COMS30030
Image Processing and Computer Vision

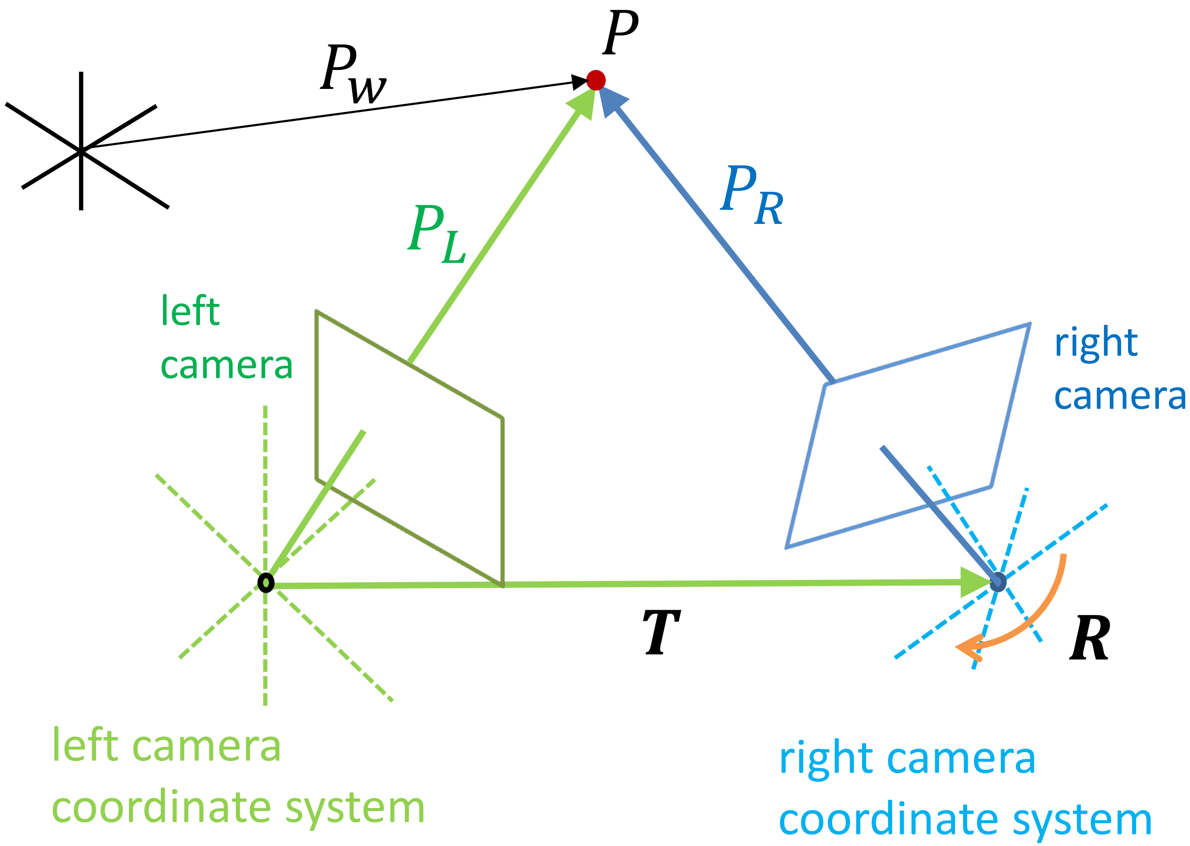
Stereo – 3-D Reconstruction

Andrew Calway

andrew@cs.bris.ac.uk

Stereo Coordinate Systems

previous lecture



$$P'_L = H_{WL}P'_W$$

$$P'_R = H_{WR}P'_W$$

$$P'_L = H_{WL}H_{WR}^{-1}P'_R$$

