COMS30030
Image Processing and Computer Vision

Stereo 2 – Epipolar Geometry

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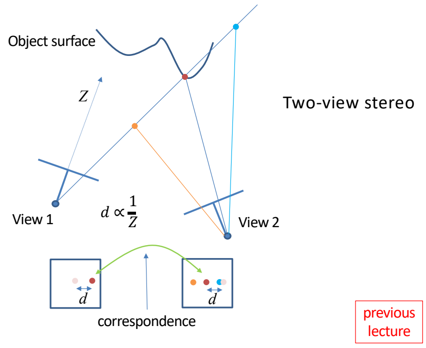
1

Stereo Computer Vision

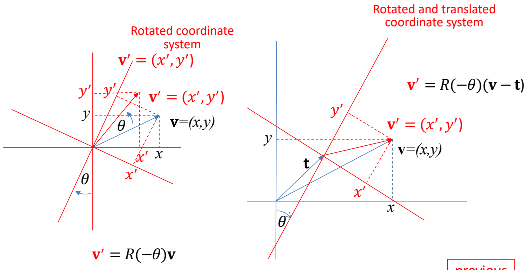


Two-view stereo

previous lecture

2

2-D Coordinate Transformations



$\mathbf{v}' = R(-\theta)\mathbf{v}$

Vector representation in rotated coordinate system

previous lecture

3

3-D Coordinate Transformations

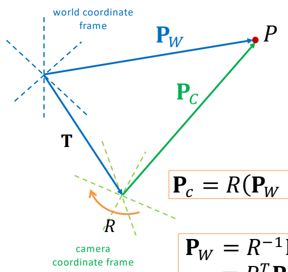
$\mathbf{P}_W$ Vector defining P in world coordinates

$\mathbf{P}_C$ Vector defining P in camera coordinates

$\mathbf{T}$: 3-D camera position vector

R : 3-D camera rotation matrix

R defines rotation to be applied to camera coordinate system to align it with world coordinate system



$\mathbf{P}_c = R(\mathbf{P}_W - \mathbf{T})$

$$\mathbf{P}_W = R^{-1}\mathbf{P}_C + \mathbf{T} = R^T\mathbf{P}_C + \mathbf{T}$$

Rotation matrices: $R^T = R^{-1}$

4

Homogeneous Coordinates

- Homogeneous coordinates allow coordinate transformations to be defined by 4x4 matrices:

$$\mathbf{P}'_W = \begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{P}_W \\ 1 \end{bmatrix} = \begin{bmatrix} R^T & \mathbf{T} \\ \mathbf{0} & 1 \end{bmatrix}\begin{bmatrix} \mathbf{P}_C \\ 1 \end{bmatrix} = H_{CW}\mathbf{P}'_c \Rightarrow \boxed{\mathbf{P}_W = R^T\mathbf{P}_C + \mathbf{T}}$$

$$H_{CW} = \begin{bmatrix} R_{00} & R_{10} & R_{20} & T_x \\ R_{01} & R_{11} & R_{21} & T_y \\ R_{02} & R_{12} & R_{22} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{P}'_C = H_{CW}^{-1}\mathbf{P}'_W = H_{WC}\mathbf{P}'_W$$

$$H_{WC} = \begin{bmatrix} R & -R\mathbf{T} \\ \mathbf{0} & 1 \end{bmatrix} \Rightarrow H_{WC}H_{CW} = I$$

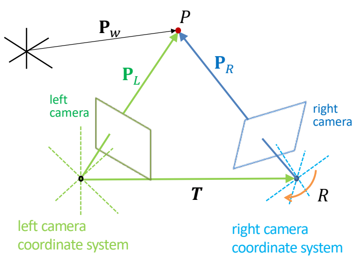
I - Identity matrix

$$\Rightarrow \boxed{\mathbf{P}_C = R\,(\mathbf{P}_W - \mathbf{T})}$$

H_{cw} : camera to world coordinate transformation matrix

5

Stereo Coordinate Systems



R defines rotation to be applied to right camera coordinate system to align it with left coordinate system

$$\mathbf{P}'_L = H_{WL}\mathbf{P}'_W$$
$$\mathbf{P}'_R = H_{WR}\mathbf{P}'_W$$
$$\mathbf{P}'_L = H_{WL}H_{WR}^{-1}\mathbf{P}'_R$$
$$\mathbf{P}'_L = H_{RL}\mathbf{P}'_R$$

$$H_{RL} = \begin{bmatrix} R^T & \mathbf{T} \\ \mathbf{0} & 1 \end{bmatrix}$$

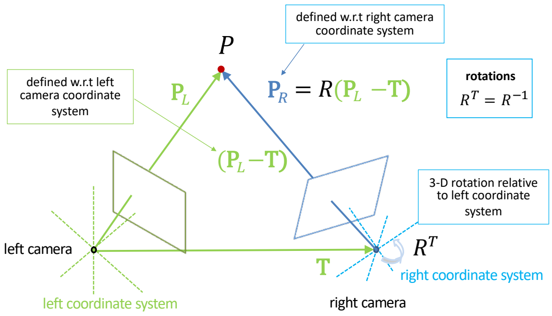
$$\mathbf{P}_L = R^T\mathbf{P}_R + \mathbf{T}$$

