Mathematics for Computer Science B: Analysis

LECTURE 1

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What’s in this section of the course

1. Differentiation
2. Partial differentiation
3. Solving differential equations
4. Optimisation
5. Taylor series and numerical solutions
6. Complex numbers and modular arithmetic

I expect that you've already seen…

Differentiation (with one variable)

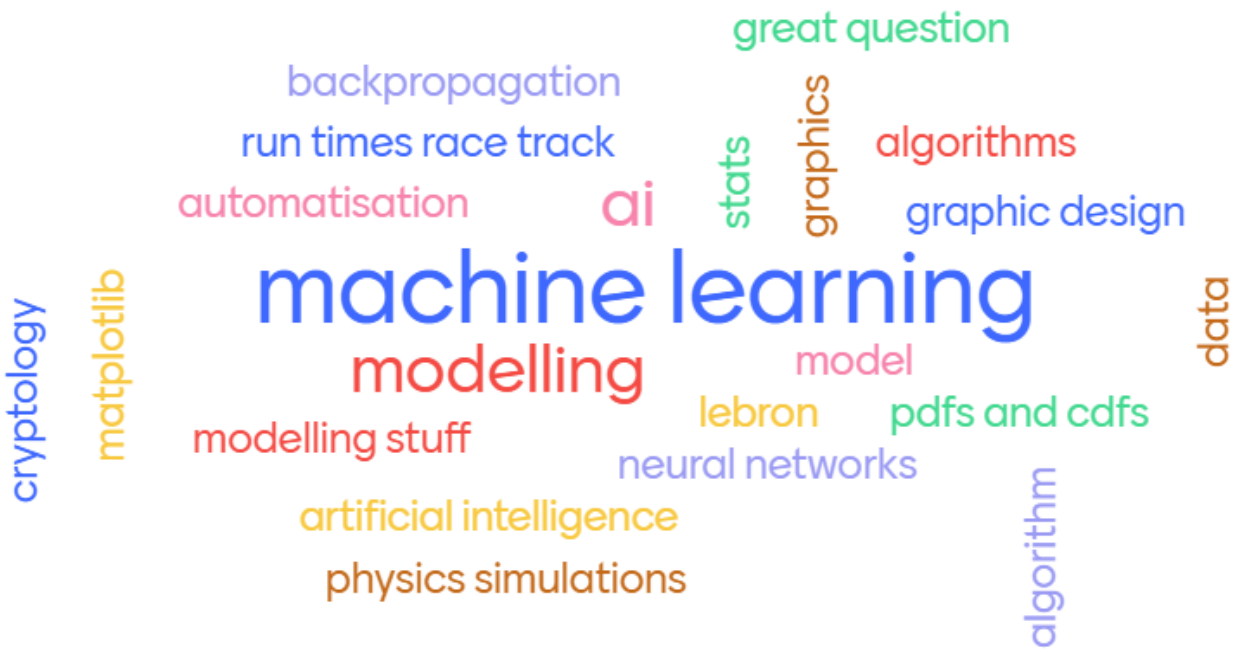
Integration

Taylor series (?)

Applications of calculus in Computer Science

Activity:
Where do you think you'll use this section of the course throughout your degree?

Applications of calculus in Computer Science



Board work: differentiation

In which we derive the formula for the derivative

Rules for differentiation

- Sums:

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

- Multiplication by a constant:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

- Products :

$$\frac{d}{dx}f(x)g(x) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

Derivatives of common functions

Polynomials:

$$\frac{d}{dx}x^n = nx^{n-1}$$

Trigonometric functions:

$$\frac{d}{dx}\sin(x) = \cos(x)$$

Exponential function:

$$\frac{d}{dx}e^x = e^x$$

Logarithmic function:

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

The Chain Rule

If $y = f(u)$ and u $= g(x)$ then

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{dg}{dx}$$

Example: [board]

Notation

Leibniz : $\frac{dy}{dx}$

Lagrange: $y'(x)$

Newton: $\dot{y}(x)$

Notation: differentiate at a point

Leibniz : $\frac{dy}{dx}|_{x=a}$

Lagrange: $y'(a)$

Newton: $\dot{y}(a)$

Summary

- Differentiation from infinitesimals perspective
- Rules for differentiation…
- … especially the Chain Rule
- Derivatives of common functions