$$P'_L = H_{RL}P'_R$$

$$H_{RL} = \begin{bmatrix} R^T & T \\ \mathbf{0} & 1 \end{bmatrix}$$

$$P_L = R^T P_R + T$$

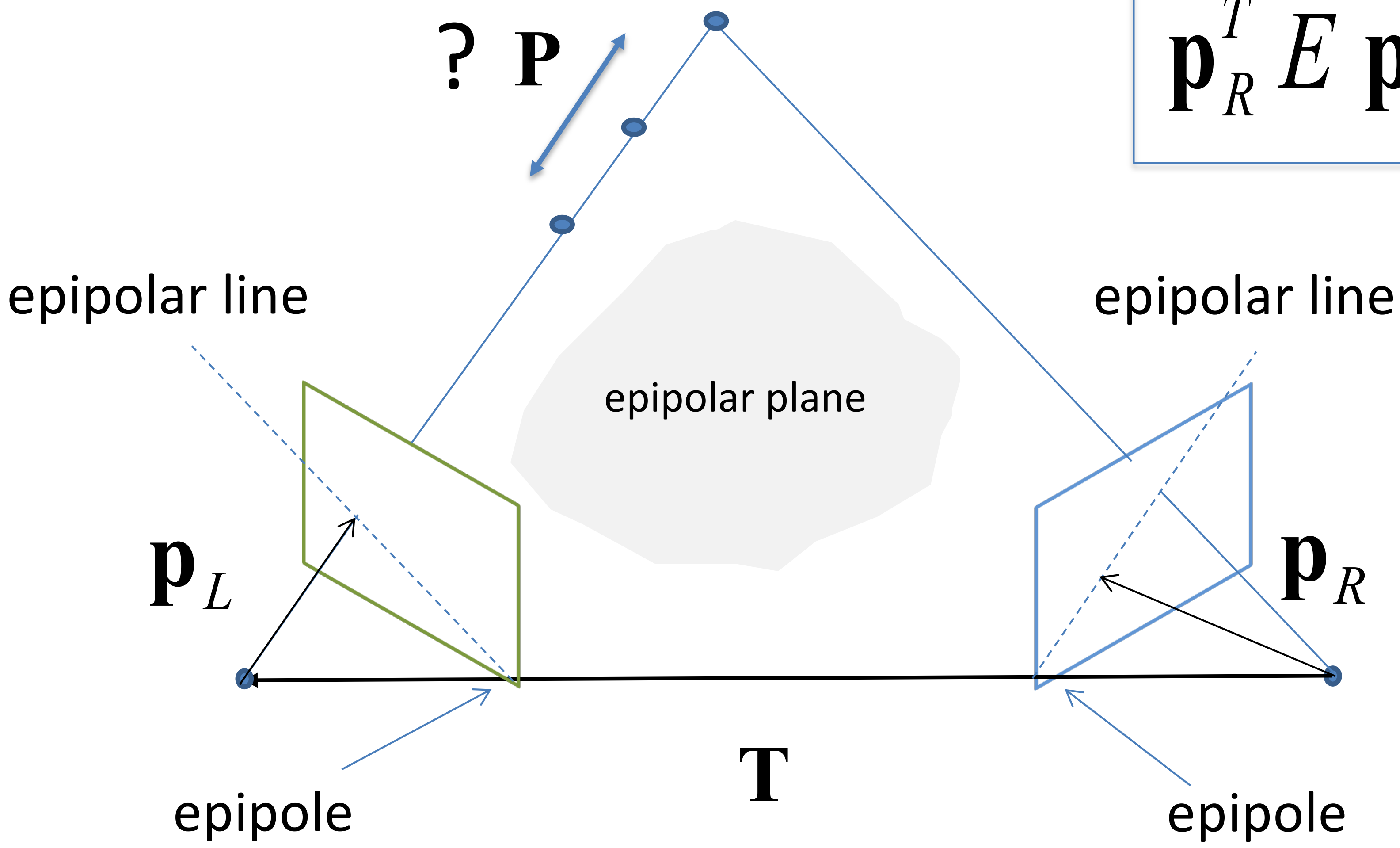
$$P_R = R(P_L - T)$$

R defines rotation to be applied to right camera coordinate system to align it with left coordinate system

