STAT 231

October 28, 2016

- <u>Tutorial</u>: Past midterm solutions
  + problems.

- Video Review → Posted Tonight.

Interval Estimation

- Recap the Student's T - distribution
- How to look up T- tables.

Applications

- Gaussian problem with unknown σ
  - Interval Estimation μ.

  . . . . . σ

- Relationship between Likelihood Interval and Confidence Interval.

T- distribution

- T a r.v. $\in (-\infty, \infty)$ is said to follow a T-distribution with n degrees of freedom. if

$$T = Z / Y$$

Z, Y independent

$$Z \sim G(0,1)$$

where $W \sim \chi_n^2$

$$Y = \sqrt{W/n}$$

Properties of the T-distribution

- T takes all possible values in the real line. for any $n = df$
- T is always symmetric around zero. $\forall n$.
- The T-distribution looks similar to the Z-distribution, but with more extreme observations

$K > 3$

• As $n \to \infty$ (df becomes large) the T-distribution approaches the Z-distribution.

CALCULATION OF T- probabilities

• Look up the T-table

T-table: Row = degrees of freedom

Entries: Quantiles corresponding to the prob. on the columns

Example:

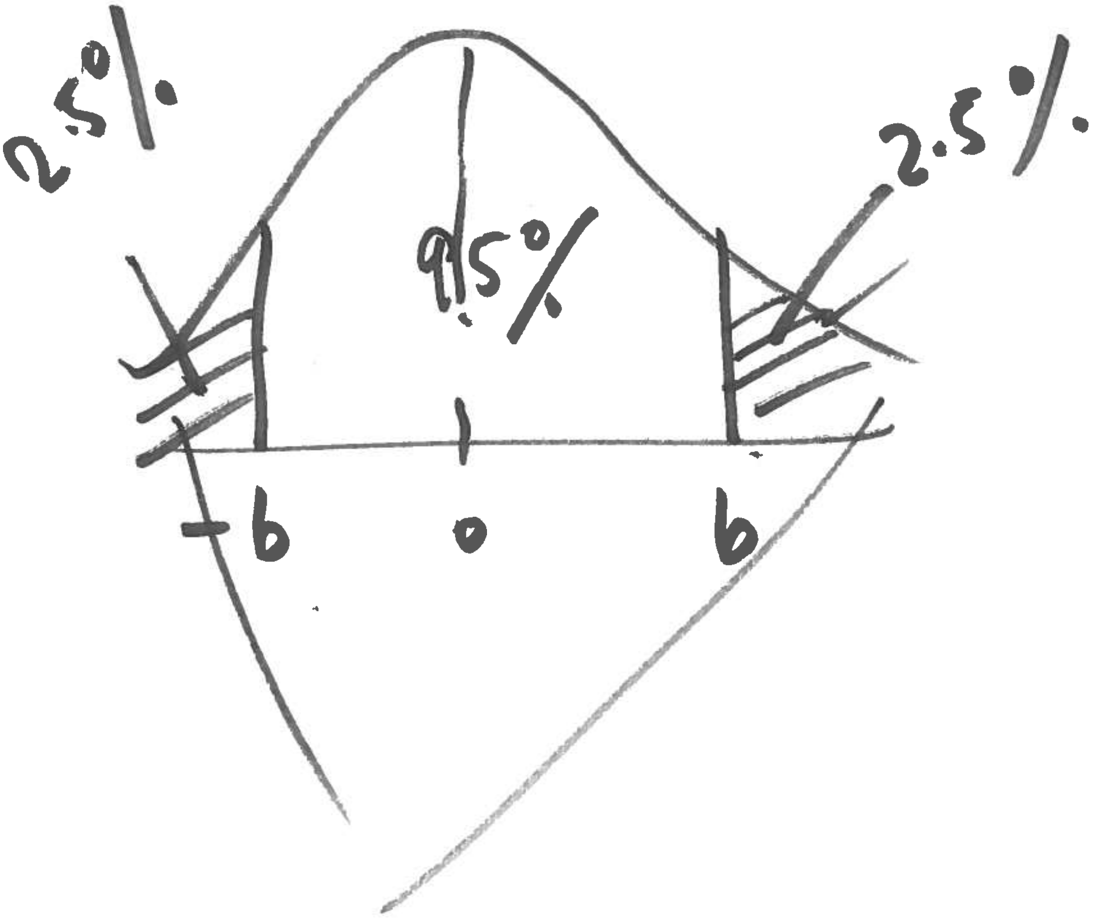
$$\left.\begin{array}{l} n = 20. \\ \text{Column} = 0.9 \end{array}\right\} \boxed{1.3253}$$

