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Πρόβλημα 1

Μαθηματικό μοντέλο:

$$m * v' = (f1 + f2) - b\text{x} * |v| * v \qquad (1)$$

$$Iz * \theta'' = \frac{d}{2}(f2 - f1) - b\theta * |\theta'| * \theta' => \theta'' = \frac{\frac{d}{2}(f2 - f1) - b_\theta * |\theta'| * \theta'}{Iz} \qquad (2)$$

$$v(0) = 0 \qquad (3)$$

$$\theta'(0) = 0 \,, \theta(0) = \theta\text{o} \qquad (4)$$

Τιμες:

$[f1, f2]^T = [4710/7000 = 0.672, 4710/7000 = 0.672]^T$ N

$[f1, f2]^T = [4710/7000 = 0.672, 4710/8000 = 0.589]^T$ N

θ_0 = 4710/20000 = 0.235

b_x = 3 − (4710/5000) = 3 - 0.942 = 2.058

b_θ = 5 − (4710/5000) = 5 - 0.942 = 4.058

m = 9 kg

d = 1 m

Iz = 0.38 kg*m2

*Εγινε στρογγυλοποιηση στα τρια δεκαδικα ψηφια .

A)

Τυποι:

Μέθοδο του Euler: ($y_{n+1} = y_n + h * y'_n$) για [f1, f2]^T = [0.672 , 0.672]^T

$$(1) => \ V_{n+1} = V_n + h * V'_n => \ V_{n+1} = V_n + h\frac{(f1+f2)-bx*|V_n|*V_n}{m} =>$$

$$V_{n+1} = \frac{V_n * m + h*(f1+f2) - bx*|V_n|*V_n}{m} => (\alpha\nu\tau\iota\kappa\alpha\tau\acute{\alpha}\sigma\tau\alpha\sigma\eta) =>$$

$$V_{n+1} = \frac{V_n * 9 + h*(0.672+0.672) - 2.058*|V_n|*V_n}{9} = \frac{V_n * 9 + h*1.344 - 2.058*|V_n|*V_n}{9}$$

$$(1),(3) => \ V_1 = \frac{V_0 * 9 + h*1.344 - 2.058*|V_0|*V_0}{9} = \frac{0 + h*1.344 - 0}{9} => V_1 = \frac{h*1.344}{9}$$

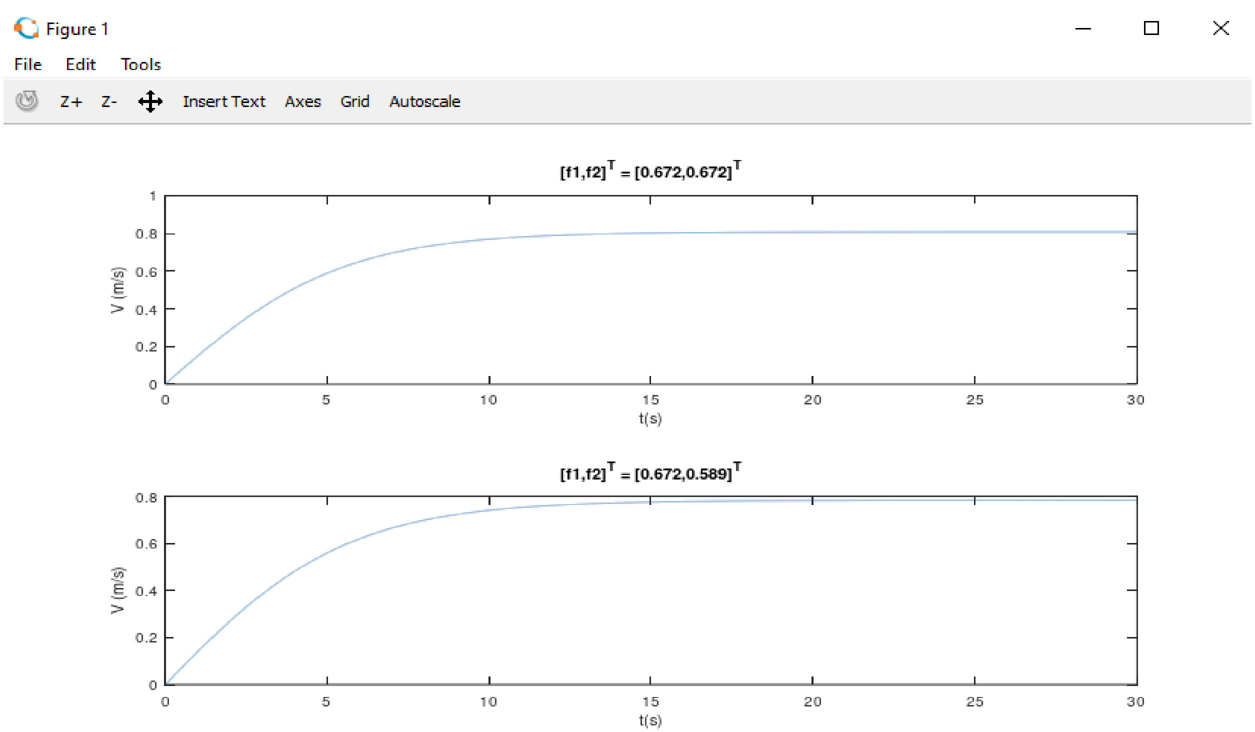
Για [f1, f2]^T = [0.672 , 0.589]^T

$$(1) => V_{n+1} = V_n + h * V'_n => \ V_{n+1} = V_n + h\frac{(f1+f2)-bx*|V_n|*V_n}{m} =>$$

$$V_{n+1} = \frac{V_n * m + h*(f1+f2) - bx*|V_n|*V_n}{m} => (\alpha\nu\tau\iota\kappa\alpha\tau\acute{\alpha}\sigma\tau\alpha\sigma\eta) =>$$

$$V_{n+1} = \frac{V_n * 9 + h*(0.672+0.589) - 2.058*|V_n|*V_n}{9} = \frac{V_n * 9 + h*1.261 - 2.058*|V_n|*V_n}{9}$$

$$(1),(3) => \ V_1 = \frac{V_0 * 9 + h*1.344 - 2.058*|V_0|*V_0}{9} = \frac{0 + h*1.261 - 0}{9} => V_1 = \frac{h*1.261}{9}$$



Για τη (2) και για [f1, f2]^T = [0.672 , 0.672]^T =>

Εστω $\theta'' = y'$ και $\theta' = y$ => (2) => $y' = \frac{\frac{d}{2}(f2-f1)-b_\theta*|y|*y}{Iz}$, $\mu\varepsilon\ \theta'(0) = y(0) = 0$

$$y_{n+1} = y + h*y_n' => (2) => y_{n+1} = y_n + h*\frac{\frac{d}{2}(f2-f1) - b_\theta*|y_n|*y_n}{Iz}$$

$$y_1 = y_0 + h*\frac{\frac{d}{2}(f2-f1) - b_\theta*|y_0|*y_0}{Iz} => (\alpha\nu\tau\iota\kappa\alpha\tau\acute{\alpha}\sigma\tau\alpha\sigma\eta) =>$$

$$y_1 = 0 + h*\frac{\frac{1}{2}(0.672-0.672) - 4.058*|0|*0}{0.38} = 0 => y_1 = 0 = \theta_1' \quad (5)$$

$$\theta_{n+1}' = \theta_n' + h*\frac{\frac{d}{2}(f2-f1) - b_\theta*|\theta_n{}'|*\theta_n{}'}{Iz} \qquad (6)$$

$$\theta_{n+1} = \theta_n + h*\theta_n' \overset{(6)}{=>} \theta_{n+1} = \theta_n + h*(\theta_{n+1}' - h\frac{\frac{d}{2}(f2-f1) - b_\theta*|\theta_n{}'|*\theta_n{}'}{Iz}) \overset{\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta}{=>}$$

$$\theta_1 = \theta_0 + h*(\theta_1' - h\frac{\frac{d}{2}(f2-f1) - b_\theta*|\theta_0{}'|*\theta_0{}'}{Iz}) \overset{(5)}{=>}$$

$$\theta_1 = \theta_0 + h*\left(0 - h\frac{\frac{1}{2}(0) - 4.058*|0|*0}{0.38}\right) => \theta_1 = \theta_0 + 0 = \theta_0 = 0.235$$

Για τη (2) και για [f1, f2]^T = [0.672 , 0.589]^T =>

Εστω $\theta'' = y'$ και $\theta' = y$ => (2) => $y' = \frac{\frac{d}{2}(f2-f1)-b_\theta*|y|*y}{Iz}$, $\mu\varepsilon\ \theta'(0) = y(0) = 0$

$$y_{n+1} = y + h*y_n' => (2) => y_{n+1} = y_n + h*\frac{\frac{d}{2}(f2-f1) - b_\theta*|y_n|*y_n}{Iz}$$

$$y_1 = y_0 + h*\frac{\frac{d}{2}(f2-f1) - b_\theta*|y_0|*y_0}{Iz} => (\alpha\nu\tau\iota\kappa\alpha\tau\acute{\alpha}\sigma\tau\alpha\sigma\eta) =>$$

$$y_1 = 0 + h*\frac{\frac{1}{2}(0.589-0.672) - 4.058*|0|*0}{0.38} = 0 => y_1 = -0.0415*h = \theta_1' \quad (5)$$

