



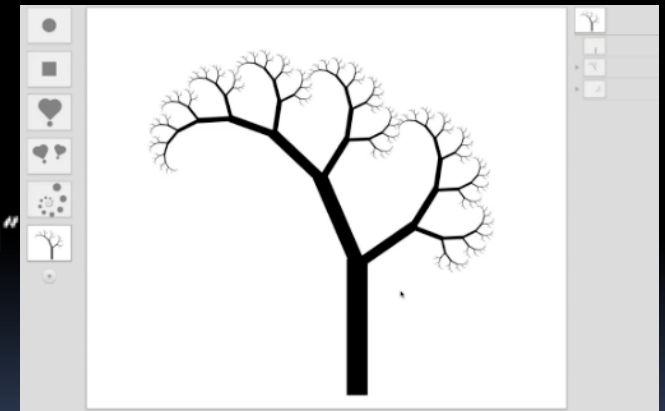
The Beauty and Joy of Computing

Lecture #10 Recursion II



Recursive Drawing

Toby Shachman created this amazing spatial programming language called "Recursive Drawing" that allows you to create drawings (even recursive ones) without typing a line of code. It's a great example of a next-generation interface...



recursivedrawing.com

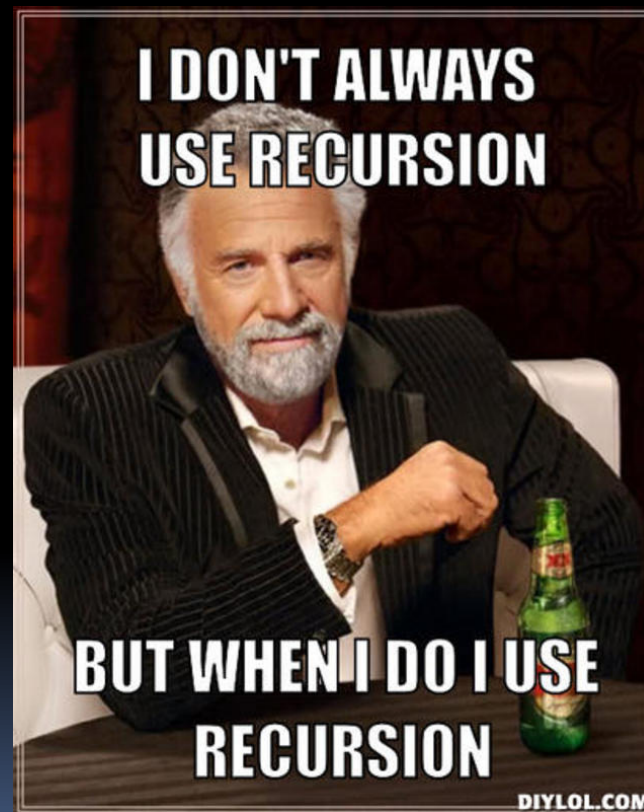


Overview

www.worldofescher.com/gallery/A13.html

■ Recursion

- Factorial Demo
- Fibonacci Demo
- Count Change



Recursion: Factorial



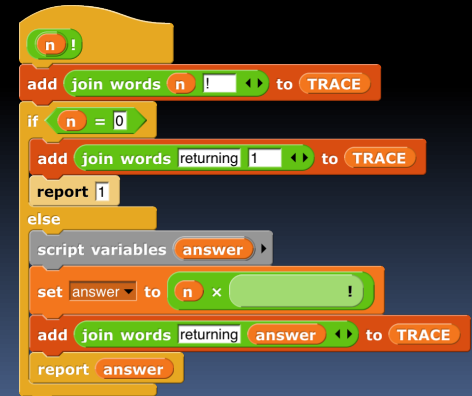
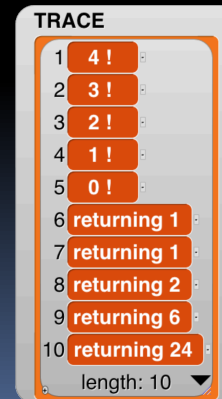
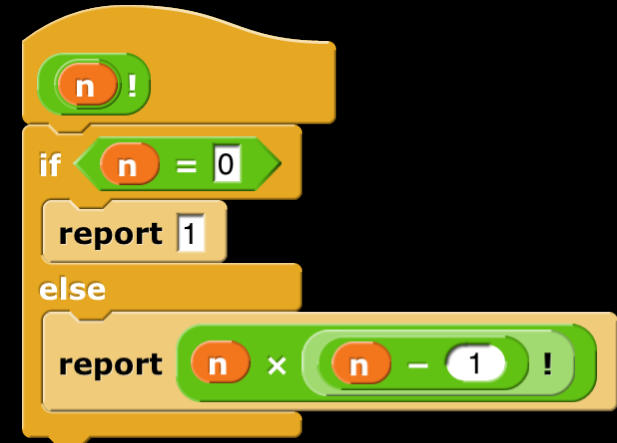
Recursion: Demonstrating $n!$

- $\text{Factorial}(n) = n!$
Inductive definition:
 - $n! = 1$, $n = 0$
 - $n! = n * (n-1)!$, $n > 0$
- Let's act $4!$ out...
 - "contractor" model

n	n!
0	1
1	1
2	2
3	6
4	24



4!



