CS6690 - Pattern Recognition
Tutorial 1

1. When an unbiased coin is tossed 100 times, what is the probability of getting exactly 50 heads?

2. On an average, how many times a dice has to be rolled to get the value 4 ?

3. A character is chosen randomly from each of the words/phrase *Pattern* and *Recognition*. What is the probability that the same character is chosen?

4. Find the probability of following events
   - A = {Getting at least 1 six when 6 dice are rolled}
   - B = {Getting at least 2 six when 12 dice are rolled}

5. The first of three urns contain a white balls and b black balls. The second contains c white balls and d black balls and third contains k white balls (and no black balls). A girl chooses an urn at random and draws a ball from it. The ball is white. What is the probability that it came from (a) the first, (b) the second or (c) the third urn?

6. You are given two identical looking coins. One of them is fair while other one comes up with heads 75% of the time. You flip one of the coins five times yielding "THTHT". What is the probability that the coin you have been flipping is the unfair one?

7. A die is selected at random from two twenty-faced dies on which the symbols **1-10** are written with nonuniform frequency as follows:

Symbol	1	2	3	4	5	6	7	8	9	10
Number of faces of die **A**	6	4	3	2	1	1	1	1	1	0
Number of faces of die **B**	3	3	2	2	2	2	2	2	1	1

   The die is rolled 7 times, with the following outcomes:

   $$5, 3, 9, 3, 8, 4, 7$$

   What is the probability that the die chosen is die **A**.

8. A constant "c" is added to a random variable "X". How does it affect: (a) the expectation, (b) the variance, (c) the second moment about the origin.

9. Let X be a discrete random variable with the following PMF:

$$P_X(x) = \begin{cases} 0.1 & \text{for } x = 0.2 \\ 0.2 & \text{for } x = 0.4 \\ 0.2 & \text{for } x = 0.5 \\ 0.3 & \text{for } x = 0.8 \\ 0.2 & \text{for } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

   Find (a) R_X, the range of the random variable X (b) $P(\leq 0.5)$ (c) $P(0.25 < X < 0.75)$ (d) $P(X = 0.2 | X < 0.6)$ (e) E[X] (f) Var(X)

10. The joint pdf of RVs X and Y is given by:

$$p_{X,Y} = k * e^{-(x^2+xy+y^2)} \tag{1}$$

Determine (a) constant k (b) $p_X(x)$ (c) $p_Y(y)$ (d) $p_{X|Y}(x,y)$

11. Let X be a random variable with PDF given by

$$f_X(x) = \begin{cases} cx^2 & |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find (a) constant c (b) $P(X \geq \frac{1}{2})$ (c) E[X] (d) Var(X)

12. If the density of a random variable X is $f_X(x)$, determine the density function of the random variable $Y = |X|$.

13. Determine whether X and Y are independent:

$$f_{XY}(x,y) = \begin{cases} 8xy & 0 < x < y < 1 \\ \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

14. n unbiased coins are tossed. Find the distribution of the counting random variable X, that counts the number of "heads."

15. n lamps are available for use, each of which may be defective with probability p. A lamp is screwed into the holder and tested, if faulty it burns and is immediately replaced. A random variable X that corresponds to the number of lamps that must be tried is considered. Construct the ordered series. Find E[X].

16. Two marksmen fire at two targets. Each fires at his target independent of the other. Let the probability of the marksman 1 hitting the target be p_1, and that of marksman 2 be p_2 respectively. Three RVs are considered:

    - X_1 - number of times marksman 1 hits the target
    - X_2 - number of times marksman 2 hits the target

    $Z = X_1 - X_2$. Construct the ordered series for$Z$, and determine its mean and variance.

17. Consider an oscillator $Acos\omega_c t$ whose frequency varies uniformly between [0,W]. Determine whether this process is stationary.

18. Determine whether the random process $X(t) = Asin(\omega t + \phi)$, where A and ω are constants and ϕ is a random variable uniformly distributed in (0,2 π) is stationary.