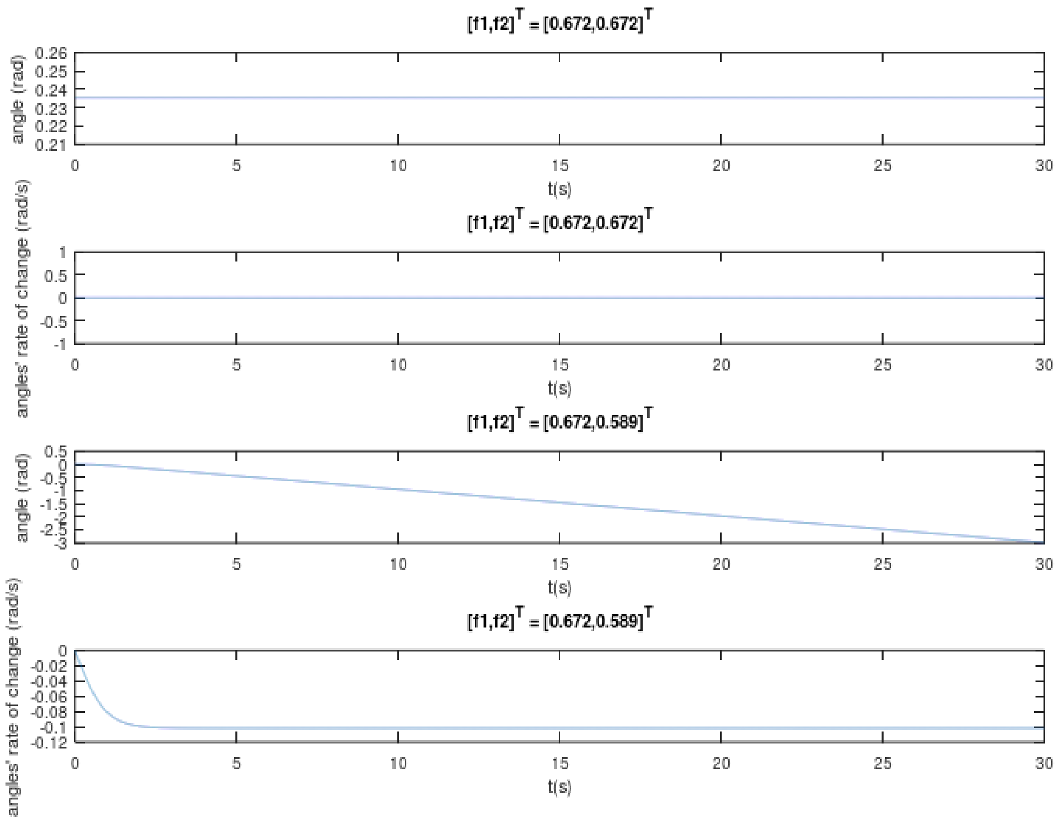
$$\theta_{n+1}' = \theta_n' + h*\frac{\frac{d}{2}(f2-f1) - b_\theta*|\theta_n{}'|*\theta_n{}'}{Iz} \qquad (6)$$

$$\theta_{n+1} = \theta_n + h*\theta_n' \overset{(6)}{=>} \theta_{n+1} = \theta_n + h*(\theta_{n+1}' - h\frac{\frac{d}{2}(f2-f1) - b_\theta*|\theta_n{}'|*\theta_n{}'}{Iz}) \overset{\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta}{=>}$$

$$\theta_1 = \theta_0 + h*\left(\theta_1' - h\frac{\frac{d}{2}(f2-f1) - b_\theta*|\theta_0{}'|*\theta_0{}'}{Iz}\right) \overset{(5)}{=>}$$

$$\theta_1 = \theta_0 + h*\left(-0.0415*h - h\frac{\frac{1}{2}(0.589-0.672) - 4.058*|0|*0}{0.38}\right) =>$$

$$\theta_1 = \theta_0 + h*(-0.0415*h - 0.109*h) = \theta_0 + h*(-0.1505*h) => \theta_1 = 0.235 - 0.1505*h^2$$


[f1,f2]T = [0.672,0.672]T
angle (rad)
0.26
0.25
0.24
0.23
0.22
0.21
0
5
10
15
20
25
30
t(s)
[f1,f2]T = [0.672,0.672]T
angles' rate of change (rad/s)
1
0.5
0
-0.5
-1
t(s)
[f1,f2]T = [0.672,0.589]T
angle (rad)
0.5
0
-0.5
-1
-1.5
-2.5
-3
t(s)
[f1,f2]T = [0.672,0.589]T
angles' rate of change (rad/s)
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
t(s)

Τροποποιημενη μέθοδο του Euler $\boldsymbol{y_{n+1} = y_n + h * f(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n))}$

Για [f1, f2]^T = [0.672 , 0.672]^T:

$$(\mathbf{1}) => \ V_{n+1} = V_n + h * f\left(x_n + \frac{h}{2}, V_n + \frac{h}{2} f(x_n, y_n)\right) =>$$

$$=> V_{n+1} = V_n + h * f\left(x_n + \frac{h}{2}, V_n + \frac{h}{2} \frac{(f1 + f2) - b\text{x} * |V_n| * V_n}{m}\right) \overset{(1)}{=>}$$

$$=> V_{n+1} = V_n + h * \frac{(f1 + f2) - b_x * \left(\left|V_n + \frac{h}{2} \frac{(f1 + f2) - b\text{x} * |V_n| * V_n}{m}\right| * \left(V_n + \frac{h}{2} \frac{(f1 + f2) - b\text{x} * |V_n| * V_n}{m}\right)\right)}{m} \underset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}{=>}$$

$$=> V_{n+1} = V_n + h * \frac{1.344 - 2.058 * \left(\left|V_n + \frac{h}{2} \frac{1.344 - 2.058 * |V_n| * V_n}{9}\right| * \left(V_n + \frac{h}{2} \frac{1.344 - 2.058 * |V_n| * V_n}{9}\right)\right)}{9}$$

$$(\mathbf{1}), (\mathbf{3}) \overset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}{=>}$$

$$=> V_1 = V_0 + h * \frac{1.344 - 2.058 * \left(\left|V_0 + \frac{h}{2} \frac{1.344 - 2.058 * |V_0| * V_0}{9}\right| * \left(V_0 + \frac{h}{2} \frac{1.344 - 2.058 * |V_0| * V_0}{9}\right)\right)}{9}$$

$$=> V_1 = 0 + h * \frac{1.344 - 2.058 * \left(\left|0 + \frac{h}{2} \frac{1.344 - 2.058 * |0| * 0}{9}\right| * \left(0 + \frac{h}{2} \frac{1.344 - 2.058 * |0| * 0}{9}\right)\right)}{9}$$

$$=> V_1 = h * \frac{1.344 - 2.058 * \left(\left|\frac{h}{2} \frac{1.344}{9}\right| * \left(\frac{h}{2} \frac{1.344}{9}\right)\right)}{9} =>$$

$$=> V_1 = h * \frac{1.344 - 2.058 * \left(|h| * \left(h * (\frac{1.344}{18})^2\right)\right)}{9} =>$$

$$=> V_1 = \frac{1.344 * h - |h| * h^2 * 0.011}{9}$$

Για [f1, f2]^T = [0.672 , 0.589]^T :

$$(\mathbf{1}) => \ V_{n+1} = V_n + h * f\left(x_n + \frac{h}{2}, V_n + \frac{h}{2} f(x_n, y_n)\right) =>$$

$$=> V_{n+1} = V_n + h * f\left(x_n + \frac{h}{2}, V_n + \frac{h}{2} \frac{(f1 + f2) - b\text{x} * |V_n| * V_n}{m}\right) \overset{(1)}{=>}$$

$$=> V_{n+1} = V_n + h * \frac{(f1 + f2) - b_x * \left(\left|V_n + \frac{h}{2} \frac{(f1 + f2) - b\text{x} * |V_n| * V_n}{m}\right| * \left(V_n + \frac{h}{2} \frac{(f1 + f2) - b\text{x} * |V_n| * V_n}{m}\right)\right)}{m} =>$$

$\overset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}{=>}$

$$=> V_{n+1} = V_n + h * \frac{1.261 - 2.058 * \left(\left|V_n + \frac{h}{2}\frac{1.261 - 2.058 * |V_n| * V_n}{9}\right| * \left(V_n + \frac{h}{2}\frac{1.261 - 2.058 * |V_n| * V_n}{9}\right)\right)}{9}$$

$$\mathbf{(1)}, \mathbf{(3)} \overset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}{=>}$$

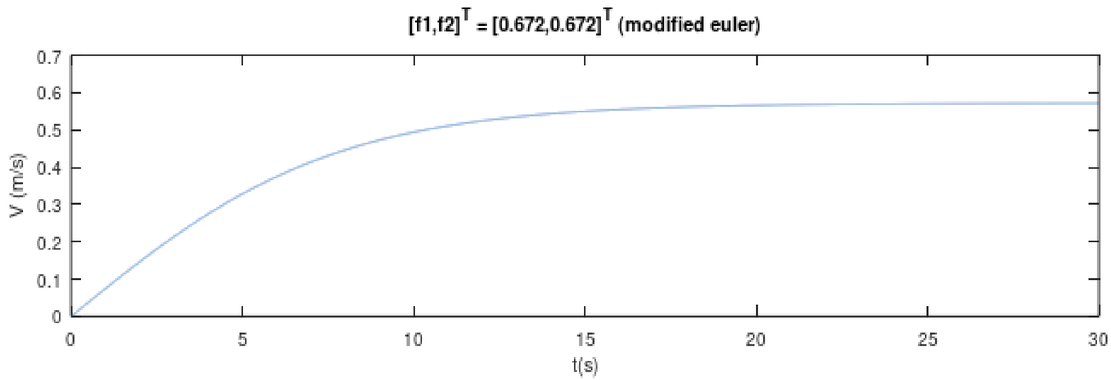
$$=> V_1 = V_0 + h * \frac{1.261 - 2.058 * \left(\left|V_0 + \frac{h}{2}\frac{1.261 - 2.058 * |V_0| * V_0}{9}\right| * \left(V_0 + \frac{h}{2}\frac{1.261 - 2.058 * |V_0| * V_0}{9}\right)\right)}{9}$$

