STAT 231

October 21, 2016.

Interval Estimation.

Roadmap

- Binomial. Confidence Interval problem.
- How to choose the optimal sample size?
- Chi-Squared distribution.

$$X \sim \chi^2_n$$

n = degrees of freedom.

- Student's T-distribution.

$$X \sim T_n$$

Interval Estimation

Given: $\{y_1, \ldots y_n\}$. ✓

Interested in θ (unknown parameter)

Model: $Y_i \sim f(y_i; \theta)$ $i = 1, \ldots n$.

Objective:

$[L, U]$ which would contain θ with a pre-specified probability.

and estimate it using our sample. $[l, u]$

$l = l(y_1, \ldots y_n)$

$u = u(y_1, \ldots y_n)$

(i) We want to find the SAMPLING DISTRIBUTION of $\tilde{\theta}$. (ESTIMATOR for θ)

(ii) "Modify" this distribution to something we know. (PIVOTAL DISTRIBUTION)

GAUSSIAN PROBLEM WITH KNOWN VARIANCE

"Estimate μ."

$\theta = \mu$. ✓

$\hat{\theta} = \bar{y}$.

$\tilde{\theta} = \bar{Y}$

$\bar{Y} \sim G(\mu, \sigma/\sqrt{n})$

$$\bar{Y} \sim G\left(\mu, \sigma/\sqrt{n}\right)$$

$\longrightarrow$ SAMPLING DISTRIBUTION

$$\frac{\bar{Y} - \mu}{\sigma/\sqrt{n}} = Z = \text{PIVOTAL DISTRIBUTION}$$

Binomial Problem

$Y \sim Bin(n, \theta)$

θ = prob. of success.

Data: n = # of trials

y = # of successes

$\hat{\theta} = y/n$ (Sample proportion)

$\tilde{\theta} = Y/n$ (ESTIMATOR)

Theorem: CLT for Binomial

(i) $$\frac{\tilde{\theta} - \theta}{\sqrt{\frac{\theta(1-\theta)}{n}}} \sim Z \sim N(0,1)$$ if n is large.

(ii) $$\frac{\tilde{\theta} - \theta}{\sqrt{\frac{\tilde{\theta}(1-\tilde{\theta})}{n}}} \sim Z \sim N(0,1)$$ if n is large.

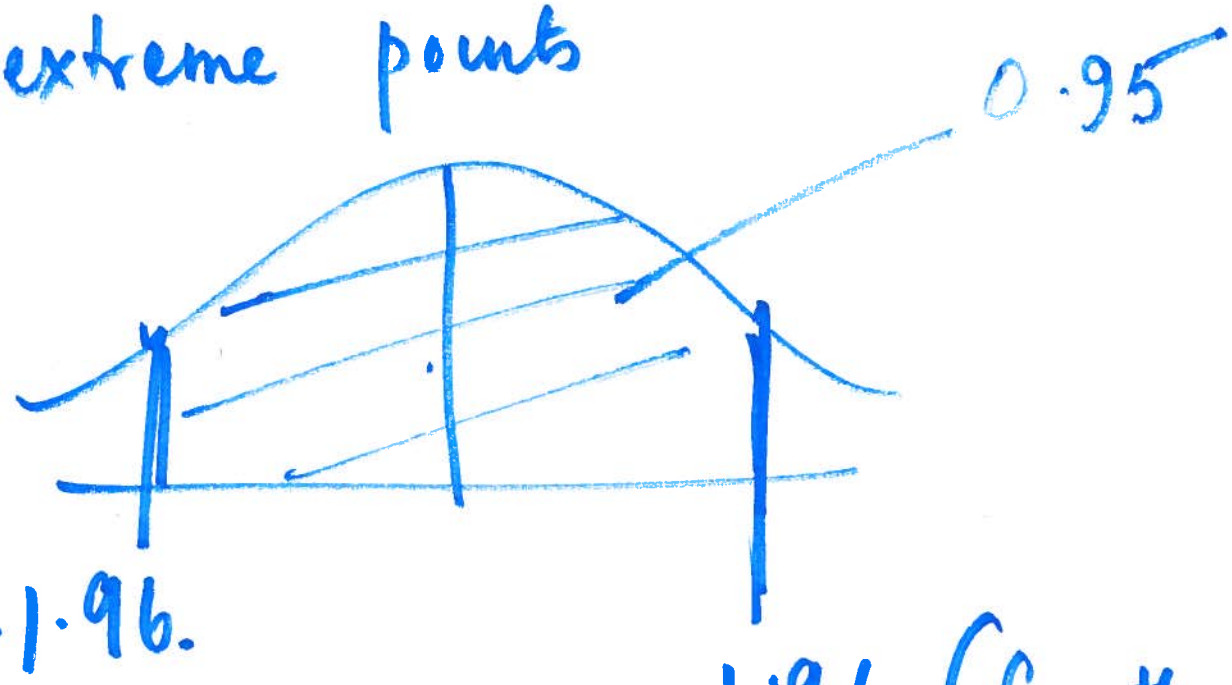
$\tilde{\theta} = Y/n$.

