COMS30030: Image Processing and Computer Vision

Stereo Problem Sheet

1. In a two view stereo system, points in the right image that may correspond to a point in the left image all lie along an epipolar line. Give two reasons why this is beneficial in a practical stereo system and state what information is needed to use it.

2. In a two view stereo system, the two image planes are coplanar and the centre of projection (COP) of the right camera is defined by the vector $\mathbf{T} = (2, 0, 0)$ in the left camera coordinate system. The focal length of each camera is 1.

   (a) If a 3-D point projects to the point $\mathbf{p}_L = (0.2, 0.1, 1)$ in the left camera and to the point $\mathbf{p}_R = (-0.2, 0.1, 1)$ in the right camera, where each vector is defined w.r.t its own camera coordinate system, determine the 3-D coordinates of the 3-D point in the left coordinate system.

   (b) For a given point in the left image, the corresponding point in the right image will lie along an epipolar line. Use the epipolar constraint equation to show that all such epipolar lines are horizontal.

3. In a two view stereo system, the image planes are parallel and the COP of the right camera is defined by the vector $\mathbf{T} = (T_x, T_y, T_z)$ in the left camera coordinate system. Use the epipolar constraint equation to derive an equation for the epipolar lines in the right image of the form $y = mx + c$. When will the epipolar lines all be parallel? You can assume that the focal length in both cameras is 1.

4. In a two view stereo system with coplanar image planes, the COP of the right camera is defined by the vector $\mathbf{T} = (T_x, 0, 0)$ in the left camera coordinate system. Consider that two pairs of corresponding points are found: $(x_{L1}, y_{L1}) \rightarrow (x_{R1}, y_{R1})$ and $(x_{L2}, y_{L2}) \rightarrow (x_{R2}, y_{R2})$.

   If $y_{L1} = y_{R1} = y_{L2} = y_{R2}$ and $x_{L2} > x_{L1}$, sketch possible scene configurations which would result in the following: (a) $x_{R2} > x_{R1}$ and (b) $x_{R2} < x_{R1}$. Provide sufficient information on your sketch to justify your answer in each case.

5. On slide 18 in Lecture 1, the relationship between the vectors defining a 3-D point P in the left and right camera coordinate systems in homogeneous coordinates is $\mathbf{P}'_L = H_{RL}\mathbf{P}'_R$, where $H_{RL} = H_{WL}H_{WR}^{-1}$. Starting from the definitions of H_{WL} and H_{WR}, prove that

$$H_{RL} = \begin{bmatrix} R^T & \mathbf{T} \\ \mathbf{0} & 1 \end{bmatrix}$$

   where R defines the rotation to be applied to the right camera coordinate system to align it with the left coordinate system and $\mathbf{T}$ defines the position of right COP in the left coordinate system.

6. In a stereo set up consisting of a left and right camera, the fundamental matrix is given by $F = M_R^T E M_L$, where E is the essential matrix and M_L and M_R are the intrinsic matrices associated with the cameras. Give the definition of one of these intrinsic matrices.

7. For points in the left image of a stereo pair, the corresponding points in the right image will lie along epipolar lines. In general, if the right camera is moved, then all of the epipolar lines will change. However, there is one form of movement that would result in all of the epipolar lines remaining unchanged. Determine that movement, justifying your answer using the epipolar constraint equation which defines the epipolar line in the right image.

8. A correspondence matching algorithm for stereo uses the epipolar constraint to identify points in the right image corresponding to points in the left image. If the Essential Matrix of the stereo set up is estimated to be:

$$E = \begin{bmatrix} 0 & -0.4 & 0 \\ 0 & 0 & -10 \\ 0 & 0.9 & 0 \end{bmatrix}$$

which of the following points in the right image would most likely correspond to point $(0.1, 0.2)$ in the left image: (a) $(0.1, 0.02)$; (b) $(0.2, 0.01)$; (c) $(0.4, 0.012)$? You can assume both cameras have a focal length of 1.

9. In a calibrated stereo system, 3-D vectors defining a scene point with respect to the left and right cameras are related by $\mathbf{P}_R = R(\mathbf{P}_L - \mathbf{T})$, where

$$R = \frac{1}{2}\begin{bmatrix} 1 & 0 & \sqrt{3} \\ 0 & 2 & 0 \\ -\sqrt{3} & 0 & 1 \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} \sqrt{3} \\ 0 \\ 1 \end{bmatrix}$$

   (a) Draw a plan view of the configuration of the cameras (showing the x-z plane, where the z axis is along the optical axis of the left camera). Indicate the relative positions and orientations of the cameras. *Hint: Note that the rotation matrix defines a rotation about the y-axis.*

   (b) For the point $(0.02, 0.01)$ (in image plane coordinates) in the left image plane, determine the equation of the epipolar line in the right image and show that the corresponding point in the right image $(-0.02, 0.01)$ lies on the line. The focal length of each camera is 0.1 units.

   (c) Determine the 3D coordinates with respect to the left camera of the associated point in the scene which projects to the two points in part (ii).

   (d) If due to noise the two corresponding points were $(0.025, 0.011)$ and $(-0.018, 0.012)$ in the left and right camera, respectively, determine the estimate of the 3D point that would result.

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October 15, 2025