Algorithm 2 Shapes

Introduction to Computer Graphics, Fall 2021

Solution Key

IInstructions: Complete this assignment by yourself without any help from anyone or anything except a current CS1230 TA, the lecture notes, official textbook, and the professor. Hand in the assignment using `cs1230_handin shapes_algo` no later than 11:59 pm on the due date. Late hand-ins are not accepted under any circumstances.

Notation: p_1 and p_2 refer to the values of Parameter 1 and Parameter 2, respectively.

1 Cube

[1 point] 1.1: Take a look at one face of the cube. Change the tessellation parameter (p_1). Notice how the number of triangles on the face changes. When $p_1 = n$, how many triangles are on one face?

Solution: $2n^2$

[1 point] 1.2: Consider a unit cube at the origin. Consider the face that lies in the +YZ plane (meaning that it is parallel to the YZ plane and is in the positive x domain). What is the normal vector of this face? (Note: when asked for a normal, you should always give a normalized vector, meaning a vector of length one.)

Solution: They are all $(1, 0, 0)$**.**

2 Cylinder

[1½ points] 2.1: The caps of the cylinder are regular polygons with N sides (an "N-gon"), where the value of N is determined by p_2. You will notice the caps are cut up like a pizza with N slices. The vertices of the N-gon lie on a perfect circle parallel to the XZ plane. How will you figure out the coordinates of these vertices in terms of the radius (0.5) and the parameter θ? (What is the equation of the circle that the vertices lie on? Given a vertex on the circle of the cap, the parameter θ is defined as the angle between the x-axis and the line segment from the center of the circle to the point on the edge of the circle.)

Solution: The circle lies on the planes $y = \pm\frac{1}{2}$ and has the equation $x^2 + z^2 = (\frac{1}{2})^2$. You can use the equations $x = r \cdot cos\theta$ and $z = r \cdot sin\theta$ to determine the x and z values of the points on the circle. Here $r = 0.5$; the angle θ is different for each vertex. We divide the full circle by parameter 2 to determine the spacing between each θ.

[1½ points] 2.2: What is the surface normal of the point $(\frac{\sqrt{2}}{4}, \frac{2}{7}, -\frac{\sqrt{2}}{4})$, which lies on the barrel of the cylinder? What about an arbitary point on the barrel of the cylinder (in terms of θ)?

Solution:$(\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2})$, and $(cos\theta, 0, sin\theta)$

3 Cone

[1 point] 3.1: Look at the cone with Y-axis rotation = 0 degrees, and X-axis rotation = 0 degrees. Note that p_2 sets the number of "sides" of the cone (i.e. faces of the cone that are not the base). Observe that when $p_1 = 2$, three triangles make up one "side" of the cone. How many triangles make up one "side" of the cone when p_1= 3? When $p_1 = 5$? When $p_1 = n$?

Solution: You can see that when p1=1, there is just one triangle that makes up one side of the cone. As you increase p1, two triangles are added each time to the side. So there are $(2 \cdot p1) - 1$ triangles on each side.

[1 point] 3.2: What are the vertex normals at the tip of the cone? This is a tricky question because a singularity does not have a normal. Observe that each of the p_2 "sides" of the cone defines a plane. You will achieve a good shading effect by creating p_2 normal vectors extending from the tip of the cone. Each of these vectors should be normal to the plane defined by the "side" of the cone it is associated with, and each of these vectors should point outward. Note that this means that there will not be a unique normal at the tip of the cone, but instead there will be p_2 different normals. Draw a simple schematic sketch illustrating the normals for all of the triangles at the tip of the cone. You may assume p_2 is any number greater than or equal to 3. As long as it is clear that you "get the idea", you will recieve full credit.

Solution:

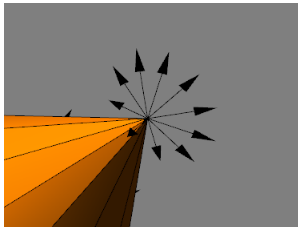


Figure 1: Halo surrounding tip of cone

[1 point] 3.3: Consider the line L in Figure 1, which is formed by the points $(0, \frac{1}{2}, 0)$ and $(\frac{1}{2}, -\frac{1}{2}, 0)$. What is the slope of this line in the XY plane (i.e. what would the slope of this line be if we ignored the z dimension)?

