University of Pittsburgh

# Algorithms and Data Structures 2
# CS 1501


Fall 2022

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(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides.)

# Announcements

- Upcoming Deadlines
  - Homework 5: this Friday @ 11:59 pm
  - Lab 3: tonight @ 11:59 pm
  - Lab 4: next Monday @ 11:59 pm
  - Assignment 1: Monday Oct 10th @ 11:59 pm
- **Live support session** for Assignment 1
  - Recording and slides on Canvas
- **Student Support Hours** of the teaching team are posted on the Syllabus page

# Previous lecture

- R-way Radix Search Tries
- De La Briandais (DLB) Tries
- The Compression Problem

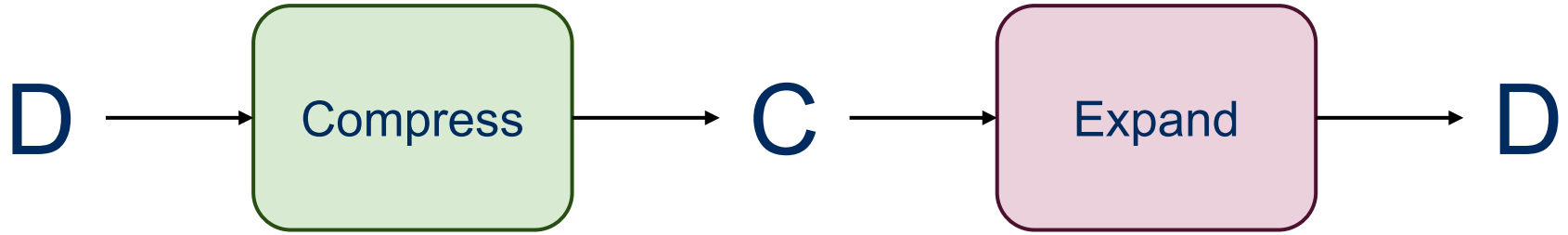
# This Lecture

- Huffman Compression
- Run-length Encoding

# Problem of the Day: Compression

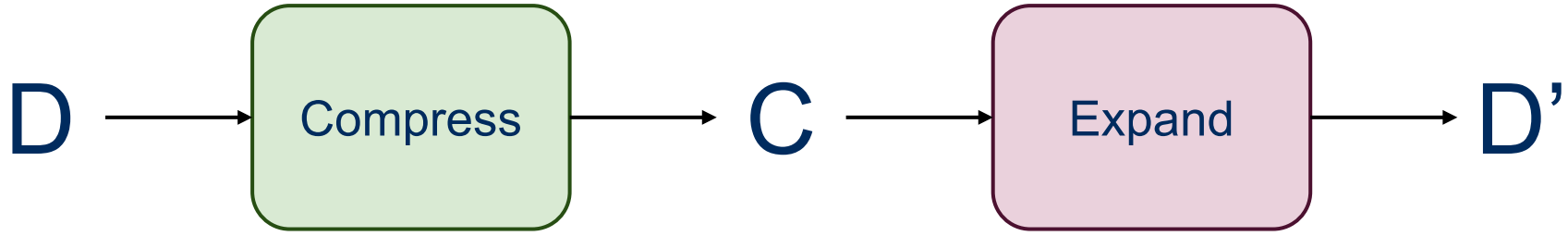
- Input: A file containing a sequence of *n* characters
  - each character encoded as an 8-bit Extended ASCII
  - total file size = *8*n* bits
- Output: A shorter bitstring
  - of bitlength < *8*n*
  - such that the original sequence can be fully restored from the shorter bitstring

# Lossless Compression




- Input can be recovered from compressed data exactly
- Examples:
  - zip files, FLAC

# Lossy Compression




- Information is permanently lost in the compression process
- Examples:
  - MP3, H264, JPEG
- With audio/video files this typically isn't a huge problem as human users might not be able to perceive the difference

# Lossy examples

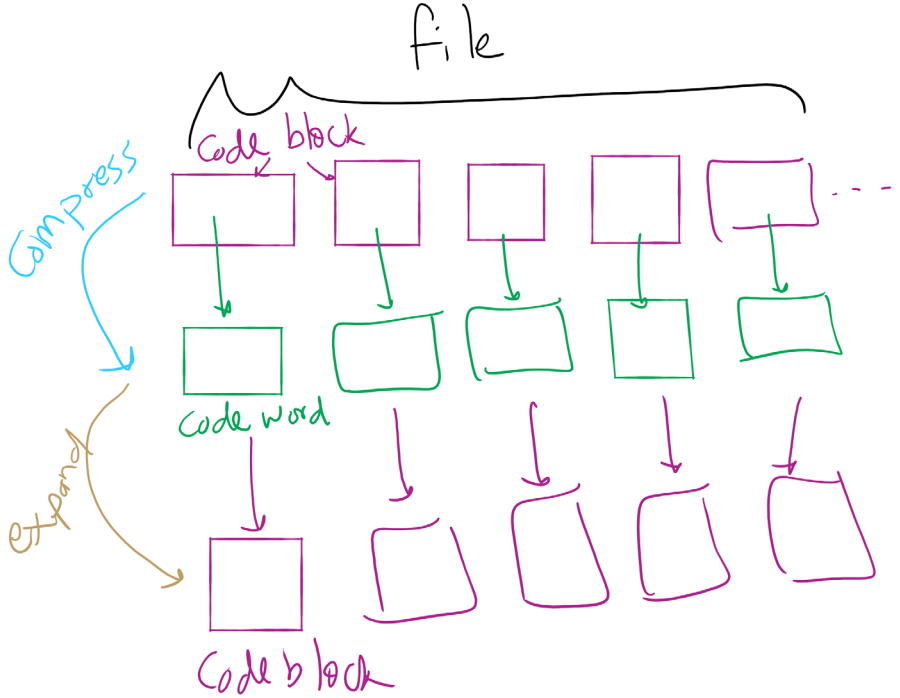
- MP3
  - “Cuts out” portions of audio that are considered beyond what most people are capable of hearing
- JPEG

40K

28K

# Lossless Compression Framework

- Compression:
  - For each code block in the input file
    - Convert into a codeword
- Expansion:
  - For each codeword in the compressed file
    - Convert into back to the code block



# Cases of the Lossless Compression Framework

- **Case 1: fixed-size code blocks, fixed-size codewords**
- To achieve compression, codeword size has to be less than code block size
  - Can be done if the used alphabet in the file is smaller than the Extended ASCII
    - e.g., a file of English letters
      - can use 5 bits per codeword instead of 8 bits
      - ***Compression ratio*** = 5/8
    - e.g., a file of 64-encoded characters
      - lower-case and upper-case English letters, digits, and two symbols
      - can use 6 bits per codeword
  - Typically, the reduced alphabet is stored at he beginning of the compresses file
    - increase the compresses file size and sthe compression ratio

# Cases of the Lossless Compression Framework

- **Case 2: fixed-size code blocks, variable-size codewords**
- To achieve compression, assign shorter codewords to more frequently occurring characters
- Total compressed file size
  - *R:* alphabet size
  - $f_i$: frequency of character *i*
  - $|codeword_i|$: bitlength of codeword corresponding to character *I*
  - compressed file size = $\sum_{i=1}^{R} f_i \, |codeword_i|$
- *Huffman compression* is an example of this case

# Cases of the Lossless Compression Framework

- **Case 3: variable-size code blocks, fixed-size codewords**
- To achieve compression, find recurring patterns that are *as long as possible* in the input file and assign fixed-size codewords to each
- *LZW compression* and *run-length encoding* are examples of this case

# Cases of the Lossless Compression Framework

- **Case 4: variable-size code blocks, variable-size codewords**
- Typically implemented as Case 3 followed by Case 2

# Solution 1: Huffman Compression

- What if we used *variable length* codewords instead of the constant 8 bits per character? Could we store the same info in less space?
  - Different characters are represented using codes of different bit lengths
  - What about different usage frequencies between characters?
    - In English, R, S, T, L, N, E are used much more than Q or X
  - Using this, we can achieve compression by:
    - Using fewer bits to represent more common characters
    - Using longer codes to represent less common characters

# But we have to worry about restoring the data!

- Decoding was easy for block codes
  - Grab the next 8 bits in the bitstring
  - How can we decode a bitstring that is made of variable length code words?
    - use delimiters?
  - BAD example of variable length encoding:

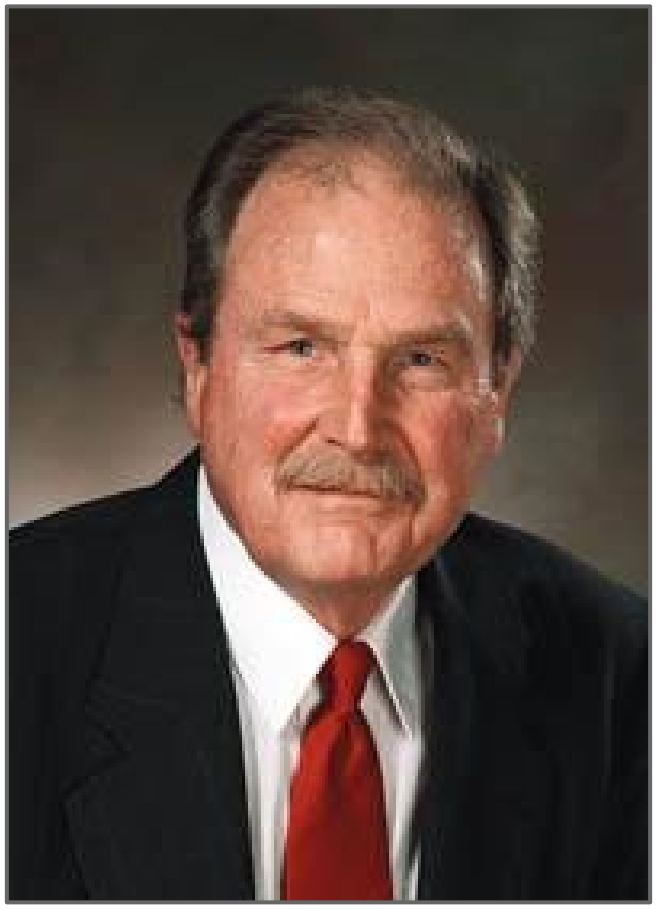
| | |
|---|---|
| 1 | A |
| 00 | T |
| 01 | K |
| 001 | U |
| 100 | R |
| 101 | C |
| 10101 | N |

# Variable length encoding for lossless compression

- Codes must be *prefix free*
  - No code can be a prefix of any other in the scheme

# How can we create these prefix-free codes?

Huffman encoding!



# Subproblem: Prefix-free Compression

- Input: A sequence of *n* characters
- Output: A codeword $h_i$ for each character *i* such that
  - No codeword is a prefix of any other
  - When each character in the input sequence is replaced with each codeword
    - the length of the compressed sequence is minimum (over other fixed-size code-block encoding schemes)
    - the original sequence can be fully restored from the compressed bitstring

# Generating Huffman codes

- Assume we have K characters that are used in the file to be compressed and each has a weight (its frequency of use)
- Create a forest, F, of K single-node trees, one for each character, with the single node storing that char's weight
- while |F| > 1:
  - Select T1, T2 ∈ F that have the smallest weights in F
  - Create a new tree node N whose weight is the sum of T1 and T2's weights
  - Add T1 and T2 as children (subtrees) of N
  - Remove T1 and T2 from F
  - Add the new tree rooted by N to F

# Example

- Build a tree for “ABRACADABRA!”
- file size: *n* = *12*
- no. of unique characters: *K* = *6*

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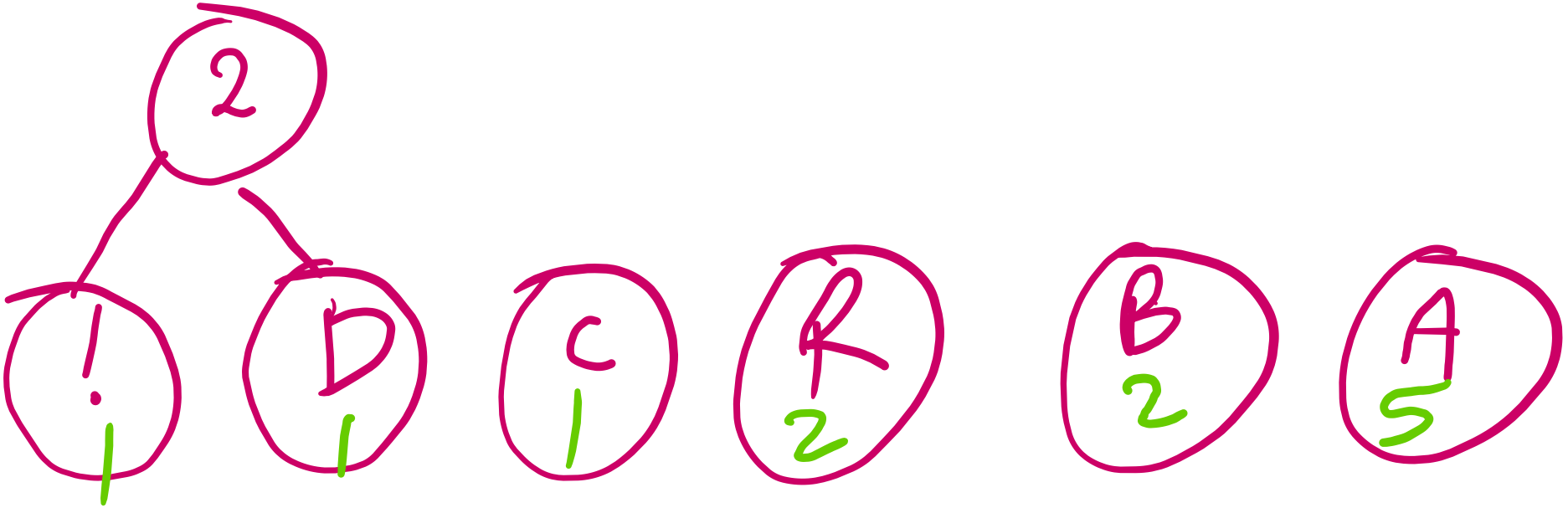

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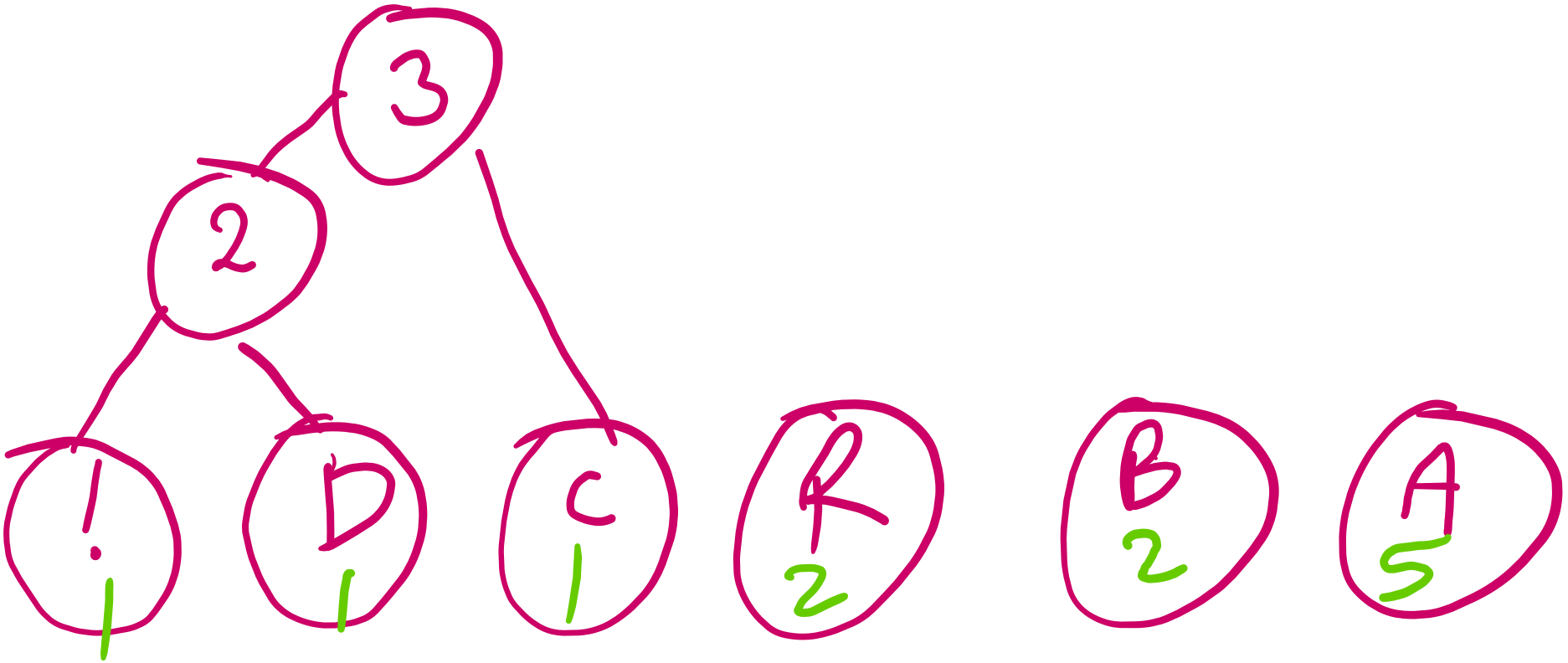