$$\mathbf{P}_R = R(\mathbf{P}_L - \mathbf{T})$$

6

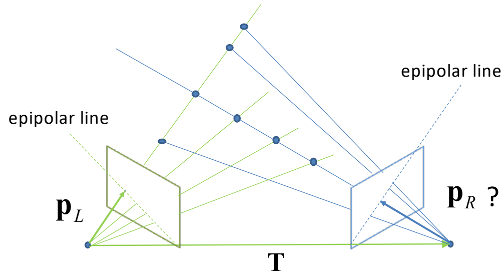
General Two-View Stereo

previous lecture



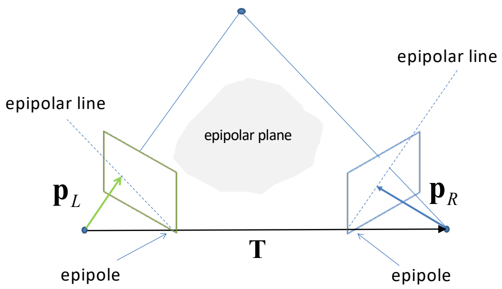
7

Epipolar Lines



8

Epipolar Planes



9

Epipolar Geometry

- Epipolar geometry defines relationship between two stereo views
- For known viewpoints:
  – it constrains matches to lie along epipolar lines
- For unknown viewpoints:
  – given matching points ....
  – it enables estimation of viewpoints

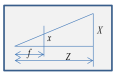
10

Epipolar Geometry - Maths

Rigid transformation between cameras:

$$\mathbf{P}_R = R(\mathbf{P}_L - \mathbf{T})$$

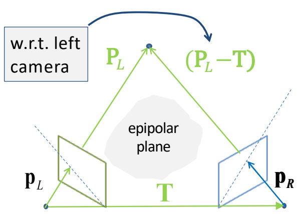
Perspective projection:



$$\mathbf{P}_L = \begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} \qquad \mathbf{p}_L = \begin{bmatrix} x_L \\ y_L \\ f \end{bmatrix} = \frac{f\mathbf{P}_L}{Z_L} \qquad \mathbf{p}_R = \begin{bmatrix} x_R \\ y_R \\ f \end{bmatrix} = \frac{f\mathbf{P}_R}{Z_R}$$

11

Epipolar Geometry - Maths

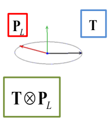


Vectors $\mathbf{P}_L$, $\mathbf{T}$ and $\mathbf{P}_L - \mathbf{T}$ all lie in epipolar plane

in plane perpendicular to plane

$$(\mathbf{P}_L - \mathbf{T})^T(\mathbf{T} \otimes \mathbf{P}_L) = 0$$

$\otimes$ cross product :



dot product = 0 for **perpendicular** vectors

12

Epipolar Geometry - Maths

$$(\mathbf{P}_L - \mathbf{T})^T(\mathbf{T}\otimes\mathbf{P}_L) = 0$$

$$(\mathbf{T}\otimes\mathbf{P}_L) = S\,\mathbf{P}_L$$

$$S = \begin{bmatrix} 0 & -T_Z & T_Y \\ T_Z & 0 & -T_X \\ -T_Y & T_X & 0 \end{bmatrix}$$

w.r.t. right camera

$$\mathbf{P}_R = R(\mathbf{P}_L - \mathbf{T})$$

$$R^T = R^{-1}$$ Rotation matrix

$$\Downarrow$$

$$R^T\mathbf{P}_R = (\mathbf{P}_L - \mathbf{T})$$

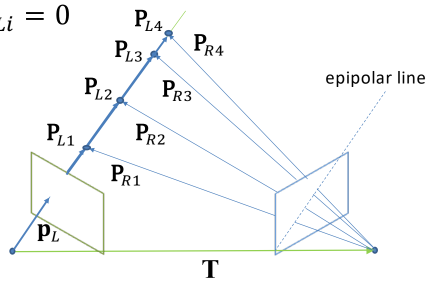
$$\mathbf{P}_R^T R = (\mathbf{P}_L - \mathbf{T})^T$$

$$\mathbf{P}_R^T\, R\, S\, \mathbf{P}_L = 0$$

13

Epipolar Line Constraint

$$\mathbf{P}_{Ri}^T R S \mathbf{P}_{Li} = 0$$



14

The Essential Matrix

$$\mathbf{P}_R^T\, \underbrace{R\, S}\, \mathbf{P}_L = 0 \Rightarrow \mathbf{P}_R^T\, E\, \mathbf{P}_L = 0$$

$$E = R\,S \Rightarrow$$ the essential matrix

$$\mathbf{p}_L = \frac{f\mathbf{P}_L}{Z_L} \quad \mathbf{p}_R = \frac{f\mathbf{P}_R}{Z_R}$$

$$\Downarrow$$

$$\mathbf{P}_L = \frac{Z_L\mathbf{p}_L}{f} \quad \mathbf{P}_R = \frac{Z_R\mathbf{p}_R}{f}$$

$$\Rightarrow \cancel{\frac{Z_R}{f}}\mathbf{p}_R^T\, E\, \cancel{\frac{Z_L}{f}}\mathbf{p}_L = 0$$

$$\Rightarrow \mathbf{p}_R^T\, E\, \mathbf{p}_L = 0$$

15