

# industrial p02 notes

cs

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## 1 Implementation Summary

The provided notebook implements a spatial agent-based simulation on a  $100 \times 100$  lattice representing the topology of Edinburgh. The map incorporates hard-coded geographic features, including parks (e.g., The Meadows), roads, and fixed amenities (schools, gyms, shops). Agents are initialized with income levels (derived from Scottish Index of Multiple Deprivation data) and heterogeneous attributes including religion, language, and age category.

The simulation evolves through two coupled dynamic processes:

1. **Price Evolution:** Housing prices  $V$  at location  $\mathbf{x}$  are updated iteratively based on occupant income  $A$  and the average price of the Moore neighborhood  $\mathcal{N}(\mathbf{x})$ :

$$V^{t+1}(\mathbf{x}) = V^t(\mathbf{x}) + A^t(\mathbf{x}) + \lambda \frac{\sum_{\mathbf{y} \in \mathcal{N}(\mathbf{x})} V^t(\mathbf{y})}{\#\mathcal{N}(\mathbf{x})}$$

2. **Agent Mobility:** Unlike the vacancy-based model described in the text, the code utilizes pairwise swapping (Kawasaki dynamics). Two randomly selected grid cells  $\mathbf{x}$  and  $\mathbf{y}$  exchange occupants if the swap yields a positive net change in utility ( $\Delta > 0$ ). The utility function balances economic affordability with social amenities:

$$\Delta = \Delta_{\text{money}} + \Delta_{\text{amenities}}$$

where  $\Delta_{\text{money}}$  minimizes the squared difference between household income and property price, and  $\Delta_{\text{amenities}}$  maximizes cultural similarity with neighbors (Schelling mechanism) and proximity to preferred facilities.

## 2 Entropy implementation

To quantify segregation, we measure how “mixed” each neighbourhood is. Let  $i$  index a habitable cell, and let  $N(i)$  be a square window of radius  $r$  around  $i$ . Within this window, we compute the local proportions of the two groups  $g \in \{0, 1\}$ :

$$p_{i,g} = \frac{\#\{j \in N(i) : \text{group}(j) = g\}}{\#N(i)}.$$

If a neighbourhood is fully mixed, these proportions are close to  $1/2$ ; if it is segregated, one of them is close to 1 and the other close to 0.

To convert these proportions into a single measure of “local disorder”, we use the Shannon entropy:

$$H_i = - \sum_g p_{i,g} \ln p_{i,g}, \quad 0 \ln 0 := 0.$$

Entropy is maximised when both groups are equally present, and minimised when one group dominates. For two groups, the maximum entropy is  $\ln 2$ , so we define a normalised entropy:

$$\tilde{H}_i = \frac{H_i}{\ln 2} \in [0, 1].$$

Here  $\tilde{H}_i \approx 1$  means a well-mixed neighbourhood, while  $\tilde{H}_i \approx 0$  indicates homogeneity.

To obtain a city-wide measure, we average  $\tilde{H}_i$  over all habitable cells  $\mathcal{H}$ :

$$\tilde{H}_{\text{mean}} = \frac{1}{|\mathcal{H}|} \sum_{i \in \mathcal{H}} \tilde{H}_i.$$

Finally, because segregation corresponds to \*low\* entropy, we define the segregation index

$$S = 1 - \tilde{H}_{\text{mean}}, \quad S \in [0, 1].$$

Thus  $S = 0$  denotes a perfectly mixed configuration, while  $S = 1$  denotes a perfectly segregated configuration.