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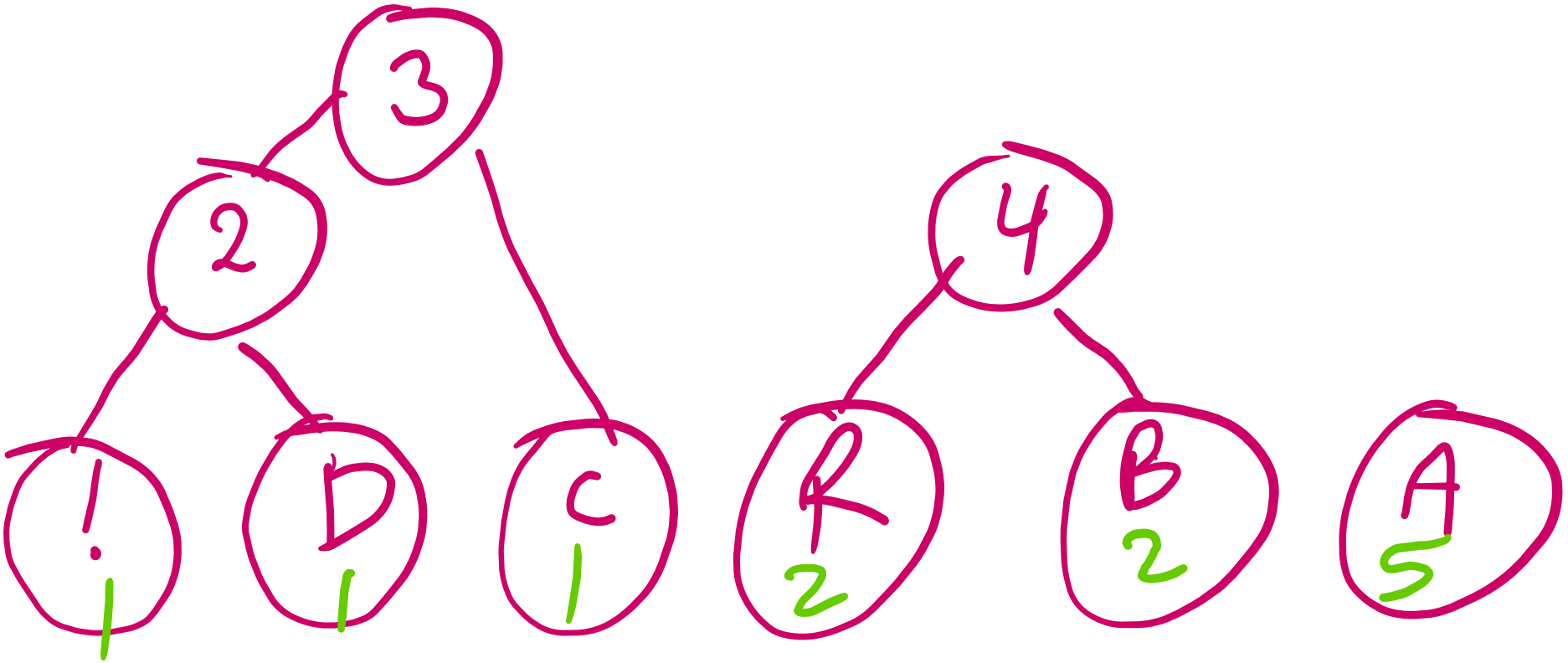

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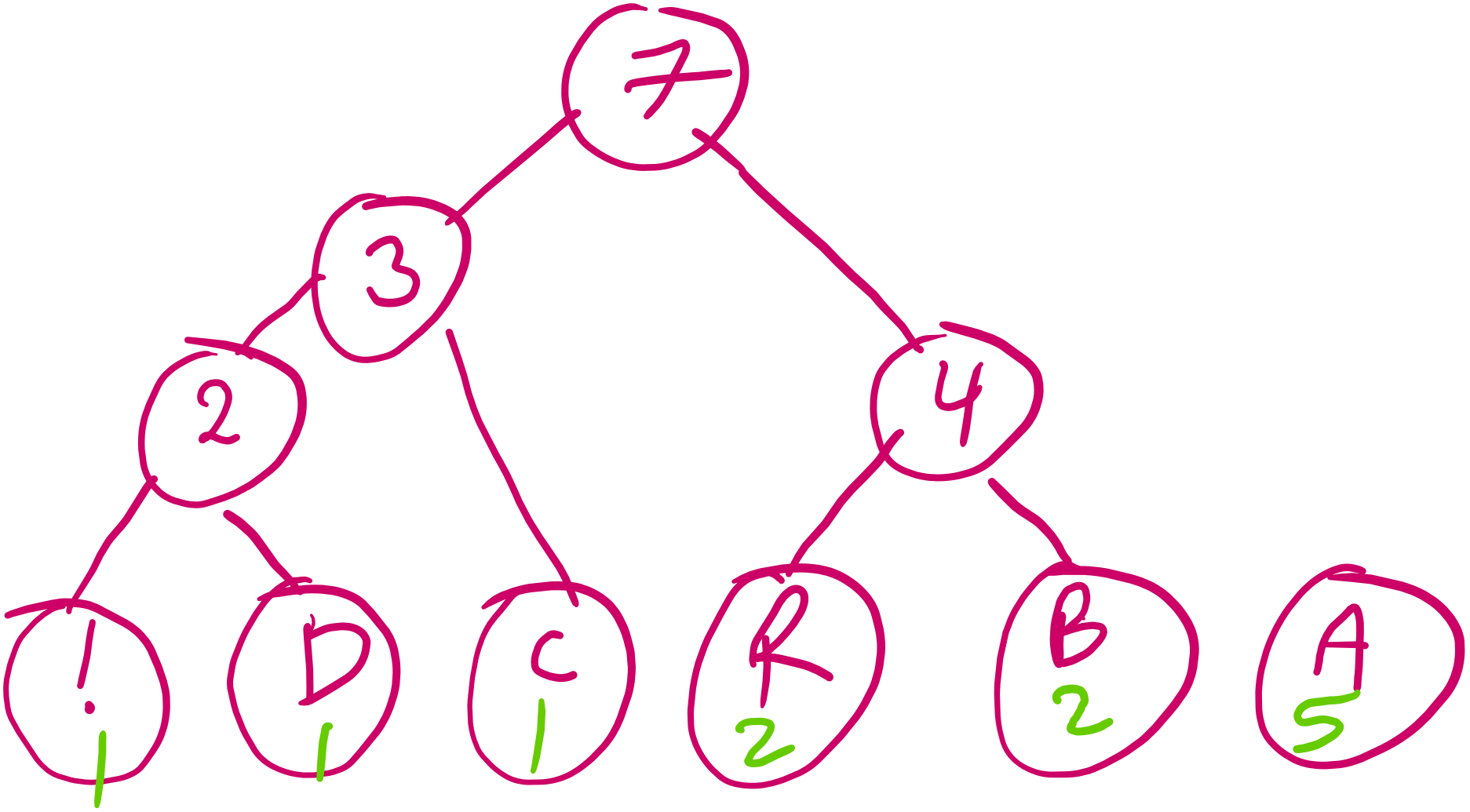

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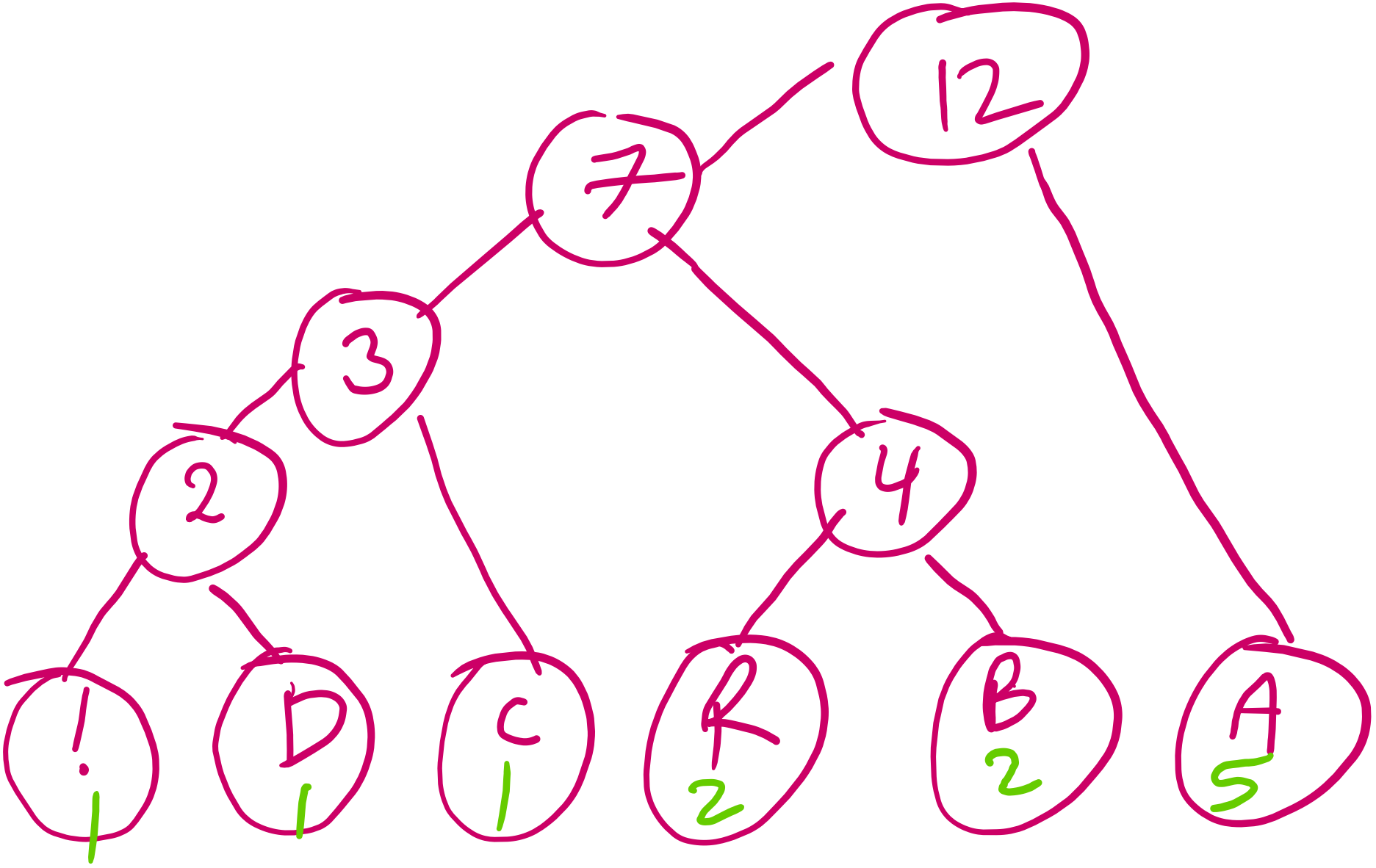

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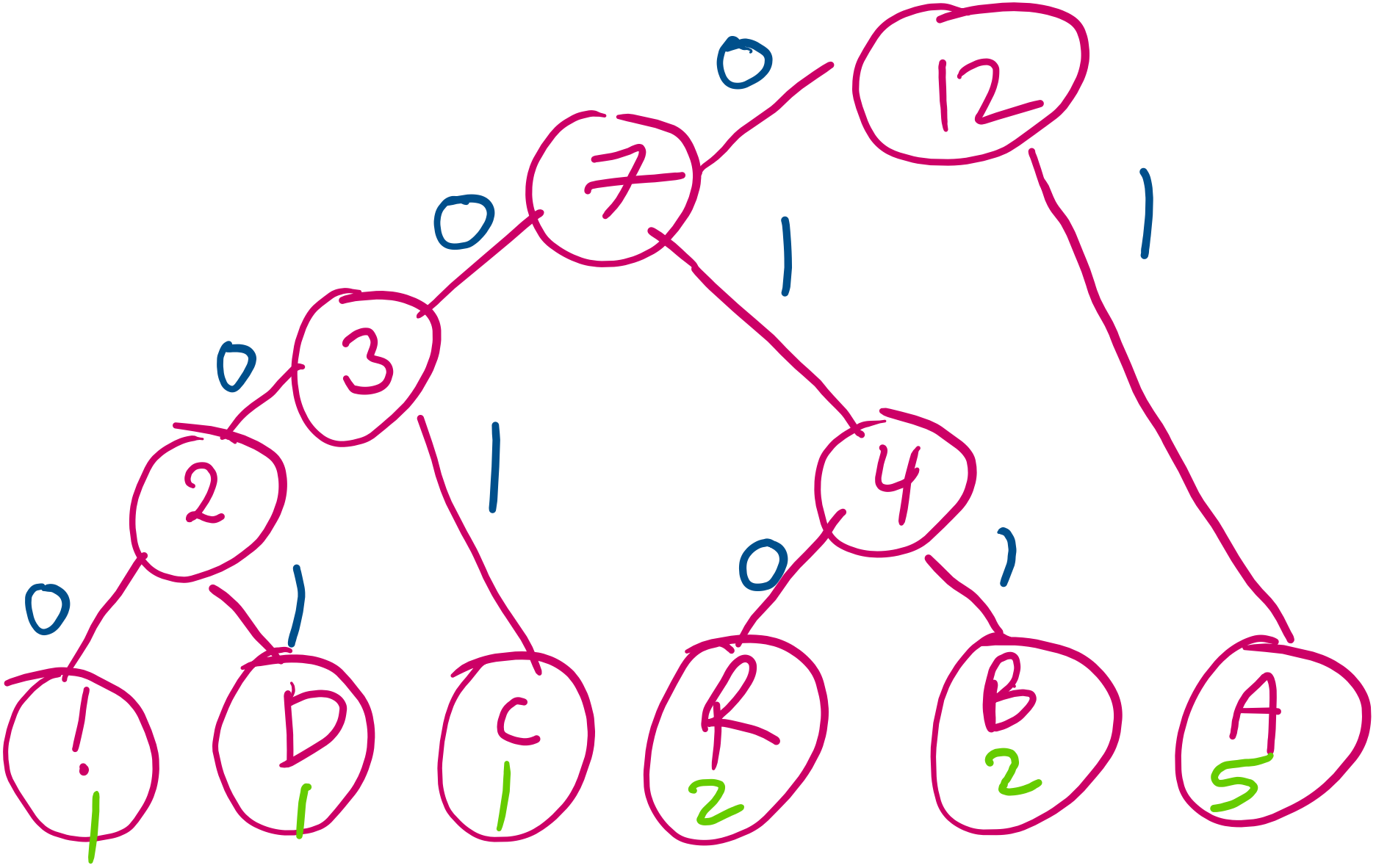

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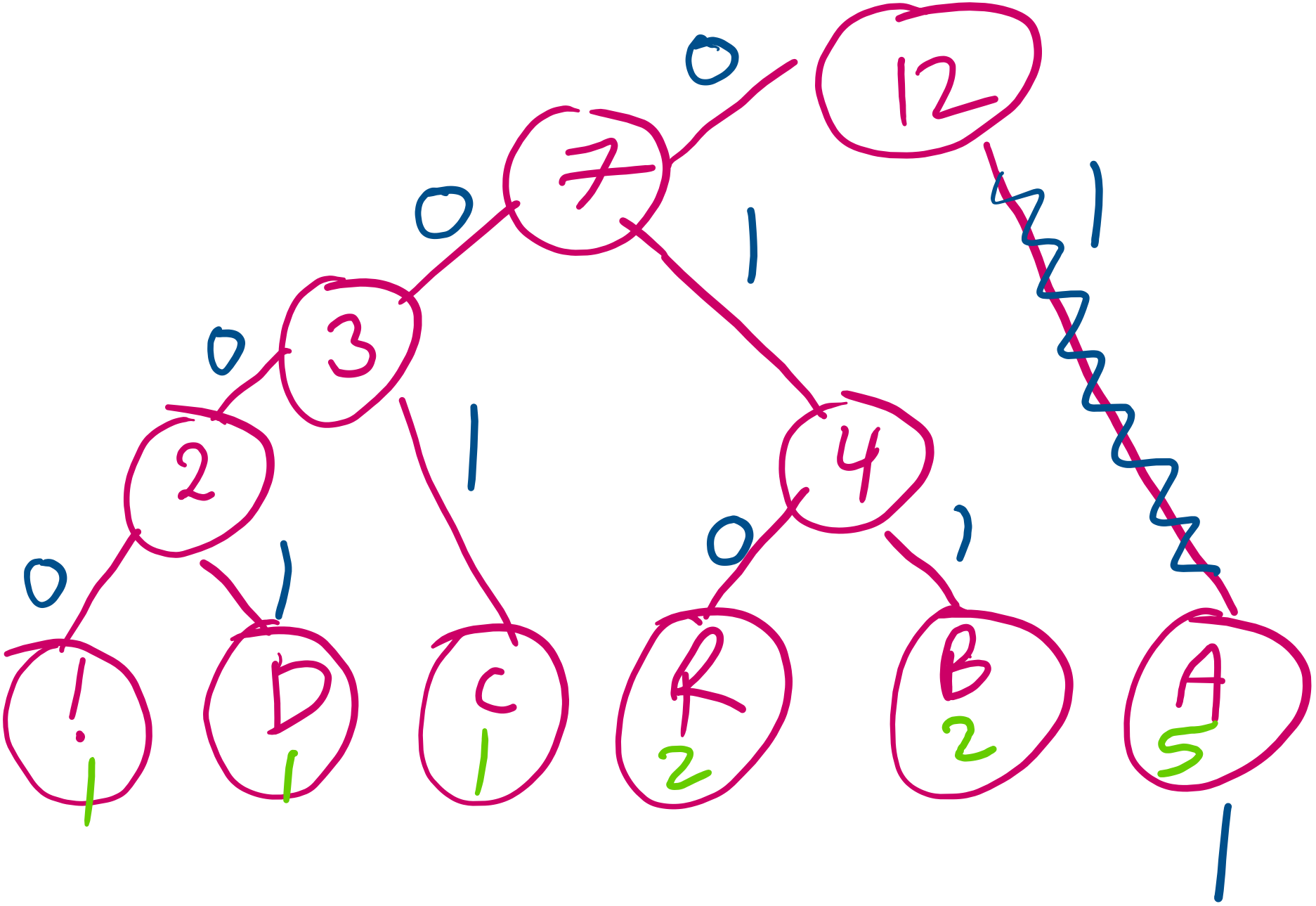

