COMS30030
Image Processing and Computer Vision

Stereo 2 – Epipolar Geometry

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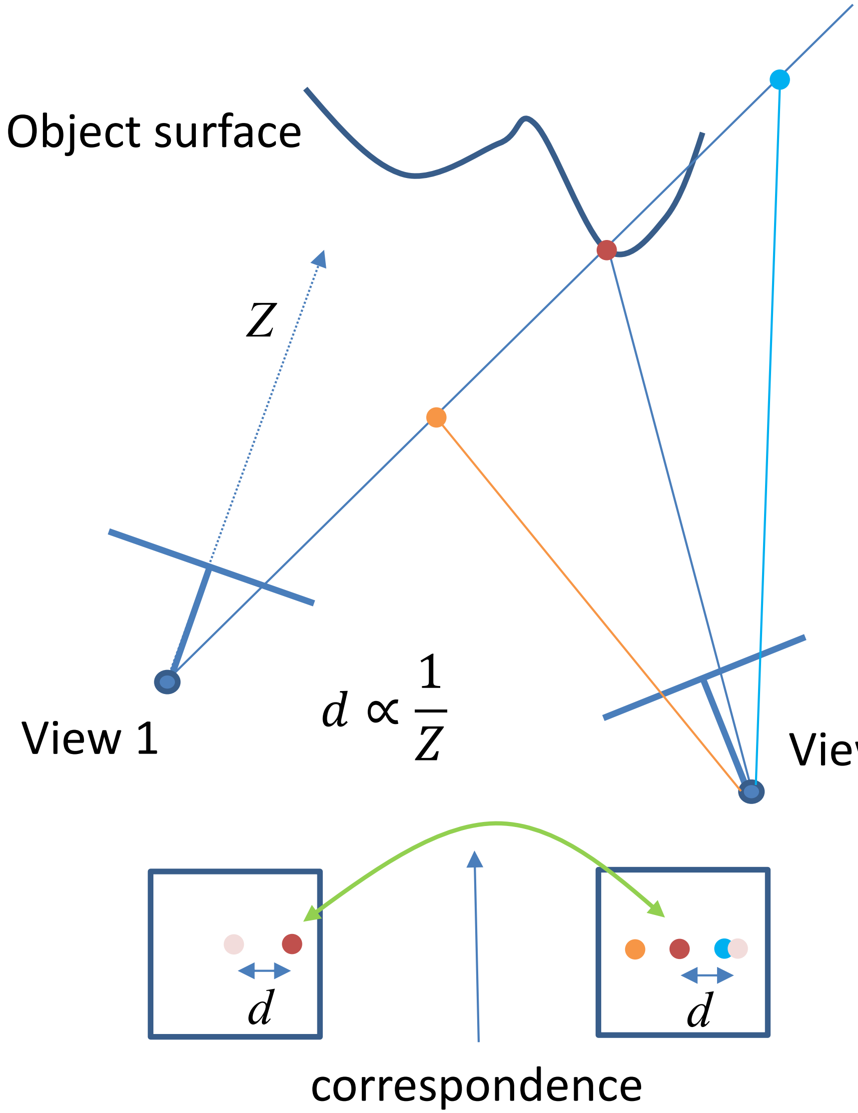
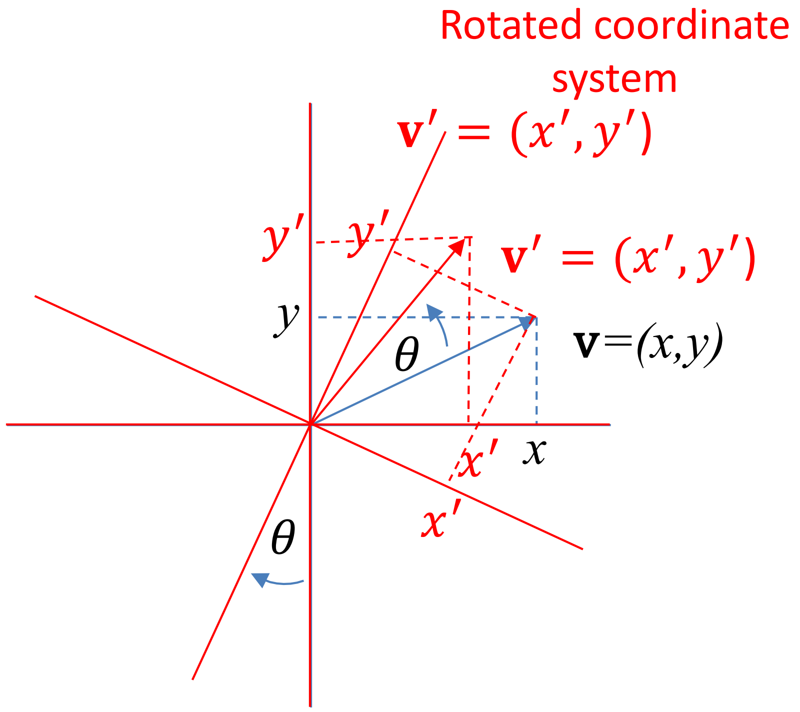
Stereo Computer Vision



