\Rightarrow$ $z^{\alpha} = 1{\cdot}96$

99% C.I $\Rightarrow$ $z^{\alpha} = 2{\cdot}58$

Confidence Interval

$$\boxed{\bar{y} \pm z^{\alpha} \frac{\sigma}{\sqrt{n}}}$$

$\bar{y}$ = sample mean.

~~Case III: Gaussian probl~~

Clicker Question

Suppose we are constructing a 95 Confidence Interval for μ when σ is known. (GAUSSIAN).

Are the following statements T/F

(a) The width of the C.I $\uparrow$ as $n \downarrow$

(b) The . . . $\uparrow$ as $\sigma \uparrow$

(c) . . . . . . $\uparrow$ as level of conf. $\uparrow$

(d) All of them are correct.

Case II : Binomial problem

$$Y \sim Bin(n, \theta)$$

n large.

PIVOTAL QUANTITY

$$\frac{\tilde{\hat{\theta}} - \theta}{\sqrt{\hat{\theta}(1-\hat{\theta})/n}}$$

$$\boxed{\tilde{\hat{\theta}} = Y/n}$$

PIVOTAL DISTRIBUTION = Z.

Coverage Interval: $\left(\tilde{\hat{\theta}} \pm z^{\alpha} \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}\right)$

Confidence " $\left(\hat{\theta} \pm z^{\alpha} \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}\right)$

$\hat{\theta} = y/n$ = sample proportion

$$\bar{y} \pm z^{\alpha} \frac{\sigma}{\sqrt{n}}.$$

Q2 : The 95% Confidence Interval for θ in a Binomail problem. with $n = 200$ is given by

$$[0.135, \ 0.279]$$

The probability that θ lies between 0.135 and 0.279 is 0.95

(a) True (b) False ✓

Case III Gaussian problem with unknown σ.

Model: $Y_i \sim G(\mu, \sigma)$

$i = 1, \dots n.$

Y_i's independent.

Sample $\{y_1, \dots y_n\}$

$\bar{y}$ = sample mean

$$s^2 = \frac{1}{n-1} \sum (y_i - \bar{y})^2$$

Objective : To find a 95% C.I. for μ.

PIVOTAL QUANTITY

$$\frac{\bar{Y} - \mu}{\frac{S}{\sqrt{n}}}$$

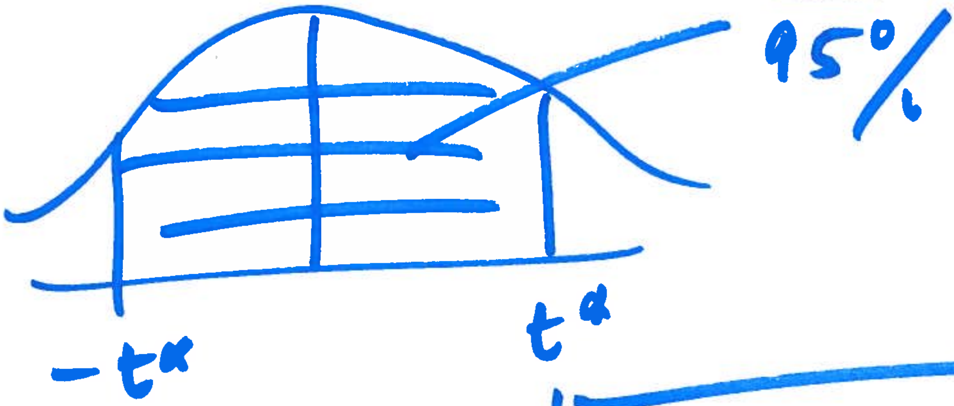
where $S = \sqrt{\frac{1}{n-1} \sum (Y_i - \bar{Y})^2}$.

= random variable.

PIVOTAL DISTRIBUTION

A student's T- distribution with n - 1 degrees of freedom.

Go to the t-table → t^{α}. df = n-1



Coverage Interval: $\boxed{\bar{Y} \pm t^{\alpha} \frac{S}{\sqrt{n}}}$

Confidence Interval $\boxed{\bar{y} \pm t^{\alpha} \frac{s}{\sqrt{n}}}$

This follows from the theorem

$$\boxed{\frac{\bar{Y} - \mu}{S / \sqrt{n}} \sim T_{n-1}}$$

If n is really large, the t-values coincide with the z-values.