Epipolar Geometry

previous lecture

$$\mathbf{p}_R^T E \, \mathbf{p}_L = 0$$



Epipolar Lines

$$\mathbf{p}_R^T E \, \mathbf{p}_L = 0$$

Let $\mathbf{u}_L = E \, \mathbf{p}_L = \begin{bmatrix} u_{L1} \\ u_{L2} \\ u_{L3} \end{bmatrix}$

$$\mathbf{p}_R^T E \, \mathbf{p}_L = \mathbf{p}_R^T \mathbf{u}_L = \underbrace{x_R u_{L1} + y_R u_{L2} + f u_{L3} = 0}_{}$$

Equation of epipolar line in right image

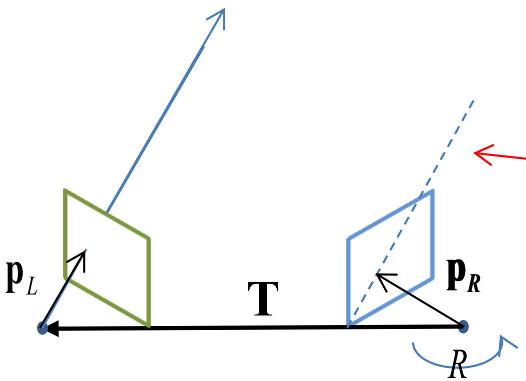
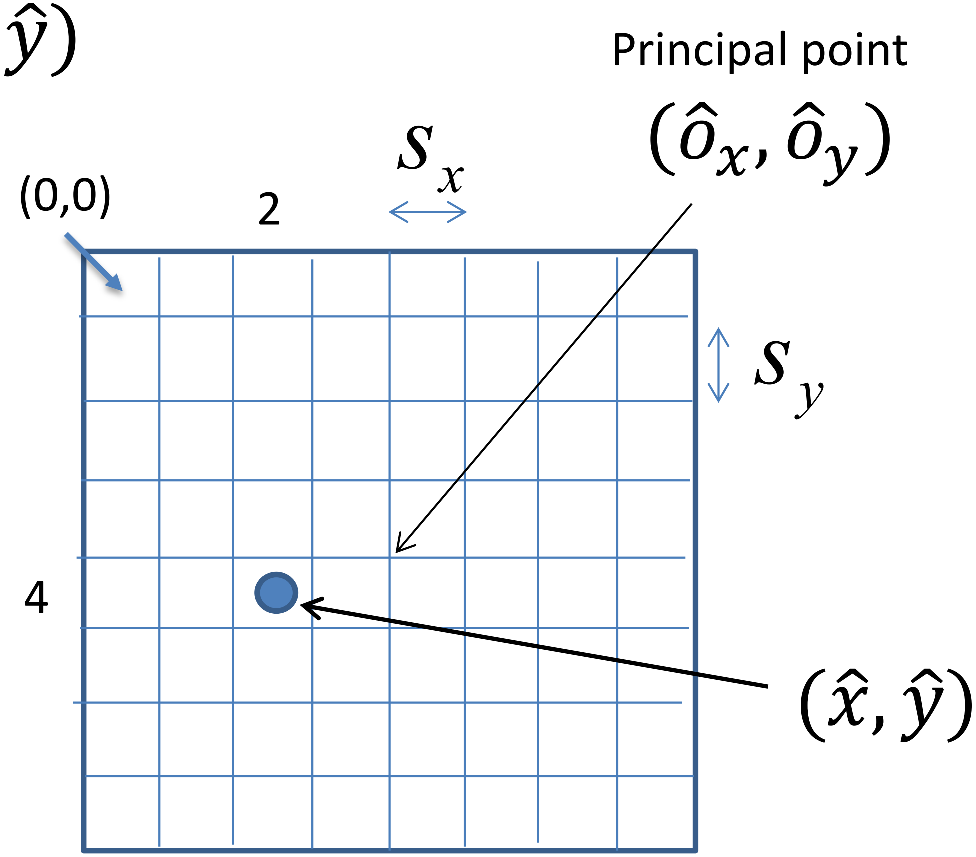


