Homework 7

Due: Nov. 10, 2022

This homework must be typed in LaTeX and handed in via Gradescope.

Please ensure that your solutions are complete, concise, and communicated clearly. Use full sentences and plan your presentation before you write. Except in the rare cases where it is indicated otherwise, consider every problem as asking you to prove your result.

Problem 1

Given an arbitrary undirected graph G, applying DFS on a given vertex will create a tree. The tree can be used to detect the separating edges and vertices of the graph in linear time.

(a) Show that the root vertex of a DFS tree is a separating vertex of G if and only if the root vertex has multiple children in the DFS tree.

(b) Show that any non-root vertex v of a DFS tree is a separating vertex of G if and only the exists a child of v, w, such that none of w's descendants in the DFS tree have a back-edge to a proper ancestor of v in the DFS tree.

A **descendant** of a vertex is any vertex reachable from v in the DFS tree.

A **proper ancestor** of a vertex v is a vertex v' such that v is a descendant of v' and $v \neq v'$.

A **child** of a vertex v is a direct descendent of v.

Problem 2

Given a flow network graph G with n nodes and m edges, assume you are given a maximum flow assigment. Propose an algorithm that finds a minimum capacity cut in time $O(m)$.

Problem 3

The city of Irvine, California, allows for residents to own a maximum of three dogs per household without a breeder's license. Imagine you are running an online pet adoption website for the city for n Irvine residents and m puppies.

Describe an efficient algorithm for assigning puppies to residents that provides for the maximum number of puppy adoptions possible while satisfying the constraints that each resident will only adopt puppies that he or she likes and that no resident can adopt more than **three** puppies.