University of Pittsburgh

Algorithms and Data Structures 2
CS 1501

Fall 2022

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(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides.)

Announcements

- Upcoming Deadlines
  - Homework 3: this Friday @ 11:59 pm
  - Lab 2: next Monday @ 11:59 pm
  - Assignment 1: Monday Oct 10th @ 11:59 pm
- Lecture recordings are available on Canvas under Panopto Video
- Please use the “Request Regrade” feature on GradeScope if you have any issues with your grades
- TAs student support hours available on the syllabus page

Previous lecture

- Binary Search Tree
  - How to delete
- Runtime of BST operations
  - add, search, delete
- Red-Black BST (Balanced BST)
  - definition and basic operations

Muddiest Points

- **Q: Was just curious of the real-life applications of red-black tree**
- A: Example applications:
  - TreeMap and TreeSet in Java
  - CPU Scheduling inside the Linux Kernel
- **Q: what does the triangle mean at the bottom of tree drawings?**
- A: It means a subtree

Muddiest Points

- **Q: when deleting interior nodes how does the parent node know where its new child is**
- A:
  - The removal method (removeFromRoot) returns the new root
  - The return value of the removal method is assigned to the right or left child of the parent
- **Q: how does the new node know where it's child is?**
- A: The new root has the same two children that the old root has; we are just replacing the data of the root node.

Muddiest Points

- **Q: Can you explain how all the different functions for the removal of a node connect together into remove(T)**
- A:
  - remove(T) creates an empty wrapper object and calls removeEntry, passing to it the entry to delete and the empty wrapper
  - removeEntry searches for *entry* by either moving left or right (depending on the comparisons between entry and the data portion of the current node)
  - Once the node that contains entry is found, the data inside the node is put inside the wrapper object
  - Then, removeFromRoot is called on the found node
  - removeFromRoot removes the node (the three cases) and may need to call findLargest and removeLargest on the left subtree of the node (if it has two children)
  - removeFromRoot returns the new root of the tree to removeEntry
  - removeEntry then sets the right or left child of current node to the return value of removeFromRoot and returns the node after this modification
  - Eventually removeEntry returns back to remove the (possibly) new root of the tree after removing entry
  - Finally, remove sets the root of the tree to the return value of removeEntry and returns the entry inside the wrapper object

Muddiest Points

- **Q: Why do you need a wrapper class for removeEntry?**
- A: Because we want removeEntry to return the root after removing entry.
- But we want it also to return a reference to the deleted data object, which is acheived using the wrapper object.

Muddiest Points

- **Q: Would the theoretical worst case BST operation be when the tree is only left nodes/right nodes?**
- A: Yes. For example, when searching for the deepest node in such a degenerate tree, the depth of the node will be n-1 and the run time is Theta(n).

Muddiest Points

- **Q: A red node can be anywhere in the right subtree just so long as it's leaning to the left side of that subtree, correct?**
- A: Correct

Muddiest Points

- **Q: what is the difference between a red node and black node?**
- A: A red node has a red edge to its parent, whereas a black node has a black edge to its parent. The root is always a black node.
- **Q: Where is this going? What utility do we get out of labelling the tree this way?**
- A: By labeling the tree nodes this way with the other constraints, the height of the tree is at most *2 log n*

Muddiest Points

- **Q: What purpose does key serve in Red-Black BST?**
- A: The data objects inside the nodes are compared based on the key field.

Muddiest Points

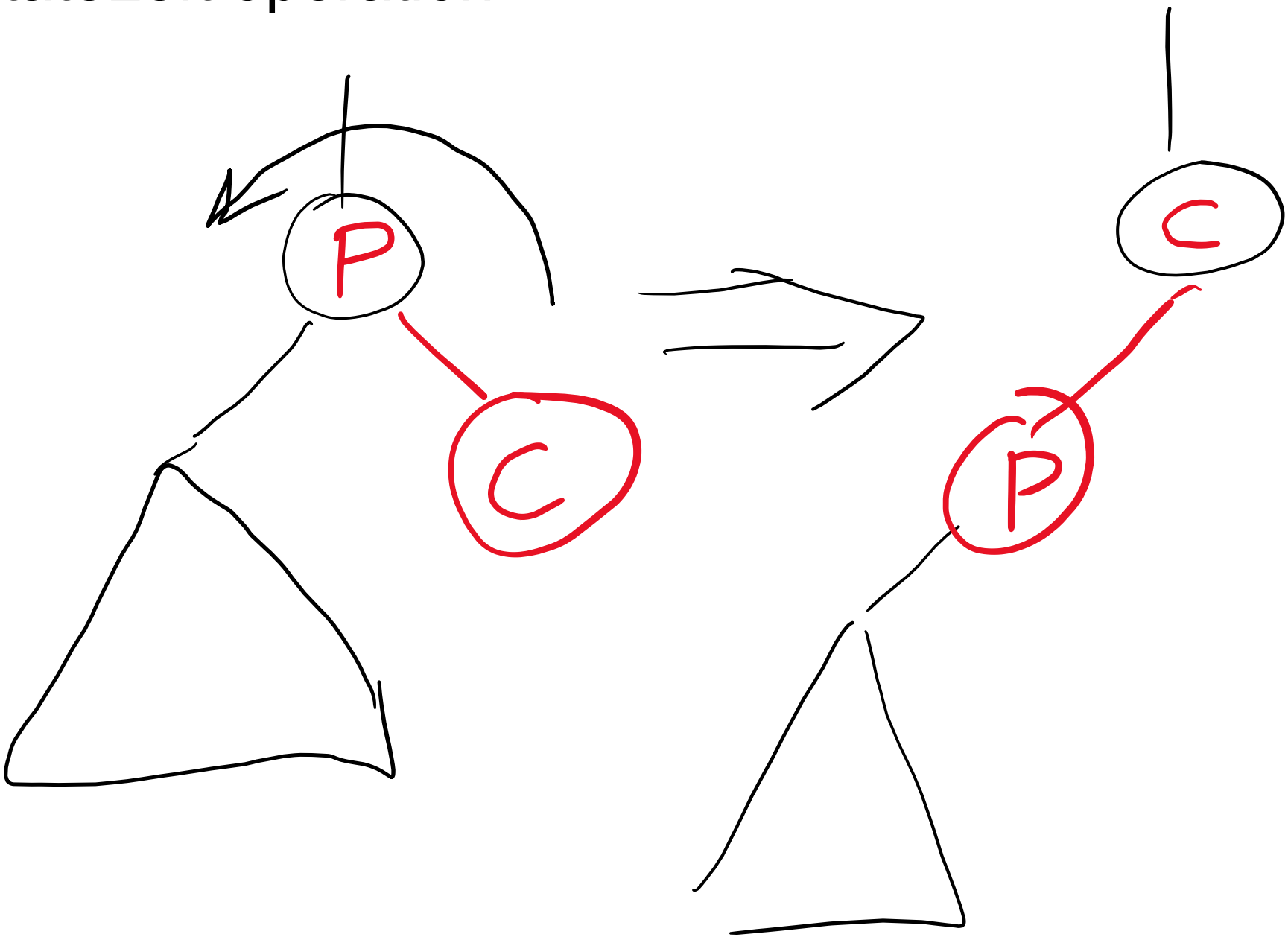
- **Q: How do you tell if a node is red/black based on the color of the link?**
- A: A node has the same color as the link to its parent.

Adding to a RB-BST

- Ok, so we add a red leaf node!
- What can go wrong then?
  - The new node is a right child
  - Fix it by left rotation

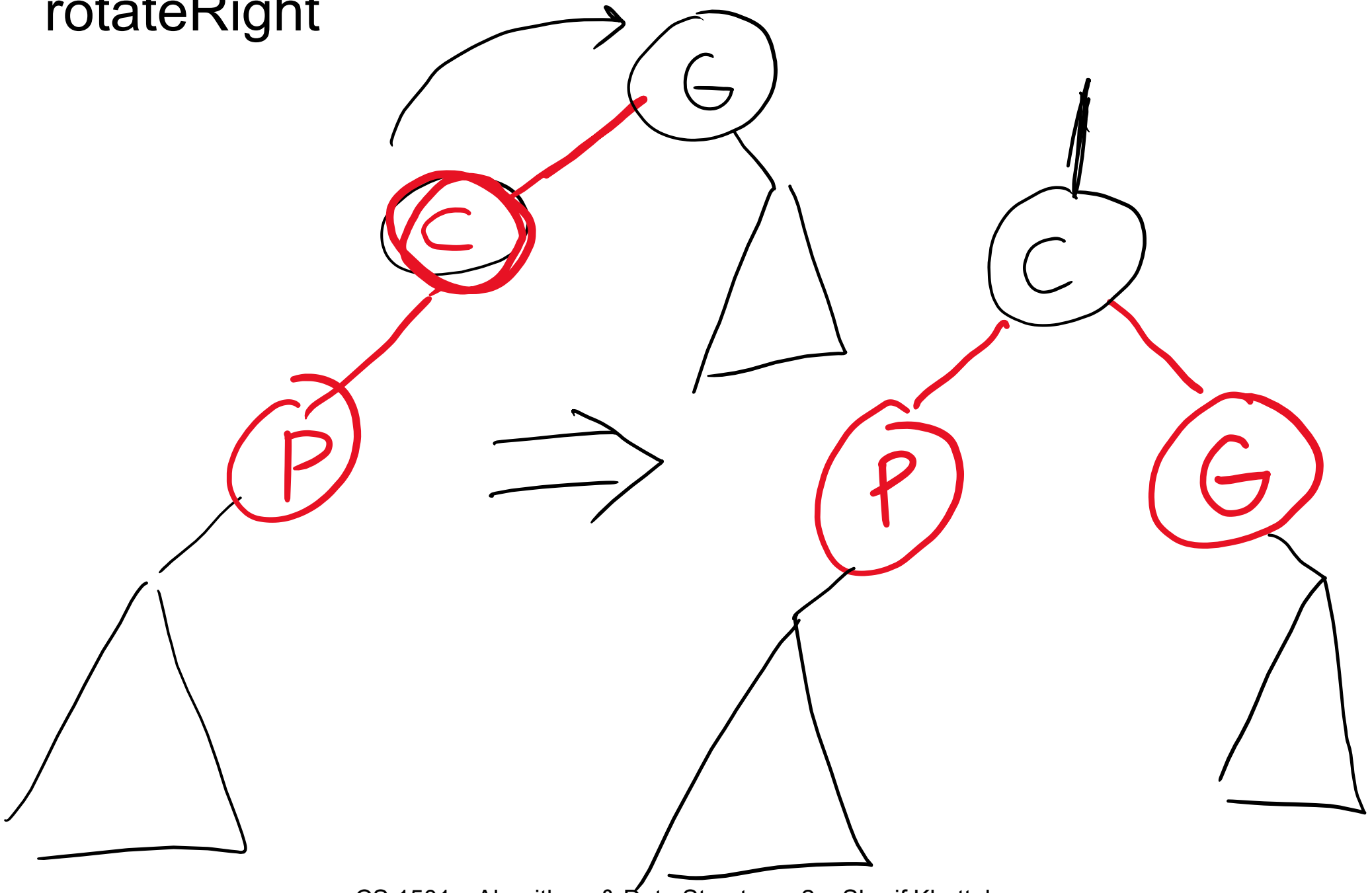
What if the new red node is a right child?

- rotateLeft operation



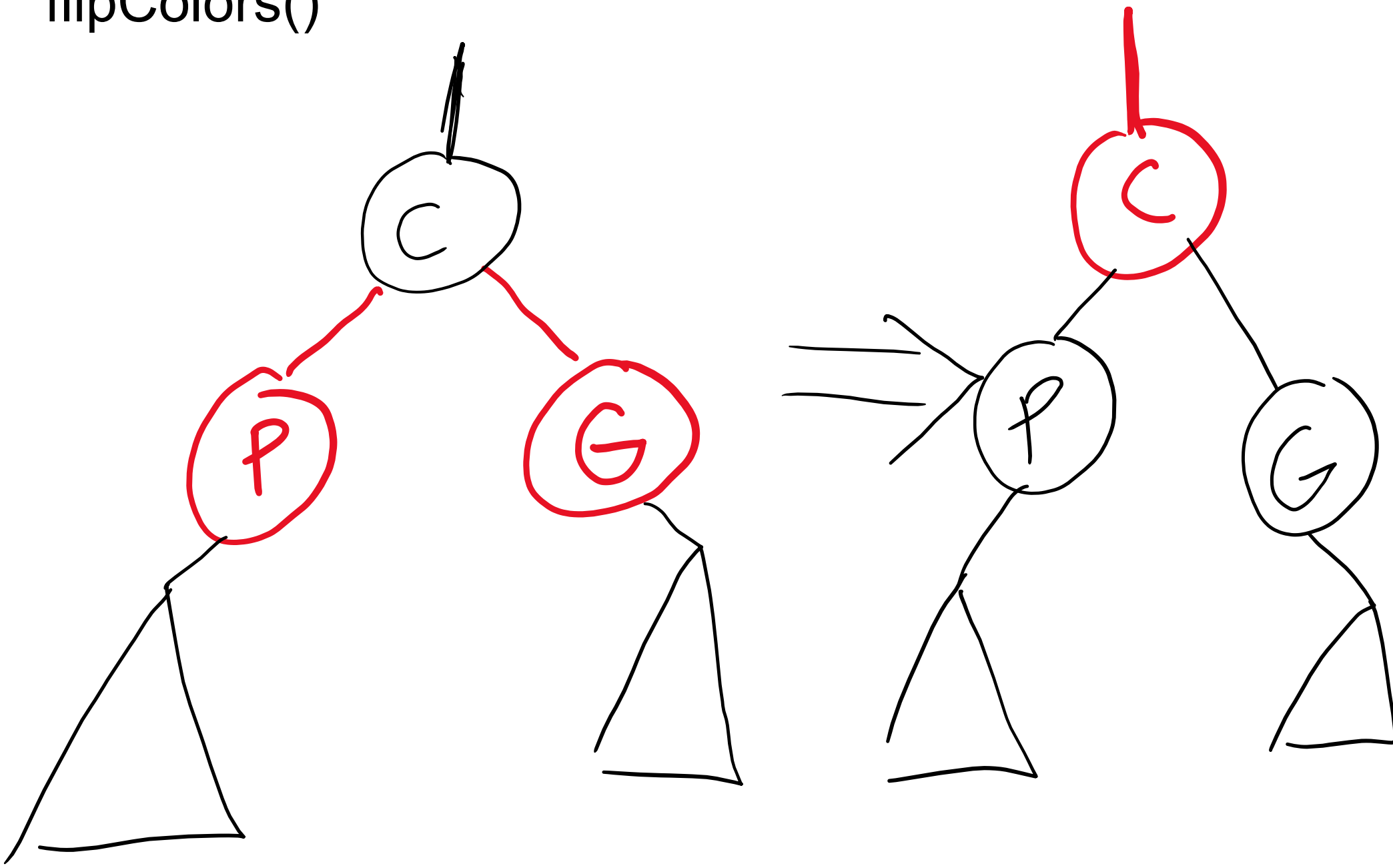
What if the new node becomes red?

- rotateRight



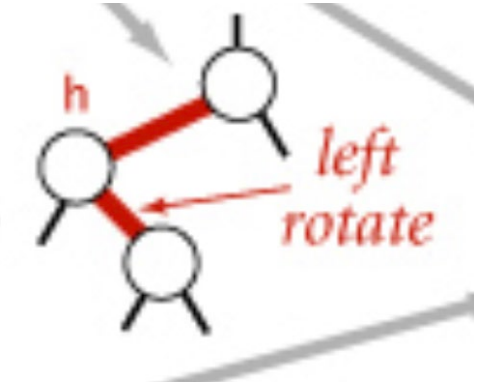
What if both children of a node are red?

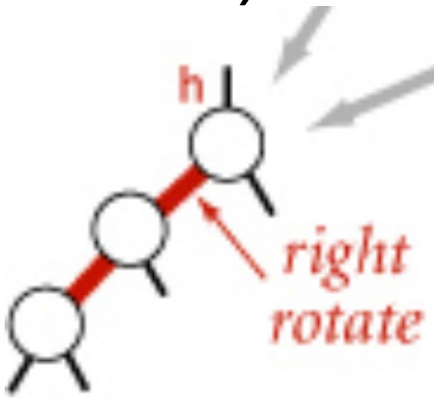
- flipColors()

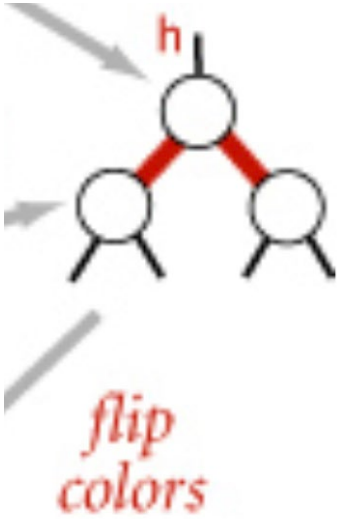


Muddiest Points

- **Q: how to fix violations in B-R BST in terms of recursive call**
- A: Violations are checked and corrected as we climb back up the tree.
- The code for violation corrections is after the recursive calls.
- Violations are detected and corrected in the following order (*h* is the current node)







Muddiest Points

- **Q: when do we add red/black nodes**
- **Q: How do we construct an RB BST if each node we add has to be red?**
- A: We always add red nodes then potentially change color of new node
- **Q: I found the transient right red edges confusing**
- **Q: I didn't quite understand how the form other than a red node being added or a rotation/color flip.**
- **Q: How does a Red black BST with red edges eventually turn into an all black BST**
- A: Let's see an example

Adding to a RB-BST: Example

- Let's add the numbers:
- 1, 2, 3, 4, 5, 6, 7

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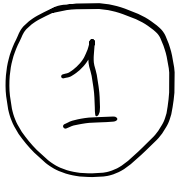

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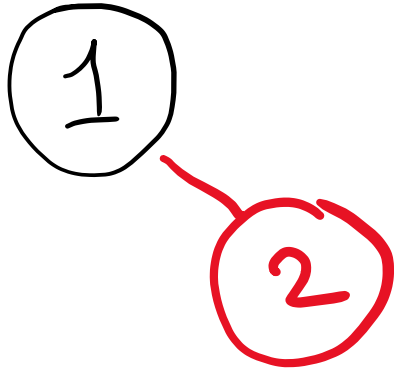

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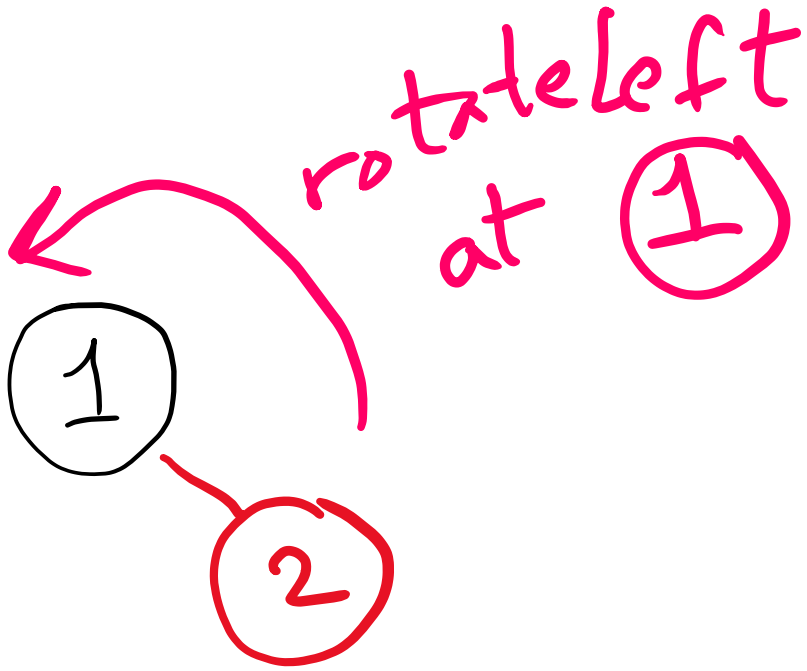

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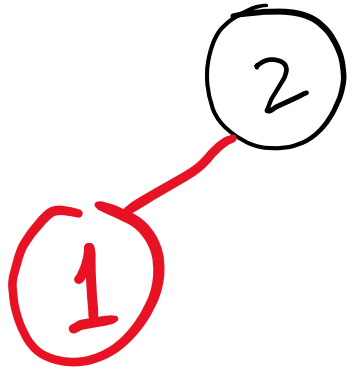

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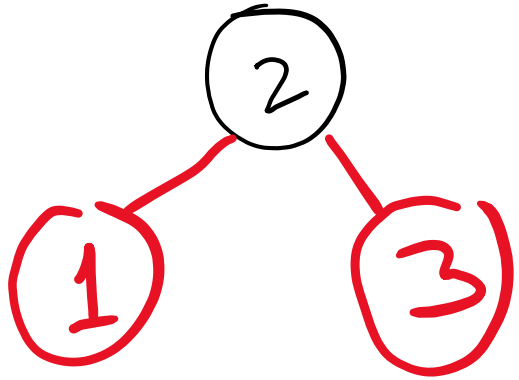

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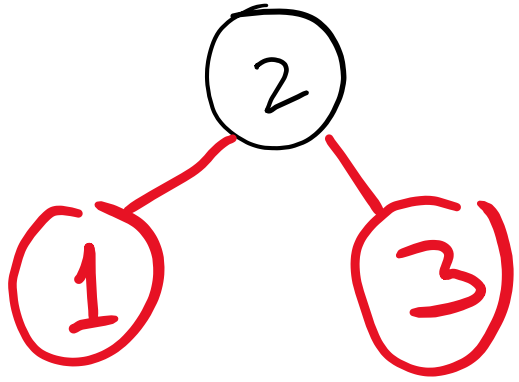

flipColors at (2)

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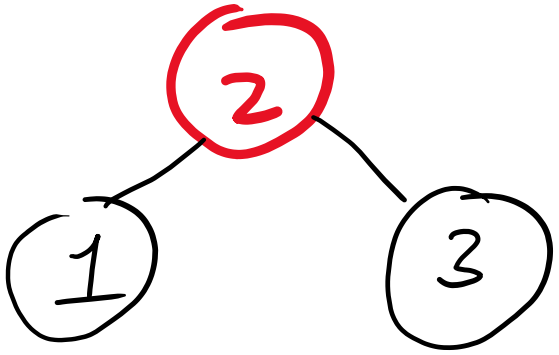

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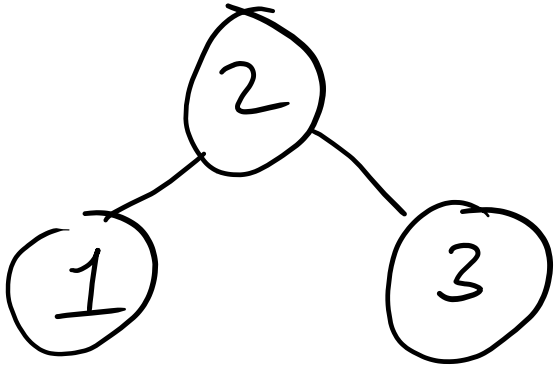

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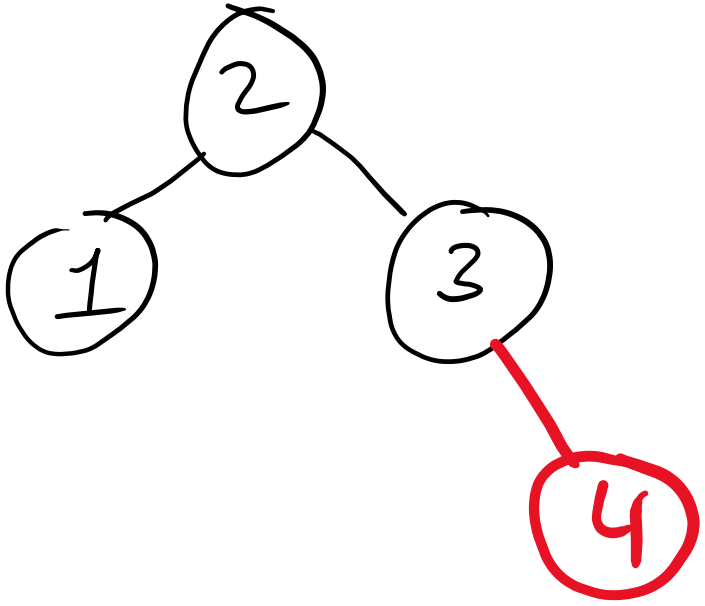

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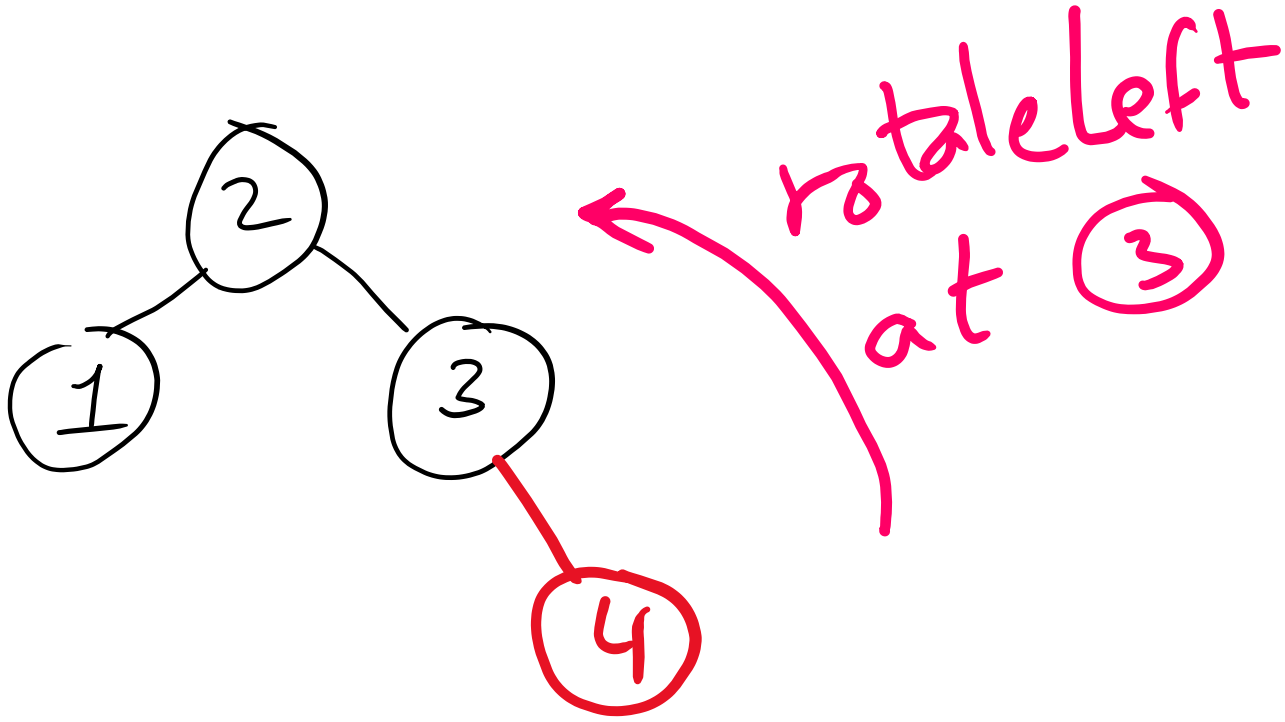

