

Section 2: Analysis

In this section on Analysis, use the Section 2 Answer Sheet for your answers. There are 8 questions. For multiple choice questions there is only one correct answer.

Question 1 (3 marks)

Determine which of the following is the derivative of $\frac{\sin x + x}{2 + \cos x}$. Enter your answer by crossing exactly one of the boxes on your answer sheet.

- a. $\frac{\cos x + 1}{(2 - \sin x)^2}$
- b. $\frac{(\cos x + 1)(2 + \cos x) - (\sin x + x) \sin x}{(2 - \sin x)^2}$
- c. $\frac{\cos x + 1}{-\sin x}$
- d. $\frac{\sin x + x}{-\sin x}$
- e. $\frac{(\cos x + 1)(2 + \cos x) + (\sin x + x) \sin x}{(2 + \cos x)^2}$

(3 marks)

Question 2 (2 marks)

If $f'(2) = 5$, $g(4) = 2$, $g(2) = 1$, $f(2) = -1$ and $g'(4) = 3$, determine the value of $\frac{d}{dx}(f(g(x)))$ at the point $x = 4$. The answer is an integer between -99 and 99. Enter your answer according to the instructions.

(2 marks)

Question 3 (2 marks)

If $f(x, y) = \cos\left(\frac{x}{y}\right)$ determine which of the following is ∇f . Enter your answer by crossing exactly one of the boxes on your answer sheet.

- a. $\left(-\frac{1}{y} \sin \frac{x}{y}, \frac{x}{y^2} \sin \frac{x}{y}\right)$
- b. $\left(-\sin \frac{x}{y}, \frac{1}{y^2} \sin \frac{x}{y}\right)$
- c. $\left(\sin \frac{x}{y}, -\frac{1}{y^2} \sin \frac{x}{y}\right)$
- d. $\left(-\frac{x}{y} \sin \frac{x}{y}, \frac{x}{y^2} \sin \frac{x}{y}\right)$
- e. $\left(\frac{1}{y} \sin \frac{x}{y}, -\frac{x}{y^2} \sin \frac{x}{y}\right)$

(2 marks)

Question 4 (3 marks)

Determine the derivative of $f(x, y)$ in the $(3, -4)$ direction at the point $(\pi, 2)$, rounded to three decimal places. The answer is a real number between -1 and 1. Enter your answer according to the instructions.

(3 marks)

Question 5 (5 marks)

Consider the pair of complex numbers $z_1 = 10 + 8i$ and $z_2 = \frac{3-(62/3)i}{1-(4/3)i}$. Determine the answers to the following questions. Enter your answer for each question by crossing exactly one box, labelled z_1 or z_2 , on the answer sheet. The allocation of marks per question is indicated in brackets.

- A. Which is closest to the origin? (1 mark)
- B. Which has rotated by the least amount in absolute terms? (2 marks)
- C. Which is closest to $z_3 = 1 + i$? (2 marks)

(5 marks)

Question 6 (2 marks)

For the 1st order ordinary differential equations (ODE) 1-5 below, classify them according to the following terms:

- A. Order, a natural number.
- B. Linear or non-linear.
- C. Homogeneous or non-homogeneous.
- D. Has constant coefficients or does not have constant coefficients.

1. $y' - e^x y + 7 = 0$
2. $y' - e^x y = 0$
3. $3y'' - 2y' = 3$
4. $2\frac{d^3y}{dx^3} + \cos x \frac{dy}{dx} = 0$
5. $2\frac{d^5y}{dx^5} + \cos y \frac{d^3y}{dx^3} = 0$

Enter your answers as follows. For each of the 5 equations, cross exactly one of the boxes in the first table, in the column representing the order of the equation. Then, cross all that apply in the second table, where L stands for linear, H stands for homogeneous, and C stands for constant coefficients.

(2 marks)

Question 7 (5 marks)

The function $y(t)$ satisfies the 2nd order ODE $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 2 = 0$ with $y(0) = 1$ and $y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}e^{\frac{\pi}{4}}$. Given that $\left(y\left(\frac{\pi}{3}\right)\right)^3 = \frac{1}{A}e^{B\pi}$, determine the values of A and B . The answers are integers between -99 and 99. Enter your answers according to the instructions.

(5 marks)

END OF QUESTIONS