Case IV: How to find the Confidence Interval for σ^2?

Example: A sample of 10 observations are drawn from a Gaussian population with mean μ and s.d σ.

$$\bar{y} = 80 \qquad s^2 = 49$$

Find the 95% C.I for σ^2.

From the theorem,

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

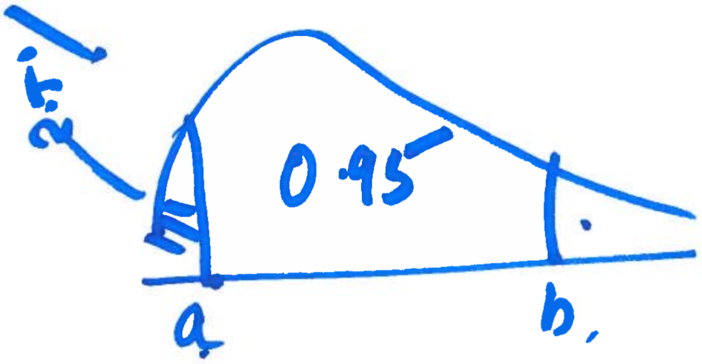
PIVOTAL QUANTITY

PIVOTAL DISTRIBUTI

PIVOTAL QTY $= \dfrac{(n-1)S^2}{\sigma^2}$

$a =$

PIVOTAL DIST.^{N} $\rightarrow \chi^2_{n-1}$



Equal tailed C.I $a = 2.7$

Row = n − 1

Columns = 0.025, 0.975

$$\left.\begin{aligned} a &= 2.7 \\ b &= 19.023 \end{aligned}\right\}$$

$$P(2.7 < \chi^2_9 < 19.023) = 0.95$$

$$P\left(2.7 < \frac{9s^2}{\sigma^2} < 19.023\right) = 0.95$$

$$\sigma^2 > \frac{9s^2}{19.023}$$ — RH inequality

$$\sigma^2 < \frac{9s^2}{2.7}$$ LH inequality

Coverage Interval:

$$\left[\frac{9s^2}{19.023}, \frac{9s^2}{2.7}\right]$$

Confidence Interval: $\left[\frac{9s^2}{19.023}, \frac{9s^2}{2.7}\right]$

Given our sample, we are 95% C.I that σ^2 will lie between

$$\left[\frac{9s^2}{19.023}, \frac{9s^2}{2.7} \right]$$

General formula:

$$\left[\frac{(n-1)s^2}{b}, \frac{(n-1)s^2}{a} \right]$$

from the χ^2 table with $n-1$ df.

Case V   Non-Gaussian problem

(a) Poisson (n is large)

$Y_1, \ldots, Y_n$ indep Poi (μ).

μ: unknown

Sample $\{y_1, \ldots, y_n\}$ $\bar{y}$ = sample mean.

Construct a 95% C.I for μ.

The CLT for Poisson

If n is large,

$$\boxed{\frac{\bar{Y}-\mu}{\sqrt{\bar{Y}/n}} \overset{\text{app.}}{=} Z \sim N(0,1)}$$

PIVOTAL QUANTITY

PIVOTAL DISTRIBUTION

$$P(-1.96 < Z < 1.96) = 0.95$$

$$P\left(-1.96 < \frac{\bar{Y}-\mu}{\sqrt{\bar{Y}/n}} < 1.96\right) = 0.95$$

Coverage Interval:

$$\bar{Y} \pm 1.96\sqrt{\bar{Y}/n}$$

Confidence Interval

$$\boxed{\bar{y} \pm 1.96\sqrt{\bar{y}/n}}$$

$\bar{y}$ = sample mean.

(b) Exponential problem:

$$Y_1, \ldots Y_n \sim Exp(\mu)$$

~~A to~~ To find a 95% C.I for μ.

Sample: $\{y_1, \ldots y_n\}$

Theorem $Y \sim Exp(\mu) \Rightarrow \frac{2Y}{\mu} \sim Exp(2)$

$\perp$

$$\boxed{2\sum Y_i/\mu \sim \chi^2_{2n}}$$