Two-view stereo

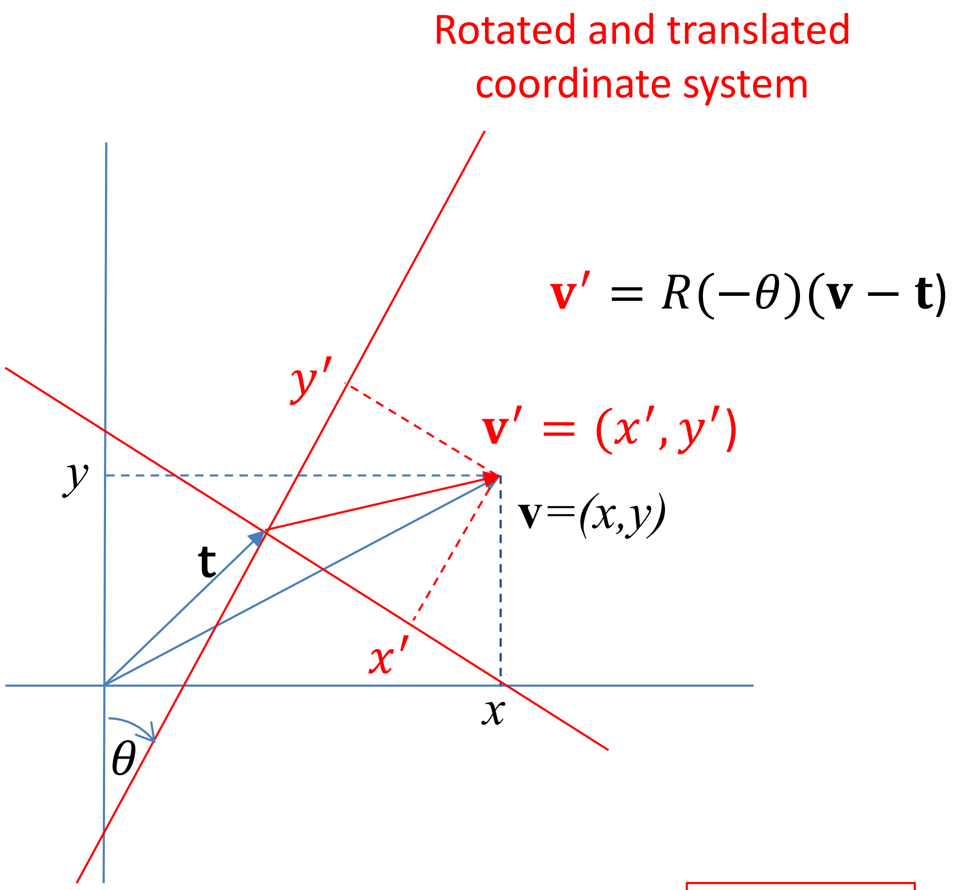
previous lecture

2-D Coordinate Transformations



$$\mathbf{v}' = R(-\theta)\mathbf{v}$$

Vector representation in rotated coordinate system

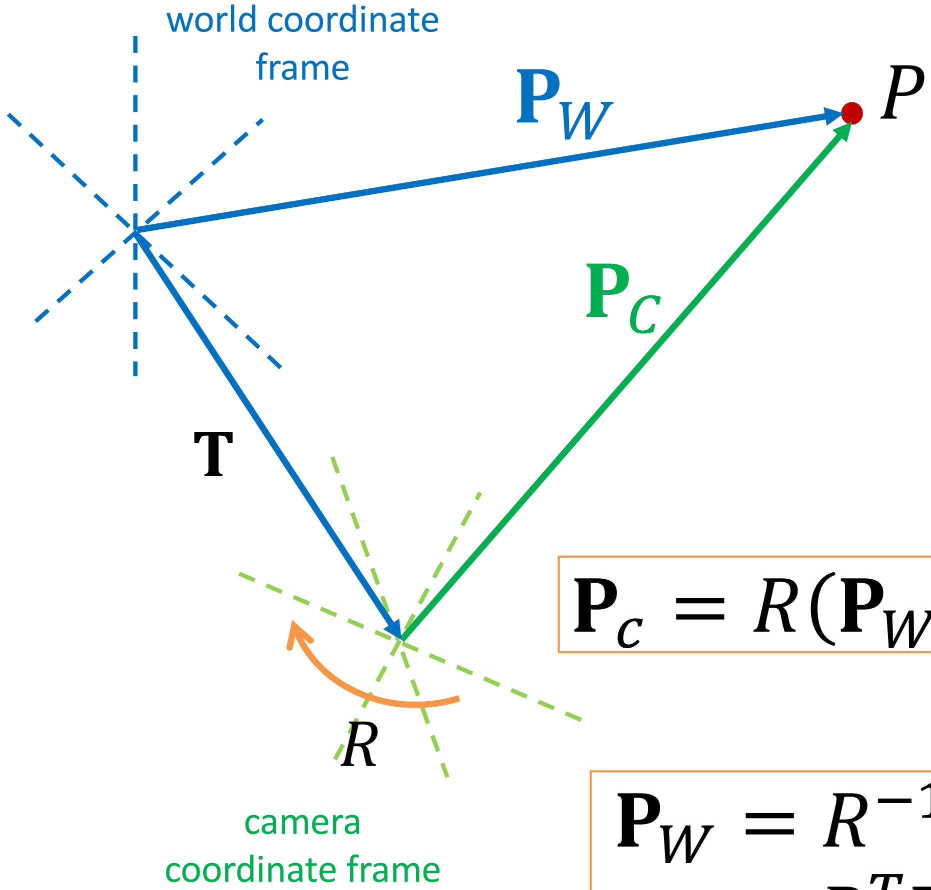


previous lecture

3-D Coordinate Transformations

$\mathbf{P}_W$ Vector defining P in world coordinates

$\mathbf{P}_C$ Vector defining P in camera coordinates



$\mathbf{T}$: 3-D camera position vector

R : 3-D camera rotation matrix

R defines rotation to be applied to camera coordinate system to align it with world coordinate system

$$\mathbf{P}_c = R(\mathbf{P}_W - \mathbf{T})$$

$$\begin{aligned}\mathbf{P}_W &= R^{-1}\mathbf{P}_C + \mathbf{T} \\ &= R^T\mathbf{P}_C + \mathbf{T}\end{aligned}$$

Rotation matrices: $R^T = R^{-1}$

