CS 33

Machine Programming (6)

Tail Recursion

```
int factorial(int x) {
  if (x == 1)
    return x;
  else
    return
      x*factorial(x-1);
}
```

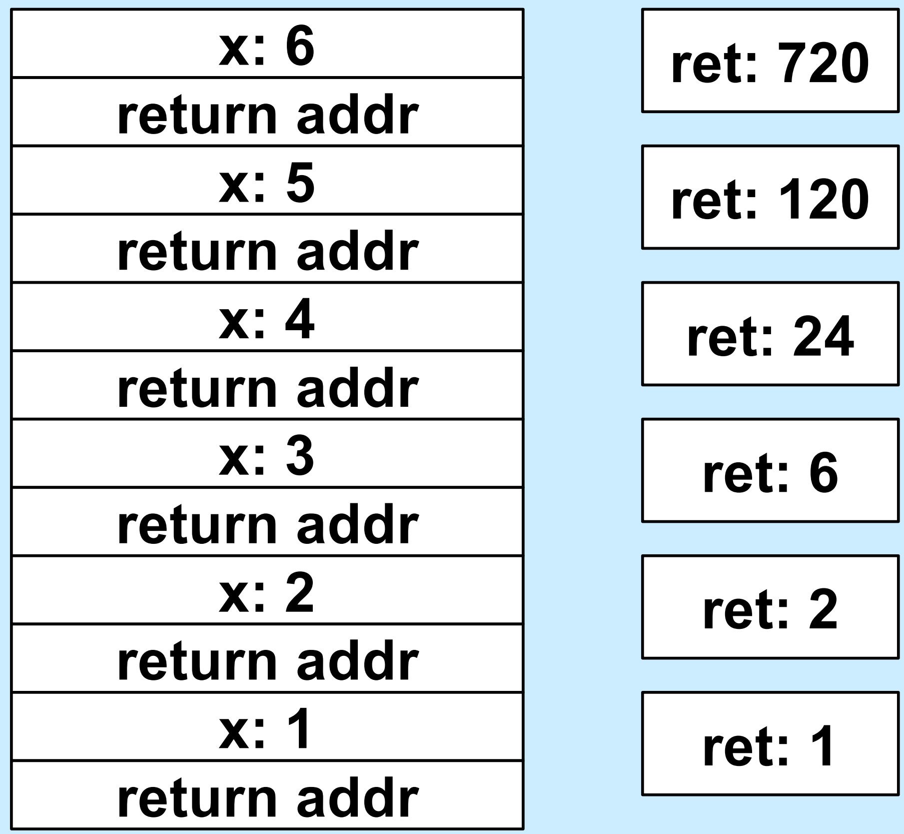
```
int factorial(int x) {
  return f2(x, 1);
}

int f2(int a1, int a2) {
  if (a1 == 1)
    return a2;
  else
    return
      f2(a1-1, a1*a2);
}
```

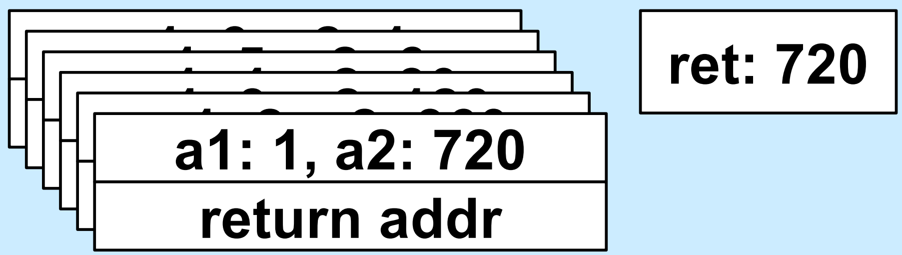
No Tail Recursion (1)

x: 6
return addr
x: 5
return addr
x: 4
return addr
x: 3
return addr
x: 2
return addr
x: 1
return addr

No Tail Recursion (2)



Tail Recursion



Code: gcc –O1

```
f2:
        movl    %esi, %eax
        cmpl    $1, %edi
        je      .L5
        subq    $8, %rsp
        movl    %edi, %esi
        imull   %eax, %esi
        subl    $1, %edi
        call    f2          # recursive call!
        addq    $8, %rsp
.L5:
        rep
        ret
```

Code: gcc –O2

```
f2:
        cmpl    $1, %edi
        movl    %esi, %eax
        je      .L8
.L12:
        imull   %edi, %eax
        subl    $1, %edi
        cmpl    $1, %edi
        jne     .L12
.L8:
        rep
        ret
```

loop!

Computer Architecture and Optimization (1)

