STAT 231

October 24, 2016

My tutorial → 6-00 pm.

INTERVAL ESTIMATION

(1) Recap the Binomial model

(2) The Chi-Squared distribution

$X \sim \chi^2_n$ n = degrees of freedom.

(3) The Student's T- distribution

$X \sim T_n$ n = degrees of freedom.

BINOMIAL ESTIMATION

$Y \sim Bin(n, \theta)$

θ = parameter of interest

(Application: Opinion Polls)

<u>Objective</u>: To construct a C. I for θ using our sample.

y = # of successes in our sample.

Using the CLT,

Coverage Interval.

$$\left[\tilde{\theta} \pm z^{\alpha} \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}} \right]$$

95%

$z^{\alpha} = 1.96$

z^{α} depends on the level of confidence.

Confidence Interval: $\boxed{\hat{\theta} \pm z^{\alpha}\sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}}$

$\hat{\theta} = y/n$

< 0.035

Hillary Beats Trump (ABC)

50 $\longrightarrow$ 38

θ = proportion of Hillary Supporters

$\hat{\theta}$ = MLE = 50% = 0.5

MOE = $\pm$ ~~0.~~ 3.5%, 19 times

out of 20. $\longrightarrow$ Level of Confidence.

$\hat{\theta}$ = Sample prop. $\quad z^{\alpha} = 1.96.$

TO SELECT THE RIGHT SAMPLE SIZE.

Before we do the survey, we want to make sure that the

MOE $\leq l$ $\quad (l = 0.03)$

To guarantee that, we need to choose n s.t

$$\boxed{n \geq \left(\frac{z^{\alpha}}{l}\right)^2 \times \frac{1}{4}}.$$

Choose n to be the next highest integer

Q level of confidence = 95%

M.O.E ± 0.03

$$\boxed{n \approx 1068}$$

For the ABC problem, the sample size (874), M.O.E = 0.035

Reducing MOE by $1/2$ we need to take 4 times the sample size

CHI-SQUARED DISTRIBUTION

Definition: Let W be a r.v. which takes values $[0, \infty)$. W is said to follow a Chi-Squared dist.^{n} with n degrees of freedom if

$$W = z_1^2 + z_2^2 + \dots + z_n^2$$

where $z_i \sim N(0,1)$

z_i's are independent.

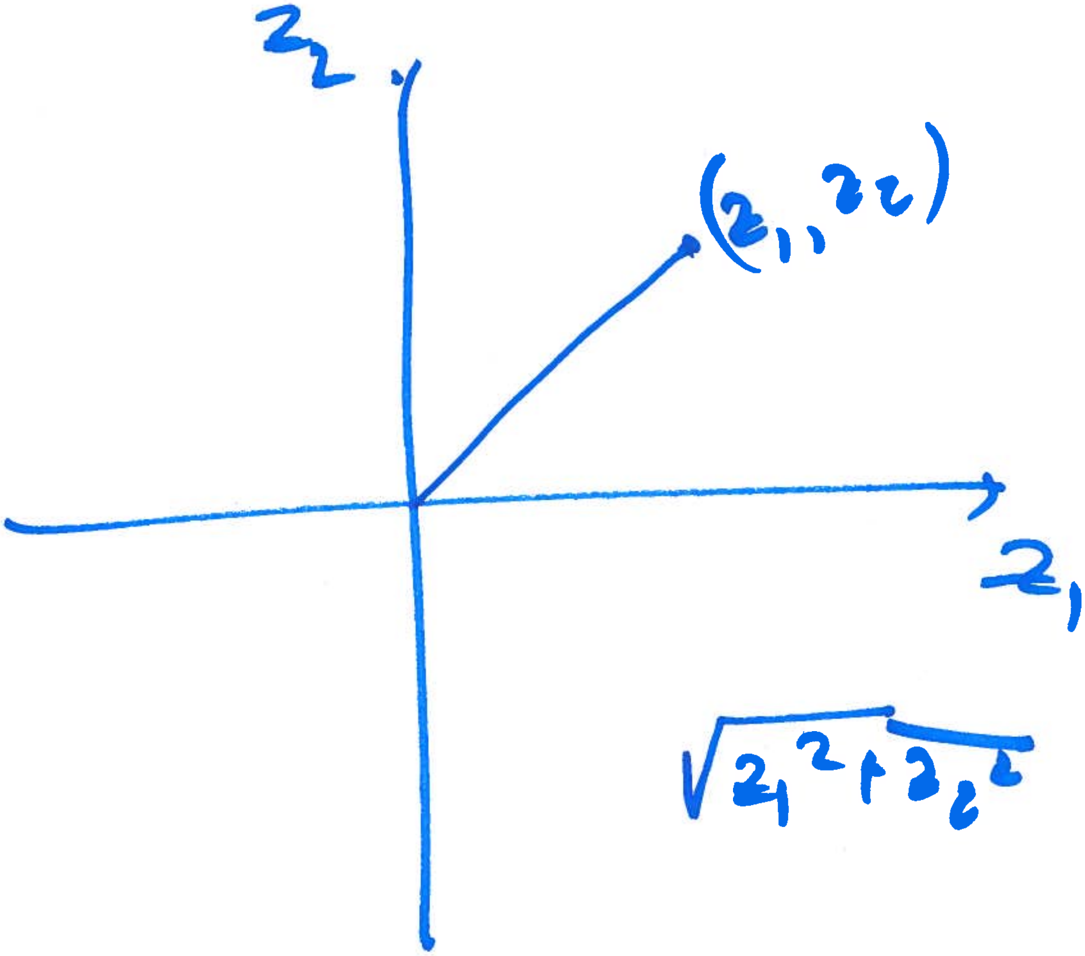
n = parameter of the Chi-Square.