Solution: $m = -2$

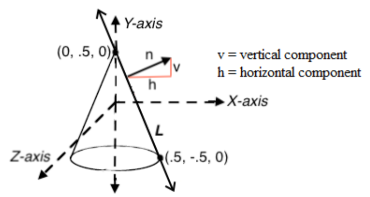


Figure 2: The line representing the sloping side of a cone. This diagram applies to questions 3.3-3.5.

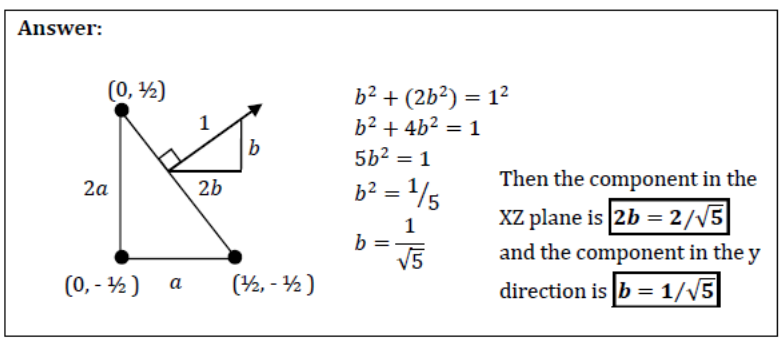
[1 point] 3.4: The line L in Figure 1 represents the sloping side of the cone. Recall that a line with slope m is perpendicular to a line with slope $-\frac{1}{m}$. Using this perpendicular slope, we can find the vertical and horizontal components of the normal n on the cone body. The vertical component is the component parallel to the Y axis. Using the slope you calculated in problem 3.3, calculate the magnitude of this vertical component in the normalized normal vector n. (Your answer should be a number)

Solution: $\frac{1}{\sqrt{5}}$

[1 point] 3.5: The horizontal component is the component parallel to the X axis. What is the magnitude of this component in the normalized normal vector n? (Your answer should be a number)

Solution: $\frac{2}{\sqrt{5}}$

(The below diagram is just to explain the solution)



Solution:

4 Sphere

[1 point] 4.1: In Shapes, you'll need to compute normals yourself. What is the normalized surface normal of the sphere at the surface point $(r, \phi, \theta) = (0.5, \pi/2, \pi/4)$? Give your answer in Cartesian coordinates. Please fully evaluate your answer.

Note that the sphere in the demo is tesselated in the latitude-longitude manner, so the points you want to calculate are straight spherical coordinates. The two parameters can be used as θ and ϕ, or longitude and latitude. The conversion from spherical to Cartesian coordinates is given by...

$$x = r \cdot sin\phi \cdot cos\theta$$

$$y = r \cdot cos\phi$$

$$z = r \cdot sin\phi \cdot sin\theta$$

Solution: $(0.7071, 0, 0.7071)$

5 Design

[1 point] 5.1: In your implementation, you will find that you will reuse functions for different shapes. There are two ways to prevent your implementation from copying code: composition and inheritance. When using inheritance, you extend a base class so that a derived class gains the base class' functionality. When using composition, you add member variables with the desired functionality to a class so that the class gains the functionality of those member variables. Suppose you have a method in an abstract `Shape` class that you would like to use in all of your concrete shape classes (i.e. `Cube`, `Cylinder`, `Sphere`, and `Cone`). In this case, should you use composition, or should you use or inheritance? Suppose you have a method that only your `Cone` and `Cylinder` classes need to use, such as drawing a circular base. In this case, should you use composition, or should you use inheritance?

Solution: Inheritance, Composition

Inheritance should be used when all subclasses require an identical set of functionality. In all other cases use composition. Avoid adding functionality that might be useful but is not common to the base class. This will make your code cleaner and extensible.