STAT 231

Roadmap

- · The Theory of Estimation
- 'rûkelihood functions and the MLE for discrete distributions.
- · Continuous distributions.

Set- ψ Model: $Y_i \sim f(y_i; \theta) - ...$ i=1,...n.

Meaning: Each data pount y.

van outcome of a. r. v. Y. which
has a distribution function f (DISCRETE)

Objective: To estimate θ = unknown.

parameter we are uterested in.

Method: METHOD OF M.L.

Likelihood function:

 $L(\theta; y_1, \dots y_n)$

= $P(Y_1 = y_1, Y_2 = y_2, ... Y_n = y_n)$

= Probability of observing your

sample

It is a function of D

Choose the value of θ (θ) that maximizes the aikelihood function

8 : Maximum dikelihood Estimate.

= MTF

D (y1>...yn) = Junction of the sample points.

Suppose all the data points are undefendently drawn.

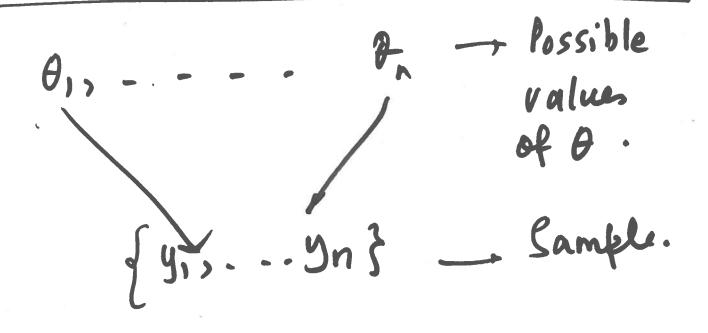
$$L(\theta;y_1,...y_N) = P(Y_1=y_1,Y_2=y_2,...Y_n=y_n)$$

$$= P(Y_1=y_1) \cdot P(Y_2=y_2) \cdot ... P(Y_n=y_n)$$

$$(g \text{ they are undependent})$$

$$(P(A \cap B) = P(A) \cdot P(B)$$

lywen an independent sample, the likelihood function: Product of the distribution function, evaluated at the sample points



8 = the value of the that has the highest prob. of generating my sample.

 $\{y_1, \dots y_n\}$ Yen f (y.) 8) The parameter o model is (a) ku unknown Constant (b) A random variable Known #, gwen the sample.

The MLE 8 is

- (x) a random vortiable
- (b) a unknown constant
 - (c) a known # given
 the sample.

Example. Suppose a data set $\{y_1, \dots, y_n\}$ is drawn from a discrete distribution with pdf. (drawn undependently) $f(y,\theta) = (1-\theta)^y$. θ

Find the MLE for θ Step 1: Construct L

L= $\pi (1-\theta)^{y'}\theta$ L= $(1-\theta)^{y}\theta$ = $(1-\theta)^{y}\theta$. $(1-\theta)^{y}\theta$. $(1-\theta)^{y}\theta$.

$$L = (1-t)^{\sum yi} n$$

Step 2: Grotruck l: log likelihood function

$$L(\theta, y_1, ... y_n) = \sum y_i \cdot ln(1-\theta) + n \cdot ln \cdot \theta$$

Step 3 Maximire le de/de = 0 and solve for 0.

$$-\frac{\sum y_{i'}}{1-\theta} + \frac{\eta}{\theta} = 0$$

$$50LVE \Rightarrow$$

2 nd order condution needs to be checked to guarantee maximum.

CONTINUOUS DISTRIBUTION

The definition in terms of probability does not quite carry over.

For a continuous distribution and an unependent sample.

f = density function of 6 Y

Example:

10 estimate the average lifetime of an electric bulb produced by a company: Y of yis - - · yn } - SAMPLE y = the lifetime of the L."
bulb vi your sample Your our data set, what is the MLE for p?

MODEL You Emp (Y) l's indépendent f(y;y)= -9/p/ yzo DENSITY FUNCTION.

Ekelhood function

L(Y; yir. yn) = -y/k --ye-y/k

The log likelihood function
$$\mathcal{L}(\gamma) = -\frac{1}{\gamma} \sum_{i=1}^{N} y_{i}$$

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$$\frac{dl}{d\gamma} = 0 \Rightarrow -\frac{n}{\gamma} + \frac{1}{\gamma^{2}} \sum_{i=1}^{N} y_{i}$$

$$\frac{n}{\gamma} = \frac{1}{\gamma^{2}} \sum_{i=1}^{N} y_{i}$$

$$\frac{n}{\gamma} = \frac{1}{\gamma^{2}} \sum_{i=1}^{N} y_{i}$$

For the Exponential, the MLE = P Example 2 Objective: To estimate p and to Population population average s.d. 1912. 9n 3 SAMPLE of STAT 231 finail grades

The appropriate " the GAUSSIAN model.

N(r, r²)
Rest of verice

Ge (14, 5-)
UW

MODEL

Yen G(4, 0)

7, 7 = 7

L=17. n
udependent

$$\frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\pi}} \frac{1}$$

The log-likelihood $l: -\frac{1}{2}\ln(2\pi) - n\ln \pi$ $-\frac{1}{2} \sum (y_{L} + y_{L})^{2}$

$$\frac{\partial \ell}{\partial r} = 0$$

$$\theta = (\gamma, \sigma) = \text{vector of unknown}$$

parameters