To Do

Read Sections 5.1 - 5.3 (Hypothesis Testing)

Do End-of-Chapter Problems 1 – 8 for Midterm Test 2.

Assignment 3 due Friday November 11.

See detailed information posted on Learn regarding material covered by Midterm Test 2 (4:40 - 6:10 on Tuesday November 15).

Last Class

(1) Testing H_0: $\mu = \mu_0$ when σ is unknown for G(μ,σ) model.

(2) Statistical significance versus practical significance.

(3) Relationship between tests of hypothesis and confidence intervals

Testing H_0: $\mu = \mu_0$ when σ is unknown

(1) Test H_0: $\mu = \mu_0$ based on observed random sample $y_1, y_2, \ldots, y_n$ from a G(μ,σ) distribution.

(2) Use the test statistic $$D = \frac{\left|\bar{Y} - \mu_0\right|}{S / \sqrt{n}}$$

with observed value $$d = \frac{\left|\bar{y} - \mu_0\right|}{s / \sqrt{n}}$$

(3) Calculate

$$p - value = 2\left[1 - P\left(T \le d\right)\right] \text{ where } T \sim t\left(n-1\right)$$

(4) Draw conclusion based on *p*-value.

Relationship Between Tests of Hypothesis and Confidence Intervals

The *p*-value for testing H_0: $\theta = \theta_0$ is greater than or equal to q

if and only if

the value $\theta = \theta_0$ is inside a 100(1-q)% confidence interval for θ

(assuming we use the same pivotal quantity).

Today's Class

(1) Testing H_0: $\sigma^2 = \sigma_0^2$ when μ is unknown.

(2) Likelihood Ratio Test for testing H_0: $\theta = \theta_0$.

H_0: $\sigma^2 = \sigma_0^2$ when μ is unknown

Suppose $Y_1, Y_2, \ldots, Y_n$ is a random sample from a G(μ,σ) distribution.

Recall the pivotal quantity

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

which is used to construct confidence intervals for σ and σ^2.

H_0: $\sigma^2 = \sigma_0^2$ when μ is unknown

To test H_0: $\sigma^2 = \sigma_0^2$ we use the test statistic

$$U = \frac{(n-1)S^2}{\sigma_0^2}$$

If H_0: $\sigma^2 = \sigma_0^2$ is true then

$$\frac{(n-1)S^2}{\sigma_0^2} \sim \chi^2(n-1)$$

H_0: $\sigma^2 = \sigma_0^2$ when μ is unknown

Large and small values of

$$U = \frac{(n-1)S^2}{\sigma_0^2}$$

provide evidence against H_0. Why?

The Chi-squared distribution is not symmetric about its mean which makes the determination of "large" and "small" values slightly more challenging.

H_0: $\sigma^2 = \sigma_0^2$ when μ is unknown

The following calculation approximates the *p*-value:

(1) Let $u = (n\text{-}1)s^2/\sigma_0^2$ denote the observed value of U from the data.

(2) If $P(U \leq u) < 0.5$ then *p*-value = $2P(U \leq u)$
If $P(U \geq u) < 0.5$ then *p*-value = $2P(U \geq u)$
where $U \sim \chi^2(n\text{-}1)$.

Note: Only one of these values will be between 0 and 1 and this is the value you want.

Female Handspan Example

Suppose we want to test the hypothesis H_0: $\sigma^2 = 1$.

For these data $s^2 = 2.055$

Previously we determined a 95% confidence interval for σ^2 which was [1.491, 3.014].

Since $\sigma^2 = 1$ is not in this interval we already know that the *p*-value ≤ 0.05.

Female Handspan Example

Following the suggested steps

(1) Calculate observed value of U:

$u = (n\text{-}1)s^2/\sigma_0^2$

$= (63)(2.055)/1 = 129.464375/1 = 129.46$

(2) p-value $= 2\mathrm{P}(U \geq u)$

$= 2\mathrm{P}(U \geq 129.46) \approx 0$

where $U \sim \chi^2(63)$.

There is very strong evidence against H_0: $\sigma^2 = 1$ based on the observed data.

Likelihood Ratio Test Statistic

Recall that, an exact pivotal quantity does not always exist for constructing a confidence interval for a parameter *θ*.

In these cases we used an approximate Normal pivotal quantity or the approximate pivotal quantity based on the likelihood ratio statistic:

$$\Lambda = -2\log\left[\frac{L(\theta)}{L(\tilde{\theta})}\right] \sim \chi^2(1) \text{ approximately}$$

Likelihood Ratio Statistic

$$\Lambda = -2\log\left[\frac{L(\theta)}{L(\tilde{\theta})}\right] \sim \chi^2(1) \text{ approximately}$$

Recall that we used this result to justify the fact that a 15% likelihood interval is an approximate 95% confidence interval.

We can also use this result to construct an approximate hypothesis test of H_0: $\theta = \theta_0$.