CS10
The Beauty and Joy of Computing

Lecture #11 : Recursion II

2011-10-10

UC Berkeley EECS
Lecturer SOE
Dan Garcia

KINECT COULD ESTIMATE AGE!

Microsoft filed a patent that proposed to use the 3D depth camera to estimate the age of the viewer (height, head-width to shoulder-width, torso length to overall height), and automatically restrict access to content. Tech-savvy kids usu override controls, we're told.
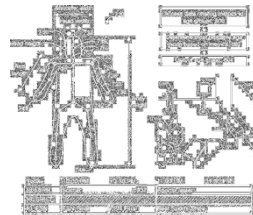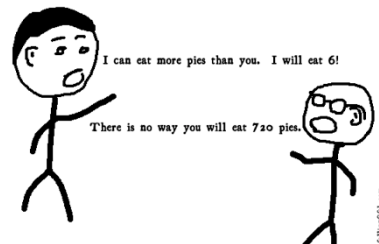
`www.geekwire.com/2011/microsoft-idea-kinect-body-scans-estimate-age-automate-parental-controls`

How the Computer Works … n!

- **Factorial(n) = n!**
  Inductive definition:
  - n! = 1   , n = 0
  - n! = n * (n-1)!, n > 0
- **Let's act it out…**
  - "Little people", or "subcontractor" model
  - **5!**

n	n!
0	1
1	1
2	2
3	6
4	24
5	120

Order of growth of # of calls of n!

(source: FallingFifth.com)

a) **Constant**
b) **Logarithmic**
c) **Linear**
d) **Quadratic**
e) **Exponential**

PIE-EATING CONTEST

I can eat more pies than you. I will eat 6!

There is no way you will eat 720 pies.

How the Computer Works … fib(n)

- **Inductive definition:**
  - fib(n) = n   , n < 2
  - fib(n) = fib(n-1)+fib(n-2), n > 1

$$F(n) := \begin{cases} 0 & \text{if } n = 0; \\ 1 & \text{if } n = 1; \\ F(n-1) + F(n-2) & \text{if } n > 1. \end{cases}$$

- **Let's act it out…**
  - "contractor" model
  - **fib(5)**

n	fib(n)
0	0
1	1
2	1
3	2
4	3
5	5

Leonardo de Pisa
aka, Fibonacci
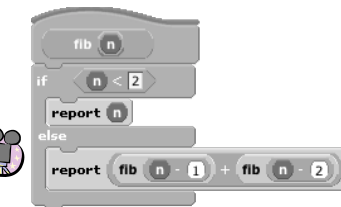
Let's now: trace… (gif from Ybungalobill@wikimedia)

Order of growth of # of calls of fib(n)
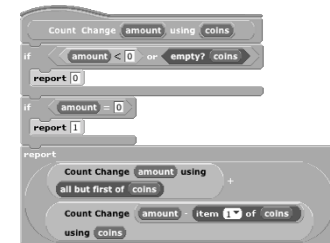
a) Constant
b) Logarithmic
c) Linear
d) Quadratic
e) Exponential

Chimney of Turku Energia, Turku, Finland featuring Fibonacci sequence in 2m high neon lights. By Italian artist Mario Merz for an environmental art project. (Wikipedia)
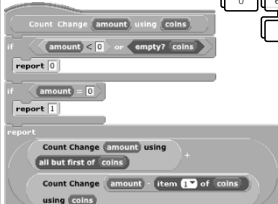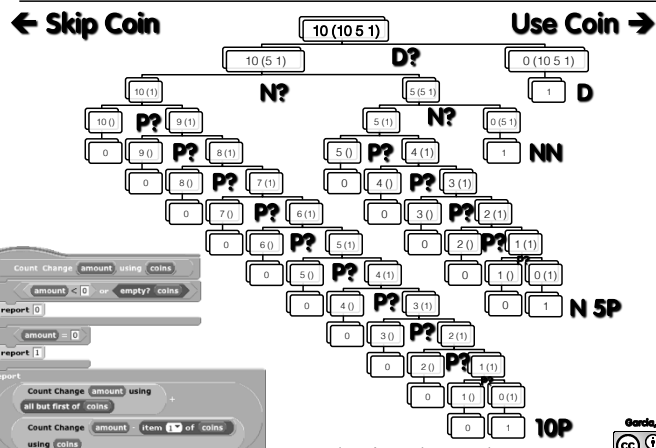

Counting Change (thanks to BH)

- Given coins {50, 25, 10, 5, 1} how many ways are there of making change?
  - 5: 2 (N,5 P)
  - 10
    - 4 (D, 2N, N 5P, 10P)
  - 15
    - 6 (DN,D5P,3N,2N5P,1N10P, 15P)
  - 100?

```
Count Change amount using coins
if   amount < 0  or  empty? coins
report 0

if   amount = 0
report 1

report
      Count Change amount using
      all but first of coins           +
      Count Change amount - item 1 of coins
      using coins
```

Call Tree for "Count Change 10 (10 5 1)"

← Skip Coin   Use Coin →

10 (10 5 1)

D?

N?

P?

NN

N 5P

10P

```
Count Change amount using coins
if   amount < 0  or  empty? coins
report 0

if   amount = 0
report 1

report
      Count Change amount using
      all but first of coins           +
      Count Change amount - item 1 of coins
      using coins
```

Summary

- It's important to understand the machine model
- It's often the cleanest, simplest way to solve many problems
  - Esp those recursive in nature!
- Recursion is a very powerful idea, and one way to separate good from great

Menger Cube by Dan Garcia