Example: A survey of 1000 US voters are taken and 53% plan ~~for~~ to vote for Hillary Clinton. Find a 95% C.I for the proportion of votes Clinton will get.

θ = proportion of Hillary voters

$\hat{\theta}$: 0.53 $\tilde{\theta} = \frac{Y}{1000}$

Go to the z-table and find the extreme points



(from the t-table)

$$P(-1.96 \leq Z \leq 1.96) = 0.95$$

$$P\left(-1.96 \leq \frac{\tilde{\theta} - \theta}{\sqrt{\frac{\tilde{\theta}(1-\tilde{\theta})}{n}}} \leq 1.96\right) = 0.95$$

Right hand inequality

$$\theta \geq \tilde{\theta} - 1.96\sqrt{\frac{\tilde{\theta}(1-\tilde{\theta})}{n}}$$

L. H. inequality

$$\theta \leq \tilde{\theta} + 1.96\sqrt{\frac{\tilde{\theta}(1-\tilde{\theta})}{n}}$$

Coverage Interval

$$\left[\tilde{\theta} \pm 1.96\sqrt{\frac{\tilde{\theta}(1-\tilde{\theta})}{n}}\right]$$

Confidence Interval

$$\hat{\theta} \pm 1.96\sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}$$

CBC-Angus Reid Institute poll: Canadians want minorities to do more to 'fit in'

Top Stories

Majority polled also said immigration policies should put Canada's economic needs first

Jason Proctor · proctor@cbc.ca · CBC News · October 3, 2016

- 1 1845 reading now Trump a 'puppet' of Putin? WikiLeaks target isn't who you may think, Russia experts say
- 2 883 reading now 'There's a big problem': Two-thirds of Tragically Hip tickets weren't sold directly to fans
- 3 871 reading now 'There's a big problem': Two-thirds of Tragically Hip tickets weren't sold directly to fans

- Toronto

Toronto

- 1 283 reading now Parents find son's lifeless body after pharmacy switches sleep medication for toxic dose of another drug
- 2 181 reading now Jason Grilli wants to finish career with Toronto Blue Jays
- 3 121 reading now Parents find son's lifeless body after pharmacy switches sleep medication for toxic dose of another drug

Change Local News Region

Select your region

- British Columbia
- Kamloops
- Calgary
- Edmonton
- Saskatchewan
- Saskatoon
- Manitoba
- Thunder Bay
- Sudbury
- Windsor
- Kitchener Waterloo
- Hamilton
- Toronto
- Ottawa
- Montreal
- New Brunswick
- Prince Edward Island
- Nova Scotia
- Newfoundland & Labrador
- North

original: Which is correct?
onald's birthday cause for reflection, not celebration for First Nations

10k shares

983 comments

As a divisive election tears Americans apart over questions of race and immigration, a CBC News poll suggests Canadians are right in believing they think very differently than their U.S. neighbours when it comes to multiculturalism.

In fact, we're more likely to think minorities should assimilate.

In a national polling partnership between CBC and the Angus Reid Institute, 68 per cent of Canadian respondents said minorities should be doing more to fit in with mainstream society instead of keeping their own customs and languages.

The same question was put to Americans, with only 53 per cent of respondents saying minorities need to better adjust.

The Canadian response represents a hardening of attitudes away from multiculturalism over time.

"It does seem like a very surprising finding, especially when you consider this is a country that has been living with 45 years of official multiculturalism as government policy," said Shachi Kurl, executive director of the Angus Reid Institute.

"It is maybe not what conventional wisdom might expect. But what these findings show is there are real limits on what Canadians — regardless of their own heritage or walk of life — are prepared to put up with in terms of accommodation and the sense of the mosaic versus the melting pot."

- **How Kellie Leitch and Justin Trudeau are defining themselves on immigration**
- **Kellie Leitch defends 'anti-Canadian values' screening for new immigrants**

The results also hint at why Conservative leadership candidate Kellie Leitch believes she may be onto a winning issue by asking supporters their thoughts on vetting would-be immigrants and refugees for "anti-Canadian values."