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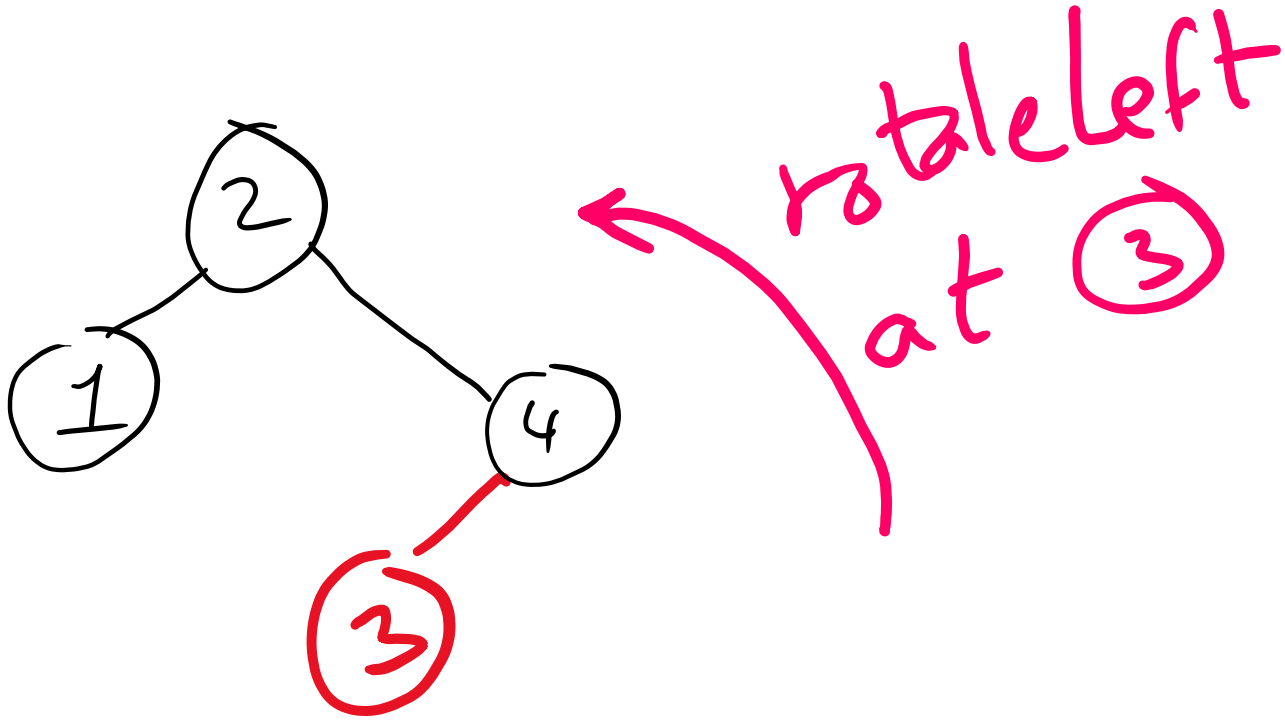

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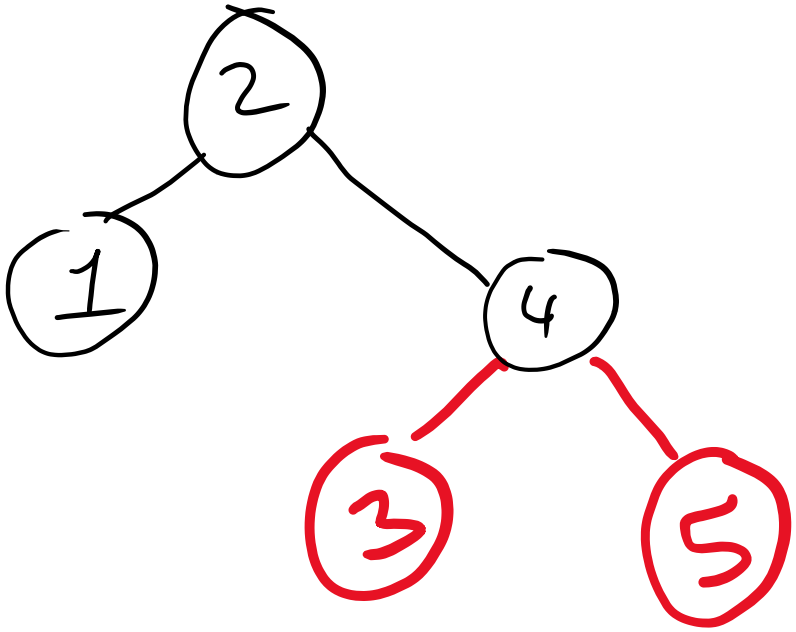

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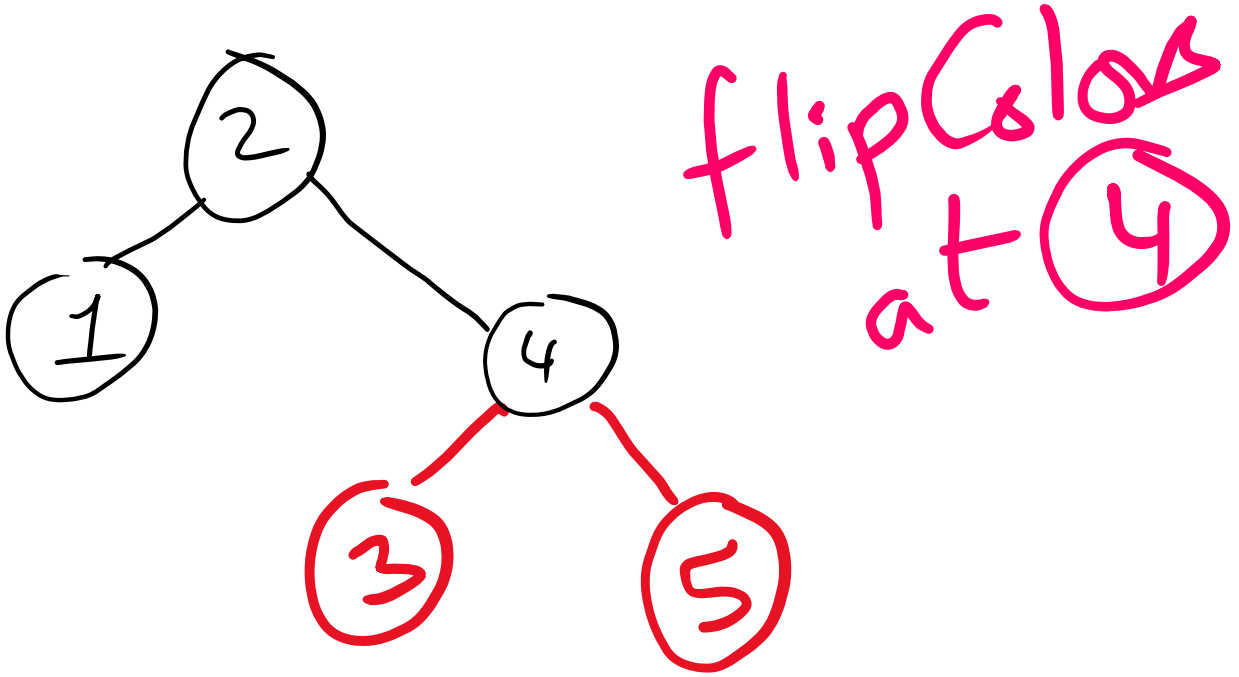

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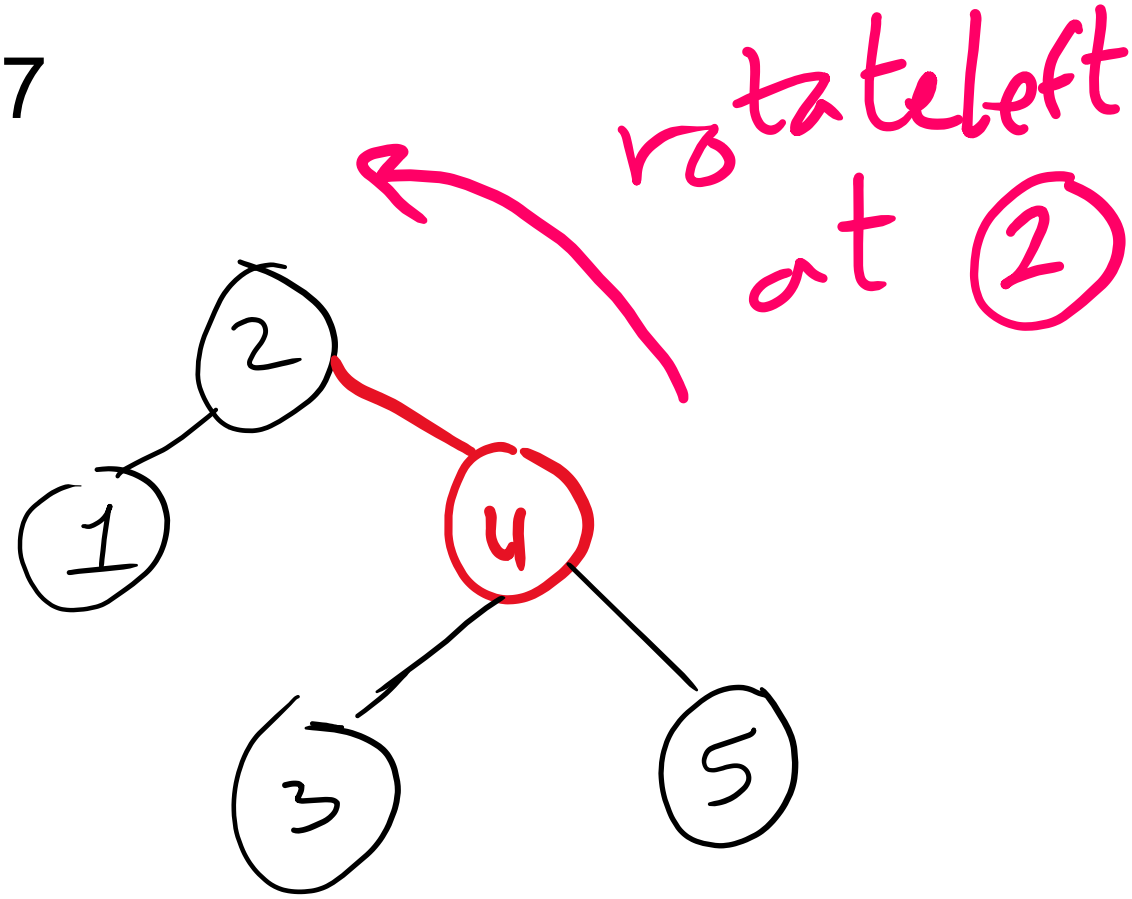

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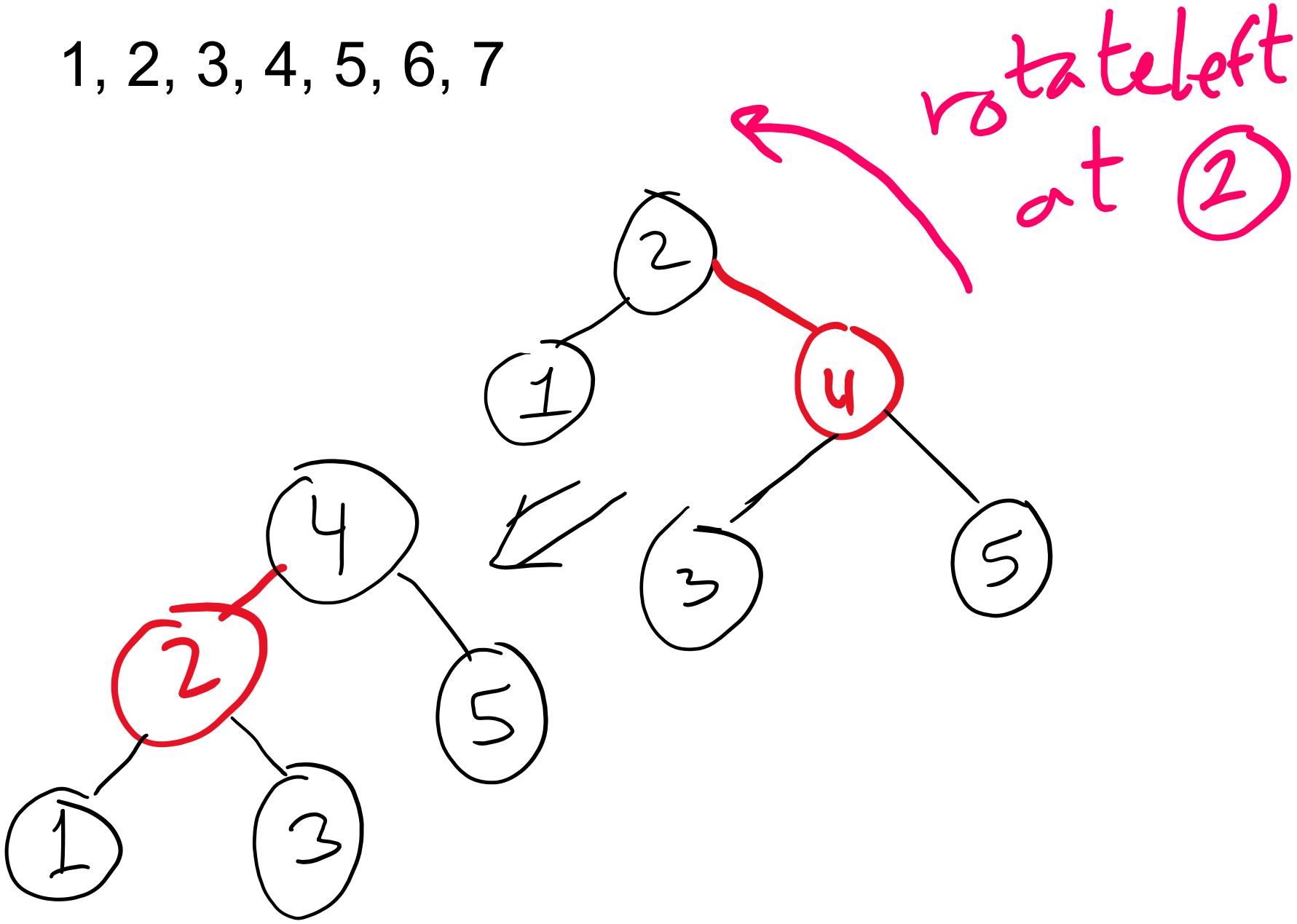

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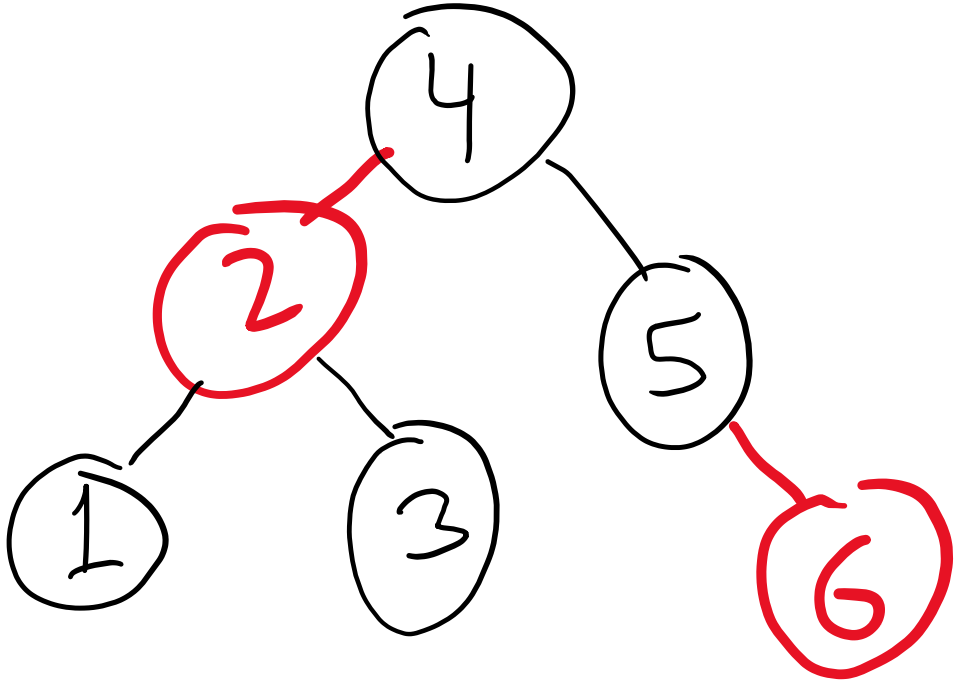

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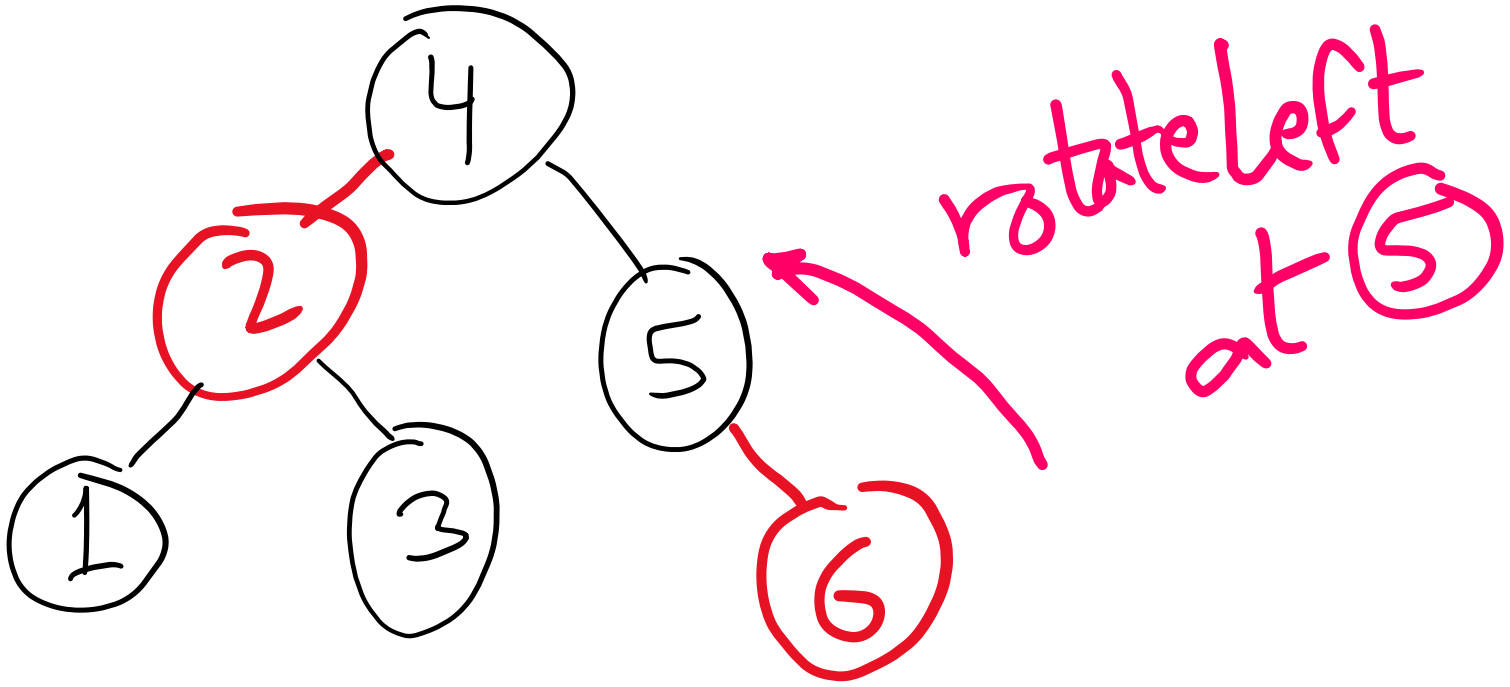

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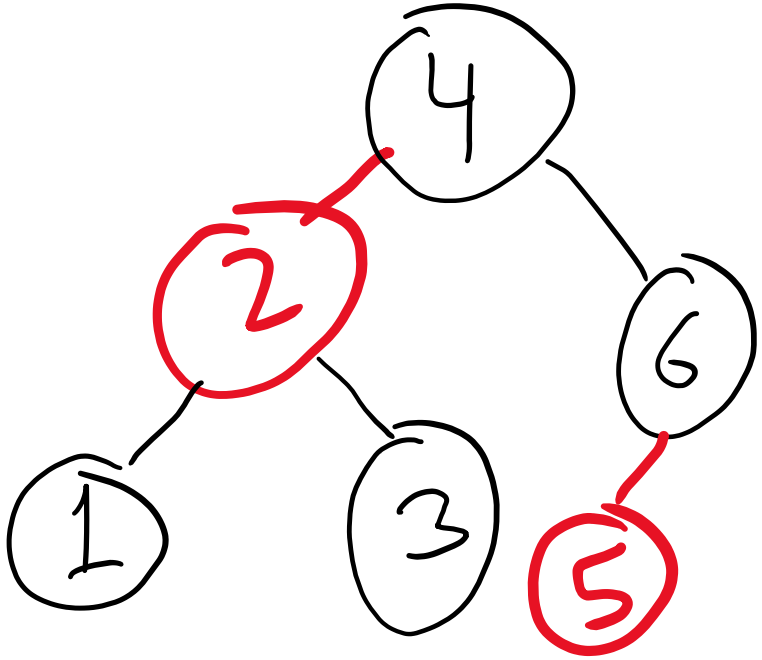

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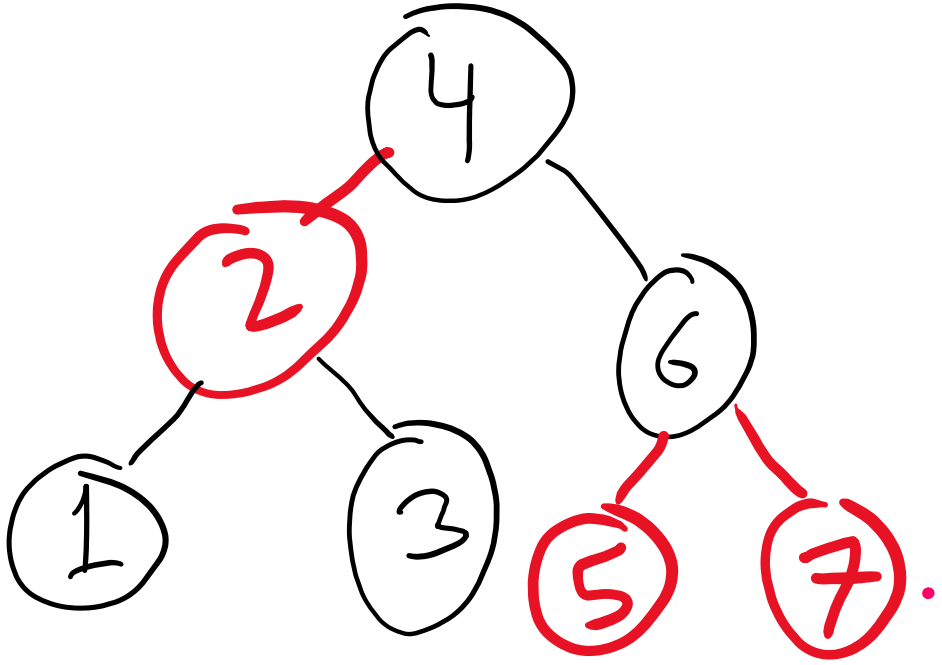

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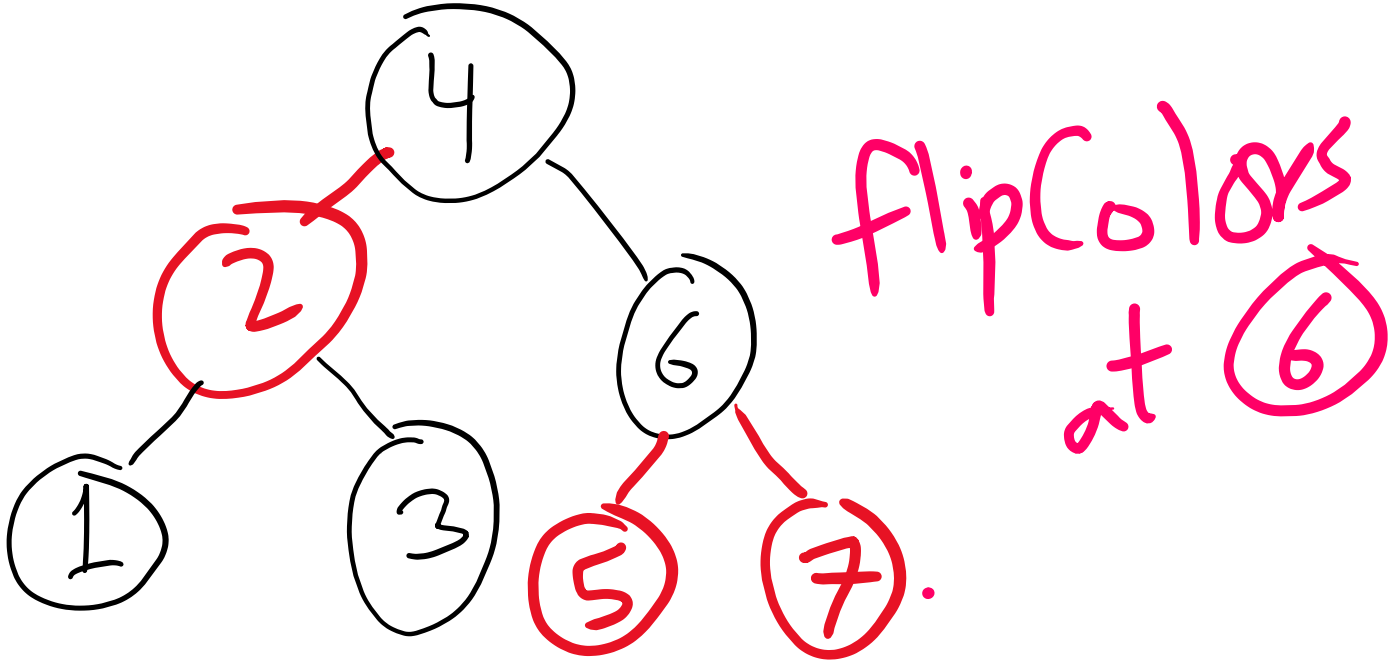