Order of growth of # of calls of $n!$

- a) Constant
- b) Logarithmic
- c) Linear
- d) Quadratic
- e) Exponential



(source: FallingFifth.com)



Recursion: Fibonacci



Recursion: Demonstrating fib(n)

- Inductive definition:

- $\text{fib}(n) = n$, $n < 2$
- $\text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2)$, $n \geq 2$

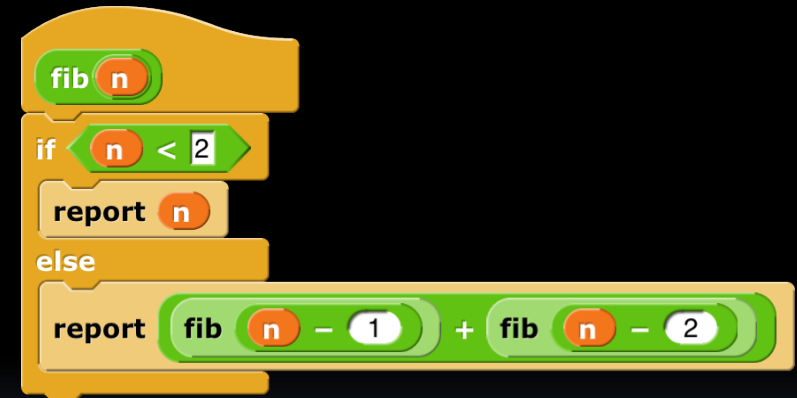
$$F(n) := \begin{cases} 0 & \text{if } n = 0; \\ 1 & \text{if } n = 1; \\ F(n-1) + F(n-2) & \text{if } n > 1. \end{cases}$$

- Let's act it out...

- "contractor" model
- fib(5)

n fib(n)

0	0
1	1
2	1
3	2
4	3
5	5



Let's now: trace... (gif from Ybungalobill@wikimedia)





Order of growth of # of calls of fib(n)

- a) Constant
- b) Logarithmic
- c) Linear
- d) Quadratic
- e) Exponential

Chimney of Turku Energia, Turku, Finland featuring Fibonacci sequence in 2m high neon lights. By Italian artist [Mario Merz](#) for an environmental art project. (Wikipedia)

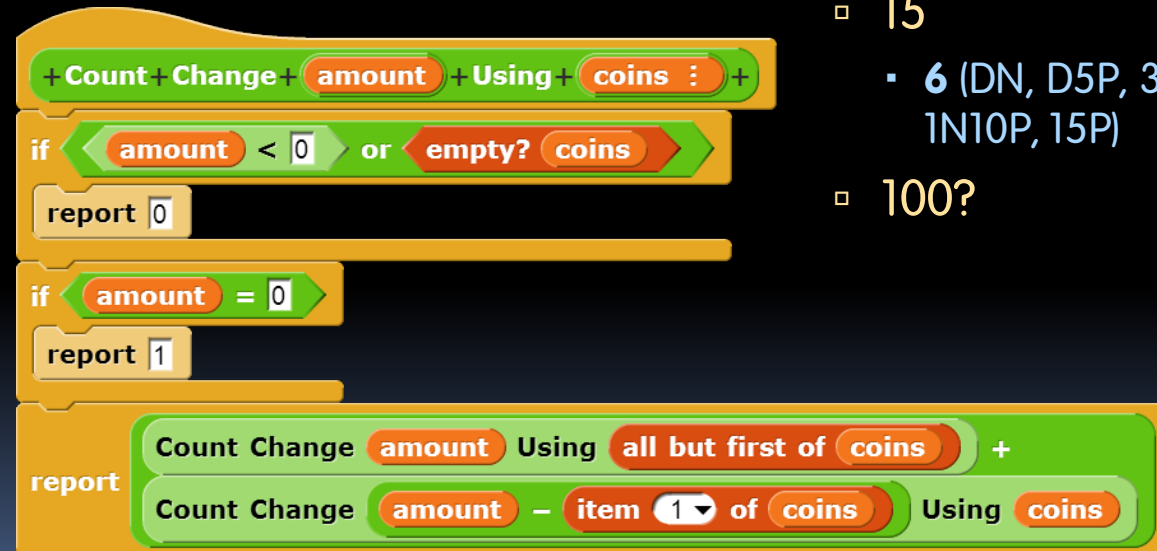


Recursion: Count Change



Counting Change (thanks to BH)

- Given coins {50, 25, 10, 5, 1} how many ways are there of making change?
 - 5
 - 2 (N, 5P)
 - 10
 - 4 (D, 2N, N5P, 10P)
 - 15
 - 6 (DN, D5P, 3N, 2N5P, 1N10P, 15P)
 - 100?