According to the poll, two-thirds of Canadians say they're "satisfied" with how well new immigrants are integrating into their communities.

That figure seems to fly in the face of another result, because an equal number said they believe "minorities should do more to fit in better with mainstream Canadian society."

Kurl compared that figure with a similar poll done in 1993 in which 57 per cent of respondents thought minority groups should be encouraged to "try to change to be more like most Canadians."

"It's not a crisis by any means" she said.

"That said, when nearly 70 per cent of people in this country are saying they would like to see minorities do more to fit in, it is something that bears watching, particularly because that view has hardened over the last 25 years."

'Unthinking or mindless multiculturalism'

Former B.C. premier and Liberal cabinet minister Ujjal Dosanjh has written and spoken extensively about the need to address concerns about equality, race and culture in the face of blind devotion to multiculturalism.

He said the poll shows Canada's political leadership needs to pay attention.

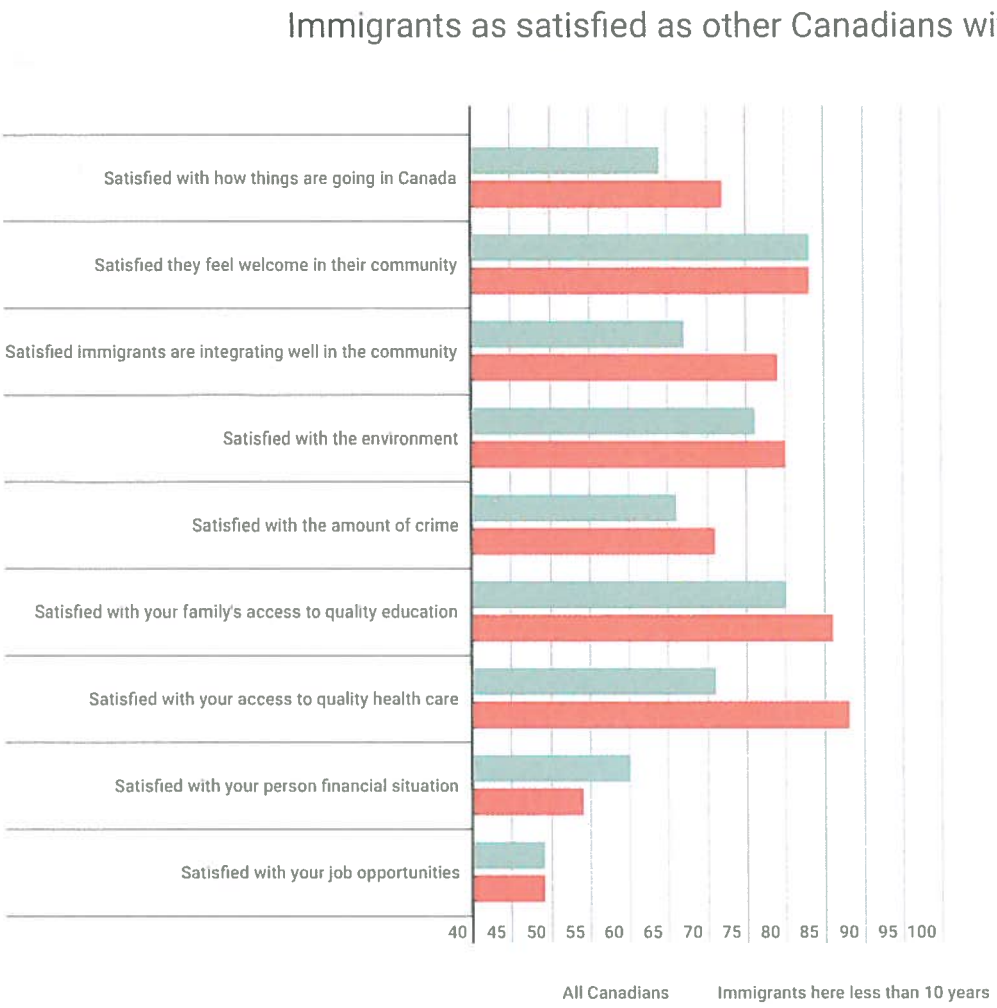
"What you want is creative multiculturalism, generous multiculturalism, but not unthinking or mindless multiculturalism where everything anybody brings to this country is acceptable," he said.

"Diversity is great if we can begin to live with each other in equality, in understanding ... but we also understand our collective obligations to building a better society. If we can't live together with each other properly and make concessions to each other, then this phrase that politicians use — that diversity is a strength — is nonsensical."

