COMS10013 - Analysis - WS5

Questions

These questions are partially taken from worksheets created by Conor Houghton and Chloe Martindale.

These are the questions you should make sure you work on in the workshop.

1. **Linear Approximation** The value of the function $\sin(x)$ is typically difficult to calculate. But let's suppose you urgently need to find the value of $\sin(0.1)$ but - catastrophe! - you've forgotten your calculator.
   Use the method of linear approximation of $\sin(x)$ at 0 to estimate $\sin(0.1)$.
   Assume that you're seeking $\sin(0.1)$ in radians.

2. **Root finding I:**
   (a) Pick an initial value and use the Newton root finding method to find the root of the equation
   $$f(x) = x^3 - 5\,,$$
   using say five iterations of the algorithm.
   (b) Compare your solution with $\sqrt[3]{5}$.

3. **Root finding II:**
   (a) Pick an initial value and use the Newton root finding method to find the root of the equation
   $$f(x) = \sin(x)x^3 + \cos(x)\,,$$
   using say five iterations of the algorithm.
   (b) Evaluate f at your root guess. Was your initial value good?

4. **Taylor series**
   (a) Compute the Taylor series of $1/(1-x)^2$ at $x = 0$.
   (b) Compute the Taylor series of $1/x$ at $x = 2$.

5. **Computing with Taylor series**.
   This exercise is to approximate $\sin(\pi/4)$ without using any trigonometric functions on your calculator. Compute, without a calculator, the Taylor series of $\sin(x)$ at $x = 0$. Compute the approximations $T_1(x)$, $T_3(x)$, $T_5(x)$, $T_7(x)$ from your series at $x = \pi/4$ to eight decimal places (you can use a calculator). (You can check how accurate your approximations are by plugging in $\sin(\pi/4)$ to your calculator and comparing your answer.)