Roadmap

- 5 min recap
- Measurement bias problem:
- Testing for variance of a Gaussian population
- Clicker question
- Other distributions

$$H_0: \theta = \theta_0$$

θ_0 given

Data: $\{y_1, \ldots, y_n\}$

- Construct the DISCREPANCY MEASURE / TEST STATISTIC D

- Calculate d = observed value of D from your sample: Obs. value of the Test statistic

- Calculate the p-value: $P(D \geq d;$ H_0 is true$)$

$$Y_1, \dots Y_n \sim G(\mu, \sigma)$$

μ, σ unknown $\qquad H_0: \mu = \mu_0$

$$D = \left| \frac{\bar{Y} - \mu_0}{\frac{S}{\sqrt{n}}} \right| \qquad d = \left| \frac{\bar{y} - \mu_0}{\frac{s}{\sqrt{n}}} \right|.$$

$$\text{p-value} = P(D \geq d)$$
$$= P(|T_{n-1}| \geq d)$$

→ We need to know the distribution of capital D.

$H_0 : \theta = \theta_0$

p-value of this test $\geq q$

θ_0 must lie in the $100(1-q)\%$ C.I

(assuming you are using the same pivot)

Application:

To test whether a scale is biased?

We will take a known weight = 10 lb,

and we take n measurements

$$\{y_1, \ldots y_n\}$$

$$Y_i = \underbrace{10 + \delta} + R_i$$

$$R_i \sim N(0, \sigma^2)$$

$$H_0 : \delta = 0$$

$$n = 36, \quad \bar{y} = 13; \quad s = 12.$$

$$Y_i \sim \cancel{N}\ G(\mu, \sigma^2)$$

$$H_0 : \mu = 10 \qquad \mu = 10 + \delta.$$

This test is equivalent to testing for the unknown mean for a Gaussian problem.

$$D = \left| \frac{\bar{Y} - \mu_0}{\frac{S}{\sqrt{n}}} \right| \qquad d = \left| \frac{\bar{y} - \mu_0}{s/\sqrt{n}} \right|$$

p-value:

$$= \left| \frac{13 - 10}{12/\sqrt{36}} \right|$$

$$P(D \geqslant d)$$

$$= 1.5.$$

$$= P(|T_{35}| \geqslant 1.5)$$

Based on your value of p, draw appropriate conclusions.

TEST FOR VARIANCE

$$Y_1, \ldots Y_n \sim G(\mu, \sigma)$$

μ, σ unknown.

$$\cancel{H_0 : \sigma^2 = \sigma_0^2}$$

$$H_0 : \sigma^2 = \sigma_0^2$$

$n = 51, \quad \bar{y} = 10 \quad s^2 = 2.035$

$H_0 : \sigma^2 = 1$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$D = \frac{(n-1)S^2}{\sigma_0^2}$$

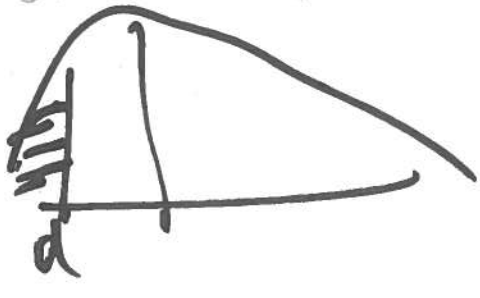
Large and small values of D are "bad news" for H_0

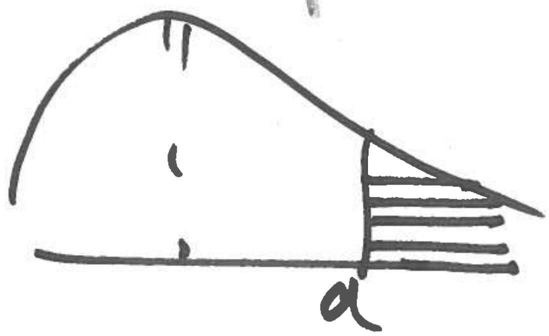
Step 1 : Calculate

$$d = \frac{(n-1)s^2}{\sigma_0^2}$$

Step 2: If d is to the right of the median of χ^2_{n-1}, then calculate the probability of the right tail to get the p-value.

If it is to the left ~~hand~~ of the median, left-tail probability and double it





Step 3: Draw appropriate conclusions based on your p-value.

<u>Example</u> $Y_1, \ldots, Y_n \sim G(\mu, \sigma)$

$n = 51$, $\bar{y} = 10$; $s^2 = 2.055$

(a) Construct the 95% C.I for σ^2.

(b) Test whether $\underline{H_0 : \sigma_0^2 = 1}$

C.I for σ^2: $\left[\dfrac{(n-1)s^2}{b}, \dfrac{(n-1)s^2}{a} \right]$

b, a are from the χ^2 table.

To get a and b

Row = n - 1 = 50

Columns = 0.025 , 0.975
a b.

$a = 32.357$

$b = 71.420.$

$$C.I = \left[\frac{50 \times 2.055}{71.420}, \frac{50 \times 2.055}{32.357}\right]$$

$= [1.44, 3.18]$

The p-value would be <u>low</u>.

<u>Step 1</u> Calculate d

$$\therefore d = \frac{(n-1)s^2}{\sigma_0^2} = \frac{50 \times 2.055}{1}$$

$$= 102.75$$

<u>Step 2</u>

Is d to the right/left of the median?

d is bigger than the median.

<u>Step 3</u> : $P(D \geq d)$

$$= P(\chi^2_{50} \geq 102.75) \simeq 0$$

Step 4 → The p-value $= 2 \times \uparrow = \simeq 0$

We have strong evidence against H_0

Clicker questions

① If you are testing $H_0 : \theta = \theta_0$

The p-value for your test $= 0.06$

Does θ_0 belong to the 90% C.I

(a) Yes

(b) No

(c) Can't Say

Example:

$Y_1, \ldots, Y_{50} \sim \text{Poi}(\mu)$

$\bar{y} = 5.$

$$\left.\begin{array}{l} H_0 : \mu = \overset{\mu_0}{6} \\ H_1 : \mu \neq 6 \end{array}\right\}$$

CLT

$$D = \left| \frac{\bar{Y} - \overset{6}{\mu_0}}{\sqrt{\mu_0 / n}} \right| = Z.$$

$$d = \left| \frac{5 - 6}{\sqrt{6/50}} \right|$$

p-value

$P(D \geq d)$

$= P(|Z| \geq d)$