University of Pittsburgh

Algorithms and Data Structures 2
CS 1501

Fall 2022

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(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides.)

Announcements

- Homework 1 is due this Friday
- Lab 1 posted on Canvas
  - Explained in recitations of this week
  - Github Classroom
- Assignment 1 will be posted this Friday

Previous lecture …

- Asymptotic analysis
- Boggle Game Problem

Muddiest Points

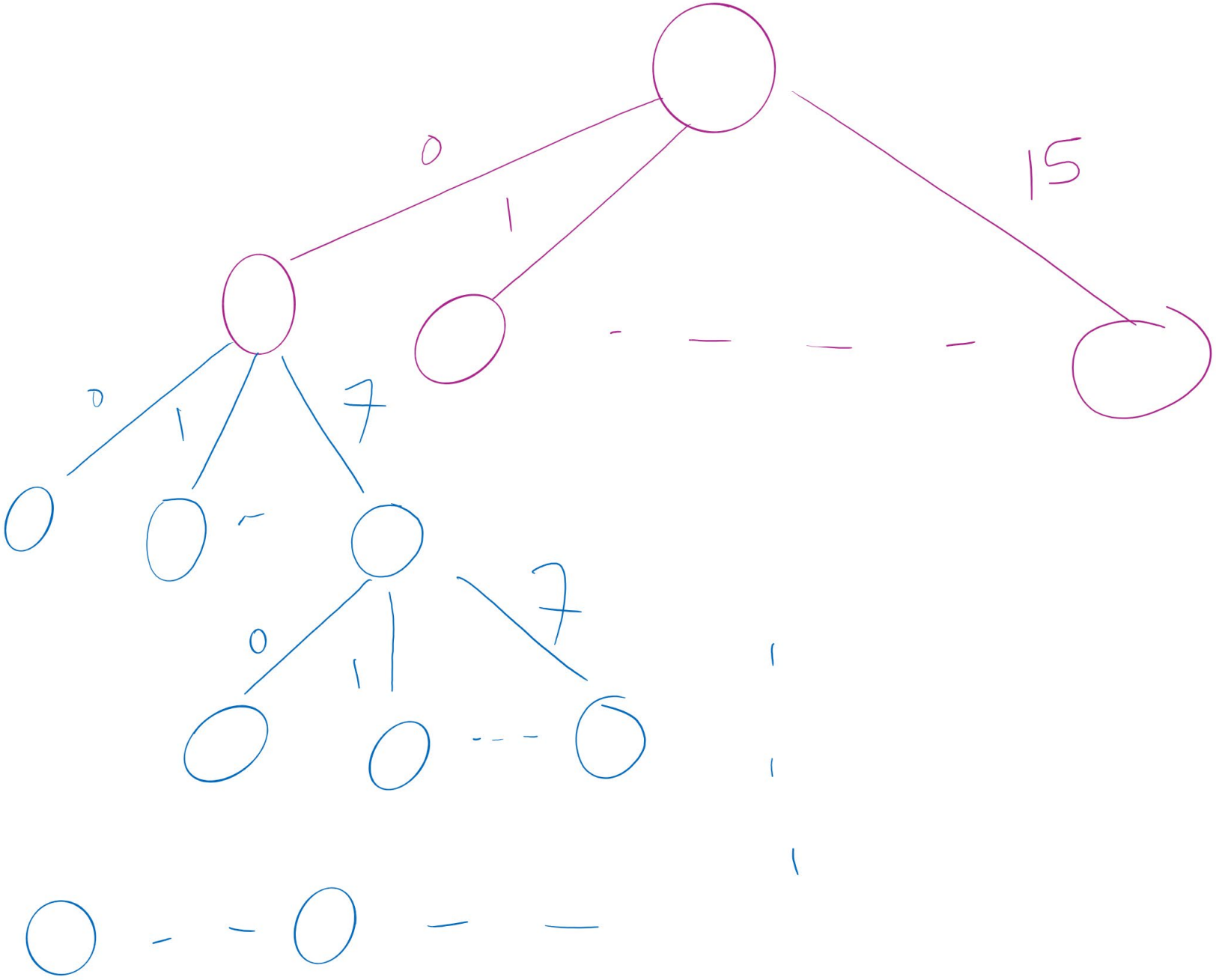
Boggle Game Problem (Recap)

- Words at least 3 adjacent letters long must be assembled from a 4x4 grid
- Adjacent letters are horizontally, vertically, or diagonally neighboring
- Any cube in the grid can only be used once per word

Backtracking Framework

```
void traverse(current decision, partial solution) {
   for each choice at the current decision {
      if choice is valid {
         apply choice to partial solution
         if partial solution a valid solution
            report partial solution as a final solution
         if more decisions possible
            traverse(next decision, updated partial solution)
          undo changes to partial solution
      }
   }
}
```

Search Space for Boggle



Moving down the tree

```
void traverse(current decision, partial solution) {
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}
```

Backtracking

```
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      }
   }
}
```

What is the running time?

- Can't really use the frequency and cost technique because of the recursive call(s)
- In the worst case, the backtracking algorithm has to visit each node in the search space
- We can use the number of nodes in the search tree as a lower bound on the worst-case runtime

Search Space Size

- How many nodes are there?
- **Maximum number of nodes** = 1 + 16 * (…)
- = 1 + 16 * (1 + 8 + 8^2 + 8^3 + … + 8^{15})
- = 1 + 16 * $\theta(largest\ term)$
- = 1 + 16 * $\theta(8^{15})$ =
- In terms of the board size (*n*)
  - **Maximum number of nodes =** 1 + n * $\theta(8^{n-1})$
  - = $\theta(n * 8^{n-1}) = \theta(n * 8)$

