University of Pittsburgh

Applied Cryptography and Network Security
CS 1653

Summer 2023

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(Slides are adapted from Prof. Adam Lee's CS1653 slides.)

Announcements

- Homework 2 due this Friday @ 11:59 pm
  - 3 attempts
- Phase 1 of Project due next Tuesday @ 11:59 pm

Example: Advanced Encryption Standard (AES)

Develop a (brief) history of modern cryptography

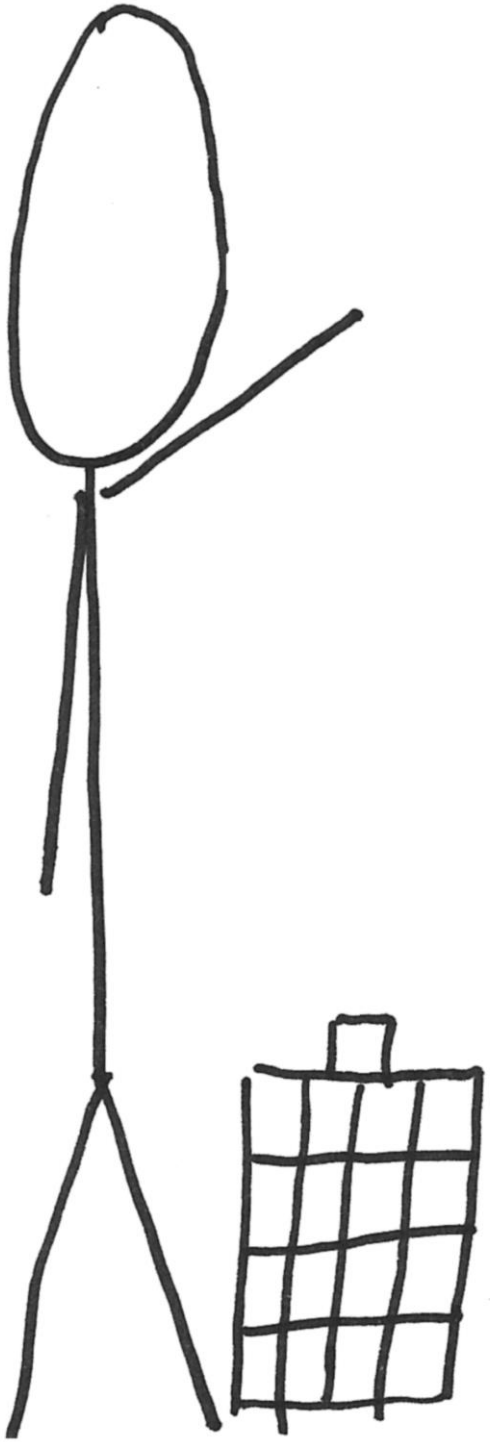
Learn about the AES standardization process

Understand AES:

-  High level: Appreciate how AES utilizes confusion/diffusion
- **Low level: Gain exposure to the mathematics behind AES**

Appreciate that implementing something as complex as AES is a very non-trivial task

A Stick Figure Guide to the Advanced Encryption Standard (AES)



http://www.moserware.com/

In the early 70s, Lucifer was created at IBM

Lucifer became DES

- Slightly modified
  - Shorter key and new, (not so) suspicious S-boxes
- standardized by NSA/NIST*

Over the years, DES suffered many breaks and brute forces

- 1977 Diffie/Hellman propose $20M machine to find key in a day
- 1993 Wiener proposed $1M machine, 7 hours
- 1997 RSA Security held contest, distributed DESCHALL broke DES
- 1998 EFF built Deep Crack, $250,000, about 2 days

1998, 3DES: **More DES!**

- Encrypt three times with DES with different keys
- Oh man, this is so slow…

Even by the late 80s, replacements started to appear

Summary of last lecture …

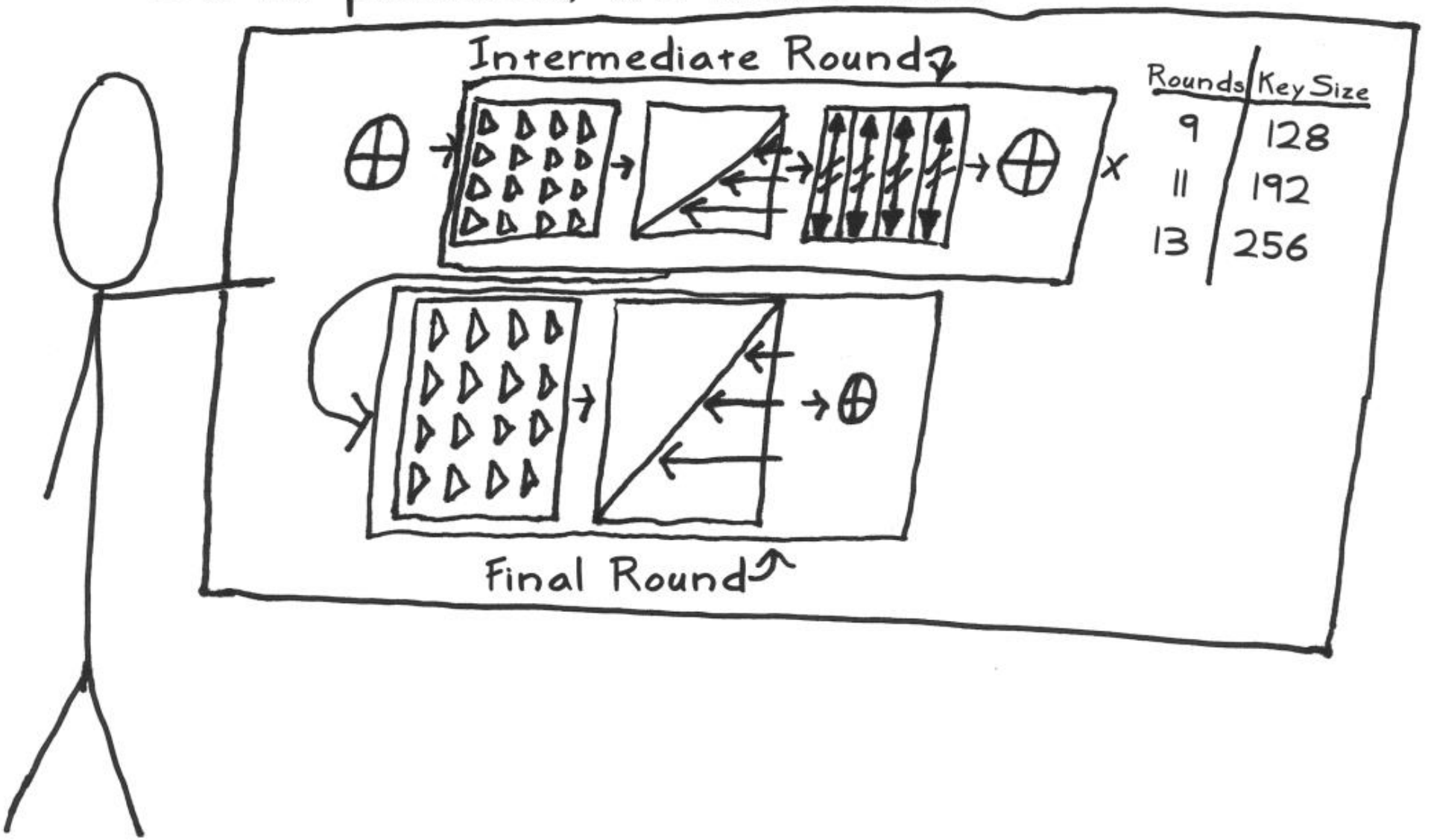
In theory, you now understand how 128-bit AES works

High level: Apply **confusion** and **diffusion** to 128-bit blocks

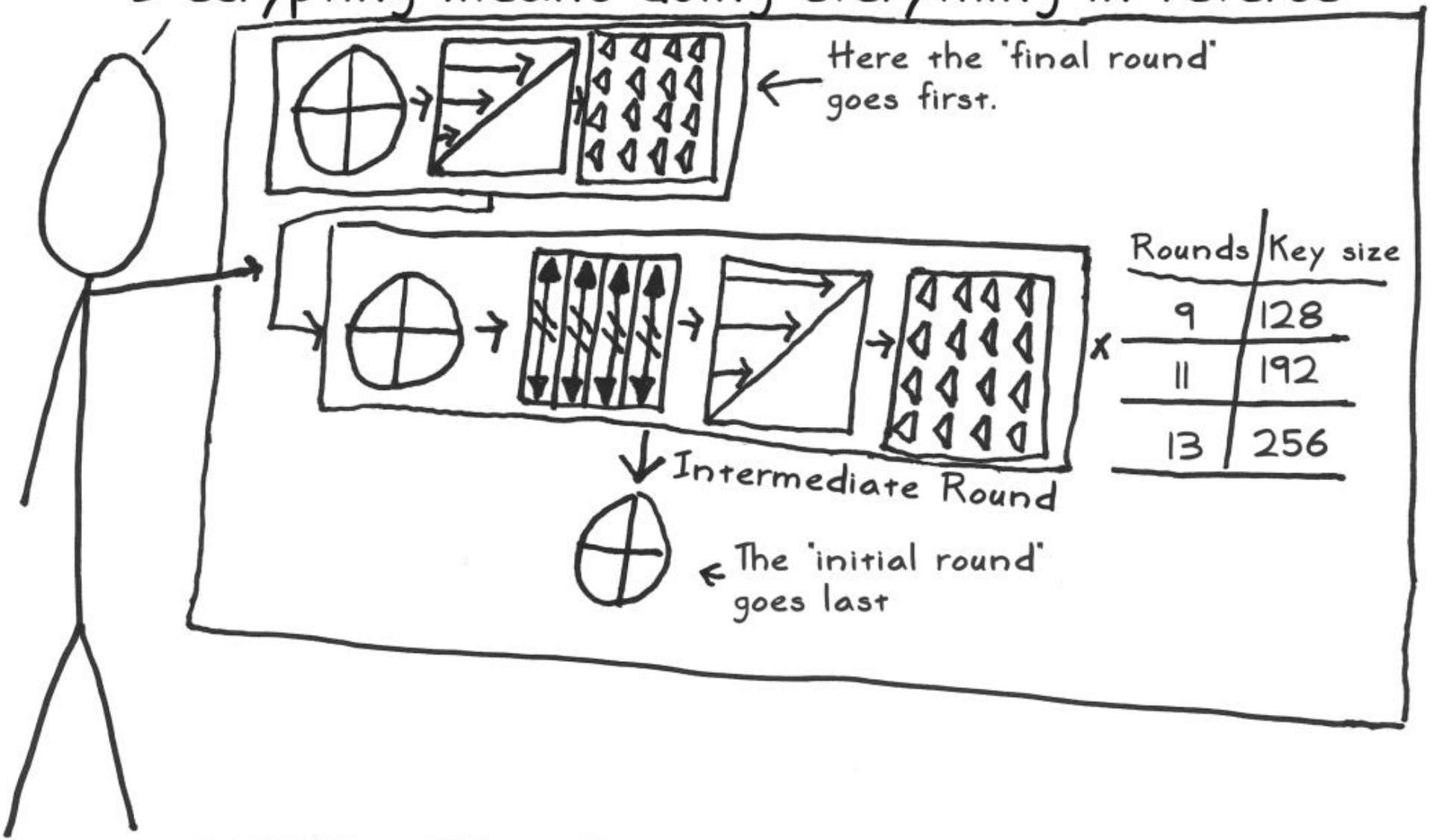
Medium level:

- Key expansion to get 10 round keys
- SubBytes via some crazy S-Box
- ShiftRows to spread around the bytes
- MixColumns to further spread around the bytes
- XOR with round key, and repeat

So in pictures, we have this:



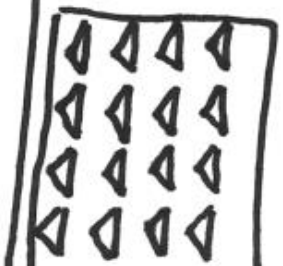
Decrypting means doing everything in reverse



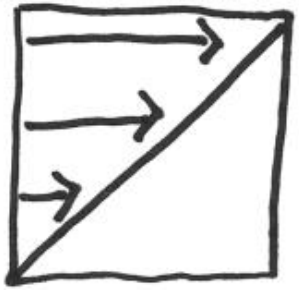
Rounds	Key size
9	128
11	192
13	256

Add Round Key Inverse

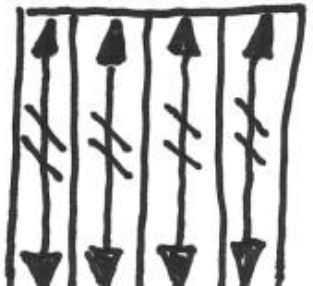
Inverse Substitute Bytes

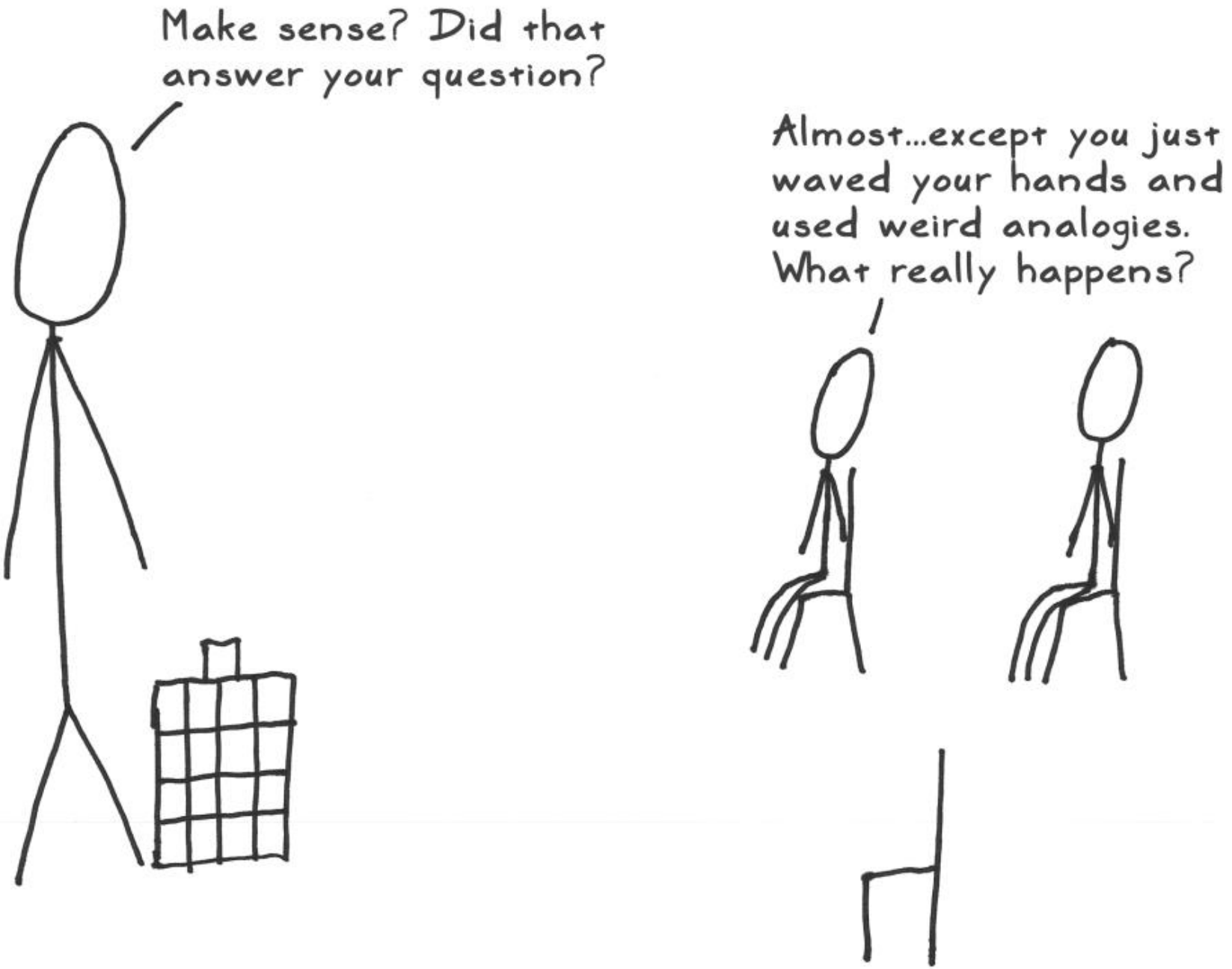


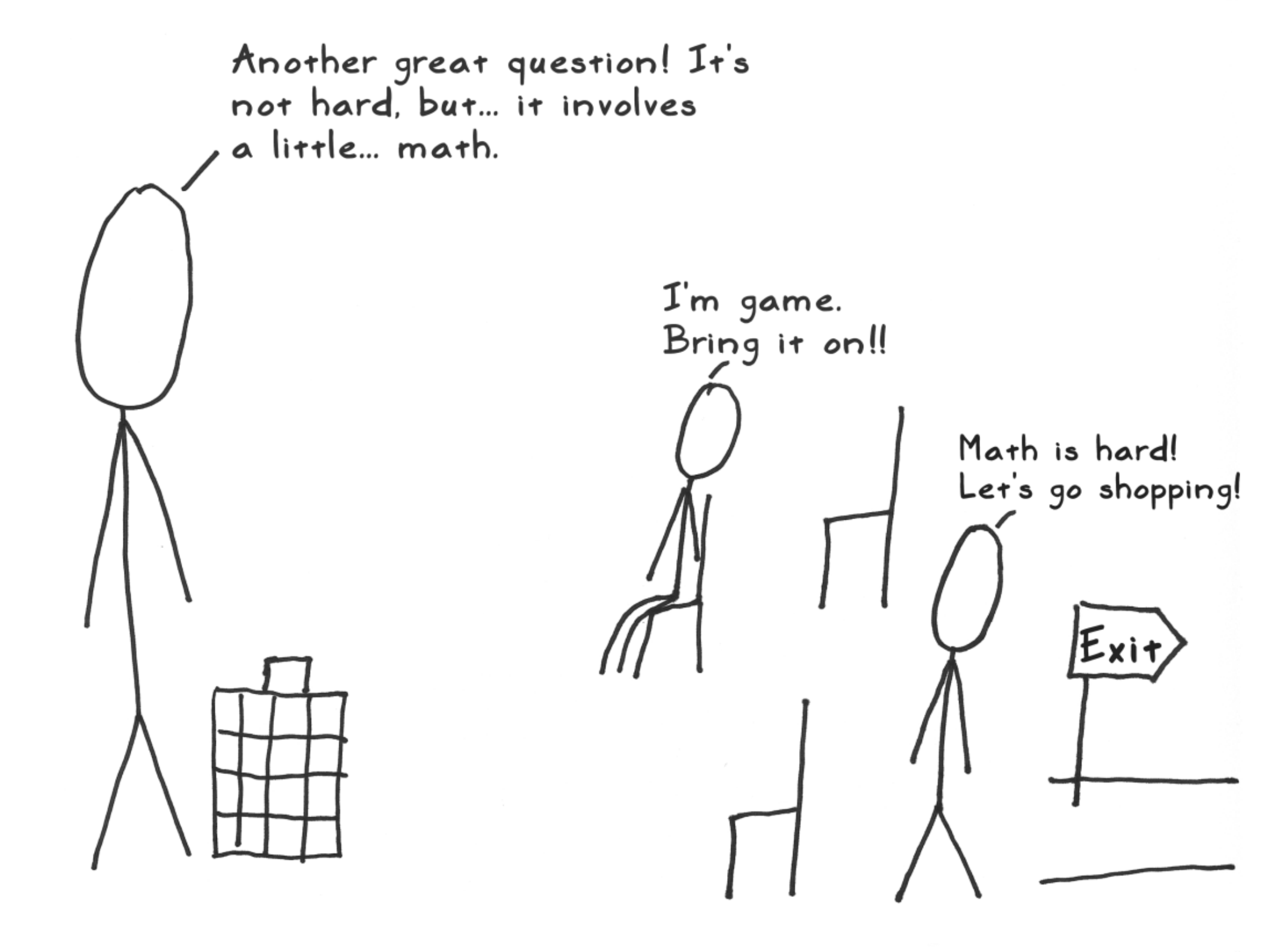
Inverse Shift Rows



Inverse Mix Columns

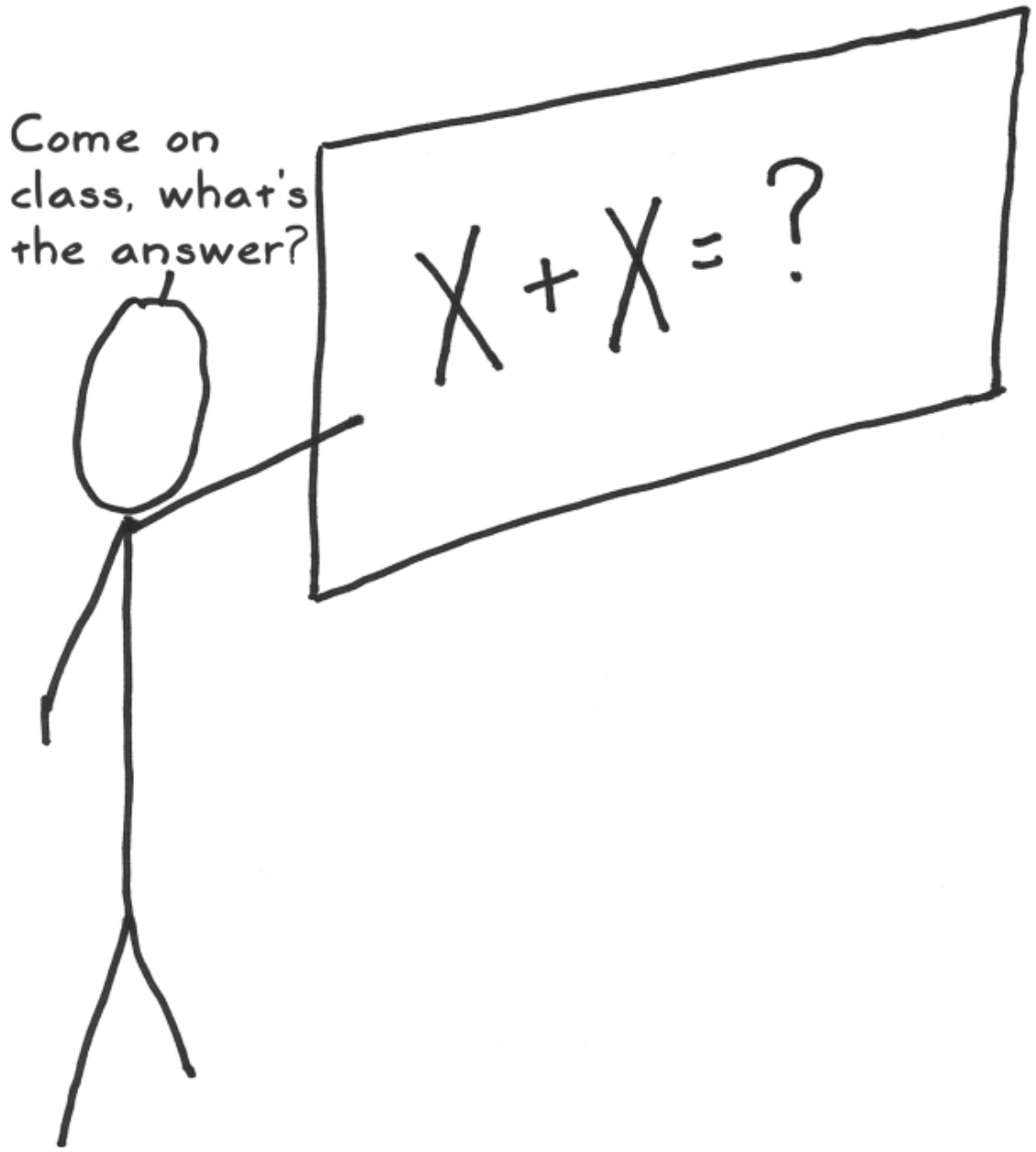


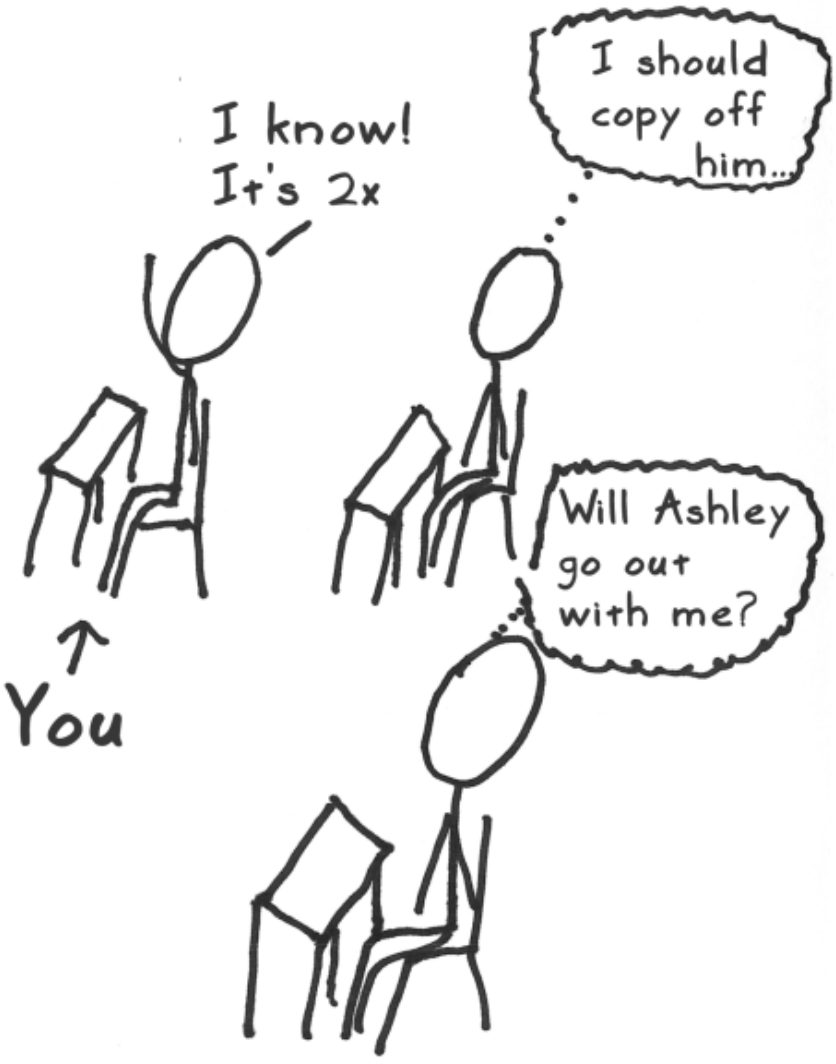

Make sense? Did that answer your question?
Almost...except you just waved your hands and used weird analogies. What really happens?


Another great question! It's not hard, but... it involves a little... math.
I'm game. Bring it on!!
Math is hard! Let's go shopping!
Exit

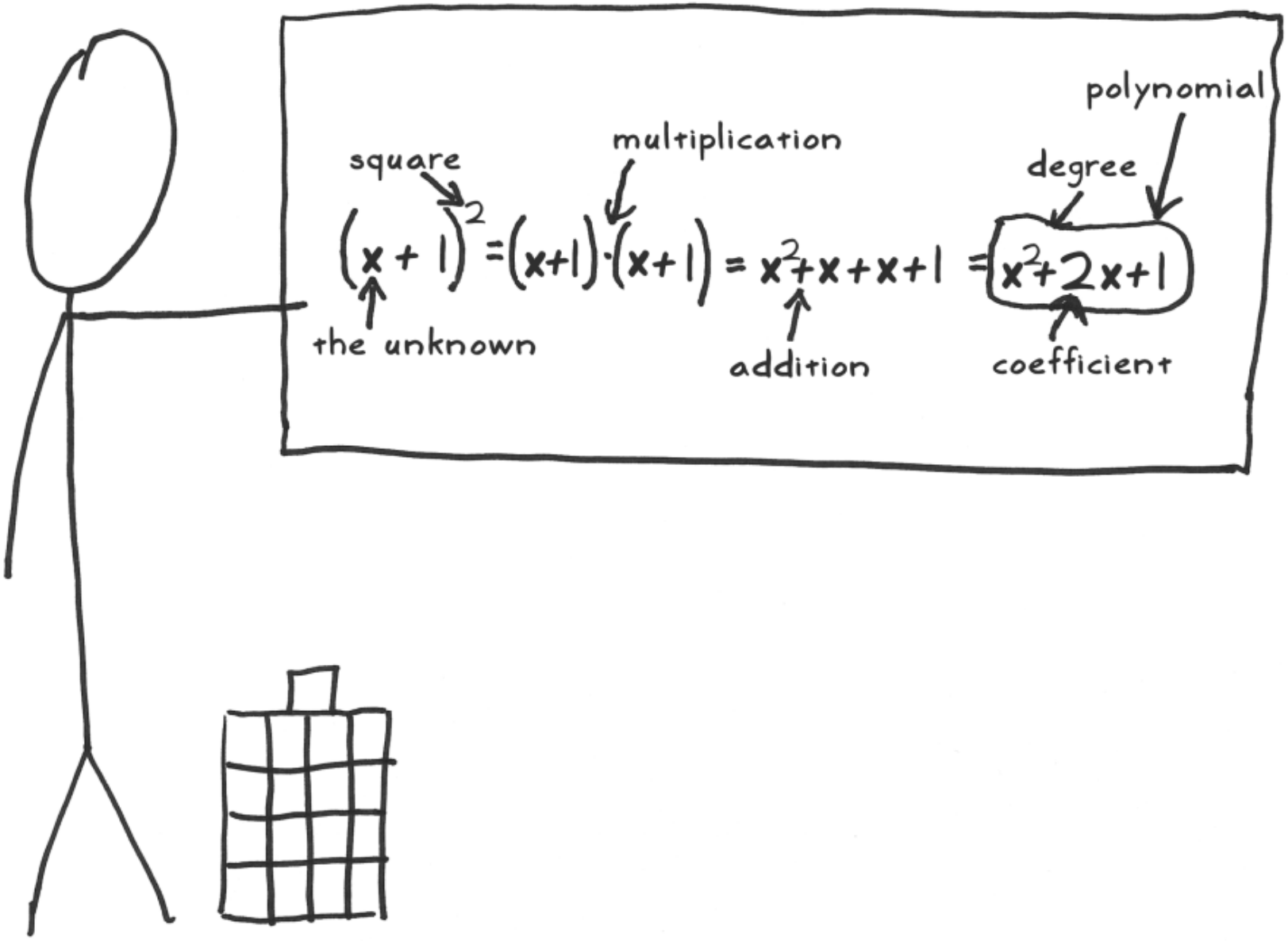
Act 4: Math!

Let's go back to your algebra class...

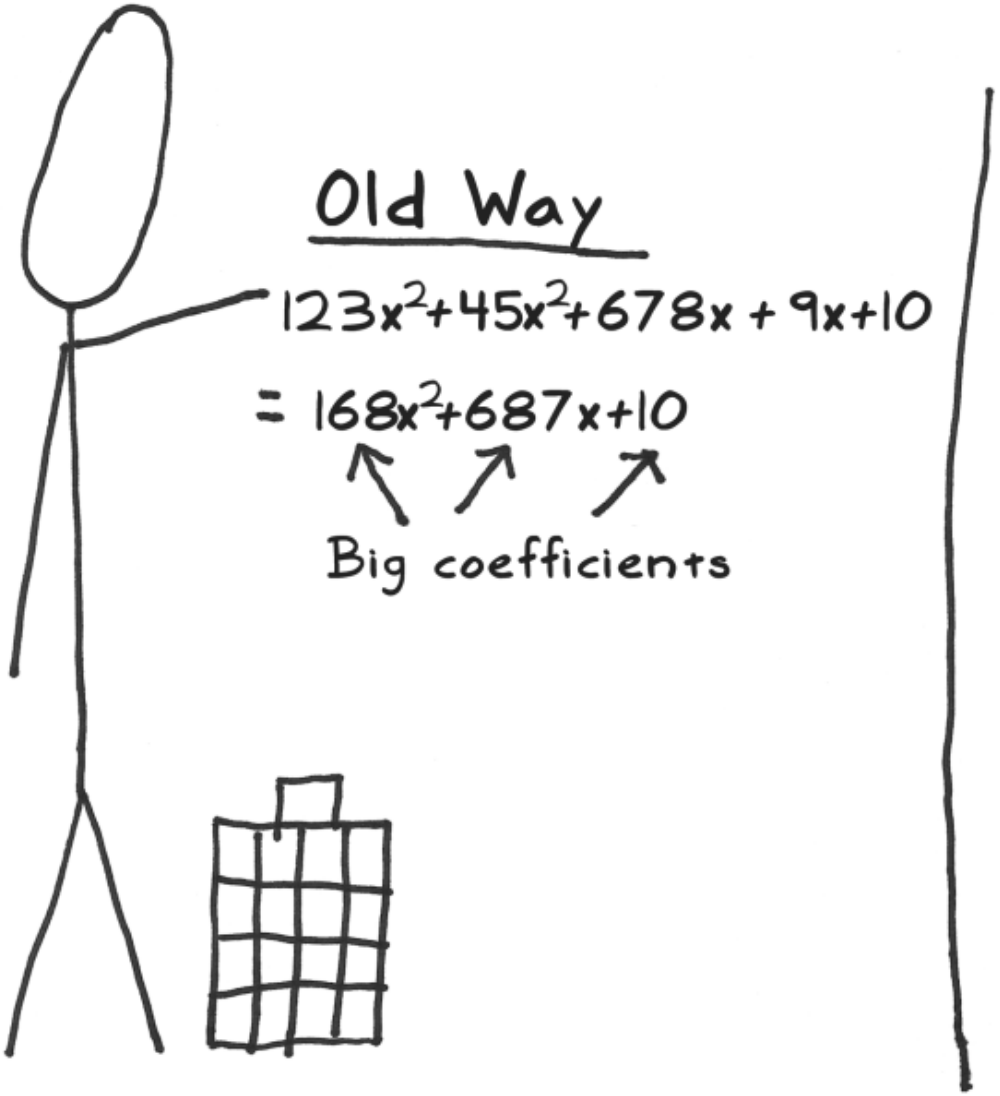




Reviewing the Basics...



We'll change things slightly. In the old way, coefficients could get as big as we wanted. In the new way, they can only be 0 or 1:



Old Way

$123x^2+45x^2+678x+9x+10$

$= 168x^2+687x+10$

Big coefficients

New Way

$x^2 \oplus x^2 \oplus x^2 \oplus x \oplus x \oplus 1$

$= x^2 \oplus 1$

The 'new' add*

Small coefficients

$x^2 \oplus x^2 \oplus x^2 = (x^2 \oplus x^2) \oplus x^2$

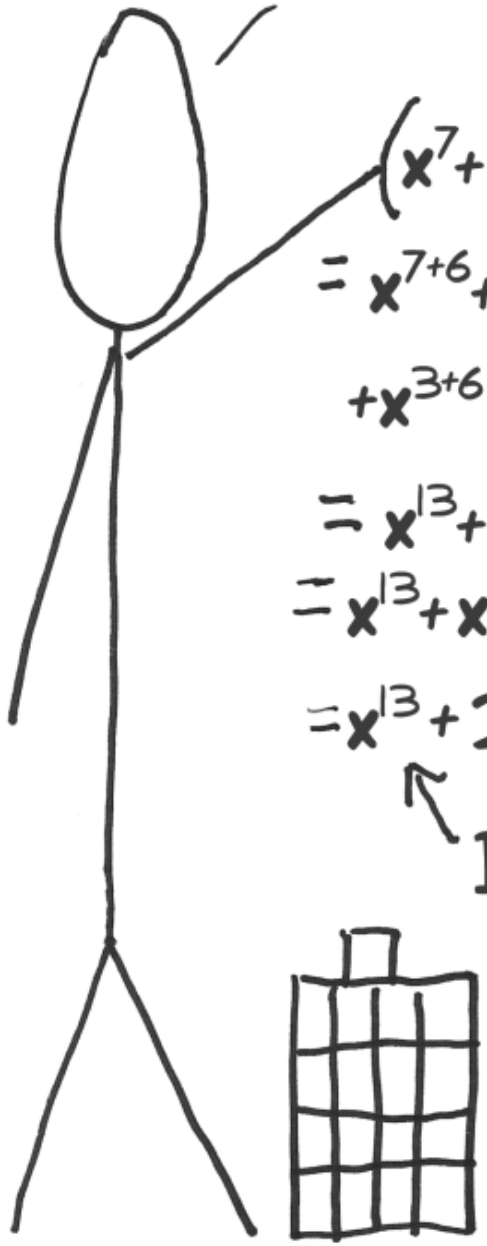
$= 0 \oplus x^2$

$= x^2$

i.e., Coefficients drawn from Z_2. You need material from CS0441 again!

*Nifty Fact: In the new way, addition is the same as subtraction (e.g. $x \oplus x = x - x = 0$)

Remember how multiplication could make things grow fast?



$$(x^7+x^5+x^3+x)\cdot(x^6+x^4+x^2+1)$$

$$= x^{7+6}+x^{7+4}+x^{7+2}+x^{7+0}+x^{5+6}+x^{5+4}+x^{5+2}+x^{5+0}$$

$$+x^{3+6}+x^{3+4}+x^{3+2}+x^{3+0}+x^{1+6}+x^{1+4}+x^{1+2}+x^{1+0}$$

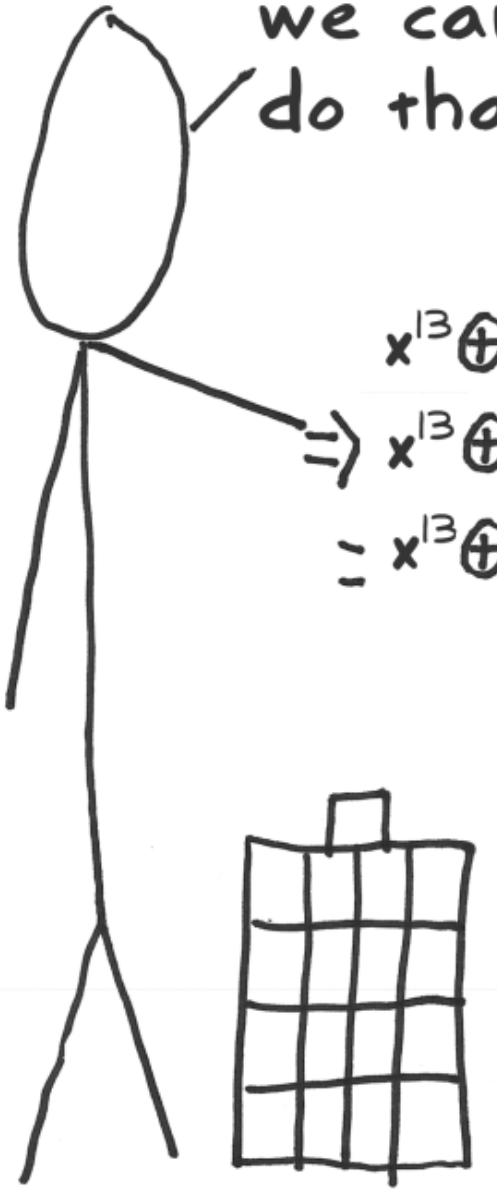
$$= x^{13}+x^{11}+x^9+x^7+x^{11}+x^9+x^7+x^5+x^9+x^7+x^5+x^3+x^7+x^5+x^3+x$$

$$= x^{13}+x^{11}+x^{11}+x^9+x^9+x^9+x^7+x^7+x^7+x^7+x^5+x^5+x^5+x^3+x^3+x$$

$$= x^{13}+2x^{11}+3x^9+4x^7+3x^5+2x^3+x$$

Big and yucky!

With the "new" addition, things are simpler, but the x^{13} is still too big. Let's make it so we can't go bigger than x^7. How can we do that?



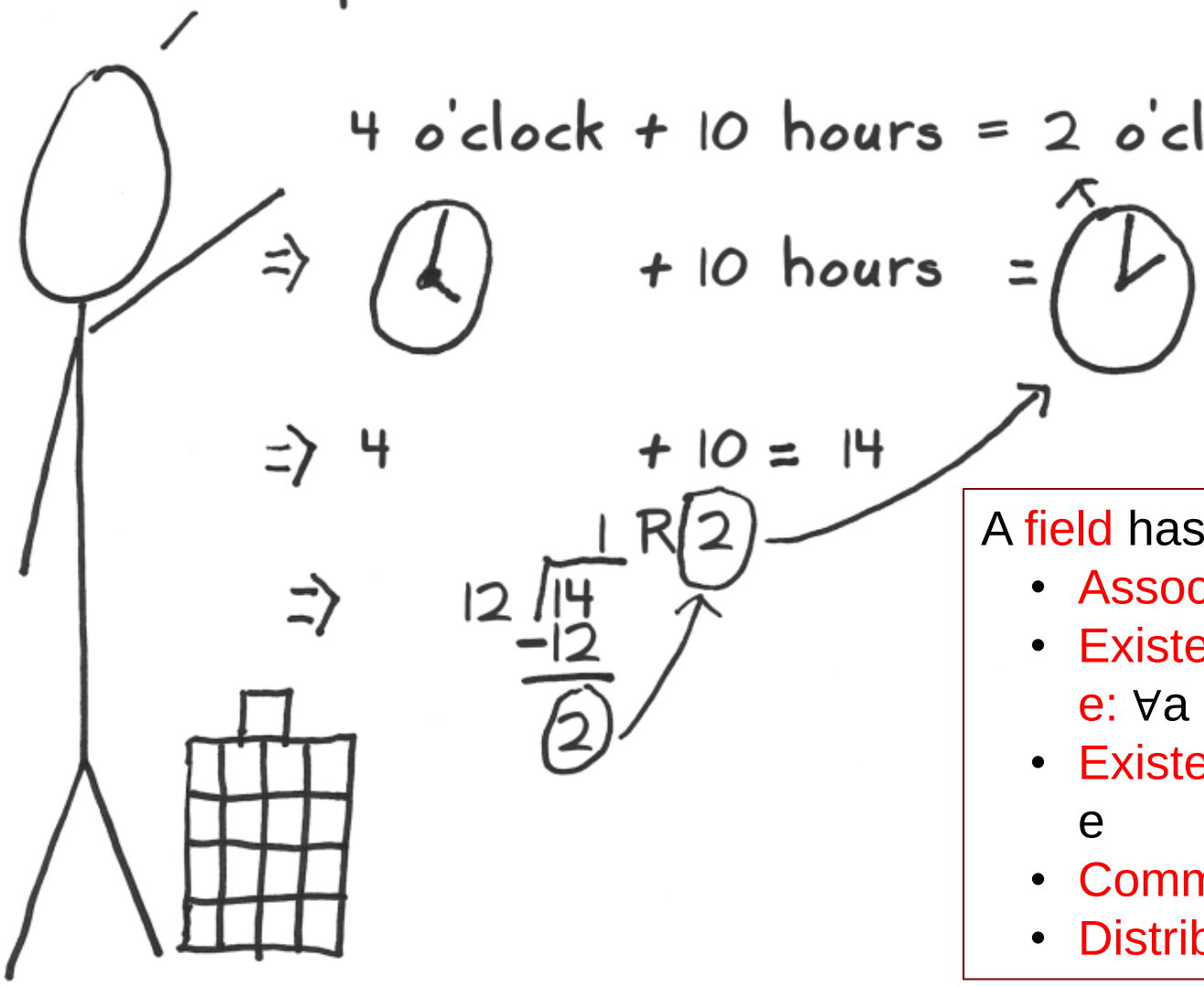
$$x^{13} \oplus 2x^{11} \oplus 3x^9 \oplus 4x^7 \oplus 3x^5 \oplus 2x^3 \oplus x$$

$$\Rightarrow x^{13} \oplus 0x^{11} \oplus x^9 \oplus 0x^7 \oplus x^5 \oplus 0x^3 \oplus x$$

$$= x^{13} \oplus x^9 \oplus x^5 \oplus x$$

Question: How might we accomplish this?

We use our friend, "clock math*," to do this. Just add things up and do long division. Keep a close watch on the remainder:



A field has the following properties:

- Associativity: (ab)c = a(bc)
- Existence of an identity element e: $\forall a : ae = a$
- Existence of inverses: $\forall a : aa^{-1} = e$
- Commutativity: ab = ba
- Distributivity: a(b+c) = ab + bc

* This is also known as "modular addition." Math geeks call this a "group." AES uses a special group called a "finite field."

We can do "clock" math with polynomials. Instead of dividing by 12, my creators told me to use $m(x) = x^8 \oplus x^4 \oplus x^3 \oplus x \oplus 1$. Let's say we wanted to multiply $x \cdot b(x)$ where $b(x)$ has coefficients $b_7 \ldots b_0$:

$$x \cdot b(x)$$

$$= x \cdot (b_7 x^7 \oplus b_6 x^6 \oplus b_5 x^5 \oplus b_4 x^4 \oplus b_3 x^3 \oplus b_2 x^2 \oplus b_1 x \oplus b_0)$$

$$= b_7 x^8 \oplus b_6 x^7 \oplus b_5 x^6 \oplus b_4 x^5 \oplus b_3 x^4 \oplus b_2 x^3 \oplus b_1 x^2 \oplus b_0 x)$$

Eeek! x^8 is too big. We must make it smaller.

Note: Rather than reducing elements of our field modulo an integer, we will reduce them modulo another polynomial. This polynomial is irreducible, which means it's kind of like a prime number (i.e., it has no other factors).

* Remember that each b_n (e.g. b_7) is either 0 or 1.

We divide it by $m(x) = x^8 \oplus x^4 \oplus x^3 \oplus x \oplus 1$ and take the remainder:

$$
\begin{array}{r|l}
 & b_7 \\
\hline
x^8 \oplus x^4 \oplus x^3 \oplus x \oplus 1 & b_7x^8 \oplus b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus b_3x^4 \oplus b_2x^3 \oplus b_1x^2 \oplus b_0x \\
\oplus & b_7x^8 \qquad\qquad\qquad\qquad \oplus b_7x^4 \oplus b_7x^3 \oplus b_7x \oplus b_7 \\
\hline
 & b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus (b_3 \oplus b_7)x^4 \oplus (b_2 \oplus b_7)x^3 \\
 & \qquad\qquad \oplus b_1x^2 \oplus (b_0 \oplus b_7)x \oplus b_7
\end{array}
$$

$m(x)$

Remainder

$$b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus b_3x^4 \oplus b_2x^3 \oplus b_1x^2 \oplus b_0x$$

$$\oplus b_7 \cdot (x^4 \oplus x^3 \oplus x \oplus 1)$$

Note how the b's are shifted left by 1 spot.

This is just b_7 multiplied by a small polynomial.

Now we're ready for the hardest blast from the past: logarithms. After logarithms, everything else is cake! Logarithms let us turn multiplication into addition:

$$\log(x \cdot y) = \log(x) + \log(y)$$

So... $\log(10 \cdot 100) = \log(10^1) + \log(10^2)$

$= 2 + 1 = 3$

In reverse:

$\log^{-1}(1) = 10^1 = 10$

$\log^{-1}(2) = 10^2 = 100$

$\log^{-1}(3) = 10^3 = 1{,}000$

$\Rightarrow 10 \cdot 100 = 1{,}000$

We can use logarithms in our new world. Instead of using 10 as the base, we can use the simple polynomial of $x \oplus 1$ and watch the magic unravel.*

$$(x \oplus 1)^1 = x \oplus 1$$

$$(x \oplus 1)^2 = (x \oplus 1)(x \oplus 1) = x^2 \oplus x \oplus x \oplus 1 = x^2 \oplus 1$$

$$(x \oplus 1)^3 = (x \oplus 1)^2 \cdot (x \oplus 1) = x^3 \oplus x^2 \oplus x \oplus 1$$

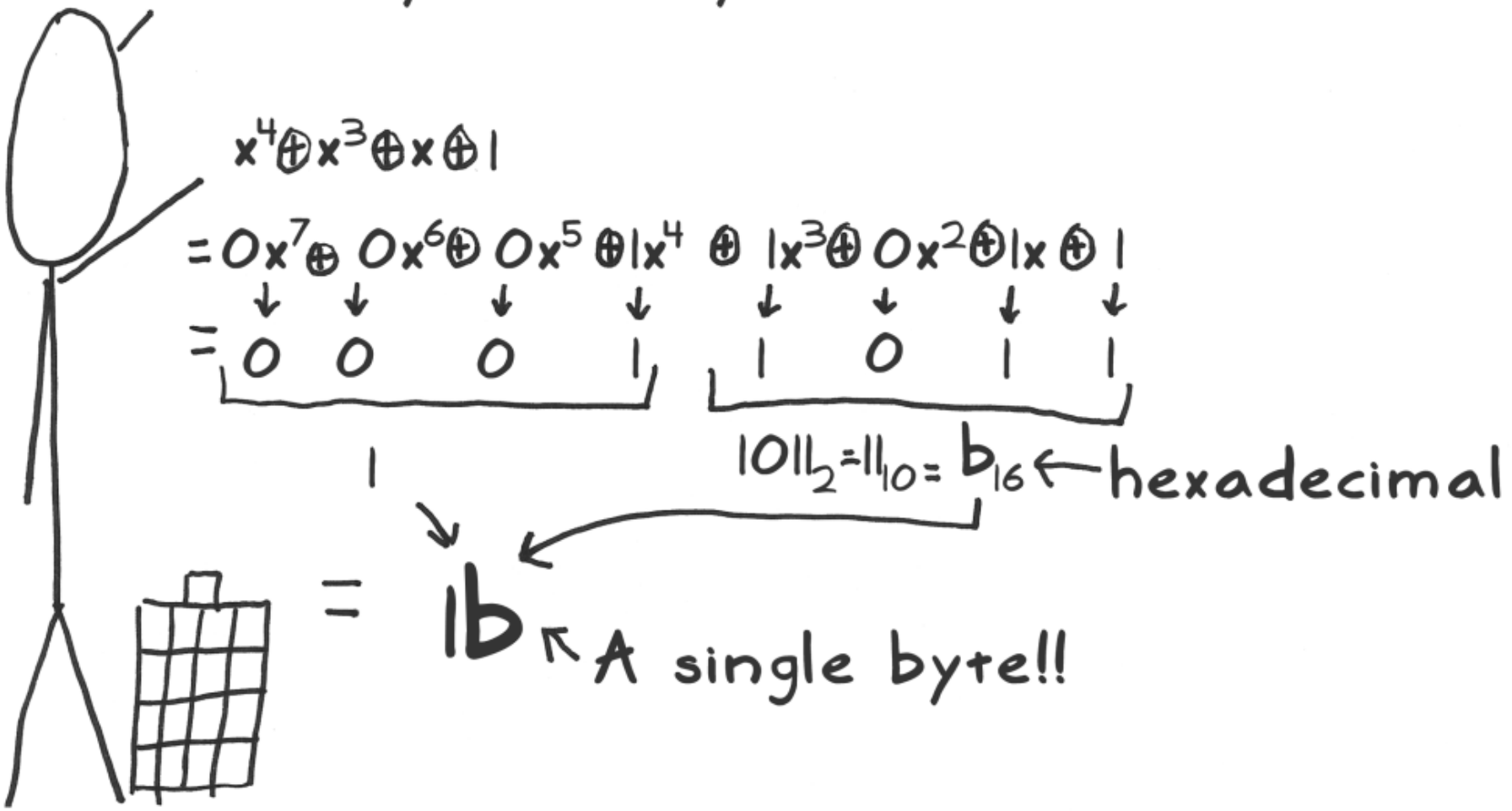
So...

$$\log_{x \oplus 1}(x \oplus 1) = 1, \log_{x \oplus 1}(x^2 \oplus 1) = 2, \log_{x \oplus 1}(x^3 \oplus x^2 \oplus x \oplus 1) = 3$$

The polynomial $(x \oplus 1)$ is called a <u>generator</u> of our field $GF(2^8)$

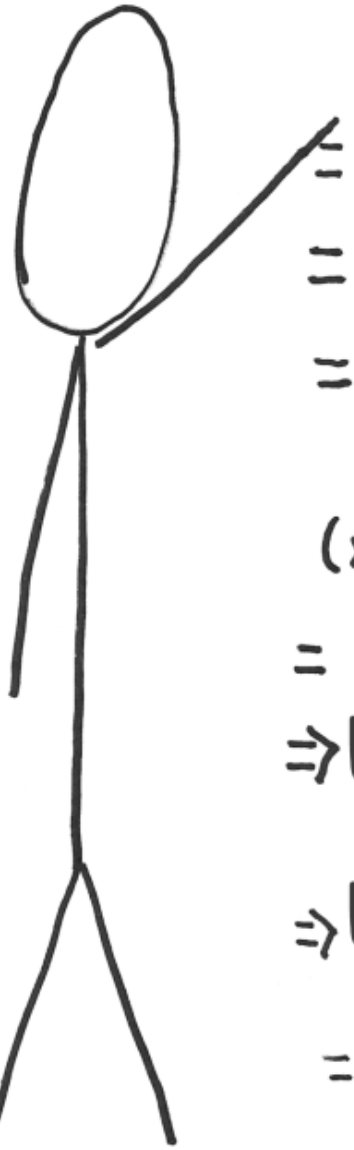
*If you keep multiplying by $(x \oplus 1)$ and then take the remainder after dividing by $m(x)$, you'll see that you generate all possible polynomial below x^8. This is very important!

Why bother with all of this math?* Encryption deals with bits and bytes, right? Well, there's one last connection: a 7^{th} degree polynomial can be represented in exactly 1 byte since the new way uses only 0 or 1 for coefficients:



*Although we'll work with bytes from now on, the math makes sure everything works out.

With bytes, polynomial addition becomes a simple xor. We can use our logarithm skills to make a table for speedy multiplication.*



$$(x^4 \oplus x^3 \oplus x \oplus 1) \oplus (x^7 \oplus x^5 \oplus x^3 \oplus x)$$

$$= 1b \oplus aa \quad \leftarrow \text{byte xor}$$

$$= b1$$

$$= x^7 \oplus x^5 \oplus x^4 \oplus 1$$

$$(x^4 \oplus x^3 \oplus x \oplus 1) \cdot (x^7 \oplus x^5 \oplus x^3 \oplus x)$$

$$= 1b \cdot aa \quad \text{logarithm table lookup}$$

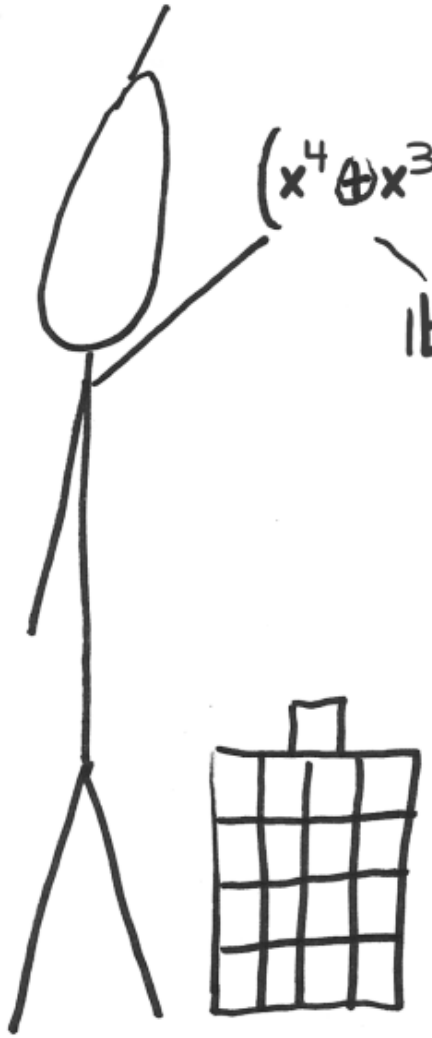
$$\Rightarrow \log(1b) + \log(aa) = c8 + 1f = e7$$

$$\Rightarrow \log^{-1}(e7) = 8c \Rightarrow 1b \cdot aa \quad \text{inverse table lookup}$$

$$= x^7 \oplus x^3 \oplus x^2$$

* We can create the table as we keep multiplying by $(x \oplus 1)$.

Since we know how to multiply, we can find the "inverse" polynomial byte for each byte. This is the byte that will undo/invert the polynomial back to 1. There are only 255* of them, so we can use brute force to find them:



$(x^4 \oplus x^3 \oplus x \oplus 1) \cdot ? = 1$

$1b \cdot cc = 1$

↖ found using a brute force for-loop

* There are only 255 instead of 256 because 0 has no inverse.

Now we can understand the mysterious s-box. It takes a byte "a" and applies two functions. The first is "g" which just finds the byte inverse. The second is "f" which intentionally makes the math uglier to foil attackers.

$$g(a) = a^{-1}$$

$$f(a) = \begin{bmatrix} 1&1&1&1&1&0&0&0 \\ 0&1&1&1&1&1&0&0 \\ 0&0&1&1&1&1&1&0 \\ 0&0&0&1&1&1&1&1 \\ 1&0&0&0&1&1&1&1 \\ 1&1&0&0&0&1&1&1 \\ 1&1&1&0&0&0&1&1 \\ 1&1&1&1&0&0&0&1 \end{bmatrix} \cdot \begin{bmatrix} a_7 \\ a_6 \\ a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Why build our S-box using properties of the inverse over $GF(2^8)$? It is known to have good non-linearity properties. This makes cryptanalysis harder!

sbox[a] = f(g(a))

sbox[58] = f(g(58))

sbox[58] = f(18) = 6a

58·18=01

Translation

Linear transformation

The S-Box, Redux

		y															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
x	0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
	1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
	2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
	3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
	4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
	5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
	6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
	7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
	8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
	9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
	a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
	b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
	c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
	d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
	e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
	f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

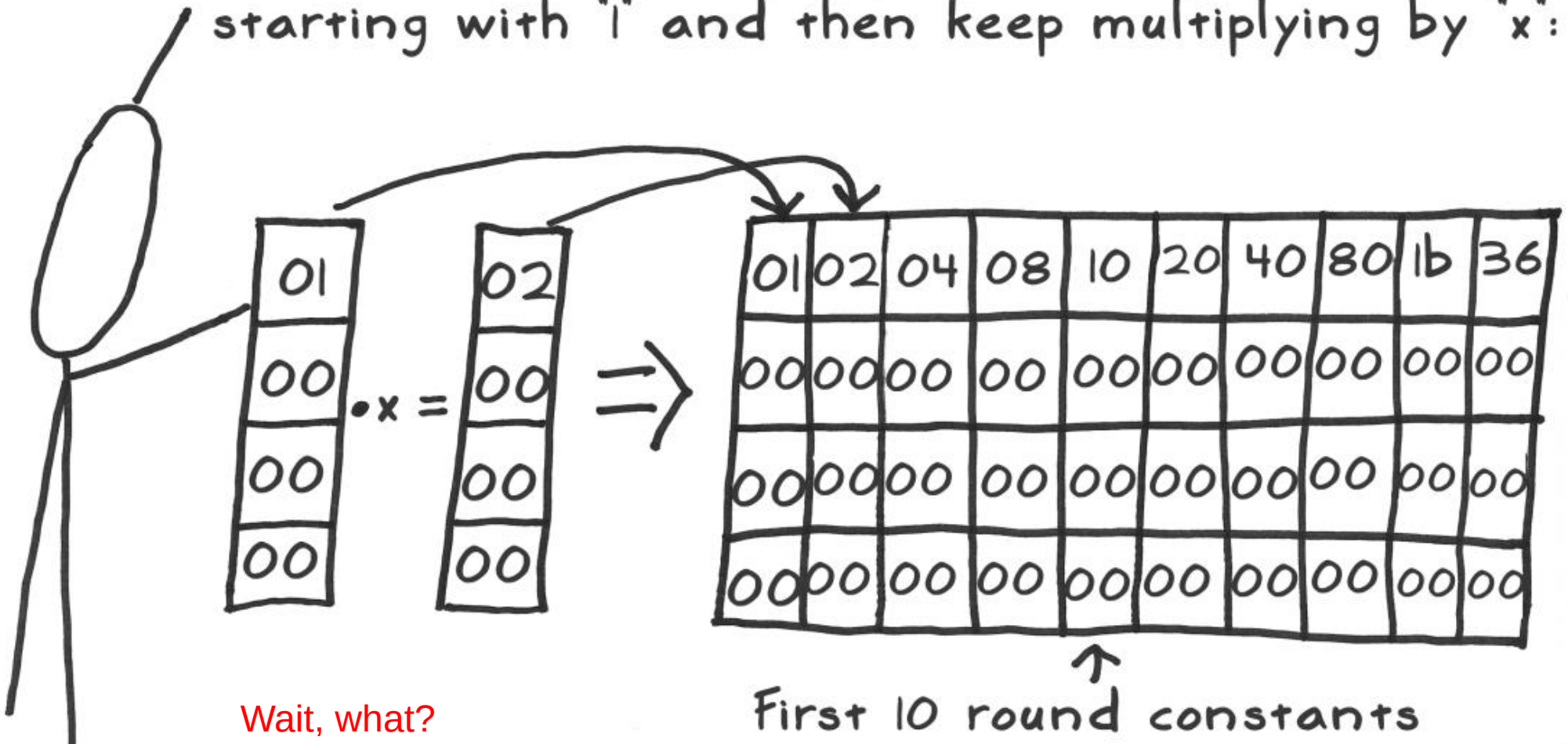
Recall: sbox[58] = f(g(58))

sbox[58] = f(18) = 6a

58·18=01

Doing the lookup is way faster than doing the actual math!

We can also understand those crazy round constants in the key expansion. I get them by starting with "1" and then keep multiplying by "x":



Wait, what?

$1 = 0x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 + 0x^2 + 0x + 1 = 0000\ 0001 = 01$

$1 \cdot x = x = 0x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 + 0x^2 + 1x + 0 = 0000\ 0010 = 02$

$x \cdot x = x^2 = 0x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 + 1x^2 + 0x + 0 = 0000\ 0100 = 04$

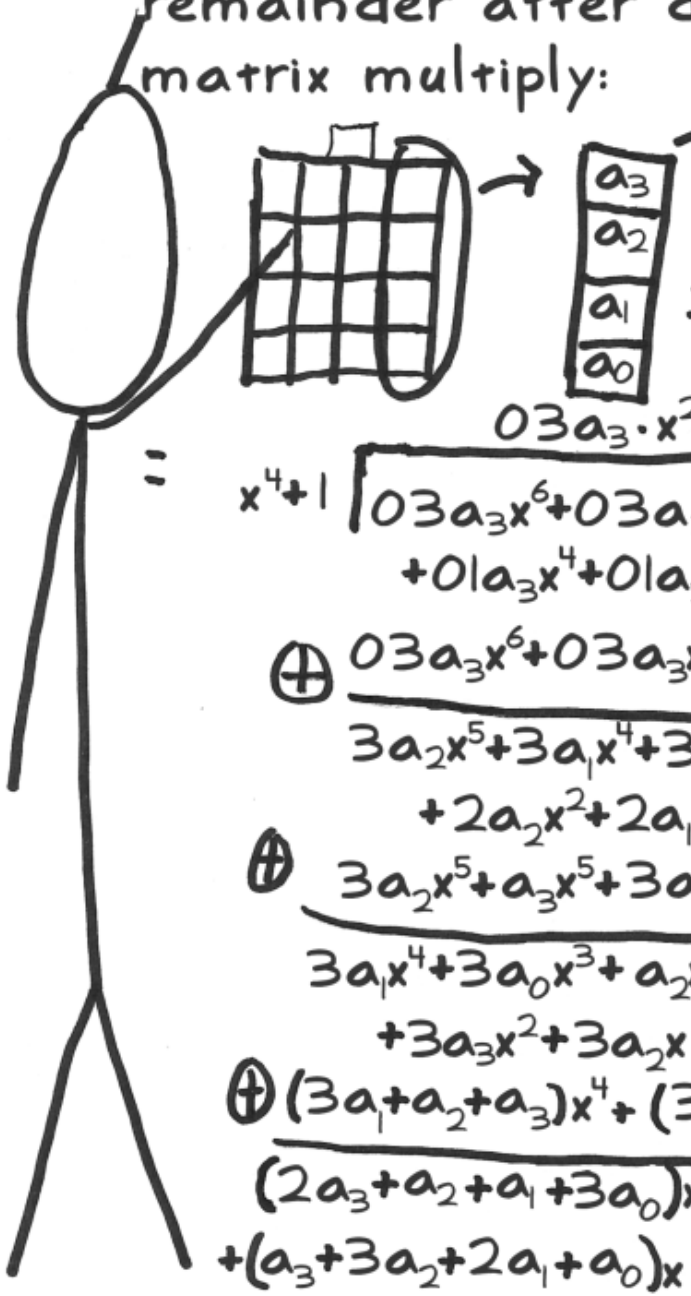
$x^2 \cdot x = x^3 = 0x^7 + 0x^6 + 0x^5 + 0x^4 + 1x^3 + 0x^2 + 0x + 0 = 0000\ 1000 = 08$

…

Observation: We're just left rotating here…

Mix Columns is the hardest. I treat each column as a polynomial. I then use our new multiply method to multiply it by a specially crafted polynomial and then take the remainder after dividing by x^4+1. This all simplifies to a matrix multiply:

i.e., Table lookups and XORs



$$b(x) = c(x) \cdot a(x) \bmod x^4+1$$

$$= (03x^3+01x^2+01x+02) \cdot (a_3x^3+a_2x^2+a_1x+a_0) \bmod x^4+1$$

$(03x^3+01x^2+01x+02)$: special polynomial

$(a_3x^3+a_2x^2+a_1x+a_0)$: the column

$$= x^4+1 \;\Big|\; \begin{array}{l} 03a_3 \cdot x^2+(3a_2+a_3)x+(3a_1+a_2+a_3) \\ \hline 03a_3x^6+03a_2x^5+03a_1x^4+03a_0x^3+01a_3x^5+01a_2x^4+01a_1x^3+01a_0x^2 \\ \quad +01a_3x^4+01a_2x^3+01a_1x^2+01a_0x+02a_3x^3+02a_2x^2+02a_1x+02a_0 \end{array}$$

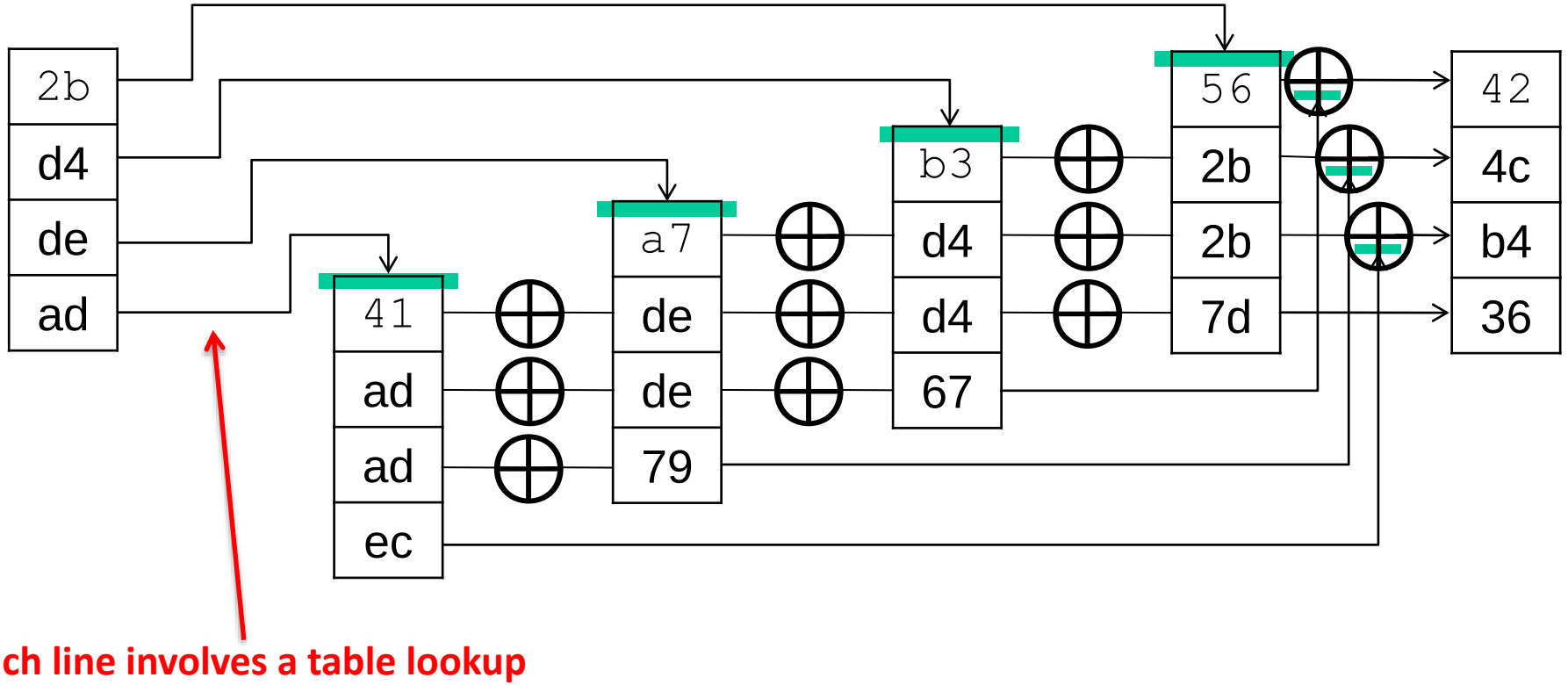
$$\oplus \; \frac{03a_3x^6+03a_3x^2}{3a_2x^5+3a_1x^4+3a_0x^3+a_3x^5+a_2x^4+a_1x^3+a_0x^2+a_3x^4+a_2x^3+a_1x^2+a_0x+2a_3x^3 \\ +2a_2x^2+2a_1x+2a_0+3a_3x^2}$$

$$\oplus \; \frac{3a_2x^5+a_3x^5+3a_2x+a_3x}{3a_1x^4+3a_0x^3+a_2x^4+a_1x^3+a_0x^2+a_3x^4+a_2x^3+a_1x^2+a_0x+2a_3x^3+2a_2x^2+2a_1x+2a_0 \\ +3a_3x^2+3a_2x+a_3x}$$

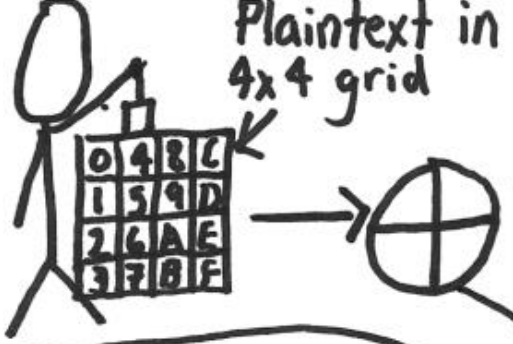
$$\oplus \; \frac{(3a_1+a_2+a_3)x^4+(3a_1+a_2+a_3)}{(2a_3+a_2+a_1+3a_0)x^3+(3a_3+2a_2+a_1+a_0)x^2 \\ +(a_3+3a_2+2a_1+a_0)x+(a_3+a_2+3a_1+2a_0)}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} b_3 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix}$$

MixColumns is essentially table lookups and XORs



AES Crib Sheet
(Handy for memorizing)

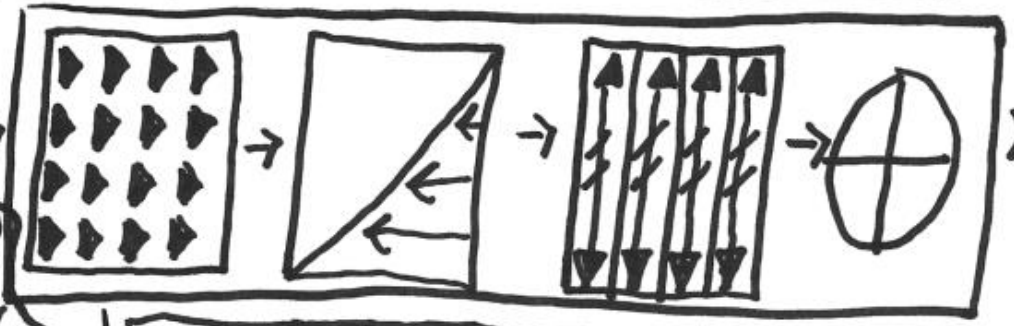


Initial Round

Intermediate Rounds

#	Key
9	128
11	192
13	256

X



General Math

11B = AES Polynomial = m(x)

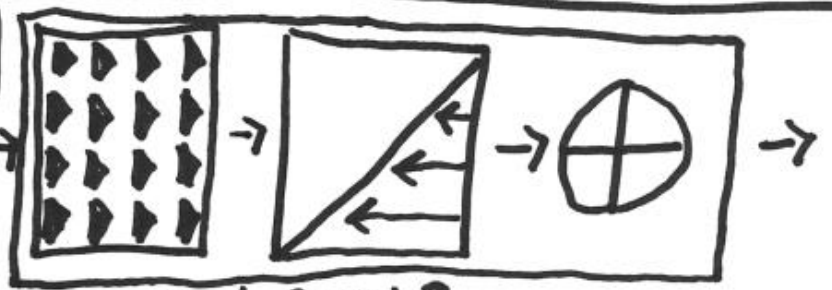
$x^8 + x^4 + x^3 + x + 1$

Fast Multiply

$x \cdot a(x) = (a << 1) \oplus (a_7 == 1)\ ?\ 1B : 00$

$\log(x \cdot y) = \log(x) + \log(y)$

Use $(x+1) = 03$ for log base



Final Round

Ciphertext

S-Box (SRD)

SRD[a] = f(g(a))

$g(a) = a^{-1} \bmod m(x)$

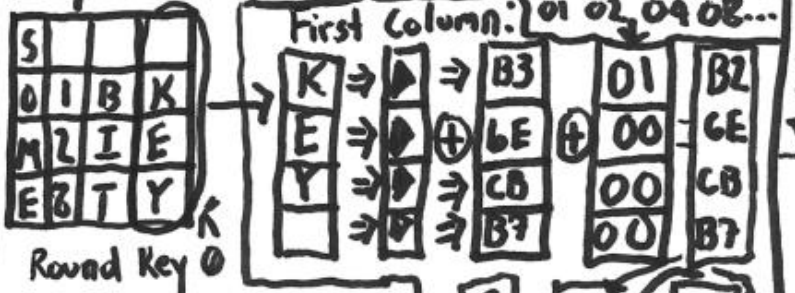
f(a) Think $\vec{5}3 \oplus 63^T$

5 1's and 3 0's $[0110\ 0011]^T$

$$\begin{bmatrix} 1&1&1&1&1&0&0&0 \\ 0&1&1&1&1&1&0&0 \\ 0&0&1&1&1&1&1&0 \\ 0&0&0&1&1&1&1&1 \\ 1&0&0&0&1&1&1&1 \\ 1&1&0&0&0&1&1&1 \\ 1&1&1&0&0&0&1&1 \\ 1&1&1&1&0&0&0&1 \end{bmatrix} \cdot \begin{bmatrix} a_7 \\ a_6 \\ a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Key Expansion:

Round Constants: 01 02 04 08 ...



Other Columns:

Prev Col ⊕ Col from Previous round key

Mix Columns:

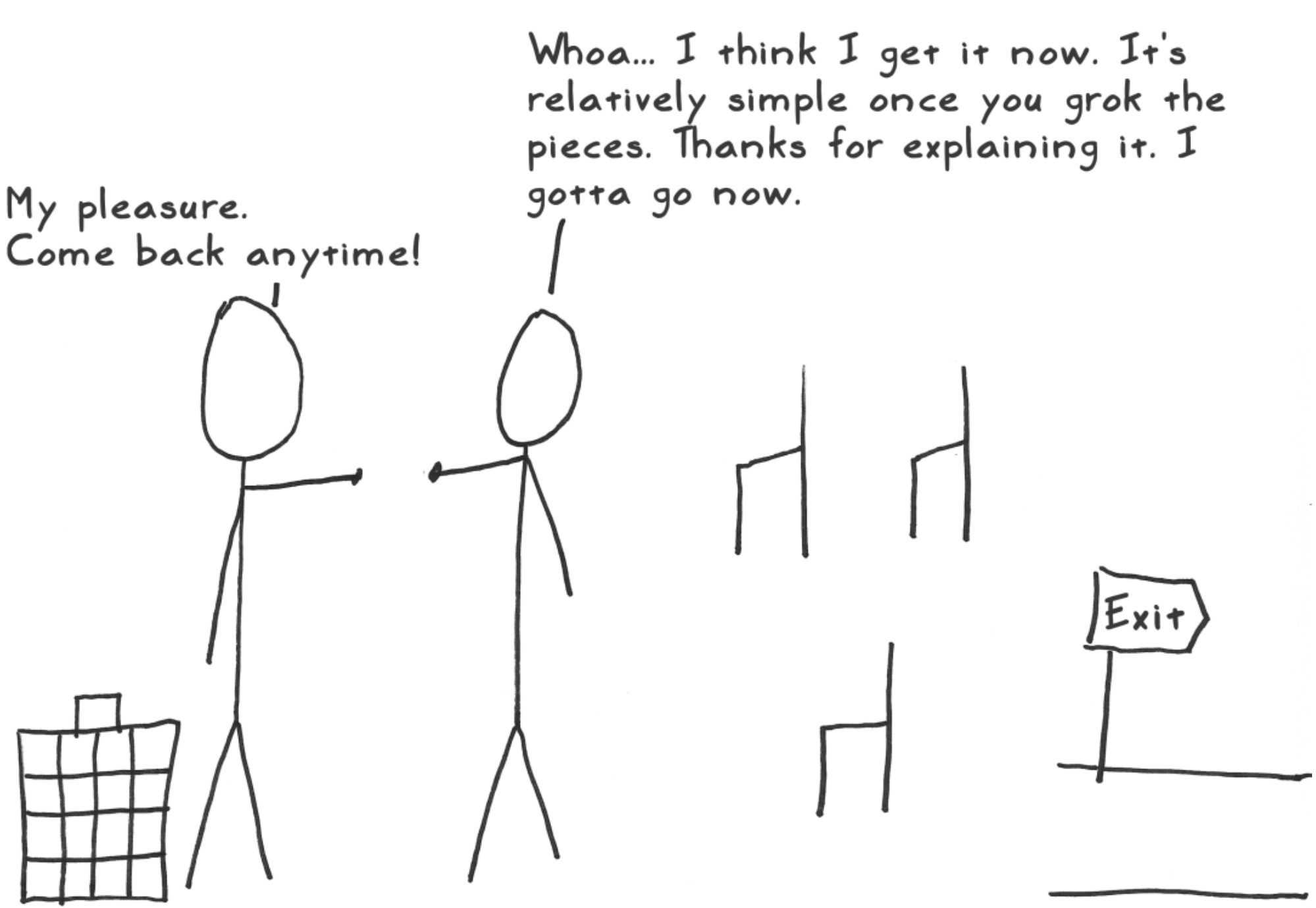
2113

$$\begin{bmatrix} 2&1&1&3 \\ 3&2&1&1 \\ 1&3&2&1 \\ 1&1&3&2 \end{bmatrix} \cdot \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

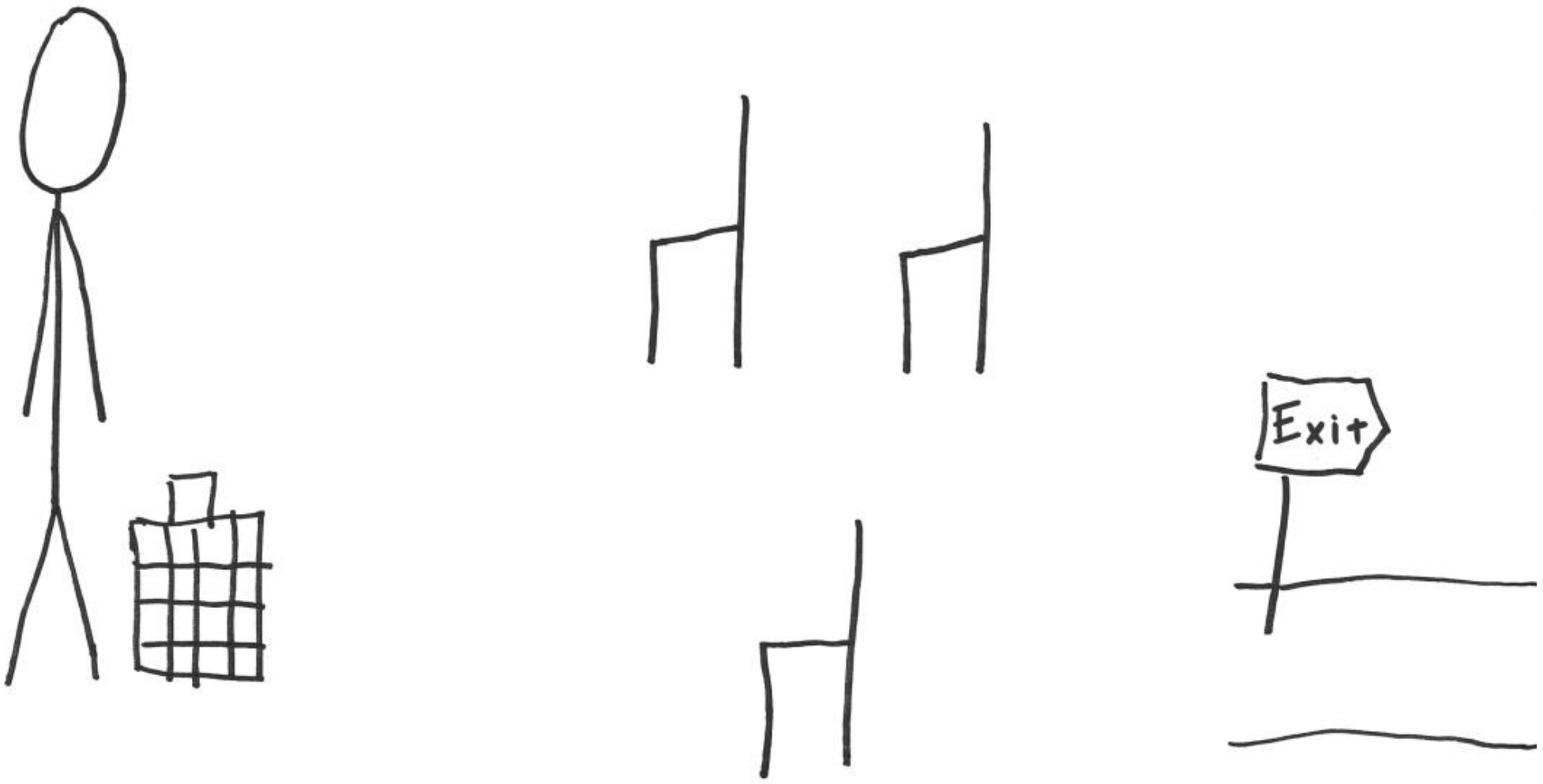
Inverse Mix

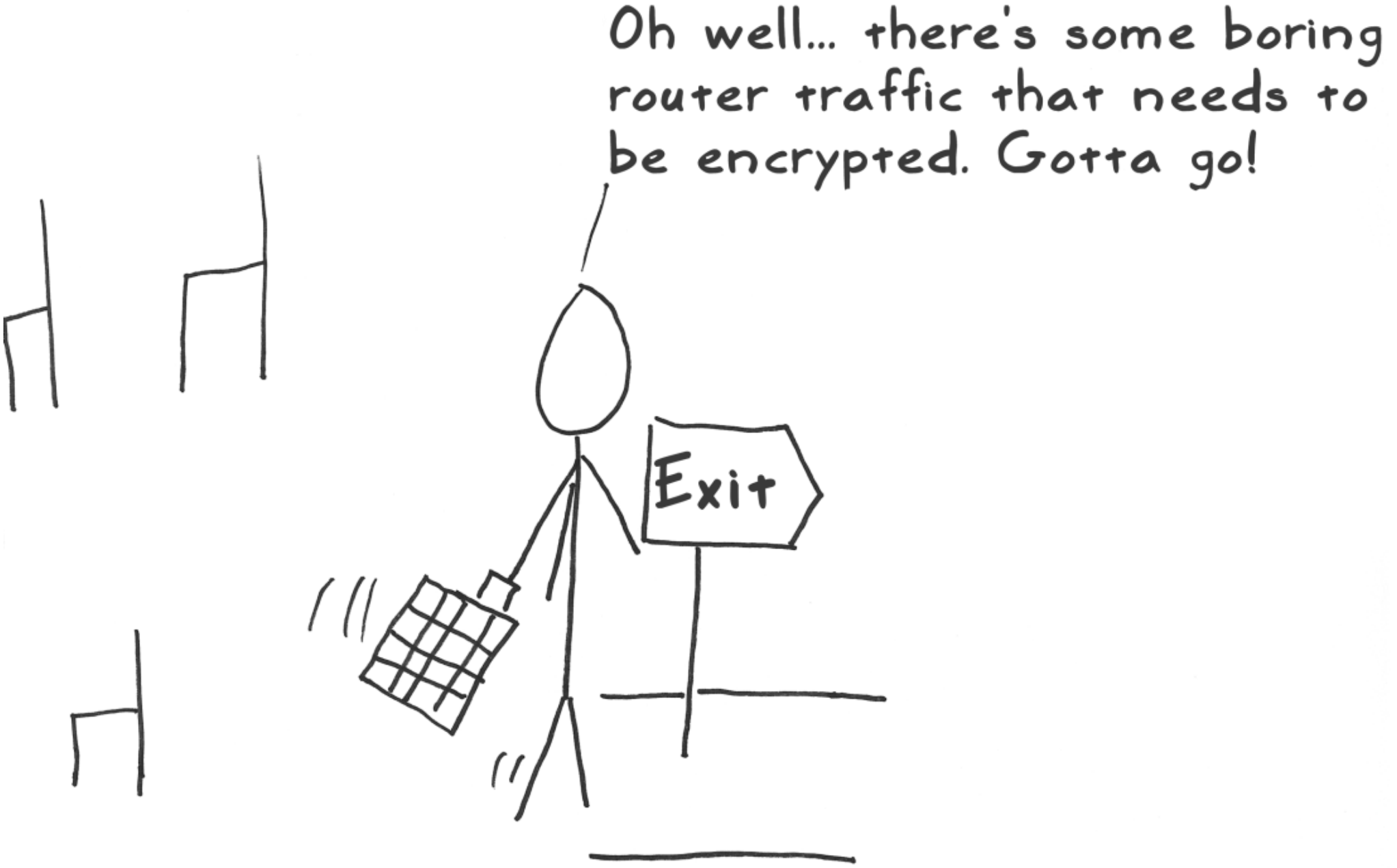
EBD9

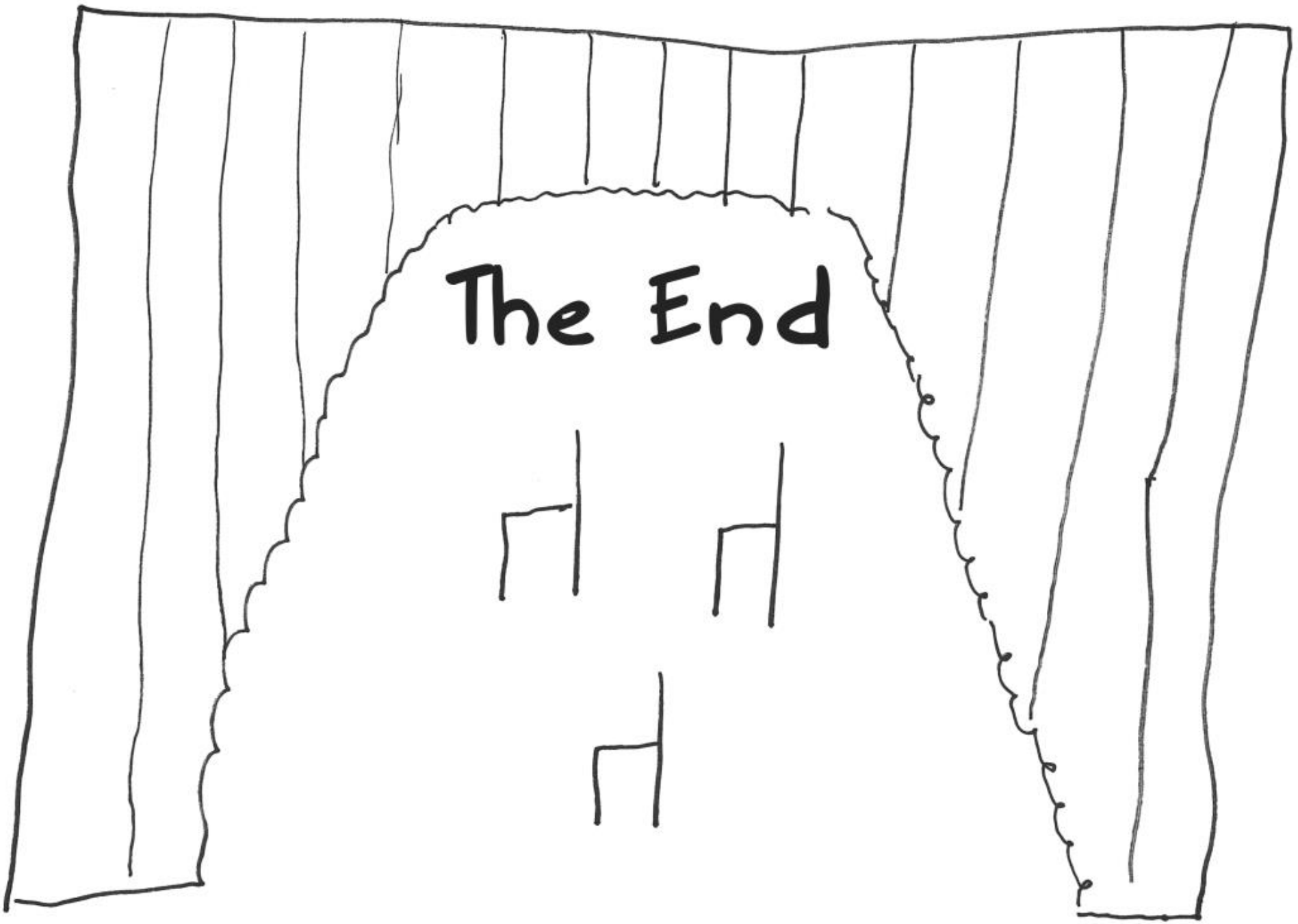
$$\begin{bmatrix} E&B&D&9 \\ 9&E&B&D \\ D&9&E&B \\ B&D&9&B \end{bmatrix} \cdot \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$


Whoa... I think I get it now. It's relatively simple once you grok the pieces. Thanks for explaining it. I gotta go now.
My pleasure.
Come back anytime!
Exit

But there's so much more to talk about: my resistance to linear and differential cryptanalysis, my Wide Trail Strategy, impractical related-key attacks, and... so much more... but no one is left.




Oh well... there's some boring router traffic that needs to be encrypted. Gotta go!
Exit


The End

Final Thoughts…

In theory, you now understand how 128-bit AES works

Low level:

Good non-linearity properties

- S-Box is not random, it's based on inverses in GF(2^8)
- Constants in MixColumns also derived from math in GF(2^8)
- All of the insane shifting, XORing, and table lookups are really additions, multiplications, and inversion of polynomials in disguise!

Take away point: Crypto is difficult to implement properly, let alone design. Unless you plan to pursue higher education in mathematics, probably best to understand it as a tool, but leave the design to experts ☺

One alternative was Blowfish, by Schneier et al.

Designed in 1993

No patents

64-bit block size

Huge subkeys

Slow to switch keys

Fast otherwise

…

Now, we'll look closely at Blowfish

Despite its long history, Blowfish is still interesting to study

Still no bad cryptographic breaks

- (Some weak keys)

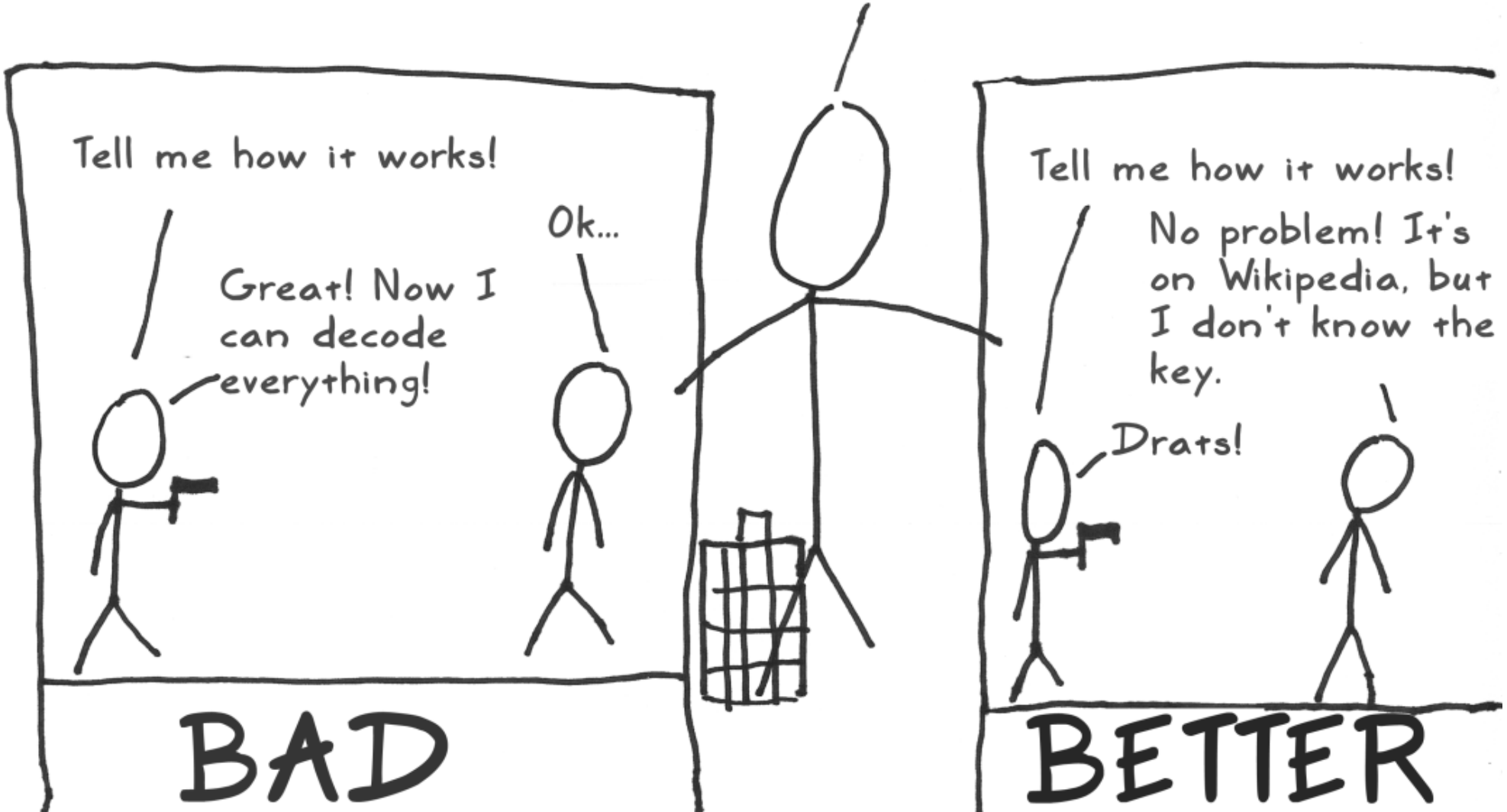
Some very interesting properties that Rijndael does not have

- Key-dependent S-boxes
- Long key schedule (computing subkeys)
- Pi (?!?)
- …

Big Idea #3: Secrecy Only in the Key

Kerchoffs's principle!

After thousands of years, we learned that it's a bad idea to assume that no one knows how your method works. Someone will eventually find that out.



Foot-Shooting Prevention Agreement

I, _____ , promise that once
Your Name

Blowfish

I see how simple ~~AES~~ really is, I will not implement it in production code even though it would be really fun.

This agreement shall be in effect until the undersigned creates a meaningful interpretive dance that compares and contrasts cache-based, timing, and other side channel attacks and their countermeasures.

X _____ _____
Signature Date

First of all, what are these subkeys you keep mentioning?

Data computed from the key, used in encryption

P-array

- 18 subkeys
- 32 bits each
- $P_1, P_2, \ldots, P_{18}$

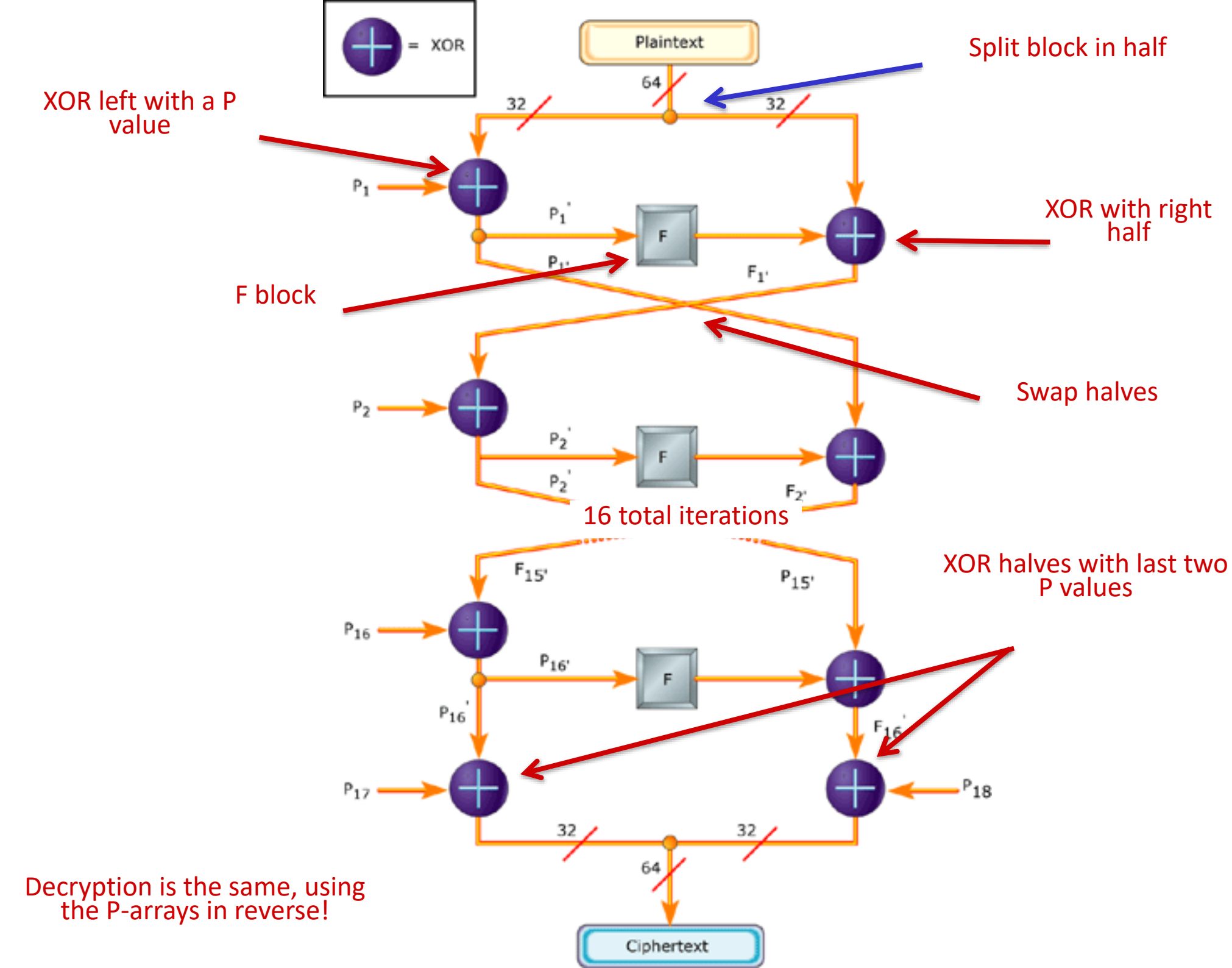
72 bytes

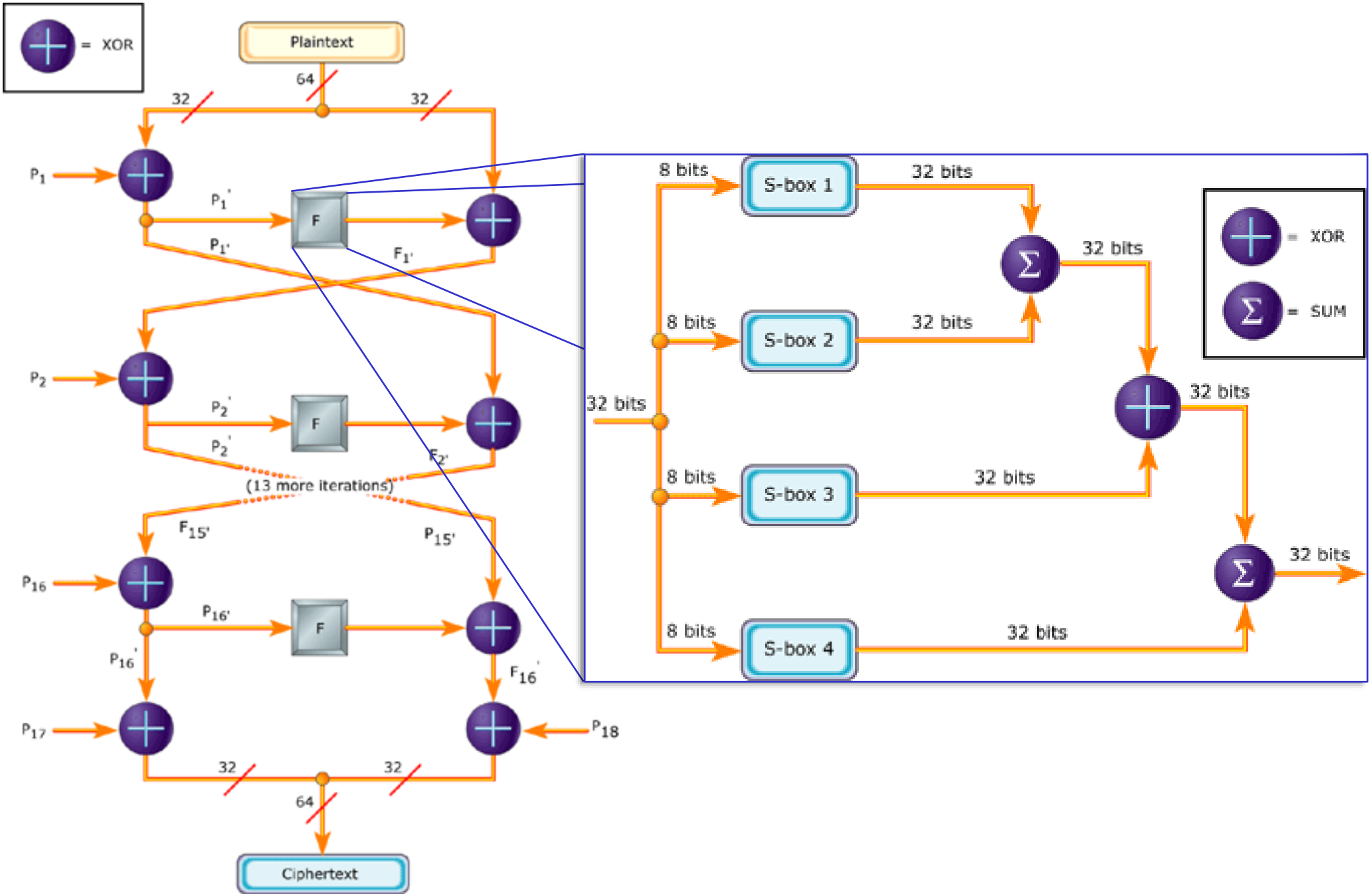
S-boxes

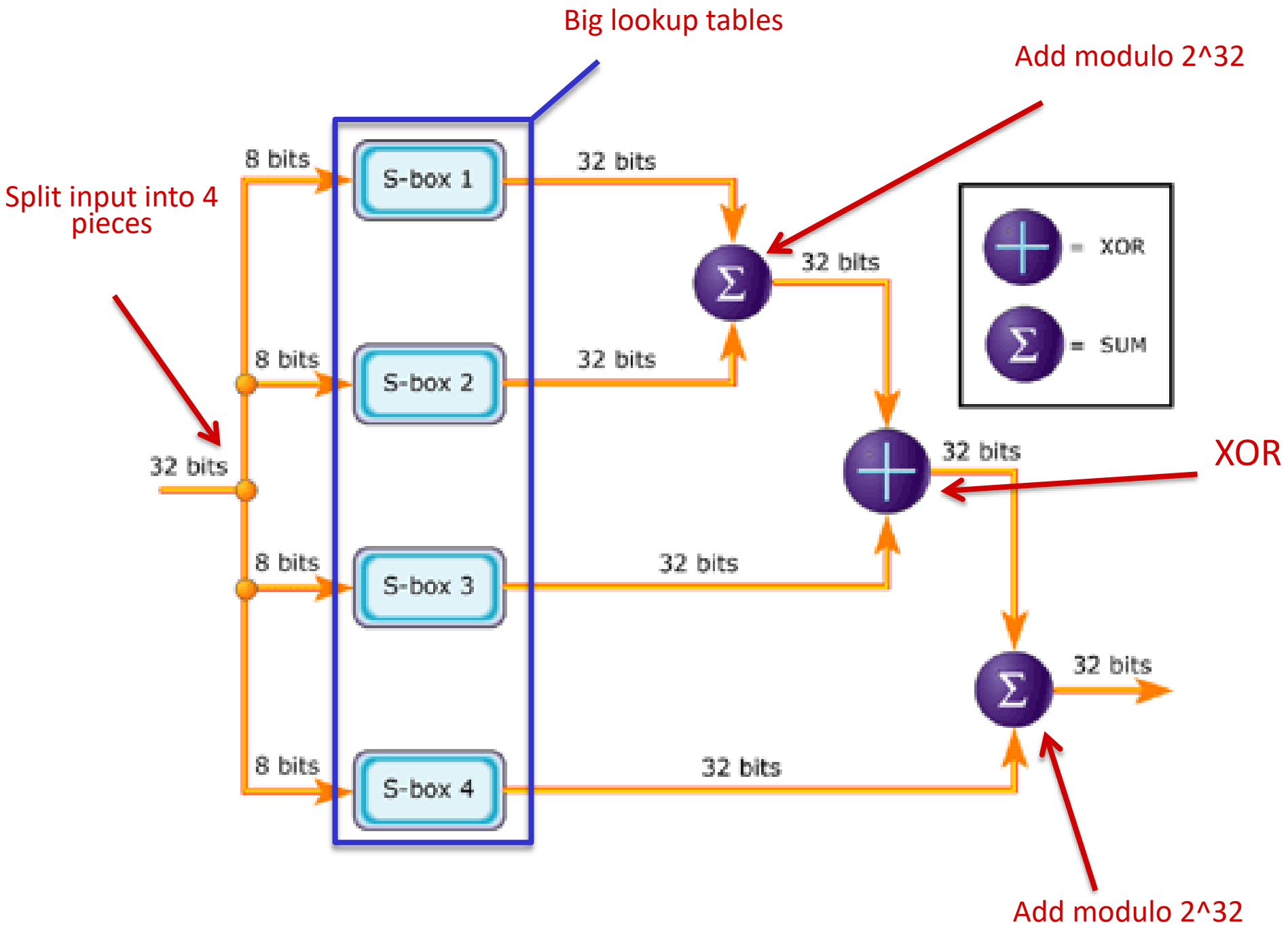
- 4 lookup tables
- 8 bits → 32 bits
- $S_{1,0}, S_{1,1}, \ldots, S_{1,255}$;
  $S_{2,0}, S_{2,1}, \ldots, S_{2,255}$;
  $S_{3,0}, S_{3,1}, \ldots, S_{3,255}$;
  $S_{4,0}, S_{4,1}, \ldots, S_{4,255}$;

4096 bytes

18 32-bit P values + four 8-bit to 32-bit S-boxes
= 4168 bytes from (max) 448-bit (56 byte) key!


= XOR
Plaintext
64
32
32
Split block in half
XOR left with a P value
P1
P1'
F
XOR with right half
F block
F1'
Swap halves
P2
P2'
F
P2'
F2'
16 total iterations
F15'
P15'
XOR halves with last two P values
P16
P16'
F
P16'
F16'
P17
P18
32
32
64
Ciphertext
Decryption is the same, using the P-arrays in reverse!


= XOR
Plaintext
64
32
32
P1
P1'
F
P1'
F1'
P2
P2'
F
P2'
F2'
(13 more iterations)
F15'
P15'
P16
P16'
F
P16'
F16'
P17
P18
32
32
64
Ciphertext
8 bits
S-box 1
32 bits
32 bits
8 bits
S-box 2
32 bits
32 bits
32 bits
8 bits
S-box 3
32 bits
32 bits
8 bits
S-box 4
32 bits
= XOR
= SUM


Big lookup tables
Add modulo 2^32
Split input into 4 pieces
8 bits
S-box 1
32 bits
32 bits
= XOR
= SUM
8 bits
S-box 2
32 bits
32 bits
XOR
32 bits
8 bits
S-box 3
32 bits
32 bits
8 bits
S-box 4
32 bits
Add modulo 2^32

Where do the P-array and S-boxes come from?

1. P-array and S-boxes (4168 bits total) depend on 448-bit (56 byte) key
2. Fill P-array and S-boxes with the (hex) digits of Pi (Π=3.14159265359……)
3. XOR the key into the P-array

   $P_1 = P_1 \oplus$ first 32 bits of key

   $P_2 = P_2 \oplus$ second 32 bits of key

   … repeat key as needed

   521 encryptions!!

4. Encrypt 0 string, replace P_1, P_2 with output (64-bit)
5. Encrypt output, replace P_3, P_4 with new output
6. Repeat until entire P-array and all S-boxes are replaced

Discussion question!

Why initialize the constants with the digits of pi?

Nothing up my sleeve!

Discussion question!

Which step in Blowfish is very inefficient?

Deriving subkeys, changing keys

In what way is that a good thing?

Makes brute forcing hard!

Blowfish Summary

You now understand how Blowfish applies confusion and diffusion to 64-bit blocks

1. Key expansion: P-array and S-boxes

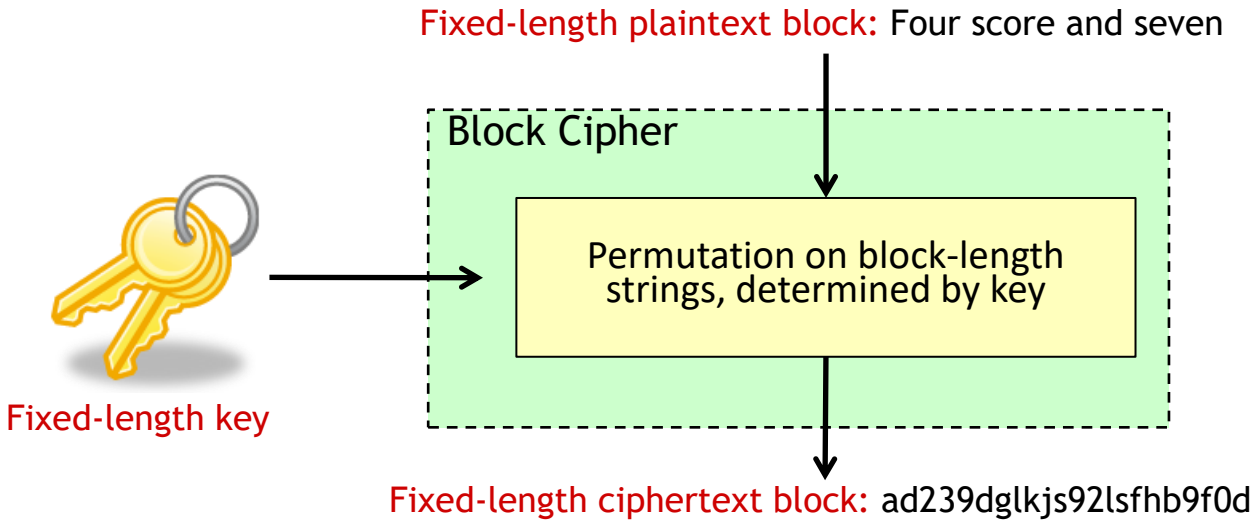
   Long key schedule, 521 encrypts (4 KB)
2. XOR with a P value
3. Break into 4 chunks, feed to S-boxes
4. Recombine with XORs and modular additions
5. Swap halves and repeat 2–4

How is that different from AES?

AES uses Additions, multiplications, and inversion of polynomials in GF(2^8) disguised as column shifts, XORs, and huge table lookups

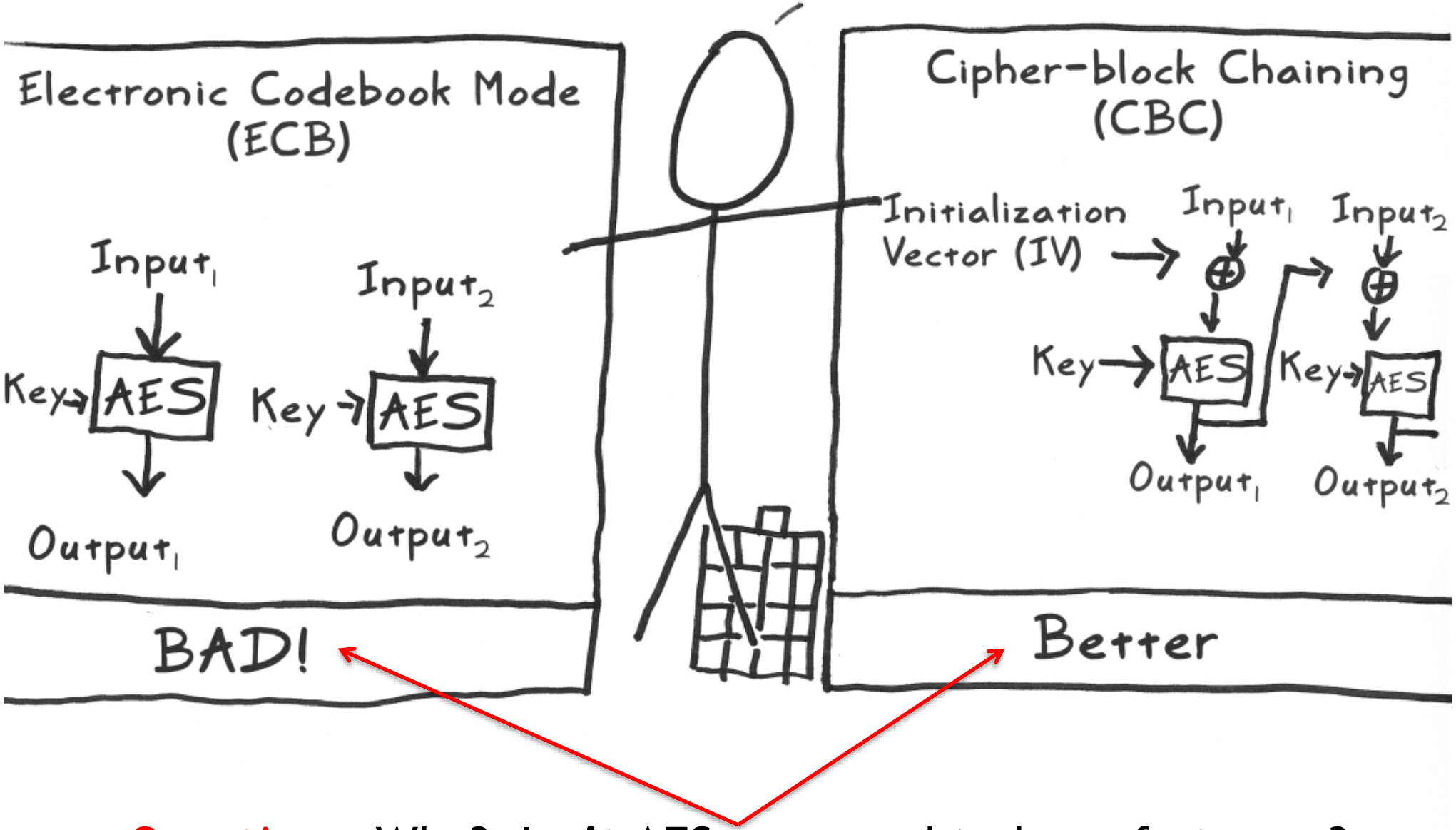
So what? Crypto is difficult to implement properly, let alone design. Understand it as a tool, but leave design/implementation to experts

Block Cipher Modes of Operation



Question: What happens if we need to encrypt more than one block of plaintext?

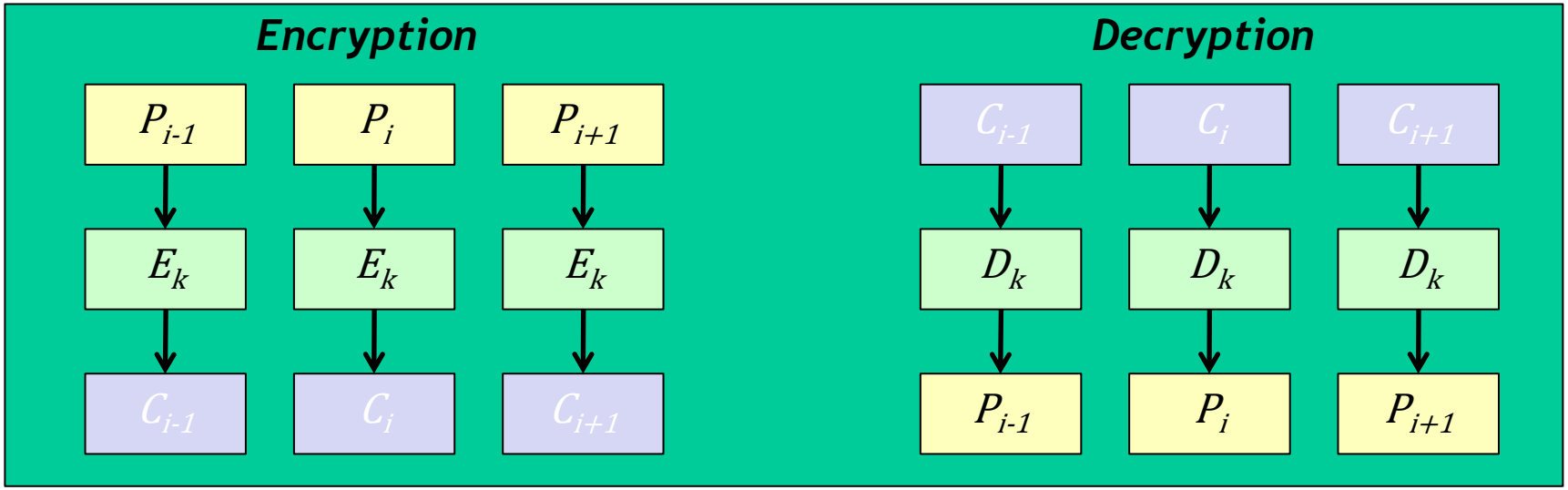
One last tidbit: I shouldn't be used as-is, but rather as a building block to a decent "mode."



Question: Why? Isn't AES supposed to be safe to use?

Block ciphers have many modes of operation

The most obvious way of using a block cipher is called electronic codebook mode (ECB)



Benefit: Errors in ciphertext do not propagate past a single block

What is wrong with ECB?

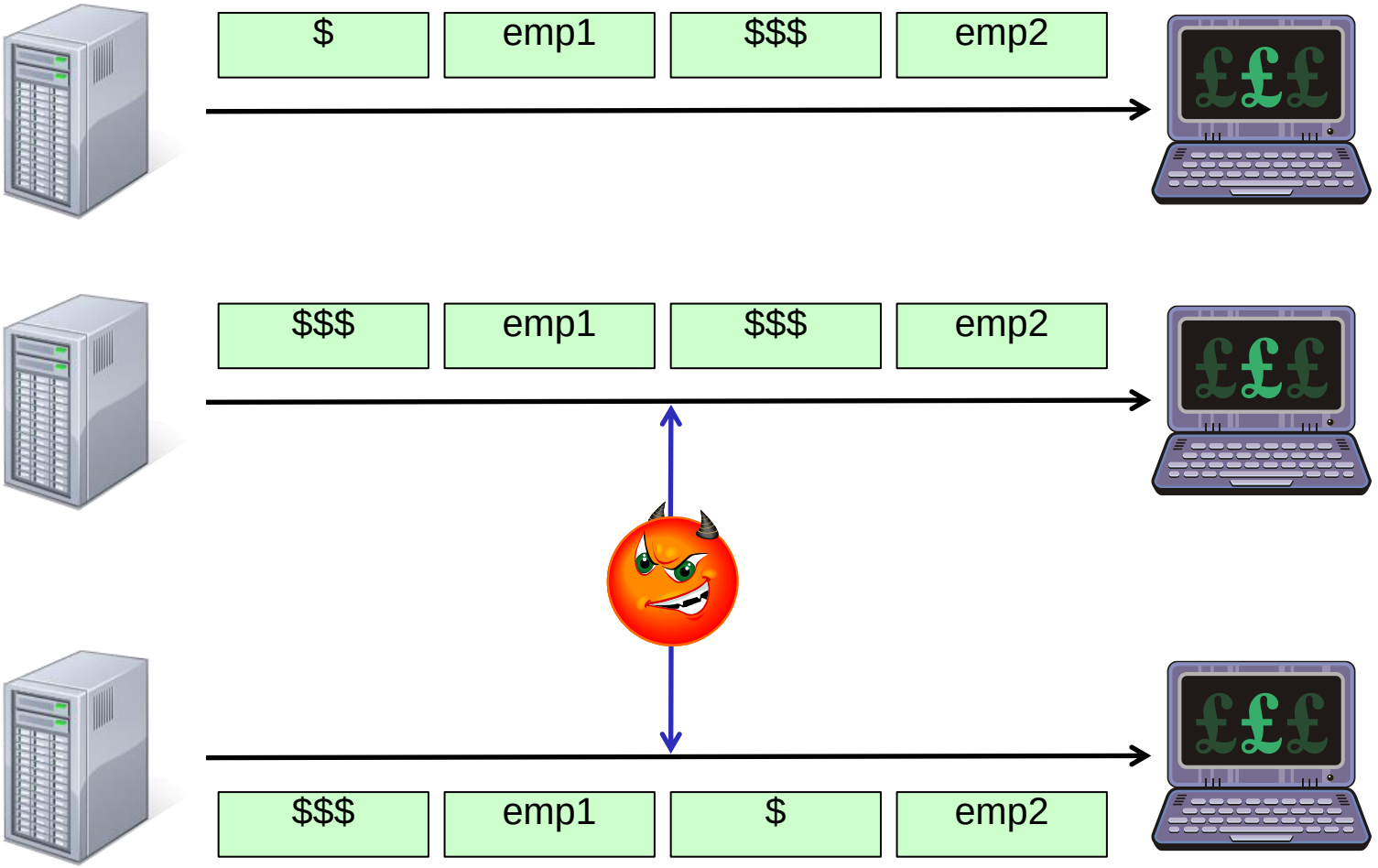
- Known plaintext/ciphertext pairings
- Block replay attacks

This is called a code book

In general, using ECB mode is not a great idea…

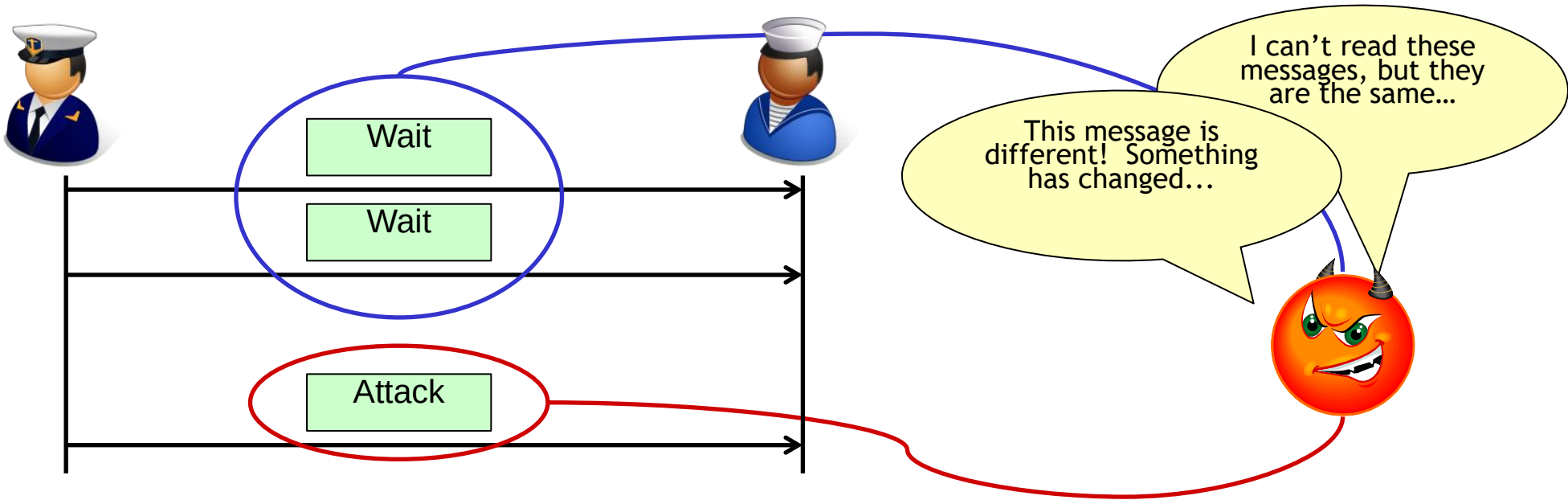
The use of ECB mode can lead to block replay or substitution attacks

Example: Salary data transmitted using ECB



Why is the ability to build a codebook dangerous?

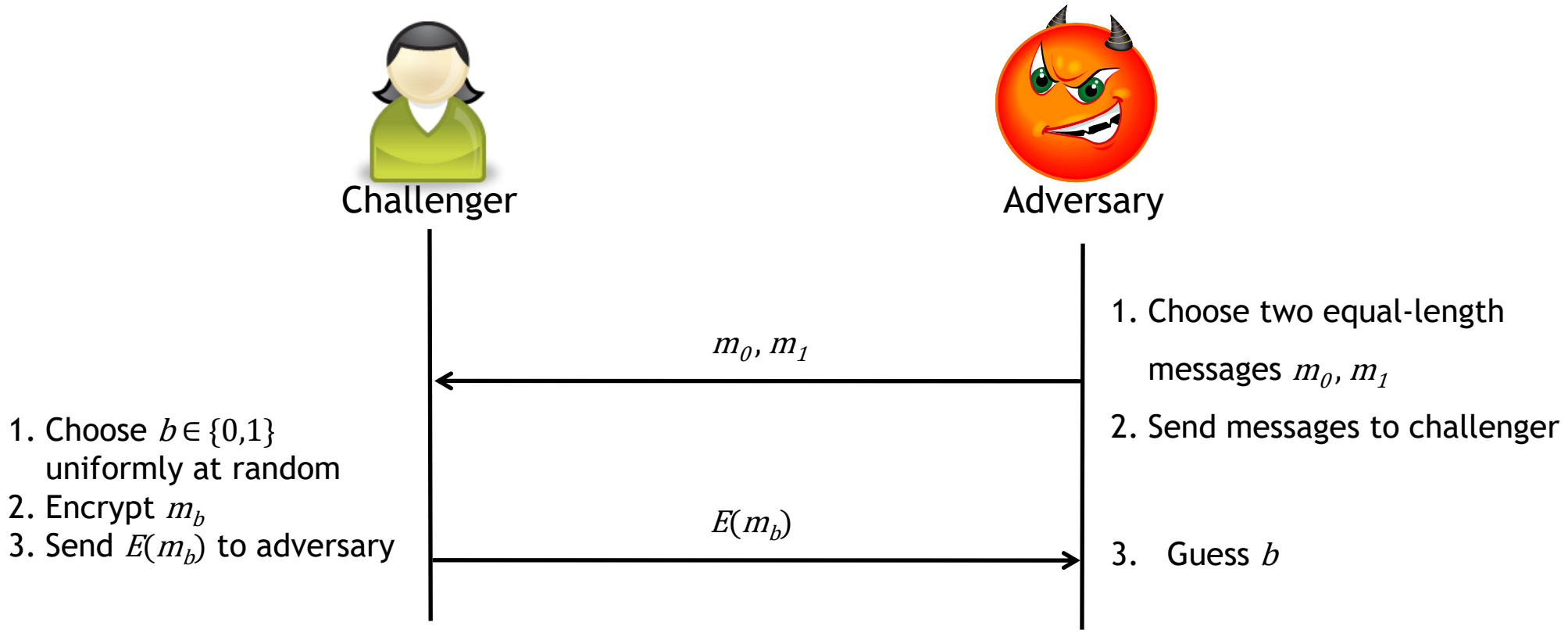
Observation: When using ECB, the same block will always be encrypted the same way



To protect against this type of guessing attack, we need our cryptosystem to provide us with semantic security.

Semantic Security

The semantic (in)security of a cipher can be established as follows:



The adversary wins if he has a non-negligible advantage in guessing b. More concretely, he wins if $P[b' = b] > ½ + \varepsilon$.

If the adversary does not have an advantage, the cipher is said to be semantically secure.

The "covert channel" attack shows up because block ciphers running in ECB mode are not semantically secure!

Question: Can you demonstrate this?

Challenger

Adversary

1. Choose $b \in \{0, 1\}$ uniformly at random
2. Encrypt m_b using ECB
3. Send $E(m_b)$ to adversary

m_0, m_1

1. m_0 = "HelloHello"
   m_1 = "HelloWorld"
2. Send messages to challenger

$E(m_b)$

3. If first block = second block, then $b = 0$.
   Otherwise, $b = 1$.

$P[b' = b] = 1$

Cipher Block Chaining (CBC) mode addresses the problems with ECB

In CBC mode, each plaintext block is XORed with the previous ciphertext block prior to encryption

$$C_i = \mathrm{E}_k(P_i \oplus C_{i-1})$$

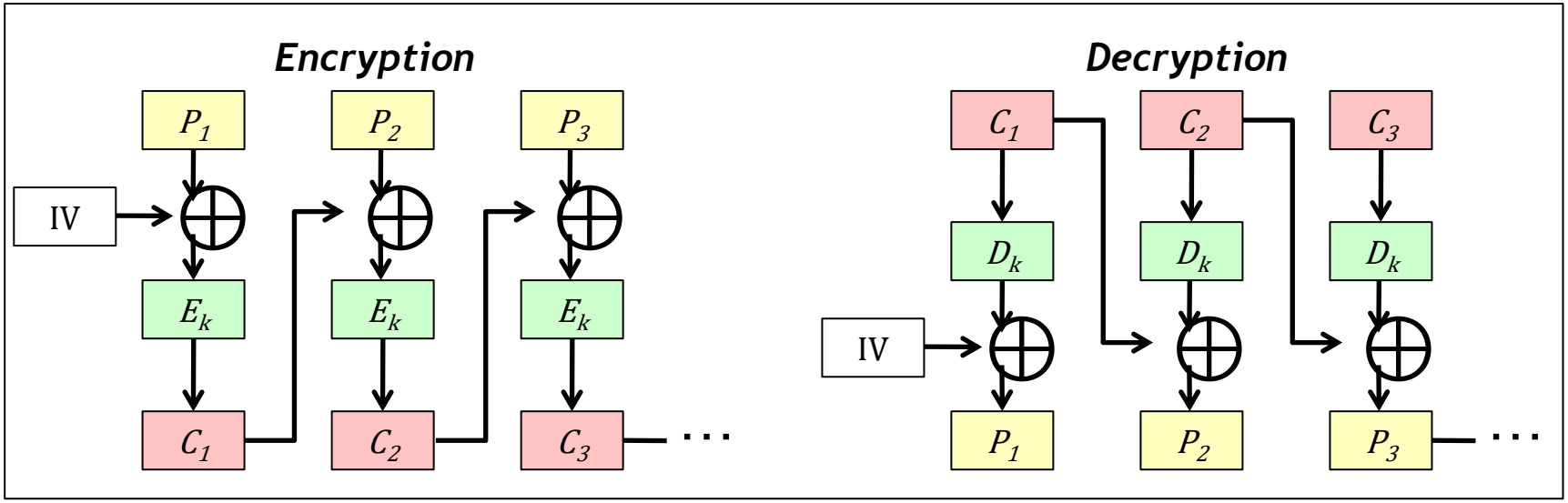
$$P_i = C_{i-1} \oplus \mathrm{D}_k(C_i)$$

Need to encrypt a random block to get things started

This initialization vector needs to be random, but not secret (Why?)

CBC eliminates block replay attacks

Each ciphertext block depends on previous block



Semantic security, redux

Note that the adversary's "trick" does not work anymore (Why?)

$$c_{01} = \mathrm{E}(\mathrm{IV} \oplus m_{01})$$

$$c_{02} = \mathrm{E}(c_{01} \oplus m_{02})$$

Essentially, the IV randomizes the output of the game, even if it is played over multiple rounds

Challenger

Adversary

1. m_0 = "HelloHello"
   m_1 = "HelloWorld"
2. Send messages to challenger

m_0, m_1

1. Choose $b \in \{0,1\}$ uniformly at random
2. Generate a random IV
3. Encrypt m_b using CBC
4. Send $E(m_b)$ to adversary

$E(m_b)$

3. If first block = second block, then $b = 0$.
   Otherwise, $b = 1$.

Cipher Feedback Mode (CFB) can be used to construct a self-synchronizing stream cipher from a block cipher

To generate an n-bit CFB based upon an n-bit block cipher algorithm, as above, we have that:

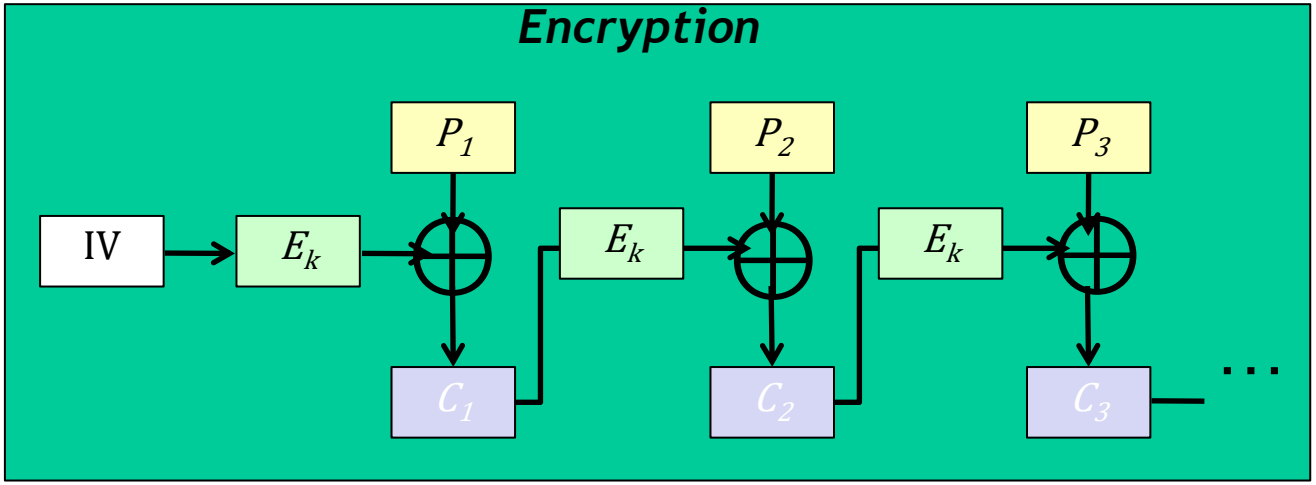
$$C_i = P_i \oplus \mathrm{E}_k(C_{i-1})$$

$$P_i = C_i \oplus \mathrm{E}_k(C_{i-1})$$

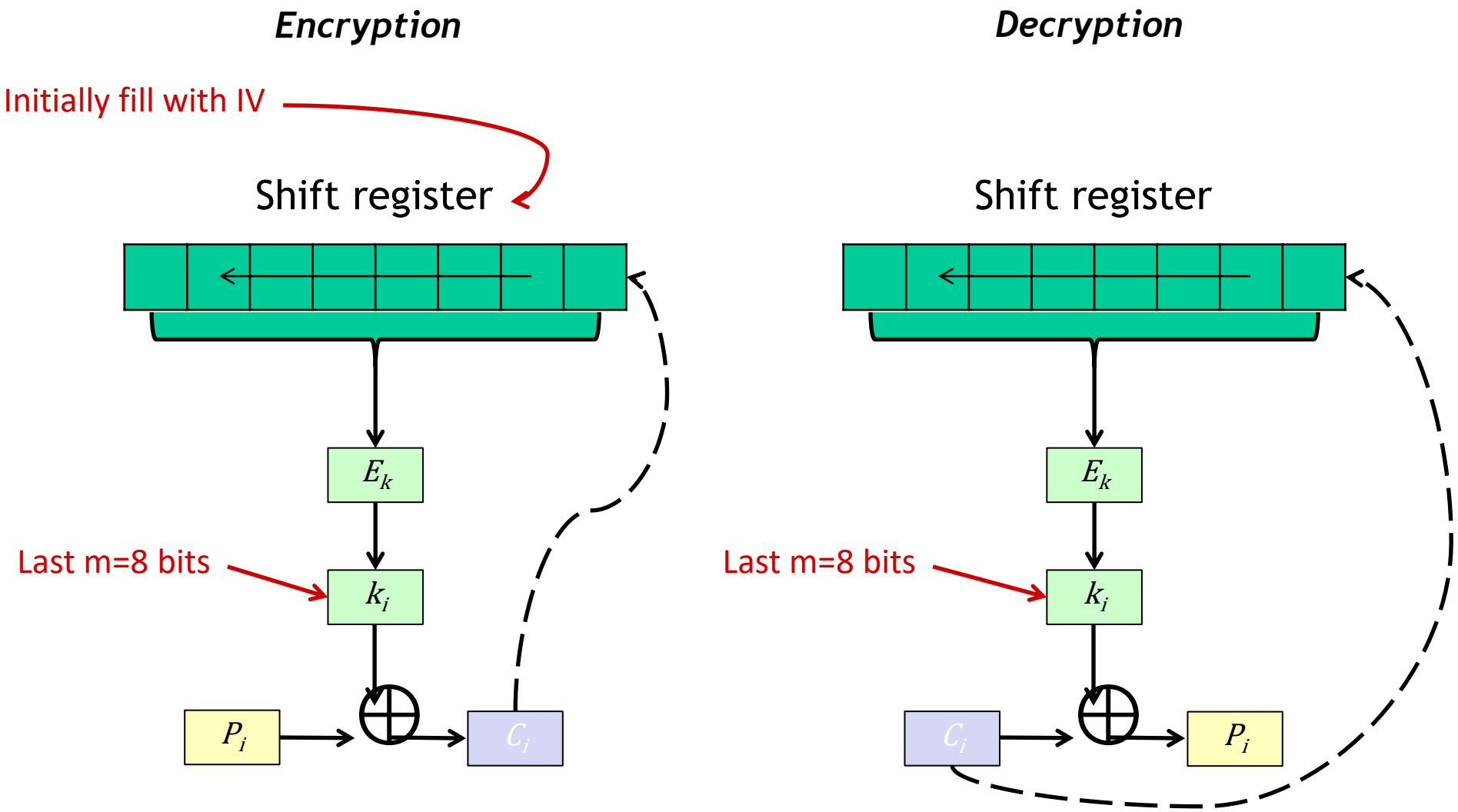
What is really interesting is that this technique can be used to develop an m-bit cipher based upon an n-bit block cipher, where $m \leq n$ by using a shift-register approach

This is great, since we don't need to wait for n bits of plaintext to encrypt!

Example: Typing at a terminal



Using an n-bit cipher to get an m-bit cipher ($m < n$)



Output Feedback Mode (OFB) can be used to construct a synchronous stream cipher from a block cipher

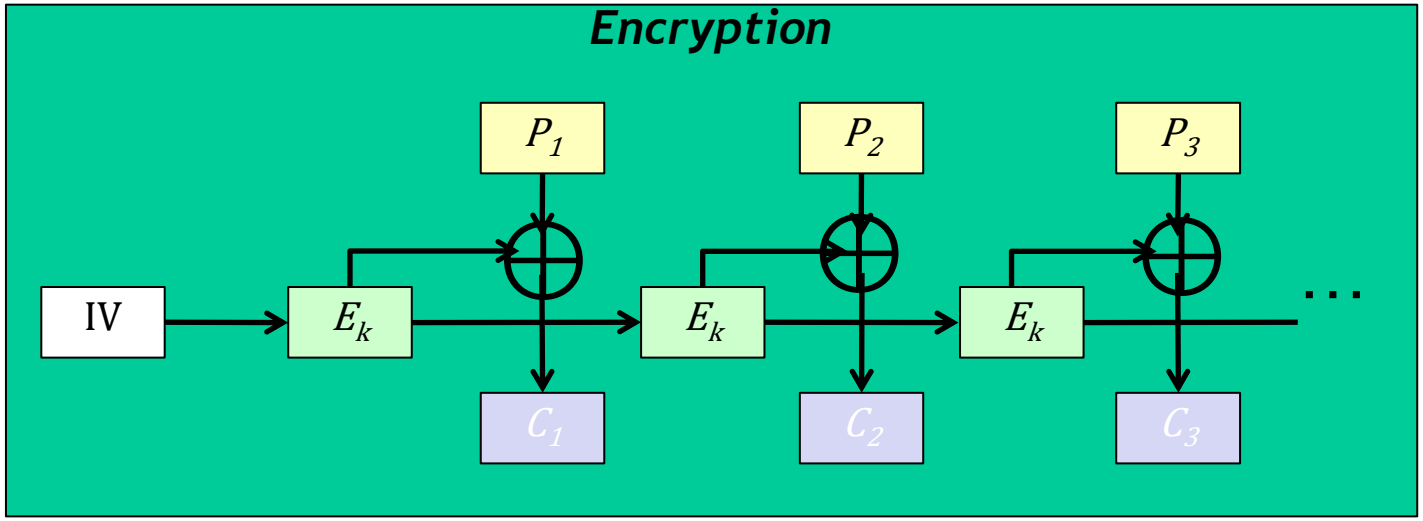
How does this work?

$$C_i = P_i \oplus S_i, \quad S_i = \mathrm{E}_k(S_{i-1})$$

$$P_i = C_i \oplus S_i, \quad S_i = \mathrm{E}_k(S_{i-1})$$

Benefit: Key stream generation can occur offline

Pitfall: Loss of synchronization is a killer…



Counter mode (CTR) generates a key stream independently of the data

Pros:

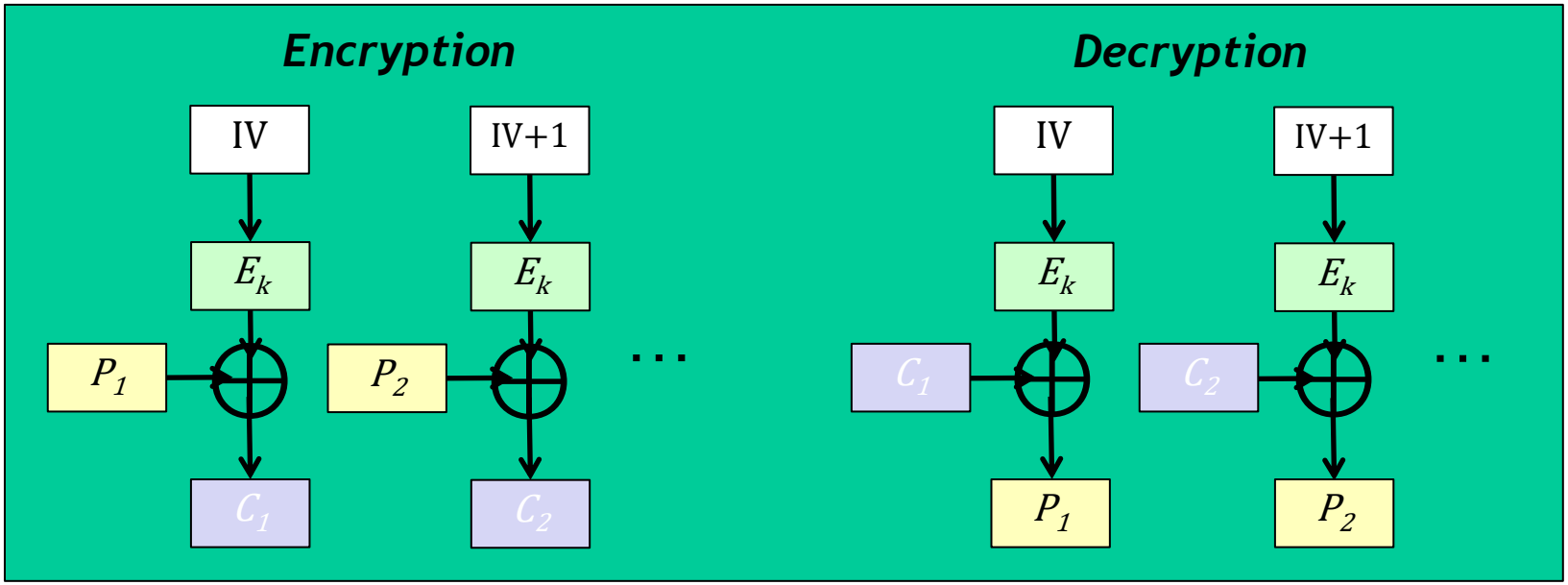
We can do the expensive cryptographic operations offline

Encryption/decryption is just an XOR

It is possible to encrypt/decrypt starting anywhere in the message

Cons:

Don't use the same (key, IV) for different files (Why?)



CTR mode has some interesting applications

Example: Accessing a large file or database

Operation: Read block number n of the file

CTR: One encryption operation is needed

$$p_n = c_n \oplus \mathrm{E}(\mathrm{IV} + n)$$

CBC: One decryption operation is needed

$$p_n = c_{n-1} \oplus \mathrm{D}(c_n)$$

In most symmetric key ciphers encryption and decryption have the same complexity

Operation: Update block k of n

If n is large, this is problematic…

CTR: One encryption operation is needed

$$c_k = p_k \oplus \mathrm{E}(\mathrm{IV} + k)$$

What about CBC?

First, we need to decrypt all blocks after k ($n - k$ decryptions)

Then, we need to encrypt blocks k through n ($n - k + 1$ encryptions)

Operation: Encrypt all n blocks of a file on a machine with c cores

CTR: $\mathrm{O}(n / c)$ time required, as cores can operate in parallel

CBC: $\mathrm{O}(n)$ time required on one core…

So… Which mode of operation should I use?

Unless you are encrypting short, random data (e.g., a cryptographic key) do not use ECB!

Use CBC if either:

- You are encrypting files, since there are rarely errors on storage devices
- You are dealing with a software implementation

CFB (usually 8-bit CFB) is the best choice for encrypting streams of characters entered at, e.g., a text terminal

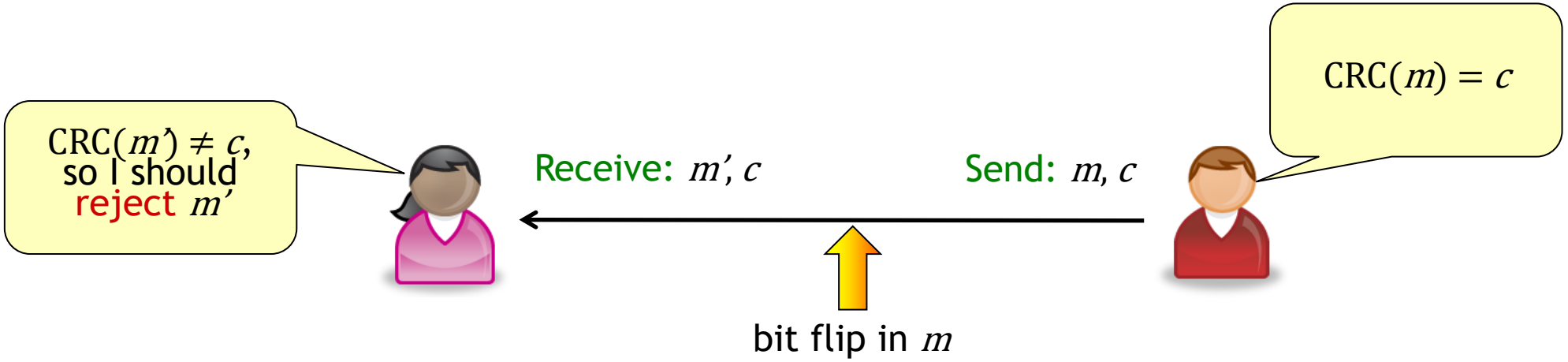
In error prone environments, OFB is probably your best choice

---

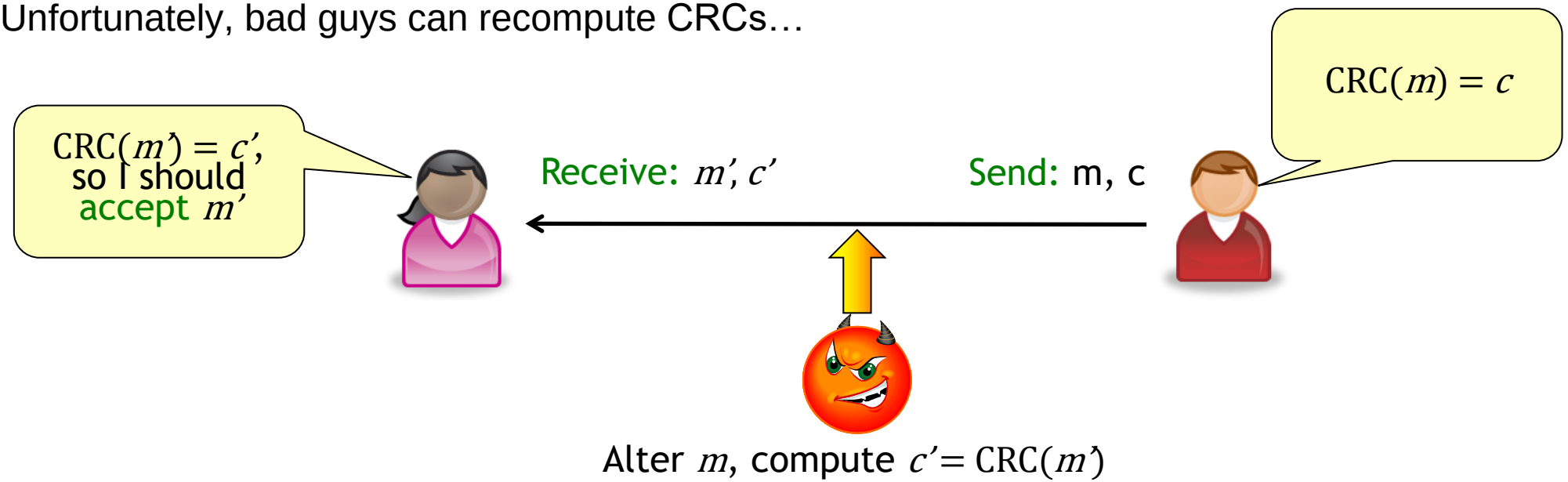
Want more information? See chapter 9 of *Applied Cryptography.*

Encryption does not guarantee integrity/authenticity

CRCs can be used to detect random errors in a message



Unfortunately, bad guys can recompute CRCs…

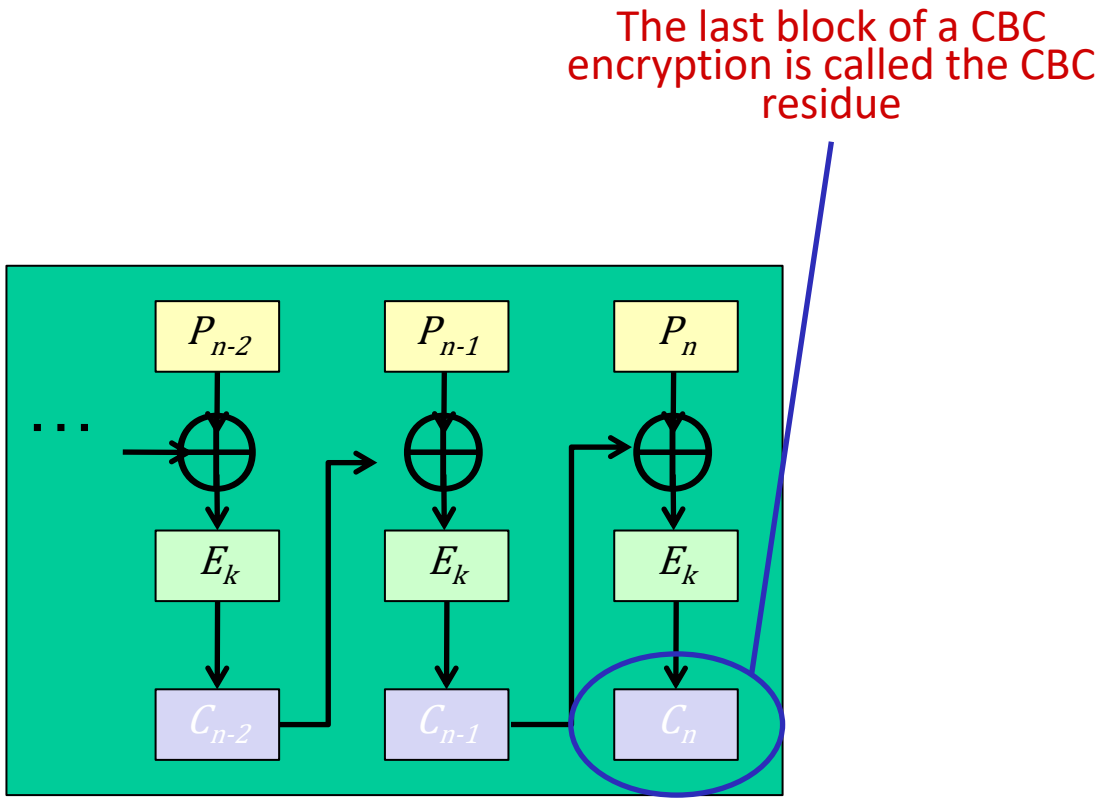


Solution: Cryptographic message authentication codes (MACs)

The CBC residue of an encrypted message can be used as a cryptographic MAC

How does this work?

- Use a block cipher in CBC mode to encrypt m using the shared key k
- Save the CBC residue r
- Transmit m and r to the remote party
- The remote party recomputes and verifies the CBC residue of m



Why does this work?

- Malicious parties can still manipulate m in transit
- However, without k, they cannot compute the corresponding CBC residue!

The bad news: Encrypting the whole message is expensive!

How can we guarantee the confidentiality and integrity of a message?

Does this mean using CBC encryption gives us confidentiality and integrity at the same time?

Unfortunately, it does not 😢

To use CBC for confidentiality and integrity, we need two keys

Encrypt the message M using k_1 to get ciphertext $C_1 = \{c_{11}, ..., c_{1n}\}$

Encrypt M using k_2 to get $C_2 = \{c_{21}, ..., c_{2n}\}$

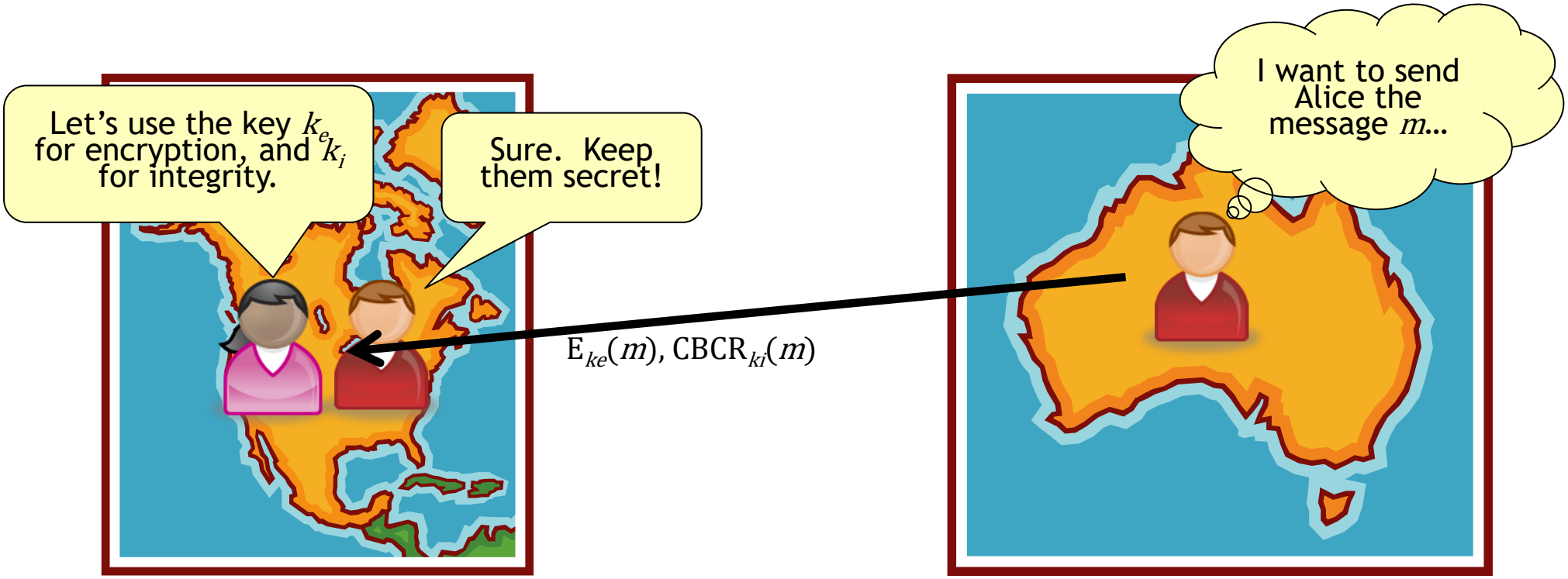
Transmit $\langle C_1, c_{2n} \rangle$

But wait, isn't that expensive?

Fix #1: Exploit parallelism if there is access to multiple cores

Fix #2: Faster hash-based MACs (next lecture)

Putting it all together…



All is well?

Today we learned how symmetric-key cryptography can protect the confidentiality and integrity of our communications

So, the security problem is solved, right?

Unfortunately, symmetric key cryptography doesn't solve everything…

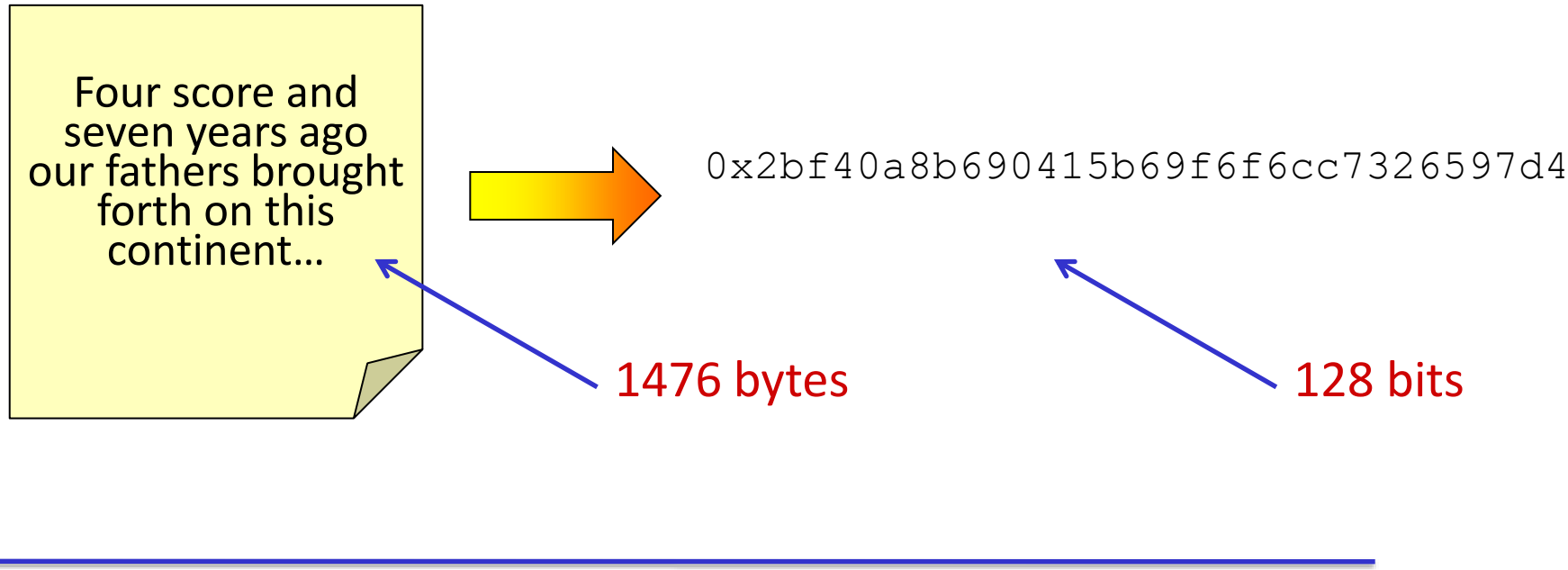
1. How do we get secret keys for everyone that we want to talk to?
2. How can we update these keys over time?

Later: Public key cryptography will help us solve problem 1

Even later in the semester, we'll look at key exchange protocols that help with problem 2

What is a hash function?

Definition: A hash function is a function that maps a variable-length input to a fixed-length code



Hash functions are sometimes called message digest functions

- SHA (e.g., SHA-1, SHA-256, SHA-3) stands for the secure hash algorithm
- MD5 stands for message digest algorithm (version 5)

In order to be useful cryptographically, a hash function needs to have a "randomized" output

For example:

- Given a large number of inputs, any given bit in the corresponding outputs should be set about half of the time
- Any given output should have half of its bits set on average
- Given two messages m and m' that are very closely related, $H(m)$ and $H(m')$ should appear completely uncorrelated

Informally: The output of an m-bit hash function should appear as if it was created by flipping m unbiased coins

Theoretical cryptographers sometimes use a more formalized notion of random oracles to model hash functions when analyzing security protocols

More formally, cryptographic hash functions should have the following three properties

Assume that we have a hash function $H : \{0,1\}^* \rightarrow \{0,1\}^m$

What does infeasible mean?

1. Preimage resistance: Given a hash output value z, it should be infeasible to calculate a message x such that $H(x) = z$

   i.e., H is a one way function

   Ideally, computing x from z should take $O(2^m)$ time

2. Second preimage resistance: Given a message x, it is infeasible to calculate a second message y such that $H(x) = H(y)$

   Note that this attack is always possible given infinite time (Why?)

   Ideally, this attack should take $O(2^m)$ time

3. Collision resistance: It is infeasible to find two messages x and y such that $H(x) = H(y)$

   Ideally, this attack should take $O(2^{m/2})$ time

Why only $O(2^{m/2})$?

The Birthday Paradox!

The gist: If there are more than 23 people in a room, there is a better than 50% chance that two people have the same birthday

Wait, what?

366 possible birthdays

To solve: Find probability p_n that n people all have *different* birthdays,

then compute 1-p_n

$$p_n = \frac{365}{366}\frac{364}{366}\frac{363}{366}\cdots\frac{367-n}{366}$$

Note: The value of n can be approximated as $1.1774 \times \sqrt{N} = 1.1774\, x\, \sqrt{366} \approx 22.525$

If $n = 22$, 1 - $p_n \approx 0.475$

If $n = 23$, 1 - $p_n \approx 0.506$

What does this have to do with hash functions?!

Note that "birthday" is just a function b : person → date

Goal: How many inputs x to the function b do we need to consider to find x_i, x_j such that $\mathrm{b}(x_i) = \mathrm{b}(x_j)$?

We're looking for collisions in the birthday function!

Now, a hash is a function $\mathrm{H} : \{0, 1\}^* \rightarrow \{0, 1\}^{\mathrm{m}}$

Note: H has 2^{m} possible outputs

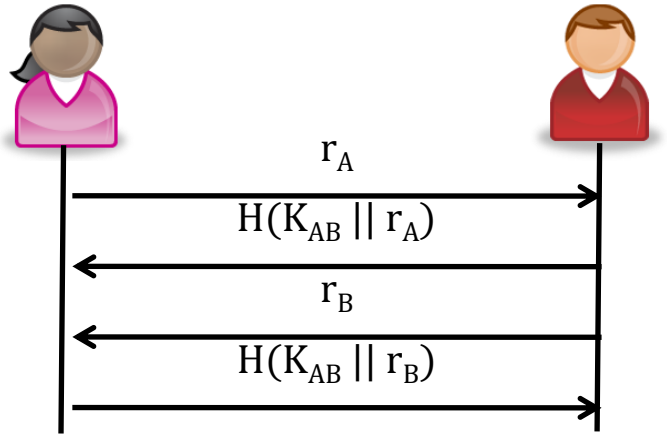
So, using our approximation from the last slide, we'd need to examine about $1.1774 \times \sqrt{2^m} = 1.1774 \times 2^{\frac{m}{2}} = O(2^{\frac{m}{2}})$ inputs to find a collision!

What are some things that we can do with a hash function?

Document Fingerprinting

Use H(D) to see if D has been modified

Example: GitHub commit hashes



Mutual Authentication

MAC Functions

- Assume a shared key K
- Sender:
  - Compute $c = E_K(H(m))$
  - Transmit m and c
- Receiver:
  - Compute $d = E_K(H(m))$
  - Compare c and d

Hash functions can even be used to generate cipher keystreams

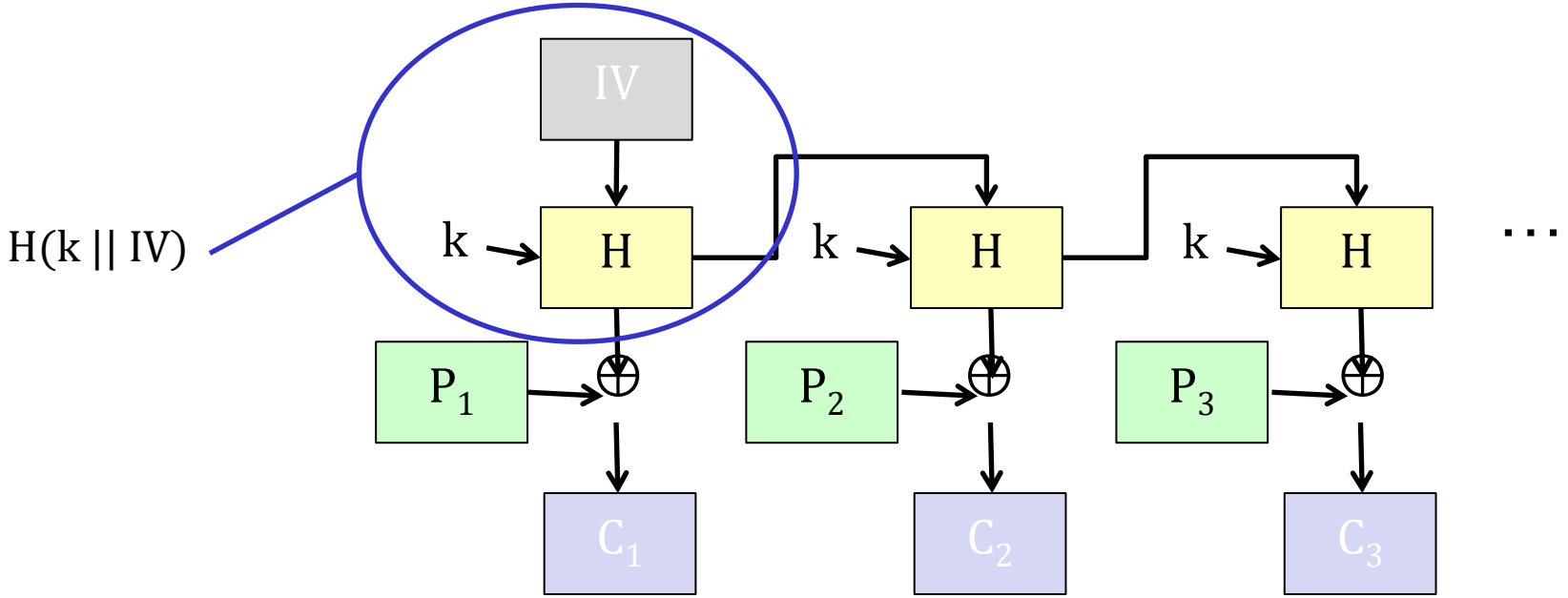
Question: What block cipher mode does this remind you of?

Output feedback mode (OFB)

Why is this safe to do?

Remember that hash functions need to behave "randomly" in order to be used in cryptographic applications

Even if the adversary knows the IV, he cannot figure out the keystream without also knowing the key, k



Hash functions also provide a means of safely storing user passwords

Consider the problem of safely logging into a computer system

Option 1: **Store 〈username, password〉 pairs on disk**

Correctness: This approach will certainly work

Safety: What if an adversary compromises the machine?

All passwords are leaked!

This probably means the adversary can log into your email, bank, etc…

Option 2: Store 〈username, H(password)〉 pairs on disk

Correctness:

Host computes H(password)

Checks to see if it is a match for the copy stored on disk

Safety: Stealing the password file is less* of an issue

The previous applications provide us with an intuitive way to understand the importance of a hash function's cryptographic properties

1. Preimage resistance: Given a hash output value z, it should be infeasible to calculate a message x such that $\mathrm{H}(x) = z$

Without this, we could recover hashed passwords!

2. Second preimage resistance: Given a message x, it is infeasible to calculate a second message y such that $\mathrm{H}(x) = \mathrm{H}(y)$

Example: File integrity checking

Say the `ls` program has a fingerprint f

We could create a malicious version of `ls` that actually executes `rm -rf *`, but has the same document fingerprint

3. Collision resistance: It is infeasible to find two messages x and y such that $\mathrm{H}(x) = \mathrm{H}(y)$

Later on, we'll see that this can lead to attacks that let us inject arbitrary content into protected documents!

Okay, enough high-level talk. How do hash functions actually work?

It is perhaps unsurprising that hash functions are effectively compression functions that are iterated many times

- Compression: Implied by the ability to map a large input to a small output
- Iteration: Helps “spread around” input perturbations

The KPS book spends a lot of time talking about the “MD” family of message digest functions developed by Professor Ron Rivest (MIT)

Bad news: the most recent MD function, MD5, was broken in 2008

- Specifically, it has been shown possible to generate MD5 collisions in $O(2^{32})$ time, which is much faster than the theoretical “best case” of $O(2^{64})$
- We’ll talk more about this later in the course (~Week 9)

We’ll focus on SHA-1 (officially deprecated by NIST in 2011)

SHA-1 is built using the Merkle-Damgård construction

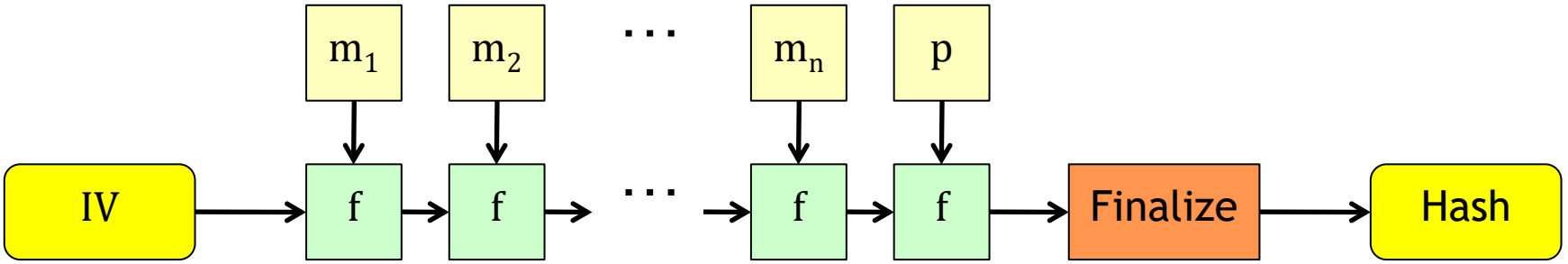
The Merkle-Damgård construction is a "template" for constructing cryptographic hash functions

- Proposed in the late '70s
- Named after Ralph Merkle and Ivan Damgård

Essentially, a Merkle-Damgård hash function does the following:

1. Pad the input message if necessary
2. Initialize the function with a (static) IV
3. Iterate over the message blocks, applying a compression function f
4. Finalize the hash block and output

Why is a static IV ok?



Merke and Damgård independently showed that the resulting hash function is secure if the compression function is collision resistant

A thousand-mile view…

Input: A message of bit length $\leq 2^{64} - 1$

Output: A 160-bit digest

Steps:

- Pad message to a multiple of 512 bits
- Process one 512 bit chunk at a time
- Expand the sixteen 32-bit words into eighty 32-bit words
- Initialize five 32-bit words of state
- For each block of five 32-bit words
  - Apply function at right
  - Add result to output
- Concatenate five 32-bit words of output state

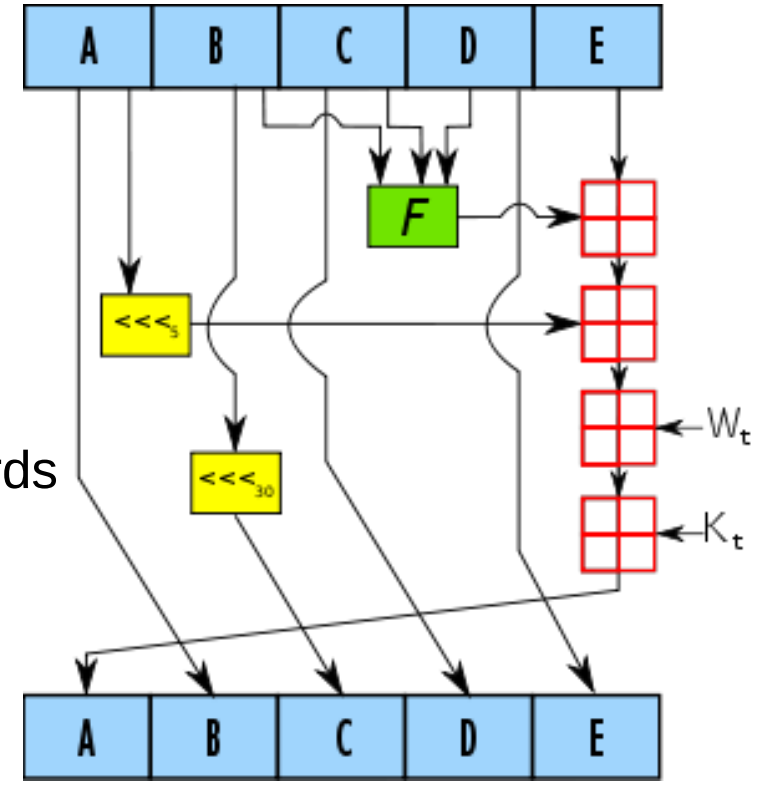


Image from Wikipedia

Initialization and Padding

Initialize variables:

```
h0 = 0x67452301
h1 = 0xEFCDAB89
h2 = 0x98BADCFE
h3 = 0x10325476
h4 = 0xC3D2E1F0
```

Note: These variables comprise the internal state of SHA-1. They are continuously updated by the compression function, and are used to construct the final 160-bit hash value.

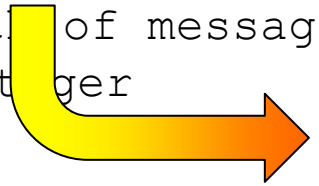
Pre-processing:

append the bit '1' to the message

append 0 ≤ k < 512 '0' bits, so that the resulting message length (in bits)

is congruent to 448 ≡ −64 (mod 512)

append length of message (before pre-processing), in bits, as 64-bit big-endian integer



Example: 0xDEADBEEF → 0xDEADBEEF8000 … 0020

32 bits

32_{10} = 0x20

Initializing the compression function

```
Process the message in successive 512-bit chunks:

break message into 512-bit chunks

for each chunk

  break chunk into sixteen 32-bit big-endian words w[i], 0 ≤ i ≤ 15


  Extend the sixteen 32-bit words into eighty 32-bit words:

  for i from 16 to 79

  w[i] = (w[i-3] xor w[i-8] xor w[i-14] xor w[i-16])  <<< 1


  Initialize hash value for this chunk:

  a = h0

  b = h1

  c = h2

  d = h3

  e = h4
```

Note: <<< denotes a left rotate.

Example: 00011000 <<< 4

10000001

Main body of the compression function

```
Main loop:

for i from 0 to 79
   if 0 ≤ i ≤ 19 then
    f = (b and c) or ((not b) and d); k = 0x5A827999
   else if 20 ≤ i ≤ 39
    f = b xor c xor d; k = 0x6ED9EBA1
   else if 40 ≤ i ≤ 59
    f = (b and c) or (b and d) or (c and d); k = 0x8F1BBCDC
   else if 60 ≤ i ≤ 79
     f = b xor c xor d; k = 0xCA62C1D6


   temp = (a <<< 5) + f + e + k + w[i]
   e = d; d = c; c = b <<< 30; b = a; a = temp


Add this chunk's hash to result so far:

h0 = h0 + a; h1 = h1 + b; h2 = h2 + c; h3 = h3 + d; h4 = h4 + e
```

Note: Sometimes, we treat state as a bit vector…

… but other times, it is treated as an unsigned integer

Finalizing the result

Produce the final hash value (big-endian):

```
output = h0 || h1 || h2 || h3 || h4
```

"||" denotes concatenation

Interesting note:

k_1 = 0x5A827999 = $2^{30} \times \sqrt{2}$

k_2 = 0x6ED9EBA1 = $2^{30} \times \sqrt{3}$

k_3 = 0x8F1BBCDC = $2^{30} \times \sqrt{5}$

k_4 = 0xCA62C1D6 = $2^{30} \times \sqrt{10}$

Question: Why might it make sense to choose the k values for SHA-1 in this manner?

SHA-1 in Practice

SHA-1 has fairly good randomness properties

- SHA1("The quick brown fox jumps over the lazy dog")
  - `2fd4e1c6 7a2d28fc ed849ee1 bb76e739 1b93eb12`
- SHA1("The quick brown fox jumps over the lazy cog")
  - `de9f2c7f d25e1b3a fad3e85a 0bd17d9b 100db4b3`

In the above example, changing 1 character of input alters 81 of the 160 bits in the output!

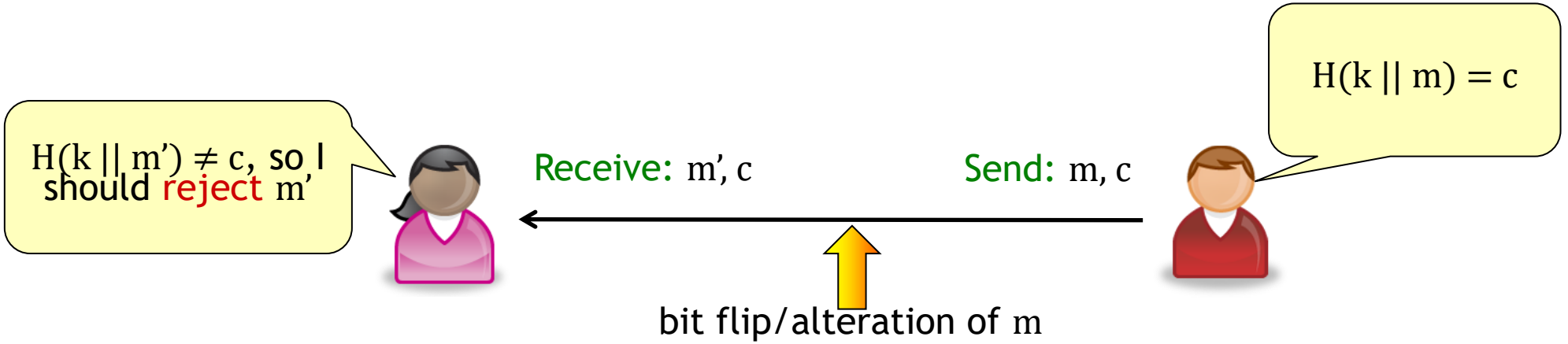
To date, the best attack on SHA-1 can find a collision with about $O(2^{61})$ steps; in theory, this attack *should* take $O(2^{80})$ steps.

As a result, NIST ran a hash function competition to design a replacement for SHA-1 (Keccak chosen as SHA-3 in Oct 2012)

Like the AES competition

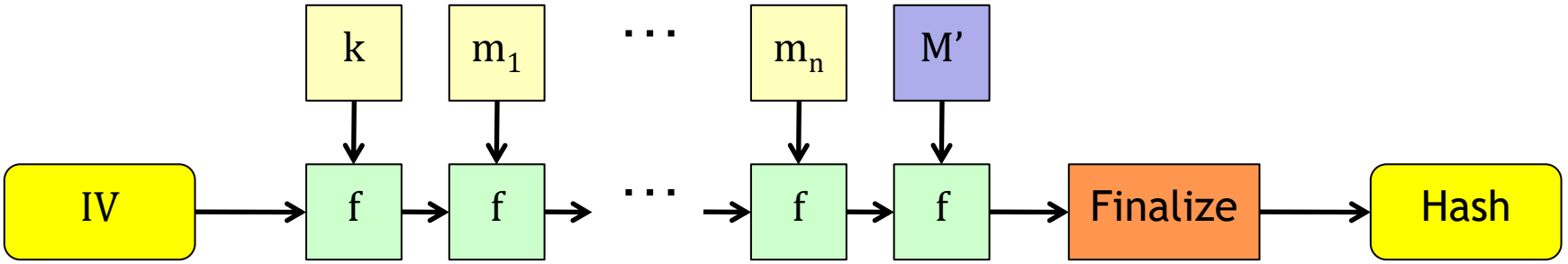
Although hashes are unkeyed functions, they can be used to generate MACs

A keyed hash can be used to detect errors in a message



Unfortunately, this isn't *totally* secure…

It's usually easy to add more data while still generating a correct MAC!



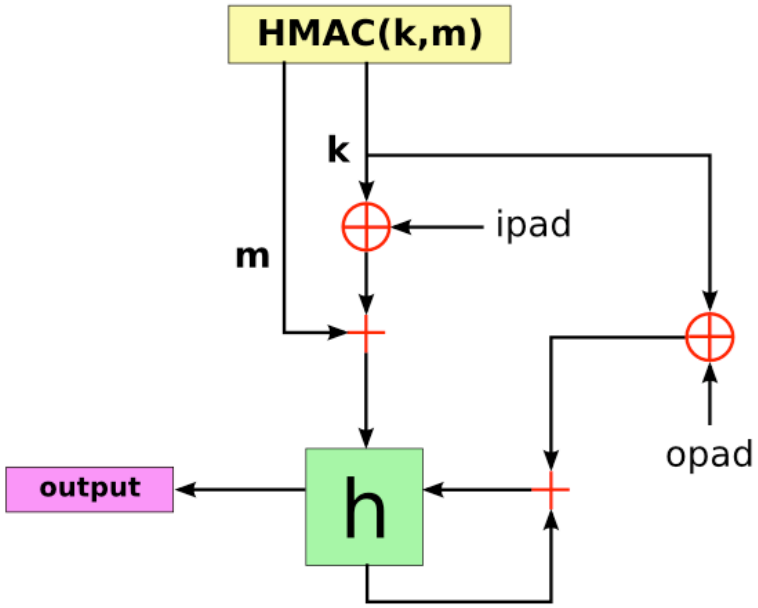
There are also attacks against $H(m \,||\, k)$ and $H(k \,||\, m \,||\, k)$!

HMAC is a construction that uses a hash function to generate a cryptographically strong MAC

$$\text{HMAC}(k, m) = H(\ (k \oplus \text{opad}) \ ||\ H(\ (k \oplus \text{ipad}) \ ||\ m)\ )$$

$\text{opad} = 01011100\ (0\text{x}5\text{c})$

$\text{ipad} = 00110110\ (0\text{x}36)$



The opad and ipad constants were carefully chosen to ensure that the internal keys have a large Hamming distance between them

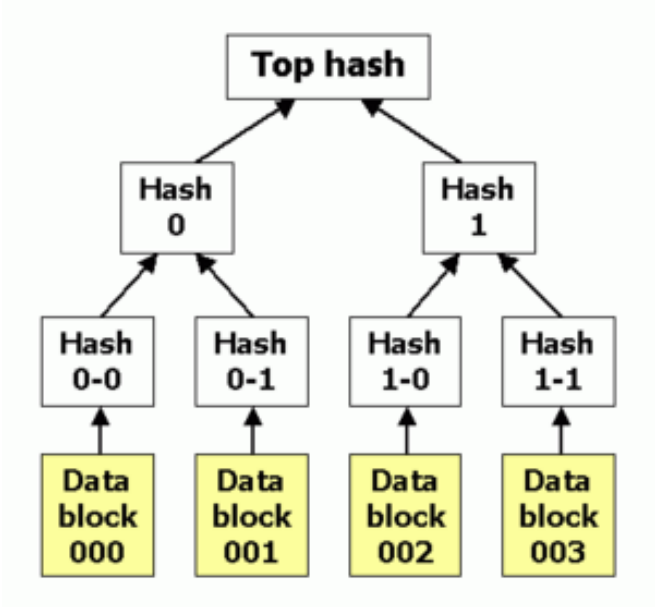
Note that *H* can be any hash function. For example, HMAC-SHA-1 is the name of the HMAC function built using the SHA-1 hash function.

Benefits of HMAC:

- Hash functions are faster than block ciphers
- Good security properties
- Since HMAC is based on an unkeyed primitive, it is not controlled by export restrictions!

Hash functions can also help us check the integrity of large files efficiently

Many peer-to-peer file sharing systems use Merkle trees for this purpose



Why is this good?

- One branch of the hash tree can be downloaded and verified at a time
- Interleave integrity check with acquisition of file data
- Errors can be corrected on the fly

BitTorrent uses hash lists for file integrity verification

- Must download full hash list prior to verification

Putting it all together…

Why compute the HMAC over $E_{ke}(m)$?

Alice doesn't need to waste time decrypting m if it was mangled in transit, since its authenticity can be checked first!

Why use two separate keys?

In general, it's a bad idea to use cryptographic material for multiple purposes