Midterm 1

Due: October 25, 2022

Reminder: Your name should not appear anywhere on your handin; each individual page of the homework should include your Banner ID only. For your digital submission, each page should include work for only one problem (i.e., make a new page/new pages for each problem).

Collaboration or any kind or use of any third-party resource is strictly **not allowed** on this assignment. The TA staff will only answer questions regarding clarifications on the statement of the problems in the assignment. Questions should be asked via private posts on Ed. Please monitor Ed, as we will post clarifications of frequently asked questions there.

Hand in your solutions by 14:29pm to Gradescope. See the course syllabus for the late policy. Usage of any materials outside of course notes, the course textbook, lecture slides and Ed posts is strictly forbidden.

Problem 1 - 10 points

Let X and Y be two sorted arrays in non-decreasing order, containing n and m elements respectively. Give an $O(\log(\max\{n, m\}))$-time algorithm to find the median of all $n + m$ elements in arrays X and Y.

(a) Provide a succinct (but clear) description of your algorithm. You may provide pseudocode.

(b) Provide a proof of the correctness of the algorithm.

(c) Analyze the running time and memory utilization of the algorithm.

Problem 2 - 10 points

Provide an efficient greedy algorithm that, given a set $\{x_1, x_2, \ldots, x_n\}$ of points on the real line, determines the smallest set of closed intervals of length three that contains all of the given points.

(a) Provide a succinct (but clear) description of your algorithm. You may provide pseudocode.

(b) Prove the correctness (optimality) of your algorithm.

(c) Analyze the running time and memory utilization of the algorithm.

Problem 3 - 10 points

You are given an $n \times m$ matrix with non-negative integer values. Count the number of distinct paths to reach the last cell, $(n-1,\ m-1)$ of the matrix from its first cell, $(0,0)$, such that the path has total cost C. The cost of a path is the sum of the entries of the cells in the path. You can only move one unit right, one unit down, or one unit right-down diagonal from any cell, i.e., from cell (i,j), we can move to $(i,j+1)$ or $(i+1,j)$ or $(i+1,j+1)$. Two paths are distinct if they have at least one different step.

(a) Provide a succinct (but clear) description of your algorithm. The running time of your algorithm should not exceed $O(nmC)$. You may provide pseudocode.

(b) Prove the correctness of your algorithm.

(c) Provide an analysis of the running time and memory utilization of the algorithm.

Problem 4 -10 points

Given an n-bit binary integer, design a divide-and-conquer algorithm to convert it into its decimal representation. For simplicity, you may assume that n is a power of 2.

(a) Provide a succinct (but clear) description of your algorithm, including pseudocode.

(b) Prove the correctness of your algorithm.

(c) Analyze the running time of your algorithm. Assume that it is possible to multiply two decimal integers numbers with at most m digits in $O(m^{\log_2 3})$ time.

Hint: An n-bit binary integer x can be expressed as $x = (x_{n-1}, x_{n-2}, \cdots, x_1, x_0)_2$ where $x_i \in \{0, 1\}$. Let $x_\ell = (x_{n/2-1}, x_{n/2-2}, \cdots, x_1, x_0)_2$ be the $(n/2)$-bit binary integer corresponding to the $(n/2)$ least significant digits of x. Let $x_m = (x_{n-1}, x_{n-2}, \cdots, x_{n/2+1}, x_{n/2})_2$ be the $(n/2)$-bit binary integer representing the $(n/2)$ most significant digits of x. Then, $x = x_\ell + 2^{n/2} \cdot x_m$. This should suggest us a way to set up a divide and conquer strategy… :) Careful about the number of subproblems!