Question 1 (5 marks)

This question is about differentiation.

(i) In one sentence explain the infinitesimal approach to differentiation and use it to show that $\frac{d(x^3)}{dx} = 3x^2$

(ii) Using the chain rule, product rule and $d\sin x/dx = \cos x$, differentiate: $f(x) = x\sin x^2$

(5 marks)

Question 2 (5 marks)

This question is about partial derivatives.

(i) Write down the definition of $\partial f(x,y)/\partial x$ and of ∇f.

(ii) If $f(x,y) = x\sin xy + y^2$, what is ∇f and what is the derivative along a unit vector in the $(1,1)$ direction?

(5 marks)

Question 3 (5 marks)

This question is about complex numbers.

(i) Write $z = \frac{1+2i}{2+i}$ in the form $x + iy$.

(ii) Write $z = 1 + \sqrt{3}i$ in the polar form.

(iii) The quaternions are a type of generalization of complex numbers. Instead of just i there are three imaginary numbers i, j and k, and these all square to minus one: $i^2 = j^2 = k^2 = -1$. In addition $ijk = -1$ and the numbers are *anti-commutative*: $ij = -ji$, $jk = -kj$ and so on. Lots of other relationships can be derived from these rules, for example if you multiply $ijk = -1$ you get $jk = i$, or if you switch it $jik = 1$ and multiply by j you get $ik = 1$. If $z = 1 + 3i + 2j$, what are zi and iz?

(5 marks)

Question 4 (5 marks)

This question is about differential equations.

(i) In one sentence explain the difference between a homogeneous and an inhomogeneous differential equation.

(ii) Solve $\frac{df}{dt} = 3f$ with $f(0) = 4$.

(iii) Solve $\frac{df}{dt} = 3f + t$ with $f(0) = 1$.

(5 marks)

Question 5 (5 marks)

This question is about optimization. Briefly describe gradient flow.

(5 marks)

END OF PAPER