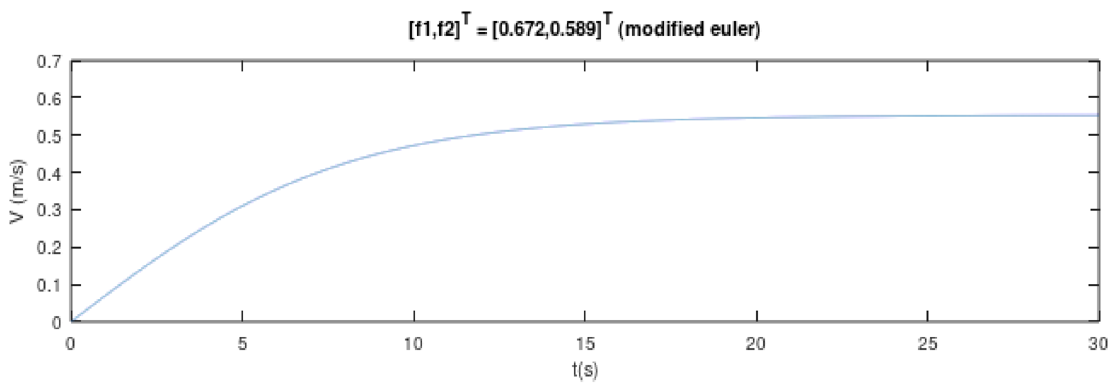
$$=> V_1 = 0 + h * \frac{1.261 - 2.058 * \left(\left|0 + \frac{h}{2}\frac{1.261 - 2.058 * |0| * 0}{9}\right| * \left(0 + \frac{h}{2}\frac{1.261 - 2.058 * |0| * 0}{9}\right)\right)}{9}$$

$$=> V_1 = h * \frac{1.261 - 2.058 * \left(\left|\frac{h}{2}\frac{1.261}{9}\right| * \left(\frac{h}{2}\frac{1.261}{9}\right)\right)}{9} =>$$

$$=> V_1 = h * \frac{1.261 - 2.058 * \left(|h| * \left(h * (\frac{1.261}{18})^2\right)\right)}{9} =>$$

$$=> V_1 = \frac{1.261 * h - |h| * h^2 * 0.005}{9}$$





Τροποποιημενη μέθοδο του Euler $\boldsymbol{y_{n+1} = y_n + h * f(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n))}$

Για [f1, f2]ᵀ = [0.672 , 0.672]ᵀ:

Εστω $\theta'' = y'$ και $\theta' = y$ => (2) => $y' = \frac{\frac{d}{2}(f2 - f1) - b_\theta * |y| * y}{Iz}$ $, \mu\varepsilon\ \theta'(0) = y(0) = 0$ $(\mathbf{5})$

$$(\mathbf{2}) => \ y_{n+1} = y_n + h * f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right) =>$$

$$=> \ y_{n+1} = y_n + h * f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\frac{\frac{d}{2}(f2 - f1) - b_\theta * |y_n| * y_n}{Iz}\right) =>$$

$$y_{n+1} = y_n + h * \frac{\frac{d}{2}(f2 - f1) - b_\theta * \left| y_n + \frac{h}{2}\frac{\frac{d}{2}(f2 - f1) - b_\theta * |y_n| * y_n}{Iz} \right| * (y_n + \frac{h}{2}\frac{\frac{d}{2}(f2 - f1) - b_\theta * |y_n| * y_n}{Iz}}{Iz}$$

$$\underset{=>}{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)} \ y_{n+1} = y_n + h * \frac{0 - 4.058 * \left| y_n + \frac{h}{2}\frac{0 - 4.058 * |y_n| * y_n}{0.38} \right| * \left(y_n + \frac{h}{2}\frac{0 - 4.058 * |y_n| * y_n}{0.38} \right)}{0.38}$$

$$\underset{(\mathbf{2}),(\mathbf{5})}{\overset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}{=>}} \ y_1 = y_0 + h * \frac{-4.058 * \left| y_0 + \frac{h}{2}\frac{-4.058 * |y_0| * y_0}{0.38} \right| * \left(y_0 + \frac{h}{2}\frac{-4.058 * |y_0| * y_0}{0.38} \right)}{0.38}$$

$$y_1 = 0 + h * \frac{-4.058 * |0 + 0| * (0 + 0)}{0.38} = 0 = \theta_1' \quad (6)\ , O\pi o\tau\varepsilon\ \varepsilon\chi o\upsilon\mu\varepsilon:$$

$$\theta'_{n+1} = \theta'_n + h * \frac{\frac{d}{2}(f2 - f1) - b_\theta * \left| \theta'_n + \frac{h}{2}\frac{\frac{d}{2}(f2 - f1) - b_\theta * |\theta'_n| * \theta'_n}{Iz} \right| * (\theta'_n + \frac{h}{2}\frac{\frac{d}{2}(f2 - f1) - b_\theta * |\theta'_n| * \theta'_n}{Iz}}{Iz}$$

$$\underset{=>}{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)} \ \theta'_{n+1} = \theta'_n + h * \frac{-4.058 * \left| \theta'_n + \frac{h}{2}\frac{-4.058 * |\theta'_n| * \theta'_n}{0.38} \right| * \left(\theta'_n + \frac{h}{2}\frac{-4.058 * |\theta'_n| * \theta'_n}{0.38} \right)}{0.38}$$

$$\theta'_{n+1} - h * \frac{-4.058 * \left| \theta'_n + \frac{h}{2}\frac{-4.058 * |\theta'_n| * \theta'_n}{0.38} \right| * \left(\theta'_n + \frac{h}{2}\frac{-4.058 * |\theta'_n| * \theta'_n}{0.38} \right)}{0.38} = \theta'_n = f(x_n, y_n)$$

$$\theta_{n+1} = \theta_n + h * f\left(x_n + \frac{h}{2}, \theta_n + \frac{h}{2}\theta'_n\right) \underset{(\mathbf{2}),(\mathbf{4}),(\mathbf{6})}{\overset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta),(\theta_0 = \ 0.235)}{=>}}$$

$$\theta_1 = \theta_0 + h * f\left(x_n + \frac{h}{2}, \theta_0 + \frac{h}{2}\theta'_0\right) = \ \theta_0 + h * f\left(x_n + \frac{h}{2}, \theta_0 + \frac{h}{2}0\right) = \theta_0 + h * f\left(x_n + \frac{h}{2}, \theta_0\right) =>$$

$$\theta_1 = \theta_0 + h * \ \theta'_1 - h * \frac{-4.058 * \left| \theta'_0 + \frac{h}{2}\frac{-4.058 * |\theta'_0| * \theta'_0}{0.38} \right| * \left(\theta'_0 + \frac{h}{2}\frac{-4.058 * |\theta'_0| * \theta'_0}{0.38} \right)}{0.38}$$

$$\theta_1 = \theta_0 + h * 0 \overset{(6)}{=>} \theta_1 = 0.235$$

Για [f1, f2]^T = [0.672 , 0.589]^T

Εστω $\theta'' = y'$ και $\theta' = y$ => (2) => $y' = \frac{\frac{d}{2}(f2-f1)-b_\theta*|y|*y}{Iz}$, $\mu\varepsilon\ \theta'(0) = y(0) = 0$ (**5**)

$$(\mathbf{2}) => \ y_{n+1} = y_n + h * f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right) =>$$

$$=> \ y_{n+1} = y_n + h * f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\frac{\frac{d}{2}(f2-f1) - b_\theta * |y_n| * y_n}{Iz}\right) =>$$

$$y_{n+1} = y_n + h * \frac{\frac{d}{2}(f2-f1) - b_\theta * \left|y_n + \frac{h}{2}\frac{\frac{d}{2}(f2-f1) - b_\theta*|y_n|*y_n}{Iz}\right| * (y_n + \frac{h}{2}\frac{\frac{d}{2}(f2-f1) - b_\theta*|y_n|*y_n}{Iz}}{Iz}$$

$$\underset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}{=>}$$

$$y_{n+1} = y_n + h * \frac{-0.0415 - 4.058 * \left|y_n + \frac{h}{2}\frac{-0.0415 - 4.058*|y_n|*y_n}{0.38}\right| * \left(y_n + \frac{h}{2}\frac{-0.0415-4.058*|y_n|*y_n}{0.38}\right)}{0.38}$$

$$\underset{(\mathbf{2}),(\mathbf{5})}{\overset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}{=>}}$$

$$y_1 = y_0 + h * \frac{-0.0415 - 4.058 * \left|y_0 + \frac{h}{2}\frac{-0.0415 - 4.058*|y_0|*y_0}{0.38}\right| * \left(y_0 + \frac{h}{2}\frac{-0.0415-4.058*|y_0|*y_0}{0.38}\right)}{0.38}$$

$$y_1 = h * \frac{-0.0415 - 4.058 * \left|\frac{h}{2} * \frac{-0.0415}{0.38}\right| * \left(\frac{h}{2} * \frac{-0.0415}{0.38}\right)}{0.38} =$$

$$y_1 = \frac{-0.042 * h + 0.012 * |h| * h^2}{0.38} = \theta_1' \quad (6)\ ,\ O\pi o\tau\varepsilon\ \varepsilon\chi o\upsilon\mu\varepsilon:$$

$$\theta'_{n+1} = \theta'_n + h * \frac{\frac{d}{2}(f2-f1) - b_\theta * \left|\theta'_n + \frac{h}{2}\frac{\frac{d}{2}(f2-f1) - b_\theta*|\theta'_n|*\theta'_n}{Iz}\right| * (\theta'_n + \frac{h}{2}\frac{\frac{d}{2}(f2-f1) - b_\theta*|\theta'_n|*\theta'_n}{Iz}}{Iz}$$

$$\underset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}{=>}$$

$$\theta'_{n+1} = \theta'_n + h * \frac{-0.0415 - 4.058 * \left|\theta'_n + \frac{h}{2}\frac{-0.0415 - 4.058*|\theta'_n|*\theta'_n}{0.38}\right| * \left(\theta'_n + \frac{h}{2}\frac{-0.0415-4.058*|\theta'_n|*\theta'_n}{0.38}\right)}{0.38}$$

$$\theta'_{n+1} - h * \frac{-0.0415 - 4.058 * \left|\theta'_n + \frac{h}{2}\frac{-0.0415 - 4.058*|\theta'_n|*\theta'_n}{0.38}\right| * \left(\theta'_n + \frac{h}{2}\frac{-0.0415-4.058*|\theta'_n|*\theta'_n}{0.38}\right)}{0.38} = \theta'_n$$

$$\theta_{n+1} = \theta_n + h * f\left(x_n + \frac{h}{2}, \theta_n + \frac{h}{2}\theta'_n\right) \underset{(\mathbf{2}),(\mathbf{4}),(\mathbf{6})}{\overset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta),(\theta_0 = 0.235)}{=>}}$$