What You Need to Know to Write Better Code

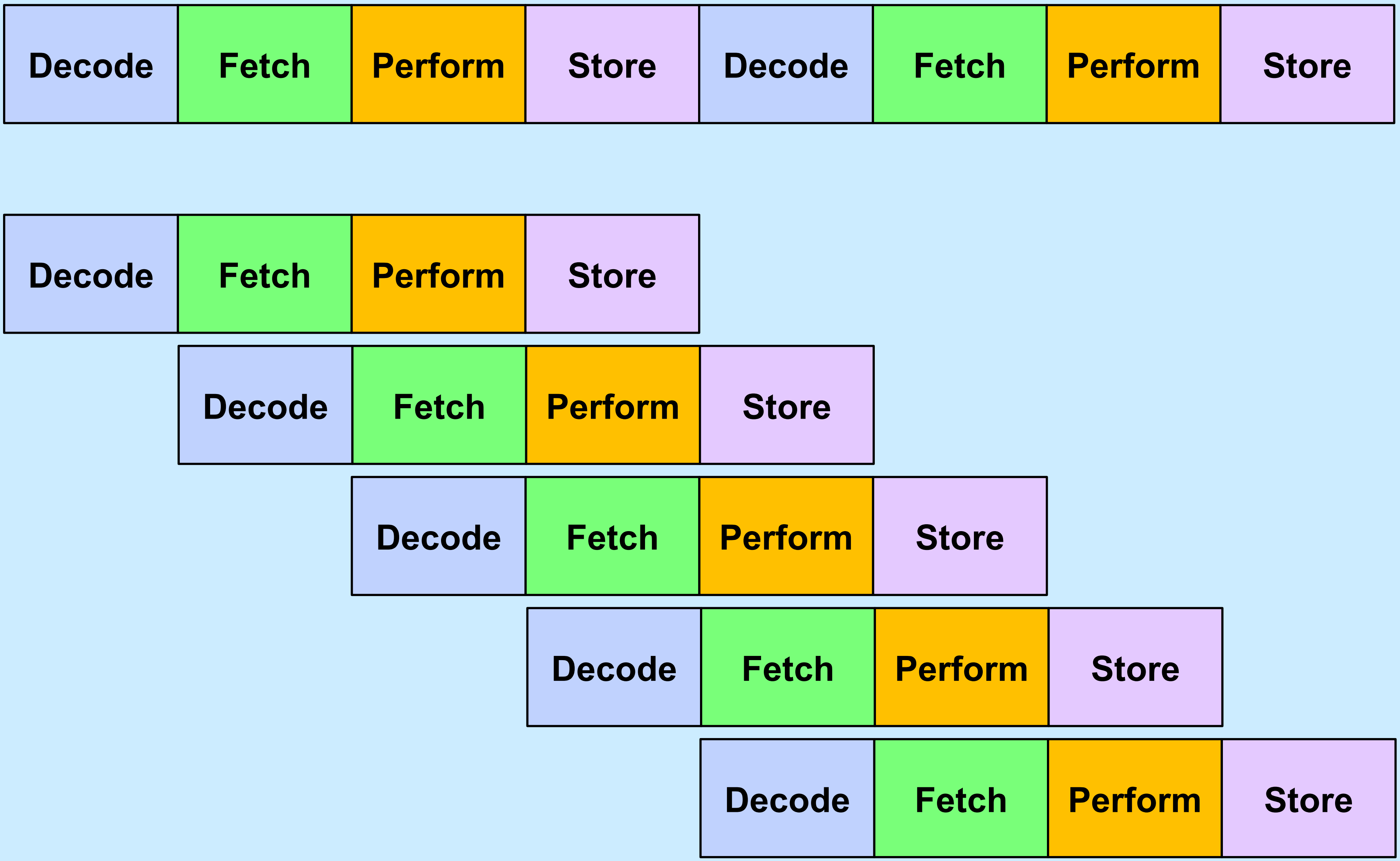
Simplistic View of Processor

```
while (true) {
  instruction = mem[rip];
  execute(instruction);
}
```

Some Details ...

```
void execute(instruction_t instruction) {
  decode(instruction, &opcode, &operands);
  fetch(operands, &in_operands);
  perform(opcode, in_operands, &out_operands);
  store(out_operands);
}
```

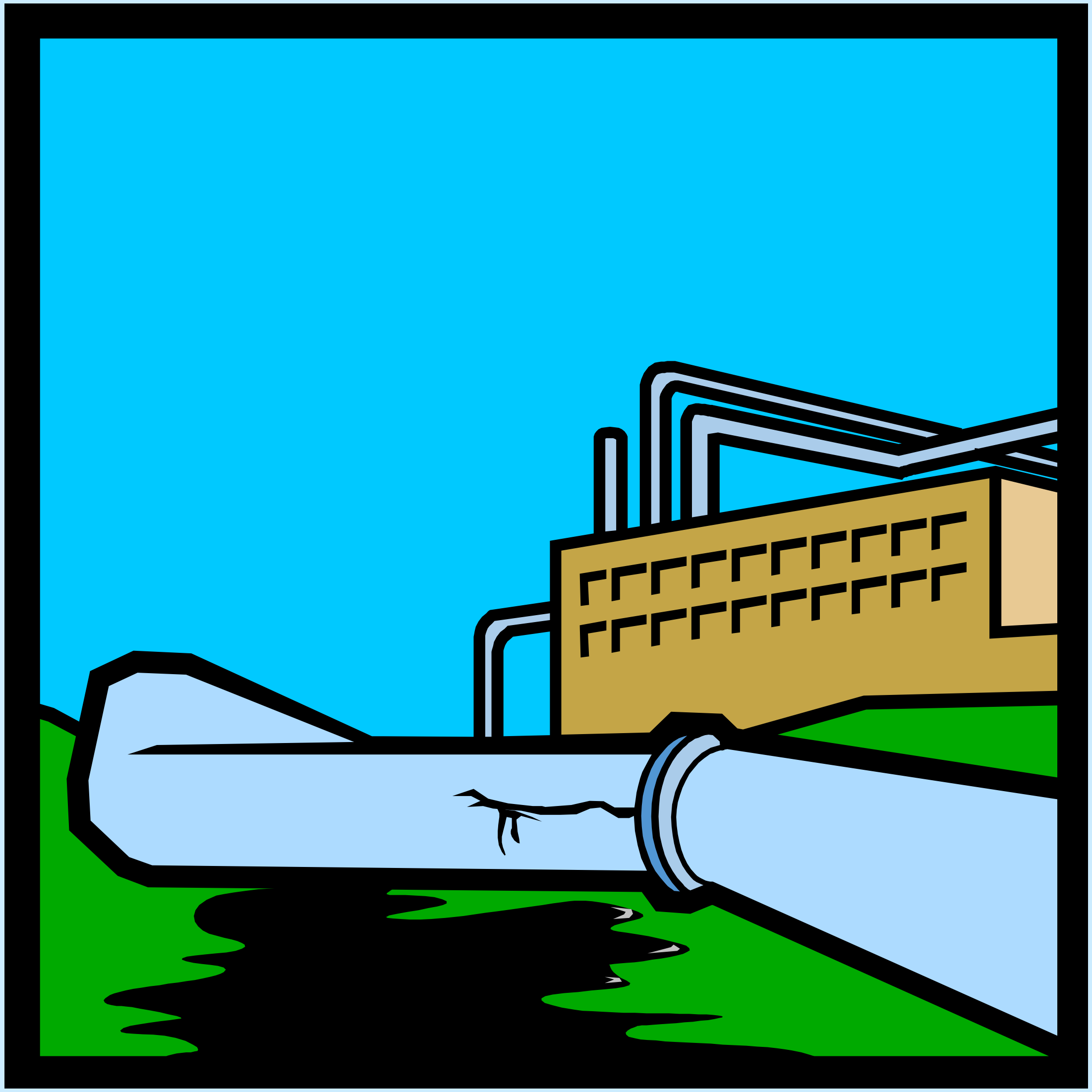
Pipelines


Decode
Fetch
Perform
Store
Decode
Fetch
Perform
Store
Decode
Fetch
Perform
Store
Decode
Fetch
Perform
Store
Decode
Fetch
Perform
Store
Decode
Fetch
Perform
Store
Decode
Fetch
Perform
Store

