STAT 231

Tutorial

Nov 9, 2016

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$$Y \sim \mathcal{E}xp\,(\theta)$$

Show that $\quad W = \dfrac{2Y}{\theta} \sim \chi^2(2)$

$$= \mathcal{E}xp\,(2)$$

cdf

$$F(w) = P(W \leq w)$$

$$= P\left(\frac{2Y}{\theta} \leq w\right) = P\left(Y \leq \frac{w\theta}{2}\right)$$

$$= 1 - e^{-w/2} \longleftarrow \; = \int_0^{w\theta/2} \frac{1}{\theta} e^{-x/\theta}\, dx$$

Winter 2014

ID Number: ______________________

NAME (Please Print): ________________________

Signature: __________________________

| Question | Mark | Maximum Mark | Marker Initials |
|---|---|---|---|
| 1 | | 18 | |
| 2 | | 20 | |
| 3 | | 17 | |
| Total | | 55 | |

(*iii*) A point estimate of $\theta$ based on the observed data is _________________.

(*iv*) An approximate 95% confidence interval for $\theta$ based on the observed data is ____________________________

(*v*) By reference to the confidence interval, indicate what you know about the $p-value$ for a test of the hypothesis $H_0 : \theta = 0.8$?

(*c*) Suppose a Binomial experiment is conducted and the observed 95% confidence interval for $\theta$ is $[0.1, 0.2]$. This means (circle the letter for the correct answer):

$A$ : The probability that $\theta$ is contained in the interval $[0.1, 0.2]$ equals 0.95.

$B$ : If the Binomial experiment was repeated 100 times independently and a 95% confidence interval was constructed each time then approximately 95 of these intervals would contain the true value of $\theta$.

(c) The maximum likelihood estimate of $\mu$ is _______________

The maximum likelihood estimate of $\sigma$ is ___________________
(You do not need to derive these estimates.)

$T_{19}$

(d) Let

$$S^2 = \frac{1}{19}\sum_{i=1}^{20}\left(Y_i - \bar{Y}\right)^2 \qquad T = \frac{\bar{Y} - \mu}{S/\sqrt{20}} \qquad \text{and} \qquad W = \frac{1}{\sigma^2}\sum_{i=1}^{20}\left(Y_i - \bar{Y}\right)^2.$$

The distribution of $T$ is ____________________.

The distribution of $W$ is ____________________.
(Be sure to specify both the distribution and its parameter(s).)

(e) The company, R.A.T. Chow, that produces the special diet claims that the mean weight gain for rats that are fed this diet is 67 grams.
The *p−value* for testing the hypothesis $H_0 : \mu = 67$ is between ______________ and ________________.

What would you conclude about R.A.T. Chow's claim?

(f) Let $a$ and $b$ be such that

$$P(W \le a) = 0.05 = P(W \ge b).$$

Then $a =$ ____________________ and $b =$ _____________________________.

A 90% confidence interval for $\sigma$ for the given data is ___________________________.

(c) Explain clearly how the pivotal quantity $U$ can be used to obtain a two-sided $100p\%$ confidence interval for $\theta$.

(d) Suppose $n = 25$ so that

$$U = \frac{2}{\theta}\sum_{i=1}^{25} Y_i \sim \chi^2(50).$$

Let $a$ and $b$ be such that

$$P(U \leq a) = 0.05 = P(U \geq b).$$

Then $a =$ ___________________ and $b =$ ____________________________.

(e) Suppose $y_1, \ldots, y_{25}$ is an observed random sample from the *Exponential* $(\theta)$ distribution with $\sum_{i=1}^{25} y_i = 560$.

The maximum likelihood estimate for $\theta$ is ______________________________________.(You do not need to derive this estimate.)

A 90% confidence interval for $\theta$ based on $U$ is _____________________________________.

(f) Suppose an experiment is conducted and the hypothesis $H_0 : \theta = \theta_0$ is tested using a test statistic $D$ with observed value $d$. If the $p - value = 0.01$ then this means (circle the letter for the correct answer):

$A$ : the probability that $H_0 : \theta = \theta_0$ is correct equals 0.01.

$B$ : the probability of observing a $D$ value greater than or equal to $d$, assuming $H_0 : \theta = \theta_0$ is true, equals 0.01.

$$F(w) = 1 - e^{-w/2}$$

$$f(w) = \frac{1}{2} e^{-w/2} - Exp(2)$$

$$\sim \chi^2(2)$$

$$\frac{1}{\mu} e^{-x/\mu}$$

$Y_1, \ldots, Y_n : \sim Exp(\theta)$

Show that

$$U = \frac{2}{\theta} \sum Y_i \sim \chi^2(2n)$$

$$Y \sim Exp(\theta)$$

$$\frac{2Y}{\theta} \sim \chi^2(2) \qquad \text{(from part 1)}$$

$$\left| \begin{array}{l} \dfrac{2Y_1}{\theta} \sim \chi^2(2) \\ \dfrac{2Y_2}{\theta} \sim \chi^2(2) \\ \vdots \\ \dfrac{2Y_n}{\theta} \sim \chi^2(2) \end{array} \right.$$

$$U \sim \chi^2(2n)$$

$$\boxed{\frac{2}{\theta}\sum Y_i \sim \chi^2(2n)}$$

PIVOTAL QUANTITY

PIVOTAL DISTRIBUTIO

Let us say $n = 10$.