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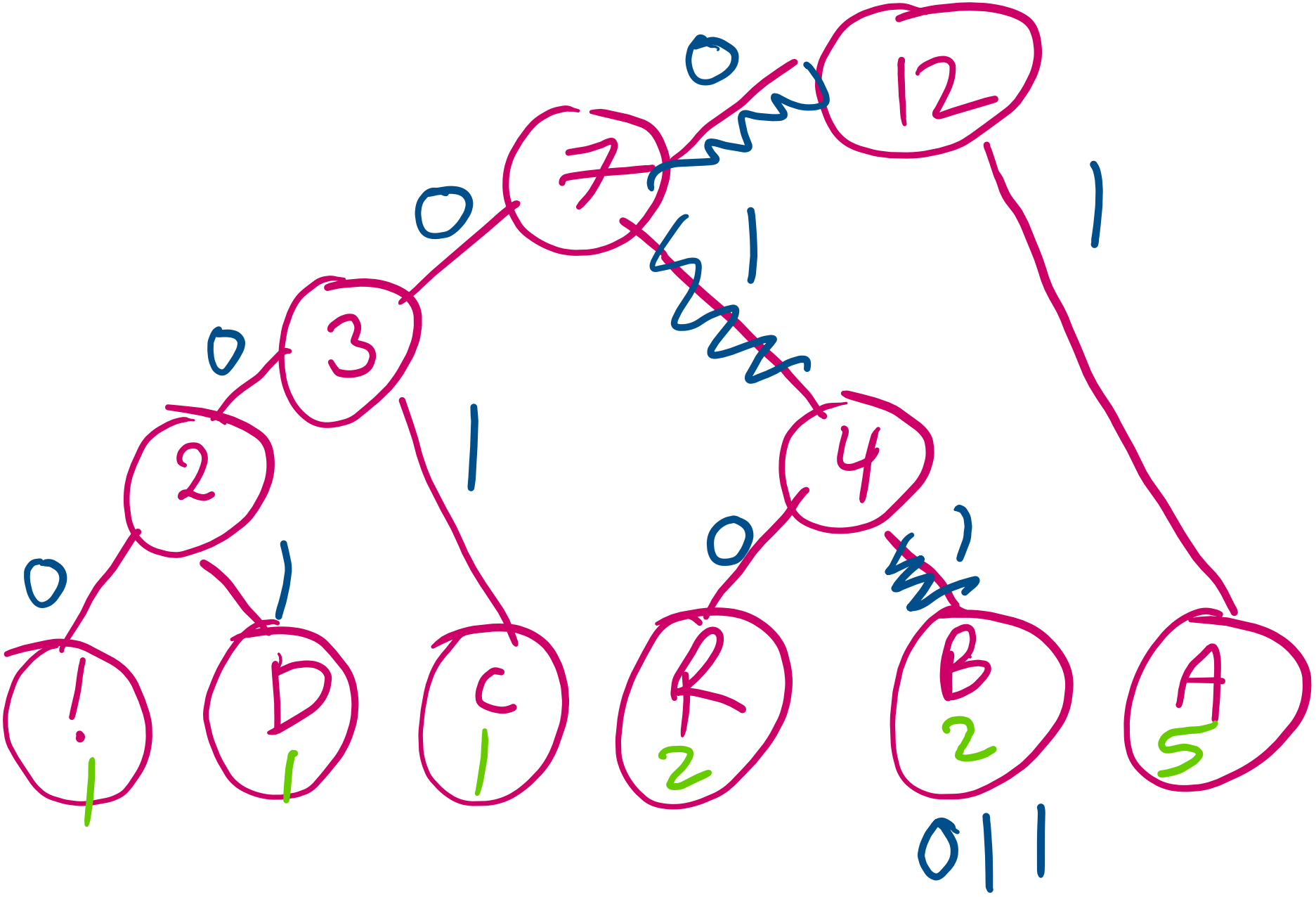

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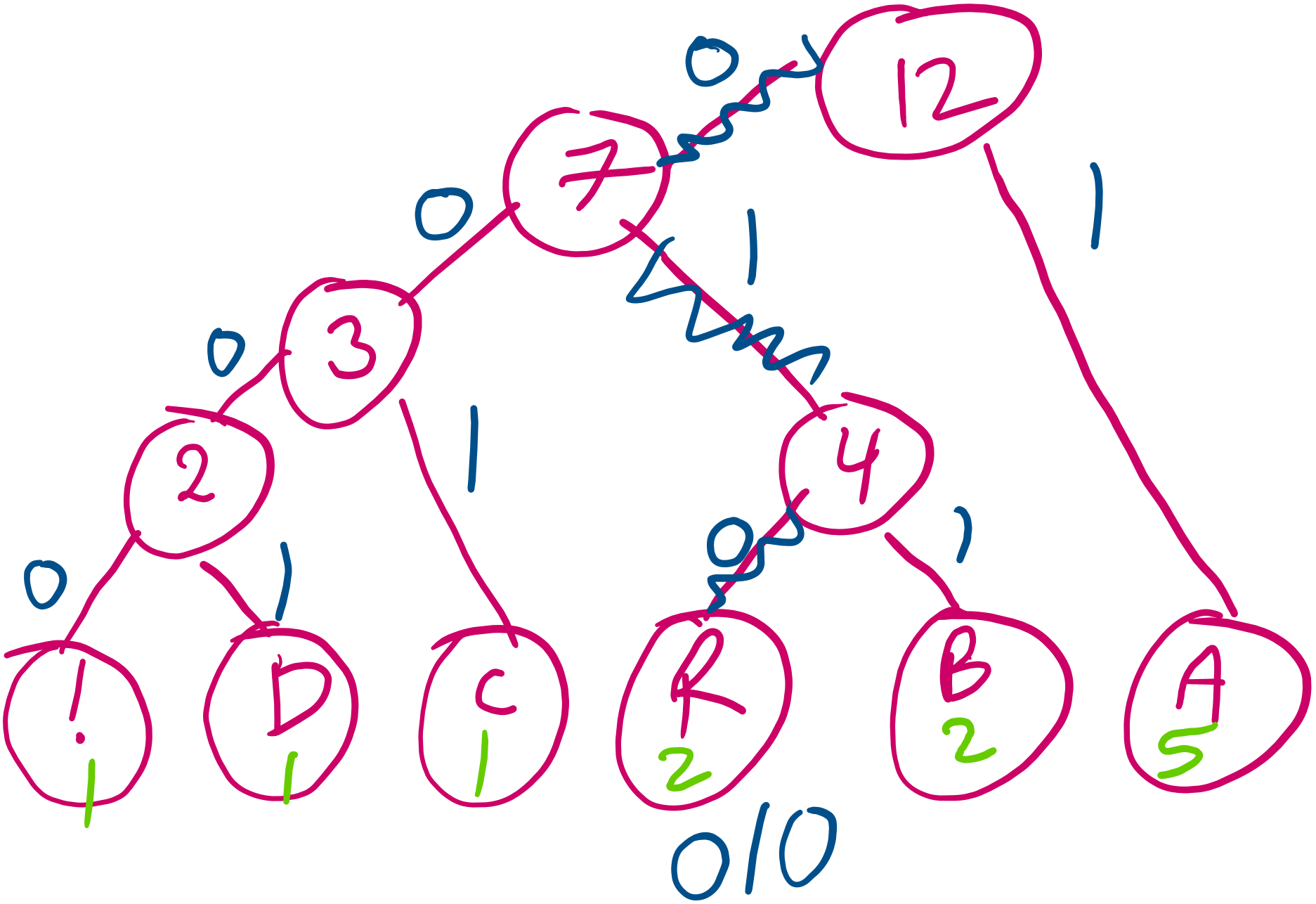

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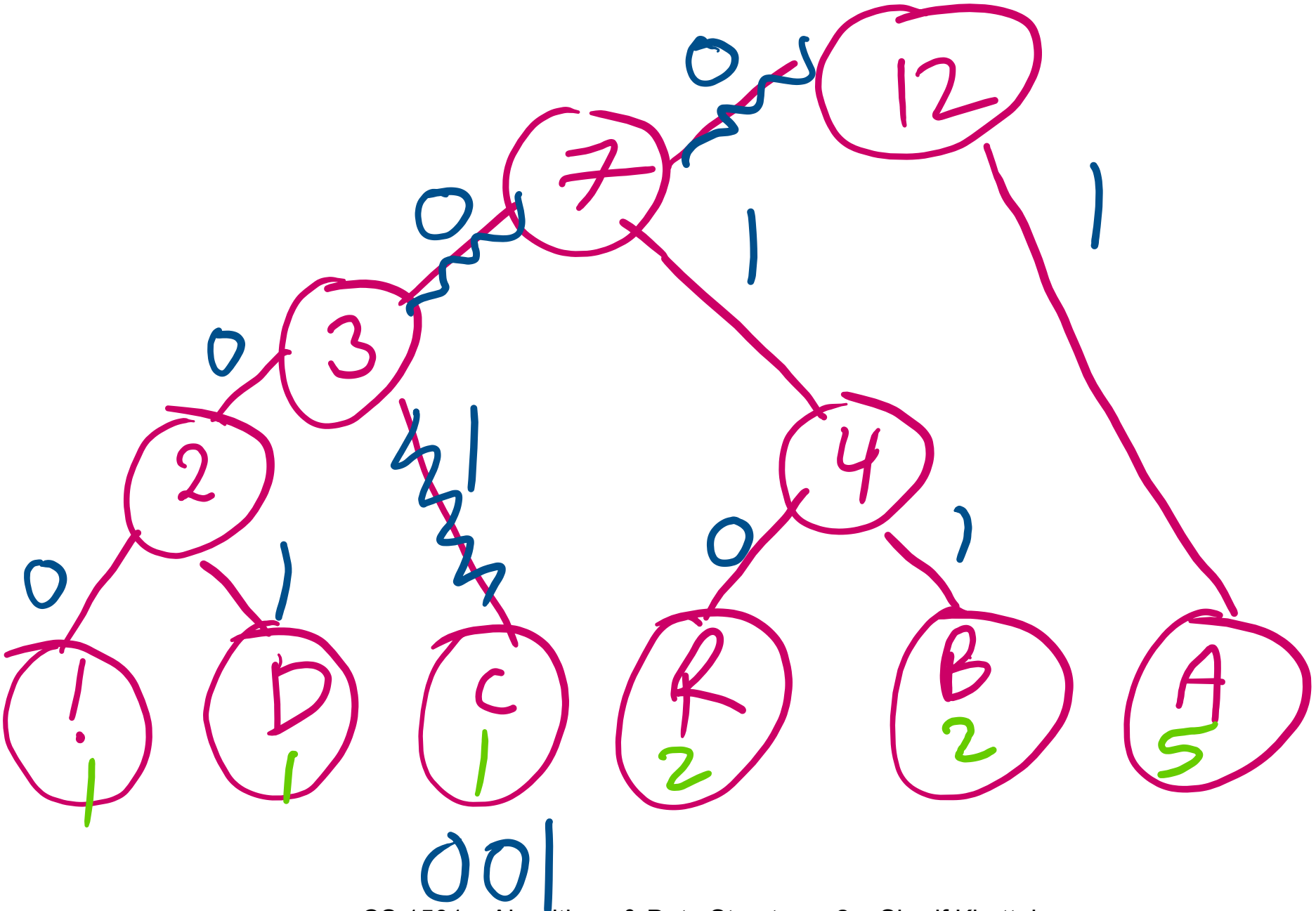

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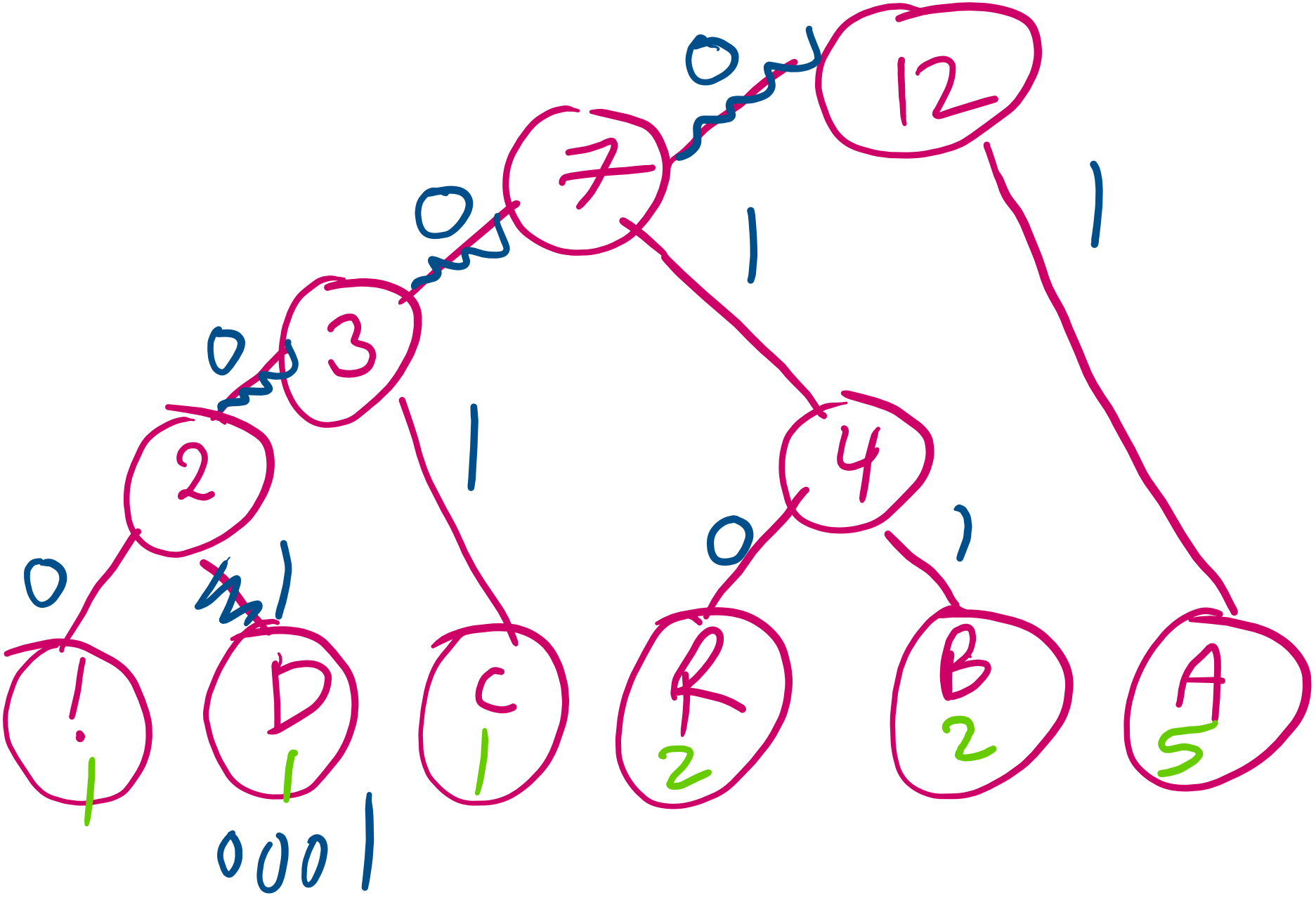

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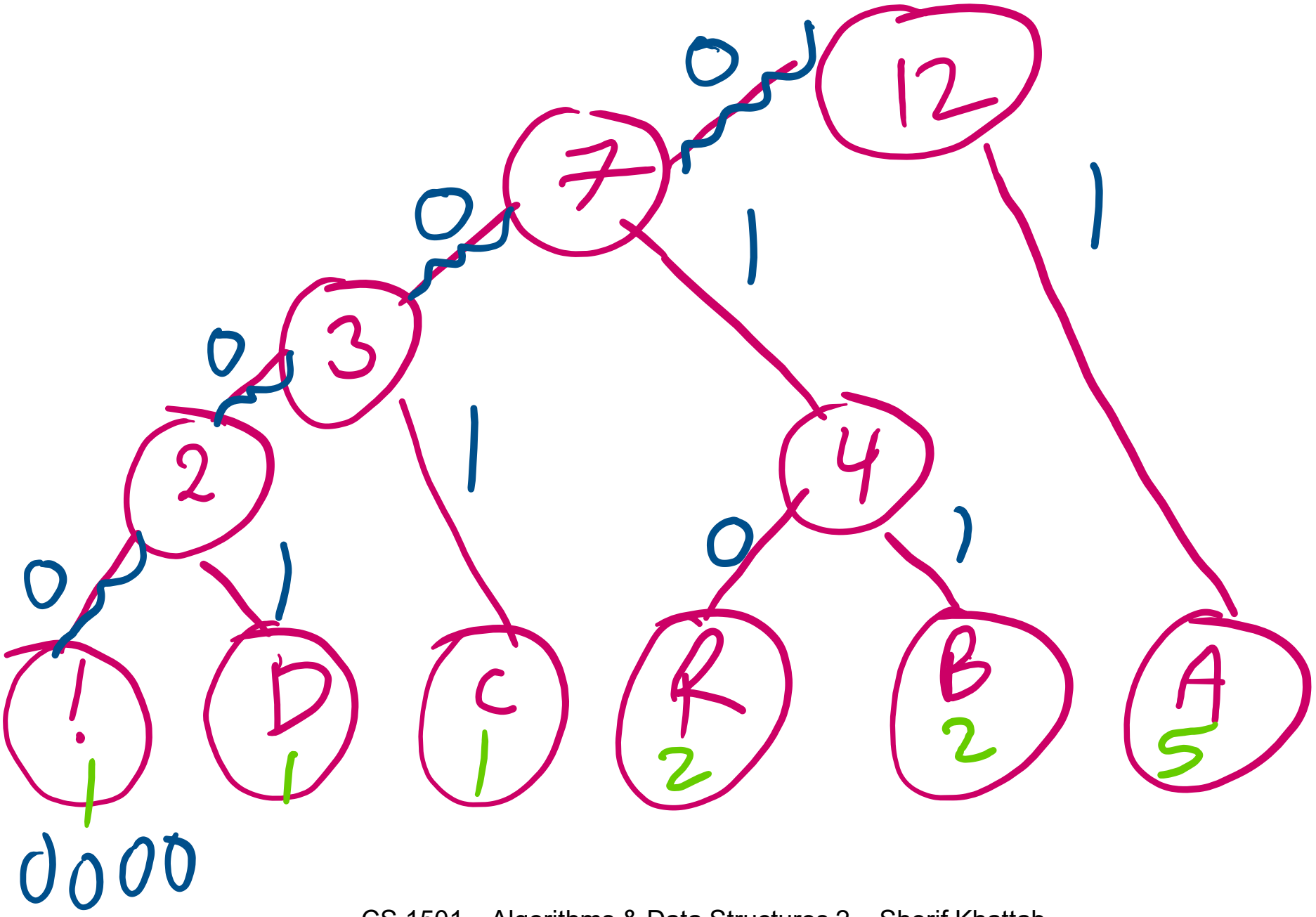