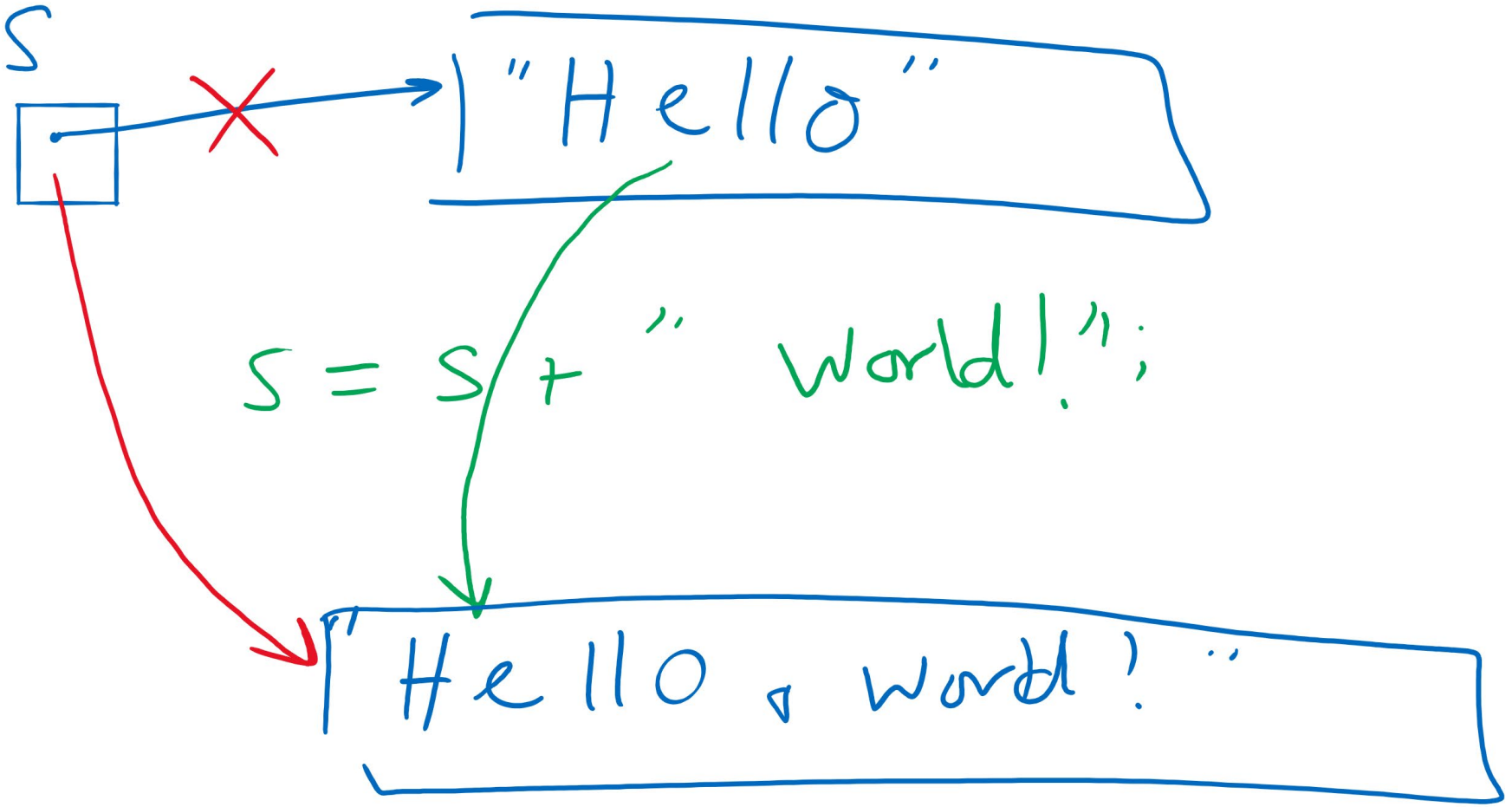
Search Space Size

- In the worst case, the backtracking algorithm must visit each node in the search space
- Worst-case runtime of Backtracking for Boggle = $\Omega(n*8^n)$, where *n* is the board size (# cells)
  - if the non-recursive work is constant, that is *O(1)*, then
    - worst-case runtime = $O(n*8^n)$
  - We will see we make it constant.
- Worst-case runtime of backtracking algorithm is exponential!
  - Pruning has practical savings in runtime, but doesn't significantly reduce the runtime
    - Still exponential

How to make the non-recursive work constant?

- Constructing the words over the course of recursion will mean building up and tearing down strings
  - Moving down the tree adds a new character to the current word string
  - Backtracking removes the most recent character
  - Basically pushing/popping to/from a string stack
- Push/Pop stack operations are generally $\Theta(1)$
  - Unless you need to resize, but that cost can be **amortized**
- What if we use String to hold the current word string?
- Java Strings are *immutable*
  - s = new String("Here is a basic string");
  - s = s + " this operation allocates and initializes all over again";
  - Becomes essentially a $\Theta(n)$ operation
    - Where n is the length() of the string

Concatenating to String Objects



StringBuilder to the rescue

- append() and deleteCharAt() can be used to push and pop
  - Back to Θ(1)!
    - Still need to account for resizing, though…
- StringBuffer can also be used for this purpose
  - Differences?

Searching Problem

- Input:
  - a (large) dynamic set of data items in the form of
    - (key, value) pairs
  - a target *key* to search for
- Output:
  - if *key* exists in the set: return the corresponding value
  - otherwise, return key *not found*
- What does dynamic mean?
- How would you implement “key not found”?

Let's create an Abstract Data Type!

- The Symbol Table ADT
  - A set of (key, value) pairs
- Operations of the ST ADT
  - insert
  - search
  - delete

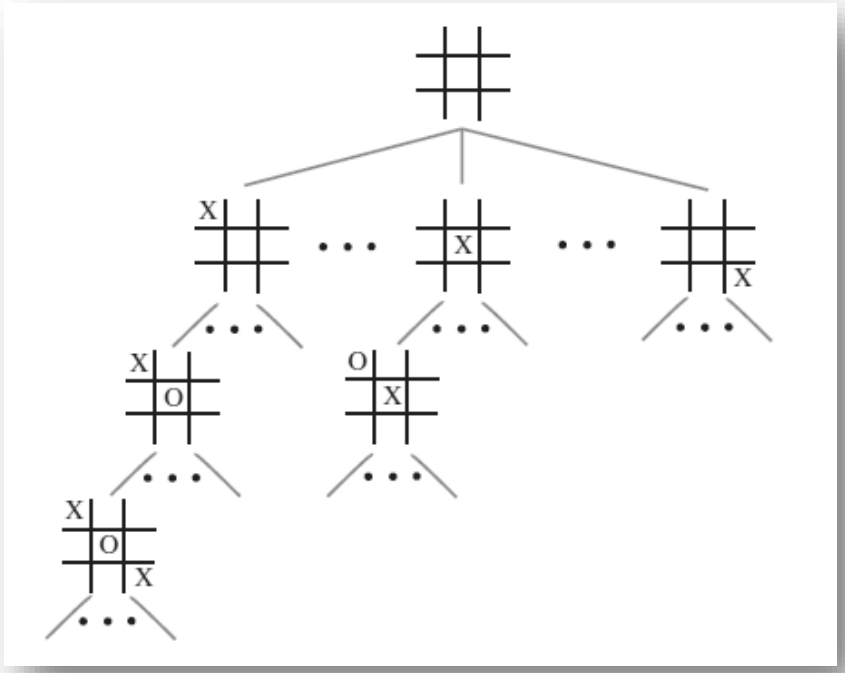
Symbol Table Implementations

Implementation	Runtime for Insert	Runtime for search	Runtime for delete
Unsorted Array			
Sorted Array			
Unsorted Linked List			
Sorted Linked List			

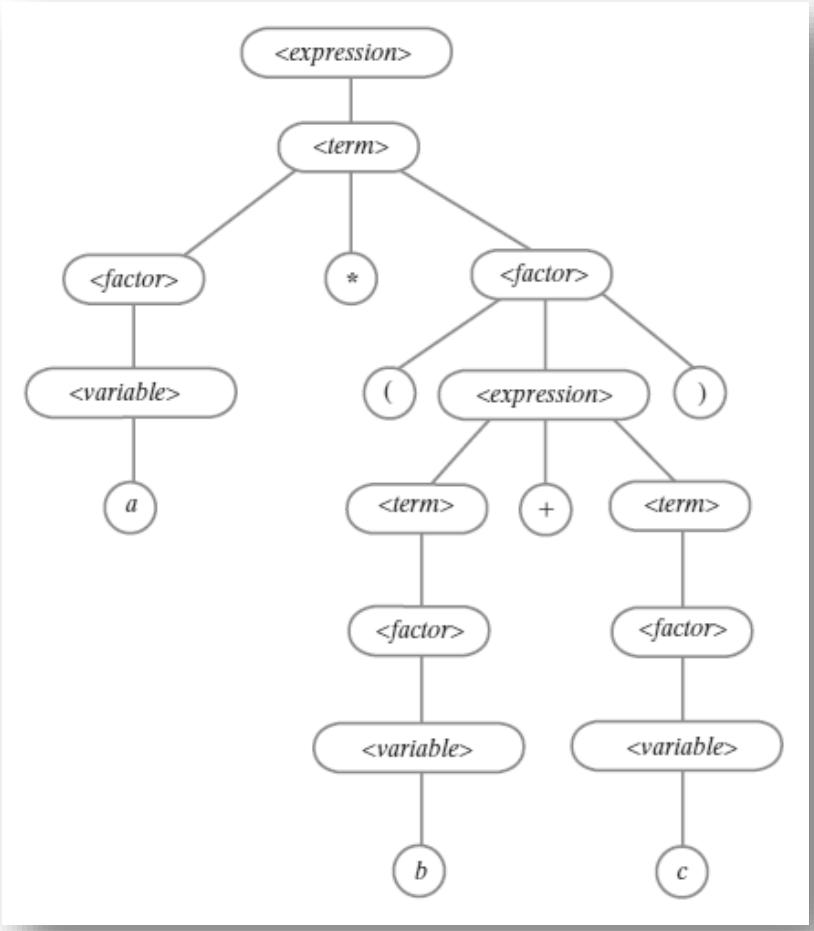
Symbol Table Implementations

- Arrays and Linked Lists are linear structures
- What if we use a non-linear data structure?
  - a Tree?