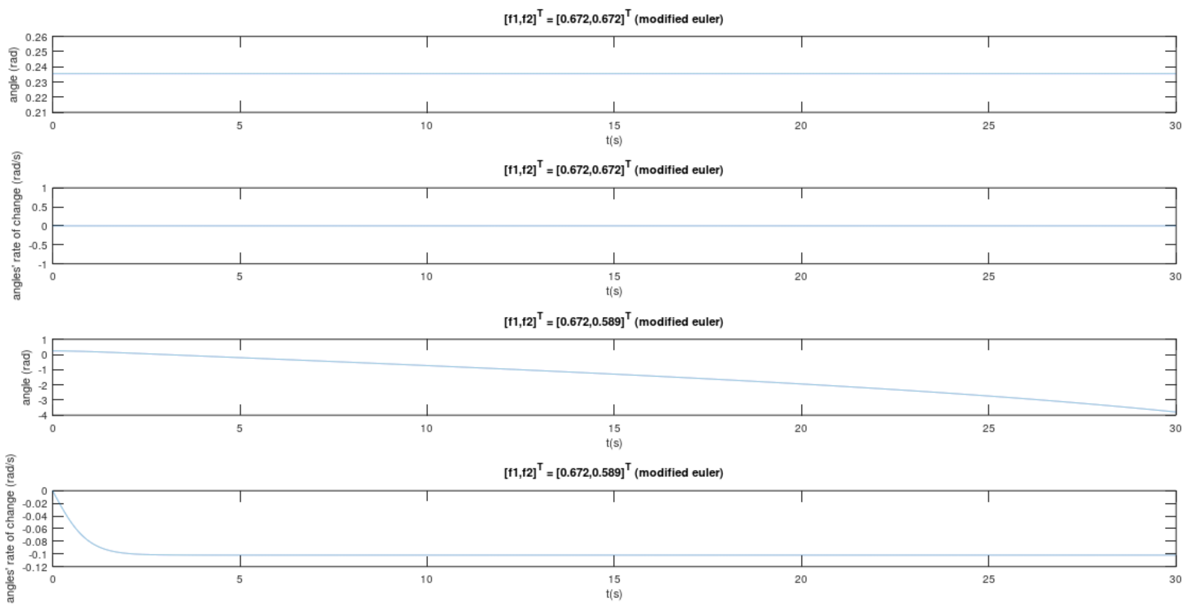
$$\theta_1 = \theta_0 + h * f\left(x_n + \frac{h}{2}, \theta_0 + \frac{h}{2}\theta'_0\right) = \theta_0 + h * f\left(x_n + \frac{h}{2}, \theta_0 + \frac{h}{2}0\right) = \theta_0 + h * f\left(x_n + \frac{h}{2}, \theta_0\right) =>$$

$$\theta_1 = \theta_0 + h * \frac{-0.0415 - 4.058 * \left|\theta_0 + \frac{h}{2}\frac{-0.0415 - 4.058 * |\theta_0| * \theta_0}{0.38}\right| * \left(\theta_0 + \frac{h}{2}\frac{-0.0415 - 4.058 * |\theta_0| * \theta_0}{0.38}\right)}{0.38}$$

$$\theta_1 = 0.235 + h * \frac{-0.0415 - 4.058 * \left|0.235 + \frac{h}{2}\frac{-0.0415 - 4.058 * 0.056}{0.38}\right| * \left(0.235 + \frac{h}{2}\frac{-0.0415 - 4.058 * 0.056}{0.38}\right)}{0.38}$$

$$\theta_1 = 0.235 + \frac{-0.0415 * h + |0.235 - 0.354 * \mathrm{h}| * (-0.954 * h + 1.436 * h^2))}{0.38} \Rightarrow$$

$$\theta_1 = 0.235 - 0.109 * h + |0.235 - 0.354 * \mathrm{h}| * (-2.511 * h + 3.779 * h^2)$$



Γ)

Τιμες:

$(f_2 - f_1) = K_{p\theta}(\theta_{des} - \theta) - K_{d\theta}(\theta') = 5(-0.471 - \theta) - 19.71(\theta')$ (5)

$K_{p\theta} = 5$

$K_{d\theta} = 15 + (4710/1000) = 19.71$

$\theta_0 = 0$

$\theta_{des} = -4710/10000 = -0.471$

$b_\theta = 5 - (4710/5000) = 4.058$

$\theta'(0) = 0$, $\theta(0) = \theta_\circ = 0$

Εξισωση:

$$(2)\ \ Iz*\theta''=\frac{d}{2}(f2-f1)-b_\theta*|\theta'|*\theta' \overset{(5)}{=>}$$

$$=>\ \theta''=\frac{\frac{d}{2}*\mathrm{K_{p\theta}}*\theta_{\mathrm{des}}-\frac{d}{2}*\mathrm{K_{p\theta}}*\theta-\frac{d}{2}*\mathrm{K_{d\theta}}*\theta'-b_\theta*|\theta'|*\theta'}{Iz}\quad(6)$$

$$\theta_0''=\frac{\frac{d}{2}*\mathrm{K_{p\theta}}*\theta_{\mathrm{des}}-\frac{d}{2}*\mathrm{K_{p\theta}}*\theta_0-\frac{d}{2}*\mathrm{K_{d\theta}}*\theta_0'-b_\theta*|\theta_0'|*\theta_0'}{Iz}=>$$

$$\theta_0''=\frac{\frac{d}{2}*\mathrm{K_{p\theta}}*\theta_{\mathrm{des}}}{Iz}=-3.1$$

Μέθοδο του Euler: ($\boldsymbol{y_{n+1}=y_n+h*y_n'}$):

Εστω

$$\Theta''=\mathrm{y}'\ \text{και}\ \theta'=\mathrm{y}\ \text{και}\ \theta=\int y dy \overset{(6)}{=>}$$

$$\mathrm{y}'=\frac{\frac{d}{2}*\mathrm{K_{p\theta}}*\theta_{\mathrm{des}}-\frac{d}{2}*\mathrm{K_{p\theta}}*\int ydy-\frac{d}{2}*\mathrm{K_{d\theta}}*\mathrm{y}-b_\theta*|\mathrm{y}|*\mathrm{y}}{Iz}\quad,\mu\varepsilon\ \theta'(0)=y(0)=0\quad(\mathbf{7})$$

$$\text{και}\ \theta=\int ydy\ \ \text{γινεται},\theta(0)=\theta\text{o}=0=>\ \frac{\mathrm{y_0}^2}{2}+C=0\overset{(7)}{=>}C=0\qquad(8)$$

$$\mathrm{y_{n+1}}=\mathrm{y_n}+\mathrm{h}*\mathrm{y_n'}=>\mathrm{y_{n+1}}=\mathrm{y_n}+\mathrm{h}*\frac{\frac{d}{2}*\mathrm{K_{p\theta}}*\theta_{\mathrm{des}}-\frac{d}{2}*\mathrm{K_{p\theta}}*\int ydy-\frac{d}{2}*\mathrm{K_{d\theta}}*\mathrm{y}-b_\theta*|\mathrm{y}|*\mathrm{y}}{Iz}\ \underset{=>}{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}$$

$$\mathrm{y_1}=\mathrm{y_0}+\mathrm{h}*\frac{\frac{d}{2}*\mathrm{K_{p\theta}}*\theta_{\mathrm{des}}-\frac{d}{2}*\mathrm{K_{p\theta}}*\frac{\mathrm{y_0}^2}{2}+\frac{d}{2}*\mathrm{K_{p\theta}}*\mathrm{C}-\frac{d}{2}*\mathrm{K_{d\theta}}*\mathrm{y_0}-b_\theta*|\mathrm{y_0}|*\mathrm{y_0}}{Iz}=>$$

$$\mathrm{y_1}=0+\mathrm{h}*\frac{\frac{d}{2}*\mathrm{K_{p\theta}}*\theta_{\mathrm{des}}-0+0-0-0}{Iz}\ \underset{(\mathbf{7}),(\mathbf{8})}{\overset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}{=>}}$$

$$\mathrm{y_1}=\mathrm{h}*\frac{\frac{d}{2}*\mathrm{K_{p\theta}}*\theta_{\mathrm{des}}}{Iz}=\mathrm{h}*\frac{\frac{1}{2}*5*(-0.471)}{0.38}=h*\frac{-1.178}{0.38}=>$$

$$=>\ \mathrm{y_1}=-3.1*h=\theta'(1)\qquad(9)$$

$$O\pi o\tau\varepsilon\ \varepsilon\chi o\upsilon\mu\varepsilon:\ \ \theta'_{n+1}=\theta'_n+h*\theta_n''=>$$

$$=>\theta'_{n+1}=\theta'_n+h*\frac{\frac{d}{2}*\mathrm{K_{p\theta}}*\theta_{\mathrm{des}}-\frac{d}{2}*\mathrm{K_{p\theta}}*\theta_n-\frac{d}{2}*\mathrm{K_{d\theta}}*\theta'_n-b_\theta*|\theta'_n|*\theta'_n}{Iz}\quad(10)$$

$$\underset{(7),(\mathbf{9}),(\mathbf{10})}{\overset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}{=>}} \theta'_1 = \theta'_0 + h * \frac{\frac{d}{2} * \mathrm{K_{p\theta}} * \theta_{\mathrm{des}} - \frac{d}{2} * \mathrm{K_{p\theta}} * \theta_0 - \frac{d}{2} * \mathrm{K_{d\theta}} * \theta'_0 - b_\theta * |\theta'_0| * \theta'_0}{Iz} =>$$

