

STAT 231

September 28, 2016

Roadmap

- The Theory of Estimation
- Likelihood function for discrete r. v. s
 - Binomial
 - Geometric
 - Poisson.
- Calculate the MLEs.

↙
Maximum Likelihood
ESTIMATORS ~~OR~~ ES.

Set-up

$\{y_1, \dots, y_n\}$ an independent sample from a population.

Parameter of Interest: $\theta = \text{constant}$
(unknown)
mean, \swarrow variance, etc.

Set up a model

$$Y_l \sim f(y_l; \theta) \quad l=1, \dots, n.$$

Samples are drawn from a r.v. with distribution function f .

θ = parameter of the r.v.

Objective: To come up with a #

$\hat{\theta}$, which is a function of your data set, that is the "most likely" value of θ

- Sample "truly random" or biased?

Your estimates will vary from sample to sample.

Post-debate poll: Hillary Clinton takes round one

By Jennifer Agiesta, CNN Polling Director

🕒 Updated 12:42 PM ET, Tue September 27, 2016

'RANDOM' SAMPLE'



Source: CNN

Poll: Clinton wins first debate 00:49

Story highlights

Poll: 62% say Clinton won, 27% said Trump did

It's a similar result to Romney topping Obama in four years ago

issues by a margin of more than 2-to-1. Clinton also was seen as having done a better job addressing concerns voters might have about her potential presidency by a 57% to 35% margin, and as the stronger leader by a 56% to 39% margin.

READ: The complete full CNN/ORC poll results

The gap was smaller on which candidate appeared more sincere and authentic, though still broke in Clinton's favor, with 53% saying she was more sincere vs. 40% who felt Trump did better on that score. Trump topped

(CNN) — Hillary Clinton was deemed the winner of Monday night's debate by 62% of voters who tuned in to watch, while just 27% said they thought Donald Trump had the better night, according to a CNN/ORC Poll of voters who watched the debate.

That drubbing is similar to Mitt Romney's dominant performance over President Barack Obama in the first 2012 presidential debate.

Voters who watched said Clinton expressed her views more clearly than Trump and had a better understanding of the

Clinton 56% to 33% as the debater who spent more time attacking their opponent.

Although the survey suggested debate watchers were more apt to describe themselves as Democrats than the overall pool of voters, even independents who watched deemed Clinton the winner, 54% vs. 33% who thought Trump did the best job in the debate.

And the survey suggests Clinton outperformed the expectations of those who watched. While pre-debate interviews indicated these watchers expected Clinton to win by a 26-point margin, that grew to 35 points in the post-debate survey.

QUIZ: Are you more like Clinton or Trump?

About half in the poll say the debate did not have an effect on their voting plans, 47% said it didn't make a difference, but those who say they were moved by it tilted in Clinton's direction, 34% said the debate made them more apt to vote for Clinton, 18% more likely to back Trump.



On the issues, voters who watched broadly say Clinton would do a better job handling foreign policy, 62% to 35%, and most think she would be the better candidate to handle terrorism, 54% to 43% who prefer Trump. But on the economy, the split is much closer, with 51% saying they favor Clinton's approach vs. 47% who prefer Trump.

Most debate watchers came away from Monday's face-off with doubts about Trump's ability to handle the presidency. Overall, 55% say they didn't think Trump would be able to handle the job of president, 43% said they thought he would. Among political independents who watched the debate, it's a near-even split, 50% say he can handle it, 49% that he can't.


CNN's Reality Check Team vets the claims in the first presidential debate

And voters who watched were more apt to see Trump's attacks on Clinton as unfair than they were to see her critiques that way. About two-thirds of debate viewers, 67%, said Clinton's critiques of Trump were fair, while just 51% said the same of Trump.

Assessments of Trump's attacks on Clinton were sharply split by gender, with 58% of men seeing them as fair compared with 44% of women who watched on Monday. There was almost no gender divide in perceptions of whether Clinton's attacks were fair.

The CNN/ORC post-debate poll includes interviews with 521 registered voters who watched the September 26 debate. Results among debate-watchers have a margin of sampling error of plus or minus 4.5 percentage points. Respondents were originally interviewed as part of a September 23-25 telephone survey of a random sample of Americans, and indicated they planned to watch the debate and would be willing to be re-interviewed when it was over.

