Homework 7

Due: Nov. 16, 2021

This homework must be typed in LaTeX and handed in via Gradescope.

Please ensure that your solutions are complete, concise, and communicated clearly. Use full sentences and plan your presentation before you write. Except in the rare cases where it is indicated otherwise, consider every problem as asking you to prove your result.

Problem 1

A **road network** is a mixed graph defined by the roads in a geographic region. Vertices in this graph are defined by road intersections and dead ends, and edges are defined by the portions of roads that connect such vertices. All of the edges are undirected.

- Present an example of a graph for which the tour that visits every node crosses exactly $n-1$ edges and each of such edges is crossed in a single direction.
- Present an example of a graph such that any tour must cross all but one of the edges in the tour in both directions.
- Describe an efficient method for designing a tour of G that starts at some vertex, v, and traverses each edge of G exactly once in each direction (with u-turns allowed). What is the running time of your algorithm?

Problem 2

Let N be a flow network with n vertices and m edges. Show how to compute an augmenting path with the largest residual capacity in $O((n+m)\log(n))$ time.

Problem 3

The city of Irvine, California, allows for residents to own a maximum of three dogs per household without a breeder's license. Imagine you are running an online pet adoption website for the city, as in the previous exercise, but now for n Irvine residents and m puppies. Describe an efficient algorithm for assigning puppies to residents that provides for the maximum number of puppy adoptions possible while satisfying the constraints that each resident will only adopt puppies that he or she likes and that no resident can adopt more than three puppies.