

## CS 1501 Biconnected Components

Output from BiconnectedTrace.java using input file classGraph.txt. Each vertex has an indent level and each has a color assignment (but some vertices have the same color). The trace code shows how each vertex is assigned a DFS # (pre[v]) and a low[v] value that are initially equal. However, a vertex can reduce its low[v] value via:

- 1) back edges to "higher up" vertices in the tree, or
- 2) a child vertex having a smaller low[v] value.

These updates are indicated in the trace code with underlines.

If the child w of vertex v ends up with a low[w] value that is  $\geq$  pre[v], it means that v is an articulation point. These are marked in **bold**.

Note that an edge from a child node "back" to a parent node in the tree is not considered a back edge and does not change the low[v] value for a vertex. To see this idea look at document bicon2.pdf. Note that vertex 8 has a low[v] value of 3 (in green), but it does have an edge to vertex 3, which has a DFS # of 2. It would seem from this that the low[v] for 8 should be 2, but since vertex 3 is the parent of vertex 8 in the tree, this edge is not considered. This does not affect the results of the execution.

```
fromWeb > java BiconnectedTrace classGraph.txt
```

```
9 11
```

```
0: 5 4 1
```

```
1: 5 0
```

```
2: 5 3
```

```
3: 8 5 2
```

```
4: 7 6 0
```

```
5: 3 2 1 0
```

```
6: 7 4
```

```
7: 6 4
```

```
8: 3
```

```
Visiting: 0
```

```
Initially: pre[v] = 0 and low[v] = 0
```

```
Recurring to child: 5
```

```
Visiting: 5
```

```
Initially: pre[v] = 1 and low[v] = 1
```

```
Recurring to child: 3
```

```
Visiting: 3
```

```
Initially: pre[v] = 2 and low[v] = 2
```

```
Recurring to child: 8
```

```
Visiting: 8
```

```
Initially: pre[v] = 3 and low[v] = 3
```

```
Final low[v] value for 8: 3
```

```
Back to 3 after recursion
```

```
Child 8 low[w]: 3
```

```
Child 8 low[w] = 3  $\geq$  pre[v] = 2
```

```
3 is an articulation point
```

```
Recurring to child: 2
```

```
Visiting: 2
```

```
Initially: pre[v] = 4 and low[v] = 4
```

```
Back edge to 5 with DFS# 1
```

```
1 < 4 so update low[v] to 1
```

```
Final low[v] value for 2: 1
```

```
Back to 3 after recursion
```

```
Child 2 low[w]: 1
```

1 < 2 so update low[v] to 1  
 Final low[v] value for 3: 1  
     Back to 5 after recursion  
     Child 3 low[w]: 1  
     **Child 3 low[w] = 1 >= pre[v] = 1**  
         **5 is an articulation point**  
     Back edge to 2 with DFS# 4  
     Recurring to child: 1

Visiting: 1  
 Initially: pre[v] = 5 and low[v] = 5  
 Back edge to 0 with DFS# 0  
0 < 5 so update low[v] to 0  
 Final low[v] value for 1: 0  
     Back to 5 after recursion  
     Child 1 low[w]: 0  
0 < 1 so update low[v] to 0  
 Final low[v] value for 5: 0

Back to 0 after recursion  
 Child 5 low[w]: 0  
 Recurring to child: 4

Visiting: 4  
 Initially: pre[v] = 6 and low[v] = 6  
 Recurring to child: 7  
     Visiting: 7  
     Initially: pre[v] = 7 and low[v] = 7  
     Recurring to child: 6  
         Visiting: 6  
         Initially: pre[v] = 8 and low[v] = 8  
         Back edge to 4 with DFS# 6  
6 < 8 so update low[v] to 6  
         Final low[v] value for 6: 6  
         Back to 7 after recursion  
         Child 6 low[w]: 6  
6 < 7 so update low[v] to 6  
         Final low[v] value for 7: 6

Back to 4 after recursion  
 Child 7 low[w]: 6  
**Child 7 low[w] = 6 >= pre[v] = 6**  
     **4 is an articulation point**  
 Back edge to 6 with DFS# 8  
 Final low[v] value for 4: 6

Back to 0 after recursion  
 Child 4 low[w]: 6  
 Back edge to 1 with DFS# 5  
**Root 0 has 2+ children**  
     **0 is an articulation point**  
 Final low[v] value for 0: 0

Articulation points

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0  
 3  
 4  
 5