Analysis

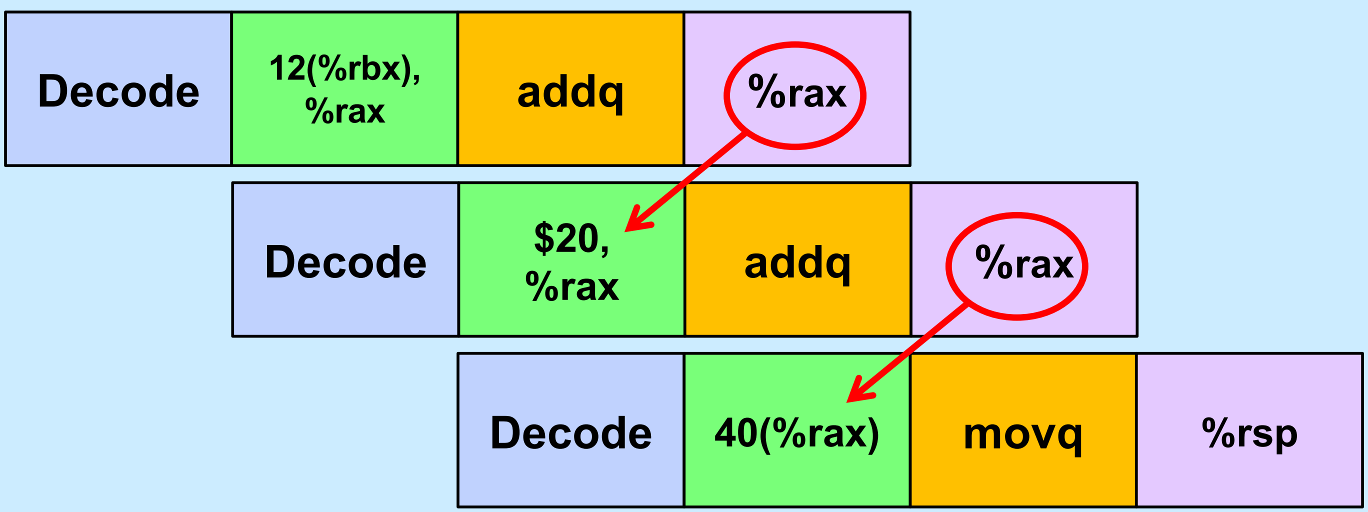
- **Not pipelined**
  - each instruction takes, say, 3.2 nanoseconds
    - » 3.2 ns latency
  - 312.5 million instructions/second (MIPS)
- **Pipelined**
  - each instruction still takes 3.2 ns
    - » latency still 3.2 ns
  - an instruction completes every .8 ns
    - » 1.25 billion instructions/second (GIPS) throughput

Hazards ...