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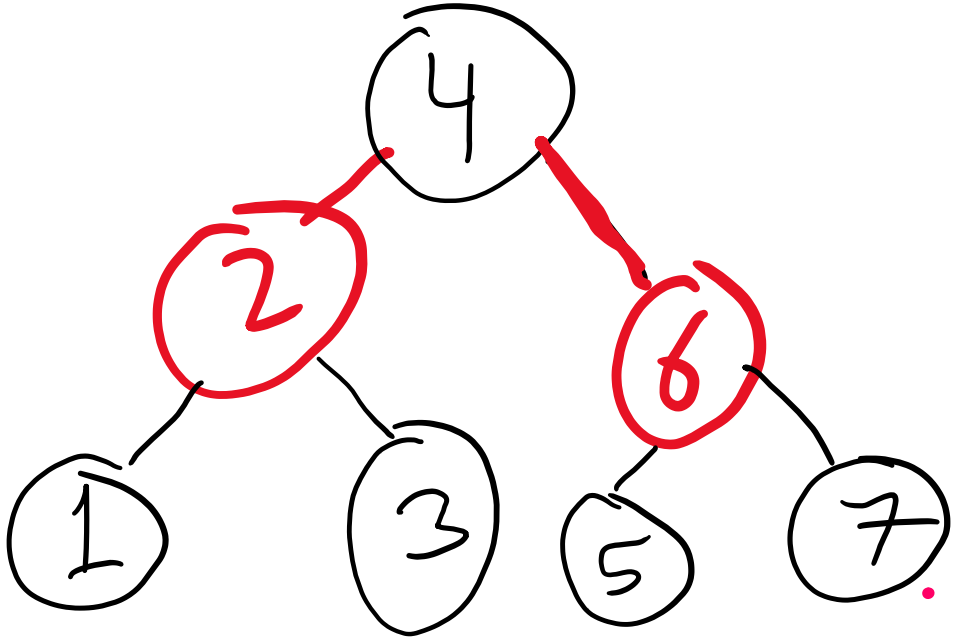

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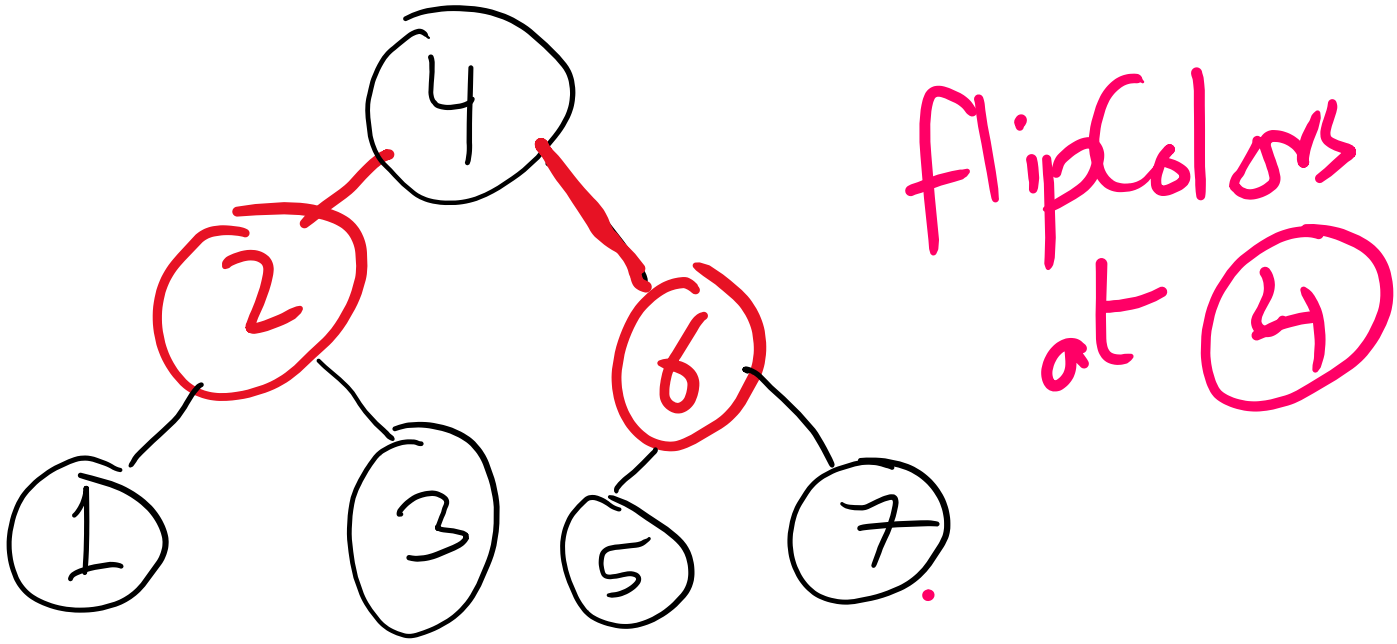

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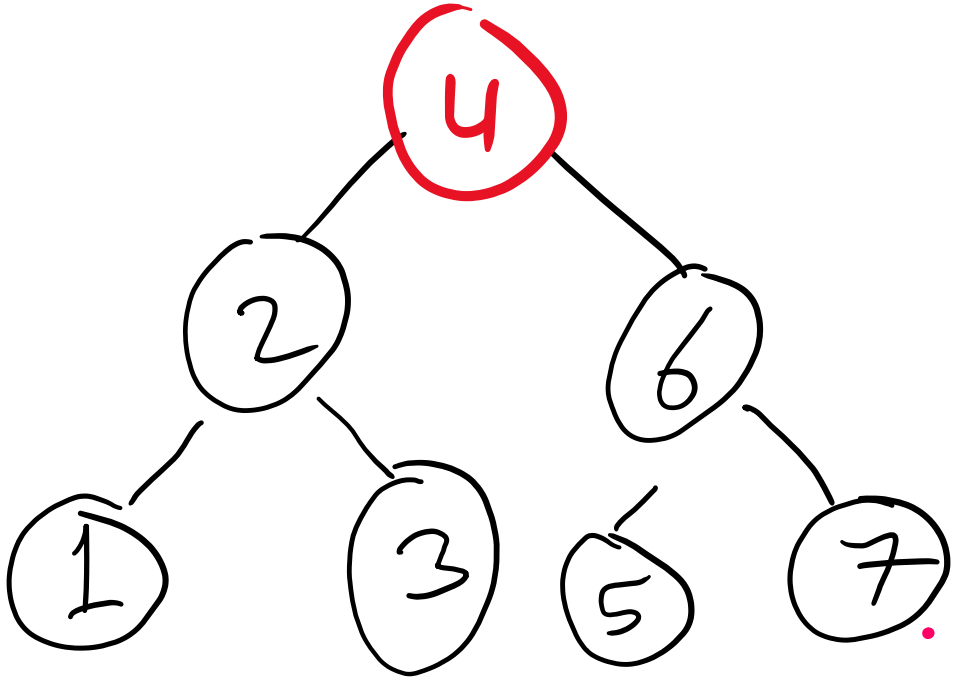

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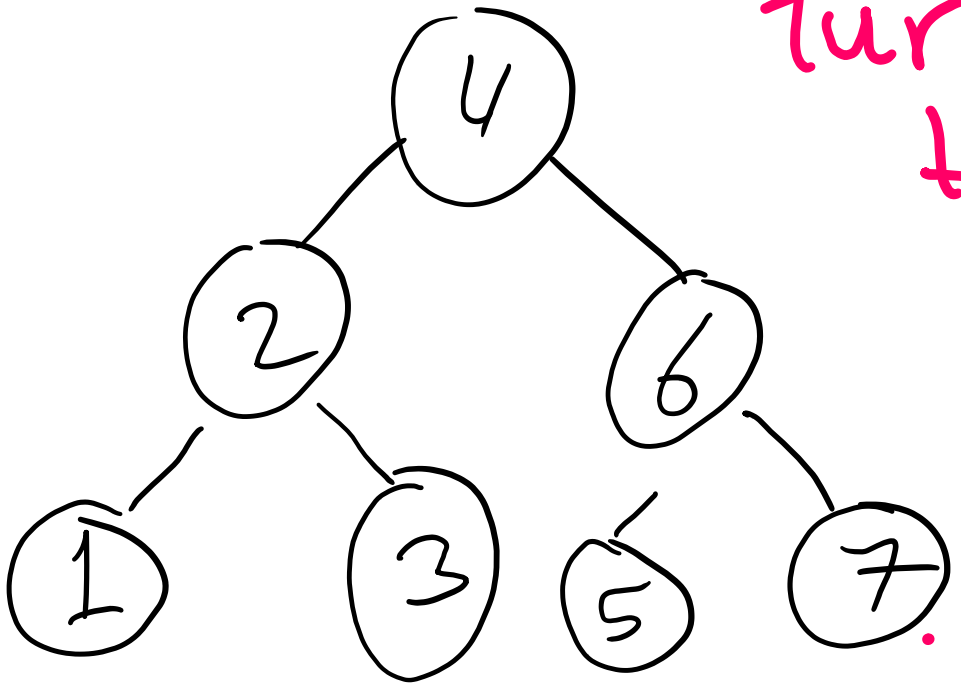
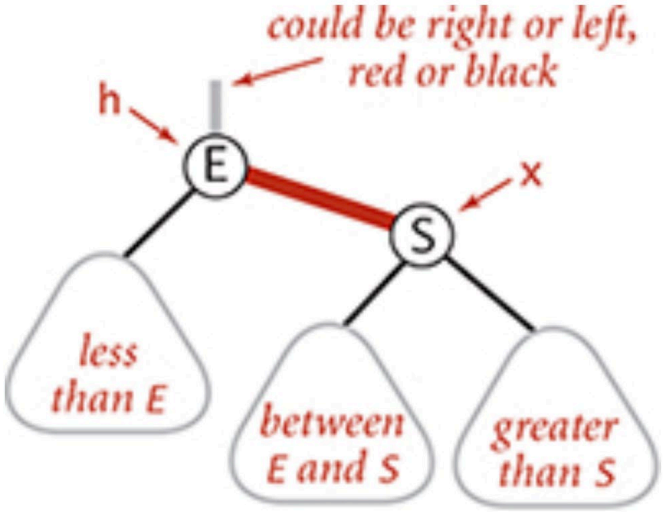
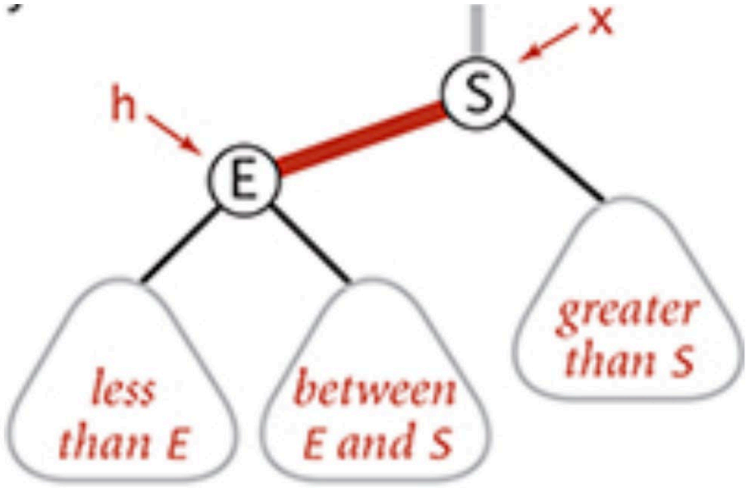


turn root to black

Left Rotate

- **Q: How to perform left rotations?**
- A: **BinaryNode<T> rotateLeft(BinaryNode<T> h)** performs left rotation at node *h*. The code emerges from contrasting the before and after pictures



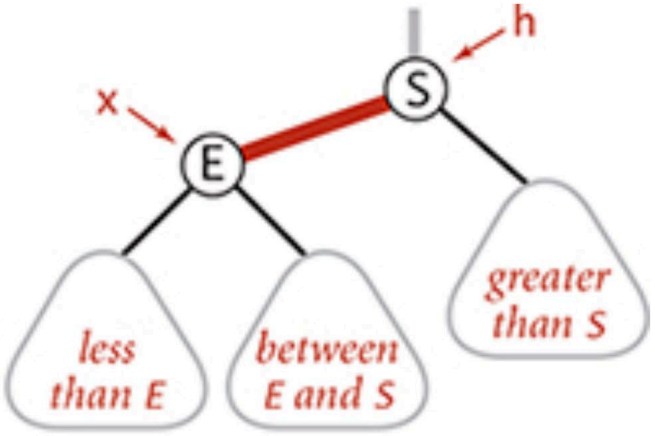


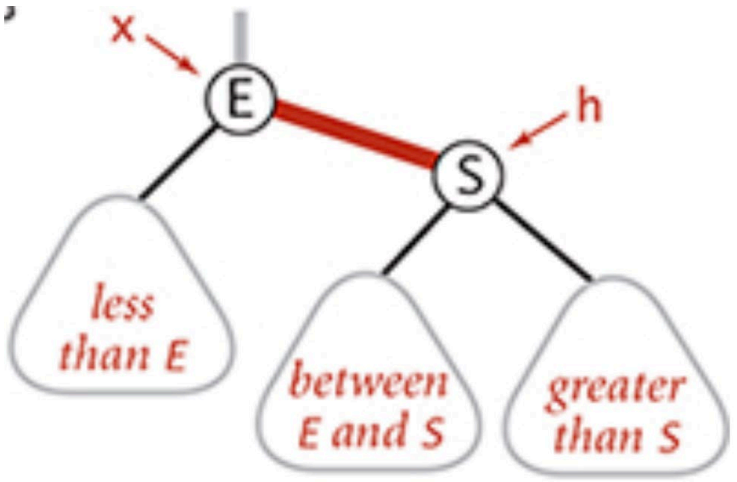

```
x = h.right;
h.right = x.left;
x.left = h;
x.color = h.color;
h.color = RED;
return x;
```

- rotateLeft maintains the search tree property
  - why?
- It also maintains the perfect balance property
  - why?

Right Rotate

- **Q: How to perform right rotations?**
- A: **BinaryNode<T> rotatRight(BinaryNode<T> h)** performs right rotation at node *h*. The code emerges from contrasting the before and after pictures



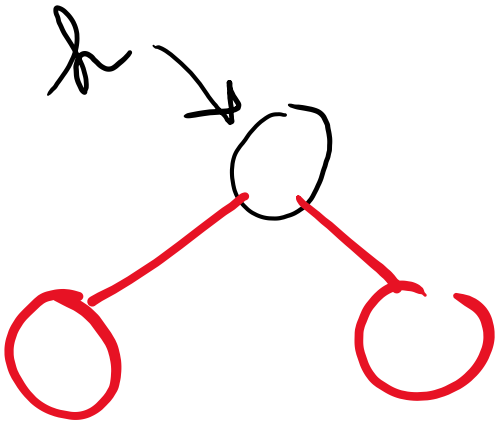


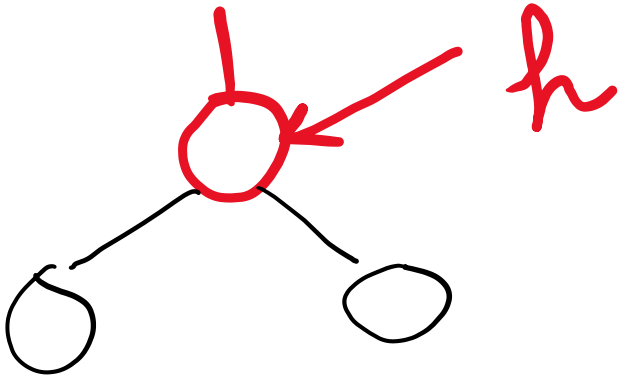

```
x = h.left;
h.left = x.right;
x.right = h;
x.color = h.color;
h.color = RED;
return x;
```

- rotateRight maintains the search tree property
  - why?
- It also maintains the perfect balance property
  - why?

Color Flip

- **Q: How to perform color flips?**
- A: **BinaryNode<T> flipColors(BinaryNode<T> h)** performs color flip at node *h*. The code emerges from contrasting the before and after pictures



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```

- flipColors maintains the search tree property
  - why?
- It also maintains the perfect balance property
  - why?

Muddiest Points

- **Q: If correcting one violation for the red-black BST, leads to another violation, how is that corrected?**
- A: One violation may lead to another violation of a different type, which may lead to another violation of a different type.
- However, this process stops after at most three corrections passing the violation to the next level up.
- Violations keep propagating up the tree until we have a violation at root, which is root node being red instead of black
  - This can be easily corrected by coloring root black again

Muddiest Points

- **Q: Confused on the benefits of adding red nodes and what that would look like in an application**
- A: Adding a black node would violate the perfect balance constraint (all paths from root to null-edges have the same number of black edges)
  - Hard to fix!

Muddiest Points

- **Q: can you explain the implementations again or point me to a resource that explains them if that would take too much time?**
- A: The code is available in the code handouts. The lecture recordings are available on Canvas.
- **Q: It is hard to visualize a red black bst without pictures and diagrams for me**
- A: I agree! When you study, use diagrams of example trees.
- **Q: PLEASE GO THROUGH THE HOMEWORKS**
- A: Please use Piazza to ask about any homework questions that you are confused about

Muddiest Points

Anonymous

a day ago

Comments 0 1

This Lecture

- Red-Black BST (self-balancing BST)
  - definition and basic operations
  - delete
  - runtime of operations
- Turning recursive tree traversals to iterative

Deleting a node

- Make sure that we are not deleting a black node
  - as we go down the tree, make sure that the next node down is red
    - using a different set of operations
  - as we go back up the tree, correct any violations
    - same as we did while adding
  - if deleting a node with 2 children
    - replace with minimum of right subtree
    - delete minimum of right subtree
    - similar trick to delete in regular BST

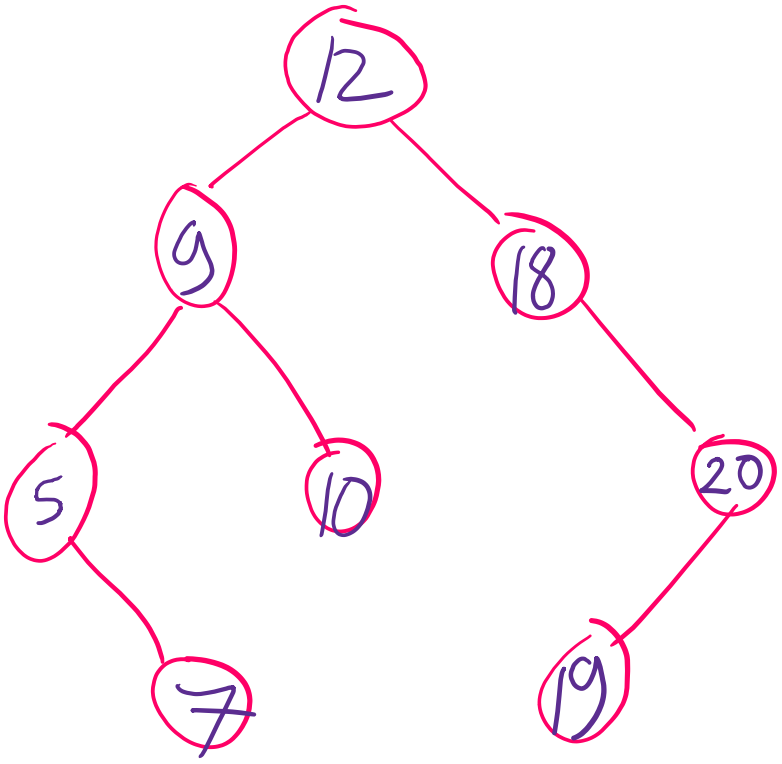
Other BST operations

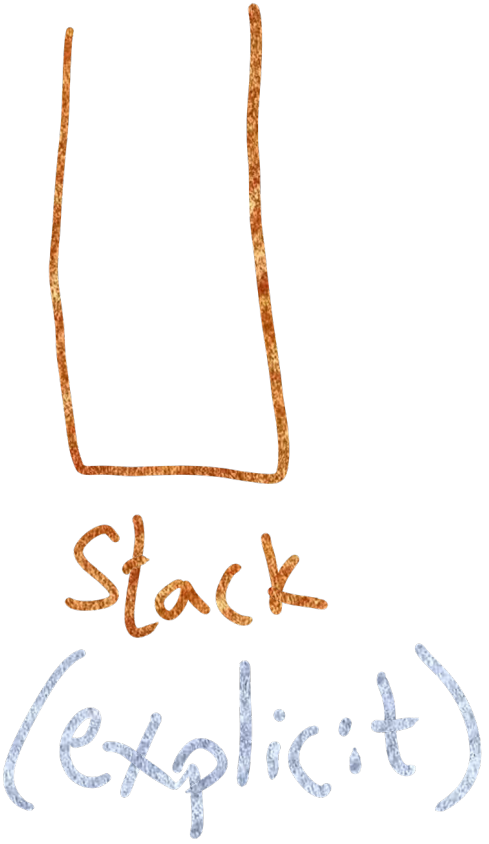
- Find successor and predecessor of an item
  - Lab 3
- Find all items within a specific range
  - Please check the keys methods in RedBlackBST.java inside the TreeADT folder in the code handouts
- Same code as regular BST!
- **worst-case runtime = Theta(log n)**

Why use iteration instead of recursion?

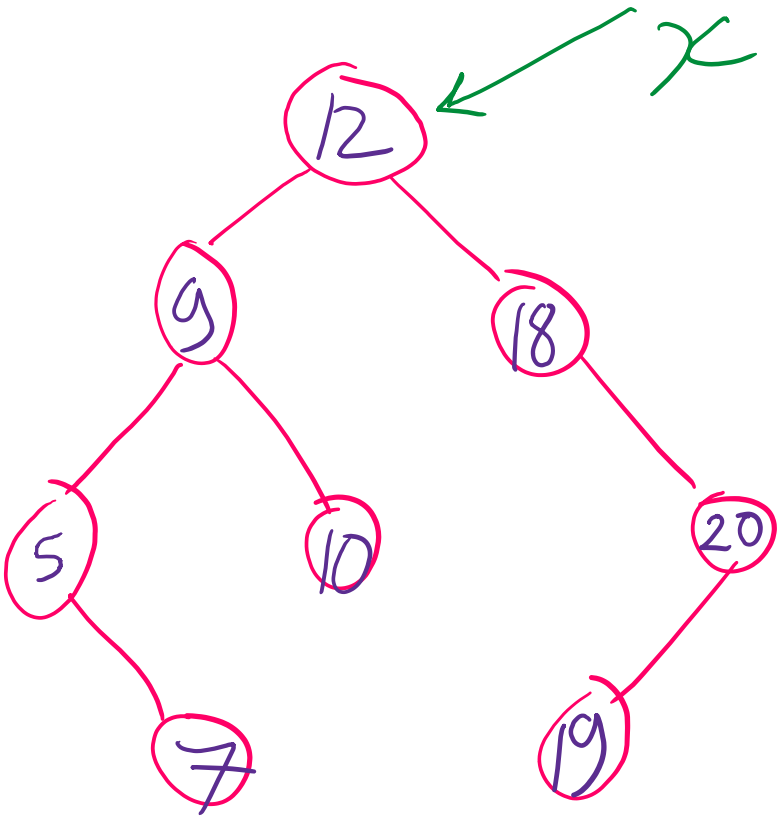
- Iteration is faster than recursion
  - No function call overhead (stack allocation)
- Especially useful for frequently used operations
  - Search is a good example of these operations

Iterative Preorder traversal





Iterative Preorder traversal



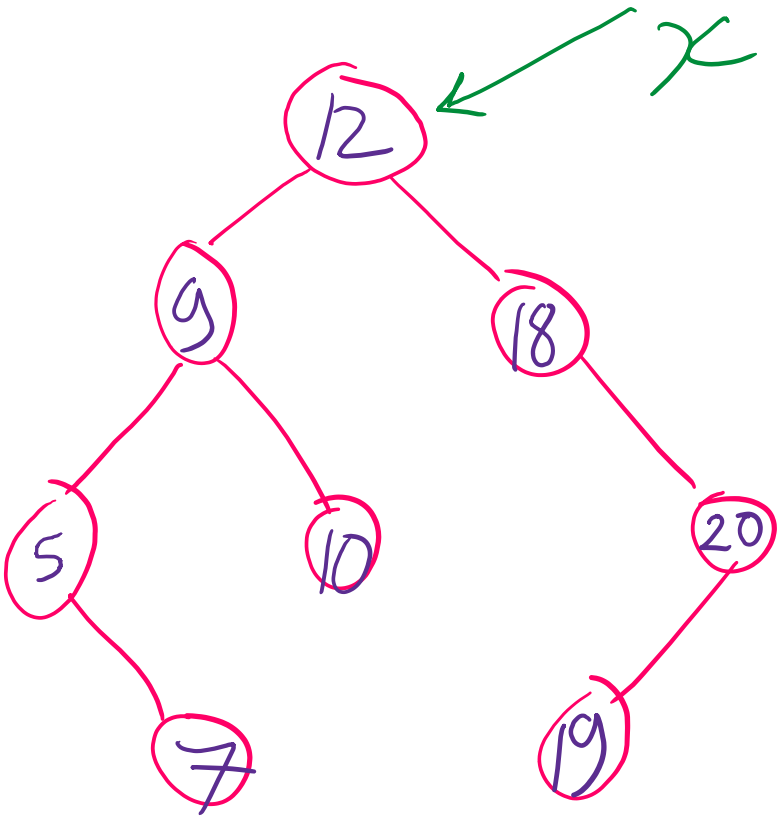
Stack (explicit)

while (stack not empty)

pop
visit
Push right
Push left

Visit order: 12

Iterative Preorder traversal



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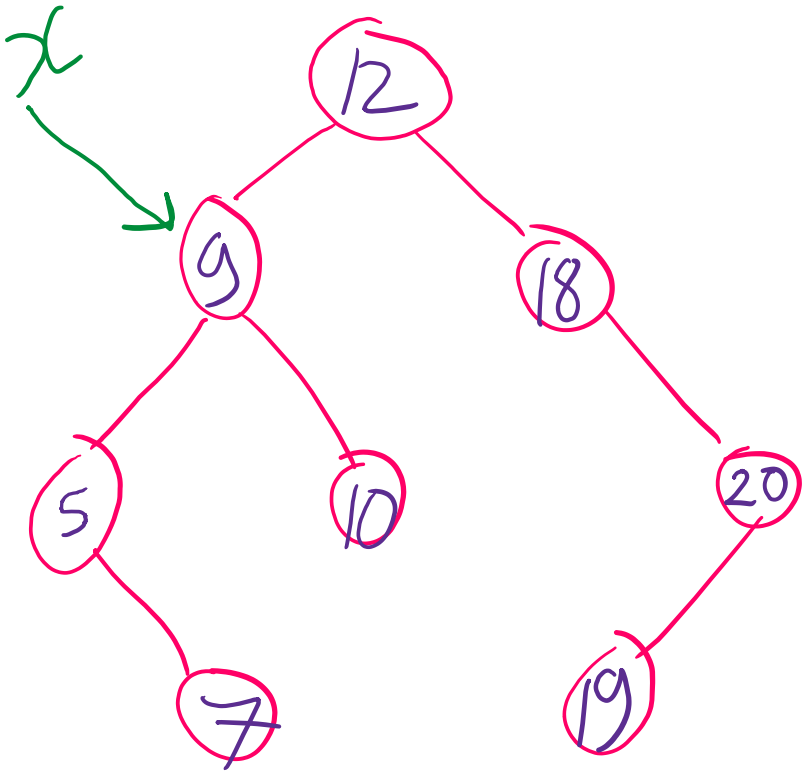
Iterative Preorder traversal



