Homework 9

Due: December 8th, 2022

This homework must be typed in LaTeX and handed in via Gradescope.

Please ensure that your solutions are complete, concise, and communicated clearly. Use full sentences and plan your presentation before you write. Except in the rare cases where it is indicated otherwise, consider every problem as asking you to prove your result.

Problem 1

In the "k-Multiple Boolean Satisfiability Problem" $kMSAT$, given a Boolean formula ϕ with m variables and $n \geq m$ literals, we want to decide whether it has at least k satisfying assignments, where $k = O(1)$. (I.e., at least k distinct truth-value assignments to its variable for which ϕ is satisfied.) Prove that $kMSAT$ is NP-complete.

Problem 2

Let BF_k denote the set of Boolean formulas in Conjunctive Normal Form such that each variable appears in at most k places (i.e., in at most k literals).

1. Show that the problem of deciding whether a Boolean Formula in BF_2 is satisfiable is in P.
2. Show that the problem of deciding whether a Boolean Formula in BF_3 is satisfiable is $NP - Complete$.