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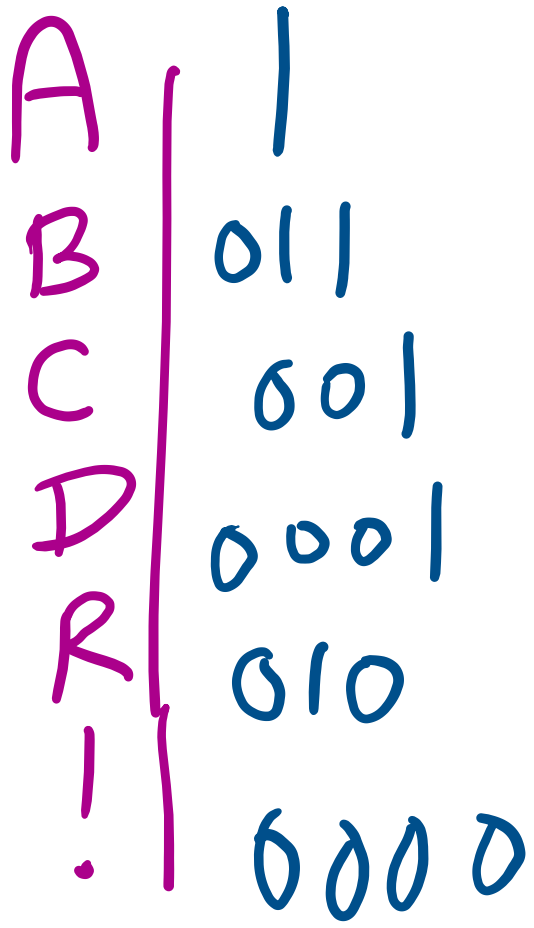

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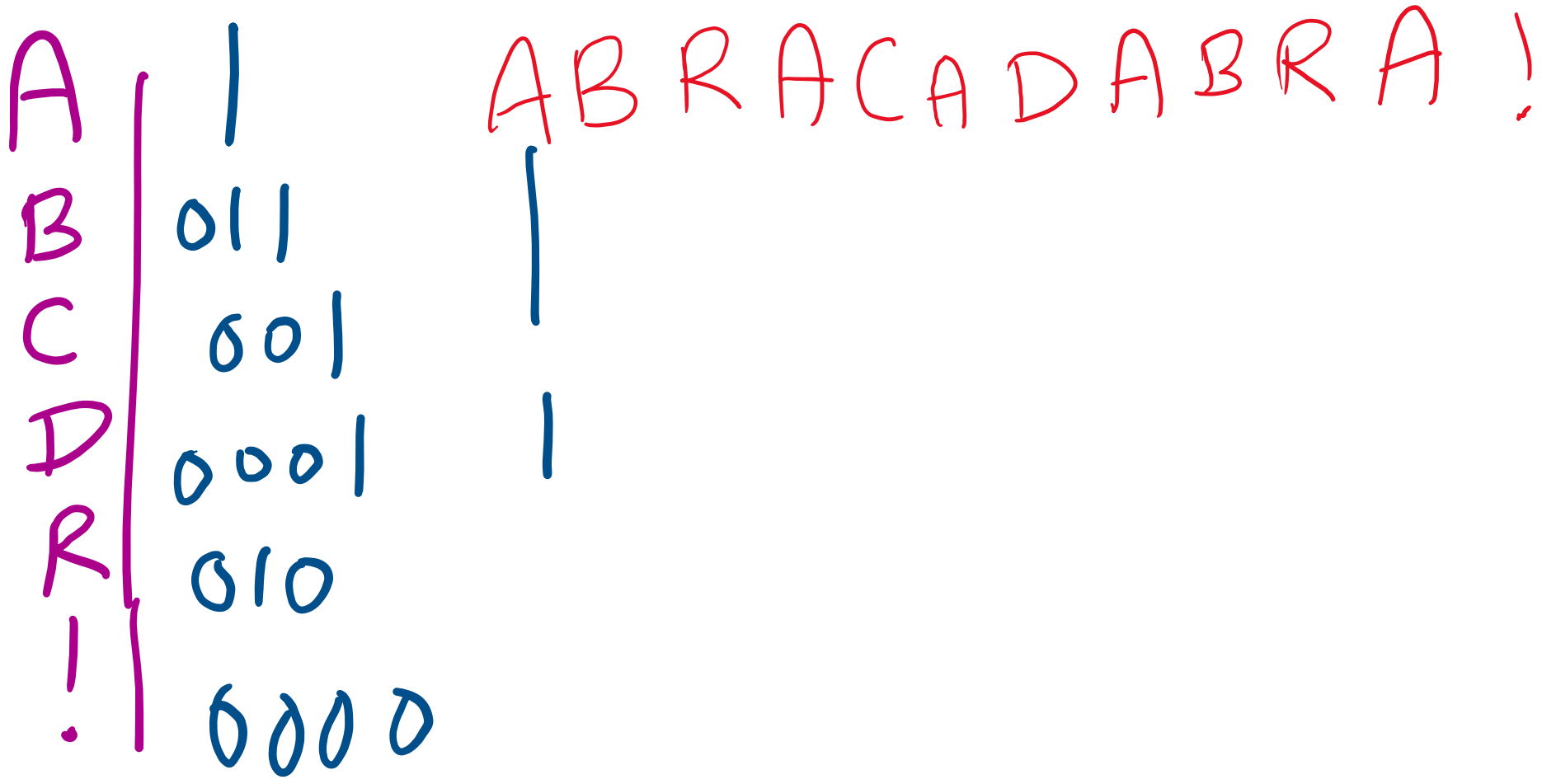

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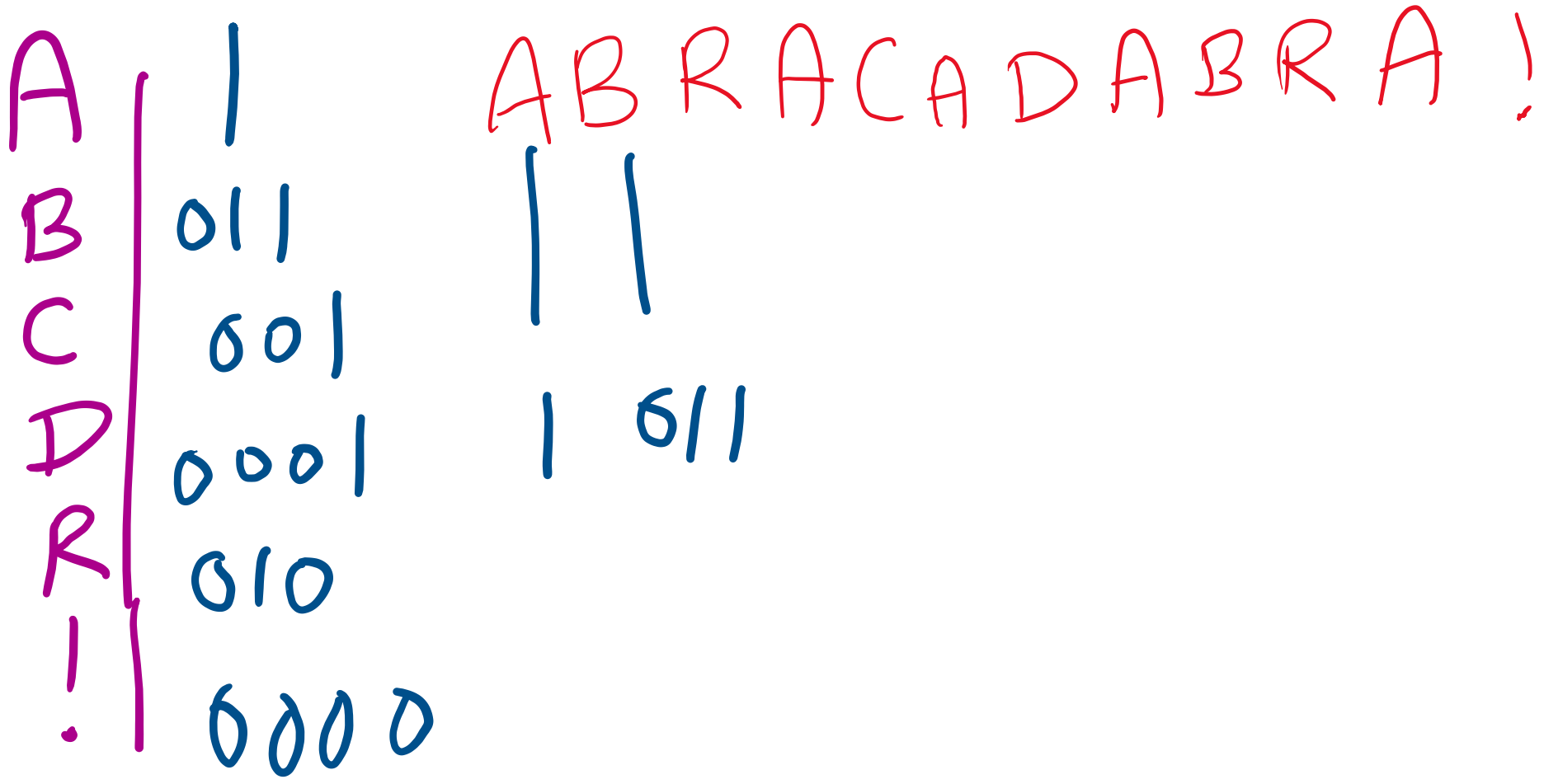

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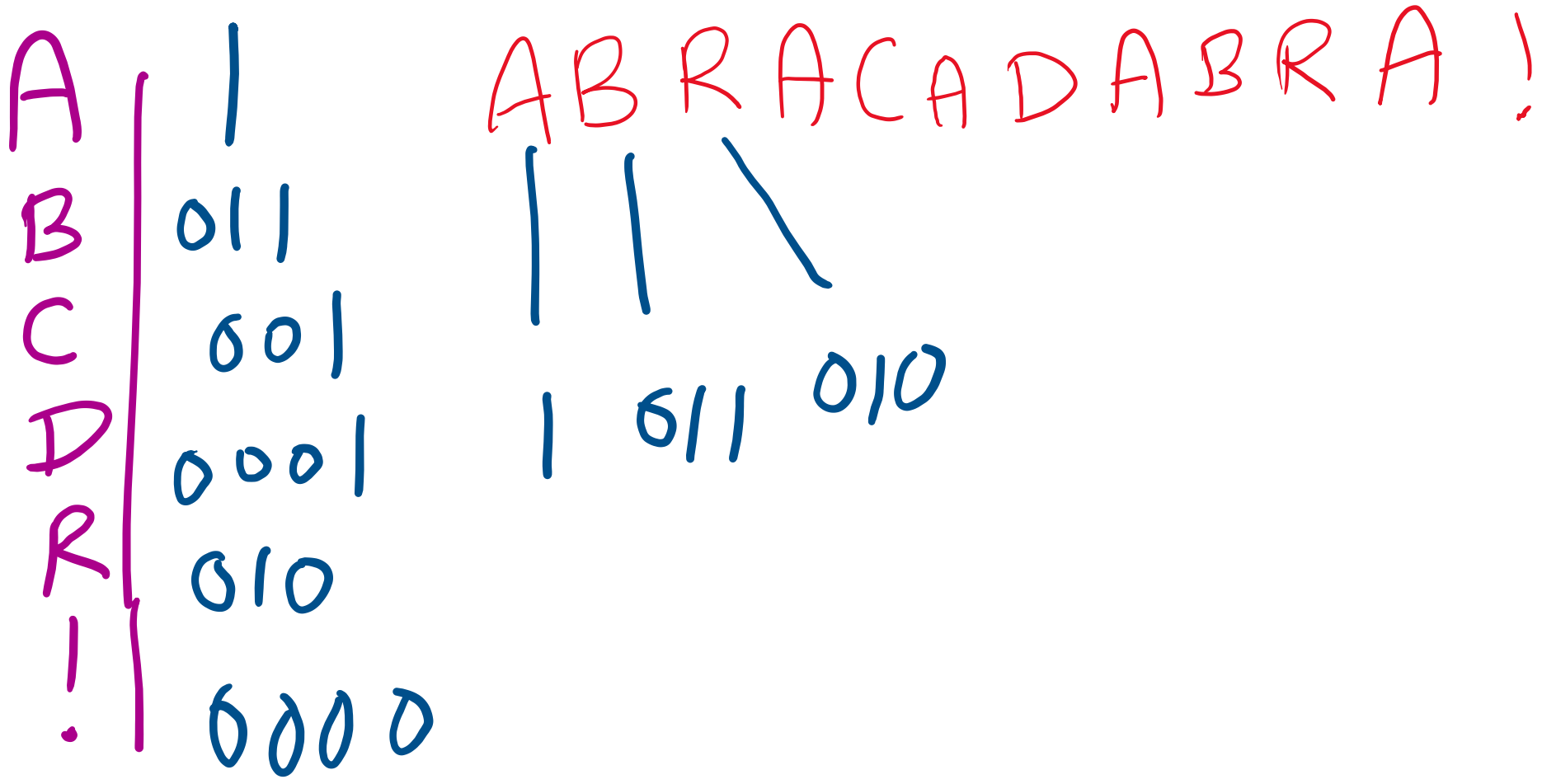

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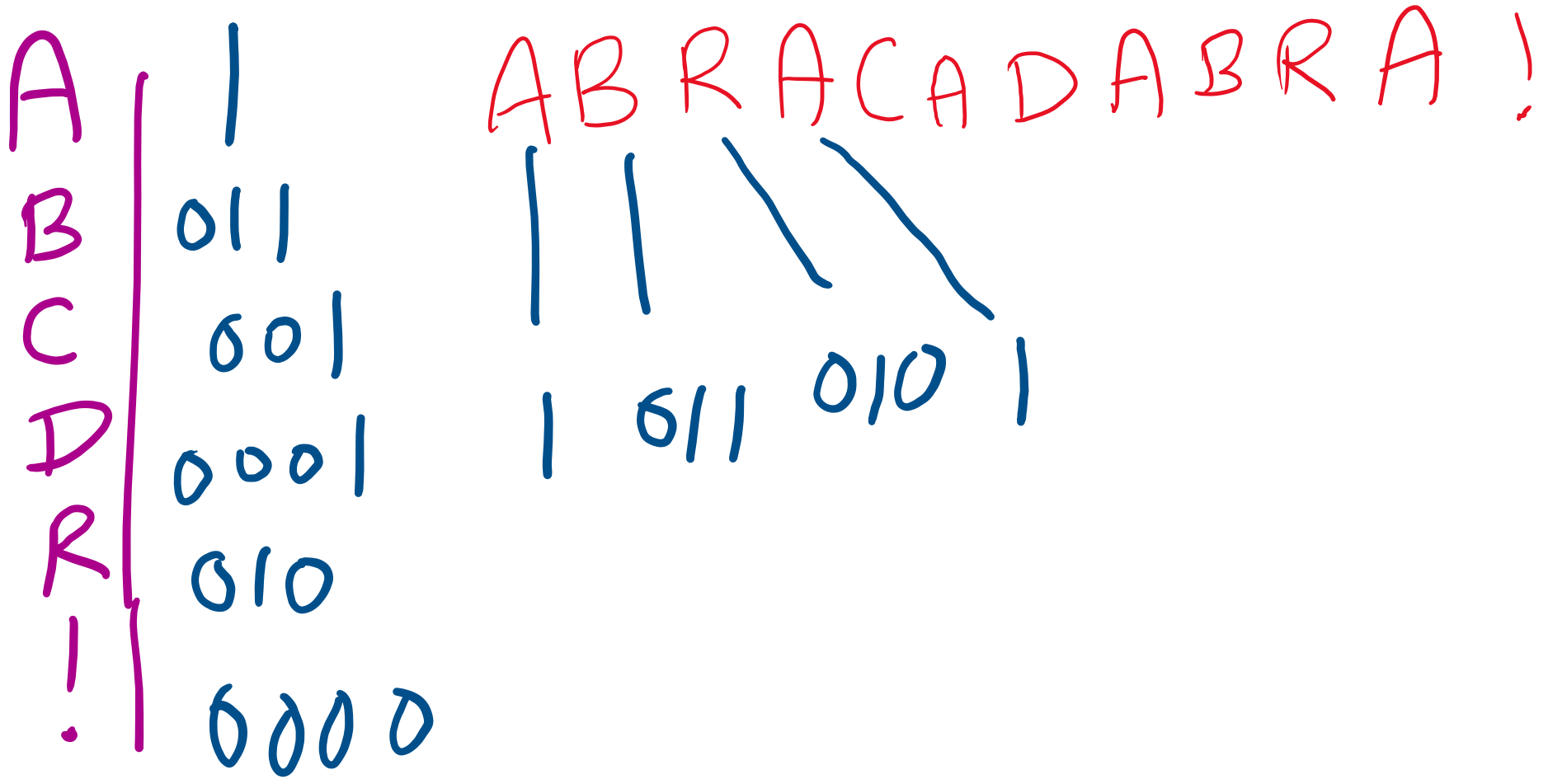