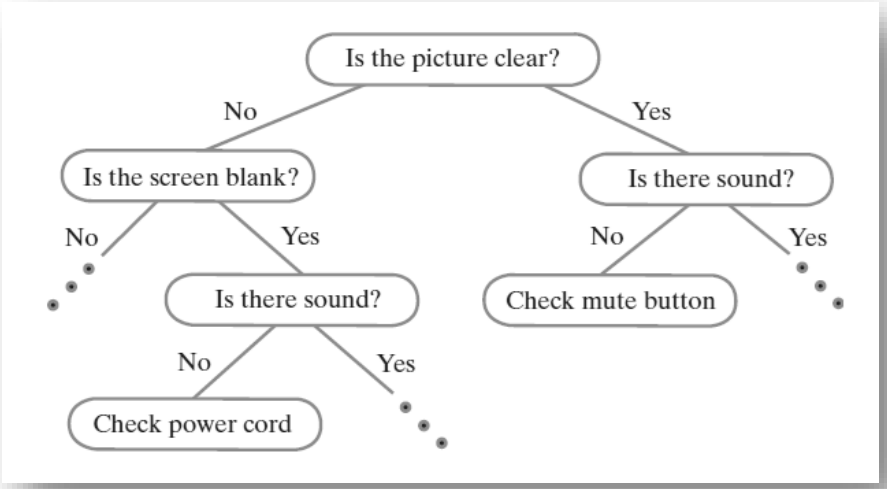
Examples of Trees



Game Tree

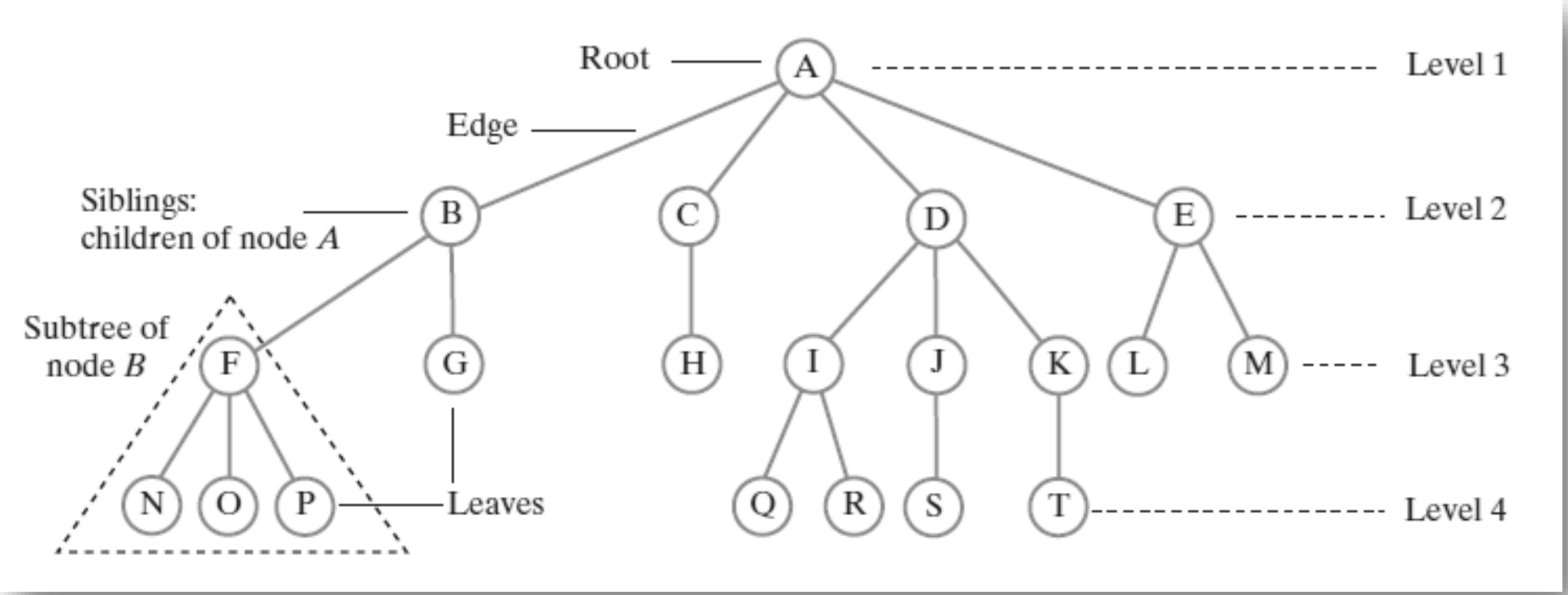


Parse Tree



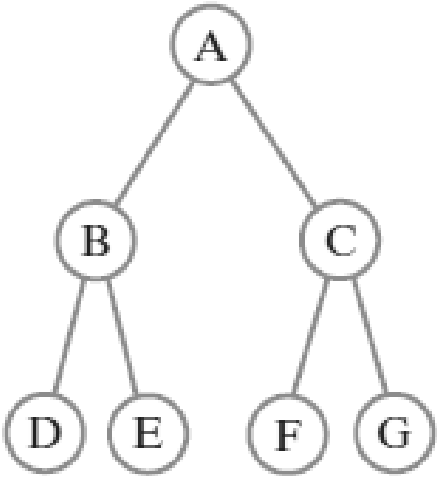
Decision Tree

Tree Terminology



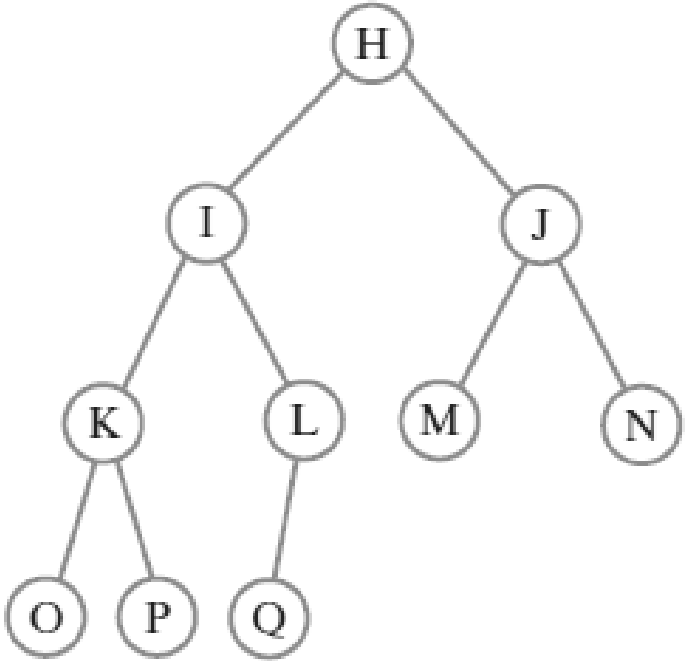
Binary Trees

(a) Full tree

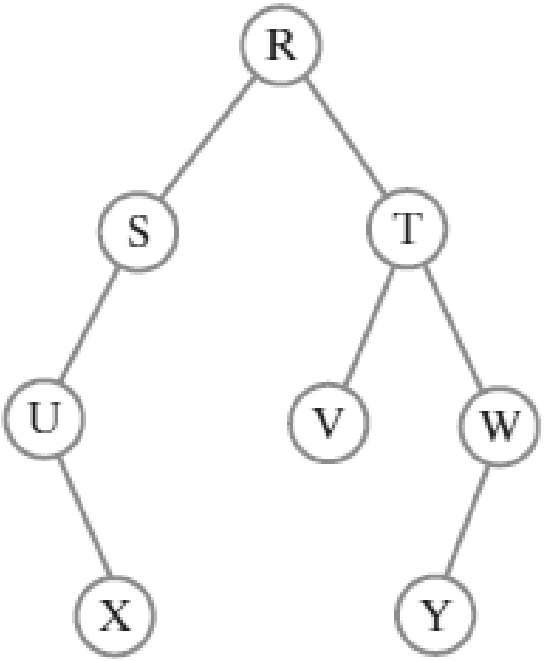


Left children: B, D, F
Right children: C, E, G

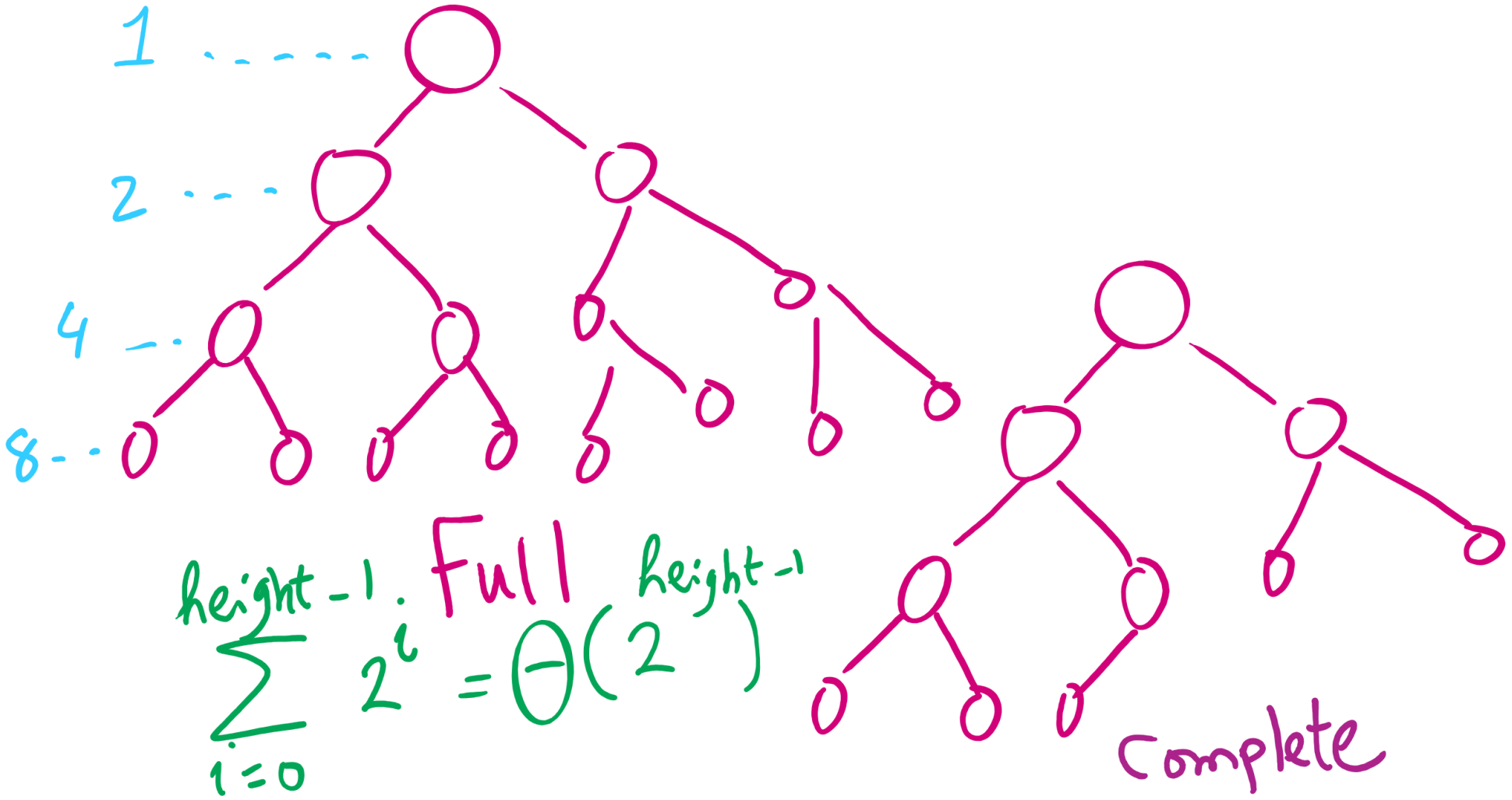
(b) Complete tree