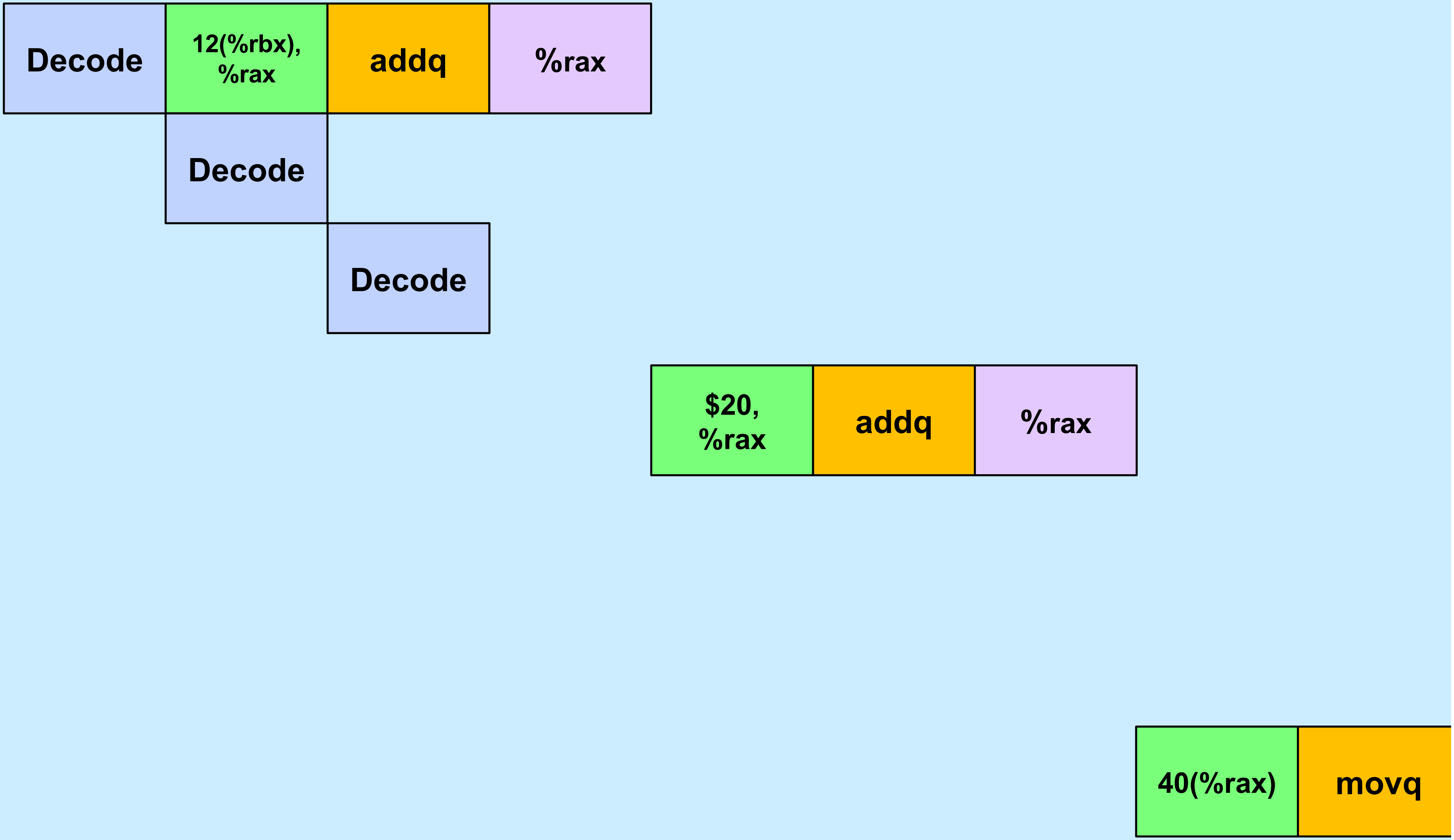


Data Hazards

```
addq 12(%rbx), %rax
addq $20, %rax
movq 40(%rax), %rsp
```



Coping



Decode
12(%rbx), %rax
addq
%rax
Decode
Decode
$20, %rax
addq
%rax
40(%rax)
movq

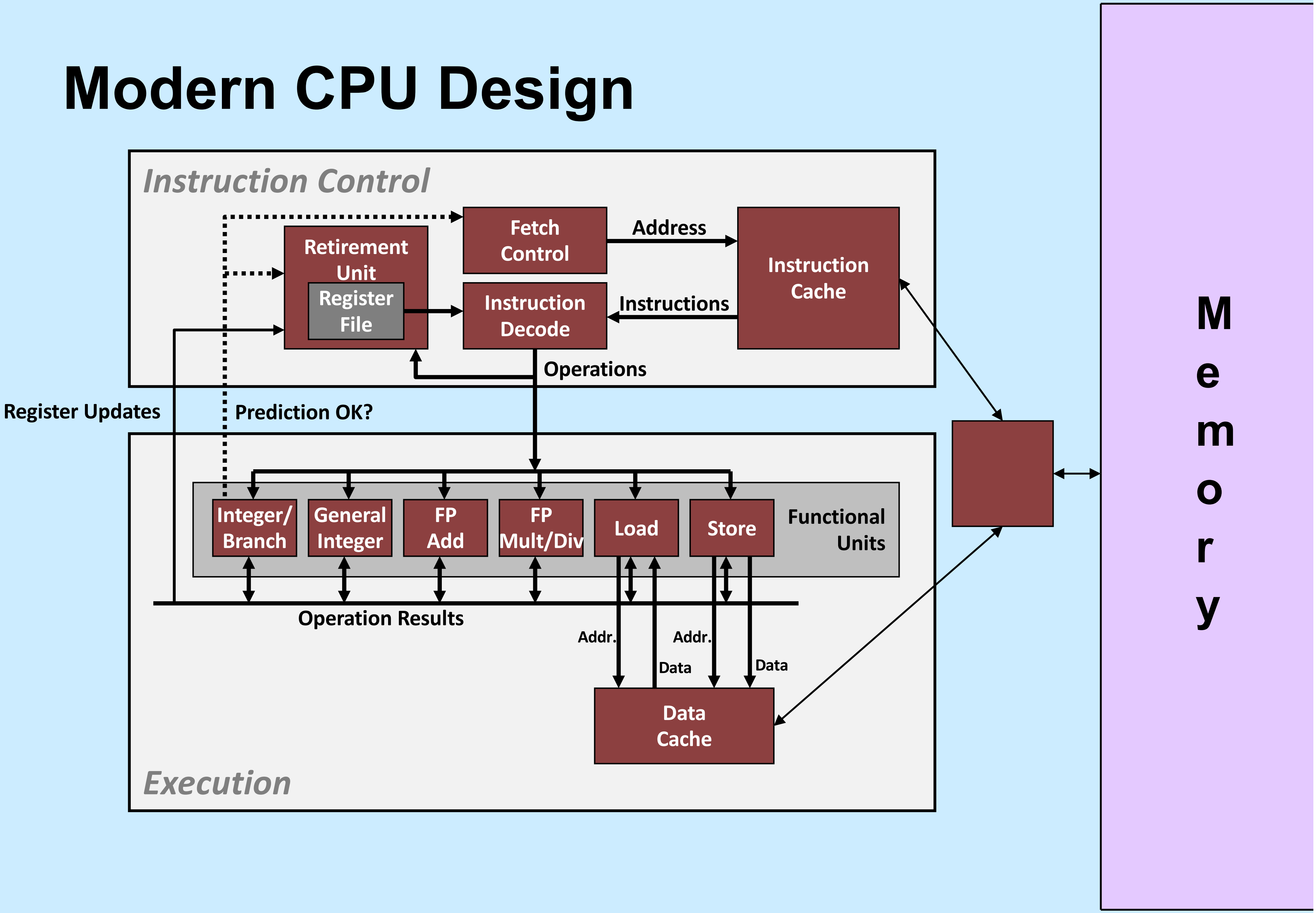
Control Hazards

```
  movl $0, %ecx
.L2:
  movl %edx, %eax
  andl $1, %eax
  addl %eax, %ecx
  shrl $1, %edx
  jne  .L2 # what goes in the pipeline?
  movl %ecx, %eax
  ...
```


Coping: Guess ...