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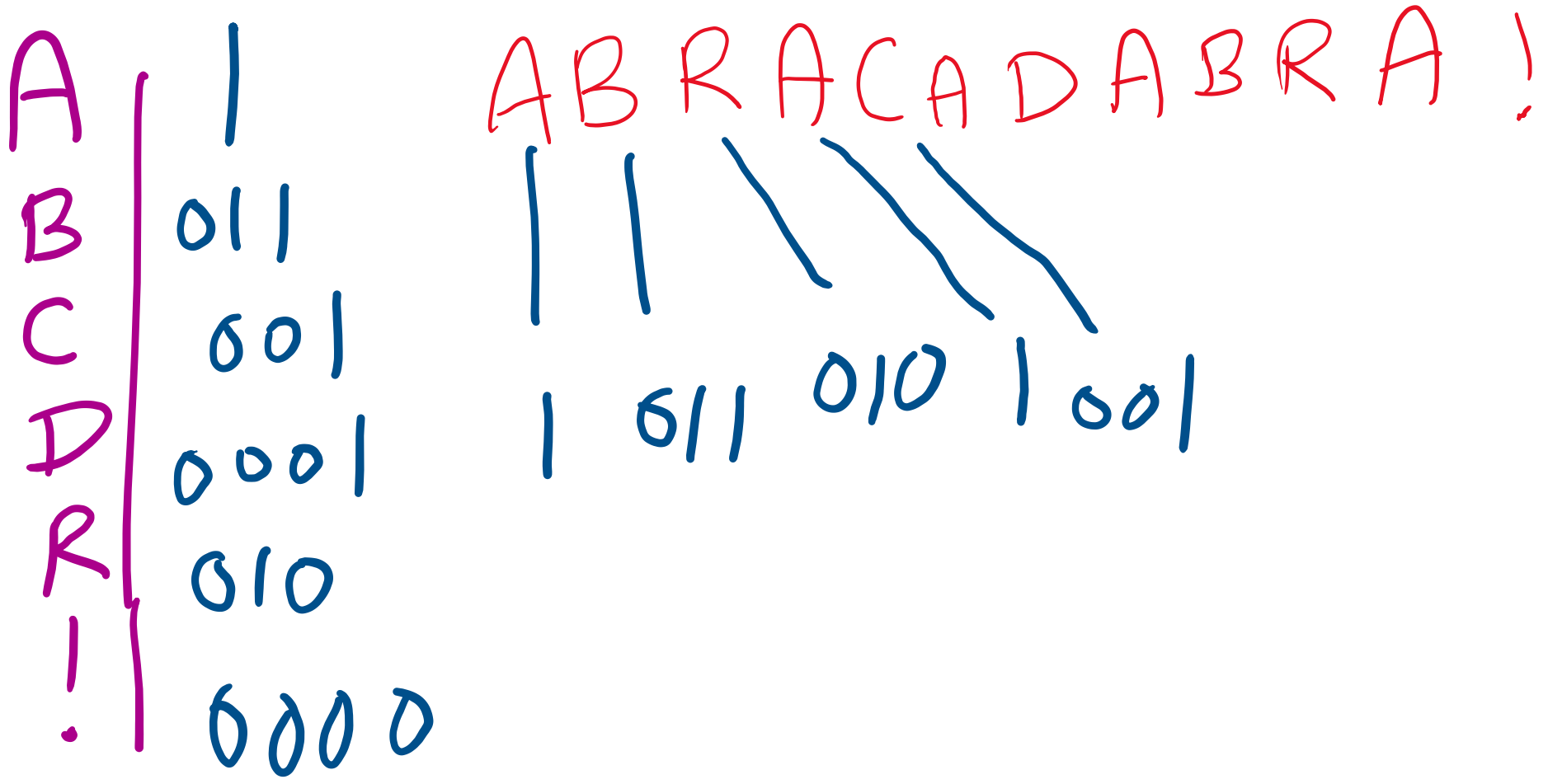

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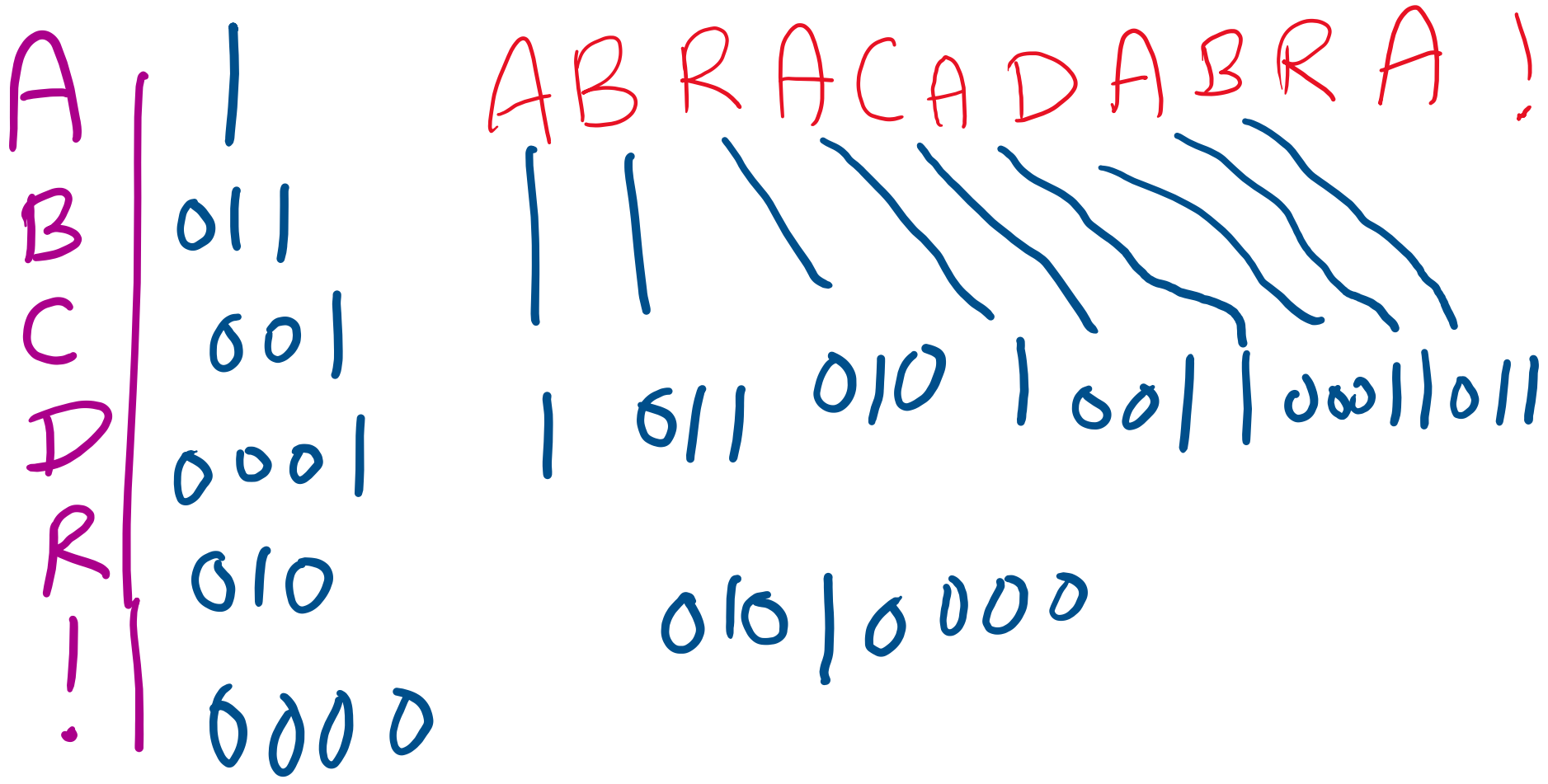

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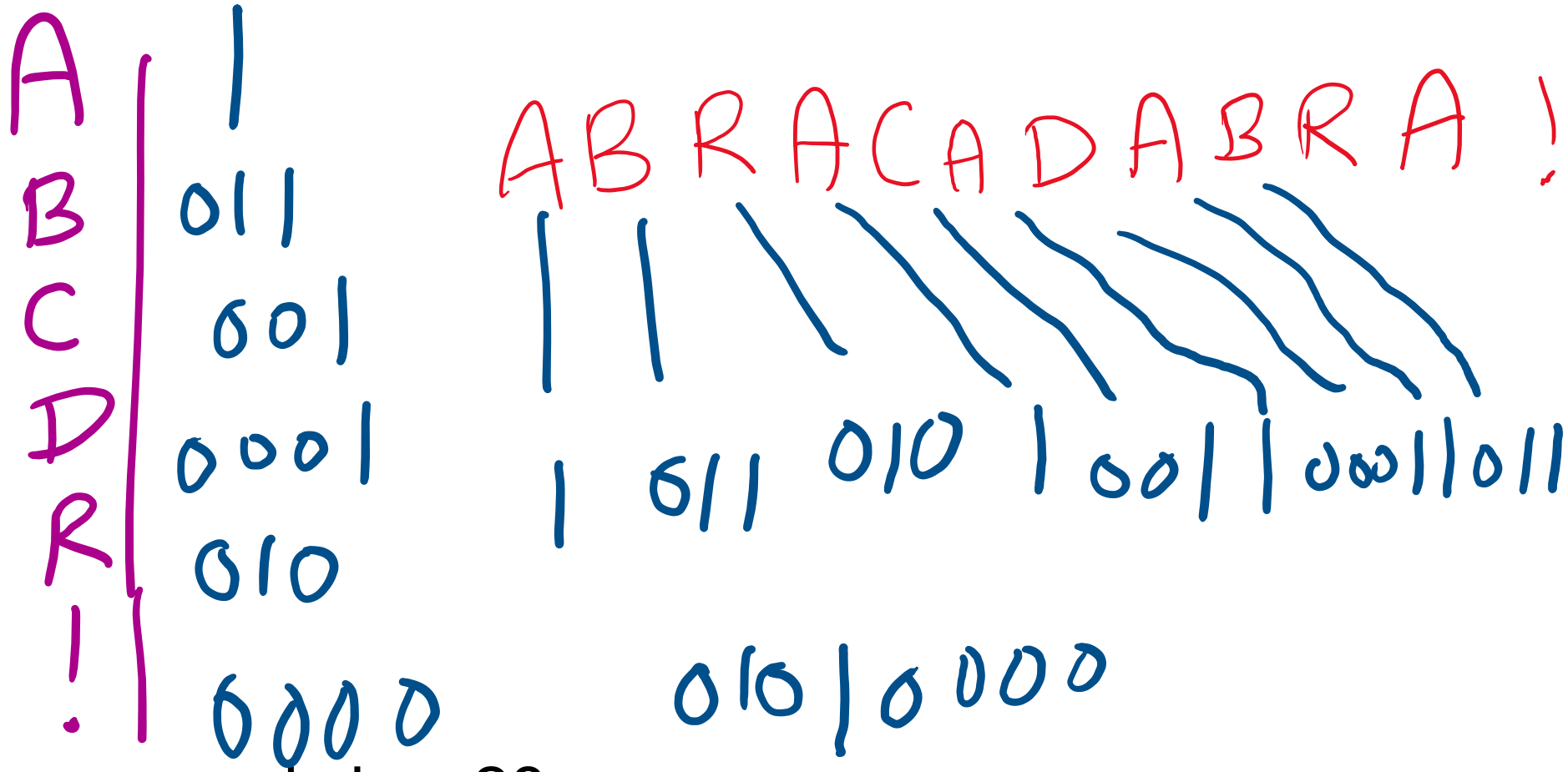
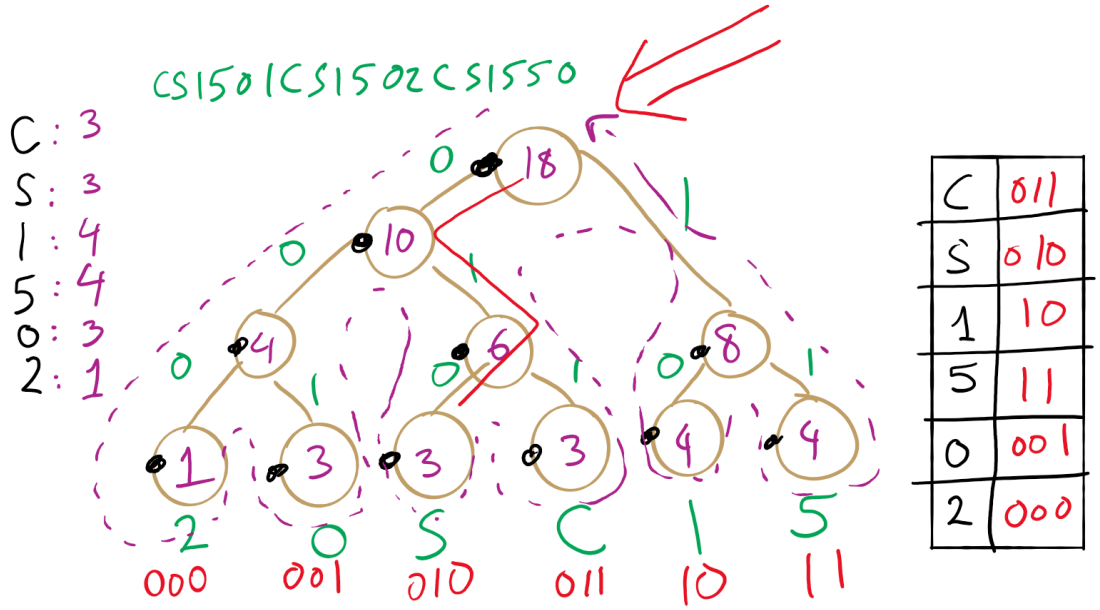



- Compressed size: 28
- Original size: 12*8 = 96 bits
- Compression ratio = 28/96

# Huffman Compression Example




| | |
|---|---|
| C | 011 |
| S | 010 |
| 1 | 10 |
| 5 | 11 |
| 0 | 001 |
| 2 | 000 |

C S 1 5 0 ! C S 1 5 0 2 C S 1 5 5 0

011 010 10 11 001 10 011 010 10 11 001 000 011 010 10 11 11 001

Trie

0001 [ASCII for 2] 1 [ASCII for 0] 1 [ASCII for S] - - — — —

CS1

# Implementation concerns

- Need to efficiently be able to select lowest weight trees to merge when constructing the trie
  - Can accomplish this using a *priority queue*
- Need to be able to read/write bitstrings!
  - Unless we pick multiples of 8 bits for our codewords, we will need to read/write *fractions* of bytes for our codewords
    - We're not actually going to do I/O on fraction of bytes
    - We'll maintain a buffer of bytes and perform bit processing on this buffer
    - See BinaryStdIn.java and BinaryStdOut.java

# We need to read and write individual bits: Binary I/O

```
private static void writeBit(boolean bit) {
      // add bit to buffer by shifting in a zero
      buffer <<= 1;
      if (bit) buffer |= 1;  //then turning it to one if needed
      // if buffer is full (8 bits), write out as a single byte
      N++;
      if (N == 8) clearBuffer();
}
```

```
writeBit(true);
writeBit(false);
writeBit(true);
writeBit(false);
writeBit(false);
writeBit(false);
writeBit(false);
writeBit(true);
```

buffer: **00000000**

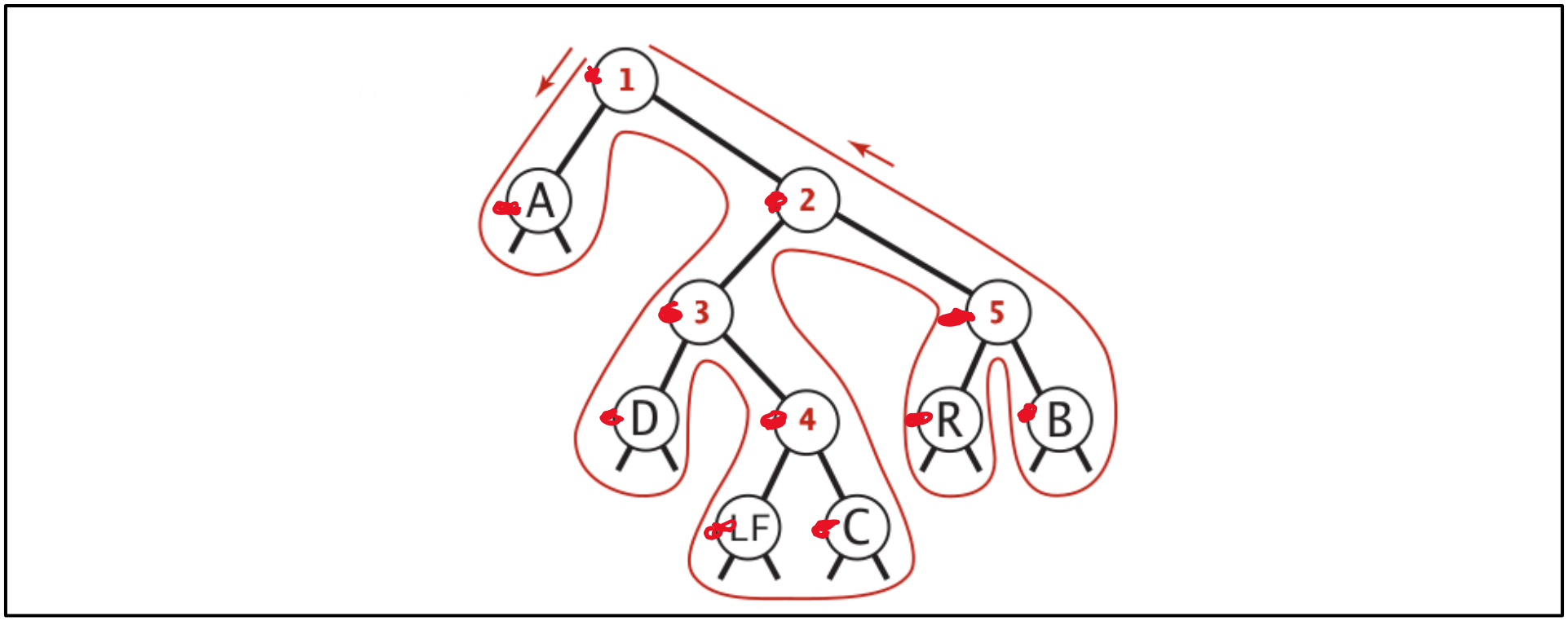
N: **0**

# Further implementation concerns

- To encode, we'll need to read in characters and output codes
- To decode, we'll need to read in codes and output characters
- Sounds like we'll need a symbol table!
  - What implementation would be best?
    - Same for encoding and decoding?
  - Note that this means we need access to the trie to expand a compressed file!
    - Let's store the trie in the compressed file
    - how?