Image Points and Pixels

- Pixel values represent light intensity within small region of image plane, e.g. of size $s_x \times s_y$
- Pixel coordinates: $(\hat{x}, \hat{y})$
- Image coordinates:

$$x = s_x(\hat{x} - \hat{o}_x)$$
$$y = s_y(\hat{y} - \hat{o}_y)$$



Example: $s_x = s_y = 2$

$$x = 2(2 - 3.5) = -3$$
$$y = 2(4 - 3.5) = 1$$

Fundamental Matrix

$$x = s_x(\hat{x} - \hat{o}_x)$$
$$y = s_y(\hat{y} - \hat{o}_y)$$

$$\Rightarrow \quad \mathbf{p}_L = \begin{bmatrix} x_L \\ y_L \\ f \end{bmatrix} = M_L \begin{bmatrix} \hat{x}_L \\ \hat{y}_L \\ f \end{bmatrix} = M_L \hat{\mathbf{p}}_L$$

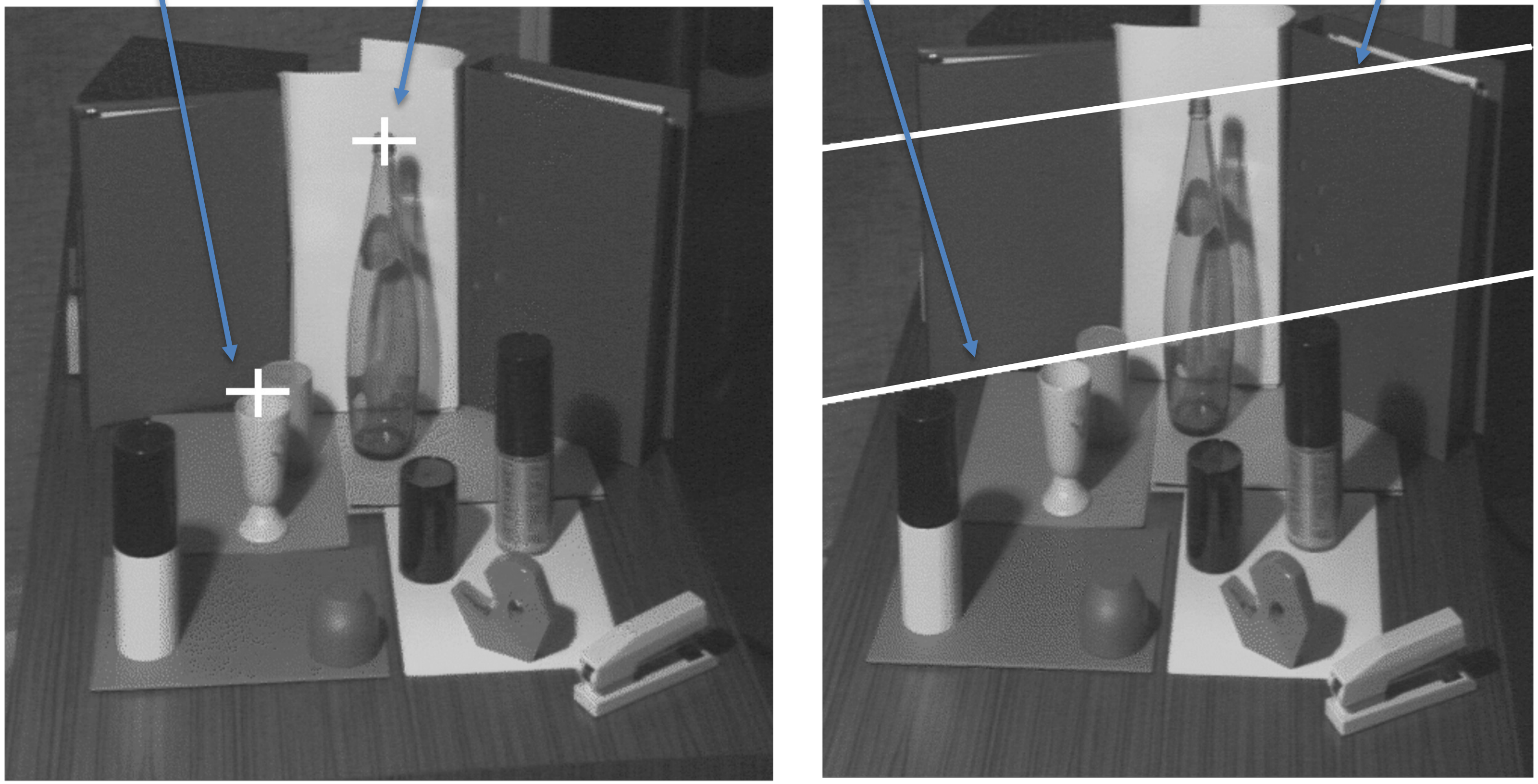
$$\mathbf{p}_R^T E \, \mathbf{p}_L = 0 \quad \Rightarrow \quad \hat{\mathbf{p}}_R^T M_R^T E M_L \hat{\mathbf{p}}_L = 0$$

