Laplace Transforms

CS 130 - Mathematical Methods in Computer Science

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Topics

Topics

1. Laplace Transforms
   - Laplace Transforms and Solving ODEs
   - Laplace Transform
   - Useful Rules and Theorems
2. Inverse Laplace Transforms
3. Solving First and Second Order ODEs
   - Examples
   - General Solutions of ODEs and Laplace Transforms
4. Solving Systems of ODEs

Laplace Transforms and Solving ODEs

(Systems of) ODEs $\Leftrightarrow$ (Inverse) Laplace Transform $\Leftrightarrow$ Algebraic System

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- Initial conditions are usually needed to solve the ODEs

Laplace Transforms and Solving ODEs

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- (System of) Linear ODEs with constant coefficients
- Initial conditions are usually needed to solve the ODEs
  - Implicitly, we are looking for a particular solution when solving ODEs using Laplace Transforms

Topics

Laplace Transform

- Laplace transform of a function f(t), $L\left[f(t)\right]$ defined for $t > 0$:

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  - Alternative notation for the Laplace transform of f(t) is F(s) or X(s)

Laplace Transforms of Some Functions

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$$L[k] = \frac{k}{s}$$

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$$L[e^{-at}] = \frac{1}{s+a}$$

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$$\Rightarrow L[\cos(at)] = \frac{s}{a^2} - \frac{s^2}{a^2}L[\cos(at)] \Rightarrow \left(1 + \frac{s^2}{a^2}\right)L[\cos(at)] = \frac{s}{a^2}$$

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$$L[\cos(at)] = \frac{s}{s^2+a^2}$$

Laplace Transforms of Some Functions

- $L[\sin(at)] = \frac{a}{s^2+a^2}$
- $L[t^n] = \frac{n!}{s^{n+1}}$
- $L[te^{-at}] = \frac{1}{(s+a)^2}$
- $L[t\cos(at)] = \frac{(s^2-a^2)}{(s^2+a^2)^2}$
- $L[t\sin(at)] = \frac{2as}{(s^2+a^2)^2}$
- $L[\sin(at) - at\cos(at)] = \frac{2a^3}{(s^2+a^2)^2}$
- $L[\cosh(at)] = \frac{s}{s^2-a^2}$
- $L[\sinh(at)] = \frac{a}{s^2-a^2}$

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$$= 2\left(\frac{5!}{s^6}\right) + 7\frac{s}{s^2+16} - \frac{1}{s}$$

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$$\Rightarrow F(s+a) = \frac{3}{(s+2)^2+9} = L[e^{-2t}\sin(3t)]$$

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$$sL[f(t)] - f(0) = s\left(\frac{6}{(s+1)^4}\right) - 0 = \frac{6s}{(s+1)^4} = L[f'(t)]$$

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- Laplace transform of a second derivative:

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Some Examples/Exercises

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- $L[(t+1)^3]$
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- Given $\frac{dx}{dt} + x = e^t$, and $x(0) = 0$, what is $L[x(t)]$?

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- (Another) Useful tip: Partial fraction decomposition can help make some functions be more familiar

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$$L^{-1}[F(s)] = f(t) = \frac{3}{2}t^2 + 4e^{2t}$$

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Examples

- $F(s) = \frac{2s+3}{s^2+3s}$

$$\frac{2s+3}{s^2+3s} \Rightarrow \frac{2s+3}{s(s+3)} \Rightarrow \frac{A}{s} + \frac{B}{s+3}$$

$$\Rightarrow \frac{1}{s} + \frac{1}{s+3}$$

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$$L^{-1}[F(s)] = f(t) = 1 + e^{-3t}$$

Examples

- $F(s) = \frac{1}{s^2+9}$

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- $F(s) = \frac{1}{s^2+9}$

$$\frac{1}{s^2 + 9}$$

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Examples

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$$\frac{1}{(s+2)^5} = G(s+2)$$

Examples

- $F(s) = \frac{1}{(s+2)^5}$

$$\frac{1}{(s+2)^5} = G(s+2) \Rightarrow G(s) = \frac{1}{s^5} = \frac{1}{24}\frac{24}{s^5}$$

Examples

- $F(s) = \frac{1}{(s+2)^5}$

$$\frac{1}{(s+2)^5} = G(s+2) \Rightarrow G(s) = \frac{1}{s^5} = \frac{1}{24}\frac{24}{s^5} \Rightarrow \frac{1}{24}L^{-1}\left[\frac{24}{s^5}\right] = \frac{1}{24}t^4$$