Stack (explicit)

while (stack not empty)

pop

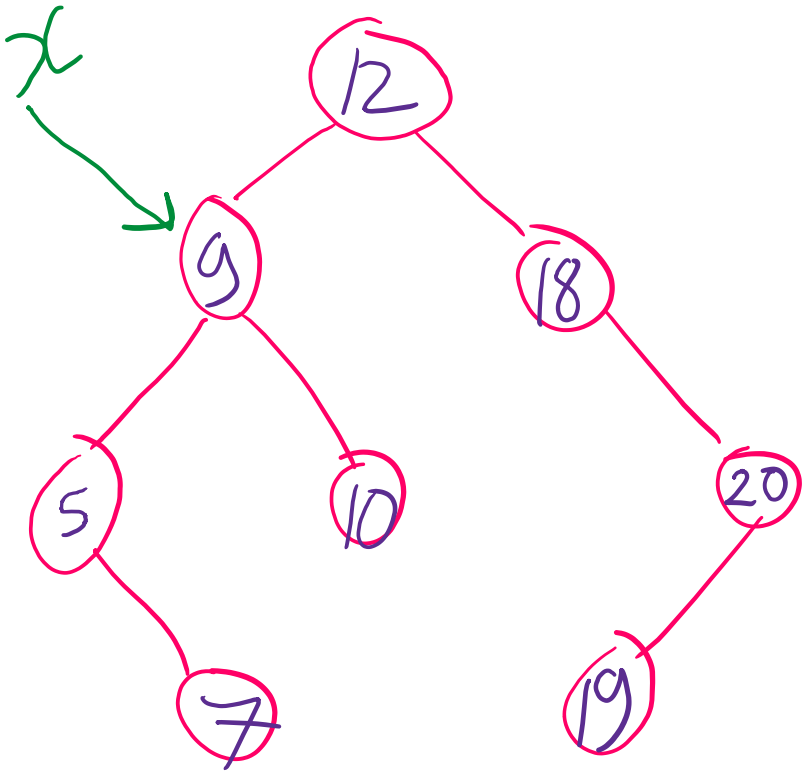
visit

Push right

Push left

Visit order: 12, 9

Iterative Preorder traversal



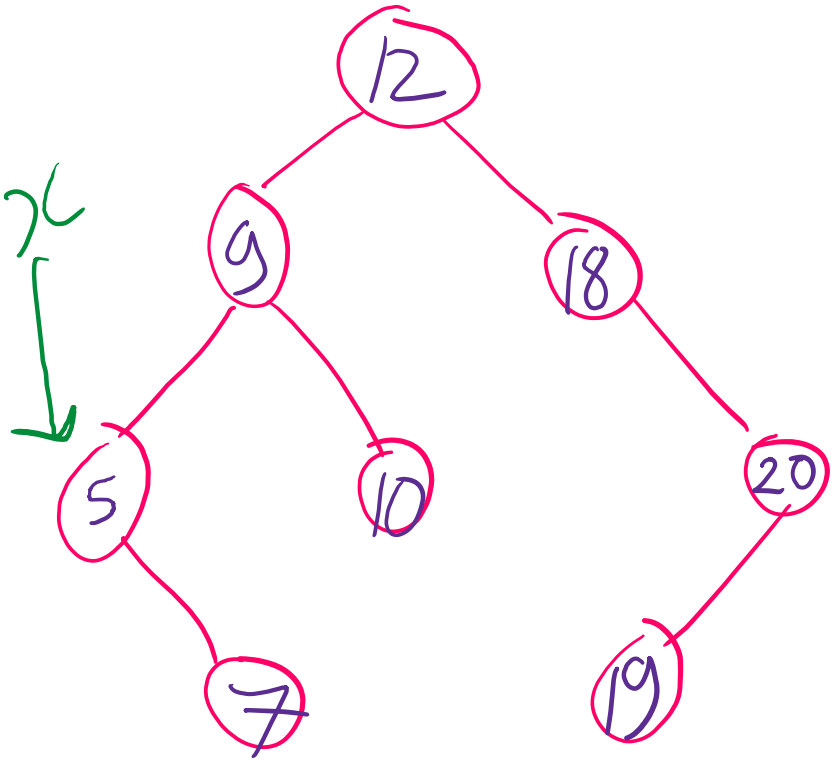
Stack (explicit)

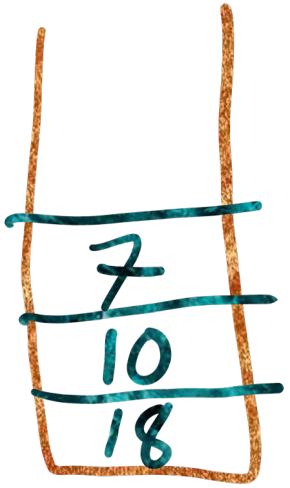
while (stack not empty)

pop
visit
Push right
Push left

Visit order: 12, 9

Iterative Preorder traversal





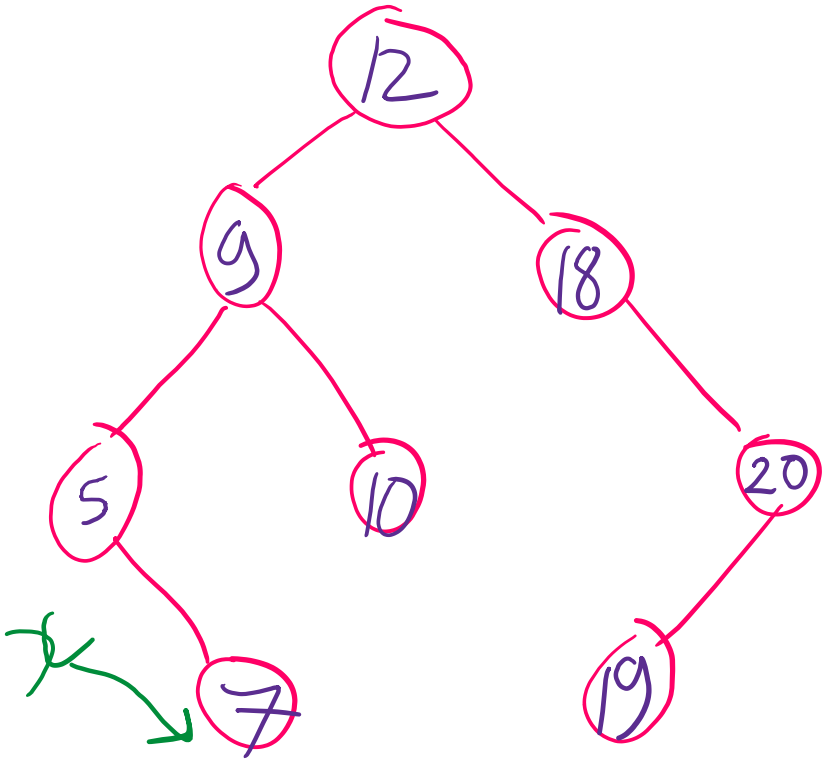
Stack (explicit)

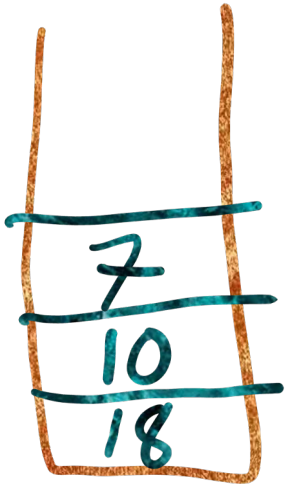
while (stack not empty)

pop
visit
Push right
Push left

Visit order: 12, 9, 5

Iterative Preorder traversal





Stack (explicit)

while (stack not empty)

pop

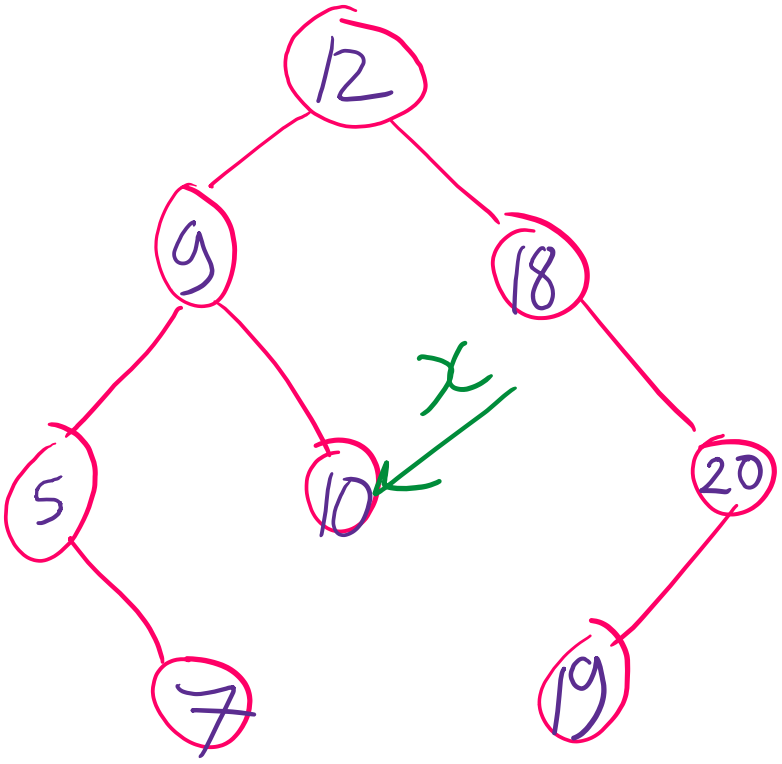
visit

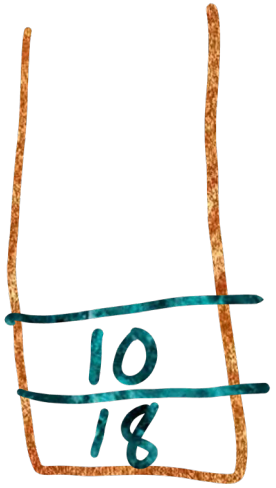
Push right

Push left

Visit order: 12, 9, 5, 7

Iterative Preorder traversal





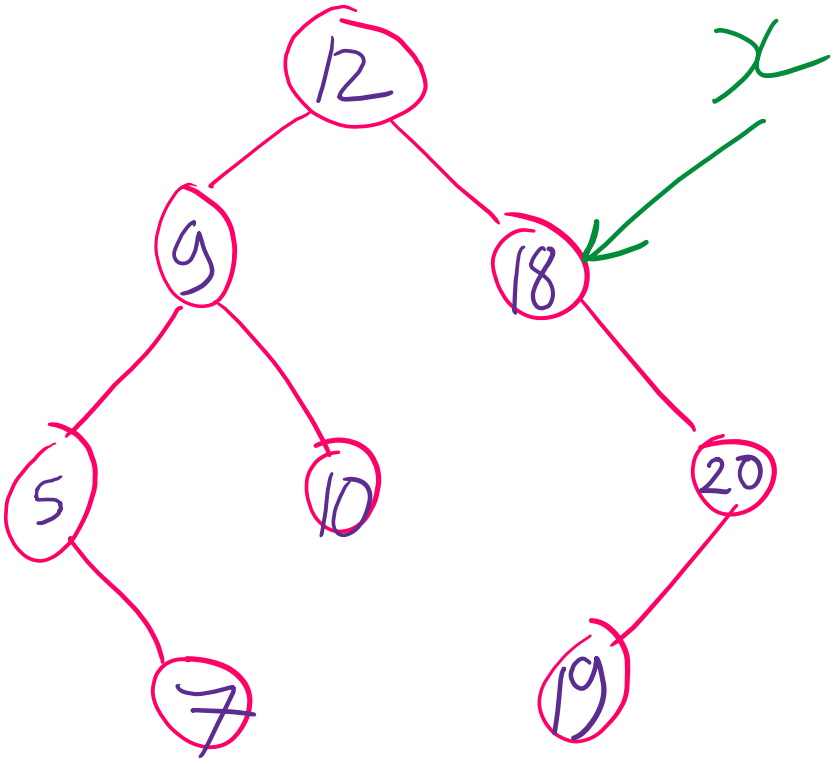
Stack (explicit)

while (stack not empty)

pop
visit
Push right
Push left

Visit order: 12, 9, 5, 7, 10

Iterative Preorder traversal



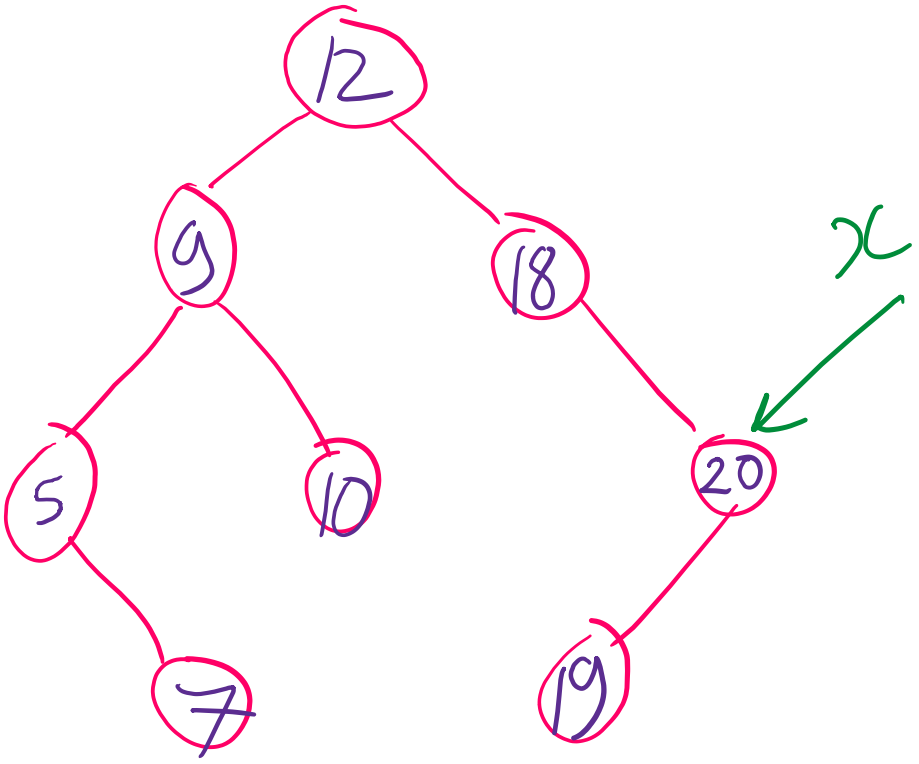
Stack (explicit)

while (stack not empty)

pop
visit
Push right
Push left

Visit order: 12, 9, 5, 7, 10, 18

Iterative Preorder traversal



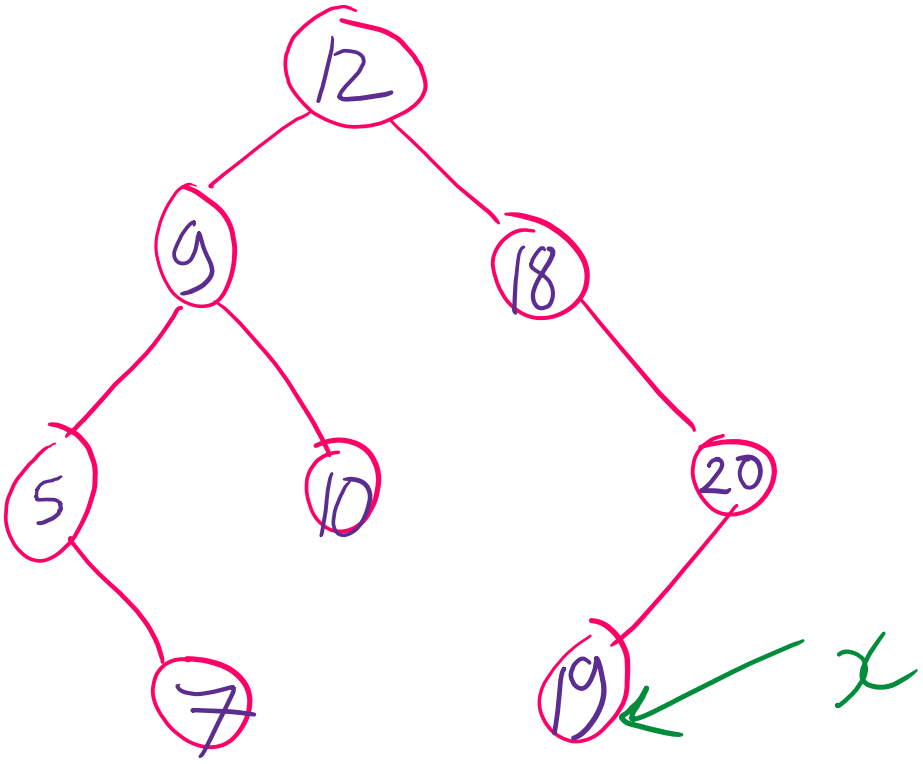
Stack (explicit)

while (stack not empty)

pop
visit
Push right
Push left

Visit order: 12, 9, 5, 7, 10, 18, 20

Iterative Preorder traversal



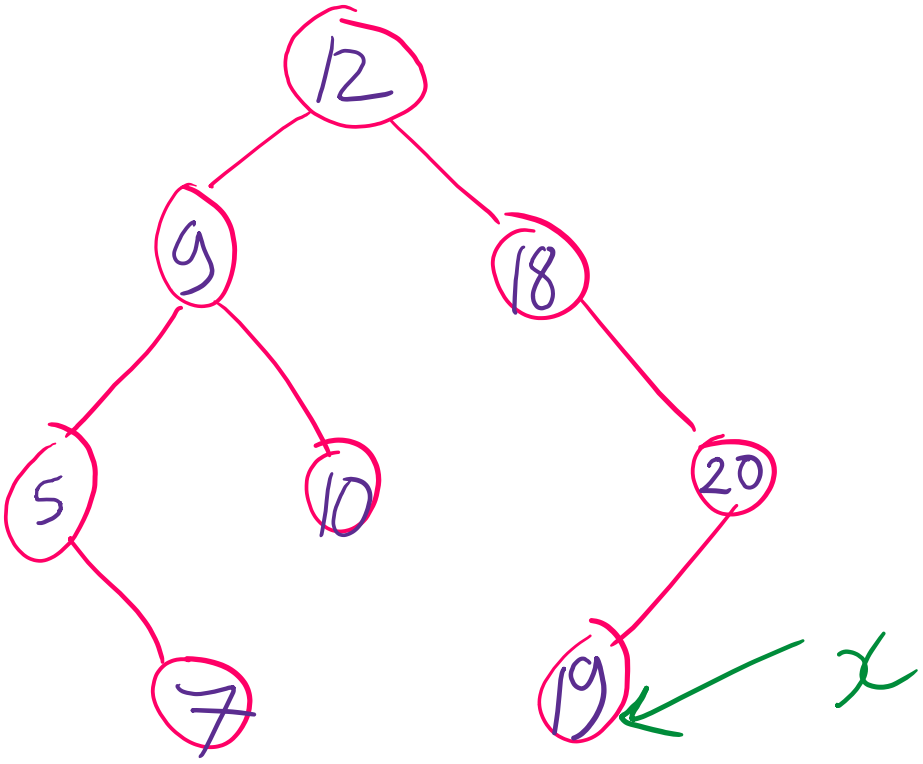
Stack (explicit)

while (stack not empty)

pop
visit
Push right
Push left

Visit order: 12, 9, 5, 7, 10, 18, 20, 19

Iterative Preorder traversal



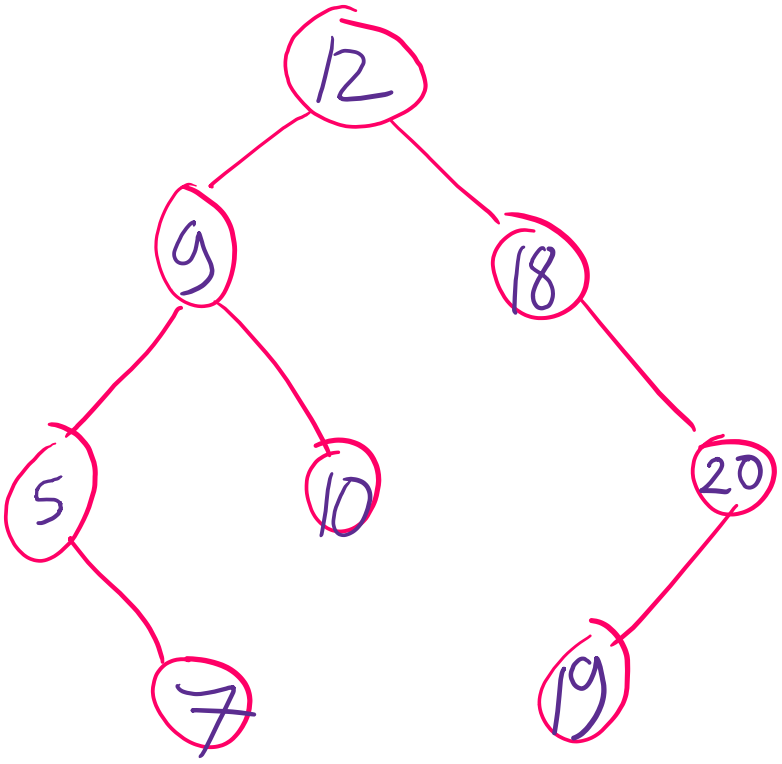
Stack
(explicit)

while (stack not empty)

pop
visit
Push right
Push left

Visit order: 12, 9, 5, 7, 10, 18, 20, 19

Iterative Inorder traversal



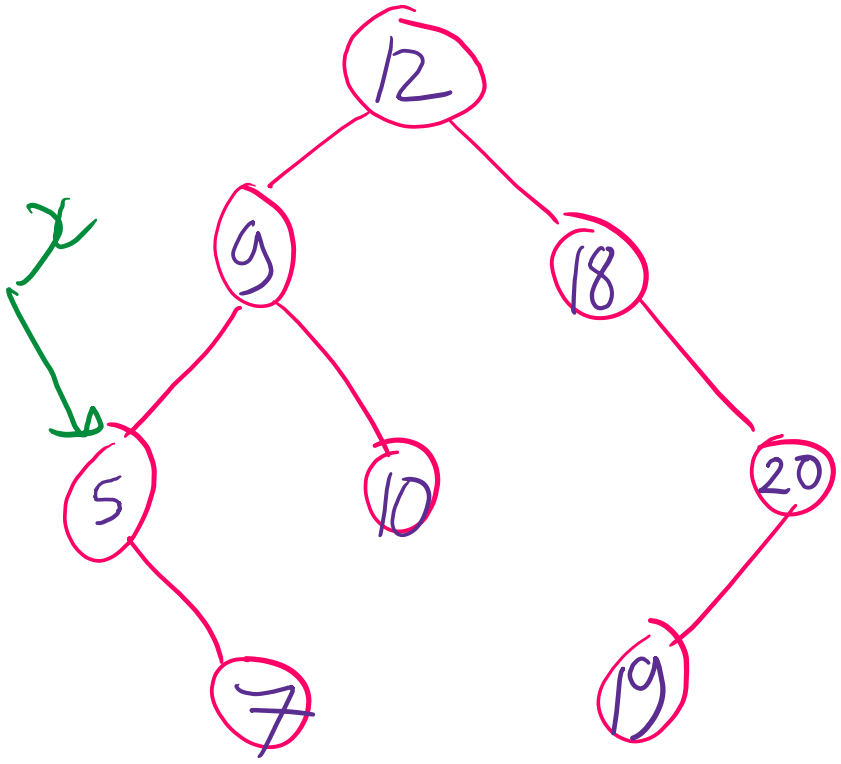
repeat until stack empty & x == null

Stack (explicit)

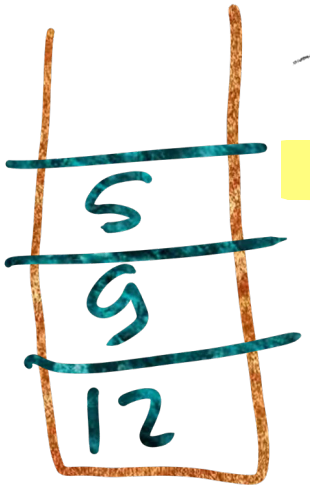
x = leftmost node
Push to stack while moving down
Pop & visit
x = x.right

Visit order:

Iterative Inorder traversal



repeat until stack empty & x == null



x = leftmost node

Push to stack while moving down

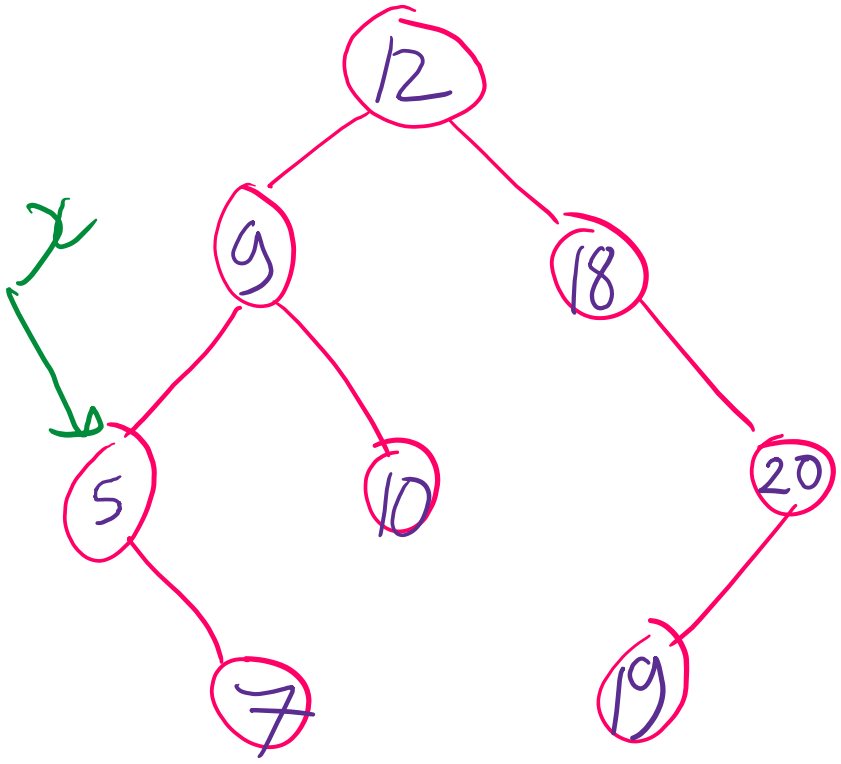
Pop & visit

x = x.right

Stack (explicit)