Homogeneous Coordinates

- Homogeneous coordinates allow coordinate transformations to be defined by 4x4 matrices:

$$\mathbf{P'}_W = \begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{P}_W \\ 1 \end{bmatrix} = \begin{bmatrix} R^T & \mathbf{T} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{P}_C \\ 1 \end{bmatrix} = H_{CW}\mathbf{P'}_c \Rightarrow \boxed{\mathbf{P}_W = R^T\mathbf{P}_C + \mathbf{T}}$$

$$H_{CW} = \begin{bmatrix} R_{00} & R_{10} & R_{20} & T_x \\ R_{01} & R_{11} & R_{21} & T_y \\ R_{02} & R_{12} & R_{22} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{P'}_C = H_{CW}^{-1}\mathbf{P'}_W = H_{WC}\mathbf{P'}_W$$

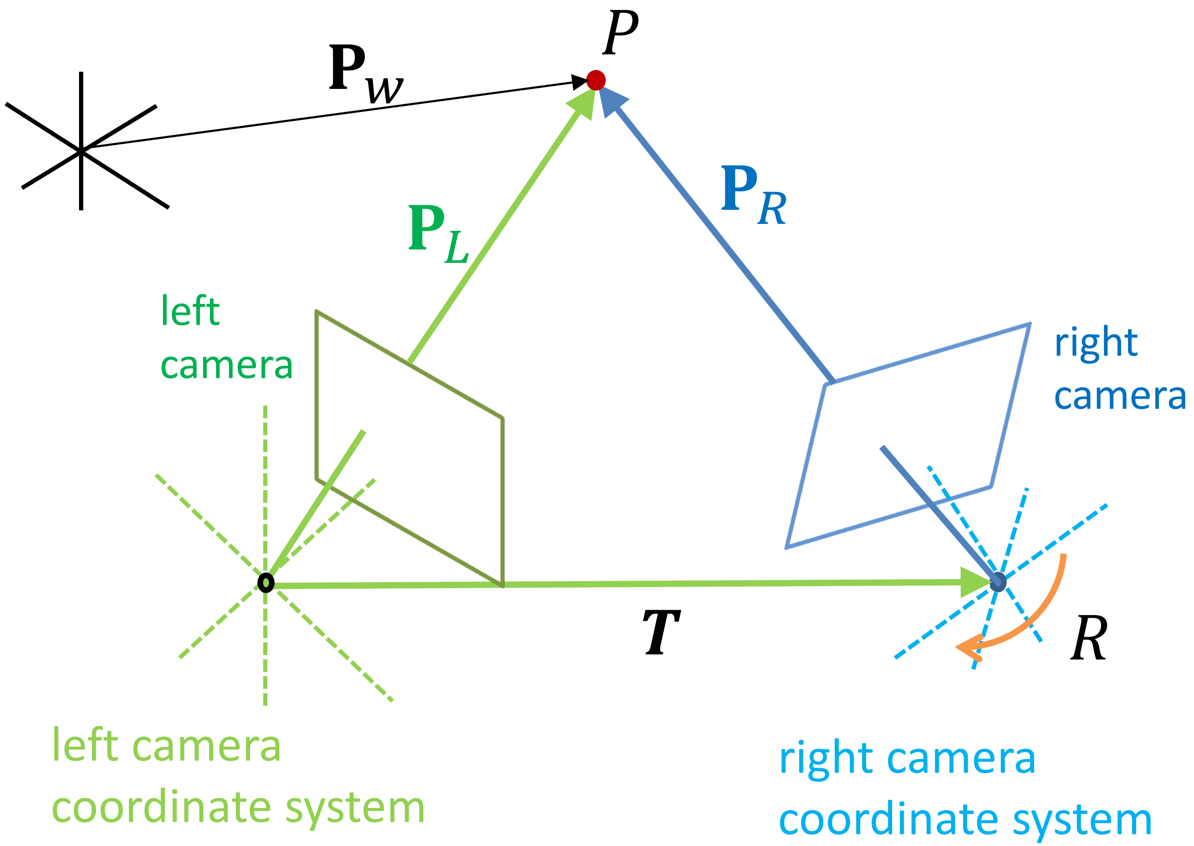
$$H_{WC} = \begin{bmatrix} R & -R\mathbf{T} \\ \mathbf{0} & 1 \end{bmatrix} \Rightarrow H_{WC}H_{CW} = I$$