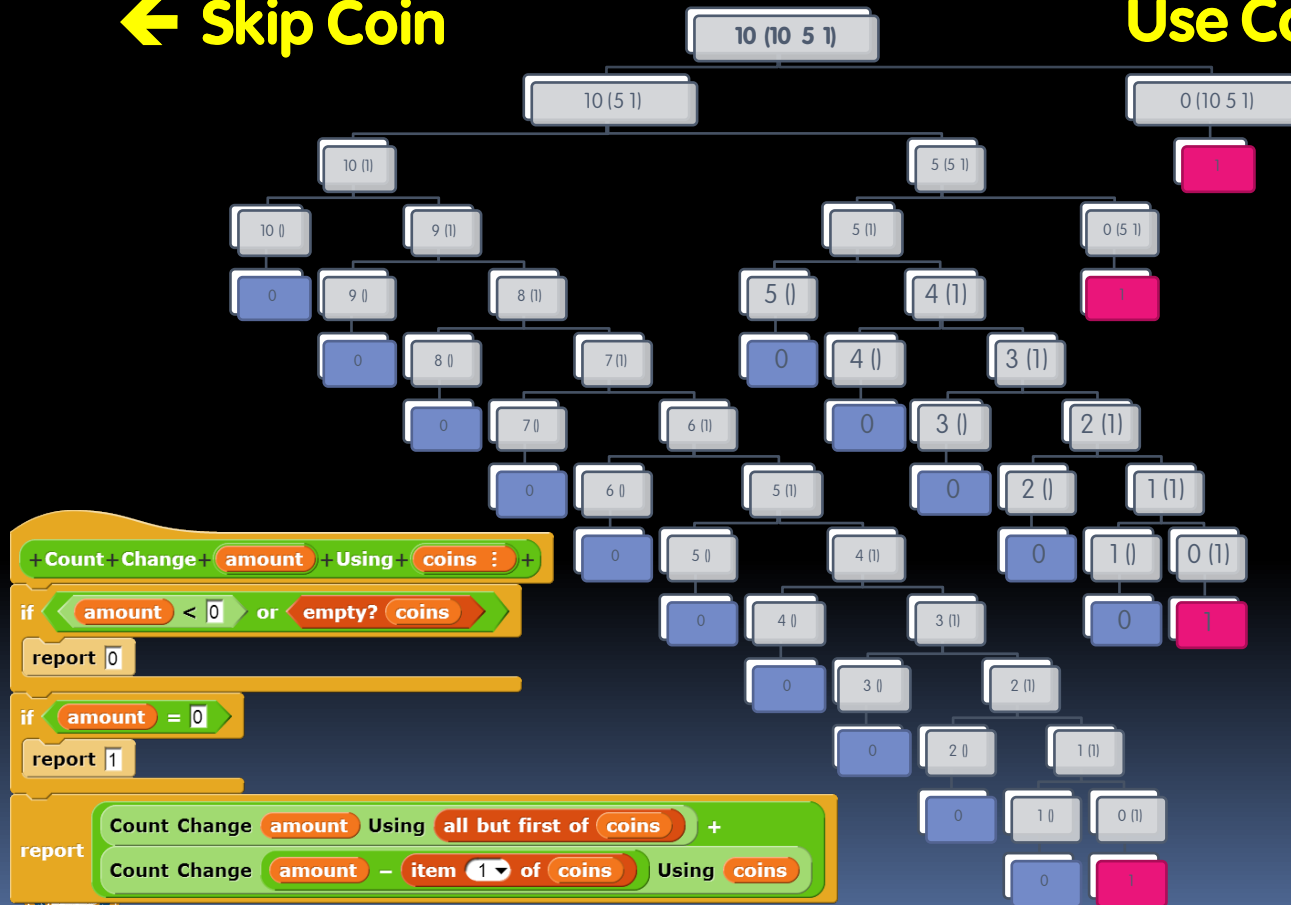




Call Tree for "Count Change 10 (10 5 1)"

← Skip Coin

Use Coin →



```
+ Count + Change + amount + Using + coins : +
if amount < 0 or empty? coins
report 0
if amount = 0
report 1
report Count Change amount Using all but first of coins +
Count Change amount - item 1 of coins Using coins
```





"I understood Count Change"

www.nilkanth.com/my-uploads/200508/dactylfractal107.jpg

- a) Strongly agree
- b) Agree
- c) Neutral
- d) Disagree
- e) Strongly disagree



Summary

- It's important to understand the machine model
- It's often the simplest way to solve many problems
 - Esp if recursive in nature!
- Remember, trust it!
- Recursion is a very powerful idea, often separates good from great (you're great!)

Menger Cube by Dan Garcia