Example: $W \sim \chi^2_2$

$$W = z_1^2 + z_2^2$$

$z_i \sim N(0,1)$

z_1, z_2 are indep.

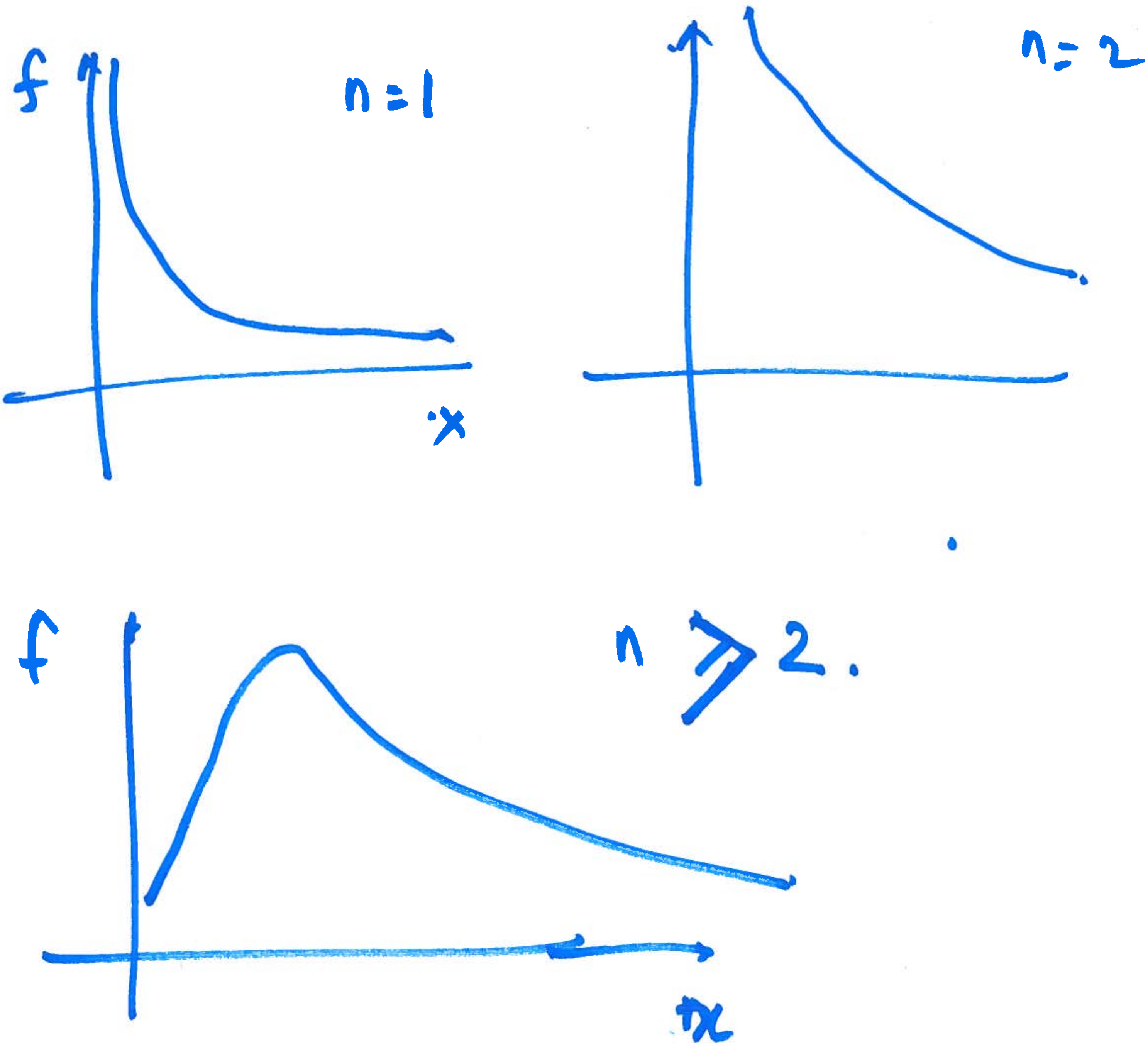


$\sqrt{z_1^2 + z_2^2}$

$z_1^2 + z_2^2.$

The plot of all the squared distances