I - Identity matrix

$$\Rightarrow \boxed{\mathbf{P}_C = R\ (\mathbf{P}_W - \mathbf{T})}$$

H_{cw} : camera to world coordinate transformation matrix

Stereo Coordinate Systems



$$\mathbf{P}'_L = H_{WL}\mathbf{P}'_W$$

$$\mathbf{P}'_R = H_{WR}\mathbf{P}'_W$$

$$\mathbf{P}'_L = H_{WL}H_{WR}^{-1}\mathbf{P}'_R$$

$$\mathbf{P}'_L = H_{RL}\mathbf{P}'_R$$

$$\boxed{H_{RL} = \begin{bmatrix} R^T & \mathbf{T} \\ \mathbf{0} & 1 \end{bmatrix}}$$

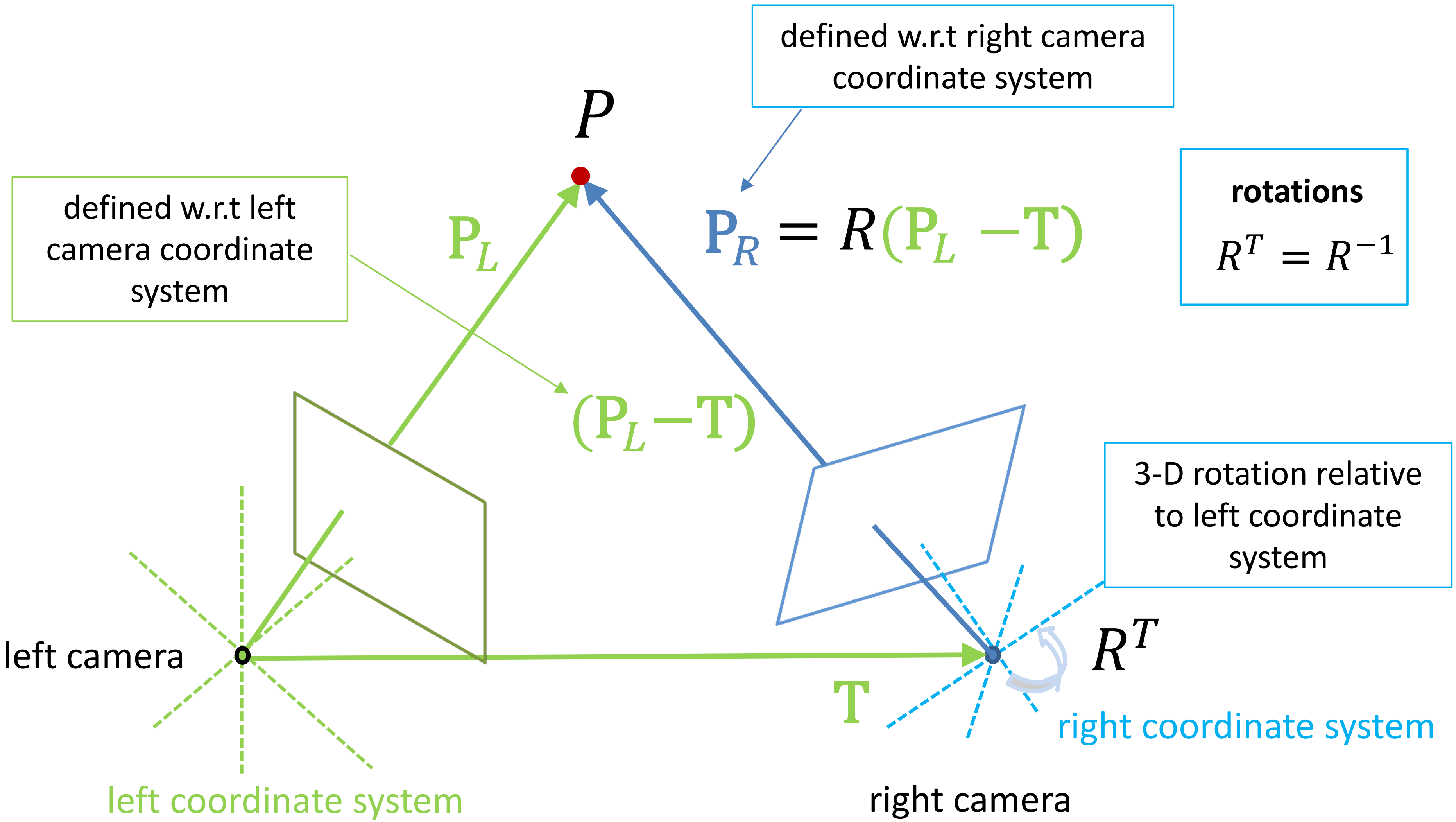
$$\boxed{\mathbf{P}_L = R^T\mathbf{P}_R + \mathbf{T}}$$