TIME

 Hillary Clinton and Donald Trump faced off at their first presidential debate Monday. Who do you think won?



409,743 Votes

SNAP Poll

Every poll \longrightarrow n : sample size.
(521)
and

$\left\{ \begin{array}{l} \text{MOE : Margin of Error} \\ \pm 4.5\% \end{array} \right.$

Ex Example We are interested
in the % age of people π who thought
Clinton won the debate

A random sample of 521 people
and 323 of them
said Hillary won.

What is your "best" estimate for π ?

$y = \# \text{ of successes}$

$$y = 323.$$

$$Y \sim \text{Bin}(521, \pi)$$

MODEL.

$$\hat{\pi} = ?$$

Construct the likelihood function.

$$L(\pi; y) = P(\text{observing your sample as a function of } \pi)$$

$$L(\pi) = {}^{521}_{323}C \pi^{323} (1-\pi)^{198}$$

The maximum likelihood estimate $\hat{\pi}$ is the value of π that maximizes L

The log-likelihood function

$$l(\pi) = \ln {}^{521}_{323}C + 323 \ln \pi + 198 \ln (1-\pi)$$

$$\frac{dl}{d\pi} = 0 \Rightarrow$$

$$\frac{323}{\pi} - \frac{198}{1-\pi} = 0$$

Solve for π

$$\boxed{\hat{\pi} = \frac{323}{521}}$$

$$= 62\%$$

The MLE for a general Binomial problem: $\hat{\pi} = \frac{y}{n} =$ SAMPLE PROPORTION OF SUCCESSES

Example 2

Jeopardy problem

Objective: To check whether
Canadians are better at Jeopardy

HYPOTHESIS TESTING PROBLEM

$$\hat{\pi} = P(\text{a Canadian contestant wins Jeopardy})$$

- (i) Estimate π
- (ii) Check whether $\hat{\pi}$ is "large enough"

$$y = \{1, 1, 1, 2, 3, 1\}$$

\downarrow \downarrow \downarrow \downarrow
 $1-\pi$ $1-\pi$ $\pi(1-\pi)$ $\pi^2(1-\pi)$

y_i = # of shows a Canadian
contestant i appeared on

Jeopardy

Y_i = # of trials before the
1st failure

$$Y_i \sim \text{Geom}(\pi)$$

π = Prob. of
Success.

Construct the likelihood function

$$L(\pi) = (1-\pi)^6 \pi^3$$

Construct the log-likelihood function

$$l(\pi) = 6 \ln(1-\pi) + 3 \ln \pi$$

Take derivatives and equate it to zero

$$dl/d\pi = -\frac{6}{1-\pi} + \frac{3}{\pi} = 0$$

$$\boxed{\hat{\pi} = 1/3}$$

General Poisson problem

of hits on a web-site every day

n days = $\{y_1, \dots, y_n\}$ indep.

Objective: To estimate μ = POPULATION
AVERAGE # of hits

Question: Estimate μ using your sample.

MODEL: $Y_i \sim \text{Poi}(\mu)$ $i=1, \dots, n$
independent

Likelihood function

$$\begin{aligned} L(\mu) &= \frac{e^{-\mu} \mu^{y_1}}{y_1!} \cdot \frac{e^{-\mu} \mu^{y_2}}{y_2!} \cdots \\ &\quad \cdots \frac{e^{-\mu} \mu^{y_n}}{y_n!} \\ &= \frac{e^{-n\mu} \mu^{\sum y_i}}{y_1! y_2! \cdots y_n!} \end{aligned}$$

$$l(\pi) = -n\mu + \sum y_i \ln \mu - \ln(y_1! y_2! \dots y_n!)$$

$$\frac{dl}{d\pi} = 0$$

$$\Rightarrow -n + \sum y_i \cdot \frac{1}{\mu} = 0$$

$$\boxed{\hat{\mu} = \sum y_i / n = \bar{y}}$$

For any Poisson problem, the sample average is the MLE for μ

This rule can be applied to any
distribution:
