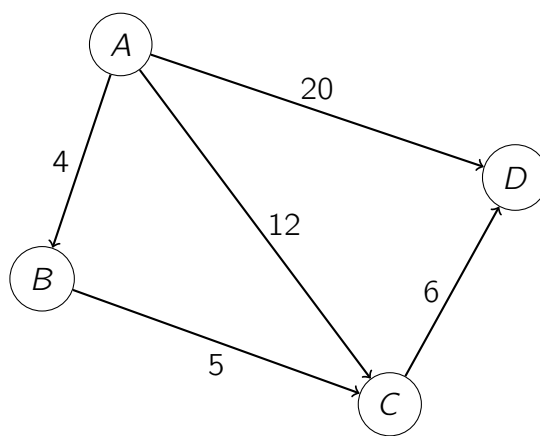


## 1 Algorithm Practice

Given the directed graph below, run the Floyd-Warshall Algorithm, processing vertices in alphabetical order. Fill in the table below which keeps track of the shortest paths. Ordered pairs of vertices with no directed path (such as  $(B, A)$ ) are omitted and their distance can be taken as  $\infty$  for updates.



$(u, v)$	$(A, A)$	$(A, B)$	$(A, C)$	$(A, D)$	$(B, B)$	$(B, C)$	$(B, D)$	$(C, C)$	$(C, D)$	$(D, D)$
$D^{(0)}$	0	4	12	20	0	5	$\infty$	0	6	0
$D^{(1)}$										
$D^{(2)}$										
$D^{(3)}$										
$D^{(4)}$										

## 2 Knapsack

Consider an instance of the knapsack problem with five items (where there is only one copy of each item):

Item	Value	Size
1	1	1
2	2	3
3	3	2
4	4	5
5	5	4

and knapsack capacity  $C = 9$ .

What are the final array entries of the Knapsack algorithm from lecture, and which items belong to the optimal solution?

### 3 Encoding

Suppose we encode lowercase letters into a numeric string as follows: we encode  $a$  as 1,  $b$  as 2,  $\dots$ , and  $z$  as 26. Given a numeric string  $S$  of length  $n$ , develop an  $O(n)$  algorithm to find how many letter strings this can correspond to. For example, for the numeric string 123, the algorithm should output 3 because the letter strings that map to this numeric string are  $abc$ ,  $lc$ , and  $aw$ .

### 4 Knight Moves

Given an  $8 \times 8$  chessboard and a knight that starts at position  $a1$ , devise an algorithm that returns how many ways the knight can end up at position  $xy$  after  $k$  moves. Knights move  $\pm 1$  squares in one direction and  $\pm 2$  squares in the other direction. In other words, knights move in a pattern similar to a "L".

Note: on a chessboard, rows are labeled from 1-8 and columns are labeled from  $a - h$ .