(c) Tree that is not full and not complete



Full vs. Complete Tree



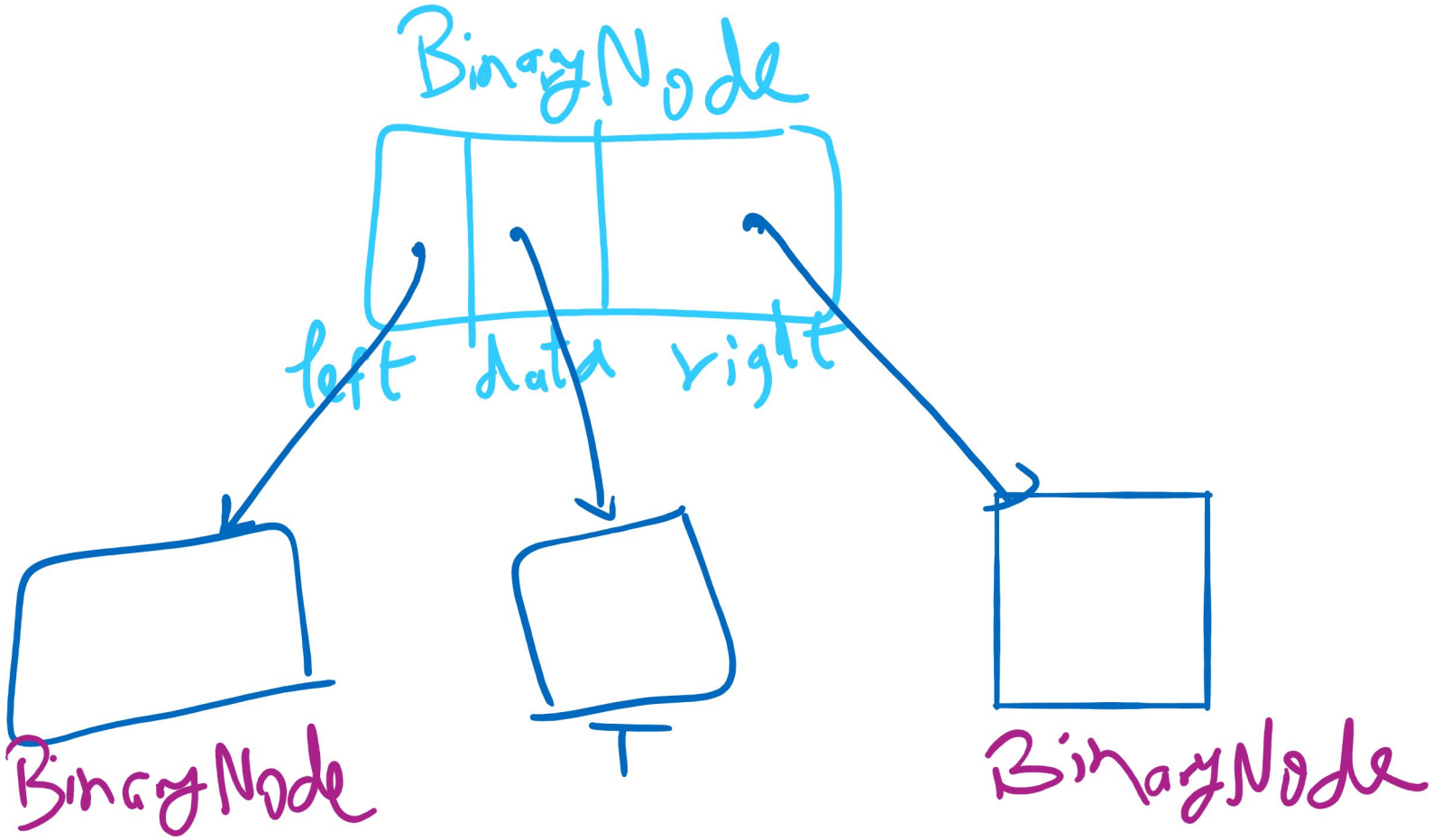
Tree Interface

```
public interface TreeInterface<T> {
  public T getRootData() throws EmptyTreeException;
  public int getHeight() throws EmptyTreeException;
  public int getNumberOfNodes() throws EmptyTreeException;
  public boolean isEmpty();
  public void levelOrderTraverse();
  public void clear();
}
```

