Homework 1

Due: September 20, 2022 at 14:30 ET

This homework must be typed in LaTeX and handed in via Gradescope.

Please ensure that your solutions are complete, concise, and communicated clearly. Use full sentences and plan your presentation before you write. Except in the rare cases where it is indicated otherwise, consider every problem as asking you to prove your result.

The goal of this first assignment is to test your proof writing skills. While correctness of an argument is paramount, conciseness is also *very* important. In general, a simple and concise proof is *better* than a long verbose one.

Problem 1

Provide a simple proof (no more than 3 lines for each) of whether each of these statements is true or false:

1. $n^{2/3} \in o(n^2)$
2. $10^{1000}n \in O(n \log n)$
3. $5000n \in \omega(n)$

The following problems require you to write simple proofs using the main fundamental proof techniques (i.e., contradiction, induction, counterexample, contraposition, construction). Identify the best proof technique and write a simple proof. Excessively long solutions will be marked down.

Problem 2

Argue whether the next statement is true or false: "Every positive integer is equal to the sum of two integer squares".

Problem 3

Provide a simple proof for the following formula using induction:

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

Problem 4

Let $f : A \longrightarrow B$ and $g : B \longrightarrow C$ be functions. Prove the following statement: "If $g \circ f$ is bijective, then f is injective and g is surjective".

Problem 5

Prove by contraposition that if $5n + 5$ is an odd integer, then n is an even integer.