Examples

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By the first shifting theorem, $L^{-1}[G(s+2)] = L^{-1}[G(s)]e^{-2t}$

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$$L^{-1}[F(s)] = f(t) = \frac{1}{24}t^4 e^{-2t}$$

Examples

- $F(s) = \frac{s}{s^2+4s+13}$

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Examples

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$$L^{-1}[G(s)] = \cos(3t) - \frac{2}{3}\sin(3t)$$

$$L^{-1}[F(s)] = f(t) = e^{-2t}\left(\cos(3t) - \frac{2}{3}\sin(3t)\right)$$

Topics

Topics

Reminder

(Systems of) ODEs $\Leftrightarrow$ (Inverse) Laplace Transform $\Leftrightarrow$ Algebraic System

- (System of) Linear ODEs with constant coefficients
- Additionally, initial conditions are also needed to solve the ODEs
  - Implicitly, we are looking for a particular solution when solving ODEs using Laplace Transforms

Examples

- $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 0$, where $x = 3$ and $\frac{dx}{dt} = 0$ when $t = 0$

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- $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 0$, where $x = 3$ and $\frac{dx}{dt} = 0$ when $t = 0$

$$L\left[\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x\right] = L[0]$$

Examples

- $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 0$, where $x = 3$ and $\frac{dx}{dt} = 0$ when $t = 0$

$$L\left[\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x\right] = L[0]$$

$$\Rightarrow s^2X(s) - sx(0) - x'(0) + 4(sX(s) - x(0)) + 13X(s) = 0$$

Examples

- $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 0$, where $x = 3$ and $\frac{dx}{dt} = 0$ when $t = 0$

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$$\Rightarrow s^2X(s) - 3s + 4sX(s) - 12 + 13X(s) = 0$$

Examples

- $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 0$, where $x = 3$ and $\frac{dx}{dt} = 0$ when $t = 0$

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$$\Rightarrow s^2X(s) + 4sX(s) + 13X(s) = 3s + 12$$

$$\Rightarrow (s^2 + 4s + 13)X(s) = 3s \Rightarrow X(s) = \frac{3s + 12}{s^2 + 4s + 13}$$

Examples

- $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 0$, where $x = 3$ and $\frac{dx}{dt} = 0$ when $t = 0$

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- $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 0$, where $x = 3$ and $\frac{dx}{dt} = 0$ when $t = 0$

$$X(s) = \frac{3s+12}{s^2+4s+13} \Rightarrow X(s) = \frac{3s+12}{(s+2)^2+9}$$

Examples

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$$X(s) = \frac{3s+12}{s^2+4s+13} \Rightarrow X(s) = \frac{3s+12}{(s+2)^2+9}$$

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Examples

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$$x(t) = 3e^{-2t}\cos(3t) + 2e^{-2t}\sin(3t)$$

Examples

- $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 50\sin t$, where $x = 1$ and $\frac{dx}{dt} = 4$ when $t = 0$

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$$s^2X(s) - s - 4 + 6(sX(s) - 1) + 9X(s) = \frac{50}{s^2+1}$$

Examples

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$$\Rightarrow s^2X(s) + 6sX(s) + 9X(s) - s - 10 = \frac{50}{s^2+1}$$

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$$\Rightarrow (s^2 + 6s + 9)X(s) = \frac{50}{s^2+1} + s + 10$$

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$$\Rightarrow X(s) = \frac{50}{(s^2+1)(s+3)^2} + \frac{s+10}{(s+3)^2}$$

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$$X(s) = 12\frac{1}{(s+3)^2} + 4\frac{1}{s+3} - 3\frac{s}{s^2+1} + 4\frac{1}{s^2+1}$$

Examples

- $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 50\sin t$, where
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$$x(t) = 12te^{-3t} + 4e^{-3t} - 3\cos t + 4\sin t$$

Examples

- $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} - 3x = 4e^t$, where $x = 1$ and $\frac{dx}{dt} = -2$ when $t = 0$

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$$\Rightarrow X(s) = \frac{2}{s-1} + \frac{-s-8}{s^2+4s-3}$$

Examples

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$$\Rightarrow X(s) = \frac{2}{s-1} + \frac{-s-2-6}{(s+2)^2-7}$$

