COMS10013 - Resit Exam Questions

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Preface

NOTE(2025): This exam was done via Blackboard due to Covid, so the ‘pool’ system of questions is different to an in-person exam. The types of exam question are representative.

One question should be selected randomly from each of the following pools of questions.

1. (a) Consider the function $z(x, y) = \sin(x)\cos(e^{-y^2})$. Which of the following statements is true?
    - A. z has infinitely many local maxima.
    - B. z has infinitely many local minima.
    - C. z has infinitely many saddle points.
    - D. All of the above.

   (b) Consider the function $z(x, y) = \cos(x)\cos(e^{-y^2})$. Which of the following statements is true?
    - A. z has infinitely many local maxima.
    - B. z has infinitely many local minima.
    - C. z has infinitely many saddle points.
    - D. All of the above.

   (c) Consider the function $z(x, y) = \sin(y)\cos(e^{-x^2})$. Which of the following statements is true?
    - A. z has infinitely many local maxima.
    - B. z has infinitely many local minima.
    - C. z has infinitely many saddle points.
    - D. All of the above.

   (d) Consider the function $z(x, y) = \cos(y)\cos(e^{-x^2})$. Which of the following statements is true?
    - A. z has infinitely many local maxima.
    - B. z has infinitely many local minima.
    - C. z has infinitely many saddle points.
    - D. All of the above.

2. (a) The function $y(t)$ satisfies $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 0$, $y(0) = 0$, $y(\ln(2)) = -2$.
Compute $y(\ln(3))$. Please write your answer as an integer, with no full stop at the end.

(b) The function $y(t)$ satisfies $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = 0$, $y(0) = 0$, $y(\ln(2)) = -12$.
Compute $y(\ln(3))$. Please write your answer as an integer, with no full stop at the end.

(c) The function $y(t)$ satisfies $\frac{d^2y}{dt^2} - 9\frac{dy}{dt} + 18y = 0$, $y(0) = 0$, $y(\ln(2)) = -56$.
Compute $y(\ln(3))$. Please write your answer as an integer, with no full stop at the end.

(d) The function $y(t)$ satisfies $\frac{d^2y}{dt^2} - 12\frac{dy}{dt} + 32y = 0$, $y(0) = 0$, $y(\ln(2)) = -240$.
Compute $y(\ln(3))$. Please write your answer as an integer, with no full stop at the end.

3. (a) How many solutions are there to the equation $\bar{z} = z^2$ in the complex numbers? Please write your answer as an integer, with no full stop at the end.

   (b) How many solutions are there to the equation $\bar{z} = z^3$ in the complex numbers? Please write your answer as an integer, with no full stop at the end.

   (c) How many solutions are there to the equation $\bar{z} = z^4$ in the complex numbers? Please write your answer as an integer, with no full stop at the end.

   (d) How many solutions are there to the equation $\bar{z} = z^5$ in the complex numbers? Please write your answer as an integer, with no full stop at the end.

4. (a) Let $f(x) = \frac{2}{(1-x)^2}$. The Taylor series for $f(x)$ around $x_0 = 0$ is of the form $\sum_{n=0}^{\infty} a_n x^n$ for some sequence a_n. By computing a formula for a_n in terms of n, or otherwise, compute a_{10}. Please write your answer as an integer, with no full stop at the end.

   (b) Let $f(x) = \frac{3}{(1-x)^2}$. The Taylor series for $f(x)$ around $x_0 = 0$ is of the form $\sum_{n=0}^{\infty} a_n x^n$ for some sequence a_n. Compute a formula for a_n, in terms of n and hence compute a_{10}. Please write your answer as an integer, with no full stop at the end.

   (c) Let $f(x) = \frac{4}{(1-x)^2}$. The Taylor series for $f(x)$ around $x_0 = 0$ is of the form $\sum_{n=0}^{\infty} a_n x^n$ for some sequence a_n. Compute a formula for a_n, in terms of n and hence compute a_{10}. Please write your answer as an integer, with no full stop at the end.

   (d) Let $f(x) = \frac{5}{(1-x)^2}$. The Taylor series for $f(x)$ around $x_0 = 0$ is of the form $\sum_{n=0}^{\infty} a_n x^n$ for some sequence a_n. Compute a formula for a_n, in terms of n and hence compute a_{10}. Please write your answer as an integer, with no full stop at the end.

5. (a) Consider the differential equation $\frac{dy}{dx} + 10y^2 = e^x$. Which of the following correctly describes this differential equation?

   A. First-order, linear, inhomogeneous, with constant coefficients
   B. First-order, non-linear, inhomogeneous, with constant coefficients
   C. First-order, non-linear, homogeneous, with constant coefficients
   D. First-order, non-linear, inhomogeneous, with non-constant coefficients

   (b) Consider the differential equation $\left(\frac{dy}{dx}\right)^2 + 10y = e^x$. Which of the following correctly describes this differential equation?

   A. First-order, linear, inhomogeneous, with constant coefficients
   B. First-order, non-linear, inhomogeneous, with constant coefficients
   C. First-order, non-linear, homogeneous, with constant coefficients
   D. First-order, non-linear, inhomogeneous, with non-constant coefficients

   (c) Consider the differential equation $\frac{dy}{dx} + 10xy = e^x$. Which of the following correctly describes this differential equation?

   A. First-order, non-linear, inhomogeneous, with non-constant coefficients
   B. First-order, linear, inhomogeneous, with non-constant coefficients
   C. First-order, linear, homogeneous, with non-constant coefficients
   D. First-order, linear, inhomogeneous, with constant coefficients

   (d) Consider the differential equation $x\frac{dy}{dx} + 10y = e^x$. Which of the following correctly describes this differential equation?

   A. First-order, non-linear, inhomogeneous, with non-constant coefficients
   B. First-order, linear, inhomogeneous, with non-constant coefficients
   C. First-order, linear, homogeneous, with non-constant coefficients
   D. First-order, linear, inhomogeneous, with constant coefficients