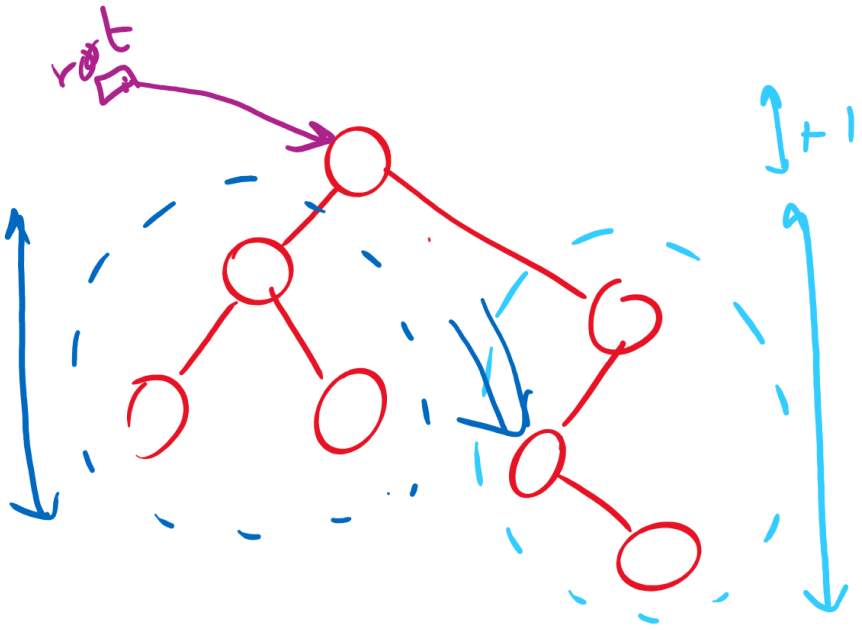
Code Walkthrough

- Available online at:
  - https://cs1501-2231.github.io/slides-handouts/CodeHandouts/TreeADT/Slides
  - The slides are under the CodeHandouts/TreeADT/slides folder in the handout repository
  - https://github.com/cs1501-2231/slides-handouts

BinaryNode

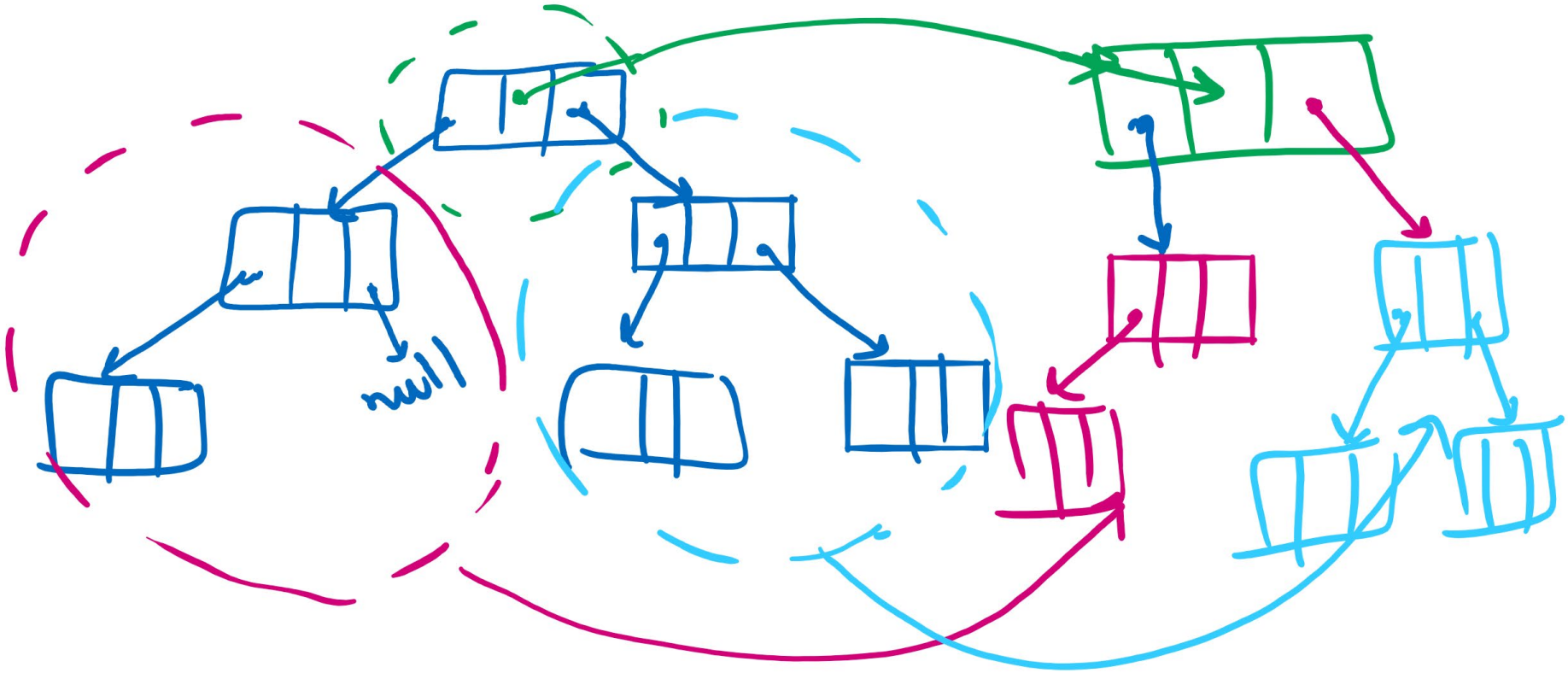


Another implementation of getHeight




```
int getHeight(BinaryNode<T> root){
    int lHeight=0;
    int rHeight=0;
    if(root.left!=null)
        lHeight=getHeight(root.left);
    if(root.right!=null)
        rHeight=getHeight(root.right);
    return Math.max(lHeight,rHeight)+1;
}
```

BinaryNode.copy

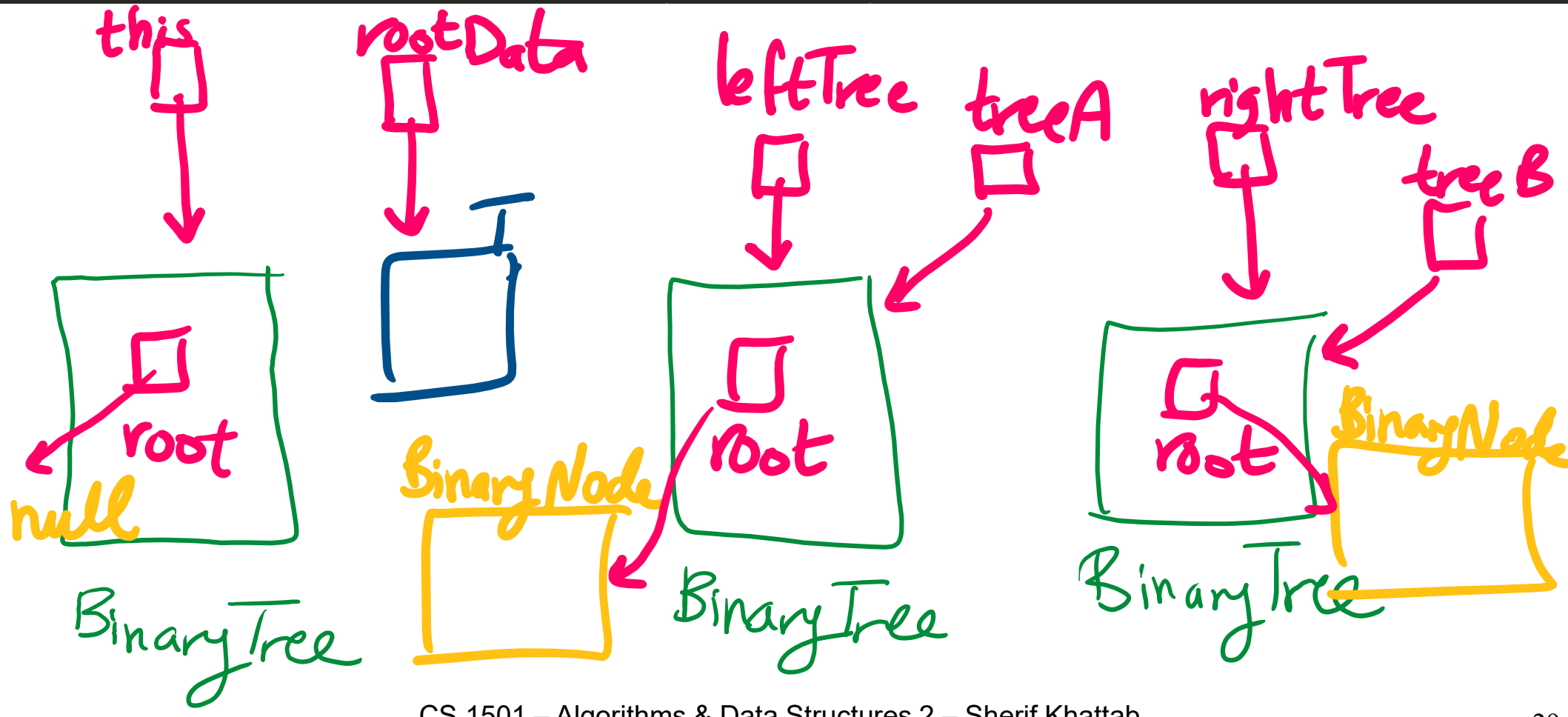


Let's draw a picture of the before state

- Given the call

```
privateBuildTree(data, treeA, treeB);
```

```
private void privateBuildTree(T rootData, BinaryTree<T> leftTree,
                              BinaryTree<T> rightTree){
```