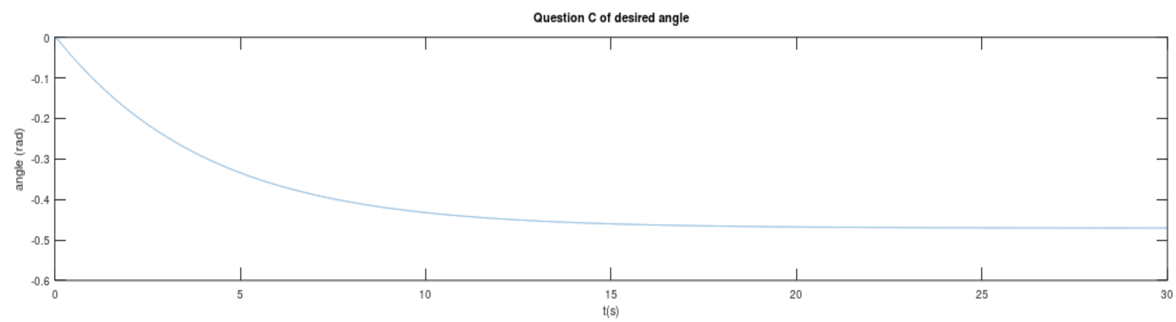
$$-3.1 * h = 0 + h * \frac{\frac{d}{2} * \mathrm{K_{p\theta}} * \theta_{\mathrm{des}} - \frac{d}{2} * \mathrm{K_{p\theta}} * \theta_0 - 0 - 0}{Iz} \overset{(h \neq 0)}{=>}$$

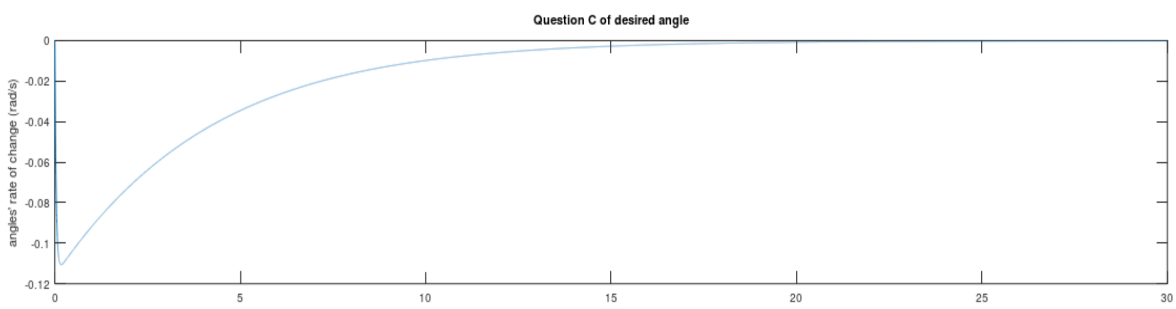
$$-3.1 = \frac{\frac{1}{2} * 5 * -0.471 - \frac{1}{2} * 5 * \theta_0}{0.38} => -3.1 = \frac{-1.178 - \frac{1}{2} * 5 * \theta_0}{0.38} =>$$

$$\theta_0 = \frac{-1.178 + 1.178}{-\frac{1}{2} * 5} = 0$$

$$\theta_{n+1} = \theta_n + h(\theta'_n) = \theta_n + h\left(\frac{\frac{d}{2} * \mathrm{K_{p\theta}} * \theta_{\mathrm{des}} - \frac{d}{2} * \mathrm{K_{p\theta}} * \theta_n - Iz * \theta_n'' - b_\theta * |\theta_n'| * \theta_n'}{\frac{d}{2} * \mathrm{K_{d\theta}}}\right) \overset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}{=>}$$

$$\theta_1 = \theta_0 + h\left(\frac{\frac{d}{2} * \mathrm{K_{p\theta}} * \theta_{\mathrm{des}} - \frac{d}{2} * \mathrm{K_{p\theta}} * \theta_0 - Iz * \theta_0'' - b_\theta * |\theta_0'| * \theta_0'}{\frac{d}{2} * \mathrm{K_{d\theta}}}\right) = h\left(\frac{\frac{d}{2} * \mathrm{K_{p\theta}} * \theta_{\mathrm{des}} - Iz * \theta_0''}{\frac{d}{2} * \mathrm{K_{d\theta}}}\right) = 0$$





Τροποποιημενη μέθοδο του Euler $\boldsymbol{y_{n+1} = y_n + h * f(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n))}$

$\Theta'' = y'$ και $\theta' = y$ και $\theta = \int y dy \overset{(6)}{=>}$

$$y' = \frac{\frac{d}{2} * K_{p\theta} * \theta_{des} - \frac{d}{2} * K_{p\theta} * \int y dy - \frac{d}{2} * K_{d\theta} * y - b_\theta * |y| * y}{Iz} \quad , \mu\varepsilon\ \theta'(0) = y(0) = 0 \quad (\mathbf{7})$$

και $\theta = \int y dy$ γινεται , $\theta(0) = \theta o = 0 => \frac{y_0^2}{2} + C = 0 \overset{(7)}{=>} C = 0$ (8)

$$y_{n+1} = y_n + h * f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right) =>$$

$$y_{n+1} = y_n + h * f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\left(\frac{\frac{d}{2} * K_{p\theta} * \theta_{des} - \frac{d}{2} * K_{p\theta} * \int y dy - \frac{d}{2} * K_{d\theta} * y_n - b_\theta * |y_n| * y_n}{Iz}\right)\right) \underset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}{=>}$$

$$y_1 = y_0 + h * f\left(x_n + \frac{h}{2}, y_0 + \frac{h}{2}\left(\frac{\frac{d}{2} * K_{p\theta} * \theta_{des} - \frac{d}{2} * K_{p\theta} * \int y dy - \frac{d}{2} * K_{d\theta} * y_0 - b_\theta * |y_0| * y_0}{Iz}\right)\right) \underset{\substack{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta) \\ (\mathbf{7}), (\mathbf{8})}}{=>}$$

$$y_1 = h * f\left(x_n + \frac{h}{2}, \frac{h}{2}\left(\frac{\frac{d}{2} * K_{p\theta} * \theta_{des}}{Iz}\right)\right) \underset{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta)}{=>} y_1 = h * f\left(x_n + \frac{h}{2}, \frac{h}{2}\left(\frac{\frac{1}{2} * 5 * (-0.471)}{0.38}\right)\right) =>$$

$$y_1 = h * f\left(x_n + \frac{h}{2}, \frac{-1.178 * h}{0.76}\right) =>$$

$$y_1 = h * \frac{\frac{d}{2} * K_{p\theta} * \theta_{des} - \frac{d}{2} * K_{p\theta} * \int \frac{-1.178 * h}{0.76} dy - \frac{1}{2} * K_{d\theta} * \frac{-1.178 * h}{0.76} - b_\theta * \left|\frac{-1.178 * h}{0.76}\right| * \frac{-1.178 * h}{0.76}}{Iz}$$

$$=> y_1 = h * \frac{\frac{d}{2} * K_{p\theta} * \theta_{des} - \frac{d}{2} * K_{p\theta} * (-\frac{31}{20} * h * y_0 + C) - \frac{1}{2} * K_{d\theta} * \frac{-1.178 * h}{0.76} - b_\theta * \left|\frac{-1.178 * h}{0.76}\right| * \frac{-1.178 * h}{0.76}}{Iz}$$

$$\underset{\substack{(\alpha\nu\tau\iota\kappa\alpha\tau\alpha\sigma\tau\alpha\sigma\eta) \\ (\mathbf{7}), (\mathbf{8})}}{=>}$$

$$y_1 = h * \frac{\frac{1}{2} * 5 * (-0.471) - \frac{1}{2} * 19.71 * \frac{-1.178 * h}{0.76} - 4.058 * \left|\frac{-1.178 * h}{0.76}\right| * \frac{-1.178 * h}{0.76}}{0.38}$$

$$y_1 = \frac{-1.178 * h + 15.275 * h^2 + 9.751 * |h| * h^2}{0.38} = -3.1 * h + 40.2 * h^2 + 25.66 * |h| * h^2 = \theta_1{}' \ (9)$$

Ακομα:

$$\theta_0{}'' = \frac{\frac{d}{2} * \mathrm{K_{p\theta}} * \theta_{\mathrm{des}} - \frac{d}{2} * \mathrm{K_{p\theta}} * \theta_0 - \frac{d}{2} * \mathrm{K_{d\theta}} * \theta_0{}' - b_\theta * |\theta_0{}'| * \theta_0{}'}{Iz} = \frac{\frac{d}{2} * \mathrm{K_{p\theta}} * \theta_{\mathrm{des}}}{Iz} = -3.1$$

Και

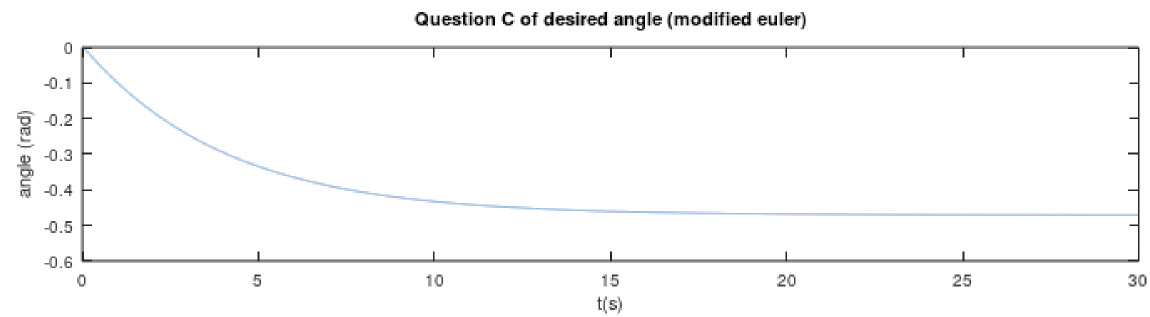
$$\theta' = \frac{\frac{d}{2} * \mathrm{K_{p\theta}} * \theta_{\mathrm{des}} - \frac{d}{2} * \mathrm{K_{p\theta}} * \theta - Iz * \theta'' - b_\theta * |\theta'| * \theta'}{\frac{d}{2} * \mathrm{K_{d\theta}}} \quad (10)$$

