Parameters Analysis of QIEA-R in Convergence Quality

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***Abstract*—QIEA-$\mathbb{R}$ (Quantum Inspired Evolutionary Algorithm with Real Codification) was proposed for solving numerical problems obtaining better results when compared with traditional EAs, DE and PSO algorithms. It is inspired on the concept of quantum superposition in order to reduce the number of evaluations. QIEA-$\mathbb{R}$ has two important steps: initialization of the quantum population and updating of the quantum population. This paper analyzes these two steps and parameters related: Size of classical population, number of iterations, over some benchmark functions using statistical measurements to evaluate their importance and effect in convergence quality. The results shows the importance of quantum population size and update frequency.**

***Index Terms*—Quantum inspired algorithm, Evolutionary algorithms, Convergence Analysis**

I. Introduction

Optimization is an area from the applied mathematics whose objective is to obtain the best or bests solutions from a function to be optimized in a known domain. These function can be linear or nonlinear, may also have one optimal and many sub-optimal solutions or many global optimal solutions. In Bio-inspired Computing, there are many techniques to solve problems with one or many solutions. Intrinsically these methods have high parallelism in search process and they have good performance in unimodal and multimodal problems [1], [2].

Quantum Inspired Evolutionary Algorithm (QIEA) are based on the concept of superposition of multiple universes from quantum physics, then it can be used for optimization process with great intrinsic parallelism. A first approach, the QIEA-$\mathbb{B}$ model [3] uses *q-bits* as the information unit creating quantum individuals behaving as generators of classical individuals and uses an update operator so probabilistically to get better individuals every iteration .

QIEA-$\mathbb{R}$ [4], [5] was proposed for numerical problems with real codification using the same principle of quantum inspiration. The updating of quantum individuals are based on uniform probability density function to generate better classic individuals for real codification [4].

FP-QIEA-$\mathbb{R}$ [6] used a mechanism inspired in Particle Filter, it was proposed to improve the behaviour in multimodal problem.

The proposal of this work is experiment using the parameters of QIEA-$\mathbb{R}$ [4] in order to analyze the impact over the overall outcomes of the algorithm under some benchmark functions commonly used in evaluation of optimization algorithms.

II. The QIEA-$\mathbb{R}$ model

The QIEA-$\mathbb{R}$ model, proposed by [4] is composed by the following steps: Generation of quantum population, generation of classic population by observing quantum population, and update of quantum population.

A. Quantum Representation

Quantum individual represent the superposition of possible states. In this model, the set of observable states is continuous.

- Let be $\{\mathbf{q}_1, \ldots, \mathbf{q}_m\}$ quantum individuals of a population $\mathbf{Q}_t$ at generation t,
- Each quantum individual $\mathbf{q}_i$ is composed by n genes, $\mathbf{q}_{ij} = \{q_{i1}, \ldots, q_{in}\}$,
- Each gen q_{ij} is defined by a pdf $p_{ij}(x)$. This pdf is initialized covering all the domain of q_{ij}.
- Under this definition, a quantum individual can be represented as:

$$\mathbf{q}_i = \{p_{i1}(x), p_{12}(x), \ldots, p_{in}(x)\} \tag{1}$$

Once initialized the quantum population $\mathbf{Q}_0$, the main loop of evolutionary process is started and continues as follow:

B. Observation

After the generation of quantum population $\mathbf{Q}_0$, the main loop of the evolutionary process is started with the process described as follows:

- Generation of classical individuals observing quantum individuals using pdf $p_{ij}(x)$, cumulative probabilities P_{ij} and a random generator $\mathbf{U}(0,1)$ doing the next procedure:
  1: **Generate** $r \sim \mathbf{U}(x)$
  2: **Find** x so that

$$P_{ij}(x) = \int_{-\infty}^{\infty} p_{ij}(\tau)d\tau \tag{2}$$

$$x = P_{ij}^{-1}(r) \tag{3}$$

  3: **Assign** $x_{ij}(t) \in \mathbf{x}_i \leftarrow x$

C. Updating

- The QIEA-$\mathbb{R}$ can reduce or increase the search space of any quantum individual according to the fitness of classical population obtained during the observation process and using the "rule of 1/5" [7]. If less 20% of classical population from current generation has fitness better than previous generation, the gen width is narrowed, if is greater than 20% the width is broaded and if it is equal to 20% the width is not modified, Eq.4, shows the 1/5 rule.

$$\sigma_{ij} = \begin{cases} \sigma\delta & \varphi < 1/5 \\ \sigma_{ij}/\delta & \varphi > 1/5 \\ \sigma_{ij} & \varphi = 1/5 \end{cases} \tag{4}$$

- Choose classical individuals to update pulses of quantum population. Every quantum individual related to quantum gen must be modified.
  By instance, suppose the center of quantum gen in generation t is $\mu_{ij}(t)$ and the value of classical gen is x_{ij} then a new position of quantum gen is calculated for generation $t+1$ by the Eq.5:

$$\mu_{ij}(t+1) = \mu_{ij}(t) + \lambda(x_{ij} + \mu_{ij}) \tag{5}$$

  where $\lambda \in$ [0,1] is the percentage of movement to the direction of classical gen.