- **Branch prediction**
  - assume, for example, that conditional branches are always taken
  - but don't do anything to registers or memory until you know for sure

Modern CPU Design



Performance Realities

There's more to performance than asymptotic complexity

- **Constant factors matter too!**
  - easily see 10:1 performance range depending on how code is written
  - must optimize at multiple levels:
    - » algorithm, data representations, functions, and loops
- **Must understand system to optimize performance**
  - how programs are compiled and executed
  - how to measure program performance and identify bottlenecks
  - how to improve performance without destroying code modularity and generality

Optimizing Compilers

- Provide efficient mapping of program to machine
  - register allocation
  - code selection and ordering (scheduling)
  - eliminating minor inefficiencies
- Don’t (usually) improve asymptotic efficiency
  - up to programmer to select best overall algorithm
  - big-O savings are (often) more important than constant factors
    - » but constant factors also matter
- Have difficulty overcoming “optimization blockers”
  - potential memory aliasing
  - potential function side-effects

Limitations of Optimizing Compilers

- **Operate under fundamental constraint**
  - must not cause any change in program behavior
  - often prevents it from making optimizations that would only affect behavior under pathological conditions
- **Behavior that may be obvious to the programmer can be obfuscated by languages and coding styles**
  - e.g., data ranges may be more limited than variable types suggest
- **Most analysis is performed only within functions**
  - whole-program analysis is too expensive in most cases
- **Most analysis is based only on *static* information**
  - compiler has difficulty anticipating run-time inputs

- **When in doubt, the compiler must be conservative**

Generally Useful Optimizations

- Optimizations that you or the compiler should do regardless of processor / compiler

- Code Motion
  - reduce frequency with which computation performed
    - » if it will always produce same result
    - » especially moving code out of loop

```
void set_row(long *a, long *b,
   long i, long n){
    long j;
    for (j = 0; j < n; j++)
        a[n*i+j] = b[j];
}
```

→

```
    long j;
    long ni = n*i;
    for (j = 0; j < n; j++)
        a[ni+j] = b[j];
```

Compiler-Generated Code Motion

```
void set_row(long *a, long *b,
   long i, long n){
    long j;
    for (j = 0; j < n; j++)
        a[n*i+j] = b[j];
}
```

```
    long j;
    long ni = n*i;
    long *rowp = a+ni;
    for (j = 0; j < n; j++)
        rowp[j] = b[j];
```

```
set_row:
        testq   %rcx, %rcx              # Test n
        jle     .L1                     # If 0, goto done
        imulq   %rcx, %rdx              # i *= n
        leaq    (%rdi,%rdx,8), %rdi     # rowp = A + n*i*8
        movl    $0, %eax                # j = 0
.L3:                                  # loop:
        movq    (%rsi,%rax,8), %rdx     # t = b[j]
        movq    %rdx, (%rdi,%rax,8)     # rowp[j] = t
        addq    $1, %rax                # j++
        cmpq    %rcx, %rax              # Compare n:j
        jg      .L3                     # If >, goto loop
.L1:                                  # done:
        rep ; ret
```

Reduction in Strength

- Replace costly operation with simpler one
- Shift, add instead of multiply or divide

  `16*x      -->    x << 4`

  – utility is machine-dependent

  – depends on cost of multiply or divide instruction

    » on some Intel processors, multiplies are 3x longer than adds
- Recognize sequence of products

```
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    a[n*i + j] = b[j];
```

→

```
int ni = 0;
for (i = 0; i < n; i++) {
  for (j = 0; j < n; j++)
    a[ni + j] = b[j];
  ni += n;
}
```

Share Common Subexpressions

- **Reuse portions of expressions**
- **Compilers often not very sophisticated in exploiting arithmetic properties**

```
/* Sum neighbors of i,j */
up =    val[(i-1)*n + j  ];
down =  val[(i+1)*n + j  ];
left =  val[i*n     + j-1];
right = val[i*n     + j+1];
sum = up + down + left + right;
```

```
long inj = i*n + j;
up =    val[inj - n];
down =  val[inj + n];
left =  val[inj - 1];
right = val[inj + 1];
sum = up + down + left + right;
```

3 multiplications: i*n, (i–1)*n, (i+1)*n

1 multiplication: i*n

```
leaq   1(%rsi), %rax  # i+1
leaq   -1(%rsi), %r8  # i-1
imulq  %rcx, %rsi     # i*n
imulq  %rcx, %rax     # (i+1)*n
imulq  %rcx, %r8      # (i-1)*n
addq   %rdx, %rsi     # i*n+j
addq   %rdx, %rax     # (i+1)*n+j
addq   %rdx, %r8      # (i-1)*n+j
```

```
imulq    %rcx, %rsi   # i*n
addq     %rdx, %rsi   # i*n+j
movq     %rsi, %rax   # i*n+j
subq     %rcx, %rax   # i*n+j-n
leaq     (%rsi,%rcx), %rcx # i*n+j+n
```

Quiz 1

The fastest means for evaluating

```
n*n + 2*n + 1
```

requires exactly:

a) 2 multiplies and 2 additions
b) one multiply and two additions
c) one multiply and one addition
d) three additions