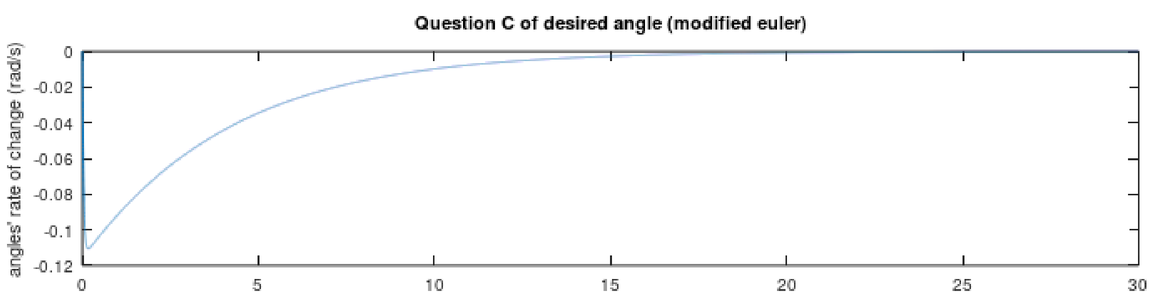
$Κατευθειαν\ αντικατασταση$:

$$\theta_1 = \theta_0 + \mathrm{h} * \mathrm{f}\left(\mathrm{x_n} + \frac{\mathrm{h}}{2}, \theta_0 + \frac{\mathrm{h}}{2}\theta_0{}'\right) = \theta_0 + \mathrm{h} * \mathrm{f}\left(\mathrm{x_n} + \frac{\mathrm{h}}{2}, \theta_0\right) \overset{(10)}{=>}$$

$$\theta_1 = \theta_0 + \mathrm{h} * \frac{\frac{d}{2} * \mathrm{K_{p\theta}} * \theta_{\mathrm{des}} - \frac{d}{2} * \mathrm{K_{p\theta}} * \theta_0 - Iz * \theta_0{}'' - b_\theta * |\theta_0| * \theta_0}{\frac{d}{2} * \mathrm{K_{d\theta}}} \overset{(αντικατασταση)}{=>}$$

$$\theta_1 = \mathrm{h} * \frac{\frac{1}{2} * 5 * -0.471 - 0.38 * (-3.1)}{\frac{1}{2} * 19.71} = \mathrm{h} * \frac{-1.178 + 1.178}{\frac{1}{2} * 19.71} = 0$$





ΠΡΟΒΛΗΜΑ 2:

Α') $Απο\ πινακα\ Laplace: U(s) = \frac{-0.471}{s}\ , εφοσον\ u(t)\ σταθερο$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{L\{y(t)\}}{L\{u(t)\}}$$

$$Iz*\theta'' + \frac{d}{2} * \mathrm{K_{d\theta}} * \theta' + b_\theta * \theta' + \frac{d}{2} * \mathrm{K_{p\theta}} * \theta = \frac{d}{2} * \mathrm{K_{p\theta}} * \theta_{des}$$

$$\underset{=>}{(L.T)} \frac{Iz*s^2}{\frac{d}{2} * \mathrm{K_{p\theta}}} * \mathrm{Y}(s) + \frac{\mathrm{K_{d\theta}} * s}{\mathrm{K_{p\theta}}} * \mathrm{Y}(s) + \frac{b_\theta * s}{\frac{d}{2} * \mathrm{K_{p\theta}}} * \mathrm{Y}(s) + \mathrm{Y}(s) = U(s)$$

$$\mathrm{Y}(s)\left(\frac{Iz*s^2}{\frac{d}{2} * \mathrm{K_{p\theta}}} + \frac{\mathrm{K_{d\theta}} * s}{\mathrm{K_{p\theta}}} + \frac{b_\theta * s}{\frac{d}{2} * \mathrm{K_{p\theta}}} + 1\right) = U(s) =>$$

$$H(s) = \frac{\mathrm{Y}(s)}{U(s)} = \frac{1}{\frac{Iz*s^2}{\frac{d}{2} * \mathrm{K_{p\theta}}} + \frac{\frac{d}{2} * \mathrm{K_{d\theta}} * s}{\frac{d}{2} * \mathrm{K_{p\theta}}} + \frac{b_\theta * s}{\frac{d}{2} * \mathrm{K_{p\theta}}} + \frac{\frac{d}{2} * \mathrm{K_{p\theta}}}{\frac{d}{2} * \mathrm{K_{p\theta}}}} = \frac{\frac{d}{2} * \mathrm{K_{p\theta}}}{Iz*s^2 + (\frac{d}{2} * \mathrm{K_{d\theta}} + b_\theta) * s + \frac{d}{2} * \mathrm{K_{p\theta}}}$$

$$\mathrm{Y}(s) = \frac{\frac{d}{2} * \mathrm{K_{p\theta}}}{Iz*s^2 + (\frac{d}{2} * \mathrm{K_{d\theta}} + b_\theta) * s + \frac{d}{2} * \mathrm{K_{p\theta}}} * \frac{\theta_{des}}{s} = \frac{\theta_{des} * \frac{d}{2} * \mathrm{K_{p\theta}}}{Iz*s^3 + \left(\frac{d}{2} * \mathrm{K_{d\theta}} + b_\theta\right) * s^2 + \frac{d}{2} * \mathrm{K_{p\theta}} * s}$$

$$\mathrm{Y}(s) = \frac{\theta_{des} * \frac{d}{2} * \mathrm{K_{p\theta}}}{(Iz*s^2 + \left(\frac{d}{2} * \mathrm{K_{d\theta}} + b_\theta\right) * s + \frac{d}{2} * \mathrm{K_{p\theta}}) * s} \ \underset{=>}{(διακρινουσα\ κλπ)}$$

$$\Delta = \left(\frac{d}{2} * \mathrm{K_{d\theta}} + b_\theta\right)^2 - 2 * d * Iz * \mathrm{K_{p\theta}}$$

Μηδενικα : $\nexists\ δεν\ υπαρχουν$

Πολοι : s1 = $\frac{-\frac{d}{2}*\mathrm{K_{d\theta}} - b_\theta + \sqrt{\left(\frac{d}{2}*\mathrm{K_{d\theta}} + b_\theta\right)^2 - 2*d*Iz*\mathrm{K_{p\theta}}}}{2*Iz}$, s2 = $\frac{-\frac{d}{2}*\mathrm{K_{d\theta}} - b_\theta - \sqrt{\left(\frac{d}{2}*\mathrm{K_{d\theta}} + b_\theta\right)^2 - 2*d*Iz*\mathrm{K_{p\theta}}}}{2*Iz}$, $s3 = 0$

$$\underset{=>}{(αντικατασταση)} \mathrm{Y}(s) = \frac{-0.471 * \frac{1}{2} * 5}{0.38*s^3 + \left(\frac{1}{2} * 15 + 3.058\right) * s^2 + \frac{1}{2} * 5 * s} = \frac{-1.178}{0.38*s^3 + 10.558 * s^2 + 2.5 * s}$$

$$\mathrm{Y}(s) = \frac{-1.178}{(s+27.545)*(s+0.239)*0.38*s}$$

$$Μερικα\ κλασματα: \frac{-1.178}{(s+27.545)*(s+0.239)*0.38*s} = \left(\frac{\alpha 1}{s+27.545} + \frac{\alpha 2}{s+0.239} + \frac{\alpha 3}{s}\right)$$

$$\alpha 1 = [(s{+}27.545)\frac{-1.178}{(s{+}27.545)*(s{+}0.239)*0.38*s}]_{s=-27.545} = [\frac{-1.178}{(s{+}0.239)*0.38*s}]_{s=-27.545} = \frac{-1.178}{285.917} = -0.0004$$

$$\alpha 2 = [(s{+}0.239)\frac{-1.17}{(s{+}27.545)*(s{+}0.239)*0.38*s}]_{s=-0.239} = [\frac{-1.178}{(s{+}27.545)*0.38*s}]_{s=-0.239} = \frac{-1.178}{-2.481} = 0.475$$

$$\alpha 3 = [s\frac{-1.178}{(s{+}27.545)*(s{+}0.239)*0.38*s}]_{s=0} = [\frac{-1.178}{0.38*(s{+}27.545)*(s{+}0.239)}]_{s=0} = \frac{-1.178}{0.38*(27.545)*(0.239)} = -0.468$$

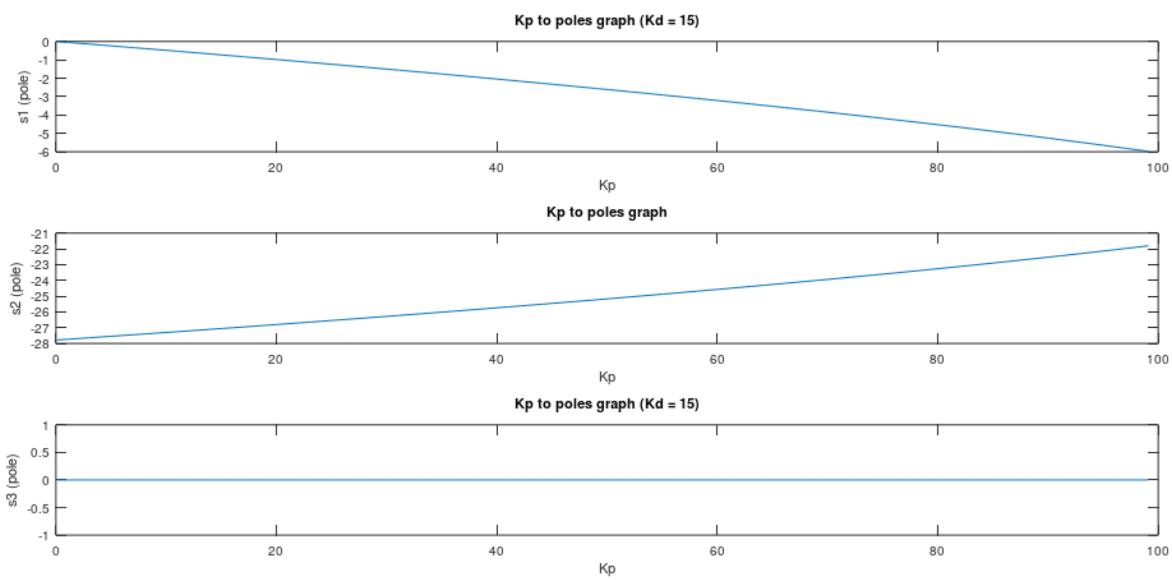
$$\mathrm{Y}(s) = \frac{-0.0004}{s{+}27.545} + \frac{0.475}{s{+}0.239} + \frac{-0.468}{s}$$