Visit order:

Iterative Inorder traversal



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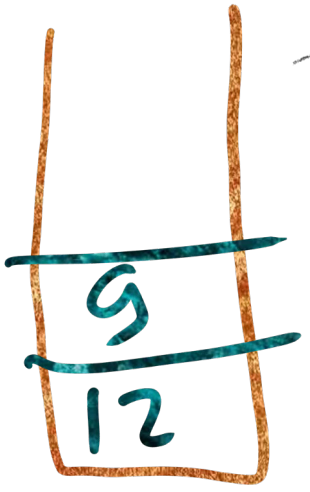


Stack (explicit)

x = leftmost node

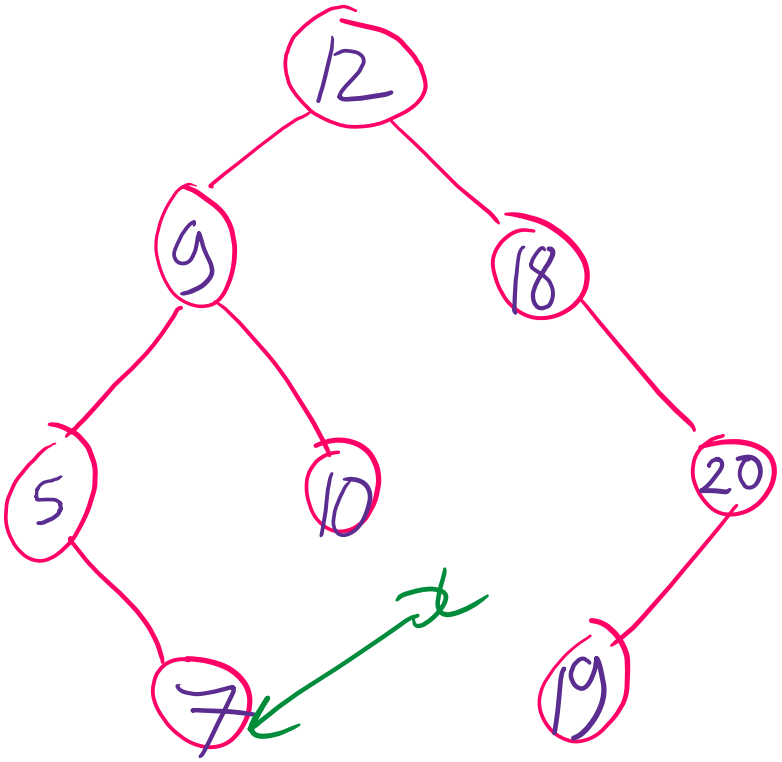
Push to stack while moving down

Pop & visit

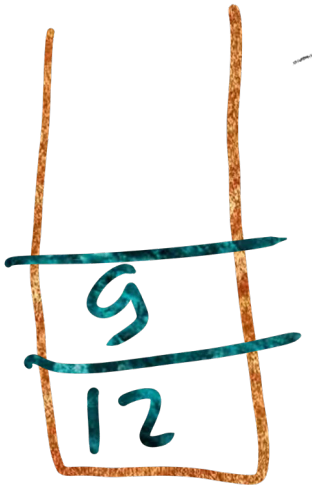
x = x.right

Visit order: 5

Iterative Inorder traversal



repeat until stack empty & x == null



Stack (explicit)

x = leftmost node

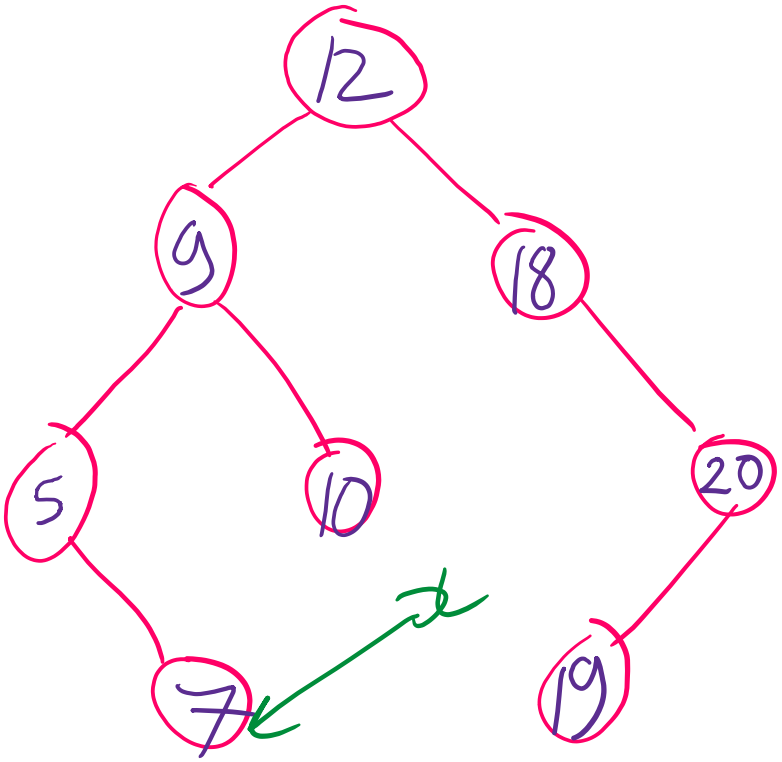
Push to stack while moving down

Pop & visit

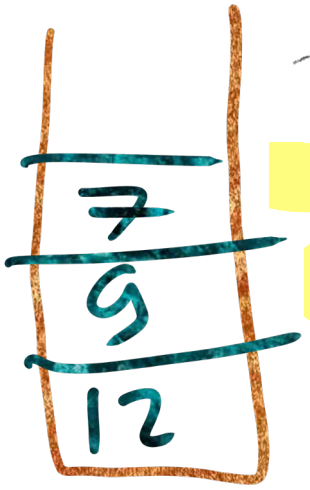
x = x.right

Visit order: 5

Iterative Inorder traversal



repeat until stack empty & x == null



Stack (explicit)

x = leftmost node

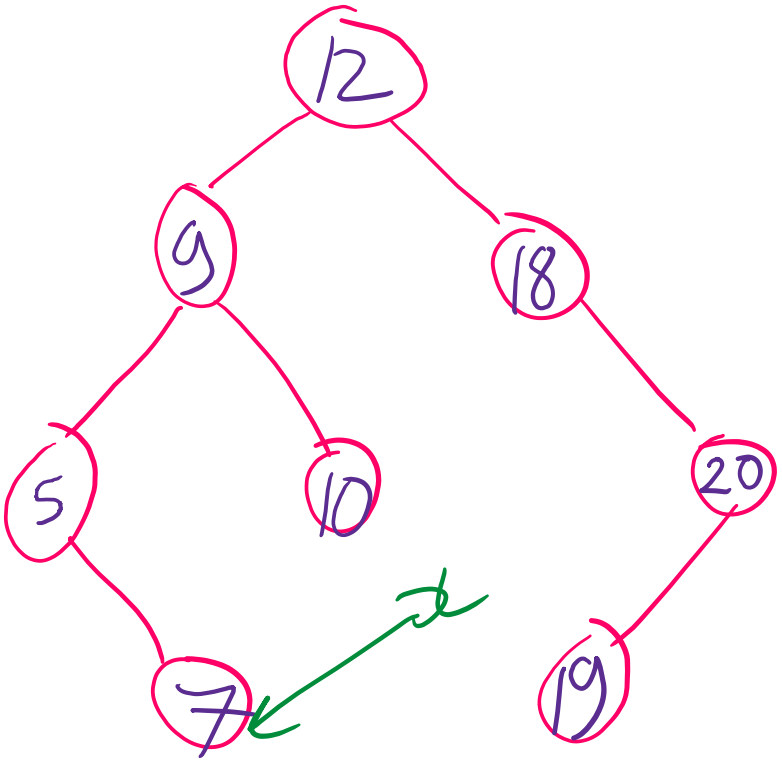
Push to stack while moving down

Pop & visit

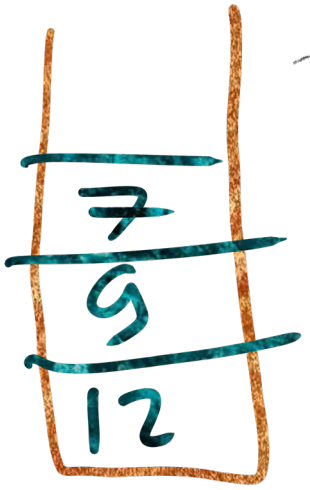
x = x.right

Visit order: 5

Iterative Inorder traversal



repeat until stack empty & x == null



x = leftmost node

Push to stack while moving down

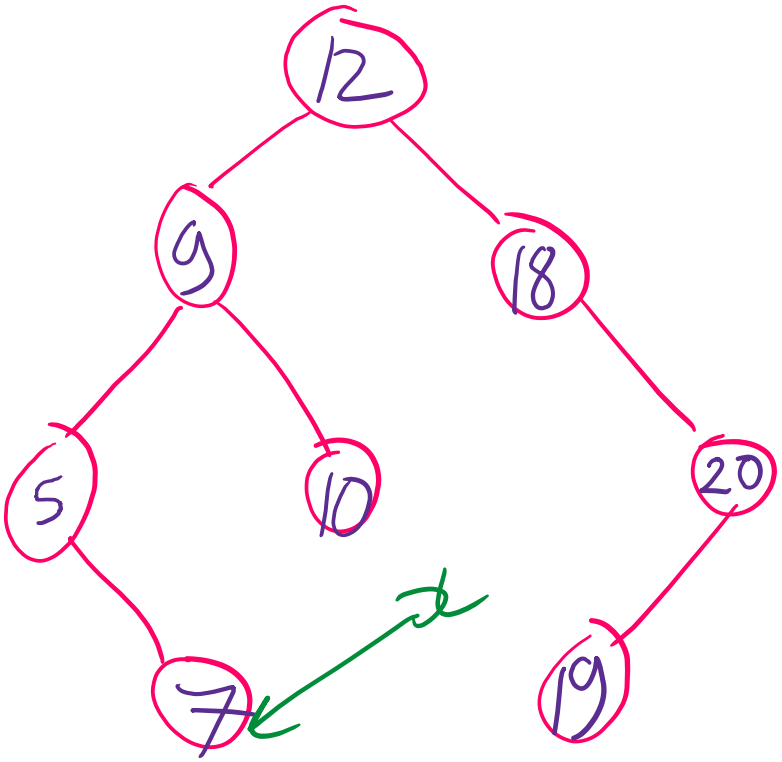
Pop & visit

x = x.right

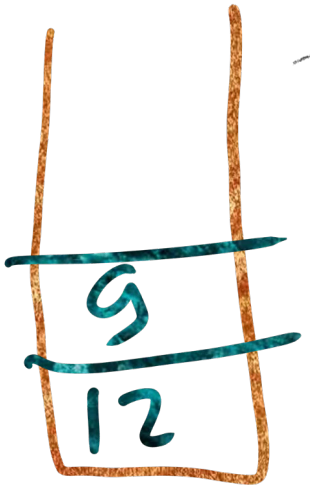
Stack (explicit)

Visit order: 5, 7

Iterative Inorder traversal



repeat until stack empty & x == null



x = leftmost node

Push to stack while moving down

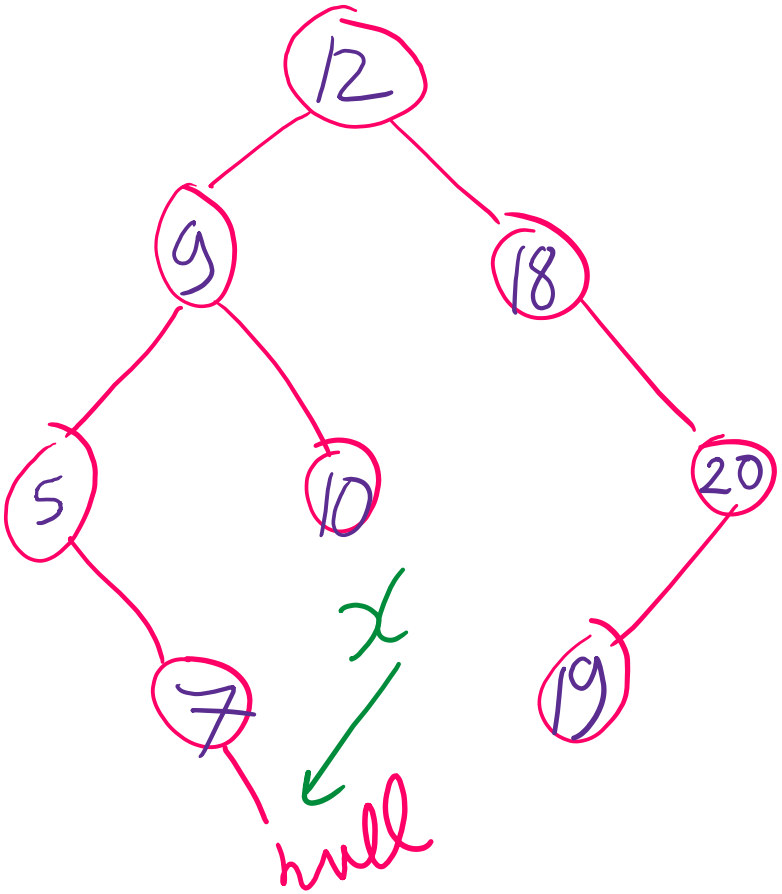
Pop & visit

x = x.right

Stack (explicit)

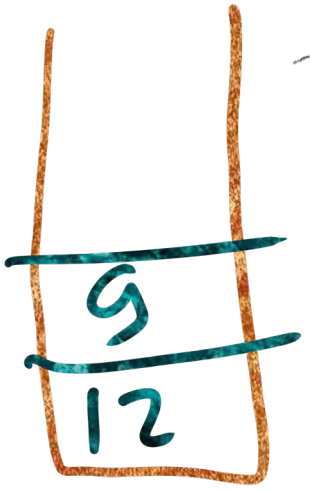
Visit order: 5, 7

Iterative Inorder traversal



Visit order: 5, 7

repeat until stack empty & x == null



x = leftmost node

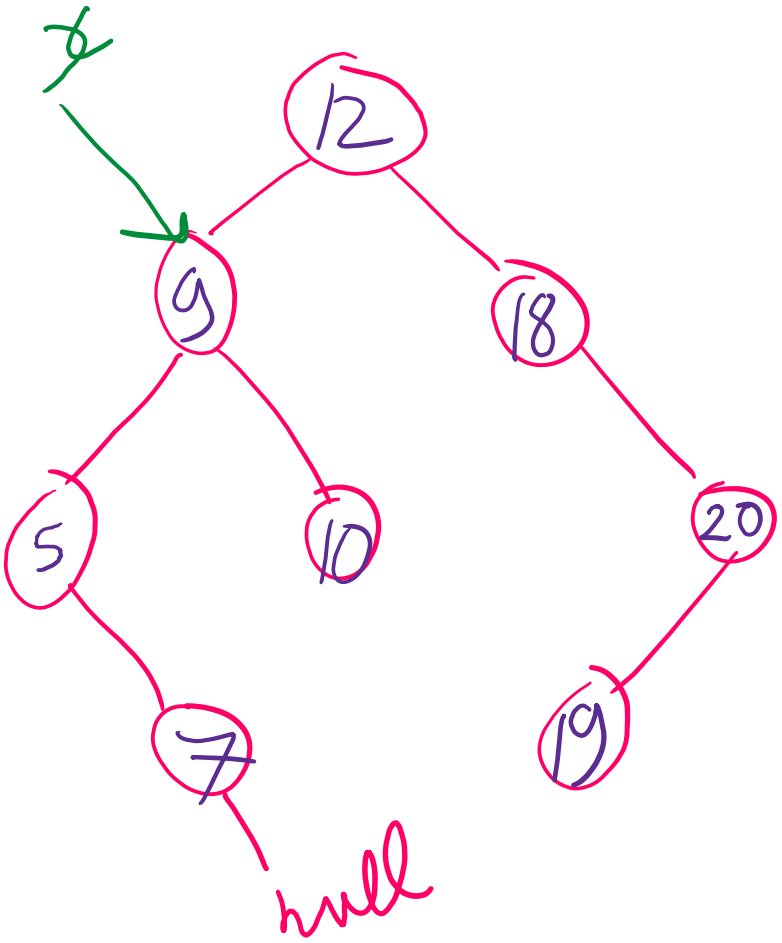
Push to stack while moving down

Pop & visit

x = x.right

Stack (explicit)

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node
push to stack while moving down

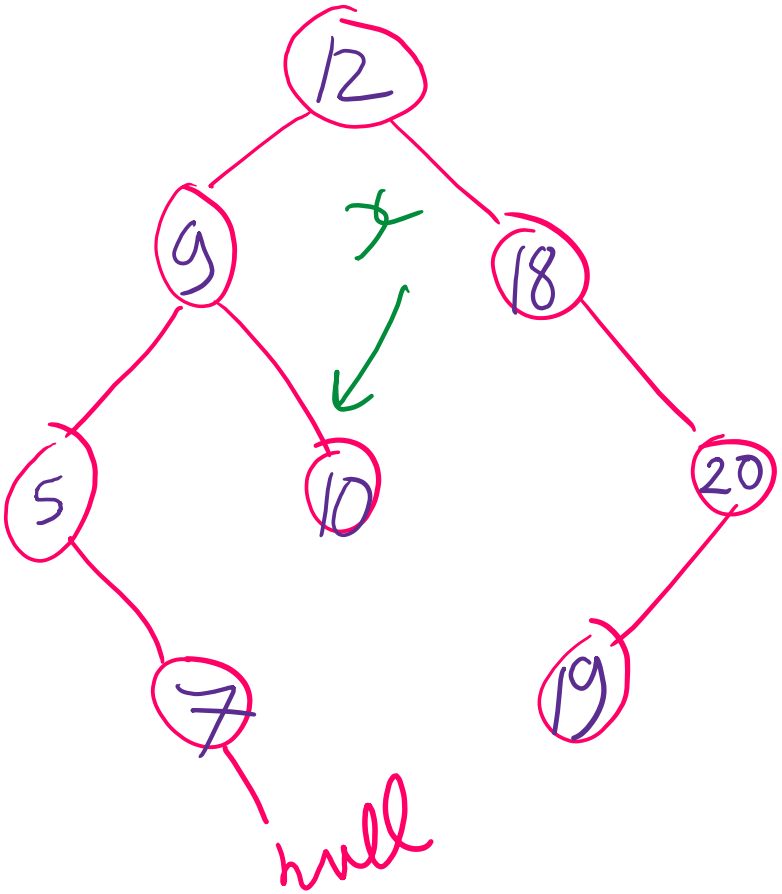
pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

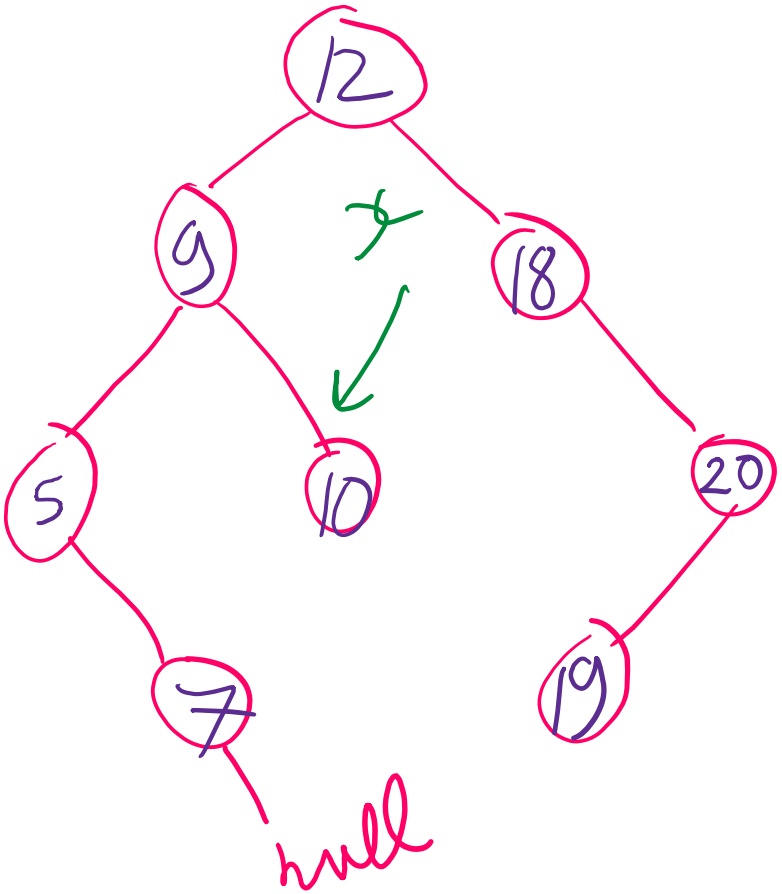
Pop & visit

x = x.right

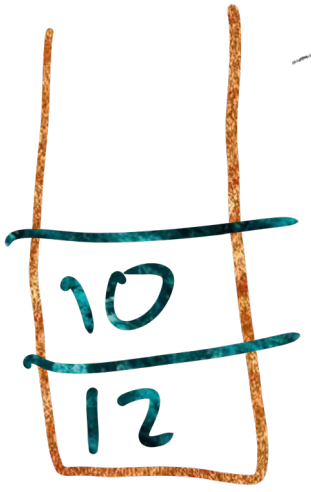
Stack (explicit)

Visit order: 5, 7, 9

Iterative Inorder traversal



repeat until stack empty & x == null



Stack (explicit)

x = leftmost node

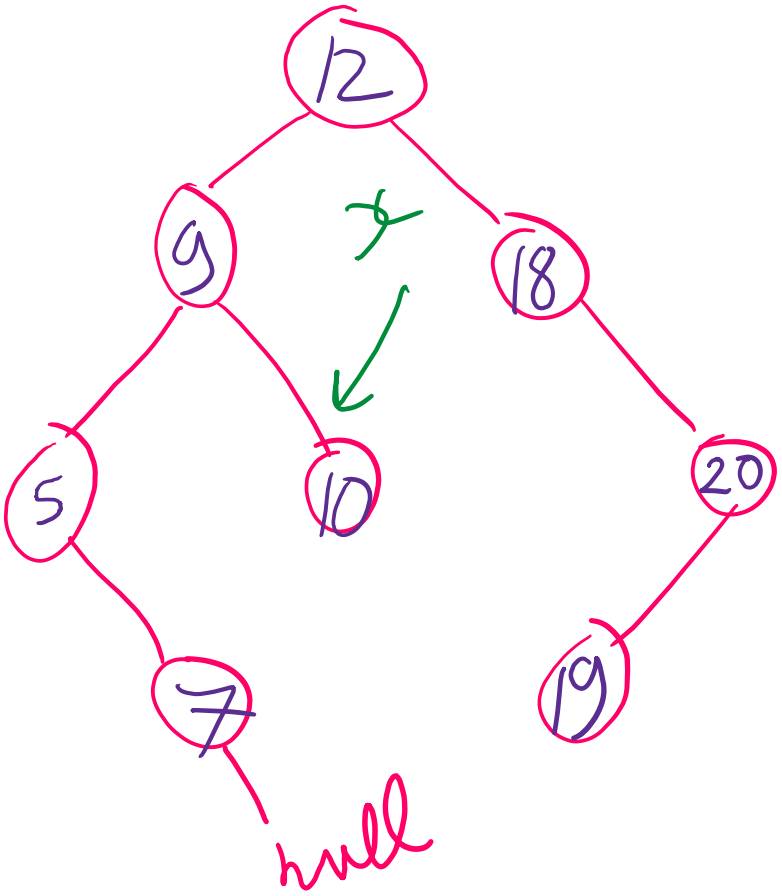
Push to stack while moving down

Pop & visit

x = x.right

Visit order: 5, 7, 9

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

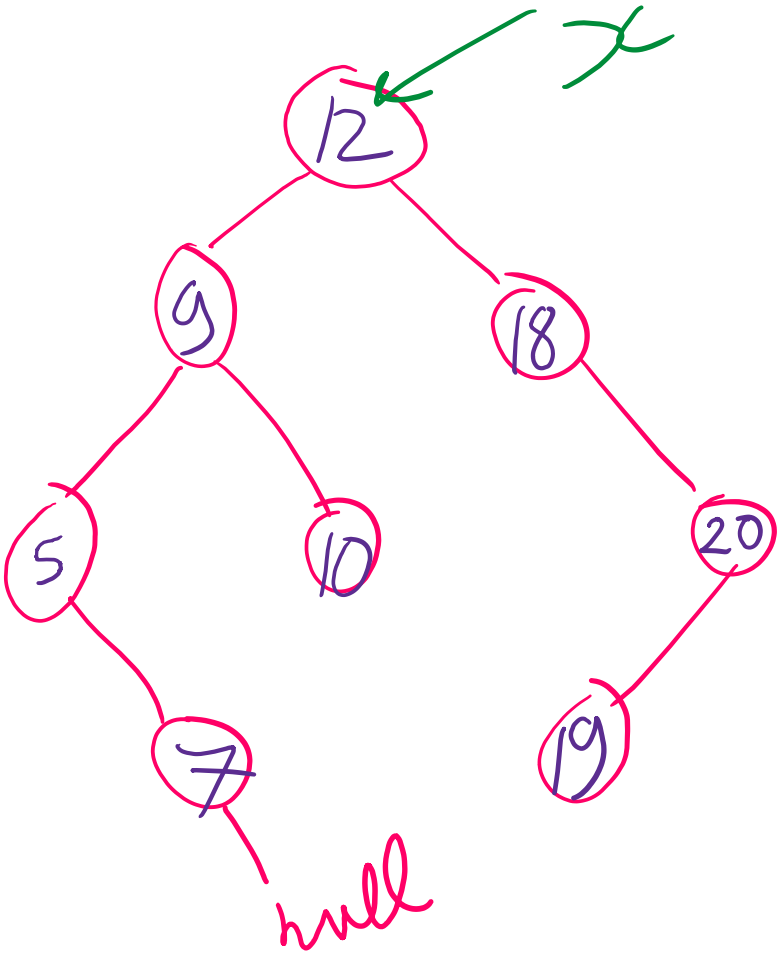
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