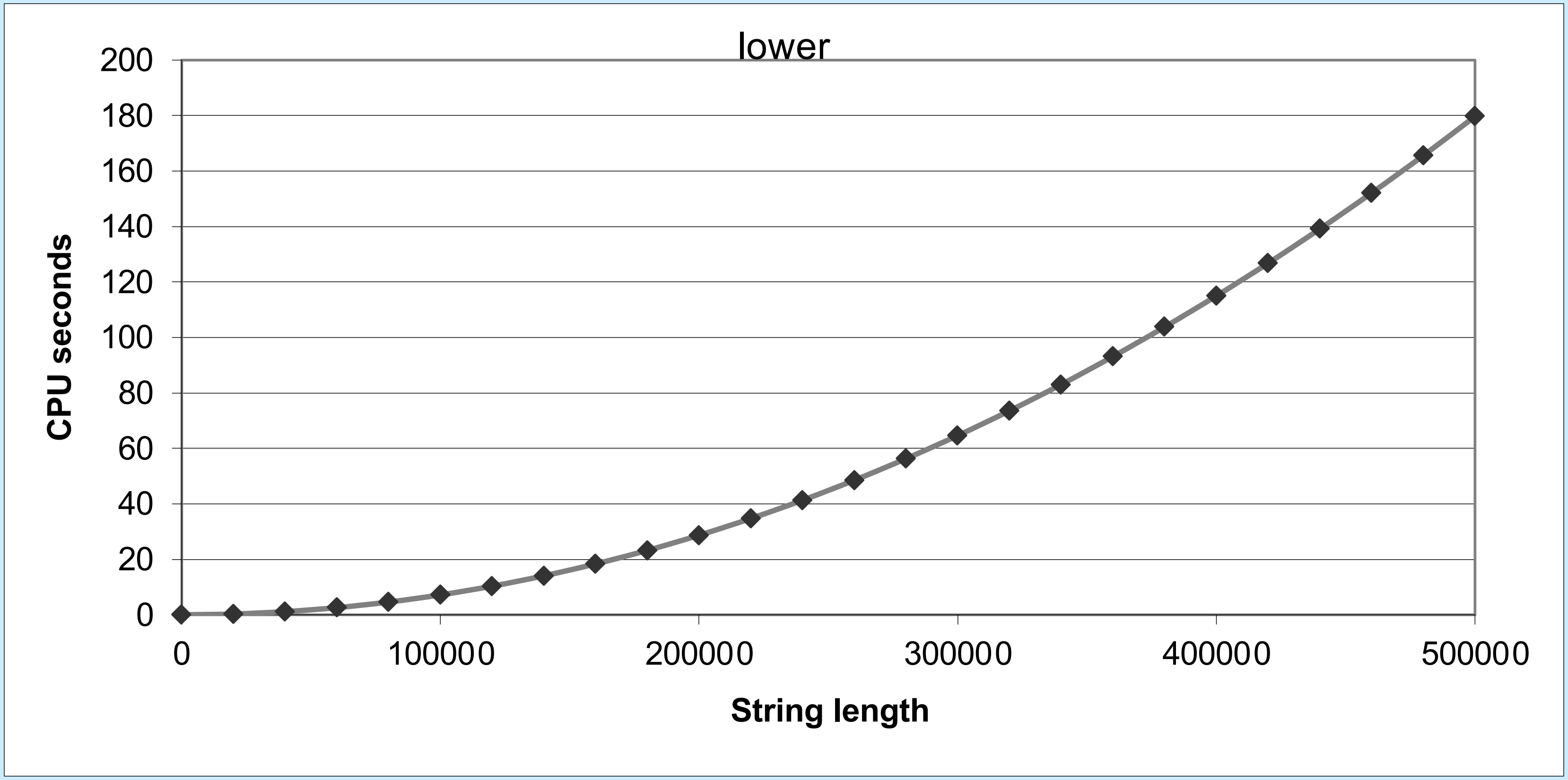
Optimization Blocker #1: Function Calls

- **Function to convert string to lower case**

```
void lower(char *s){
  int i;
  for (i = 0; i < strlen(s); i++)
    if (s[i] >= 'A' && s[i] <= 'Z')
      s[i] -= ('A' - 'a');
}
```

Lower Case Conversion Performance

- **Time quadruples when string length doubles**
- **Quadratic performance**



Convert Loop To Goto Form

```
void lower(char *s){
   int i = 0;
   if (i >= strlen(s))
     goto done;
 loop:
   if (s[i] >= 'A' && s[i] <= 'Z')
       s[i] -= ('A' - 'a');
   i++;
   if (i < strlen(s))
     goto loop;
 done:
}
```

- `strlen` **executed every iteration**

Calling Strlen

```
size_t strlen(const char *s){
    size_t length = 0;
    while (*s != '\0') {
       s++;
       length++;
    }
    return length;
}
```

- **strlen performance**
  - only way to determine length of string is to scan its entire length, looking for null character
- **Overall performance, string of length N**
  - N calls to strlen
  - overall $O(N^2)$ performance

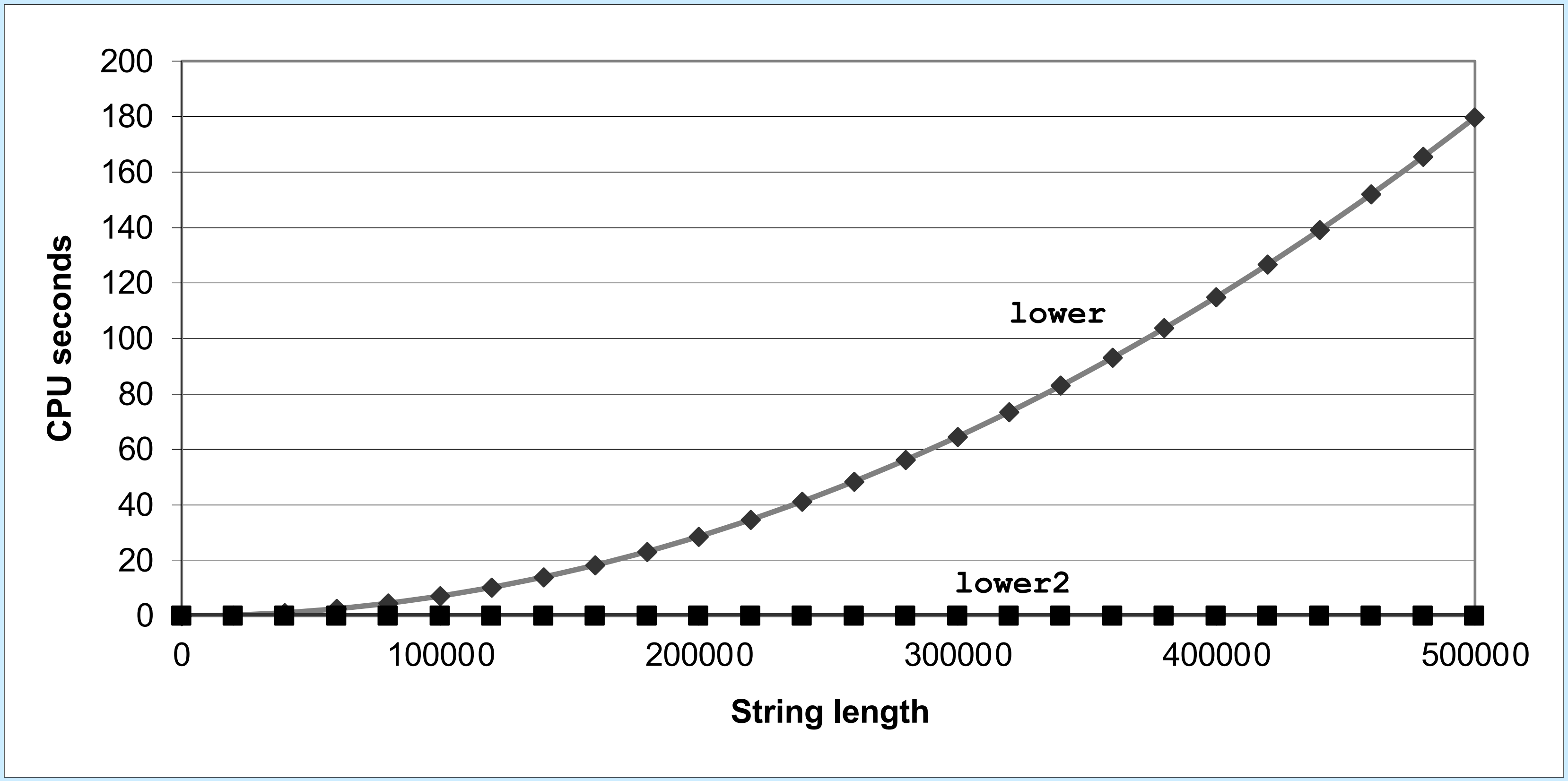
Improving Performance

```
void lower2(char *s){
  int i;
  int len = strlen(s);
  for (i = 0; i < len; i++)
    if (s[i] >= 'A' && s[i] <= 'Z')
      s[i] -= ('A' - 'a');
}
```