of (z_1, z_2) from the origin $\sim \chi^2_2$

What does the density of Chi-Squared look like?



Properties of the Chi-Squared distribution:

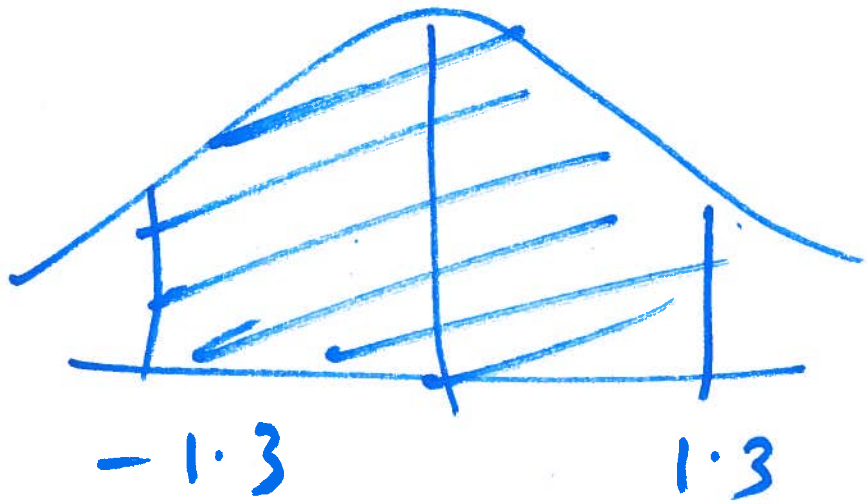
<u>Case I</u> df = 1:

<u>Example</u>: Suppose $W \sim \chi^2_1$

find $P(W \leq 1.69) = ?$

$$W = Z^2 \qquad Z \sim N(0,1)$$

$$P(W \leq 1.69)$$
$$= P(Z^2 \leq 1.69) = P(-1.3 \leq Z \leq 1.3)$$

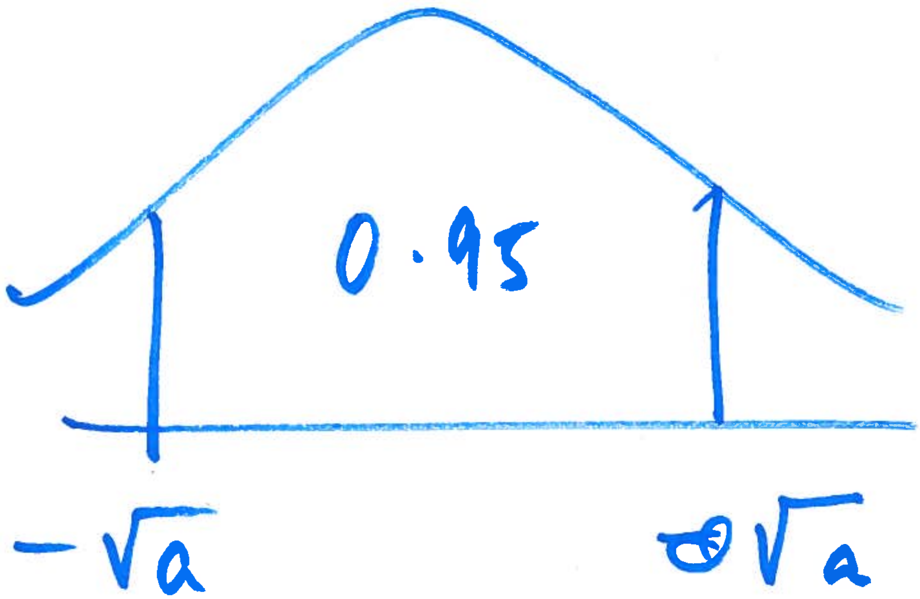


<u>Example</u>: Find the 95^{th} percentile of $W = \chi^2_1$ $\quad a = ?$

$$P(W \le a) = 0.95$$

$$\Rightarrow P(Z^2 \le a) = 0.95$$

$$\Rightarrow P(-\sqrt{a} \le Z \le \sqrt{a}) = 0.95$$



$\therefore$ $\sqrt{a}$ can be calculated from the z-table.

Case II $\quad n = 2$ df.

If $W \sim \chi^2_2$ it is the same as Exponential distribution with mean 2.

Density of Exponential

Example: Suppose $W \sim \chi^2_2$

Find $P(W \leq 2.5)$

$$\int_0^{2.5} \frac{1}{2} e^{-x/2}\, dx. \qquad \boxed{\frac{1}{\mu} e^{-x/\mu}}$$

Case III : n is "large"

If $W \sim \chi^2_n$ and $\underline{\underline{n}}$ is large,

$$W \underset{app}{\sim} N(n, 2n)$$

Example : Suppose

$W \sim \chi^2_{72}$. find $P(W \geq 96)$

$= ?$

$$W \sim \chi^2_{72}$$

$$W \overset{\text{app}}{\sim} N(\overset{\mu}{72}, \overset{\sigma^2}{144})$$

$$P(W \geq 96)$$

$$= P\left(\frac{W-72}{12} \geq \frac{96-72}{12}\right)$$

$$= P(Z \geq 2)$$

Case $\underline{IV}$ n lies between 2 and "large"

For this, we need to look up tables.

Row = df

Columns: Percentile probabil<u>ity</u>.