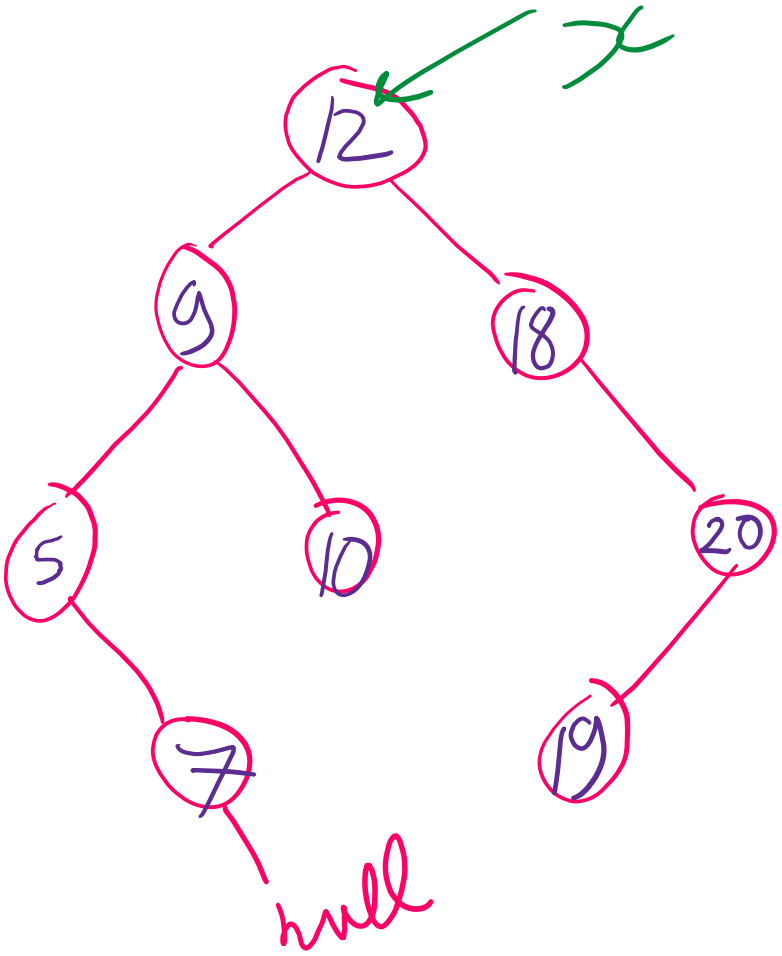
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

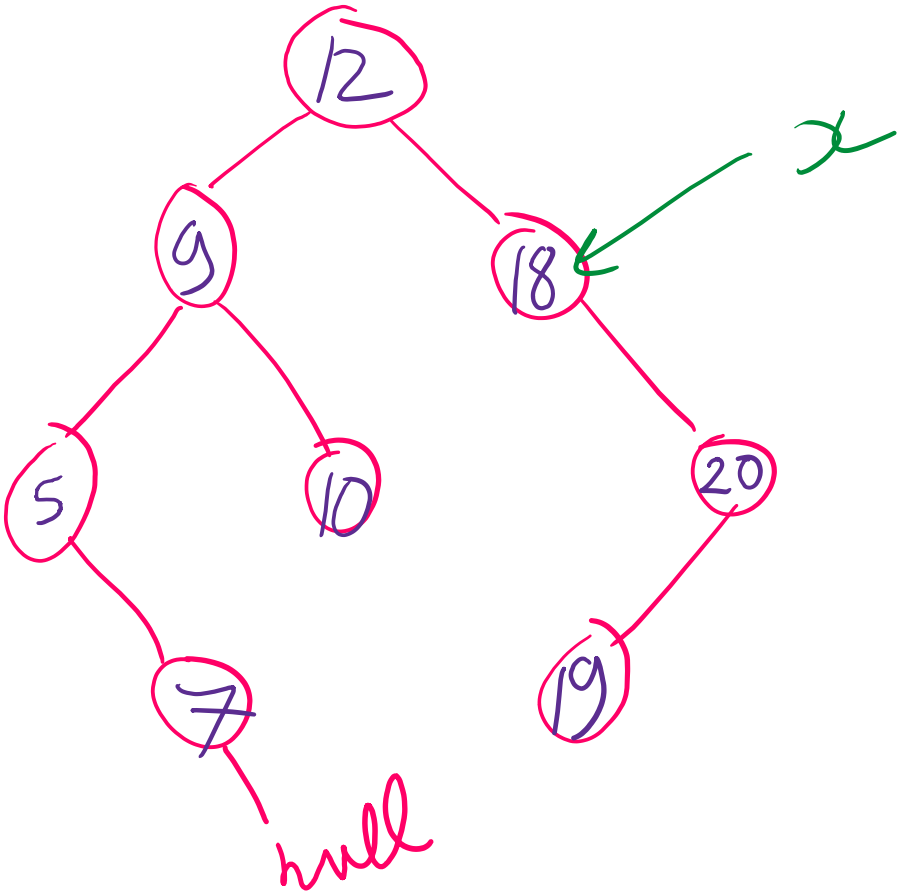
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

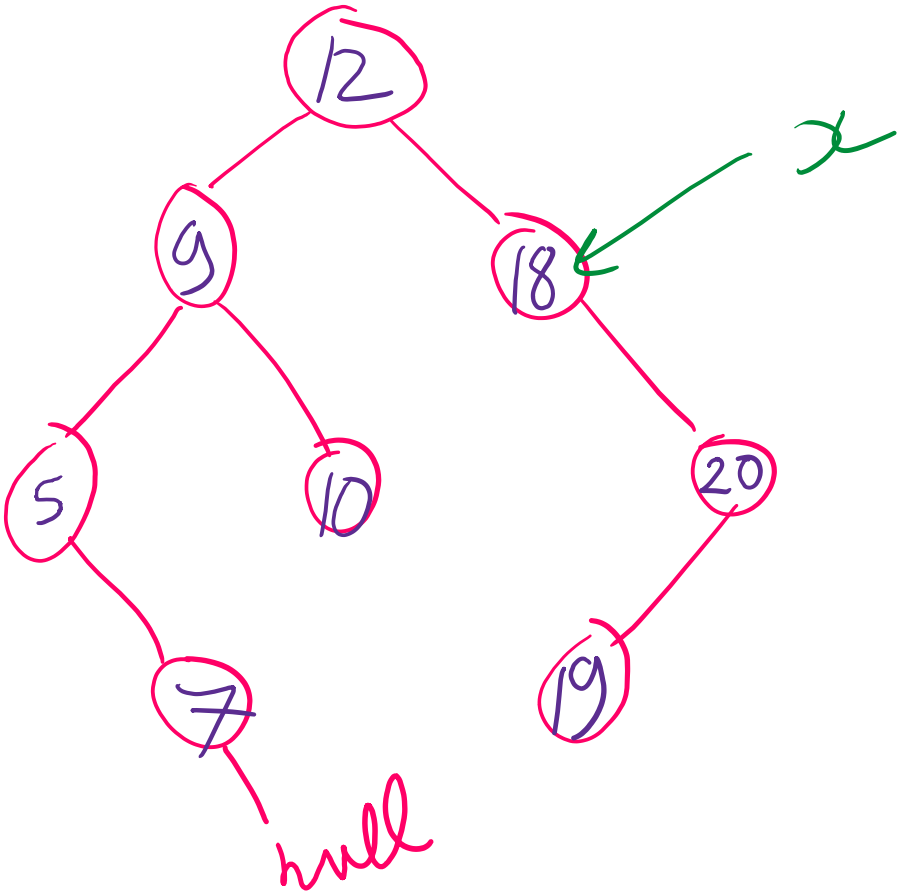
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12

Iterative Inorder traversal



repeat until stack empty & x == null

Stack (explicit)

x = leftmost node

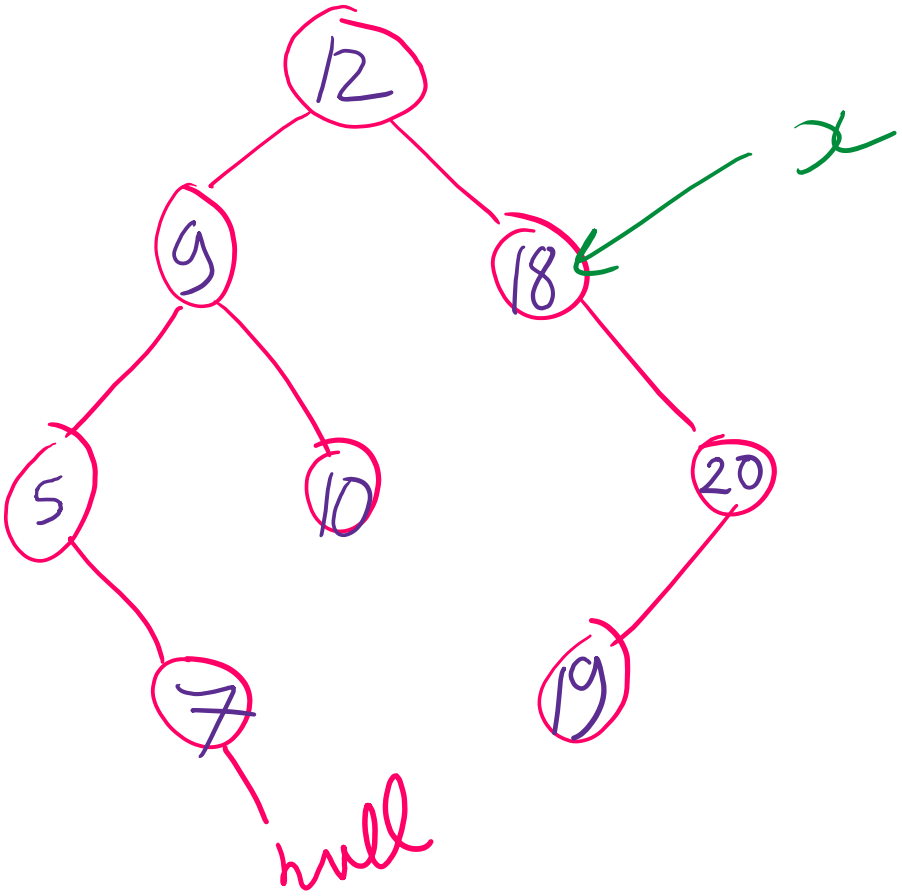
Push to stack while moving down

Pop & visit

x = x.right

Visit order: 5, 7, 9, 10, 12

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

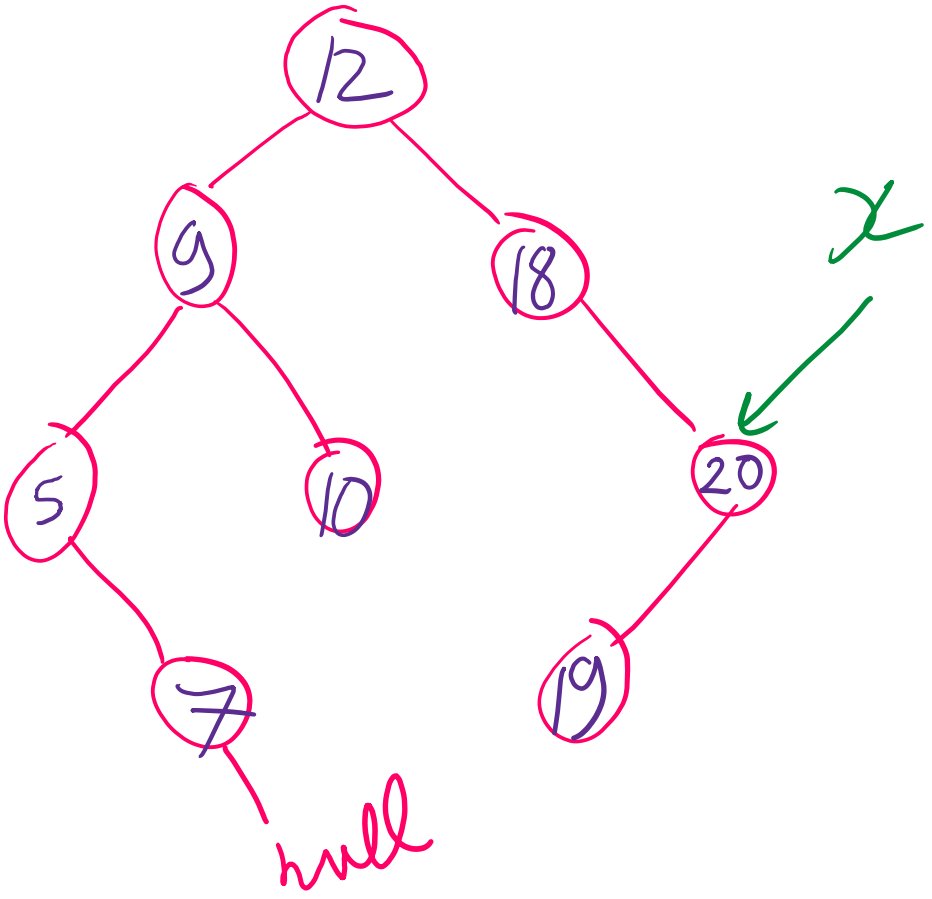
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12, 18

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

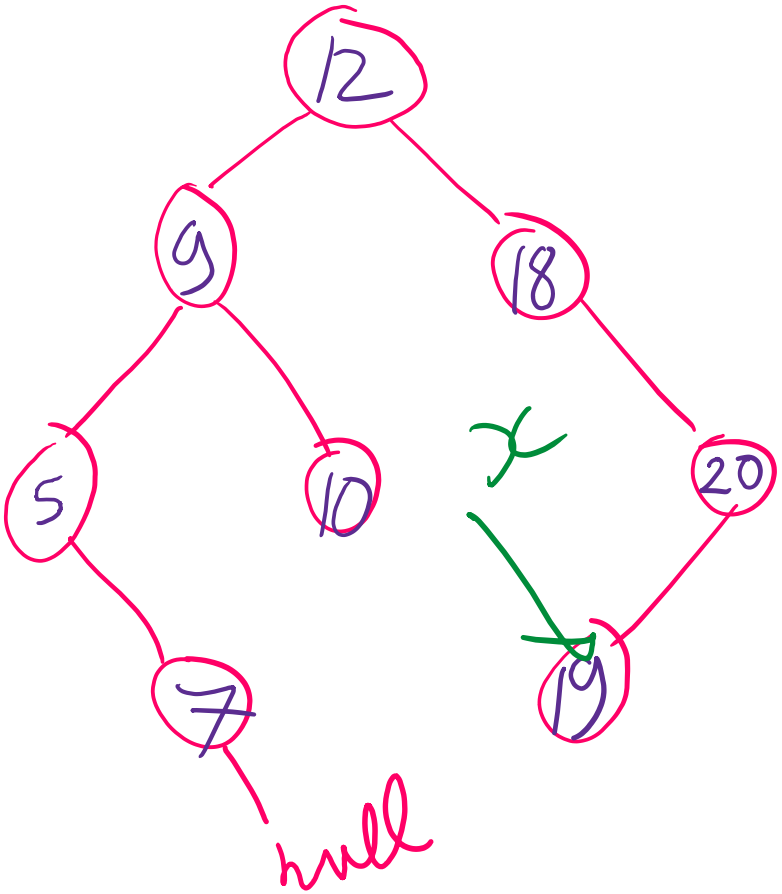
Pop & visit

x = x.right

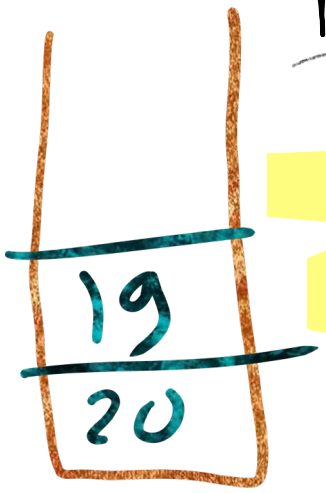
Stack (explicit)

Visit order: 5, 7, 9, 10, 12, 18

Iterative Inorder traversal



repeat until stack empty & x == null



x = leftmost node

Push to stack while moving down

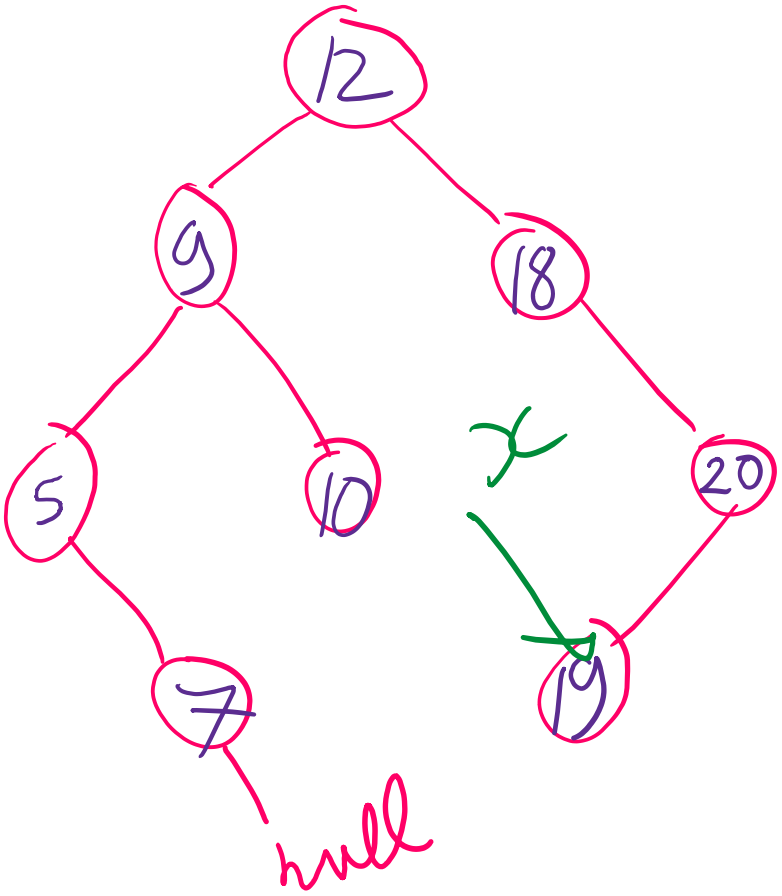
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12, 18

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

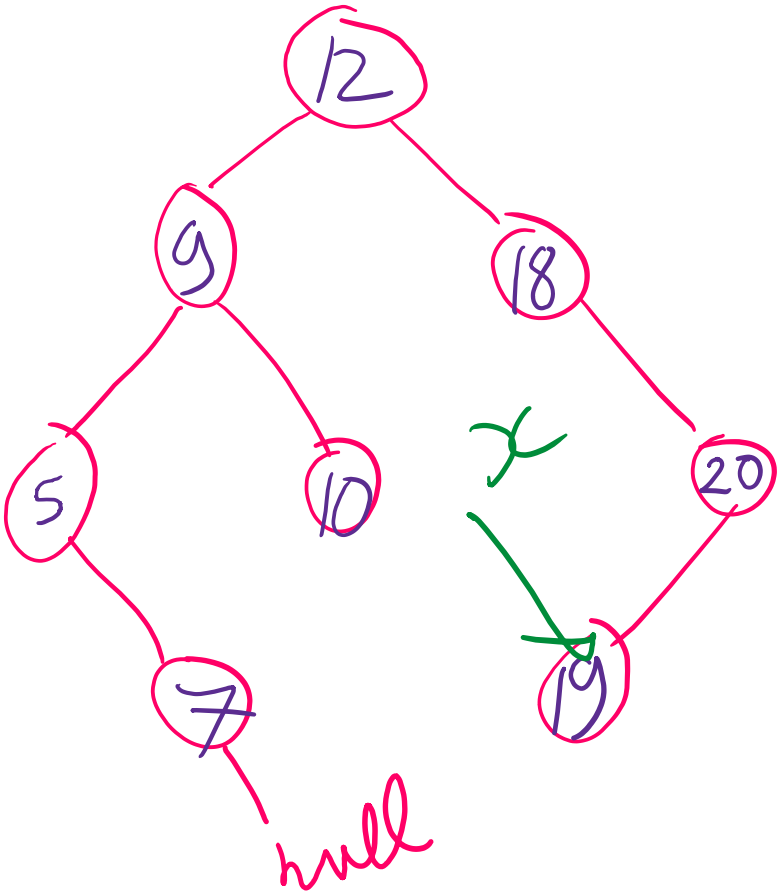
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12, 18, 19

Iterative Inorder traversal



repeat until stack empty & x == null

Stack (explicit)

x = leftmost node

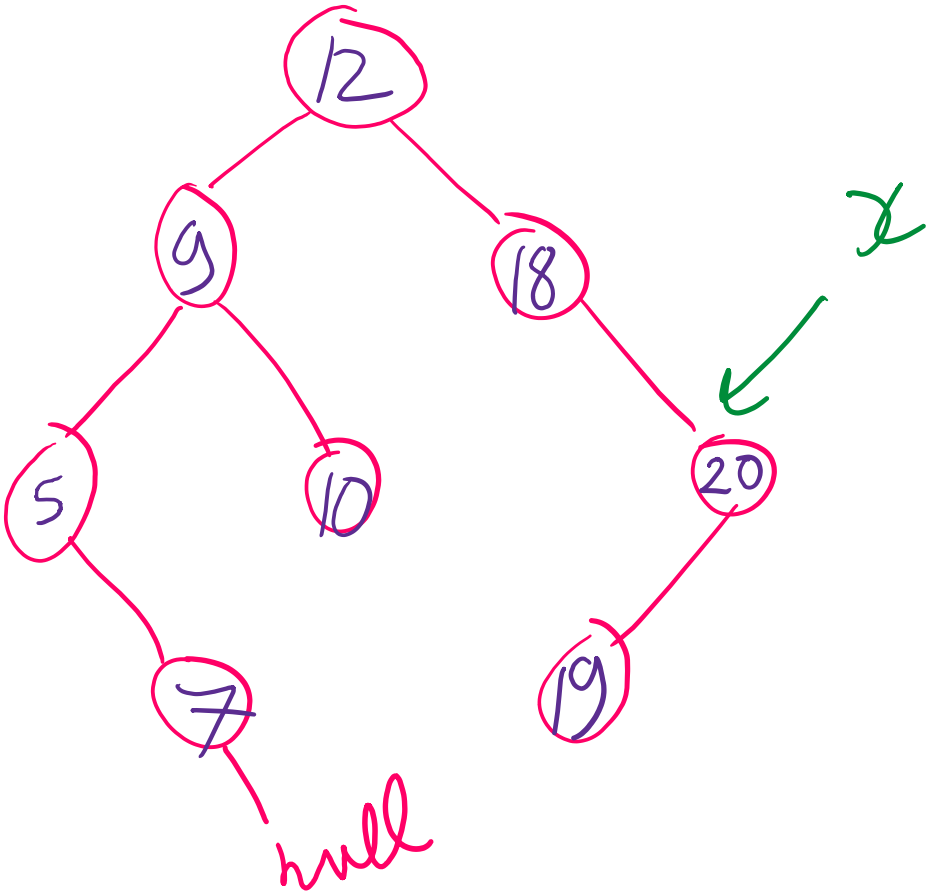
Push to stack while moving down

Pop & visit

x = x.right

Visit order: 5, 7, 9, 10, 12, 18, 19

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

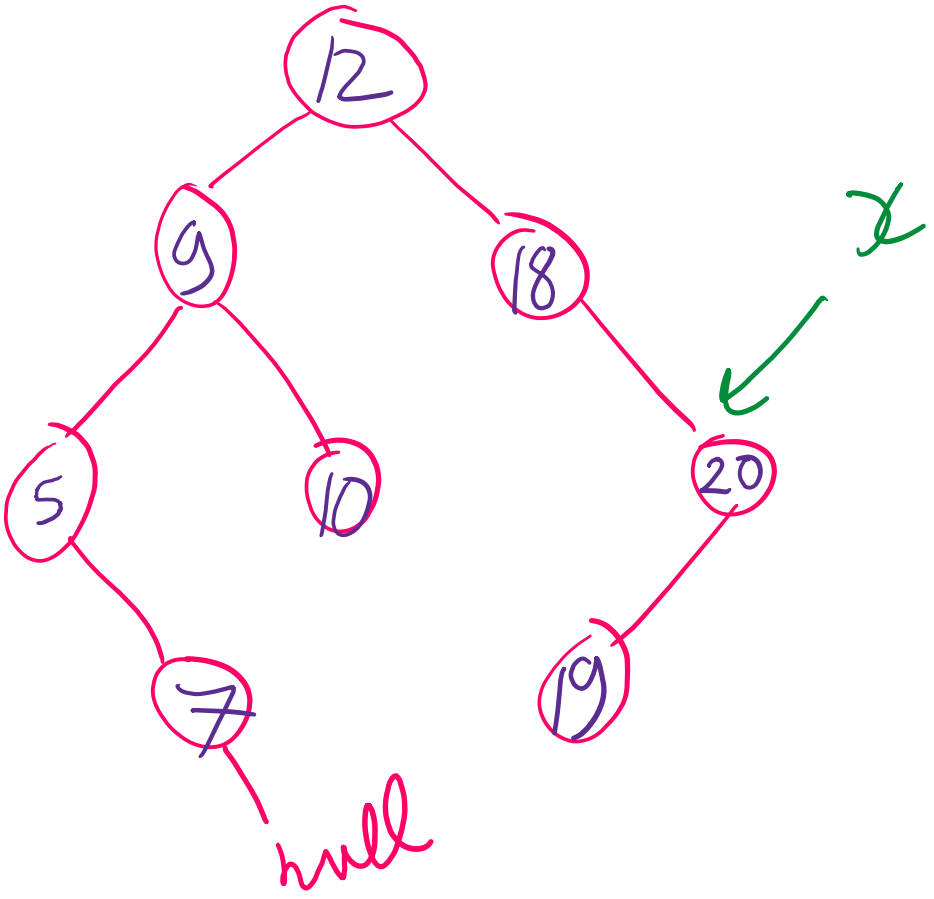
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12, 18, 19, 20

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

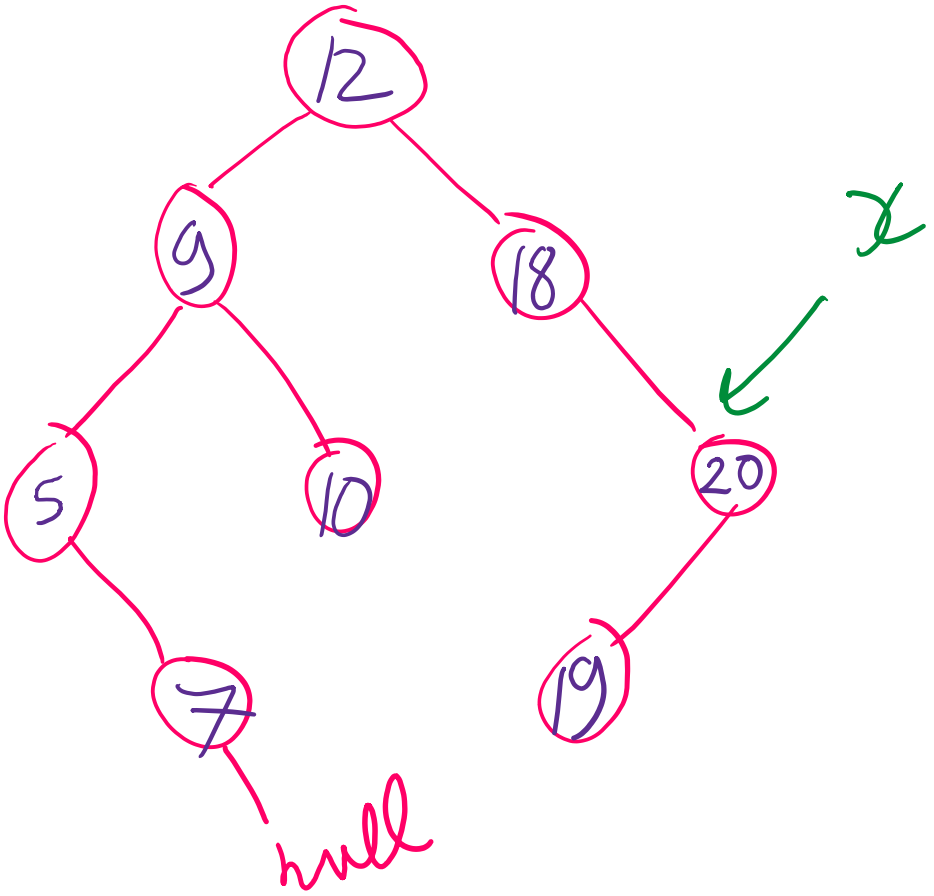
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12, 18, 19, 20