$$P(T_{20} \le 1.3253) = 0.9$$

Examples: 1) T

- Suppose a r.v. ~~with~~ follows a T- distribution with $df = 15$

Find b such that $P(|T| \geq b) = 0.05$



0.05

b = 97.5 th percentile

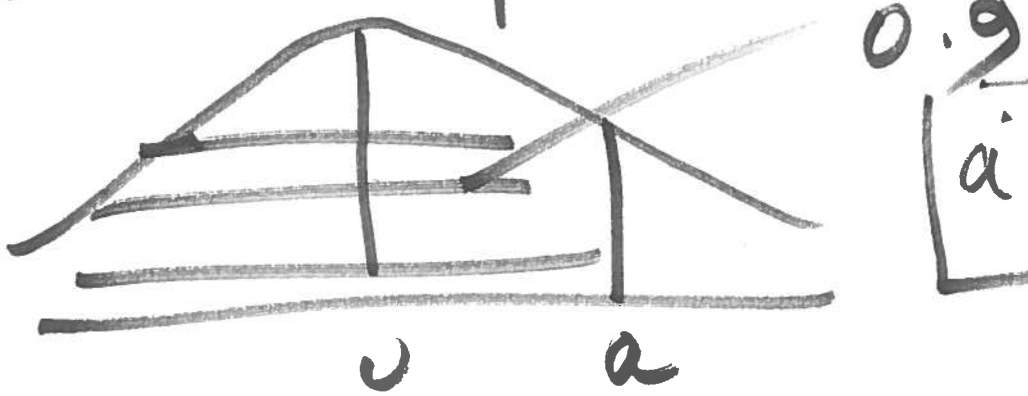
Row = 15; Column = 0.975

t_0 = b = 2.1314

Example 2

Suppose T follows a T distribution with $n = 200$ df. Find the 90^{th} percentile of T.

Since df = large, the T-problem is ~~equ~~ approximately equivalent to the Z-problem.



$\boxed{a = 1.645}$

Example Let $Y \sim G(2,3)$

$T \sim T_{400}$

$400 = n = df$

Y and T independent

Let $$W = \left(\frac{Y-2}{3}\right)^2 + T^2$$

What does W follow?

$$W \sim \chi^2(2).$$

Theorem:

§ Let $Y_1, \dots Y_n$ be Gaussian r.v.s (independent) with mean μ (unknown) and s.d σ (unknown)

Sample: $\{y_1, \dots y_n\}$

(i) $$\frac{\bar{Y} - \mu}{\frac{S}{\sqrt{n}}} \sim T_{n-1}$$

(ii) $$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

where $\bar{Y} = \frac{1}{n} \sum Y_i$

$$S^2 = \frac{1}{n-1} \sum (Y_i - \bar{Y})^2$$

<u>Suppose σ was known</u>.

$$\bar{Y} \sim G\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

(SAMPLING)

$$\frac{\bar{Y} - \mu}{\frac{\sigma}{\sqrt{n}}} = Z$$

If σ is unknown, we cannot use the Z-pivot any more. because C.I will have σ in it.

So we use the theorem and construc the T-pivot.

Example : The IQs of UW profs are have a Gaussian distribution with mean μ and s.d σ.