Let's draw a picture of the after state

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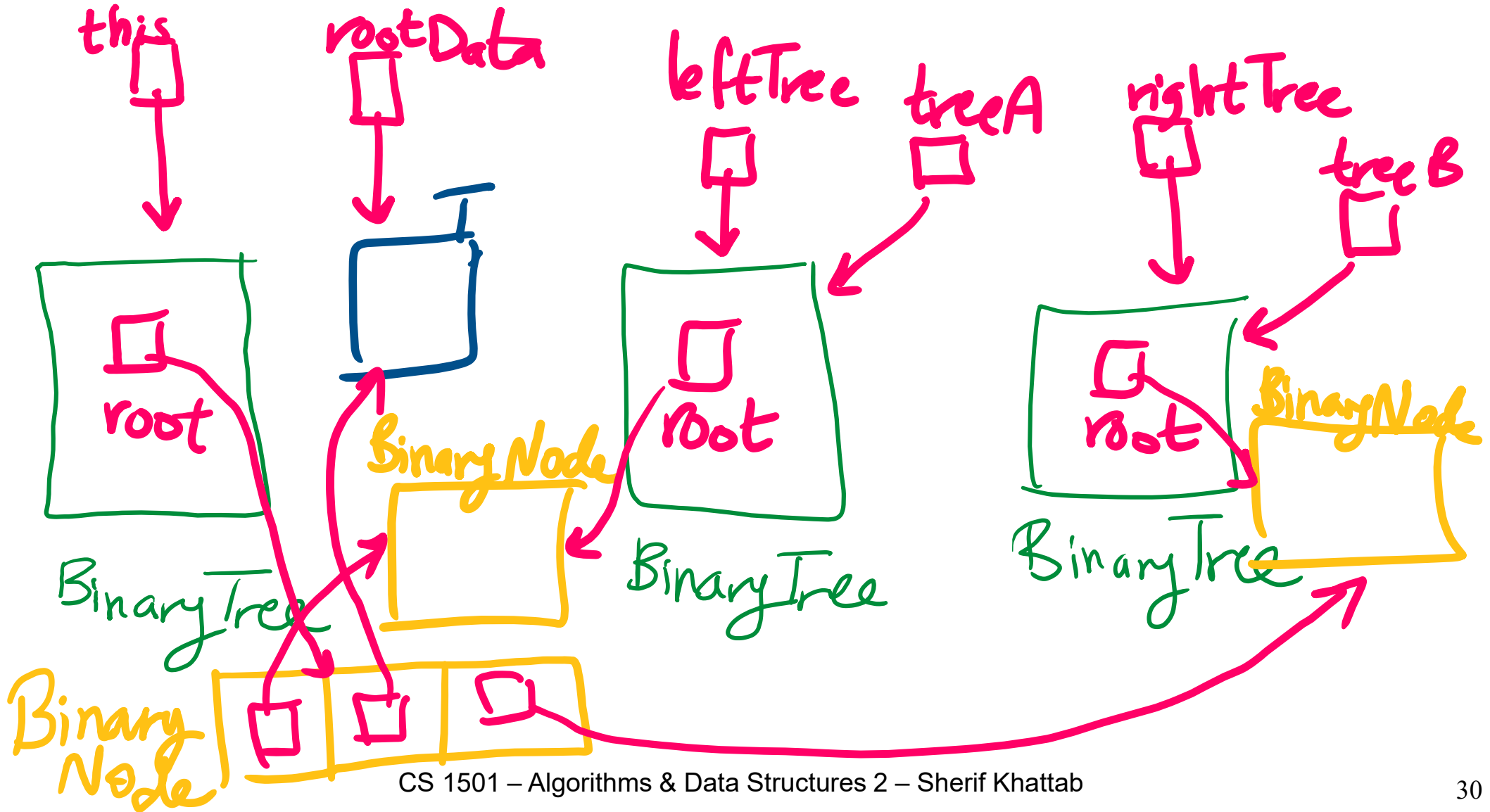
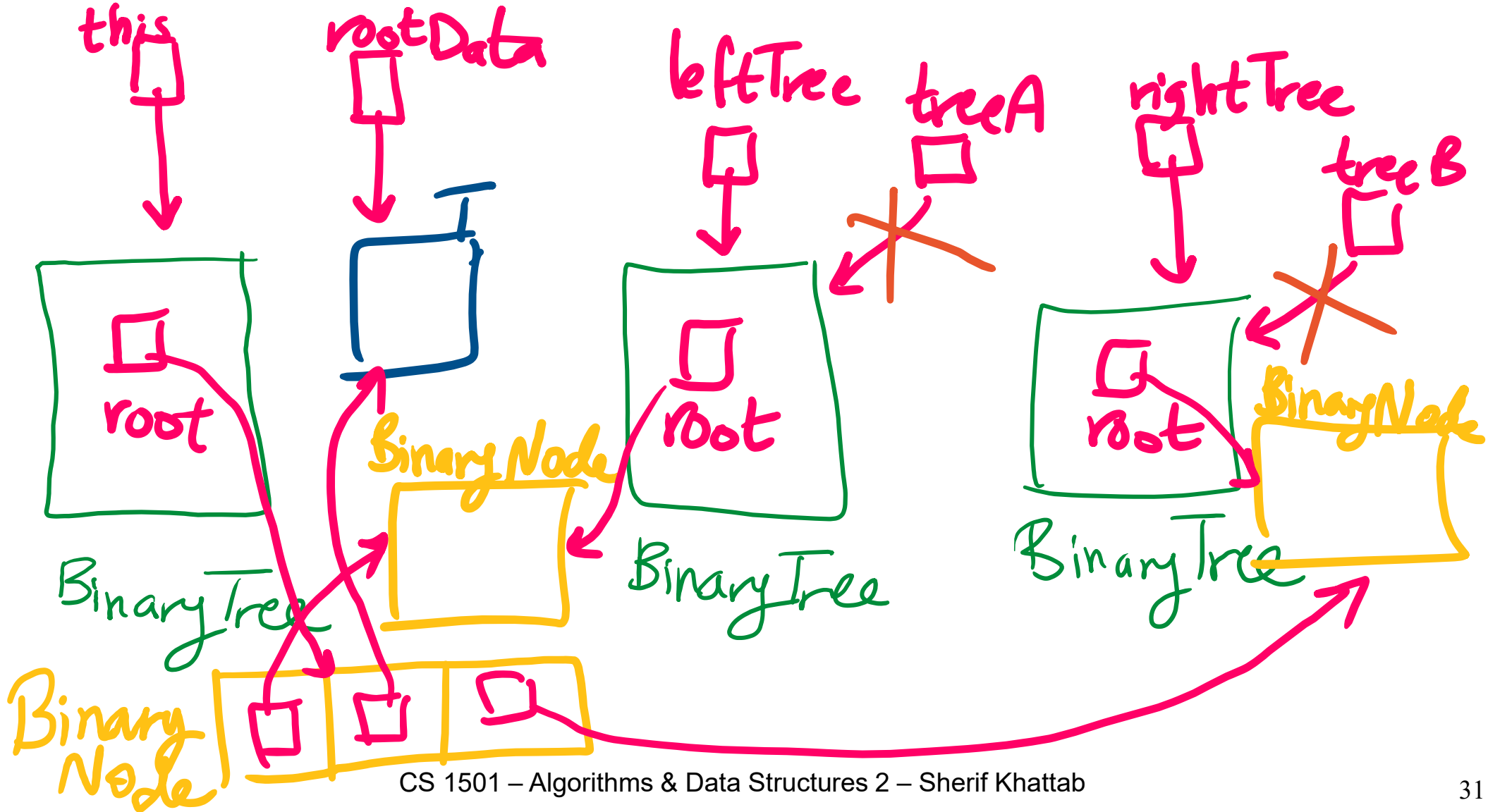