Πολοι : s1 = 27.545 , $s2 = 0.239$, $s3 = 0$

Μηδενικα : $\nexists$ *δεν υπαρχουν*

Β')

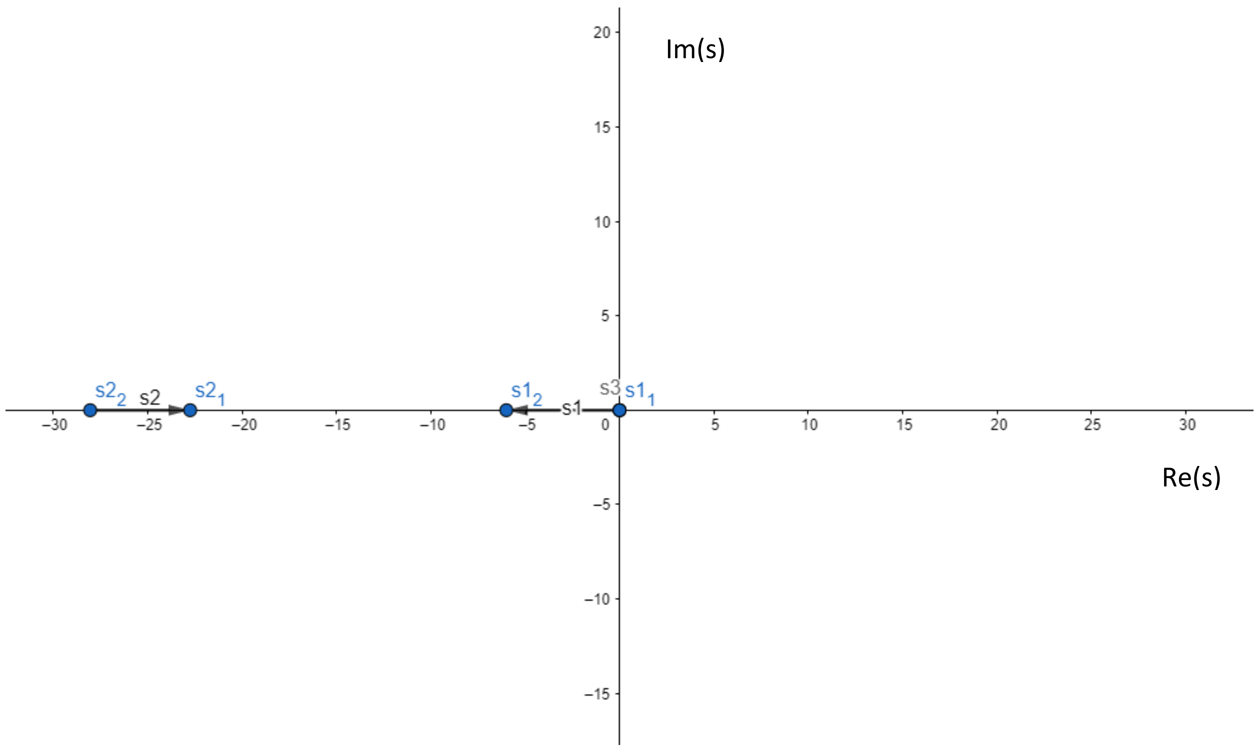
Πραγματικο επιπεδο:



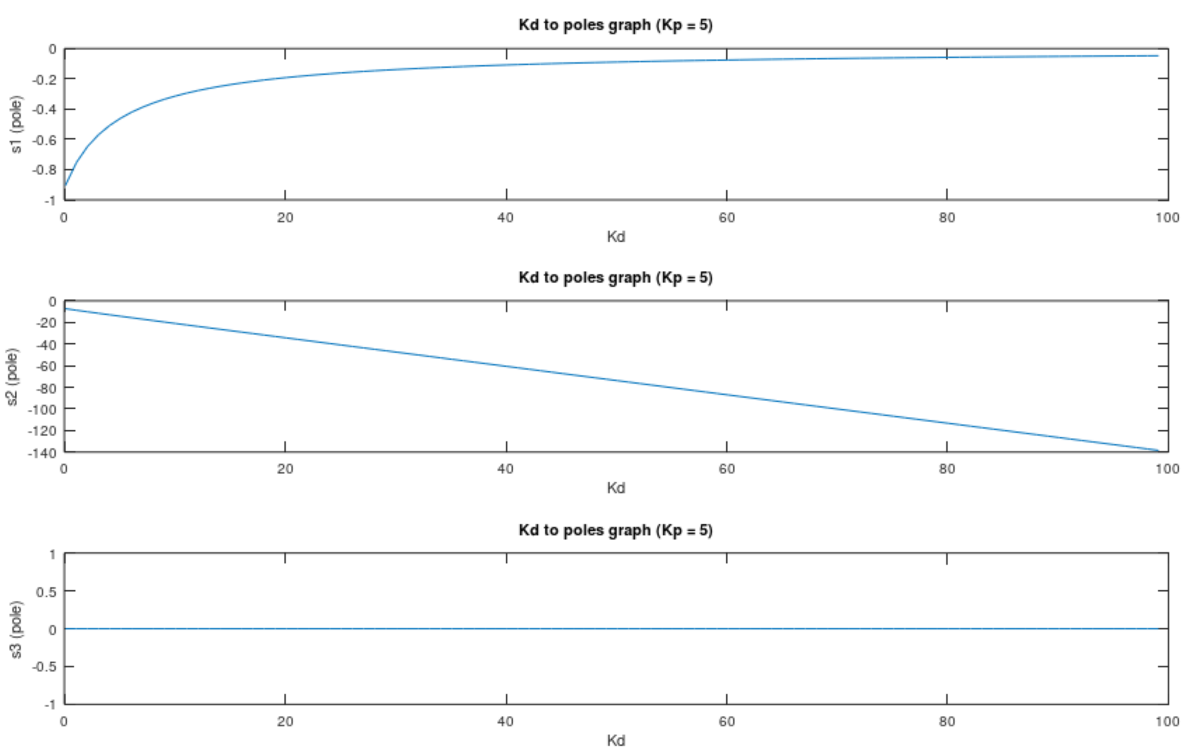
Μορφή της απόκρισης:

Και οι τρεις τιμες παραμενουν πραγματικες και μικροτερες ή ισες του μηδενος αρα εχουμε υπεραπόσβεση

Μιγαδικο επιπεδο:



Πραγματικο επιπεδο:

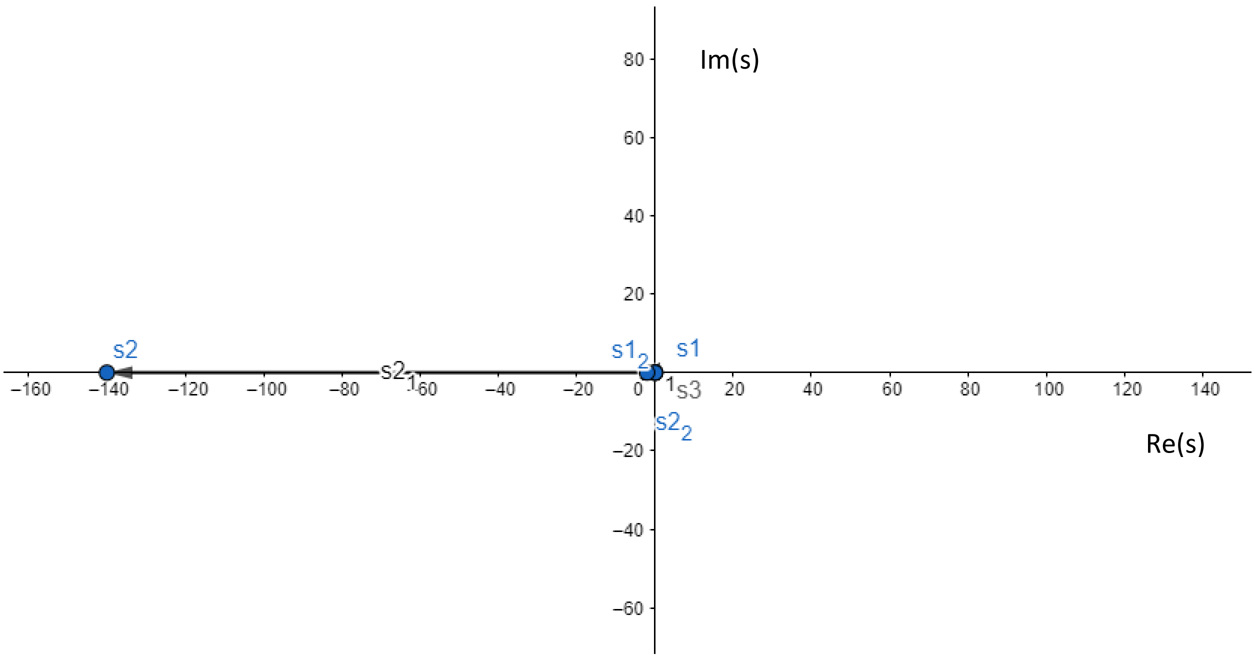


Μορφή της απόκρισης:

Και οι τρεις τιμες παραμενουν πραγματικες και μικροτερες ή ισες του μηδενος αρα εχουμε υπεραπόσβεση