Iterative Inorder traversal



repeat until stack empty & x == null

Stack (explicit)

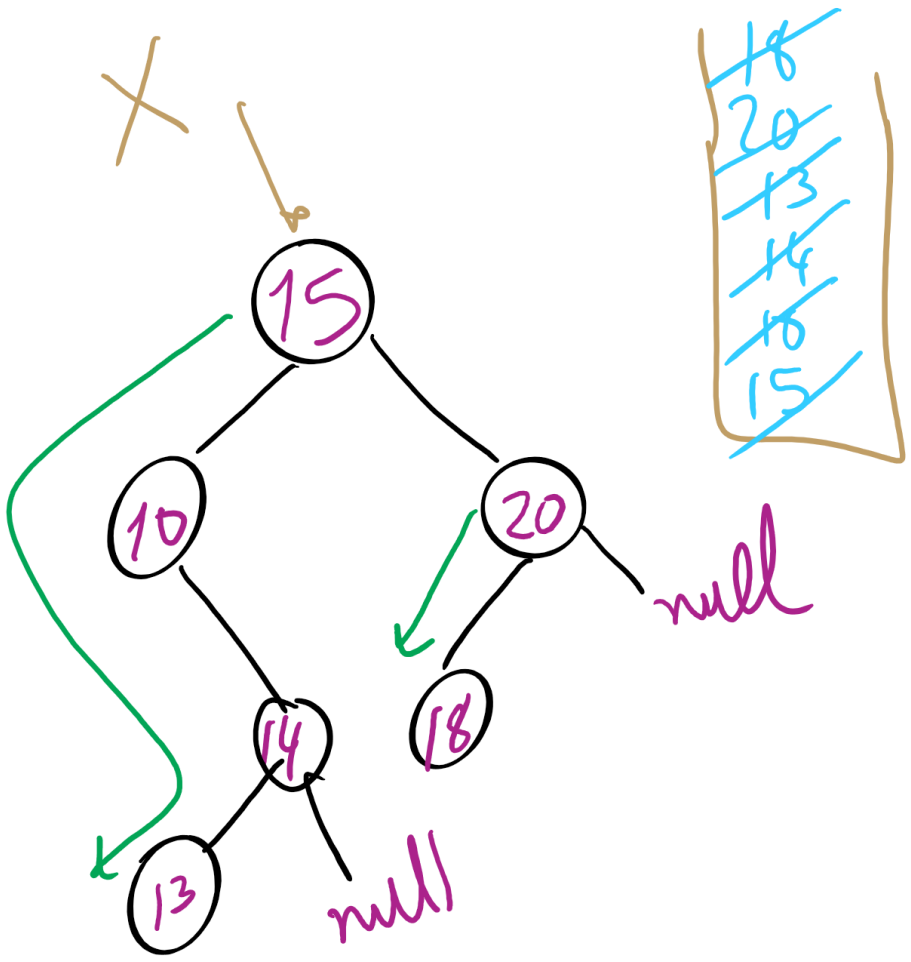
x = leftmost node
Push to stack while moving down
Pop & visit
x = x.right

Visit order: 5, 7, 9, 10, 12, 18, 19, 20

Traversals of a Binary Tree

- Preorder traversal
  - Visit root before we visit root's subtrees
- Inorder traversal
  - Visit root of a binary tree between visiting nodes in root's subtrees.
- Postorder traversal
  - Visit root of a binary tree after visiting nodes in root's subtrees
  - left then right then root

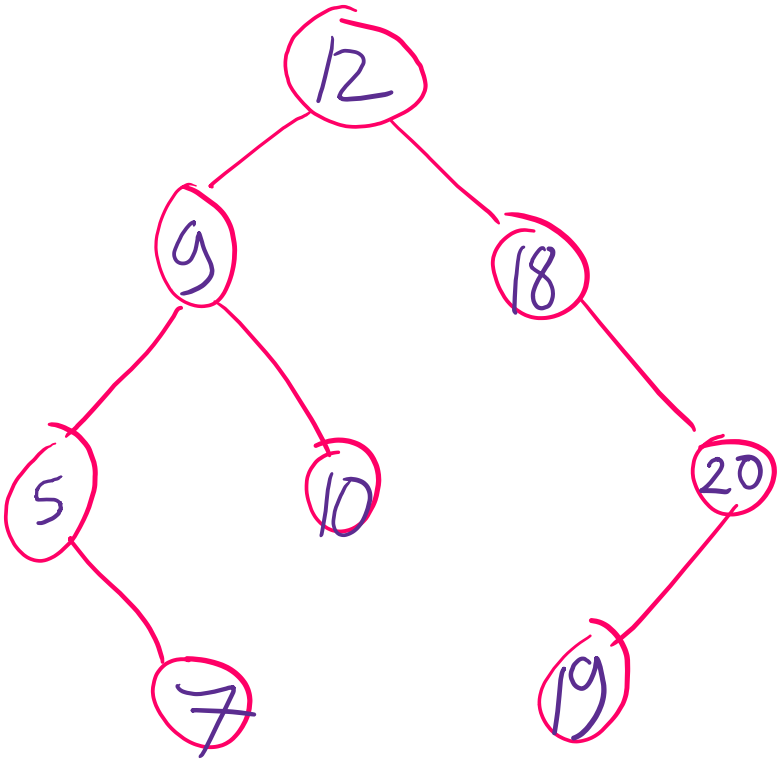
Iterative Post-order Traversal

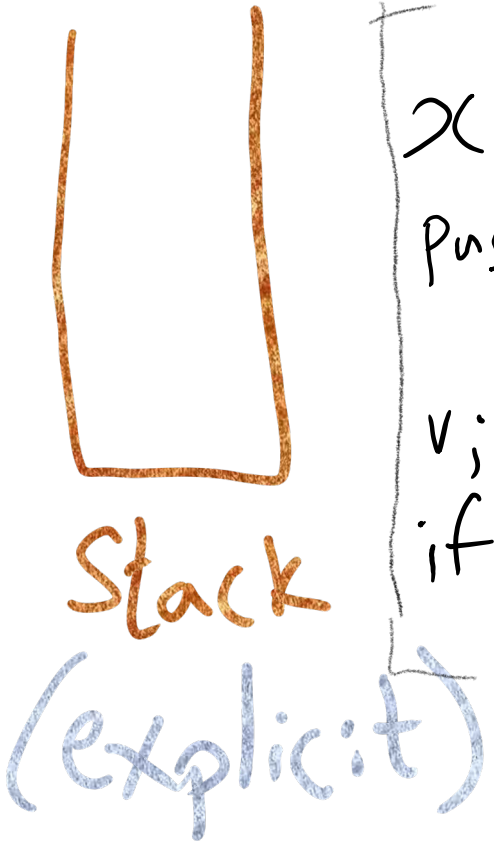


13, 14, 10, 18, 20, 15

```
stack = empty stack
x = root
While( x != null ||
       stack is not empty){
  -find leftmost leaf
  -push nodes as we
     go down the tree
  - x = pop() & visit x
  - if x is a left child
       x = right sibling
  else
       x = null
}
```

Iterative Postorder traversal





repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is right child

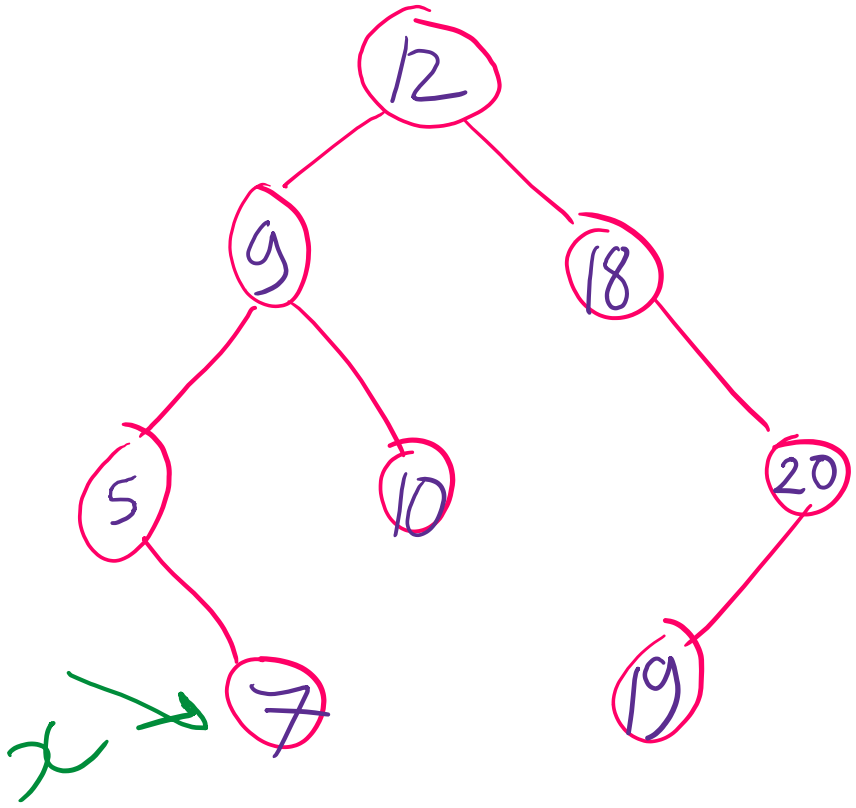
pop & visit

skip leftmost leaf

else x = parent.right

Visit order:

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf
Push to stack while moving down

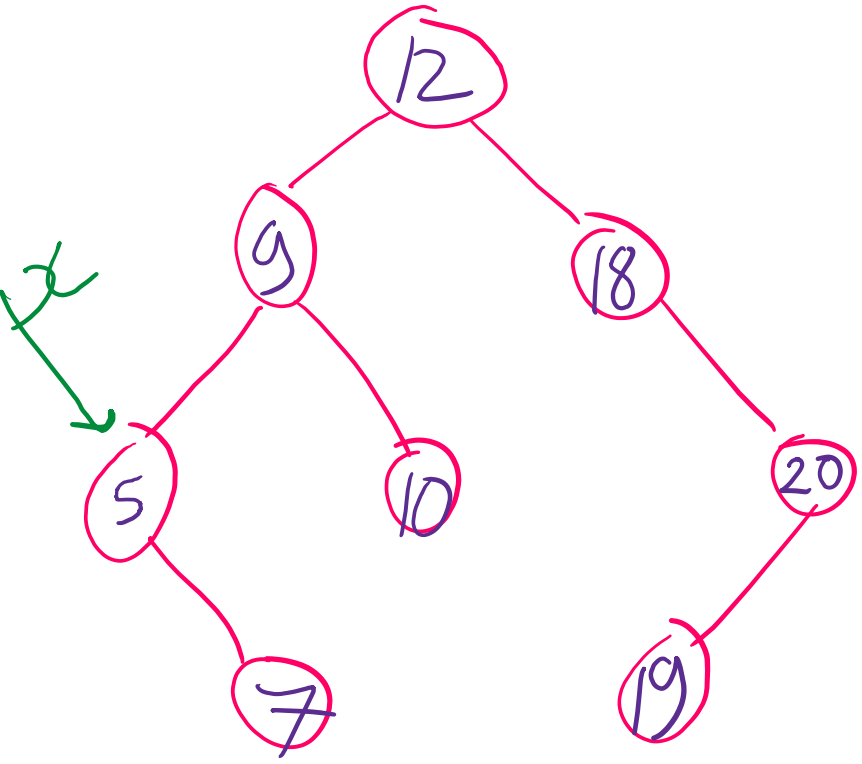
visit x

if x is right child
pop & visit
skip leftmost leaf

else x = parent.right

Visit order: 7

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is rightchild

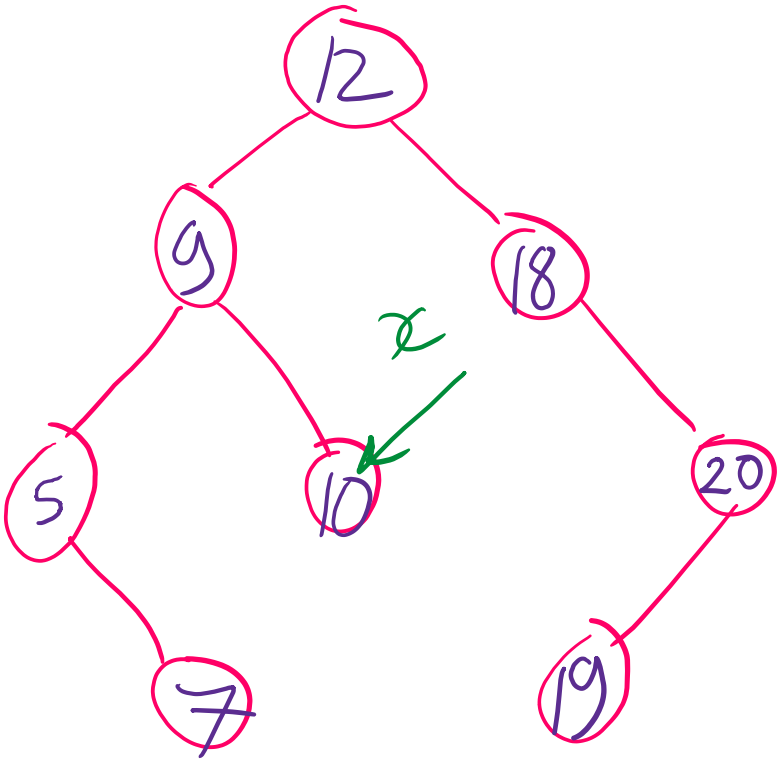
pop & visit

skip leftmost leaf

else x = parent.right

Visit order: 7, 5

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is right child

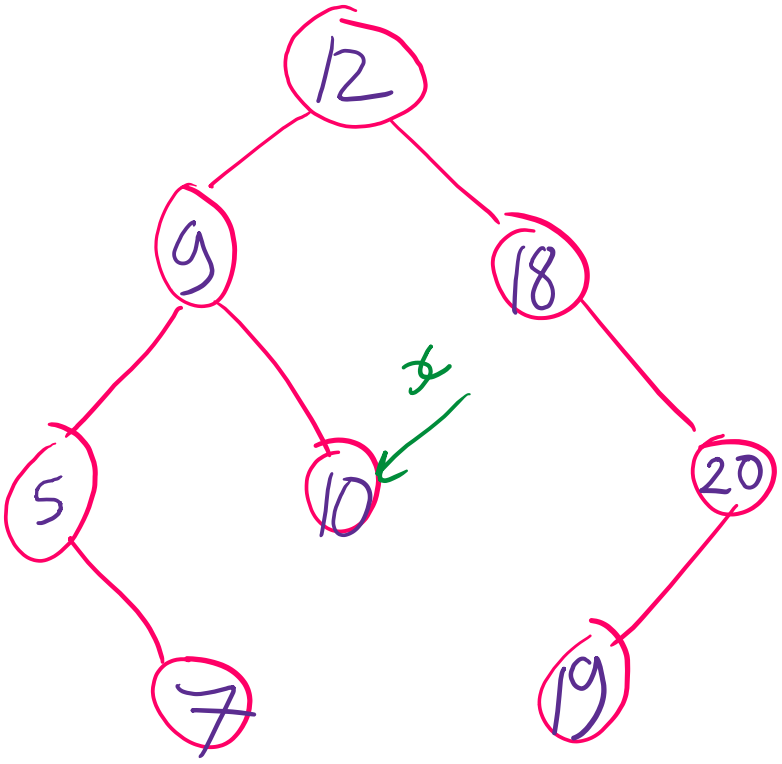
pop & visit

skip leftmost leaf

else x = parent.right

Visit order: 7, 5

Iterative Postorder traversal



repeat until stack empty & x == null

Stack (explicit)

x = leftmost leaf

Push to stack while moving down

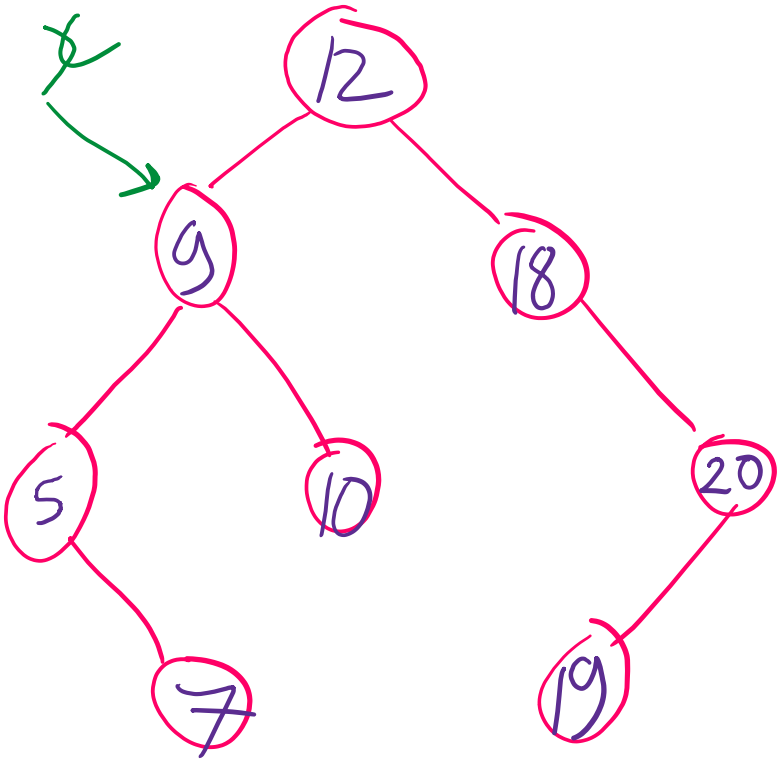
visit x

if x is right child
pop & visit
skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is right child

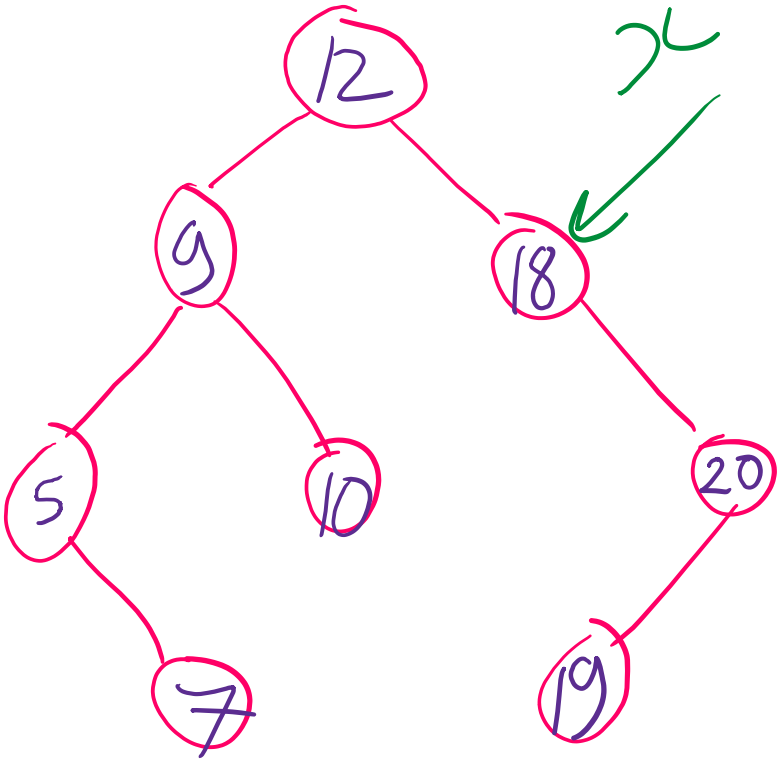
pop & visit

skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is rightchild

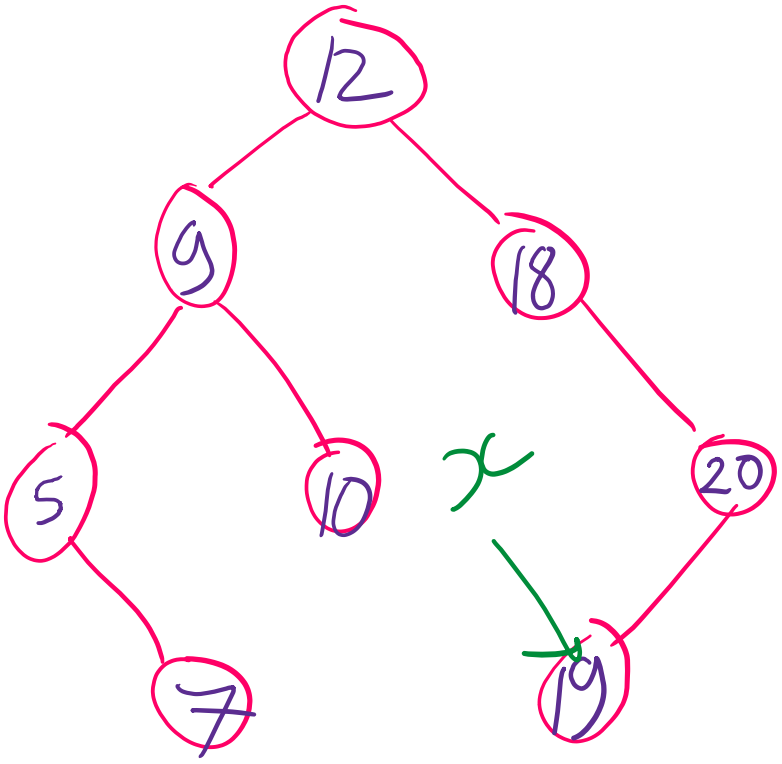
pop & visit

skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is right child

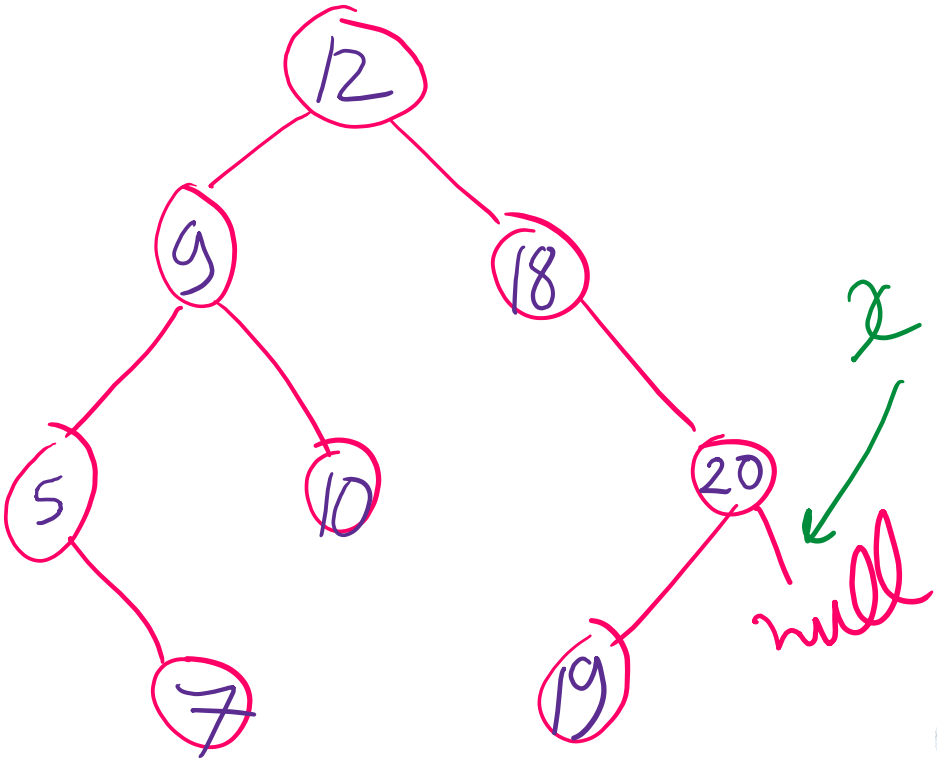
pop & visit

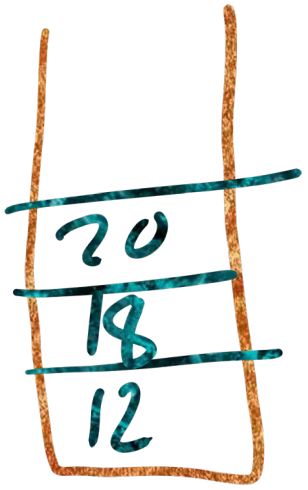
skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9, 19

Iterative Postorder traversal





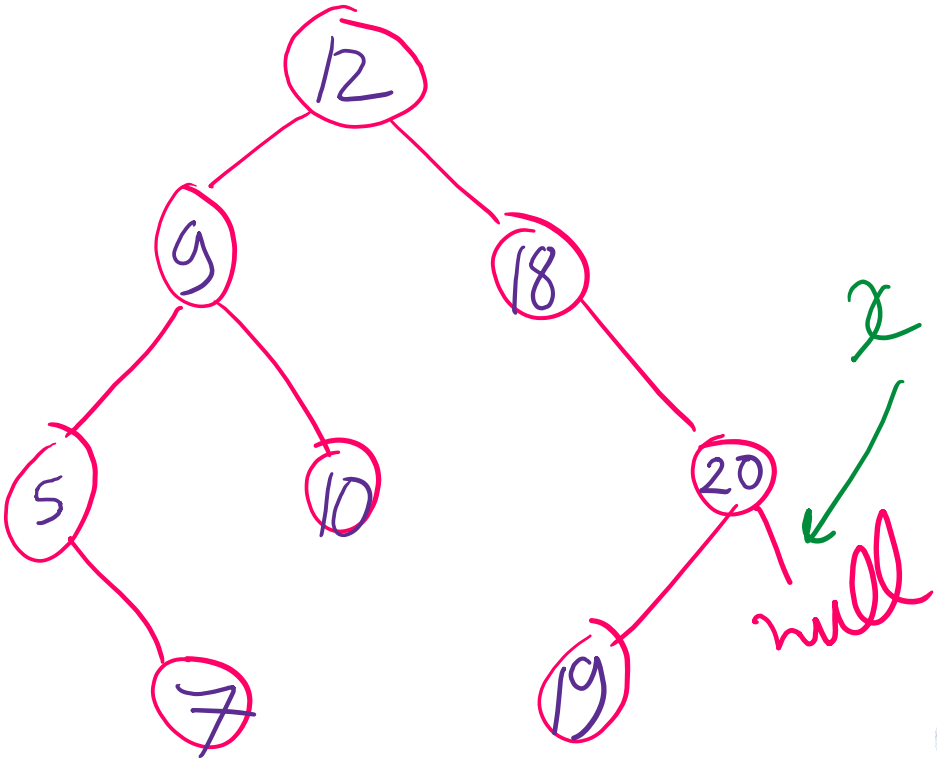
Stack (explicit)