30^{th} percentile of χ^2_7 = ? x

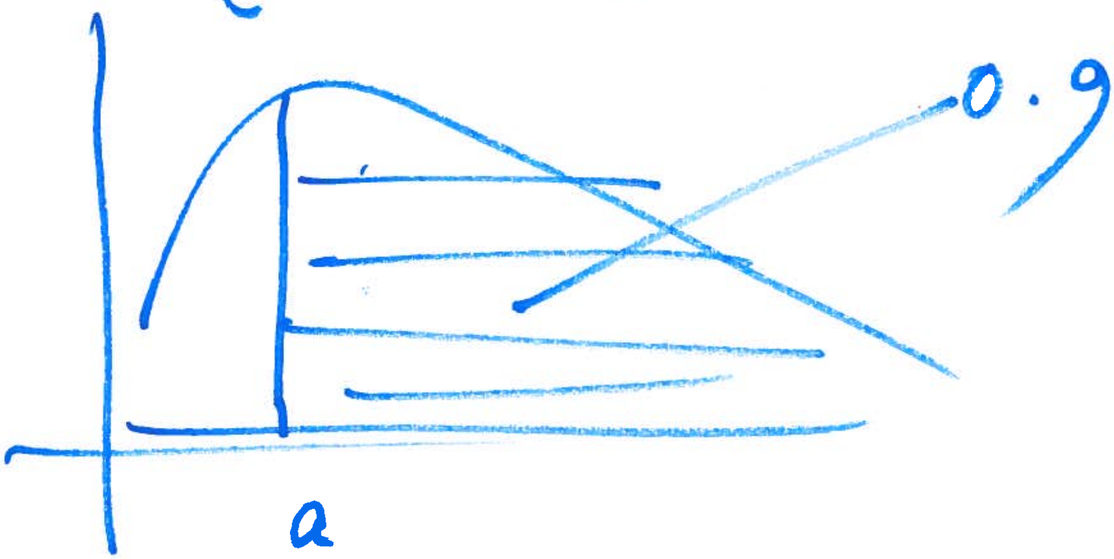
Row = 7; Column = 0.3 / $x = 4.671$

Example

- Suppose $W \sim \chi^2_5$. Find a.

such that

$$P(W \geq a) = 0.9$$



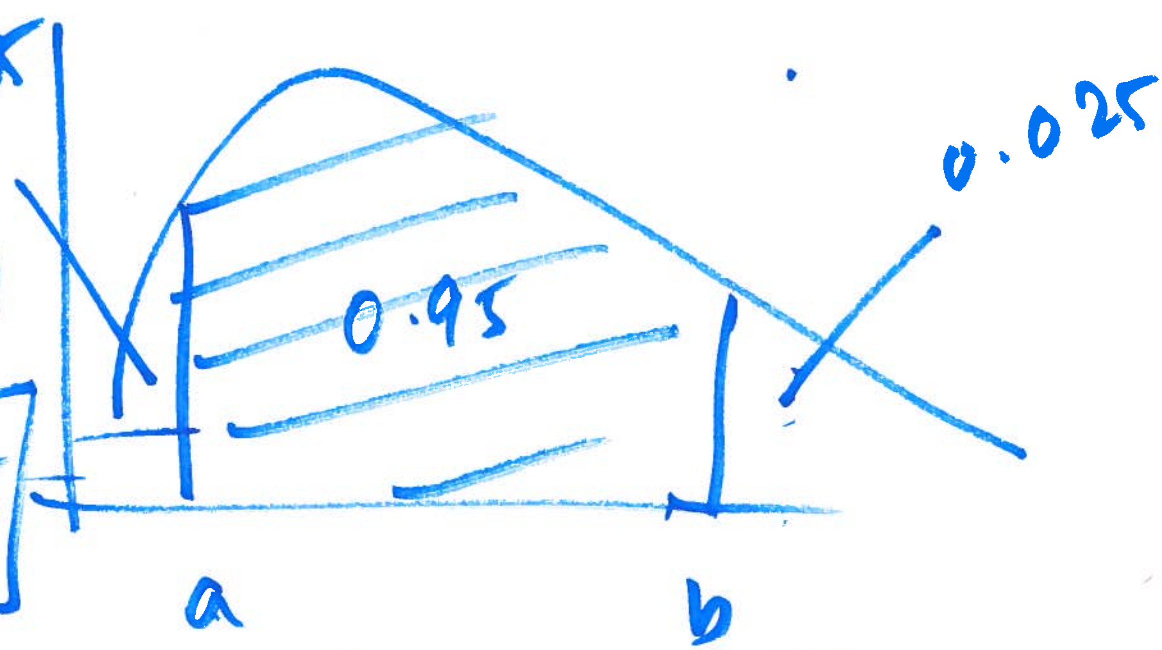
Row = 5 Column = 0.1

$$\boxed{a = 1.610}$$

Example:

Suppose $W \sim \chi^2_8$. Find a and b such that

$$P(a \le W \le b) = 0.95$$



$\boxed{a = 2.180}$

$\boxed{b = 17.535}$

To find $b \rightarrow$ Row $= 8$ Column $= 0.975$

$a \rightarrow$ Row $= 8$ Column $= 0.025$

Properties of Chi-Squared

① Let $W \sim \chi^2_n$, $E(W) = n$
$V(W) = 2n$.

② Let $W_1 \sim \chi^2_{k_1}$
$W_2 \sim \chi^2_{k_2}$

W_1 and W_2 are independent,

$W_1 + W_2 \sim \chi^2_{k_1 + k_2}$.