$$\boxed{\mathbf{P}_R = R(\mathbf{P}_L - \mathbf{T})}$$

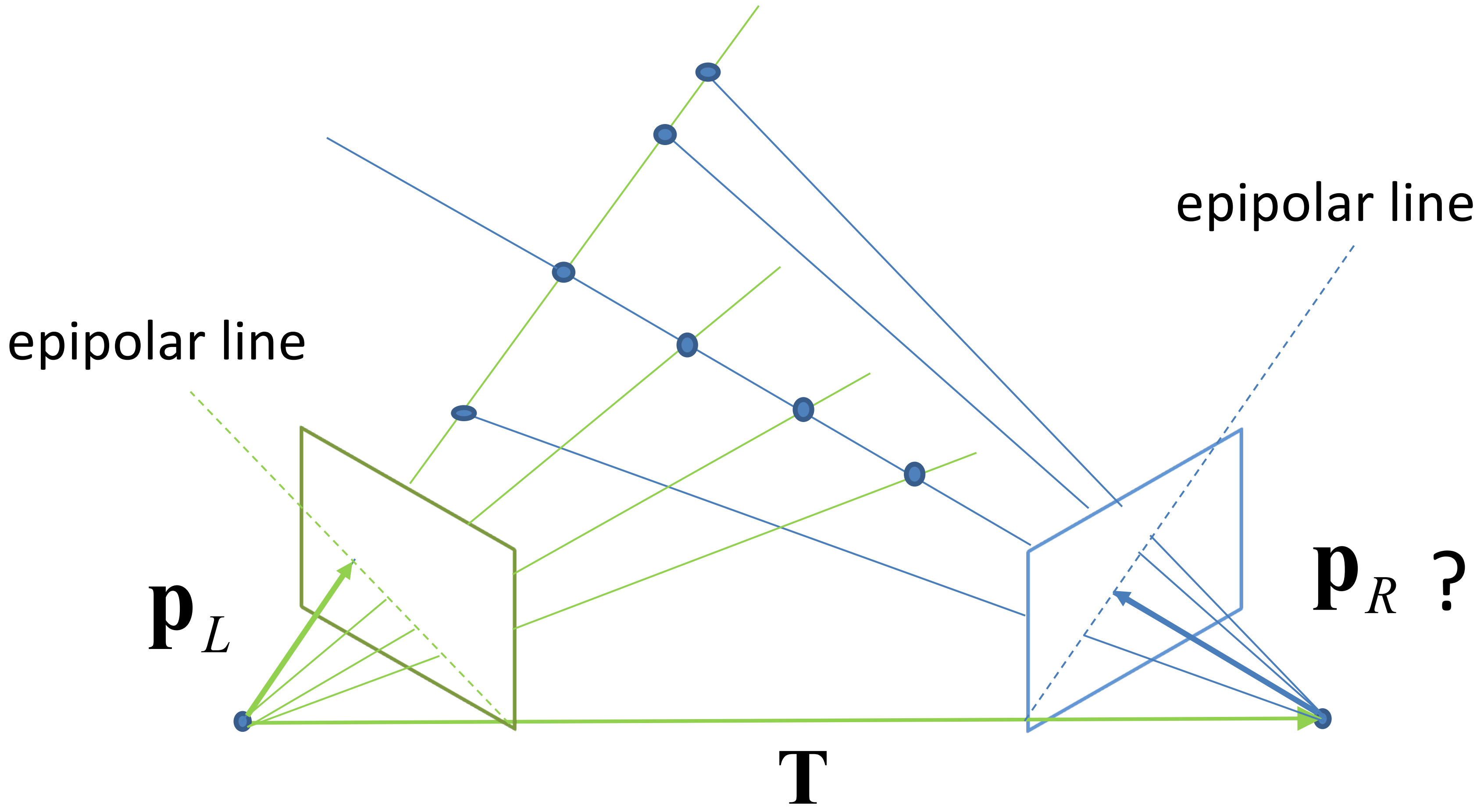
R defines rotation to be applied to right camera coordinate system to align it with left coordinate system