- **Move call to `strlen` outside of loop**
  - since result does not change from one iteration to another
  - form of code motion

Lower-Case Conversion Performance

- **Time doubles when string-length doubles**
  - **linear performance of lower2**



Optimization Blocker: Function Calls

- ***Why couldn't compiler move* `strlen` *out of inner loop?***
  - function may have side effects
    - » alters global state each time called
  - function may not return same value for given arguments
    - » depends on other parts of global state
    - » function `lower` could interact with `strlen`
- **Warning:**
  - compiler treats function call as a black box
  - weak optimizations near them
- **Remedies:**
  - use of `inline` functions
    - » gcc does this with –O2
  - do your own code motion

```
int lencnt = 0;
size_t strlen(const char *s){
    size_t length = 0;
    while (*s != '\0') {
       s++; length++;
    }
    lencnt += length;
    return length;
}
```

Memory Matters

```
/* Sum rows of n X n matrix a
   and store result in vector b  */
void sum_rows1(long n, long a[][n], long *b) {
    long i, j;
    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i][j];
    }
}
```

```
# sum_rows1 inner loop
.L3:
   movq    (%r8,%rax,8), %rcx   # rcx = a[i][j]
   addq    %rcx, (%rdx)         # b[i] += rcx
   addq    $1, %rax             # j++
   cmpq    %rax, %rdi           # if i<n
   jne     .L3                  # goto .L3
```

- Code updates `b[i]` on every iteration
- Why couldn't compiler optimize this away?

Memory Aliasing

```
/* Sum rows of n X n matrix a
   and store result in vector b  */
void sum_rows1(long n, long a[][n], long *b) {
    long i, j;
    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i][j];
    }
}
```

```
int A[3][3] =
  {{ 0,   1,   2},
   { 4,   8,  16},
   {32,  64, 128}};

int *B = &A[1][0];

sum_rows1(3, A, B;
```

Value of B:

```
init:  [4, 8, 16]
```

```
i = 0: [3, 8, 16]
```

```
i = 1: [3, 22, 16]
```

```
i = 2: [3, 22, 224]
```

- **Code updates `b[i]` on every iteration**
- **Must consider possibility that these updates will affect program behavior**

Removing Aliasing

```
/* Sum rows of n X n matrix a
   and store result in vector b  */
void sum_rows1(long n, long a[][n], long *b) {
    long i, j;
    for (i = 0; i < n; i++) {
        long val = 0;
        for (j = 0; j < n; j++)
            val += a[i][j];
        b[i] = val;
    }
}
```

```
# sum_rows2 inner loop
.L4:
   addq    (%r8, %rax, 8)), %rcx
   addq    $1, %rdi
   cmpq    %rcx, %rdi
   jne     .L4
```

- **No need to store intermediate results**

Optimization Blocker: Memory Aliasing

- Aliasing
  - two different memory references specify single location
  - easy to have happen in C
    - since allowed to do address arithmetic
    - direct access to storage structures
  - get in habit of introducing local variables
    - accumulating within loops
    - your way of telling compiler not to check for aliasing

C99 to the Rescue

- **New attribute**
  - **restrict**
    - » **applied to a pointer, tells the compiler that the object pointed to will be accessed only via this pointer**
    - » **compiler thus doesn't have to worry about aliasing**
    - » **but the programmer does ...**
    - » **syntax**

```
int *restrict pointer;
```

Pointers and Arrays

- **`long`** `a[][n]`
  - **a is a 2-D array of longs, the size of each row is n**
- **`long`** `(*b)[n]`
  - **b is a pointer to a 1-D array of size n**

- **a and b are of the same type**

Memory Matters, Fixed

```
/* Sum rows of n X n matrix a
   and store result in vector b  */
void sum_rows1(long n, long (*restrict a)[n], long *restrict b) {
    long i, j;
    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i][j];
    }
}
```

```
# sum_rows1 inner loop
.L3:
   addq    (%rdi), %rax
   addq    $8, %rdi
   cmpq    %rcx, %rdi
   jne     .L3
```

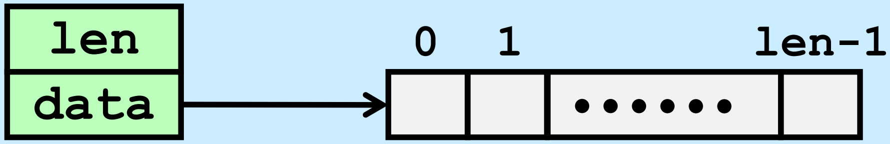
- **Code doesn't update `b[i]` on every iteration**

Exploiting Instruction-Level Parallelism

- Need general understanding of modern processor design
  - hardware can execute multiple instructions in parallel
- Performance limited by data dependencies
- Simple transformations can have dramatic performance improvement
  - compilers often cannot make these transformations
  - lack of associativity and distributivity in floating-point arithmetic