Former B.C. premier Ujjal Dosanjh says Canada's political leadership needs to pay attention to people's attitudes on cultural diversity. (CBC News)

Dosanjh says he's not surprised the results had a higher percentage of Canadians than Americans indicating they favour better assimilation.

He said the difference may simply come down to the fact that more Americans believe immigrants are integrated anyway, that newcomers and old stock alike are united in



Report Typo

Popular in News

- 1 1845 reading now Trump a 'puppet' of Putin? WikiLeaks target isn't who you may think, Russia experts say
- 2 883 reading now 'There's a big problem': Two-thirds of Tragically Hip tickets weren't sold directly to fans
- 3 871 reading now 'There's a big problem': Two-thirds of Tragically Hip tickets weren't sold directly to fans
- 4 730 reading now Trump a 'puppet' of Putin? WikiLeaks target isn't who you may think, Russia experts say
- 5 499 reading now How Netflix may be crushing border hoppers — and ways to get around it

More from CBC News

Analysis
Want cheaper access to American e-commerce? Better tell it to the feds: Neil Macdonald
Analysis

Neil Macdonald
•
3 H Ago
Politics
Waiting to become Canadian: Citizenship ceremonies delayed by judge shortage

Elizabeth Thompson
•
3 H Ago
Politics
New
Jason Grilli wants to finish career with Toronto Blue Jays
New
55 M Ago
Toronto
Bombardier to cut 2,000 Canadian jobs over next 2 years

3 M Ago
Montreal
How Netflix may be crushing border hoppers — and ways to get around it

4 M Ago
Business
'There's a big problem': Two-thirds of Tragically Hip tickets weren't sold directly to fans

David Common
•
1 H Ago
Business

Tell us what you think

Learn about our redesign project opens new window
Share your thoughts here opens new window

Explore CBC

CBC Home
TV
Radio
News
Sports
Music
Arts
Kids
Local
Documentaries
Comedy
Books

$0.53 \pm 1.96\sqrt{\frac{0.53 \times 0.47}{1000}}$

$\therefore$

0.025

Margin of Error.

~~53 ± 2~~

0.53 ± 0.03

How to choose the sample size?

(95%

M.O.E: ~~$\hat{\theta} \pm \sqrt{\quad}$~~

C.I $\hat{\theta} \pm 1.96 \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}$

MOE.

We want

$1.96 \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}} \leq 0.03$ ← 0.03

$$n \geq \left(\frac{1.96}{0.03}\right)^2 \hat{\theta}(1-\hat{\theta}) \quad \cdots (1)$$

We will choose that (1) is true for the maximum value of

$\hat{\theta}(1-\hat{\theta})$

$1 - 2\hat{\theta} = 0$

$\hat{\theta} = 1/2$.

$$n \geq \left(\frac{1.96}{0.03}\right)^2 \times \frac{1}{2} \times \frac{1}{2} \approx 1067.1$$

$= 1068$.

For the MOE to be ± 0.03,
we need ≈ 1068 people.

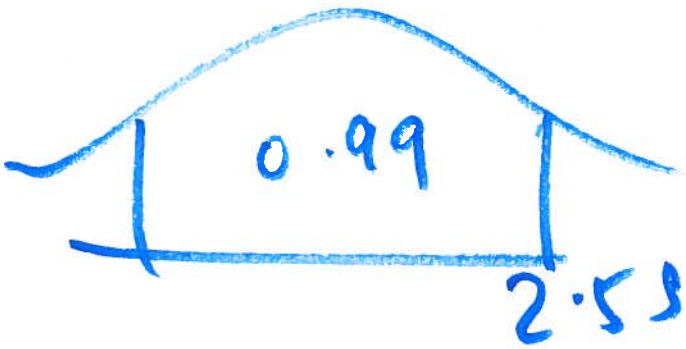
$$n \geq \left(\frac{z^*}{MOE}\right)^2 \times \frac{1}{4}$$

0.02

General Rule.

99% C.I

$MOE \leq 0.02$



Gaussian model with unknown variance.

Problem $Y_1, \ldots Y_n \sim G_c(\mu, \sigma)$

μ = unknown. mean

σ = " s.d.

Objective: To construct a 95% C.I. for μ based on my data $\{y_1, \ldots y_n\}$

$\hat{\mu} = \hat{\theta} = \bar{y}$ (MLE for Gaussian)

ESTIMATE

$\bar{Y} \sim \mathcal{N}(\mu, \sigma/\sqrt{n})$

ESTIMATOR

$$\frac{\bar{Y} - \mu}{\frac{\sigma}{\sqrt{n}}} = Z \sim N(0,1)$$