# Representing tries as bitstrings

Preorder traversal




internal node → 0

leaf node → 1 followed by ASCII code of char inside

# Representing tries as bitstrings

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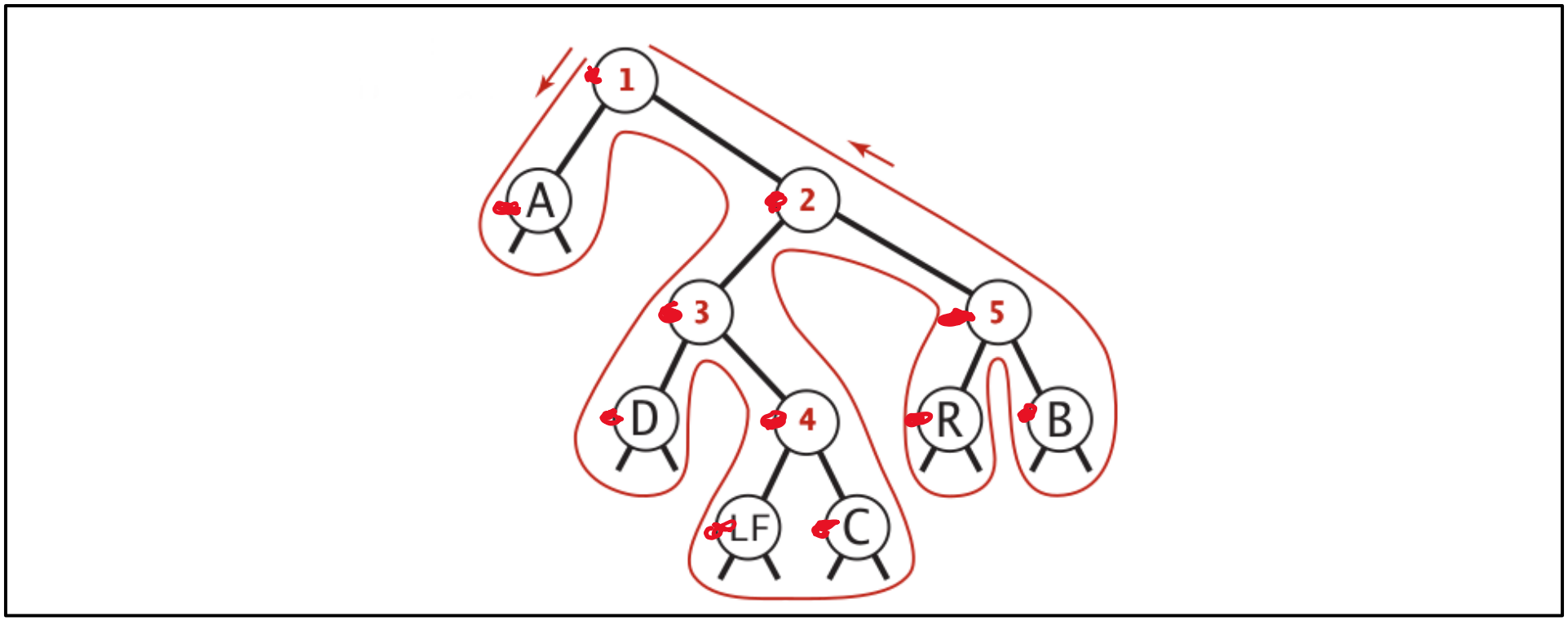


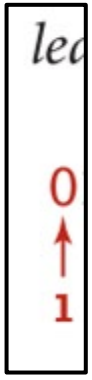

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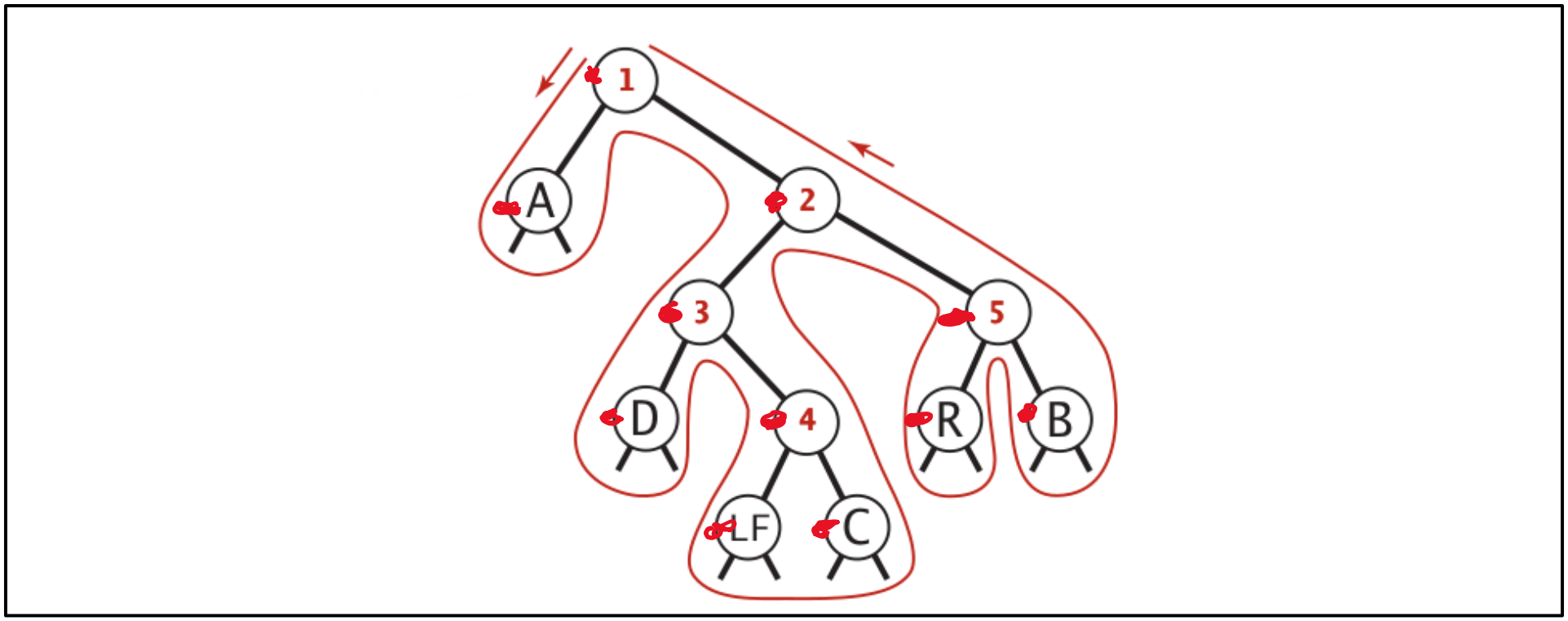


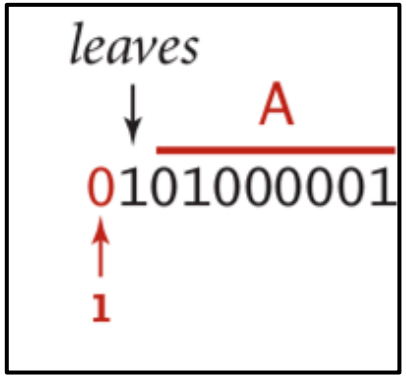

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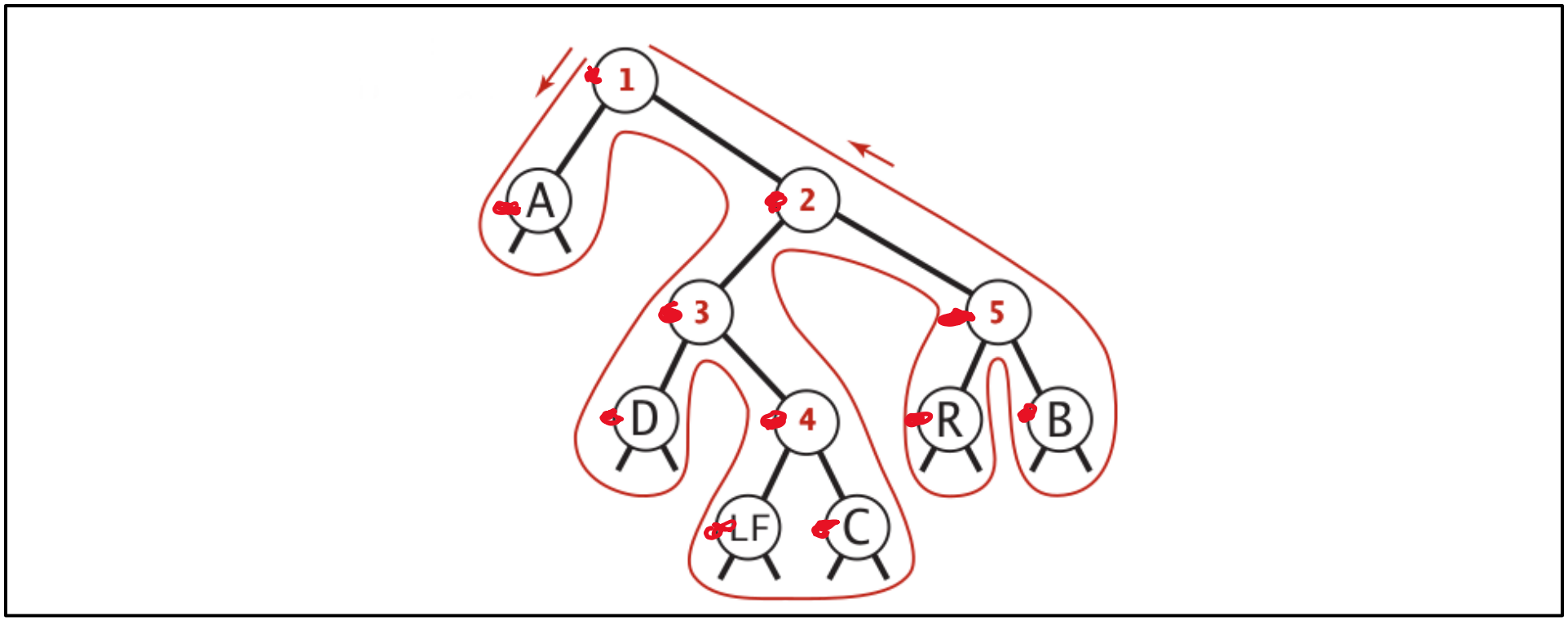


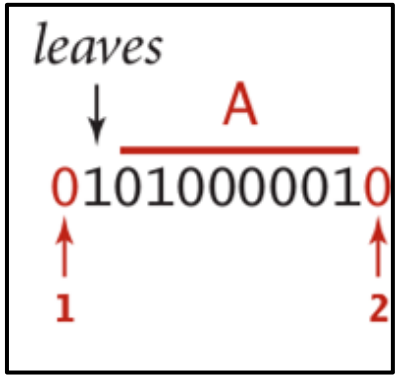

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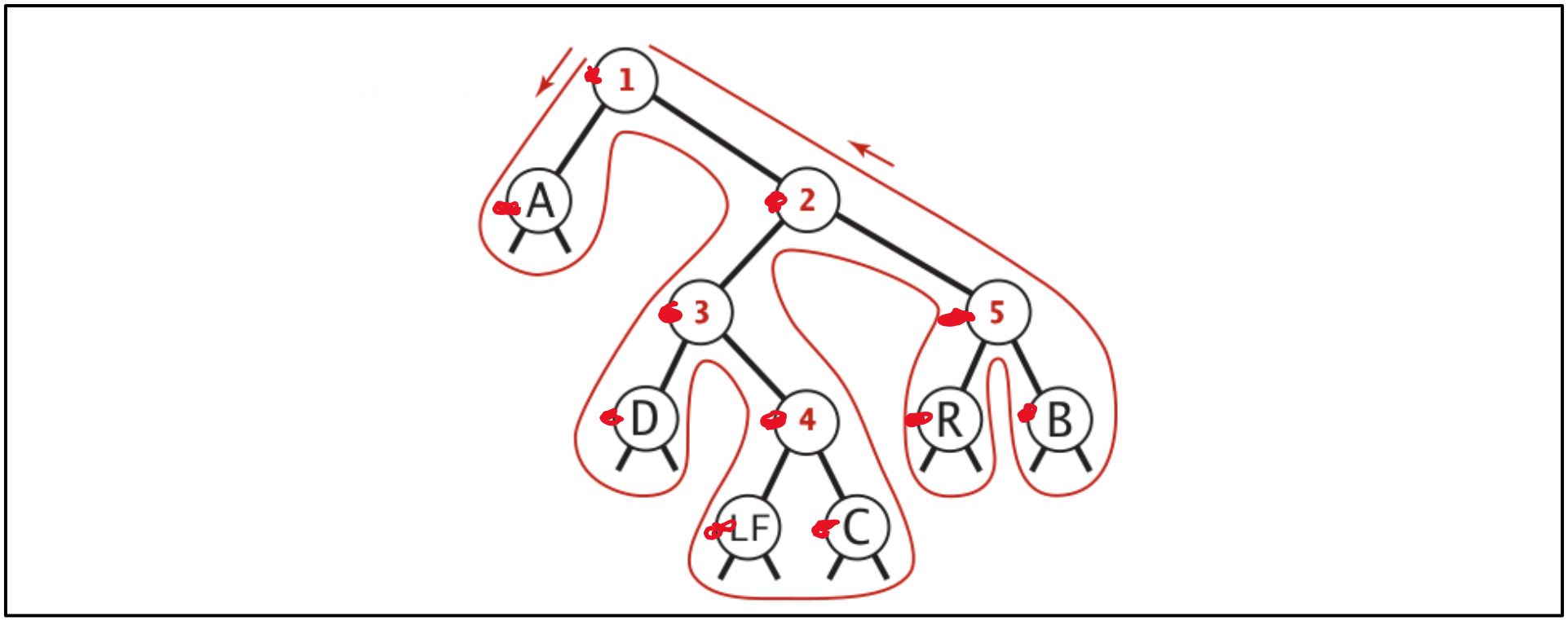


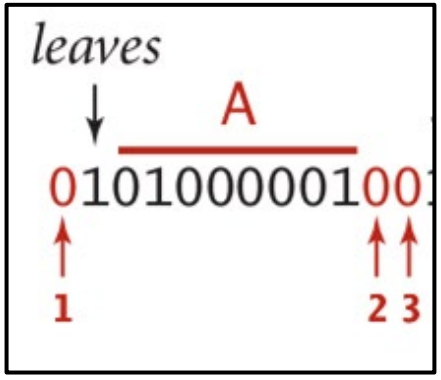

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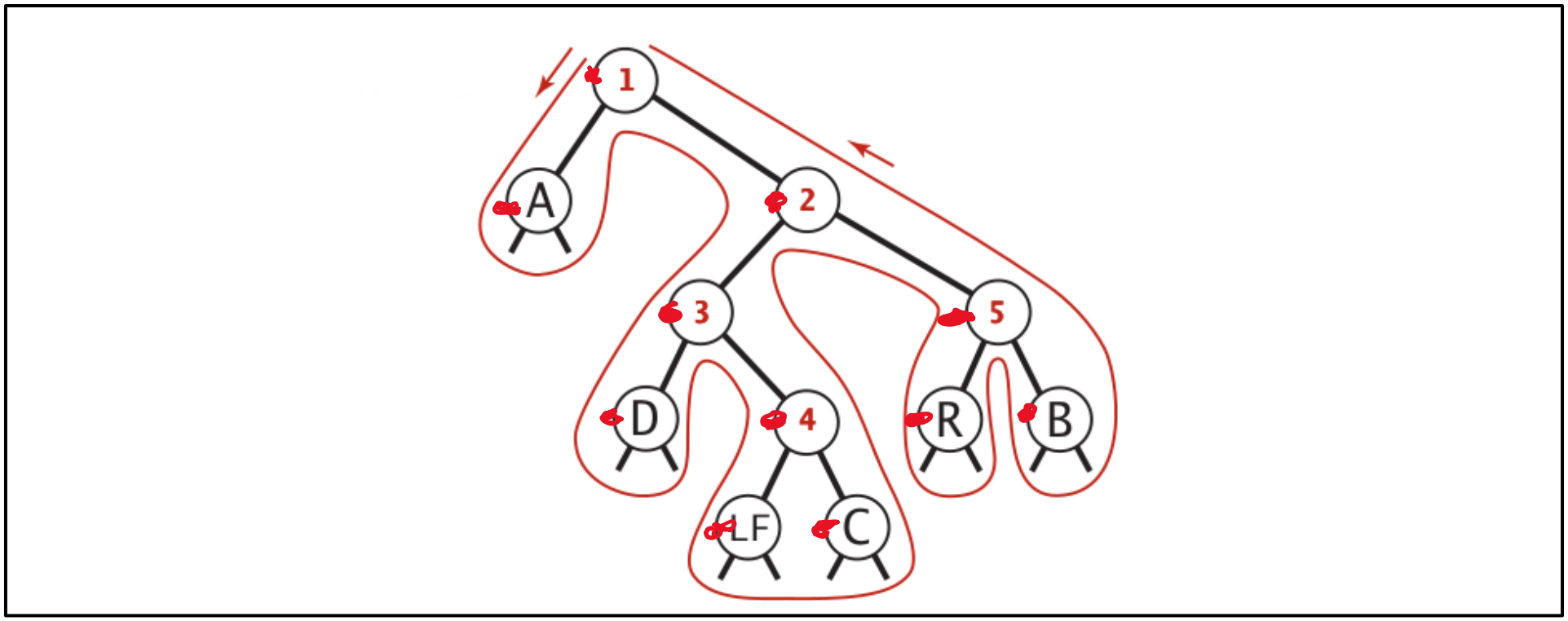