Let's draw a picture of the after state

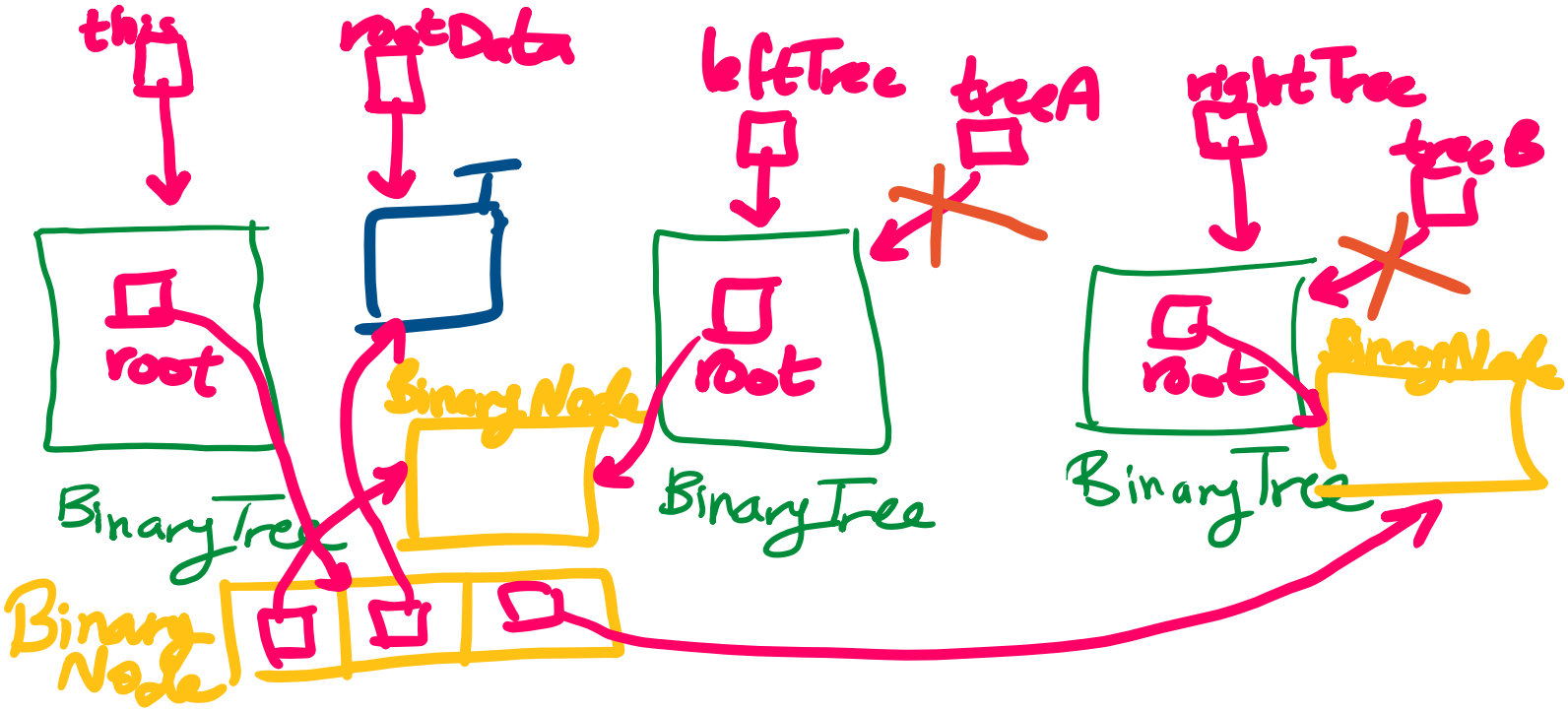
Need to also Prevent client direct access to this

`treeA` shouldn't have access this.root.left (same for treeB)



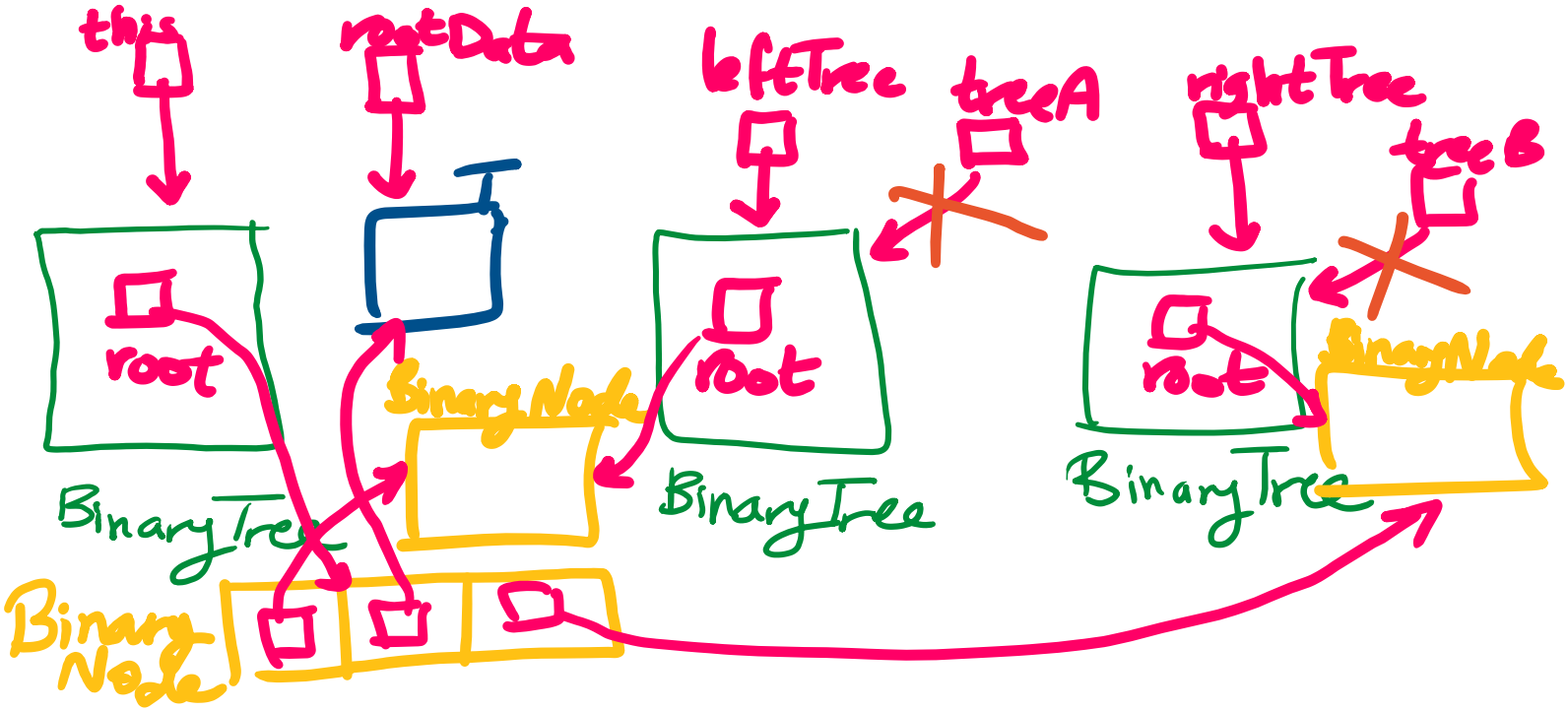
Main logic

- `root = new BinaryNode<>(rootData);`
- `root.left = leftTree.root;`
- `root.right = rightTree.root;`
- How to prevent client access?