Examples

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$$\Rightarrow X(s) = \frac{2}{s-1} - \frac{s+2}{(s+2)^2-7} - \frac{6}{(s+2)^2-7}$$

$$\Rightarrow X(s) = 2\frac{1}{s-1} + (-1)\frac{s+2}{(s+2)^2-7} + \left(-\frac{6}{\sqrt{7}}\right)\frac{\sqrt{7}}{(s+2)^2-7}$$

Examples

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$$x(t) = 2e^t - e^{-2t}\cosh\left(\sqrt{7}t\right) - \frac{6}{\sqrt{7}}e^{-2t}\sinh\left(\sqrt{7}t\right)$$

Topics

General Solution of an ODE using (Inverse) Laplace Transforms

- In the event that no boundary conditions are given, we can still solve an ODE using Laplace transforms by setting $x(0) = A$ and $x'(0) = B$, where A and B are arbitrary constants.

General Solution of an ODE using (Inverse) Laplace Transforms

- In the event that no boundary conditions are given, we can still solve an ODE using Laplace transforms by setting $x(0) = A$ and $x'(0) = B$, where A and B are arbitrary constants.
- In actuality, we could solve the previous examples by solving first for the general solution, then plugging the boundary conditions to get the particular solution.
    - Obviously, that would take a little bit more work (which we usually **DO NOT** like), 'no?

Example

- $\frac{d^2x}{dt^2} + 4x = 0$

Example

- $\frac{d^2x}{dt^2} + 4x = 0$

$$s^2X(s) - As - B + 4X(s) = 0$$

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$$s^2X(s) - As - B + 4X(s) = 0 \Rightarrow X(s) = \frac{As + B}{s^2 + 4}$$

Example

- $\frac{d^2x}{dt^2} + 4x = 0$

$$s^2X(s) - As - B + 4X(s) = 0 \Rightarrow X(s) = \frac{As + B}{s^2 + 4}$$

$$\Rightarrow X(s) = A\frac{s}{s^2 + 4} + B\frac{1}{s^2 + 4}$$

Example

- $\frac{d^2x}{dt^2} + 4x = 0$

$$s^2X(s) - As - B + 4X(s) = 0 \Rightarrow X(s) = \frac{As + B}{s^2 + 4}$$

$$\Rightarrow X(s) = A\frac{s}{s^2 + 4} + B\frac{1}{s^2 + 4}$$

$$x(t) = A\cos(2t) + B\sin(2t)$$

Topics

Solving Systems of ODEs using Laplace Transforms

- Firstly, take the Laplace Transform of each equations.

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- Solve the resulting linear system algebraically.
  - One dependent variable gets solved

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Examples

$$\frac{dy}{dt} + 2x = e^t$$

$$\frac{dx}{dt} - 2y = 1 + t$$

$$x(0) = 1 \text{ and } y(0) = 2$$

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$$x(0) = 1 \text{ and } y(0) = 2$$

- Firstly, take the Laplace Transform of each equations.

$$sY(s) - 2 + 2X(s) = \frac{1}{s-1}$$

$$sX(s) - 1 - 2Y(s) = \frac{1}{s} + \frac{1}{s^2}$$

Examples

$$\frac{dy}{dt} + 2x = e^t$$
$$\frac{dx}{dt} - 2y = 1 + t$$
$$x(0) = 1 \text{ and } y(0) = 2$$

- Solve the resulting linear system algebraically.

Examples

$$\frac{dy}{dt} + 2x = e^t$$
$$\frac{dx}{dt} - 2y = 1 + t$$
$$x(0) = 1 \text{ and } y(0) = 2$$

- Solve the resulting linear system algebraically.

$$sY(s) + 2X(s) = \frac{1}{s-1} + 2$$
$$sX(s) - 2Y(s) = \frac{1}{s} + \frac{1}{s^2} + 1$$

Examples

$$\frac{dy}{dt} + 2x = e^t$$
$$\frac{dx}{dt} - 2y = 1 + t$$
$$x(0) = 1 \text{ and } y(0) = 2$$

- Solve the resulting linear system algebraically.

$$sY(s) + 2X(s) = \frac{1}{s-1} + 2$$
$$sX(s) - 2Y(s) = \frac{1}{s} + \frac{1}{s^2} + 1$$
$$\Rightarrow (4 + s^2)X(s) = \frac{2}{s-1} + 4 + 1 + \frac{1}{s} + s$$

Examples

$$\frac{dy}{dt} + 2x = e^t$$

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$$x(0) = 1 \text{ and } y(0) = 2$$

- Solve the resulting linear system algebraically.