A random sample of 10 UW professors are taken

$$\{y_1, \ldots y_{10}\}$$

$$\bar{y} = 90 \qquad s^2 = \frac{1}{n-1} \sum (y_i - \bar{y})^2$$

$$= 64$$

Based on this data, we want to construct a $\underline{\underline{95\%}}$ C.I for μ

How to construct the right pivot?

$$\text{Is } \sigma \text{ known?} \begin{cases} \text{Yes} \rightarrow Z\text{-pivot} \\ \text{No} - T\text{-pivot.} \end{cases}$$

By the theorem,

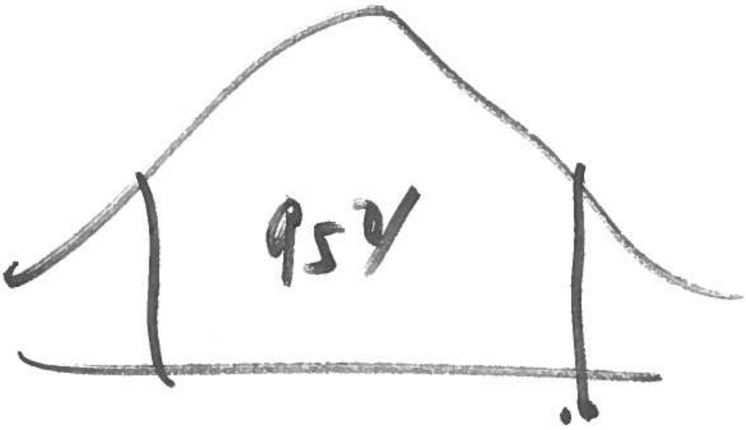
$$\frac{\bar{Y}-\mu}{\frac{S}{\sqrt{n}}} \sim T_{n-1}$$

In our example

$$\frac{\bar{Y}-\mu}{\frac{S}{\sqrt{10}}} \sim T_9$$

$\therefore$

$$\left.\begin{array}{l}\text{Row} = 9 \quad (n-1)\\ \text{Column} = 0.975\end{array}\right\} \boxed{2.2622} \rightarrow t^{\alpha}$$



9

$$P(-2.2622 < T_9 < 2.2622) = 0.95$$

$$P\left(-2.2622 < \frac{\bar{Y} - \mu}{\frac{S}{\sqrt{n}}} < 2.2622\right) = 0.95$$

$$\mu > \bar{Y} - 2.2622\frac{S}{\sqrt{n}}$$

$$\mu < \bar{Y} + 2.2622\frac{S}{\sqrt{n}}$$

Coverage Interval.

$$\left(\bar{Y} - 2.2622\frac{S}{\sqrt{n}}, \bar{Y} + 2.262\frac{S}{\sqrt{n}}\right)$$

Confidence Interval

$$\left(\bar{y} - 2.2622\frac{s}{\sqrt{n}}, \bar{y} + 2.2622\frac{s}{\sqrt{n}}\right)$$

$$\left(90 - 2.2622 \cdot 8/\sqrt{10}, 90 + 2.2622\frac{8}{\sqrt{10}}\right)$$

Gaussian problem

Interested in μ.

C.I: $$\bar{y} \pm z^{\alpha} \frac{\sigma}{\sqrt{n}}$$

if σ is known.

OR $$\bar{y} \pm t^{\alpha} \frac{s}{\sqrt{n}}$$ if σ is unknown.

(ii) $\dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

What is $E(S^2) = ?$

$$E\left[\frac{(n-1)S^2}{\sigma^2}\right] = n-1$$

$[E(\chi^2) = df]$

$$\frac{\cancel{(n-1)}}{\sigma^2} E(S^2) = \cancel{n-1}$$

$$\boxed{E(S^2) = \sigma^2}$$

$$S^2 = \frac{1}{n-1} \sum \quad)$$

$$= \frac{n}{n-1} \underbrace{\left[\frac{1}{n} \left[\quad \right] \right]}$$

$$S^2 = \frac{n}{n-1} \left[\frac{1}{n} \sum (y_i - \bar{y})^2 \right]$$

$$E(S^2) = \sigma^2$$