$$\Rightarrow \quad \hat{\mathbf{p}}_R^T F \hat{\mathbf{p}}_L = 0 \qquad F = M_R^T E \, M_L$$

The fundamental matrix

Epipolar Lines - Example

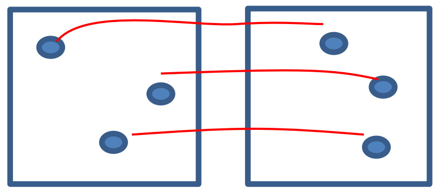
$\hat{p}_{L1}$ $\hat{p}_{L2}$ $\widehat{\mathbf{p}}_{R1}^{T} F \widehat{\mathbf{p}}_{L1} = 0$ $\widehat{\mathbf{p}}_{R2}^{T} F \widehat{\mathbf{p}}_{L2} = 0$



F from Correspondences

- Given set of correspondences, $i = 1 \ldots N$, we can also estimate the fundamental matrix :

$$\hat{\mathbf{p}}_{Ri}^T F \hat{\mathbf{p}}_{Li} = 0 \quad i = 1..N$$



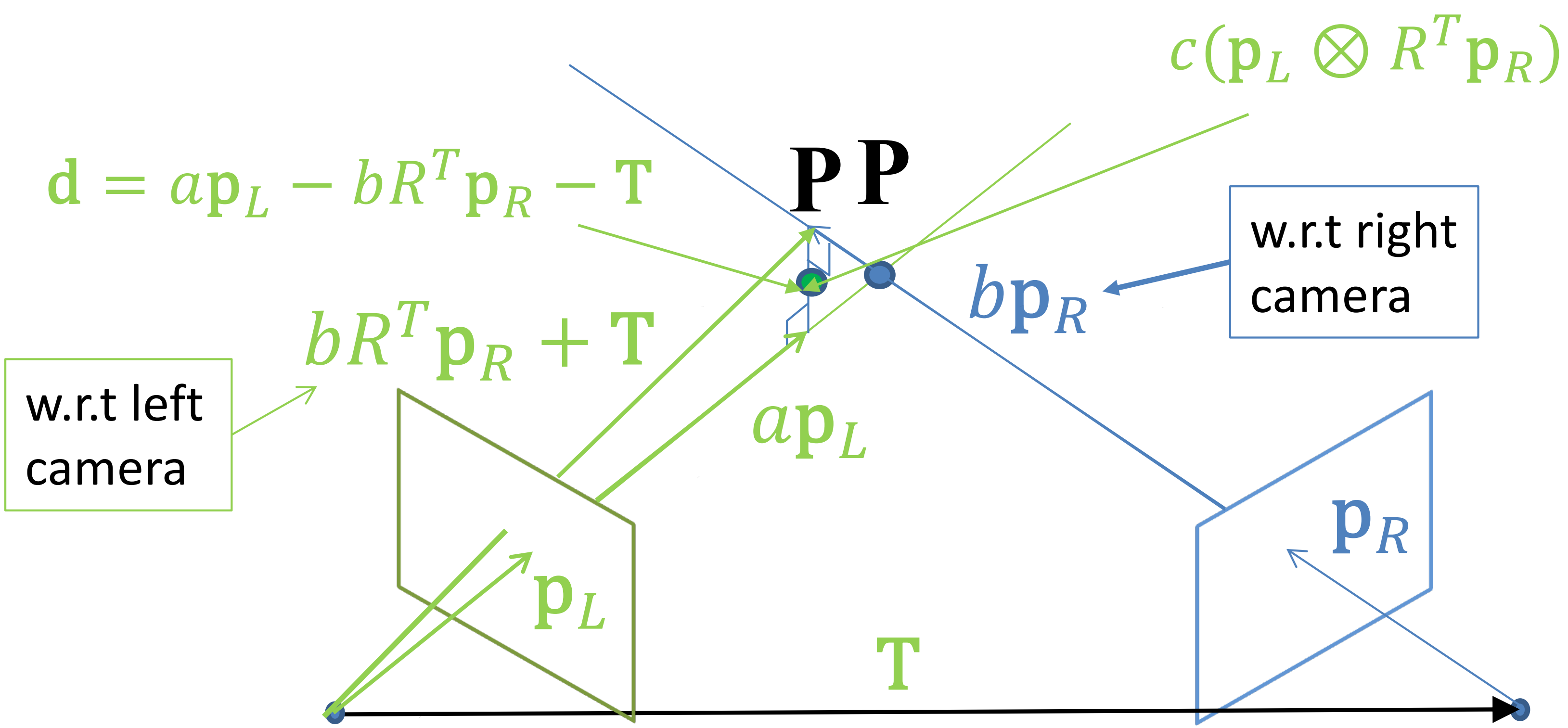
$$\Longrightarrow \quad A\mathbf{v} = 0$$

$N x 9$ matrix defined by correspondence vectors $\hat{\mathbf{p}}_{Li}$ and $\hat{\mathbf{p}}_{Ri}$

Components of F

Solve for $\mathbf{v}$ using Singular Value Decomposition

3-D Reconstruction



find a, b, c s.t: $a\mathbf{p}_L - bR^T\mathbf{p}_R - \mathbf{T} - c(\mathbf{p}_L \otimes R^T\mathbf{p}_R) = 0$

3-D Reconstruction

find a, b, c s.t: $a\,\mathbf{p}_L - b\,R^T\mathbf{p}_R - \mathbf{T} - c\,(\mathbf{p}_L \otimes R^T\mathbf{p}_R) = 0$

Given **corresponding points**, we know : $\mathbf{p}_L, \mathbf{p}_R$

Given **calibrated views**, we know : $R, \mathbf{T}$

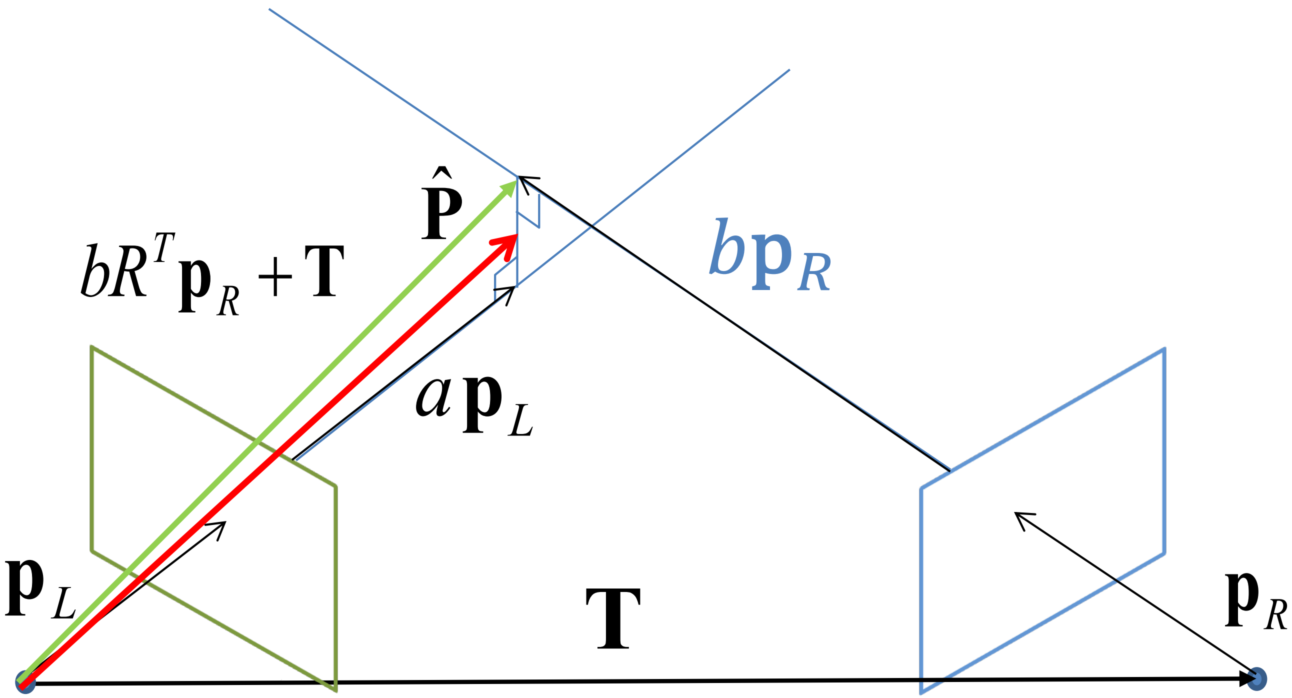
$$a \underbrace{\begin{bmatrix} \bullet \\ \mathbf{p}_L \\ \bullet \end{bmatrix}}_{3\text{x}1} - b \underbrace{\begin{bmatrix} R^T\mathbf{p}_R \end{bmatrix}}_{3\text{x}1} - c \underbrace{\begin{bmatrix} \mathbf{p}_L \otimes R^T\mathbf{p}_R \end{bmatrix}}_{3\text{x}1} = \underbrace{\begin{bmatrix} \mathbf{T} \end{bmatrix}}_{3\text{x}1}$$

3-D Reconstruction

$$a\underbrace{\begin{bmatrix} \bullet \\ \mathbf{p}_L \\ \bullet \end{bmatrix}}_{3\text{x}1} - b\underbrace{\begin{bmatrix} R^T\mathbf{p}_R \\ \\ \end{bmatrix}}_{3\text{x}1} - c\underbrace{\begin{bmatrix} \mathbf{p}_L \otimes R^T\mathbf{p}_R \\ \\ \end{bmatrix}}_{3\text{x}1} = \underbrace{\begin{bmatrix} \mathbf{T} \\ \\ \end{bmatrix}}_{3\text{x}1}$$

$$\Rightarrow \quad H\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \mathbf{T} \quad \Rightarrow \quad \begin{bmatrix} a \\ b \\ c \end{bmatrix} = H^{-1}\,\mathbf{T}$$

3-D Reconstruction



$$\Rightarrow \quad \hat{\mathbf{P}} = (a\,\mathbf{p}_L + b\,R^T\mathbf{p}_R + \mathbf{T})/2$$