$$(4 + s^2)X(s) = \frac{2}{s-1} + 4 + 1 + \frac{1}{s} + s$$

$$\Rightarrow X(s) = \frac{2}{(s-1)(4+s^2)} + \frac{5}{4+s^2} + \frac{1}{s(4+s^2)} + \frac{s}{4+s^2}$$

Examples

$$\frac{dy}{dt} + 2x = e^t$$

$$\frac{dx}{dt} - 2y = 1 + t$$

$$x(0) = 1 \text{ and } y(0) = 2$$

- Apply inverse Laplace transform on the solution for one variable.

$$X(s) = \frac{2}{(s-1)(4+s^2)} + \frac{5}{4+s^2} + \frac{1}{s(4+s^2)} + \frac{s}{4+s^2}$$

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$$\frac{dy}{dt} + 2x = e^t$$
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- Apply inverse Laplace transform on the solution for one variable.

$$X(s) = \frac{2}{(s-1)(4+s^2)} + \frac{5}{4+s^2} + \frac{1}{s(4+s^2)} + \frac{s}{4+s^2}$$

$$\Rightarrow X(s) = \frac{2}{5}\frac{1}{s-1} + \frac{7}{20}\frac{s}{s^2+4} + \frac{23}{5}\frac{1}{s^2+4} + \frac{1}{4}\frac{1}{s}$$

Examples

$$\frac{dy}{dt} + 2x = e^t$$
$$\frac{dx}{dt} - 2y = 1 + t$$
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$$\frac{dy}{dt} + 2x = e^t$$

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$$x(t) = \frac{2}{5}e^t + \frac{7}{20}\cos(2t) + \frac{23}{10}\sin(2t) + \frac{1}{4}$$

Examples

$$\frac{dy}{dt} + 2x = e^t$$
$$\frac{dx}{dt} - 2y = 1 + t$$
$$x(0) = 1 \text{ and } y(0) = 2$$

- "Back substitute" to solve for the other variable(s) using the obtained solution for one dependent variable.
$$x(t) = \frac{2}{5}e^t + \frac{7}{20}\cos(2t) + \frac{23}{10}\sin(2t) + \frac{1}{4}$$

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$$\frac{dy}{dt} + 2x = e^t$$
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- "Back substitute" to solve for the other variable(s) using the obtained solution for one dependent variable.

$$x(t) = \frac{2}{5}e^t + \frac{7}{20}\cos(2t) + \frac{23}{10}\sin(2t) + \frac{1}{4}$$

$$y = \frac{1}{2}\left(\frac{dx}{dt} - 1 - t\right)$$

Examples

$$\frac{dy}{dt} + 2x = e^t$$

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$$x(0) = 1 \text{ and } y(0) = 2$$

- "Back substitute" to solve for the other variable(s) using the obtained solution for one dependent variable.

$$y = \frac{1}{2}\left(\frac{dx}{dt} - 1 - t\right)$$

Examples

$$\frac{dy}{dt} + 2x = e^t$$

$$\frac{dx}{dt} - 2y = 1 + t$$

$$x(0) = 1 \text{ and } y(0) = 2$$

- "Back substitute" to solve for the other variable(s) using the obtained solution for one dependent variable.

$$y = \frac{1}{2}\left(\frac{dx}{dt} - 1 - t\right)$$

$$y(t) = \frac{1}{5}e^t + \frac{23}{10}\cos(2t) - \frac{7}{20}\sin(2t) - \frac{1}{2} - \frac{t}{2}$$

Examples

$$\frac{dx}{dt} - y = \sin t$$

$$\frac{dy}{dt} + x = \cos t$$

$$x(0) = 3 \text{ and } y(0) = 4$$

Examples

$$\frac{dx}{dt} - y = \sin t$$

$$\frac{dy}{dt} + x = \cos t$$

$$x(0) = 3 \text{ and } y(0) = 4$$

- Firstly, take the Laplace Transform of each equations.

$$sX(s) - 3 - Y(s) = \frac{1}{s^2+1}$$

$$sY(s) - 4 + X(s) = \frac{s}{s^2+1}$$

Examples

$$\frac{dx}{dt} - y = \sin t$$
$$\frac{dy}{dt} + x = \cos t$$
$$x(0) = 3 \text{ and } y(0) = 4$$

- Solve the resulting linear system algebraically.

$$sX(s) - Y(s) = \frac{1}{s^2+1} + 3$$
$$sY(s) + X(s) = \frac{s}{s^2+1} + 4$$

Examples

$$\frac{dx}{dt} - y = \sin t$$
$$\frac{dy}{dt} + x = \cos t$$
$$x(0) = 3 \text{ and } y(0) = 4$$