III. Experiments and Results

In order to evaluate QIEA-$\mathbb{R}$, some experiments were performed under the following benchmark functions: Ackley, Rastrigin, Rosenbrock, Schwefel and Sphere. One feature of these benchmark functions is that can be configured for $n > 2$ dimensional variables. Thus, in this work, the applied dimensionality was $n = 30$.

To minimize the effect of randomness inherent in this type of algorithm, 1000 experiments for each parametrization were conducted. The curves shown in the results are averaged. Thus, for each parametrization is possible to display behaviors more approximate to the expected value.

Table I shows the parameters that were used for experiments using QIEA-$\mathbb{R}$ model.

TABLE I
Parameters used on experiments

Parameter	Acronym
Quantum population	QP
Dimension	N
Iterations	IQ
Classical population	CI
Update frequency	UF

In this work, instead use fixed parameters like DaCruz [5] experiments, a domain of parameters is defined. For example: If the parameter QP (Quantum population) for Ackley function is 5, we define a domain as [5, 10, …,50] with an increment of 5.

The experiments were performed using multi-threading implementation considering the 8 cores from an Intel i7 architecture.

Each of the figures shows, for each used benchmark function [8], the obtained behavior for the QIEA-$\mathbb{R}$ model for each value in parameter range. Following sections show the obtained results.

A. Function description

- *Ackley:* This function is characterized by a nearly flat outer region and a large hole at the center. Ackley function poses a risk for optimization algorithms, particularly hill climbing based algorithms, which can be trapped in one of its local optima.
- *Rastrigin:* The Rastrigin function has several local minima. It is highly multi modal, but locations of the minimal are regularly distributed.
- *Rosenbrock:* This function is unimodal, and the global minimum lies in a narrow, parabolic valley. However, even though this valley is easy to find, convergence to the minimum is difficult.
- *Schwefel:* The Schwefel function is very complex, with many local minima.
- *Sphere:* It is continuous, convex and unimodal function.

B. Basic parameters

The experiments performed in [5] uses the configuration of Table II:

TABLE II
Parameters used on experiments

Function	QP	D	IQ	CP	UF
Ackley Rastrigin Sphere	5	30	25	100	1
Rosenbrock Schweffel	5	30	25	100	10

C. Averaged curves of experiments

The next figures show the results of 1000 experiments averaged of every benchmark function to notice the convergence using the domains for each parameter of the Table III.

TABLE III
Domains for each parameter

Parameter	Range
QP	$[5, 10, \ldots, 50]$
NE,CP	$[50, 55, \ldots, 100]$
UF	$[1, 2, \ldots, 10]$

1) Quantum population: The quantum population is related to domain, having a bigger quantum population can we have a better cover of the domain to have a better classical generation. In the next figures, the convergence is presented with size variation of quantum population.

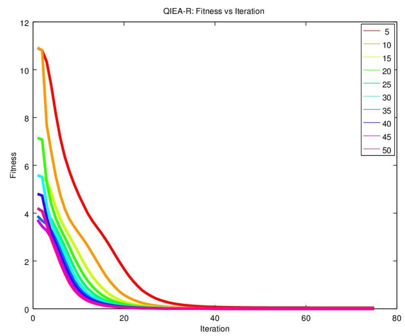

Fig. 1. Ackley

2) Number of experiments: The number of experiments is the number of iterations, the author in [5] suggested that 75 iterations was enough for the convergence. The next figures shows the influence of increasing number iterations in the algorithm.

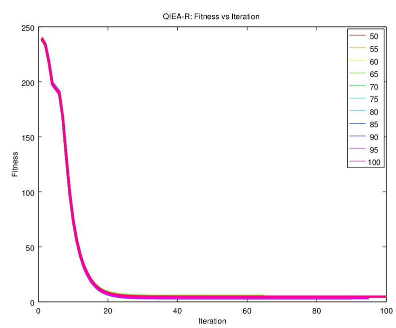

Fig. 2. Rastrigin.

3) Classical Population: The number of classical individuals generated from observing quantum individuals. The next images show the variation of convergence using different sizes of classical population.

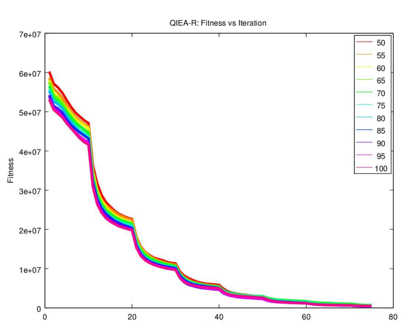

Fig. 3. Rosenbrock.

4) Update frequency: This step is related to the frequency of updating quantum population.

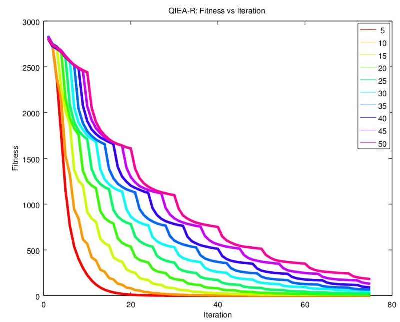

Fig. 4. Sphere

D. Statistical measurements

The minimal value(VM) and standard deviation(SD).