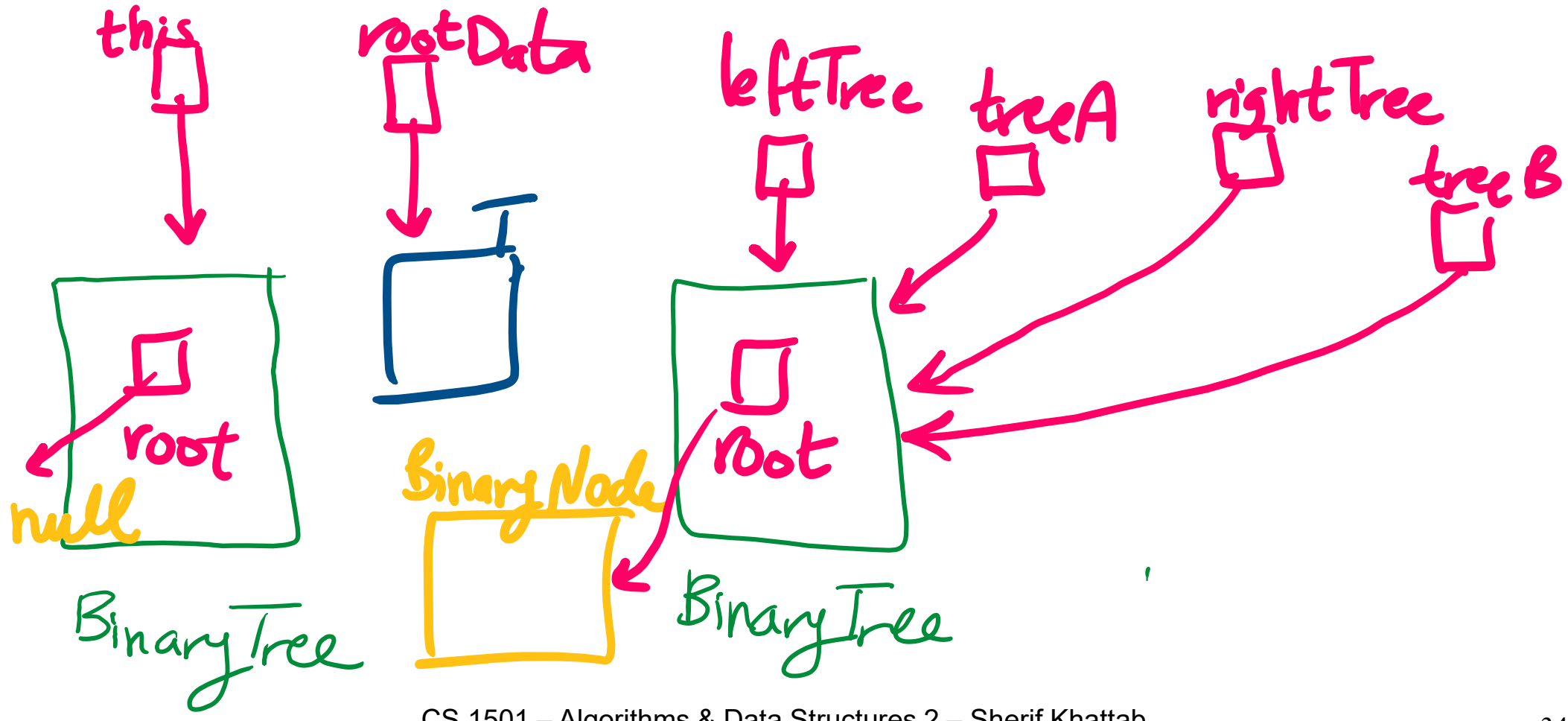
How to prevent client access?

- treeA = treeB = null; //is that possible?
- leftTree = rightTree = null; //would that work?
- leftTree.root = null; rightTree.root = null;



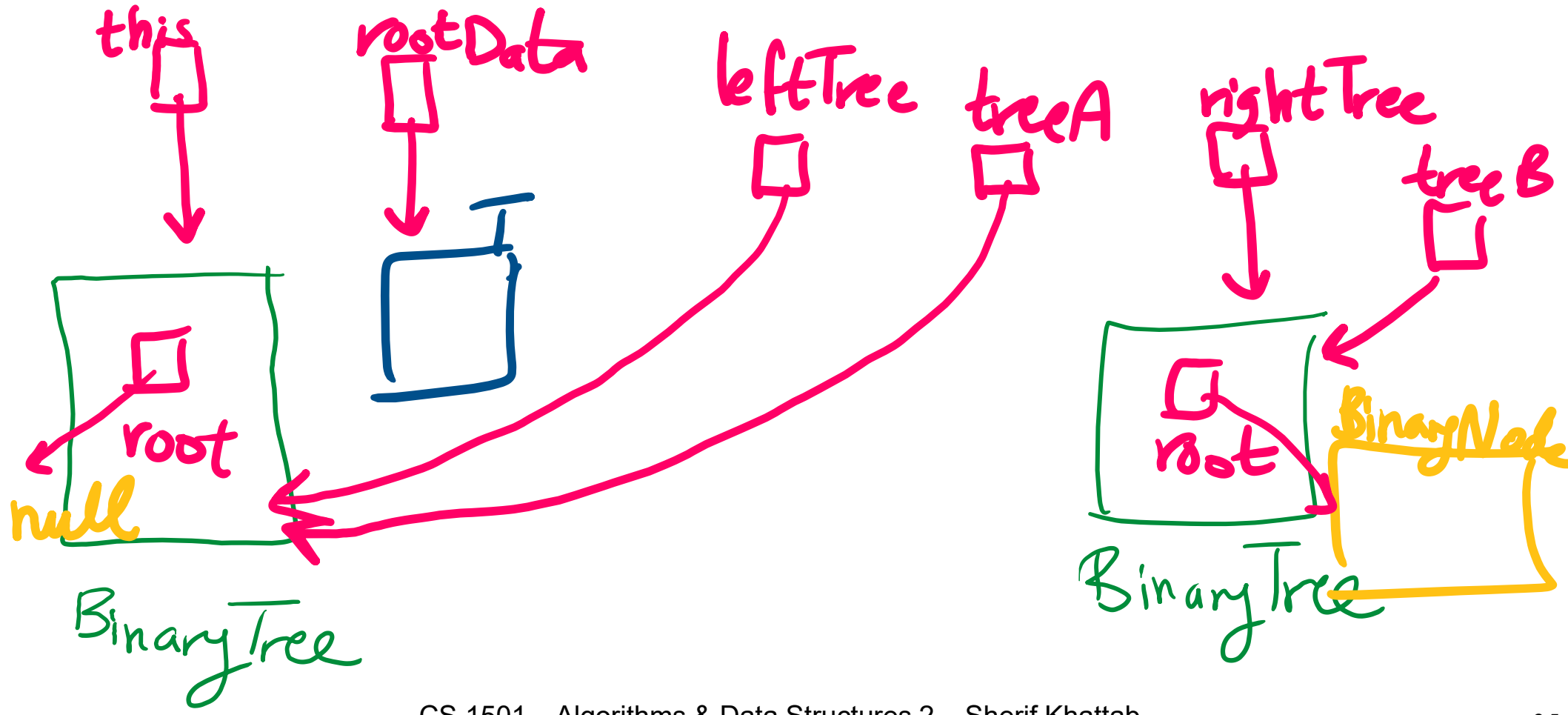
Special case: treeA == treeB

Need to make a copy of leftTree.root



Special case: treeA == this or treeB == this

Need to be careful before leftTree.root = null and rightTree.root = null



Tree Search Take 1

- *Traverse* every node of the tree
  - Is the key inside the node equal to the target *key*?
- How can we traverse the tree?

Tree Search Take 1

What is the runtime?

Can we do better?

Can we traverse the tree more intelligently?

Tree Search Take 2: Binary Search Tree

- Search Tree Property
  - left.data < root.data < right.data
  - Holds for each subtree
  - In Java:
    - root.data.compareTo(left.data) > 0 &&
    - root.data.compareTo(right.data) < 0