industrial p02 notes

cs

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1 Implementation Summary

The provided notebook implements a spatial agent-based simulation on a 100×100 lattice representing the topology of Edinburgh. The map incorporates hard-coded geographic features, including parks (e.g., The Meadows), roads, and fixed amenities (schools, gyms, shops). Agents are initialized with income levels (derived from Scottish Index of Multiple Deprivation data) and heterogeneous attributes including religion, language, and age category.

The simulation evolves through two coupled dynamic processes:

1. **Price Evolution:** Housing prices V at location $\mathbf{x}$ are updated iteratively based on occupant income A and the average price of the Moore neighborhood $\mathcal{N}(\mathbf{x})$:

$$V^{t+1}(\mathbf{x}) = V^t(\mathbf{x}) + A^t(\mathbf{x}) + \lambda \frac{\sum_{\mathbf{y} \in \mathcal{N}(\mathbf{x})} V^t(\mathbf{y})}{\# \mathcal{N}(\mathbf{x})}$$

2. **Agent Mobility:** Unlike the vacancy-based model described in the text, the code utilizes pairwise swapping (Kawasaki dynamics). Two randomly selected grid cells $\mathbf{x}$ and $\mathbf{y}$ exchange occupants if the swap yields a positive net change in utility ($\Delta > 0$). The utility function balances economic affordability with social amenities:

$$\Delta = \Delta_{\text{money}} + \Delta_{\text{amenities}}$$

where Δ_{money} minimizes the squared difference between household income and property price, and $\Delta_{\text{amenities}}$ maximizes cultural similarity with neighbors (Schelling mechanism) and proximity to preferred facilities.

2 Entropy implementation

To quantify segregation, we measure how "mixed" each neighbourhood is. Let i index a habitable cell, and let $N(i)$ be a square window of radius r around i. Within this window, we compute the local proportions of the two groups $g \in \{0, 1\}$:

$$p_{i,g} = \frac{\#\{j \in N(i) : \text{group}(j) = g\}}{\# N(i)}.$$

If a neighbourhood is fully mixed, these proportions are close to $1/2$; if it is segregated, one of them is close to 1 and the other close to 0.

To convert these proportions into a single measure of "local disorder", we use the Shannon entropy:

$$H_i = -\sum_g p_{i,g} \ln p_{i,g}, \qquad 0 \ln 0 := 0.$$

Entropy is maximised when both groups are equally present, and minimised when one group dominates. For two groups, the maximum entropy is $\ln 2$, so we define a normalised entropy:

$$\tilde{H}_i = \frac{H_i}{\ln 2} \in [0, 1].$$

Here $\tilde{H}_i \approx 1$ means a well-mixed neighbourhood, while $\tilde{H}_i \approx 0$ indicates homogeneity.

To obtain a city-wide measure, we average $\tilde{H}_i$ over all habitable cells $\mathcal{H}$:

$$\tilde{H}_{\text{mean}} = \frac{1}{|\mathcal{H}|} \sum_{i \in \mathcal{H}} \tilde{H}_i.$$

Finally, because segregation corresponds to *low* entropy, we define the segregation index

$$S = 1 - \tilde{H}_{\text{mean}}, \qquad S \in [0, 1].$$

Thus $S = 0$ denotes a perfectly mixed configuration, while $S = 1$ denotes a perfectly segregated configuration.

Implementation summary of sch_{002}

3 Environment Initialization

The simulation environment is discretized into an $N \times M$ grid (100×100), represented by variables `N` and `M`. Each cell is classified via a `city_mask` as `RESIDENTIAL`, `PARK`, `AMENITY`, or `EMPTY`.

3.1 Topology Generation

The map topology approximates Edinburgh using vectorized geometric constraints. Parks are defined by boolean unions of rotated ellipses:

$$\left(\frac{x'}{r_x}\right)^2 + \left(\frac{y'}{r_y}\right)^2 \leq 1 \tag{1}$$

implemented in the helper `rotated_oval`. Roads are defined by linear inequalities. Static amenity scores ($A_{i,j}$) are precomputed by convolving binary amenity locations with an inverse-distance kernel $K(d) = (1 + d)^{-1}$, stored in `amenity_scores`.

3.2 Agent Properties

Habitable cells are populated with agents possessing:

- **Income (I):** Drawn from `INCOME_VALUES` $\{0.1, 0.5, 1.0\}$ with probability distribution `INCOME_PROBS`.
- **Group (g):** Binary classification stored in `groups`, initialized with a 70/30 split.
- **Price (P):** Initialized proportional to income plus Gaussian noise, stored in `prices`.

4 System Dynamics

The simulation proceeds for `NUM_STEPS` iterations. Each step involves economic updates and agent relocation attempts.

4.1 Price Evolution

Housing prices evolve based on intrinsic agent income and the average price of the Moore neighborhood ($\mathcal{N}$). The update rule in `update_prices` is:

$$P_{t+1} = P_t + I + \lambda \cdot \langle P \rangle_{\mathcal{N}} \tag{2}$$

where λ corresponds to `LAMBDA_PRICE` (0.01). This simulates gentrification where high-income neighbors drive up local property values.

4.2 Agent Relocation (Swaps)

In `attempt_moves`, we randomly sample pairs of agents at locations x and y. A swap occurs if the utility change is favorable. The utility change ΔU considers financial, amenity, and social factors.

The financial incentive (ΔM), stored as `d_money`, prevents poorer agents from moving to expensive areas:

$$\Delta M = 2(P_x - P_y)(I_x - I_y) \tag{3}$$

Amenity gain is the difference in static scores A. The social score H (homophily) is the fraction of similar neighbors, calculated via `get_social_score`. A swap is executed if financial feasibility is met ($\Delta M > -500$) and the combined utility increases.

5 Metrics: Segregation Entropy

Segregation is measured using spatial Shannon entropy in `segregation_entropy`. For a local neighborhood with proportion p of group 1, the local entropy is:

$$H_{local} = -\sum_{g \in \{0,1\}} p_g \log_2(p_g) \tag{4}$$

The global segregation index S is defined as $1 - \bar{H}_{local}$, where $\bar{H}$ is the mean entropy over habitable cells.