General Two-View Stereo

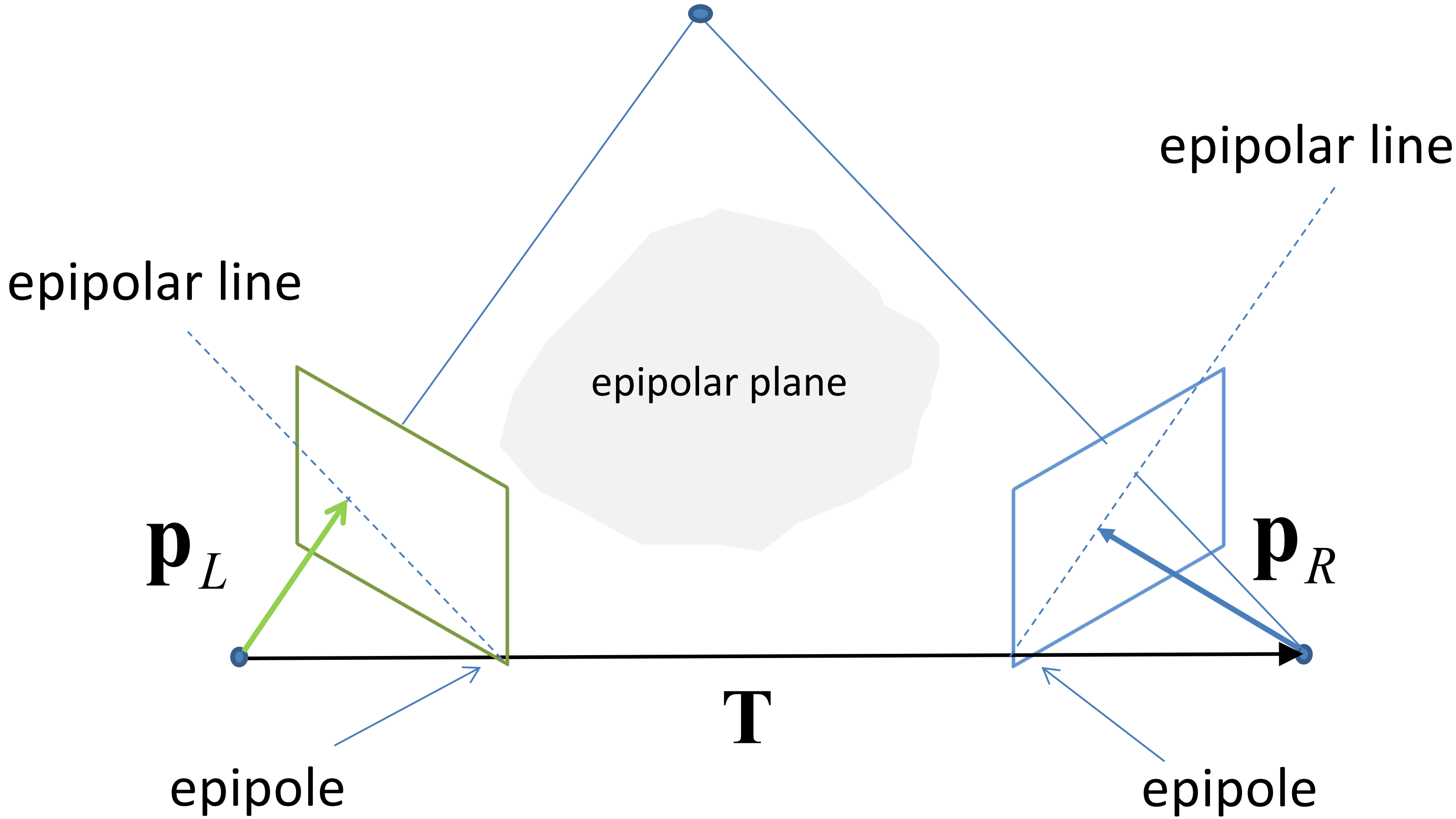
previous lecture



Epipolar Lines



Epipolar Planes



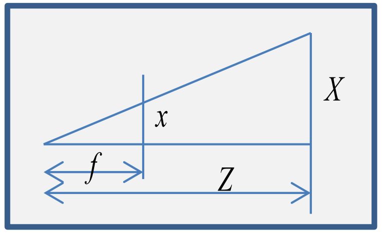
Epipolar Geometry

- Epipolar geometry defines relationship between two stereo views
- For known viewpoints:
  - it constrains matches to lie along epipolar lines
- For unknown viewpoints:
  - given matching points ....
  - it enables estimation of viewpoints

Epipolar Geometry - Maths

Rigid transformation between cameras:

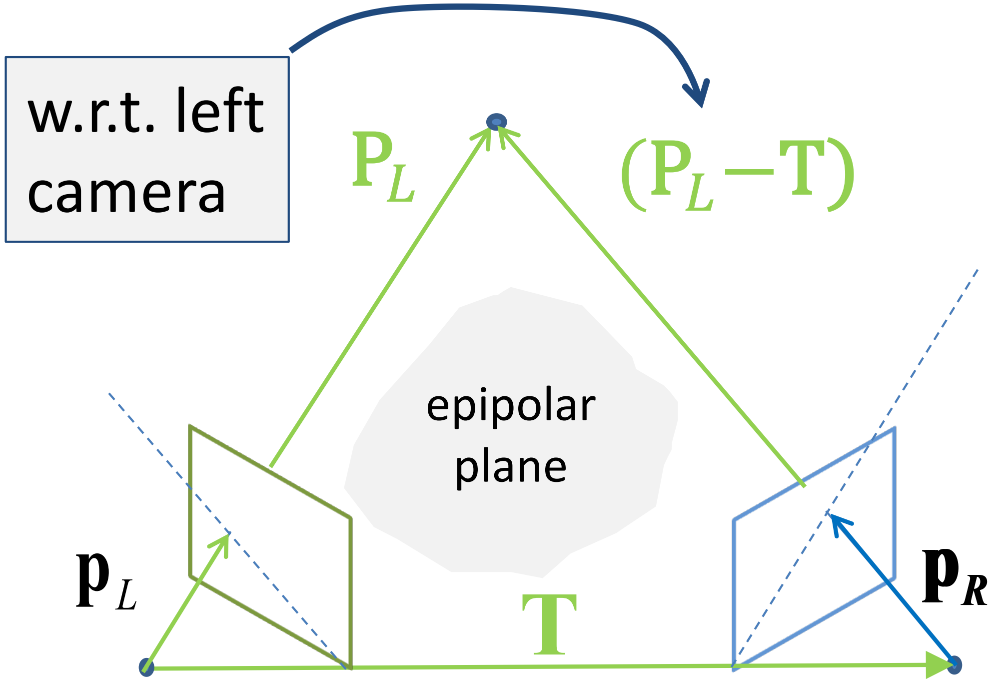
$$\mathbf{P}_R = R(\mathbf{P}_L - \mathbf{T})$$



Perspective projection:

$$\mathbf{P}_L = \begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} \qquad \mathbf{p}_L = \begin{bmatrix} x_L \\ y_L \\ f \end{bmatrix} = \frac{f\mathbf{P}_L}{Z_L} \qquad \mathbf{p}_R = \begin{bmatrix} x_R \\ y_R \\ f \end{bmatrix} = \frac{f\mathbf{P}_R}{Z_R}$$

Epipolar Geometry - Maths



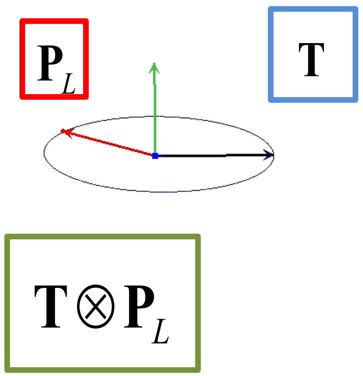
Vectors $\mathbf{P}_L$, $\mathbf{T}$
and $\mathbf{P}_L - \mathbf{T}$
all lie in epipolar plane

in plane

perpendicular to plane

$$\underbrace{(\mathbf{P}_L - \mathbf{T})^T}\underbrace{(\mathbf{T} \otimes \mathbf{P}_L)} = 0$$

$\otimes$ cross product :



