

# Homework 9

Due: November 23, 2021

This homework must be typed in  $\text{\LaTeX}$  and handed in via Gradescope.

Please ensure that your solutions are complete, concise, and communicated clearly. Use full sentences and plan your presentation before you write. Except in the rare cases where it is indicated otherwise, consider every problem as asking you to prove your result.

## Problem 1

In the “Double Boolean Satisfiability Problem” DOUBLESAT, given a Boolean formula  $\phi$  with  $n$  literals, we want to verify that it has at least 2 satisfying assignments. (I.e., at least two distinct truth-value assignments to its variables for which  $\phi$  is satisfied.) Prove that DOUBLESAT is NP-complete.

**Problem 2**

Let  $BF_k$  denote the set of Boolean formulas in Conjunctive Normal Form such that each variable appears in at most two places (i.e., in at most two literals).

1. Show that the problem of deciding whether a Boolean Formula in  $BF_2$  is satisfiable is in  $P$ .
2. Show that the problem of deciding whether a Boolean Formula in  $BF_2$  is satisfiable is  $NP$  – *Complete*.