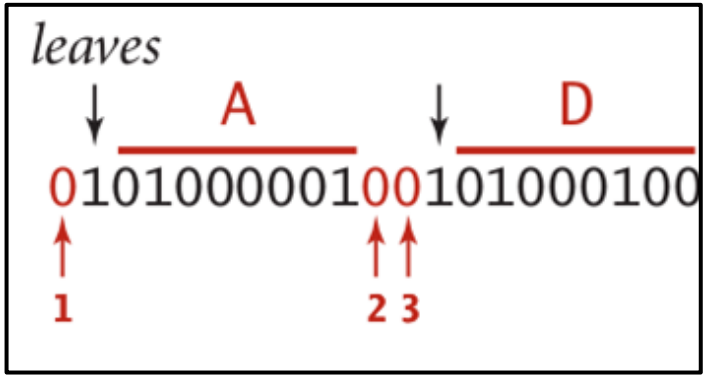

# Representing tries as bitstrings

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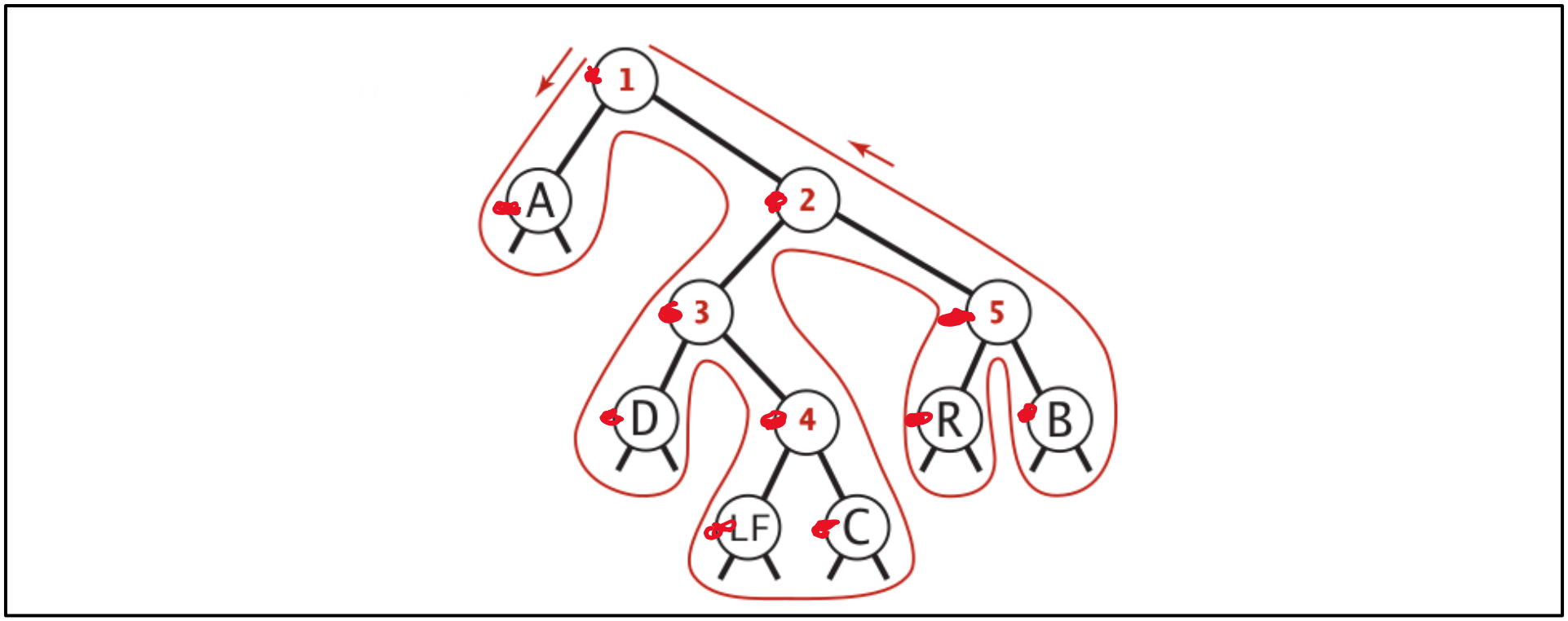


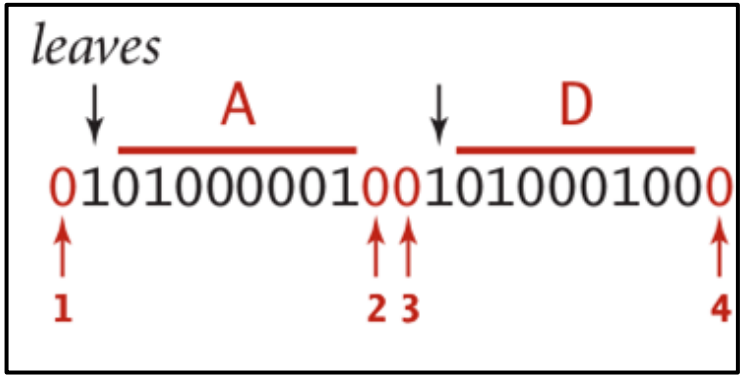

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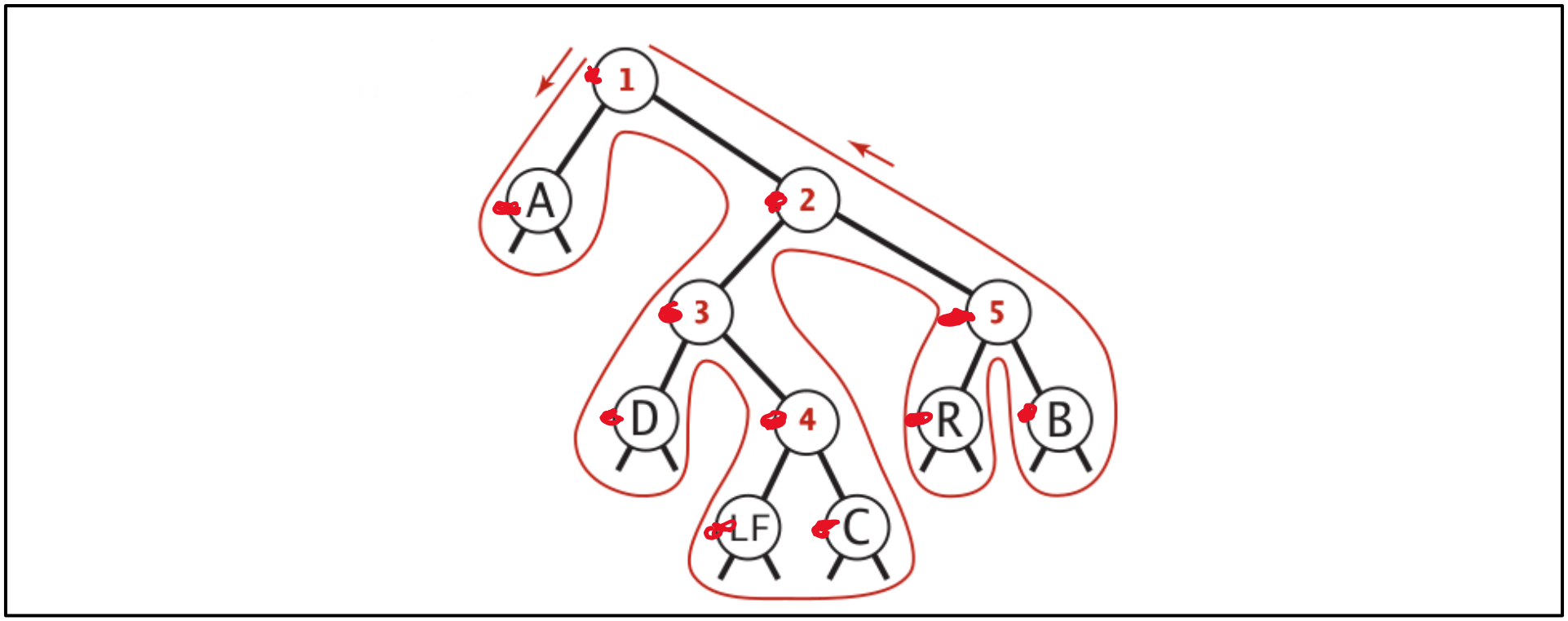
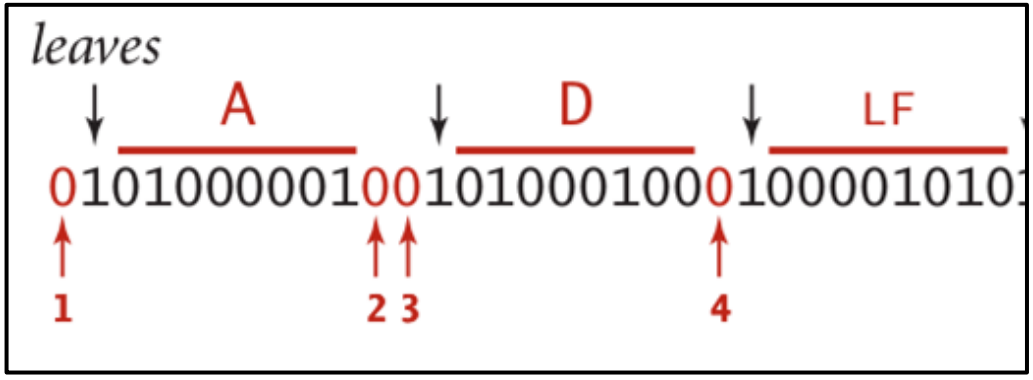
## Preorder traversal



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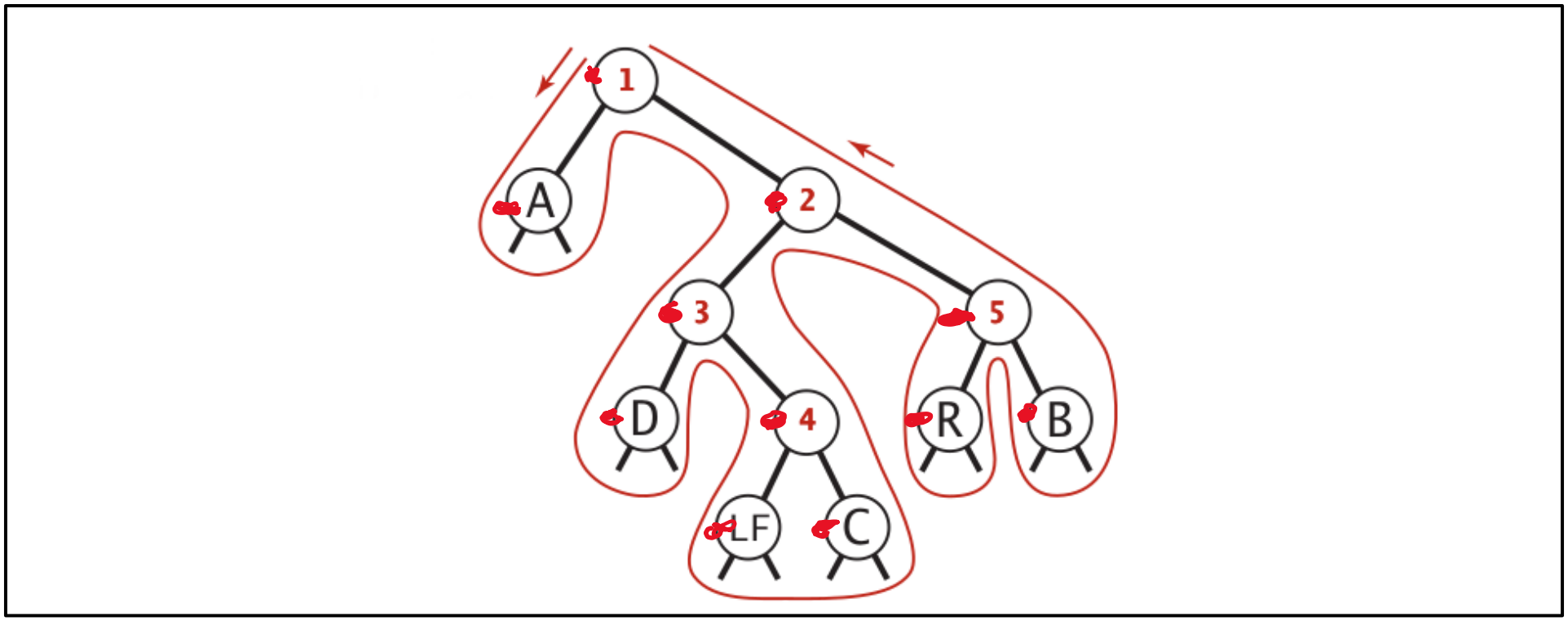
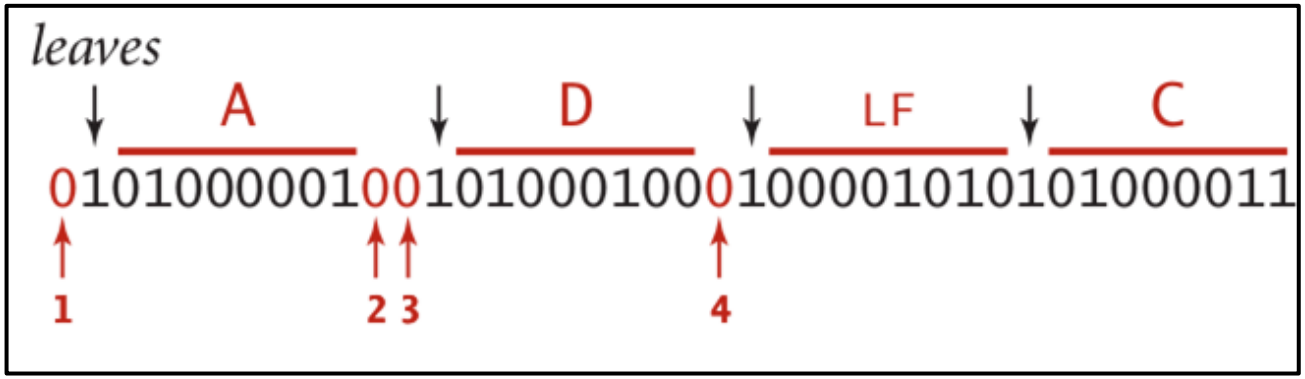
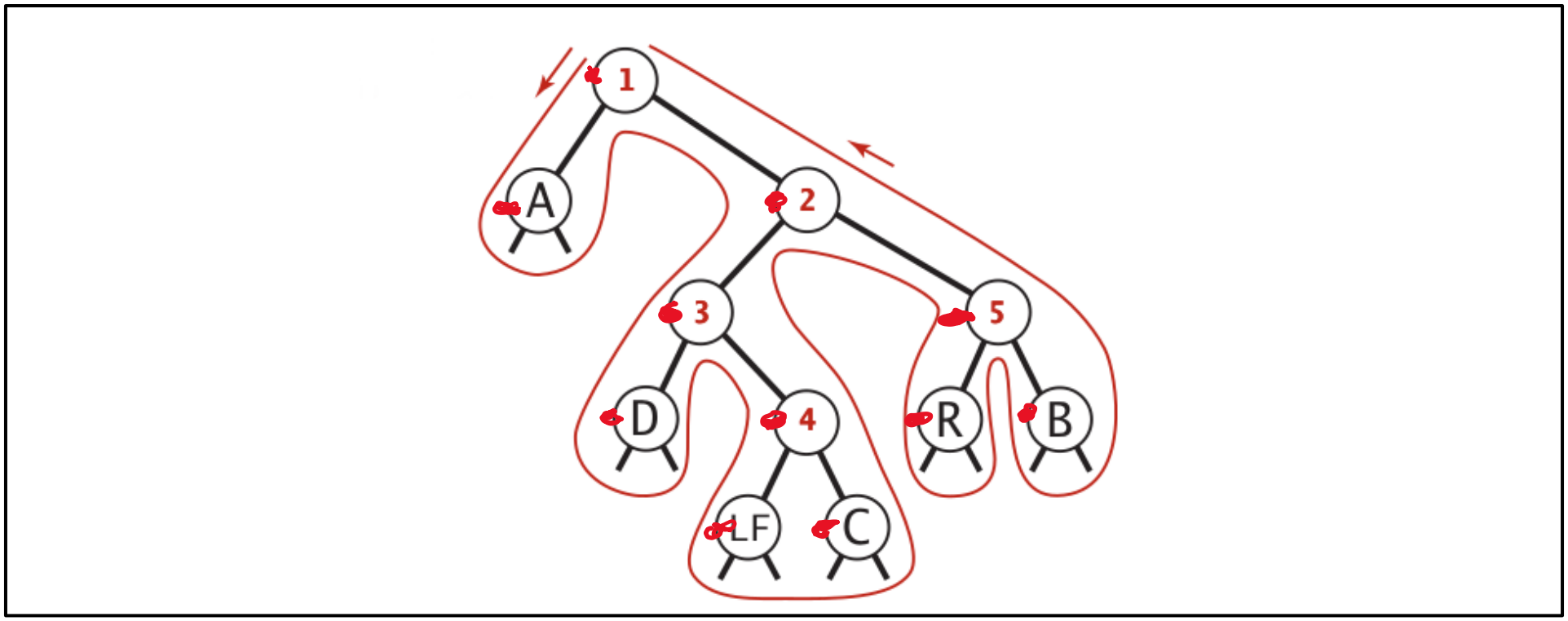
## Preorder traversal

