UC Berkeley EECS
Sr Lecturer SOE
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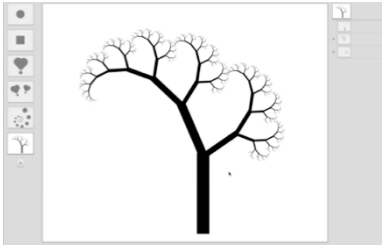
The Beauty and Joy of Computing

Lecture #10
Recursion II

Recursive Drawing

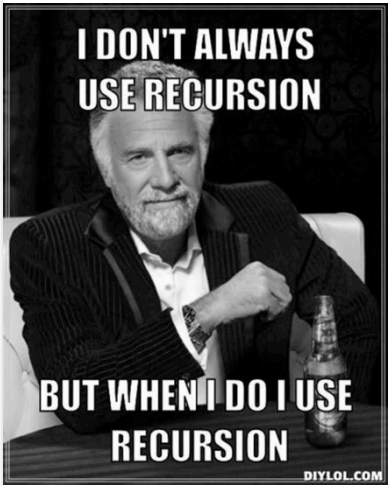
Toby Shachman created this amazing spatial programming language called "Recursive Drawing" that allows you to create drawings (even recursive ones) without typing a line of code. It's a great example of a next-generation interface…



recursivedrawing.com

(Cal) Overview

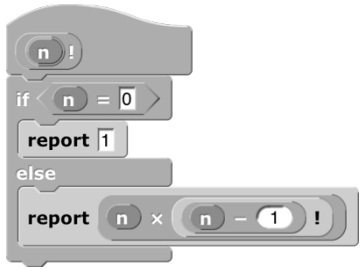
- **Recursion**
  - Factorial Demo
  - Fibonacci Demo
  - Count Change



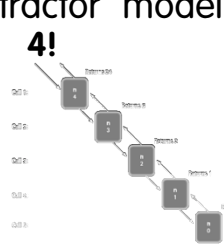
Recursion: Factorial

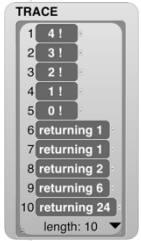
Recursion: Demonstrating n!

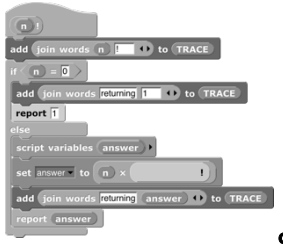
- **Factorial(n) = n!**
  Inductive definition:
  - n! = 1 , n = 0
  - n! = n * (n-1)! , n > 0
- **Let's act 4! out…**
  - "contractor" model



n	n!
0	1
1	1
2	2
3	6
4	24







(Cal) Order of growth of # of calls of n!

(source: FallingFifth.com)

a) Constant
b) Logarithmic
c) Linear
d) Quadratic
e) Exponential

Recursion: Fibonacci

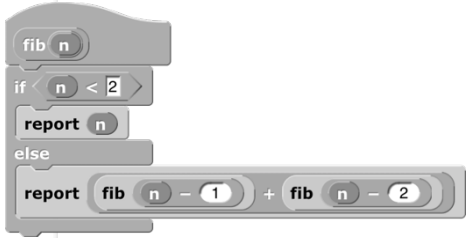
Recursion: Demonstrating fib(n)

- **Inductive definition:**
  - fib(n) = n , n < 2
  - fib(n) = fib(n-1)+fib(n-2) , n ≥ 2

$$F(n) := \begin{cases} 0 & \text{if } n = 0; \\ 1 & \text{if } n = 1; \\ F(n-1) + F(n-2) & \text{if } n > 1. \end{cases}$$

- **Let's act it out…**
  - "contractor" model
  - fib(5)

n	fib(n)
0	0
1	1
2	1
3	2
4	3
5	5



Let's now: trace… (gif from Ybungalobill@wikimedia)

(Cal) Order of growth of # of calls of fib(n)

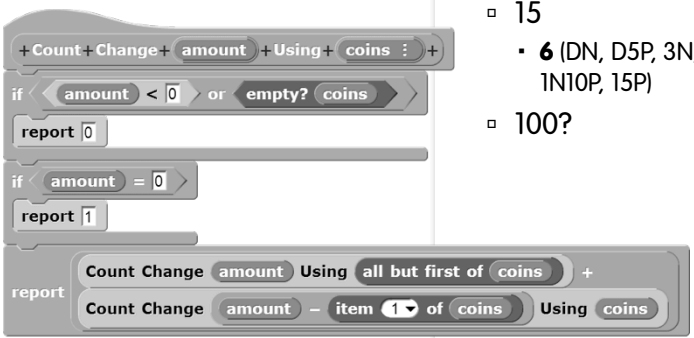
a) Constant
b) Logarithmic
c) Linear
d) Quadratic
e) Exponential

Chimney of Turku Energia, Turku, Finland featuring Fibonacci sequence in 2m high neon lights. By Italian artist Mario Merz for an environmental art project. (Wikipedia)

Recursion: Count Change

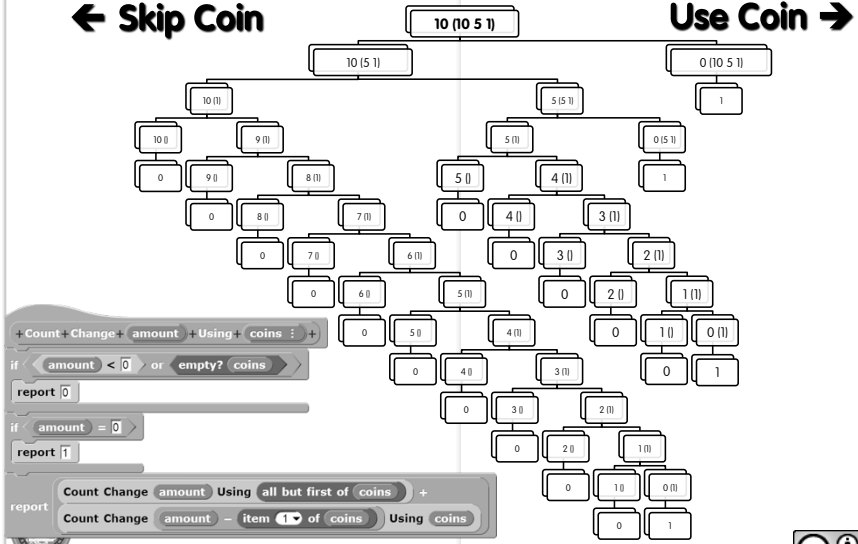
Counting Change (thanks to BH)

- Given coins {50, 25, 10, 5, 1} how many ways are there of making change?



- 5
  - **2** (N, 5P)
- 10
  - **4** (D, 2N, N5P, 10P)
- 15
  - **6** (DN, D5P, 3N, 2N5P, 1N10P, 15P)
- 100?

Call Tree for "Count Change 10 (10 5 1)"



(Cal) "I understood Count Change"

a) Strongly agree
b) Agree
c) Neutral
d) Disagree
e) Strongly disagree

www.nilkanth.com/my-uploads/200508/dactylfractal107.jpg

Summary

- It's important to understand the machine model
- It's often the simplest way to solve many problems
  - Esp if recursive in nature!
- Remember, trust it!
- Recursion is a very powerful idea, often separates good from great (you're great!)

Menger Cube by Dan Garcia