$$\frac{2}{\theta}\sum_{i=1}^{10} Y_i \sim \chi^2_{20}$$


Find a and b such that

$$P(a < \chi^2_{20} < b) = 0.95$$

a = Column = 0.025

b = Column = 0.975

$a = 9.591 \qquad b = 34.170.$

$$P\left(9.591 \leq \underline{\underline{\chi^2_{20}}} \leq 34.170\right) = 0.95$$

$$P\left(9.591 \leq \frac{2}{\theta}\sum Y_i \leq 34.170\right) = 0.9$$

$$\theta \geq \frac{2\sum Y_i}{34.170}$$

$$\theta \leq \frac{2\sum Y_i}{9.591}$$

Coverage Interval.

$$\left[ \frac{2\Sigma Y_i}{34.170}, \quad \frac{2\Sigma Y_i}{9.591} \right]$$

Confidence Interval

$$\left[ \frac{2\Sigma y_i}{34.170}, \quad \frac{2\Sigma y_i}{9.591} \right].$$

(d) MLE : $\hat{\theta} = \bar{y} = 560/25 =$

Q2. $\mu, \sigma$ unknown

Gaussian.

$$\boxed{\bar{y} = \frac{1273.8}{20}}$$

$$s^2 = \frac{1}{n-1} \sum (y_i - \bar{y})^2$$

$$\boxed{s^2 = \frac{1}{19} \times 665.718}$$

Normality = reasonable!

$\mu, \sigma$ ? unknown constants

attribute of the population

$\mu$

MLE for $\mu$ : $\bar{y}$

MLE for $\sigma$ : $\sqrt{\frac{1}{n}\sum(y_i-\bar{y})^2}$

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$$\frac{\bar{Y}-\mu}{S/\sqrt{n}} \sim T_{n-1}$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$\frac{\cancel{(n-1)}\frac{1}{\cancel{n-1}}\sum(Y_i-\bar{Y})^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$\sim \chi^2_{19}$$

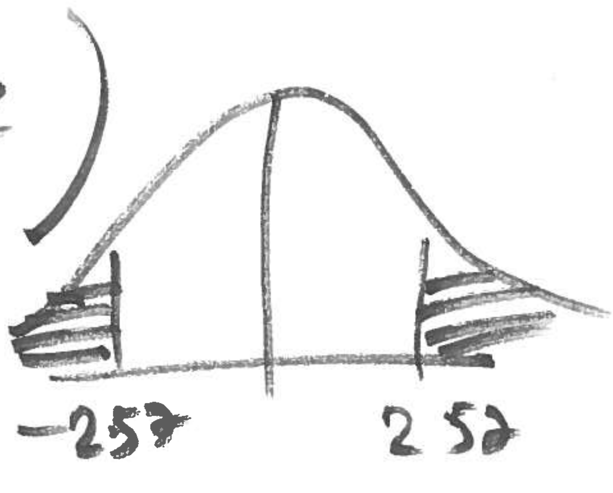
$$D = \left| \frac{\bar{Y} - \mu}{\frac{S}{\sqrt{n}}} \right|$$

$$d = \left| \frac{\bar{y} - 67}{s/\sqrt{n}} \right| = 2.57.$$

p-value.

