CS6690 - Pattern Recognition
Tutorial 2

1. Find the eigen value and eigen vectors of the following matrices:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 6 & 3 \\ 2 & 4 & 3 \\ 3 & 6 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$$

2. Show that if A is positive definite, A^{-1} exists.

3. Show that the least squares solution for regression is equivalent to the solution obtained using pseudo inverse.

4. Determine the coefficients of the polynomial that passes through the points (1,2), (2,3). (3,4). Determine the matrix A, the solution vector w, and the b vector where $Aw = b$.
The following assumptions can be made for the polynomial:
$p(x) = a_0 + a_1 x + a_2 x^2$
$w = [a_0, a_1, a_2]^t$
$p(1) = 2$
$p(2) = 3$
$p(3) = 4$

5. Find the singular values and singular vectors using SVD.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

6. Consider the following matrix

$$A = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Reconstruct the above matrix using the eigen vectors corresponding to that of the top 2 eigen values and calculate the squared error. Determine if the eigen vectors are orthogonal.

7. Let $\mathbf{u} = [1, 3, 2]^t$, $\mathbf{v} = [2, -1, 4]^t$ and $\mathbf{w} = [-3, 26, -6]^t$. Determine whether they are linearly independent.

8. Let $f_X(x) =$ unimodal, univariate Gaussian density function with $\mathcal{N}(\mu, \sigma^2)$. Determine (using integration), the values of the $E[X]$, and $E[(X - \mu)^2]$.

9. Why is orthonormal basis better than (a) linearly independent basis (b) orthogonal basis ?