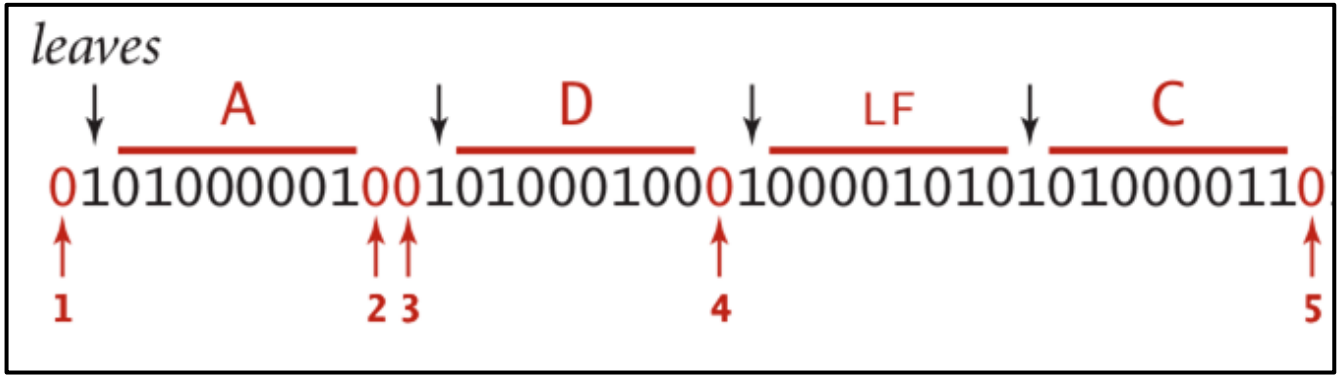


# Representing tries as bitstrings

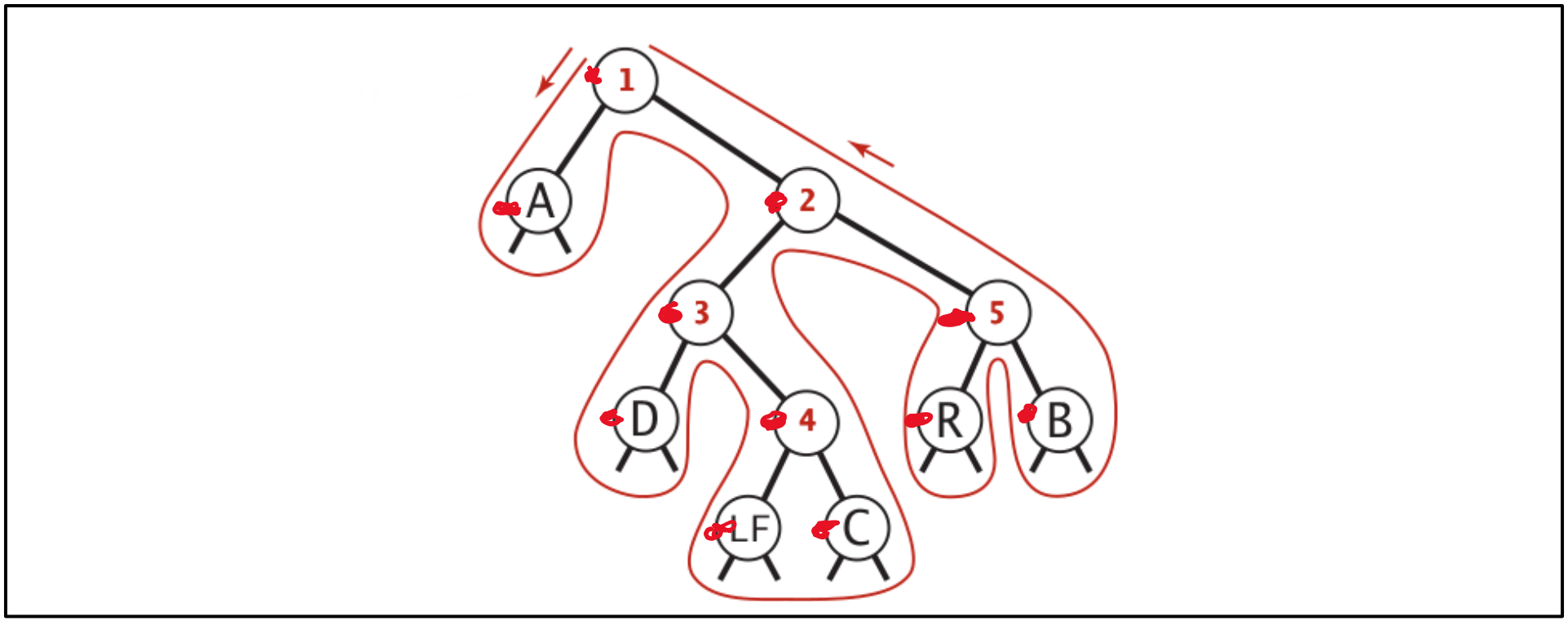
## Preorder traversal

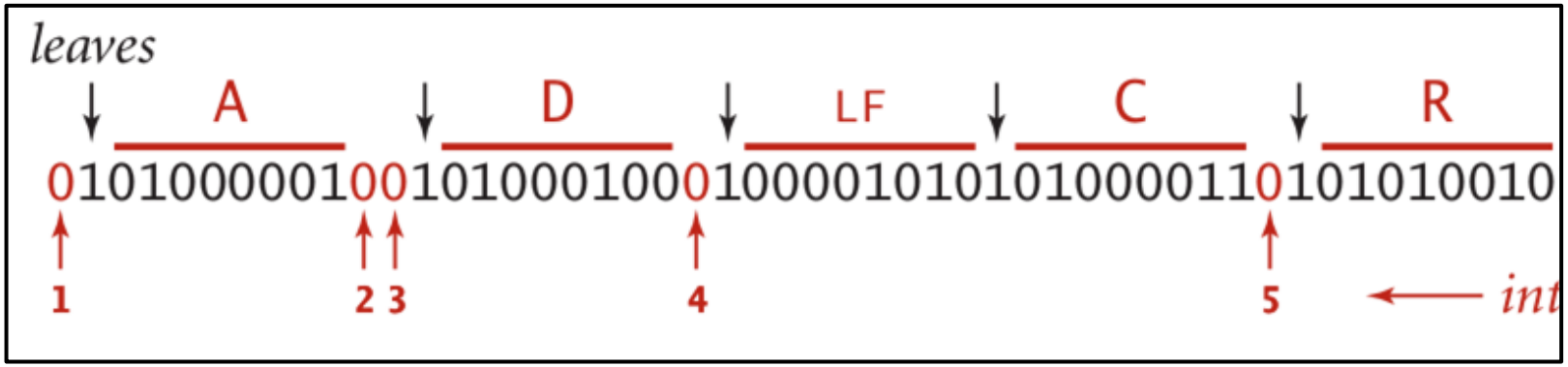


# Representing tries as bitstrings

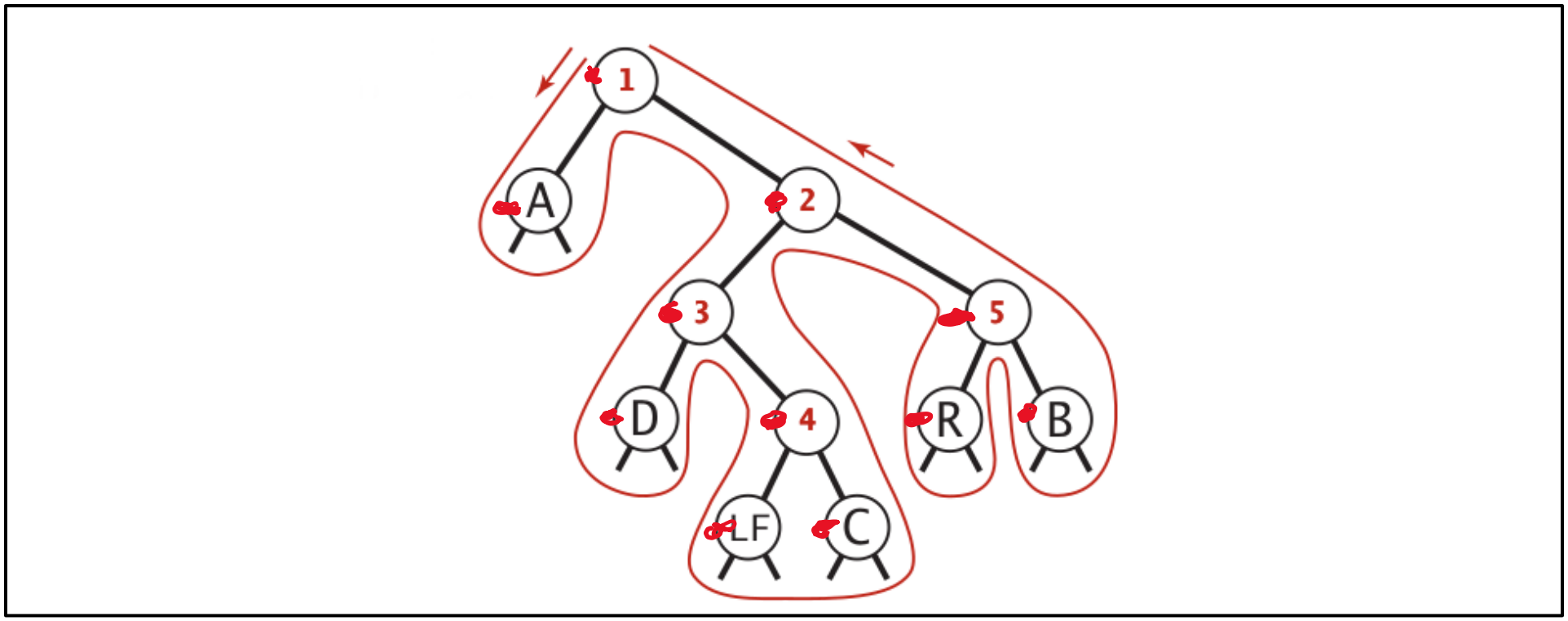
## Preorder traversal

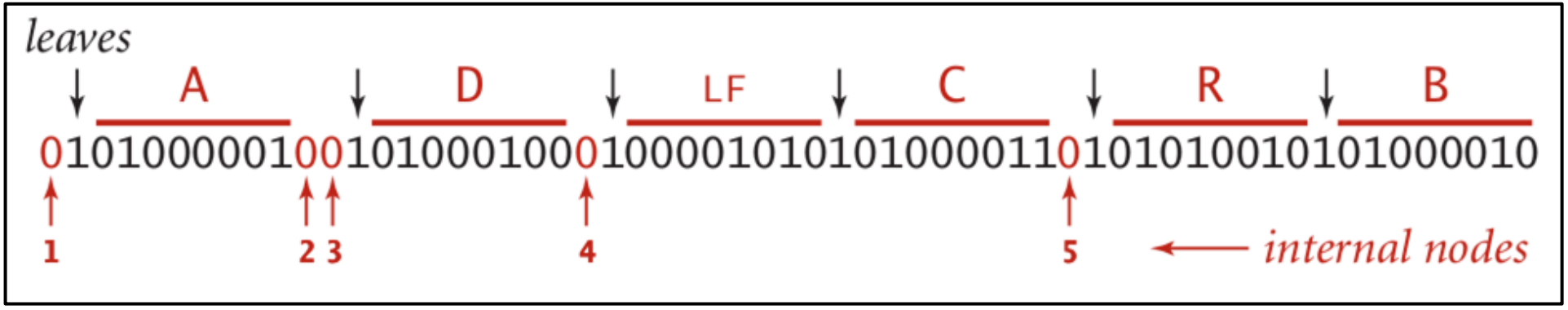


# Representing tries as bitstrings

## Preorder traversal



# Writing a trie

```
private static void writeTrie(Node x){
    if (x.isLeaf()) {
        BinaryStdOut.write(true);
        BinaryStdOut.write(x.ch);
        return;
    }
    BinaryStdOut.write(false);
    writeTrie(x.left);
    writeTrie(x.right);
}
```

# Reading a trie back

Similar to the read tree lab!

```
    private static Node readTrie() {
        if (BinaryStdIn.readBoolean())
           return new Node(BinaryStdIn.readChar(), 0, null, null);
        return new Node('\0', 0, readTrie(), readTrie());
    }
```

# Huffman pseudocode

- Encoding approach:
  - Read input
  - Compute frequencies
  - Build trie/codeword table
  - Write out trie as a bitstring to compressed file
  - Write out character count of input (**why is that necessary**?)
  - Use table to write out the codeword for each input character
- Decoding approach:
  - Read trie
  - Read character count
  - Use trie to decode bitstring of compressed file

# How do we determine character frequencies?

- Option 1: Preprocess the file to be compressed
  - Upside: Ensure that Huffman's algorithm will produce the best output for the given file
  - Downsides:
    - Requires two passes over the input, one to analyze frequencies/build the trie/build the code lookup table, and another to compress the file
    - Trie must be stored with the compressed file, reducing the quality of the compression
      - This especially hurts small files
      - Generally, large files are more amenable to Huffman compression
        - Just because a file is large, however, does not mean that it will compress well!

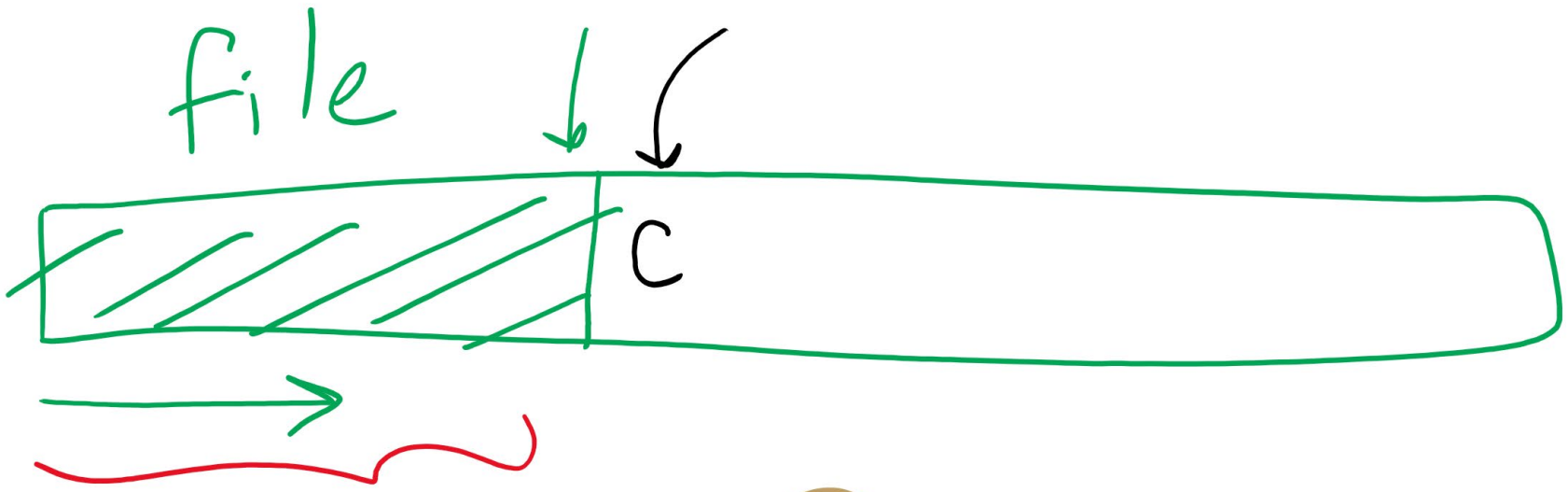
# How do we determine character frequencies?

- Option 2: Use a static trie
  - Analyze multiple sample files, build a single tree that will be used for all compressions/expansions
  - Saves on trie storage overhead…
  - But in general not a very good approach
    - Different character frequency characteristics of different files means that a code set/trie that works well for one file could work very poorly for another
      - Could even cause an increase in file size after "compression"!

# How do we determine character frequencies?

- Option 3:  Adaptive Huffman coding
  - Single pass over the data to construct the codes and compress a file with no background knowledge of the source distribution
  - Not going to really focus on adaptive Huffman in the class, just pointing out that it exists...

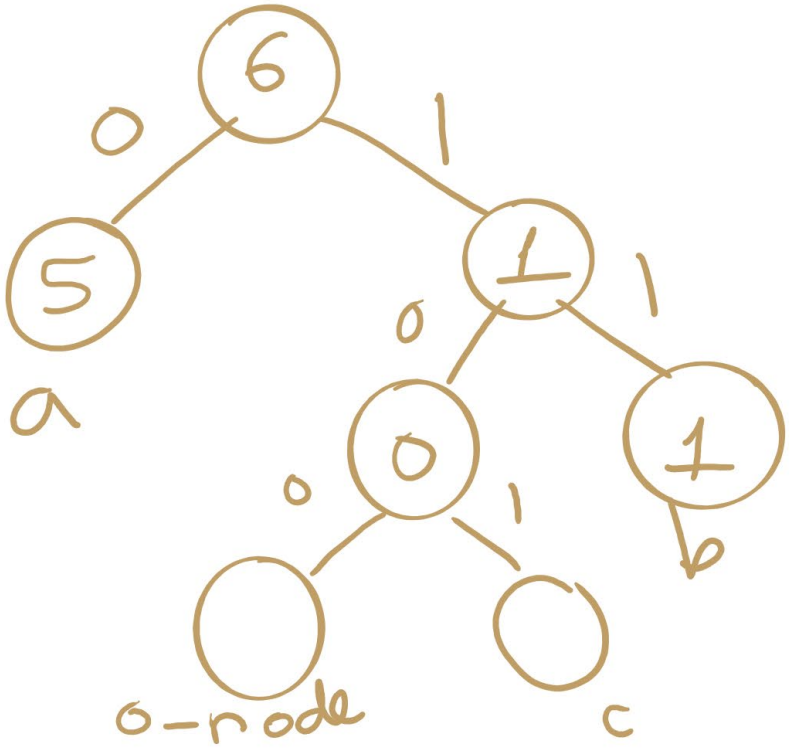
# Adaptive Huffman




a 5

b 1

0_node 0



# Ok, so how good is Huffman compression

- ASCII requires 8n bits to store n characters
- For a file containing c different characters
  - Given Huffman codes $\{h_0, h_1, h_2, \ldots, h_{(c-1)}\}$
  - And frequencies $\{f_0, f_1, f_2, \ldots, f_{(c-1)}\}$
  - Sum from 0 to c-1: $|h_i| * f_i$
- Total storage depends on the differences in frequencies
  - The bigger the differences, the better the potential for compression
- Huffman is optimal for character-by-character prefix-free encodings
  - Proof in Propositions T and U of Section 5.5 of the text
- If all characters in the alphabet have the same usage frequency, we can't beat block code (fixed-size codewords)
  - Huffman reduces to fixed-size codewords
  - On a character by character basis…