Benchmark Example: Datatype for Vectors

```
/* data structure for vectors */
typedef struct{
    int len;
    data_t *data;
} vec_t, *vec_ptr_t;
```




```
/* retrieve vector element and store at val */
int get_vec_element(vec_ptr_t v, int idx, data_t *val){
     if (idx < 0 || idx >= v->len)
          return 0;
     *val = v->data[idx];
     return 1;
}

/* return length of vector */
int vec_length(vec_ptr_t v) {
      return v->len;
}
```

Benchmark Computation

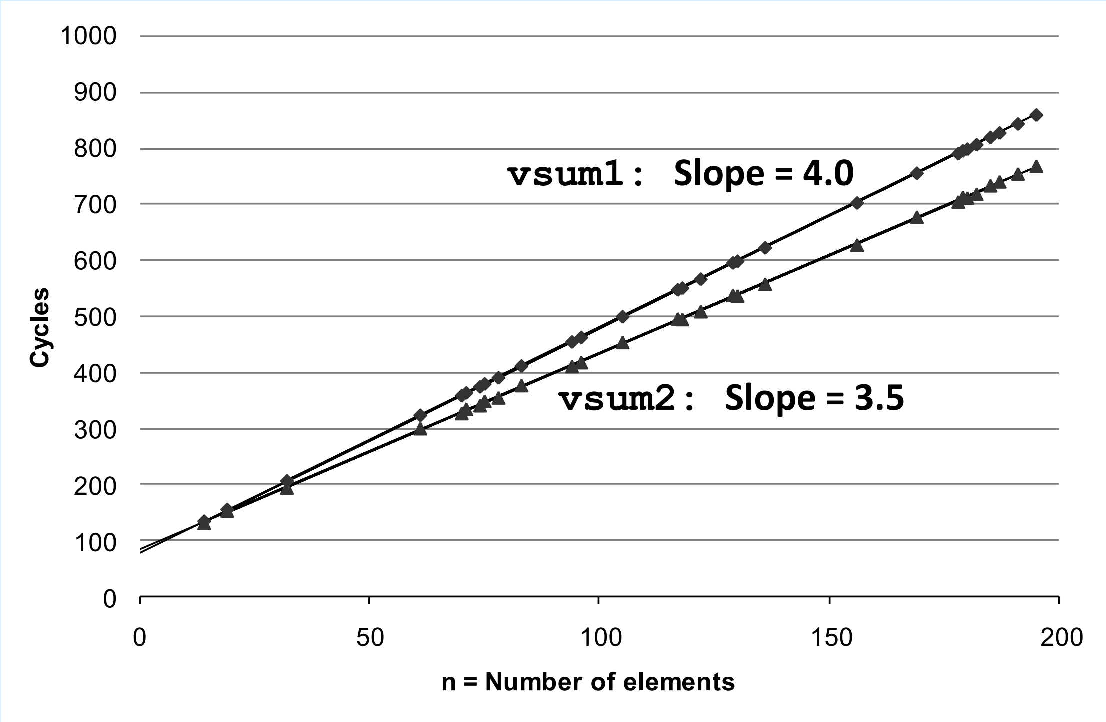
```
void combine1(vec_ptr_t v, data_t *dest){
    long int i;
    *dest = IDENT;
    for (i = 0; i < vec_length(v); i++) {
      data_t val;
      get_vec_element(v, i, &val);
      *dest = *dest OP val;
    }
}
```

Compute sum or product of vector elements

- **Data Types**
  - use different declarations for `data_t`
    - » `int`
    - » `float`
    - » `double`
- **Operations**
  - use different definitions of `OP` and `IDENT`
    - » `+, 0`
    - » `*, 1`

Cycles Per Element (CPE)

- **Convenient way to express performance of program that operates on vectors or lists**
- **Length = n**
- **T = CPE*n + Overhead**
  - **CPE is slope of line**



Benchmark Performance

```
void combine1(vec_ptr_t v, data_t *dest){
    long int i;
    *dest = IDENT;
    for (i = 0; i < vec_length(v); i++) {
       data_t val;
       get_vec_element(v, i, &val);
       *dest = *dest OP val;
    }
}
```

Compute sum or product of vector elements

Method	Integer		Double FP	
Operation	Add	Mult	Add	Mult
Combine1 unoptimized	29.0	29.2	27.4	27.9
Combine1 –O1	12.0	12.0	12.0	13.0

Move vec_length

```
void combine2(vec_ptr_t v, data_t *dest){
    long int i;
    long int length = vec_length(v);
    *dest = IDENT;
    for (i = 0; i < length; i++) {
       data_t val;
       get_vec_element(v, i, &val);
       *dest = *dest OP val;
    }
}
```

Method	Integer		Double FP	
Operation	Add	Mult	Add	Mult
Combine1 unoptimized	29.0	29.2	27.4	27.9
Combine1 –O1	12.0	12.0	12.0	13.0
Combine2	8.03	8.09	10.09	12.08

Eliminate Function Calls

```
void combine3(vec_ptr_t v, data_t *dest){
    long int i;
    long int length = vec_length(v);
    data_t *data = get_vec_start(v);
    *dest = IDENT;
    for (i = 0; i < length; i++) {
       *dest = *dest OP data[i];
    }
}
```

```
data_t *get_vec_start(
      vec_ptr v) {
  return v->data;
}
```

Method	Integer		Double FP	
Operation	Add	Mult	Add	Mult
Combine2	8.03	8.09	10.09	12.08
Combine3	6.01	8.01	10.01	12.02

Eliminate Unneeded Memory References

```
void combine4(vec_ptr_t v, data_t *dest){
  int i;
  int length = vec_length(v);
  data_t *d = get_vec_start(v);
  data_t t = IDENT;
  for (i = 0; i < length; i++)
    t = t OP d[i];
  *dest = t;
}
```

Method	Integer		Double FP	
Operation	Add	Mult	Add	Mult
Combine1 –O1	12.0	12.0	12.0	13.0
Combine4	2.0	3.0	3.0	5.0

Quiz 2

Combine4 is pretty fast; we've done all the "obvious" optimizations. How much faster will we be able to make it? (Hint: it involves taking advantage of pipelining and multiple functional units on the chip.)

a) 1× (it's already as fast as possible)
b) 2× – 4×
c) 16× – 64×