Οσον αφορα την ευσταθεια του συστηματος παρατηρουμε οτι ειναι ασταθες αφου για μικρες αλλαγες στις αρχικες συνθηκες μπορουν να επιφερουν μεγαλες αλλαγες στην λυση καθως προσεγγιζουμε το απειρο

Μιγαδικο επιπεδο:



Γ')

(f2 − f1) = Kpθ(θdes − θ) − Kdθ(θ′) = 5(-0.471 - θ) - 19.71(θ′)   (5)

$K_{p\theta} = 5$

$K_{d\theta} = 15 + (4710/1000) = 19.71$

$\theta_0 = 0$

$\theta_{des} = -4710/10000 = -0.471$

$b_\theta = 5 - (4710/5000) = 4.058$

$\theta'(0) = 0$, $\theta(0) = \theta_\circ = 0$

$$\theta = \frac{-Iz*\theta'' - (\frac{d}{2} * K_{d\theta} + b_\theta) * \theta' + \frac{d}{2} * K_{p\theta} * \theta_{des}}{\frac{d}{2} * K_{p\theta}}$$

$$Iz*\theta'' + (\frac{d}{2} * K_{d\theta} + b_\theta) * \theta' + \frac{d}{2} * K_{p\theta} * \theta = \frac{d}{2} * K_{p\theta} * \theta_{des} =>$$

$0.38*\theta'' + 13.913*\theta' + 2.5*\theta = -1.178 =>$

Αρχικα λυνουμε την ομογενη: $0.38*r^2 + 13.913*r + 2.5 = 0 => (\Delta = 189.772) =>$

$$r_1 = \frac{-13.913 + \sqrt{189.772}}{0.76} = \frac{-13.913 + 13.775}{0.76} = -0.181$$

$$r_2 = \frac{-13.913 - \sqrt{189.772}}{0.76} = \frac{-13.913 - 13.775}{0.76} = -36.431$$

Υποθετουμε οτι η μερικη λυση Θ(t) = α ,αρα:

Θ''(t) = Θ'(t) = 0

Αντικατασταση:

$0.38*\theta'' + 13.913*\theta' + 2.5*\theta = -1.178 => 0 + 0 + 2.5*\alpha = -1.178 => \alpha = \theta_{des} = -0.471$

Αρα η Γ.Λ. ειναι:

$\theta(t) = c_1 * e^{r_1 * t} + c_2 * e^{r_2 * t} - 0.471 = c_1 * e^{-0.181*t} + c_2 * e^{-36.431*t} - 0.471$

$\theta'(t) = -0.181 * c_1 * e^{-0.181*t} - 36.431 * c_2 * e^{-36.431*t}$

$\theta(0) = 0 => c_1 * e^0 + c_2 * e^0 = 0.471 => c_1 + c_2 = 0.471$ (1)

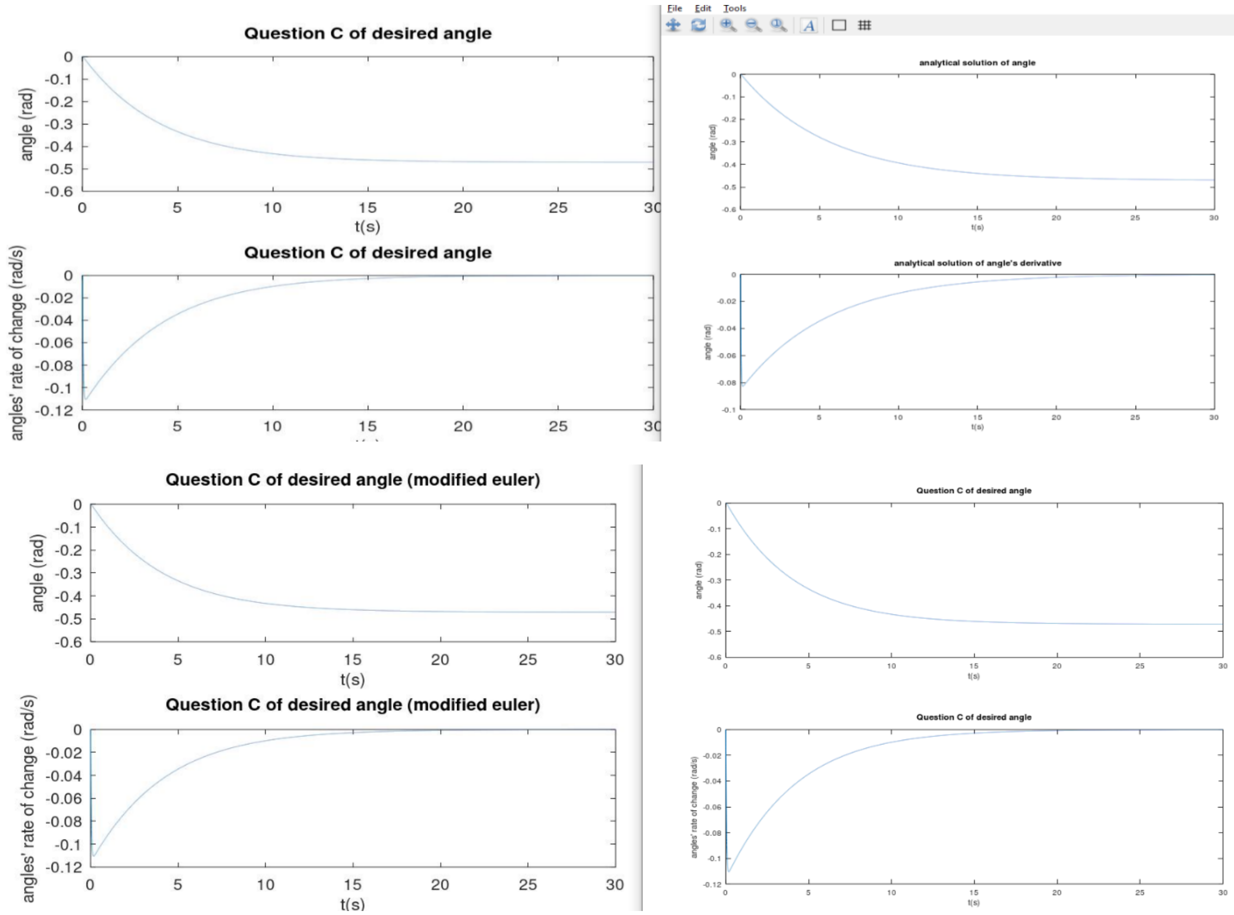
$\theta'(0) = 0 => 0 = -0.181 * c_1 * e^0 - 36.431 * c_2 * e^0 => -0.181 * c_1 - 36.431 * c_2 = 0$ (2)

$$(1),(2) => \begin{cases} c_1 + c_2 = 0.471 \\ -0.181 * c_1 - 36.431 * c_2 = 0 \end{cases} => \begin{cases} c_1 = -c_2 + 0.471 \\ 0.181 * c_2 - 0.085 - 36.431 * c_2 = 0 \end{cases} =>$$

$$\begin{cases} c_1 = -c_2 + 0.471 \\ 36.25 * c_2 = -0.085 \end{cases} => \begin{matrix} c_1 = 0.4687 \\ c_2 = -0.0023 \end{matrix}$$

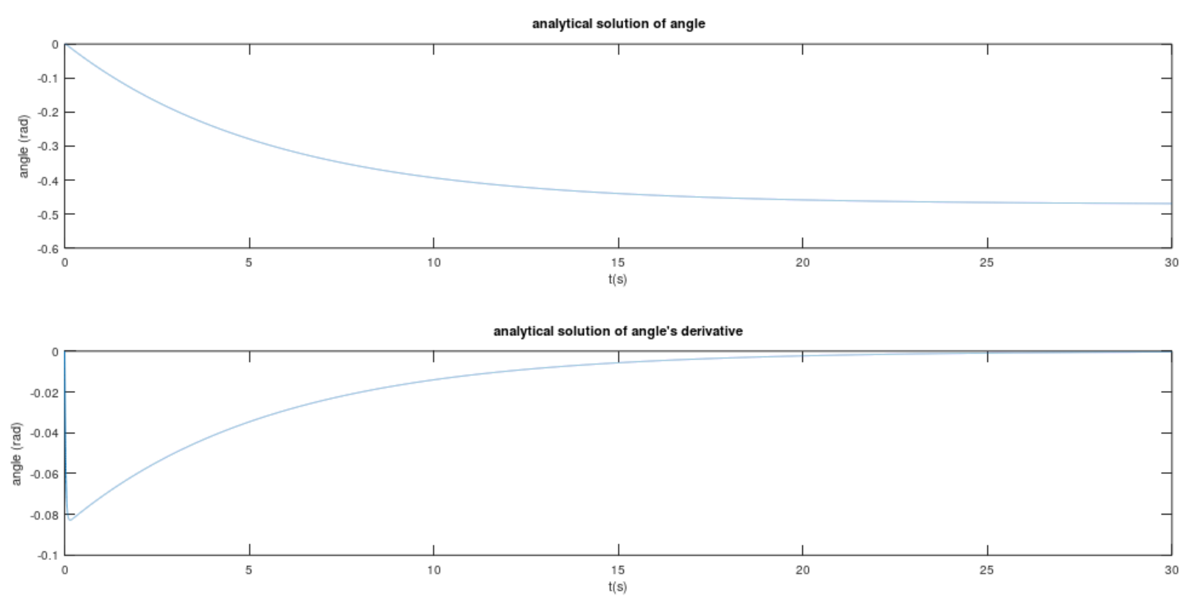
Η γενικη λυση του ΠΑΤ ειναι $\theta(t) = 0.4687 \ * e^{-0.181*t} + -0.0023 * e^{-36.431*t} - 0.471$

Δ’)



Παρατηρουμε οτι ειναι παρομοιες με την Euler να εχει ταξη ακριβειας 1 ενω η τροποποιημενη εχει ταξη ακριβειας 2 σε σχεση με την αναλυτικη λυση.

Ε’)



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