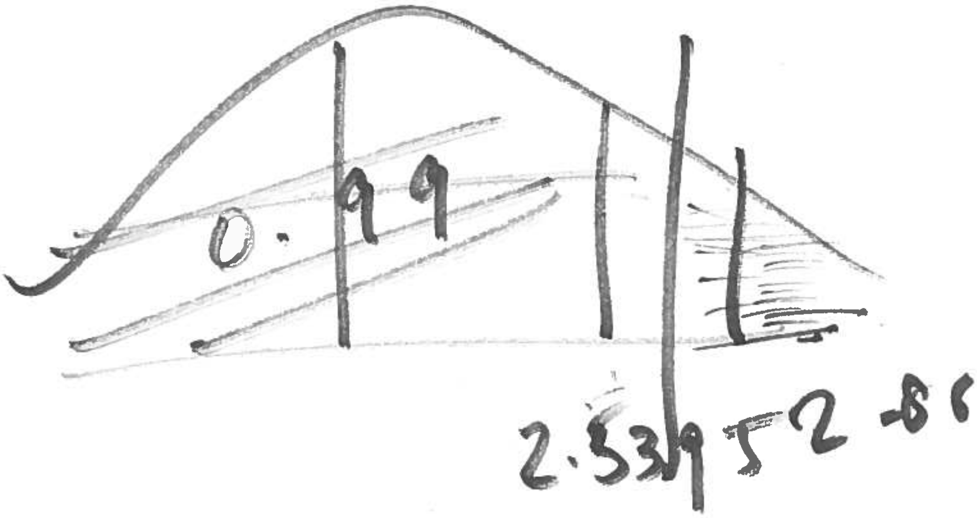
$$P(D \geq d)$$

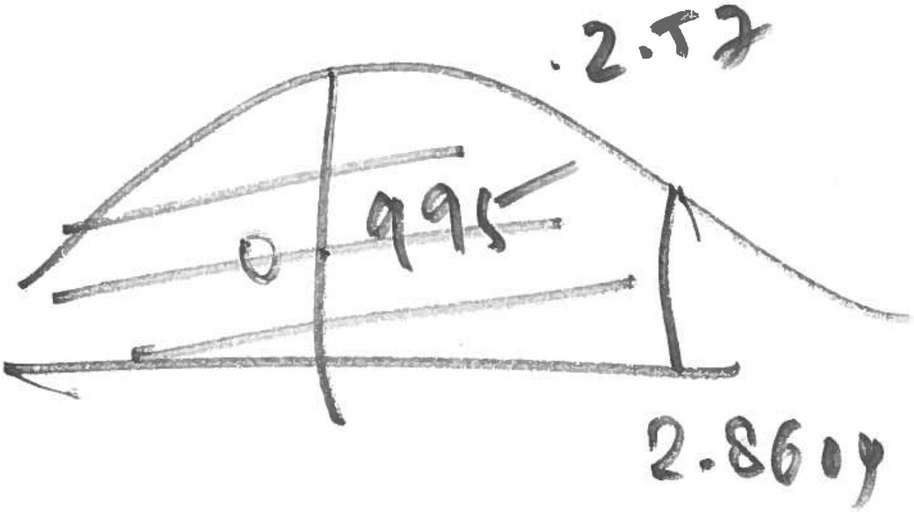
$$= P(|T_{19}| \geq 2.57)$$



2.5395 2.8609

99% 99.5%

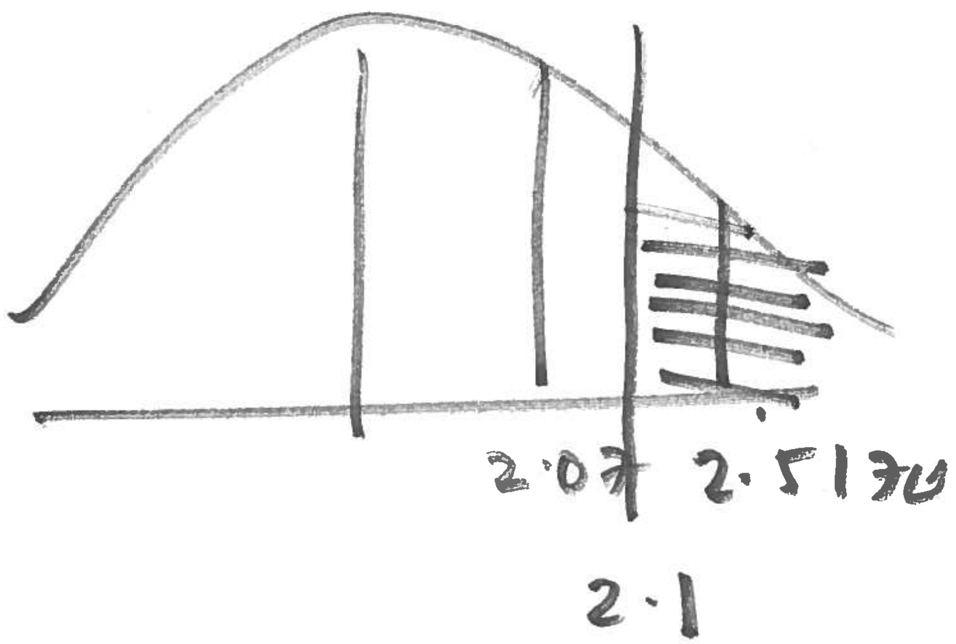


$$P(|T_{21}| \geq 2.1)$$

$2.07 \longrightarrow .975$

$2.5176 \longrightarrow 0.99.$



Confidence Interval for $\sigma$.

$$\left[\sqrt{\frac{(n-1)s^2}{b}}, \sqrt{\frac{(n-1)s^2}{a}}\right]$$

$b > a \quad \chi^2 \; (n-1)$ table.

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Binomial

sample prop

$$\left[\hat{\theta} \pm Z^* \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}\right]$$

$Z$.

$[0.2, 0.7]$