- Solve the resulting linear system algebraically.

$$sX(s) - Y(s) = \frac{1}{s^2+1} + 3$$
$$sY(s) + X(s) = \frac{s}{s^2+1} + 4$$

$$\Rightarrow s^2X(s) + X(s) = \frac{2s}{s^2+1} + 3s + 4$$

Examples

$$\frac{dx}{dt} - y = \sin t$$
$$\frac{dy}{dt} + x = \cos t$$
$$x(0) = 3 \text{ and } y(0) = 4$$

- Solve the resulting linear system algebraically.

$$s^2X(s) + X(s) = \frac{2}{s^2+1} + 3s + 4$$

Examples

$$\frac{dx}{dt} - y = \sin t$$
$$\frac{dy}{dt} + x = \cos t$$
$$x(0) = 3 \text{ and } y(0) = 4$$

- Solve the resulting linear system algebraically.

$$s^2X(s) + X(s) = \frac{2}{s^2+1} + 3s + 4$$

$$\Rightarrow X(s) = \frac{2s}{(s^2+1)^2} + \frac{3s}{s^2+1} + \frac{4}{s^2+1}$$

Examples

$$\frac{dx}{dt} - y = \sin t$$

$$\frac{dy}{dt} + x = \cos t$$

$$x(0) = 3 \text{ and } y(0) = 4$$

- Apply inverse Laplace transform on the solution for one variable.

$$X(s) = \frac{2s}{(s^2+1)^2} + \frac{3s}{s^2+1} + \frac{4}{s^2+1}$$

Examples

$$\frac{dx}{dt} - y = \sin t$$
$$\frac{dy}{dt} + x = \cos t$$
$$x(0) = 3 \text{ and } y(0) = 4$$

- Apply inverse Laplace transform on the solution for one variable.

$$X(s) = \frac{2s}{(s^2+1)^2} + \frac{3s}{s^2+1} + \frac{4}{s^2+1}$$

$$x(t) = t\sin t + 3\cos t + 4\sin t$$

Examples

$$\frac{dx}{dt} - y = \sin t$$
$$\frac{dy}{dt} + x = \cos t$$
$$x(0) = 3 \text{ and } y(0) = 4$$

- "Back substitute" to solve for the other variable(s) using the obtained solution for one dependent variable.

$$x(t) = t\sin t + 3\cos t + 4\sin t$$

Examples

$$\frac{dx}{dt} - y = \sin t$$
$$\frac{dy}{dt} + x = \cos t$$
$$x(0) = 3 \text{ and } y(0) = 4$$

- "Back substitute" to solve for the other variable(s) using the obtained solution for one dependent variable.

$$x(t) = t \sin t + 3 \cos t + 4 \sin t$$

$$y = \frac{dy}{dt} - \sin t$$

Examples

$$\frac{dx}{dt} - y = \sin t$$
$$\frac{dy}{dt} + x = \cos t$$
$$x(0) = 3 \text{ and } y(0) = 4$$

- "Back substitute" to solve for the other variable(s) using the obtained solution for one dependent variable.

$$x(t) = t\sin t + 3\cos t + 4\sin t$$

$$y = \frac{dy}{dt} - \sin t$$

$$y(t) = t\cos t - 4\sin t + 4\cos t$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$

$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$

$$x(0) = 3 \text{ and } y(0) = 0$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$

$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$

$$x(0) = 3 \text{ and } y(0) = 0$$

- Firstly, take the Laplace Transform of each equations.

$$sX(s) - 3 + 4sY(s) + 6Y(s) = 0$$

$$5sX(s) - 15 + 2sY(s) + 6X(s) = 0$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$
$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$
$$x(0) = 3 \text{ and } y(0) = 0$$

- Solve the resulting linear system algebraically.

$$sX(s) + (4s + 6)Y(s) = 3$$
$$(5s + 6)X(s) + 2sY(s) = 15$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$
$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$
$$x(0) = 3 \text{ and } y(0) = 0$$

- Solve the resulting linear system algebraically.

$$sX(s) + (4s+6)Y(s) = 3$$
$$(5s+6)X(s) + 2sY(s) = 15$$

$$\Rightarrow \left(\left(-\frac{5s+6}{s}\right)(4s+6) + 2s\right) Y(s) = -3\frac{5s+6}{s} + 15$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$