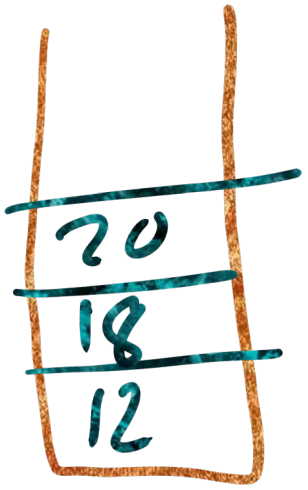
repeat until stack empty & x == null

x = leftmost leaf
push to stack while moving down
visit x
if x is right child
pop & visit
skip leftmost leaf
else x = parent.right

Visit order: 7, 5, 10, 9, 19

Iterative Postorder traversal





Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf
push to stack while moving down

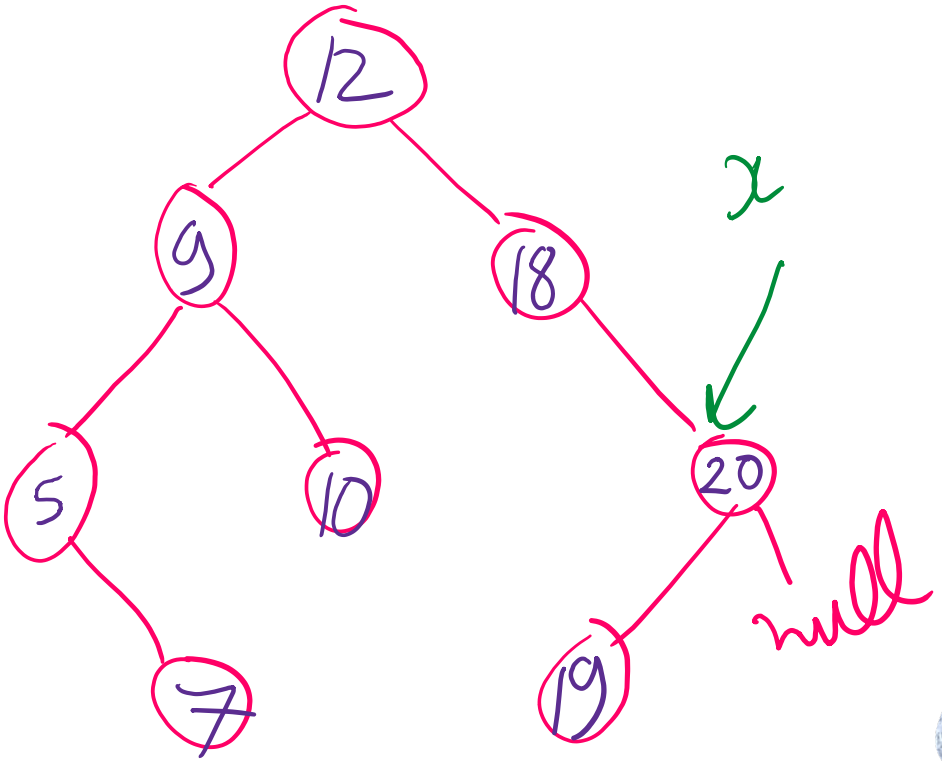
visit x

if x is right child
pop & visit
skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9, 19

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is right child

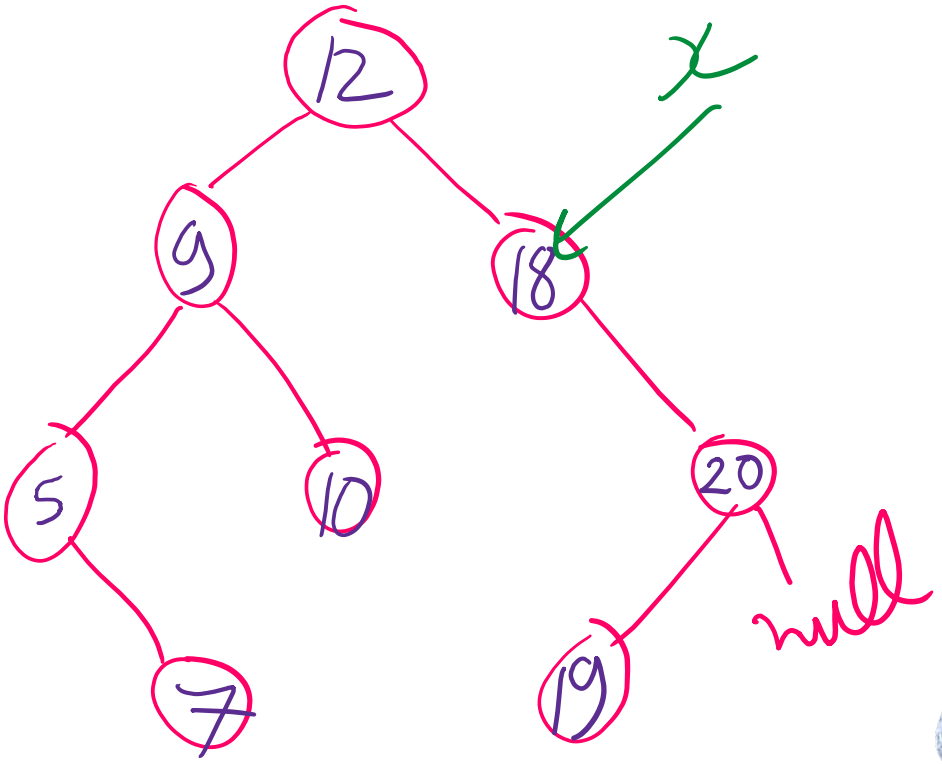
pop & visit

skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9, 19, 20

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is right child

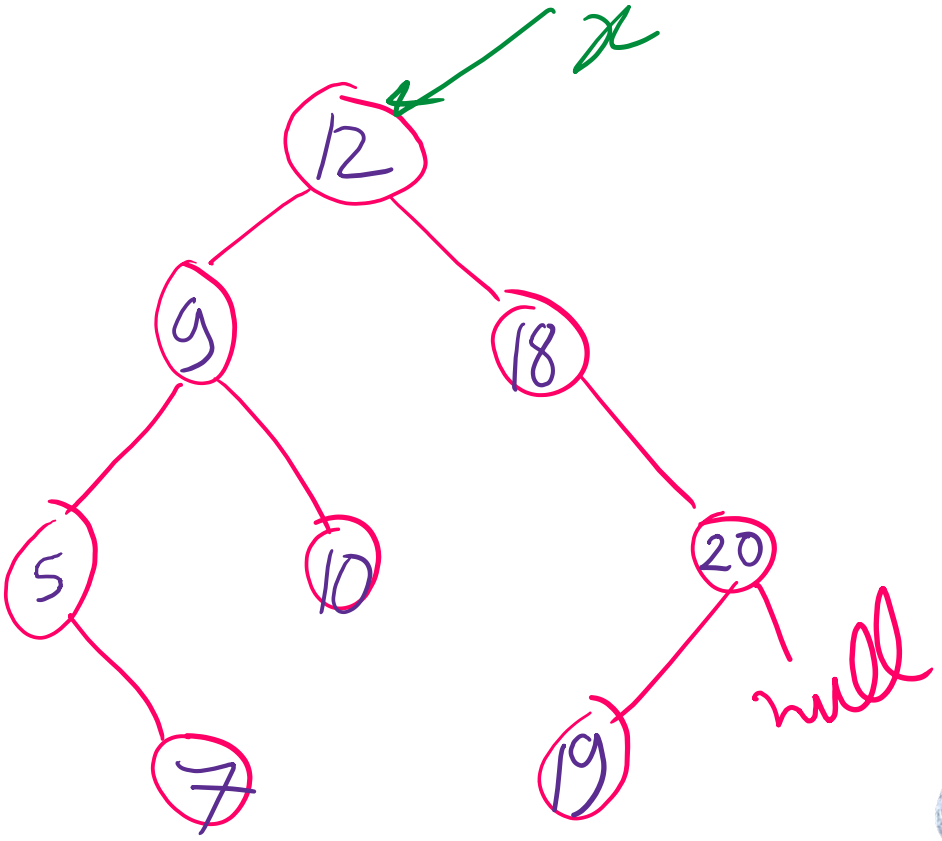
pop & visit

skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9, 19, 20, 18

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf
push to stack while moving down

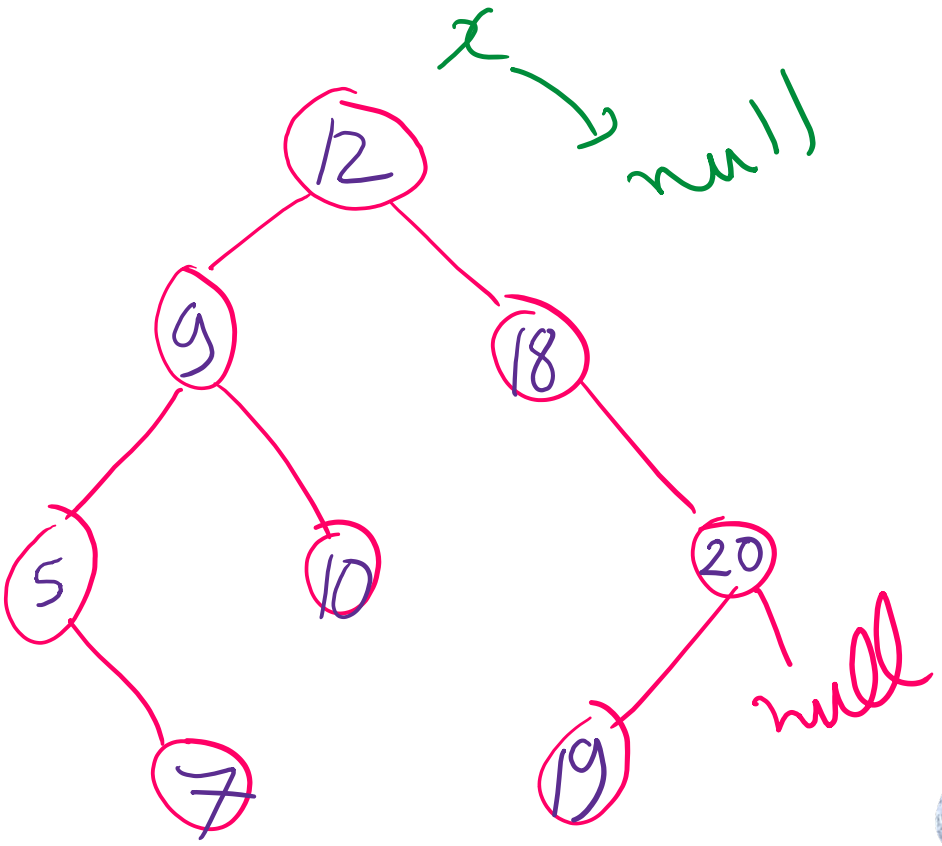
visit x

if x is right child
pop & visit
skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9, 19, 20, 18, 12

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf
Push to stack while moving down

visit x

if x is right child
pop & visit
skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9, 19, 20, 18, 12

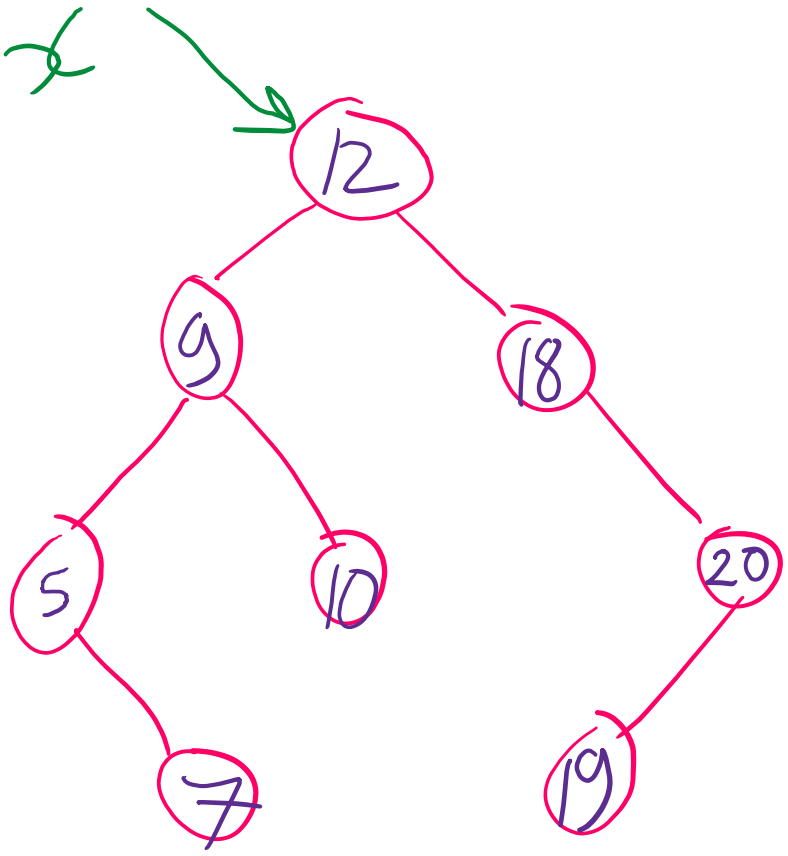
Traversals of a Binary Tree

- Preorder traversal
  - Visit root before we visit root's subtrees
- Inorder traversal
  - Visit root of a binary tree between visiting nodes in root's subtrees.
- Postorder traversal
  - Visit root of a binary tree after visiting nodes in root's subtrees
- Level-order traversal
  - Begin at root and visit nodes one level at a time
  - We will see the implementation when we learn Breadth-First Search of Graphs

Traversals of Binary Search Trees

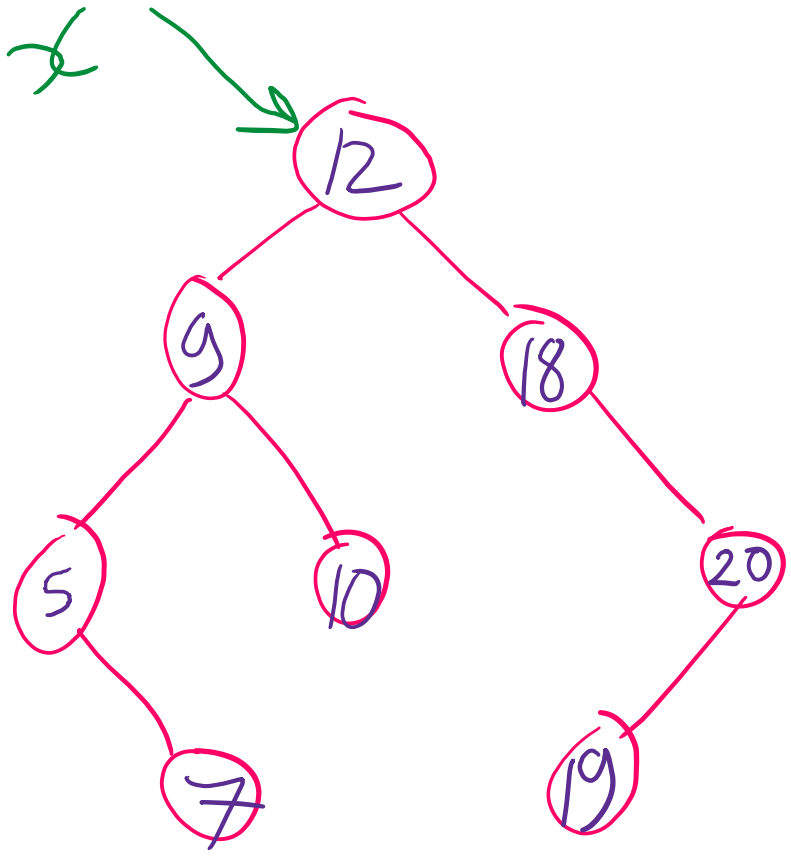
- Traversal for enumerating all items
- Traversal for finding an item

BST search using iteration




```
x = root
while(x != null){
    if equal break;
    if < x = x.left
    if > x = x.right
}
```

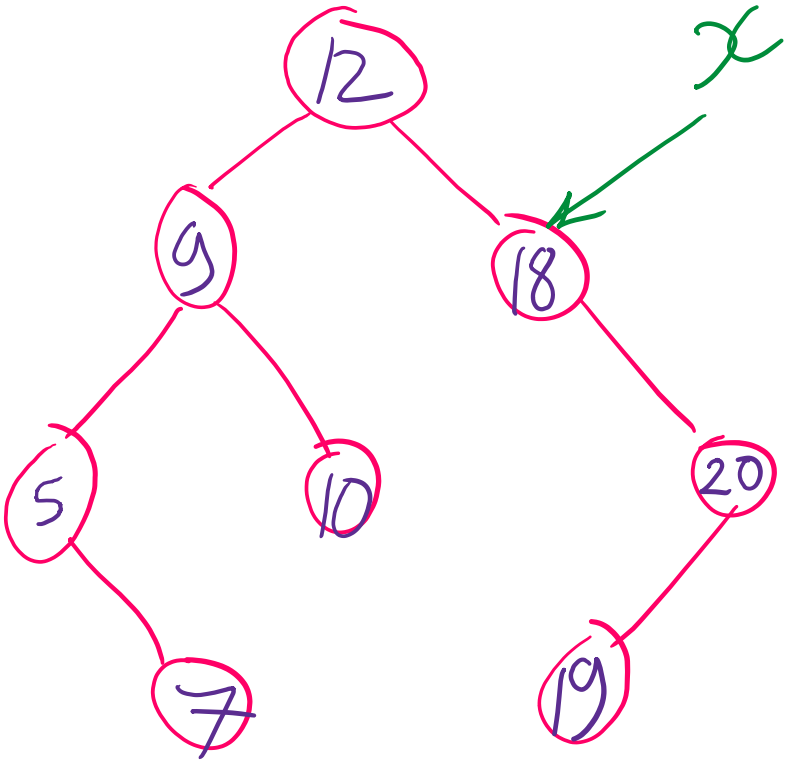
BST search using iteration




```
x = root
while(x != null){
    if equal break;
    if < x = x.left
    if > x = x.right
}
```

Search for 19

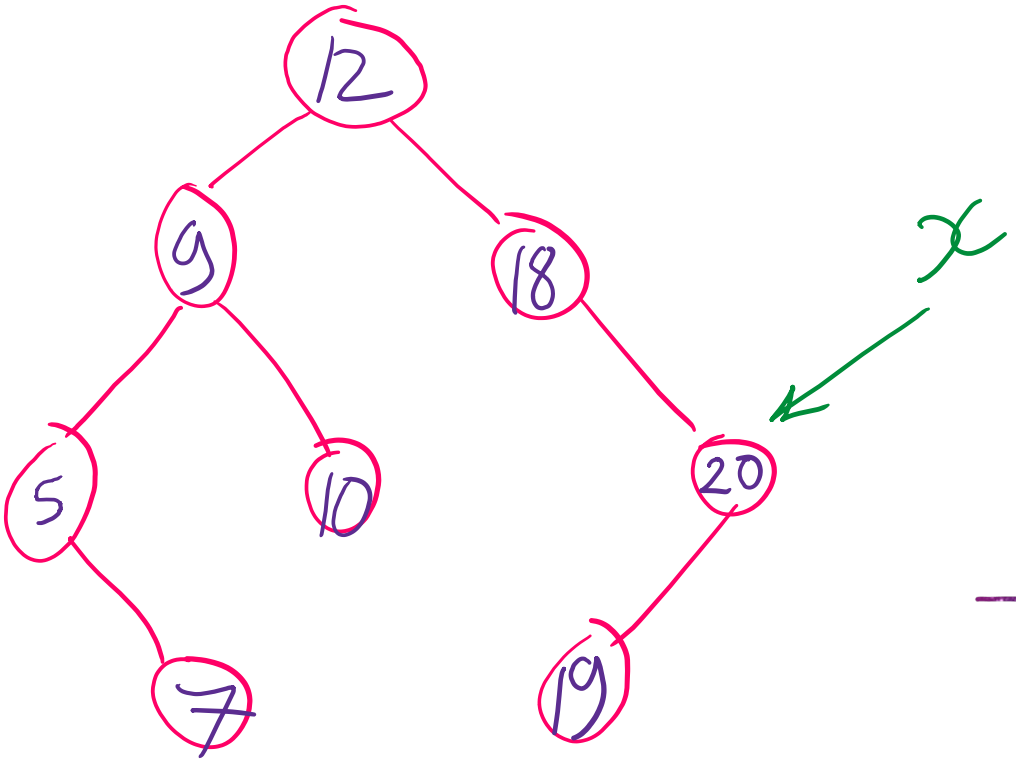
BST search using iteration




```
x = root
while(x != null){
    if equal break;
    if < x = x.left
    if > x = x.right
}
```

Search for 19

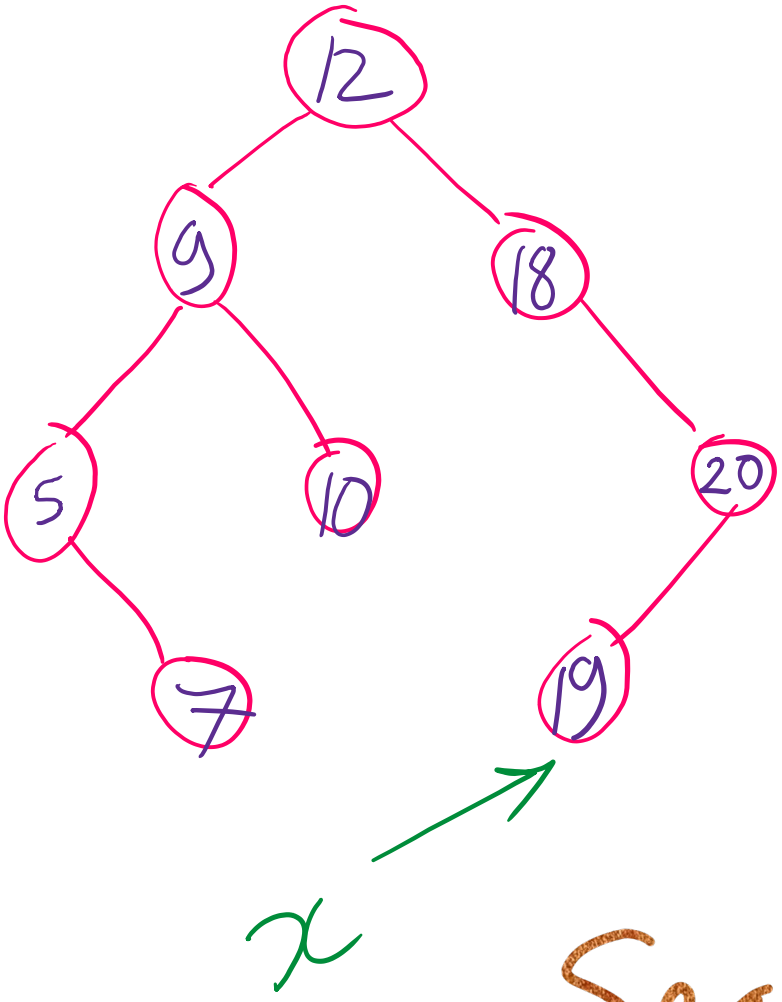
BST search using iteration




```
x = root
while(x != null){
    if equal break;
    if < x = x.left
    if > x = x.right
}
```

Search for 19

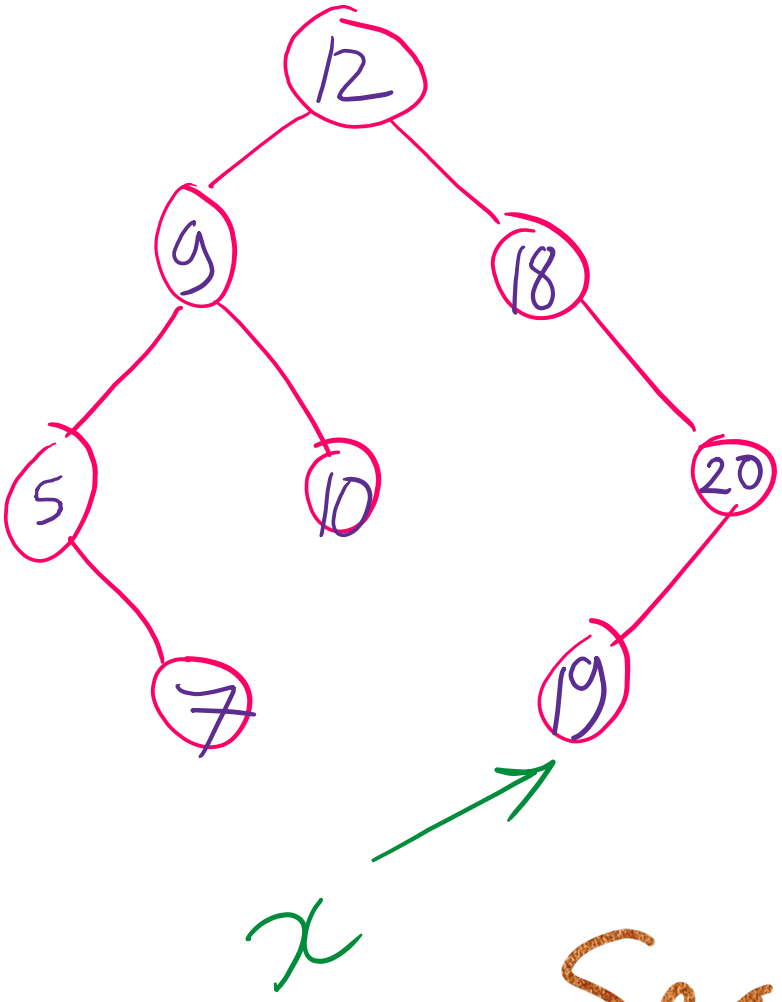
BST search using iteration




```
x = root
while(x != null){
    if equal break;
    if < x = x.left
    if > x = x.right
}
```

Search for 19

BST search using iteration




```
x = root
while(x != null){
    if equal break;
    if < x = x.left
    if > x = x.right
}
```

Search for 19

Symbol Table Implementations