dot product = 0 for **perpendicular** vectors

Epipolar Geometry - Maths

$$(\mathbf{P}_L - \mathbf{T})^T (\mathbf{T} \otimes \mathbf{P}_L) = 0$$

w.r.t. right camera

$$\mathbf{P}_R = R(\mathbf{P}_L - \mathbf{T})$$

$$R^T = R^{-1}$$ Rotation matrix

$$\Downarrow$$

$$R^T \mathbf{P}_R = (\mathbf{P}_L - \mathbf{T})$$

$$\mathbf{P}_R^T R = (\mathbf{P}_L - \mathbf{T})^T$$

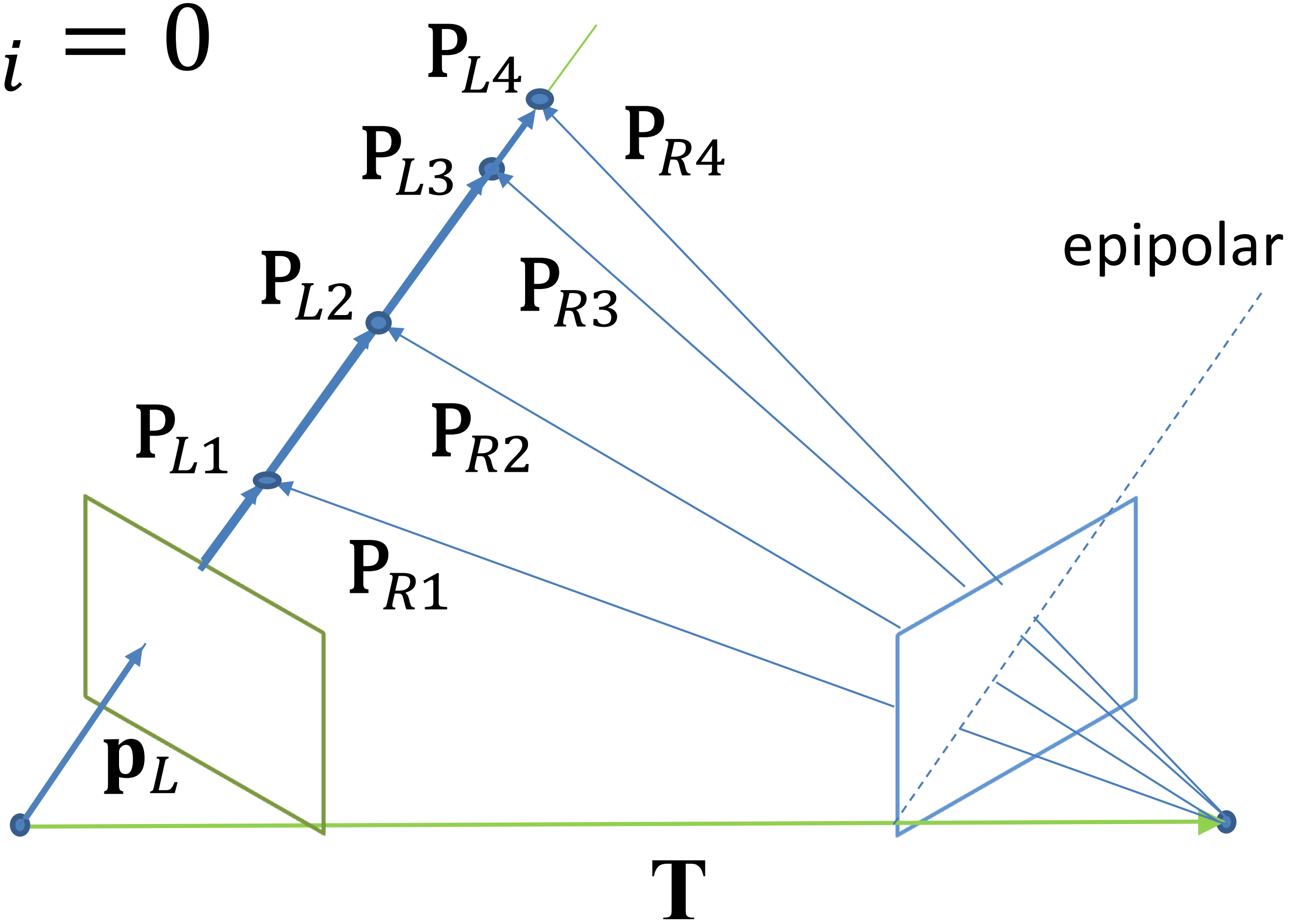
$$(\mathbf{T} \otimes \mathbf{P}_L) = S\,\mathbf{P}_L$$

$$S = \begin{bmatrix} 0 & -T_Z & T_Y \\ T_Z & 0 & -T_X \\ -T_Y & T_X & 0 \end{bmatrix}$$

$$\mathbf{P}_R^T \, R \, S \, \mathbf{P}_L = 0$$

Epipolar Line Constraint

$$\mathbf{P}_{Ri}^{T} R S \mathbf{P}_{Li} = 0$$



The Essential Matrix

$$\mathbf{P}_R^T \underbrace{R\,S}\,\mathbf{P}_L = 0 \quad \Rightarrow \quad \mathbf{P}_R^T E\,\mathbf{P}_L = 0$$

$$E = R\,S \quad \Rightarrow$$ the essential matrix

$$\boxed{\mathbf{p}_L = \frac{f\mathbf{P}_L}{Z_L} \qquad \mathbf{p}_R = \frac{f\mathbf{P}_R}{Z_R}}$$

$$\Downarrow$$

$$\boxed{\mathbf{P}_L = \frac{Z_L\mathbf{p}_L}{f} \qquad \mathbf{P}_R = \frac{Z_R\mathbf{p}_R}{f}}$$

$$\Rightarrow \quad \frac{\cancel{Z_R}}{\cancel{f}}\mathbf{p}_R^T E \frac{\cancel{Z_L}}{\cancel{f}}\mathbf{p}_L = 0$$

$$\Rightarrow \quad \mathbf{p}_R^T E\,\mathbf{p}_L = 0$$