$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$

$$x(0) = 3 \text{ and } y(0) = 0$$

- Solve the resulting linear system algebraically.

$$\left(\left(\frac{-5s-6}{s}\right)(4s+6) + 2s\right) Y(s) = -3\frac{5s+6}{s} + 15$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$
$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$
$$x(0) = 3 \text{ and } y(0) = 0$$

- Solve the resulting linear system algebraically.

$$\left(\left(\frac{-5s-6}{s}\right)(4s+6) + 2s\right) Y(s) = -3\frac{5s+6}{s} + 15$$

$$\Rightarrow \left(\frac{-18s^2-30s-36}{s}\right) Y(s) = \frac{-18}{s}$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$

$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$

$$x(0) = 3 \text{ and } y(0) = 0$$

- Solve the resulting linear system algebraically.

$$\left(\frac{-18s^2 - 54s - 36}{s}\right) Y(s) = \frac{-18}{s}$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$

$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$

$$x(0) = 3 \text{ and } y(0) = 0$$

- Solve the resulting linear system algebraically.

$$\left(\frac{-18s^2 - 54s - 36}{s}\right) Y(s) = \frac{-18}{s}$$

$$Y(s) = \frac{18}{18s^2 + 54s + 36} = \frac{1}{s^2 + 3s + 2}$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$
$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$
$$x(0) = 3 \text{ and } y(0) = 0$$

- Apply inverse Laplace transform on the solution for one variable.

$$Y(s) = \frac{1}{s^2 + 3s + 2}$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$
$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$
$$x(0) = 3 \text{ and } y(0) = 0$$

- Apply inverse Laplace transform on the solution for one variable.

$$Y(s) = \frac{1}{s^2 + 3s + 2} = \frac{-1}{s+2} + \frac{1}{s+1}$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$

$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$

$$x(0) = 3 \text{ and } y(0) = 0$$

- Apply inverse Laplace transform on the solution for one variable.

$$Y(s) = \frac{1}{s^2 + 3s + 2} = \frac{-1}{s+2} + \frac{1}{s+1}$$

$$y(t) = -e^{-2t} + e^{-t}$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$

$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$

$$x(0) = 3 \text{ and } y(0) = 0$$

- "Back substitute" to solve for the other variable(s) using the obtained solution for one dependent variable.

$$y(t) = -e^{-2t} + e^{-t}$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$

$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$

$$x(0) = 3 \text{ and } y(0) = 0$$

- "Back substitute" to solve for the other variable(s) using the obtained solution for one dependent variable.

$$y(t) = -e^{-2t} + e^{-t}$$

$$x = \frac{1}{6}\left(5\frac{dx}{dt} + 2\frac{dy}{dt}\right)$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$
$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$
$$x(0) = 3 \text{ and } y(0) = 0$$

- "Back substitute" to solve for the other variable(s) using the obtained solution for one dependent variable.

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Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$
$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$
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- "Back substitute" to solve for the other variable(s) using the obtained solution for one dependent variable.
$$y(t) = -e^{-2t} + e^{-t}$$

$$\frac{dx}{dt} = -4\frac{dy}{dt} - 6y$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$
$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$
$$x(0) = 3 \text{ and } y(0) = 0$$

- "Back substitute" to solve for the other variable(s) using the obtained solution for one dependent variable.

$$y(t) = -e^{-2t} + e^{-t}$$

$$\frac{dx}{dt} = -4\frac{dy}{dt} - 6y = -4(2e^{-2t} - e^{-t}) - 6(-e^{-2t} + e^{-t})$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$
$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$
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$$y(t) = -e^{-2t} + e^{-t}$$

$$\frac{dx}{dt} = -4\frac{dy}{dt} - 6y = -4(2e^{-2t} - e^{-t}) - 6(-e^{-2t} + e^{-t}) = -2e^{-2t} - 2e^{-t}$$

Examples

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 6y = 0$$
$$5\frac{dx}{dt} + 2\frac{dy}{dt} + 6x = 0$$
$$x(0) = 3 \text{ and } y(0) = 0$$

- "Back substitute" to solve for the other variable(s) using the obtained solution for one dependent variable.

$$y(t) = -e^{-2t} + e^{-t}$$

$$\frac{dx}{dt} = -4\frac{dy}{dt} - 6y = -4(2e^{-2t} - e^{-t}) - 6(-e^{-2t} + e^{-t}) = -2e^{-2t} - 2e^{-t}$$

$$x(t) = e^{-2t} + 2e^{-t}$$

END OF LESSON 10