TABLE IV
RESULTS OF QUANTUM POPULATION FOR ACKLEY, RASTRIGIN

	Ackley		Rastrigin	
	MV	SD	MV	SD
5	5.72954E-005	0.373453	1.14003E-008	14.7026
25	3.04133E-006	0.000002312	7.41807E-011	2.58776
50	9.90937E-007	0.000000625	5.45697E-012	0.0347425

TABLE V
RESULTS OF NUMBER OF EXPERIMENTS FOR ROSENBROCK, SCHWEFFEL

	Rosenbrock		Schweffel	
	MV	SD	MV	SD
50	781718	3965850	10698.2	24.3844
75	80030.2	770143	10673.8	25.2063
100	14129.4	1877300	10666.4	23.685

TABLE VI
RESULTS OF CLASSICAL POPULATION OF SPHERE

	Sphere	
	MV	SD
50	6.165E-008	58.2244
75	9.13329E-008	16.6721
100	4.65481E-008	16.9427

TABLE VII
RESULTS OF UPDATE FREQUENCY FOR ACKLEY, RASTRIGIN

	Ackley		Rastrigin	
	MV	SD	MV	SD
1	5.89441E-005	0.444165	2.09245E-008	10.5379
2	0.00669825	0.416428	0.000309765	12.5382
3	0.100411	0.546638	0.0608351	12.3269
4	0.476041	0.842906	0.627045	14.3013
5	1.32403	0.780234	2.99819	13.9536
6	2.05591	0.69284	5.86599	13.5309
7	2.63816	0.621467	12.8228	12.0415
8	2.88146	0.821827	21.781	15.282
9	3.05356	0.619884	31.7256	14.0985
10	3.8226	0.555829	43.4965	13.7429

TABLE VIII
RESULTS OF UPDATE FREQUENCY FOR ROSENBROCK, SCHWEFFEL

	Rosenbrock		Schweffel	
	MV	SD	MV	SD
1	0.000057974	374685	10660.4	6.13395
2	0.432918	37039.4	10660.4	8.47867
3	39.8974	235202	10660.5	9.84271
4	141.855	509067	10660.6	16.8186
5	645.072	245661	10661.2	15.3371
6	2122.79	154580	10662.3	11.7036
7	8422.65	1616910	10664.7	23.7141
8	17451.1	2038670	10666.4	22.0281
9	44393	1827470	10669.2	22.3847
10	52508	2163400	10670.8	25.2073

TABLE IX
SPHERE

	Sphere	
	MV	SD
1	4.62541E-008	8.29122
2	0.000806941	34.6485
3	0.109334	98.3168
4	1.04979	94.0546
5	4.25553	123.046
6	13.5568	184.989
7	31.0921	151.858
8	43.3905	142.334
9	65.5283	143.724
10	89.9107	185.27

IV. DISCUSS ABOUT RESULTS

We used quantum population, number of experiments, classical population, update frequency to analyze the impact on the convergence of QIEA-$\mathbb{R}$. The Figures 1 and 4 show the great influence of the size of quantum population because during the step of movement they offer a better domain coverage, thus, using the best of this population then it impacts of the algorithm. The update frequency has a great influence because the square pulse is narrow or wider according to the condition of 1/5th-rule and it impacts in the next classical generations.

The algorithm has a fast convergence because it's following the best, figure 2 shows us there is no meaningful difference is the number of iterations is increased, it could be a problem in some applications. The impact of size of classical population shown in 3 is barely, it means the size of quantum population is more important because of classical individuals are using quantum individuals if quantum individuals are located in a good place of the domain the classical generation will be better.

Visually, we have analyzed the impact of the five parameters we used to experiment and it is found that the size of quantum population and update frequency have the greater influence in the whole algorithm of QIEA-$\mathbb{R}$.
Using statistical measurements like minimal value and standard deviation confirm the last affirmation, the impact of quantum population size is observed in the table IV, showing a better minimal and standard deviations. A size of 50 quantum individuals produces better results than just five. Regarding the number of experiments shows in the table V. The minimal value is improved with more experiments in Ackley, Rastrigin but it is not meaningful in Schweffel because of this function has many local optimals. A good number of experiments is 70. According to the table VI, the size of classical population could get better individuals. Using the results obtained modifying the parameter of update frequency is shown in the tables VII,VIII, IX and it confirms the great impact of this parameter. It is found the standard deviation can be change hugely because the update of the square pulse was delayed maintaining the classical individuals more disperse.

V. CONCLUSIONS

In this work, experiments were performed changing the following parameters: size of quantum population, update frequency have a greater impact to the QIEA-$\mathbb{R}$ algorithm than size of classical population and it is confirmed using statistical measurements and increasing number of iterations or classical population is not enough meaningful.

VI. FUTURE WORK

The analysis of QIEAR was performed only using benchmark functions, it is possible to expand the analysis to other kind of problems to analyze the behavior in these contexts.

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