CS 130

Problem Set 1

Due: January 10, 2014

General Instructions

- Answer the items completely. Show your solutions. You may use Scilab or any equivalent tool to verify your answers, but you must show the steps to manually come up with the answer itself. No merits will be given if no solutions are provided.
- Express non-whole numbers (i.e. those with decimal parts) in your answers as fractions. Failure to do so will merit a deduction.
- If you have consulted references (books, journal articles, online materials, other people), cite them as footnotes to the specific item where you used the resource/s as reference.
- Save your answer sheet as a PDF file. The answer sheet can be made using any word processing program (although preferably, one uses LaTeXto make it).
- Include as part of your submissions the Scilab script source codes (.sci files) made as part in some of the items in the problem set.
- Create a .zip file containing the PDF file (answer sheet) and the .R files. Use your 9-digit student number (without the dash) as the name of the .zip file.
- Submission of the problem set answers should be done via e-mail. Attach the .zip file, and write as the subject header of the e-mail: [CS 130] $<$ *Student Number* $>$ $-$ $<$ *Last Name, First Name* $>$ $-$ Problem Set 1. For example, [CS 130] 201300011 - Kapayapaan, Reynaldo - Problem Set 1. Send your answers to janmichaelyap@gmail.com.
- If you have any questions regarding an item (EXCEPT the answer and solution) in the problem set, do not hesitate to e-mail me to ask them.

Questions

1. Consider the following matrix:

$$A = \begin{pmatrix} 5 & 0 & -3 & 2 & 4 \\ 5 & 9 & 7 & 8 & -9 \\ 1 & 1 & 6 & 1 & 1 \\ 0 & 0 & -0 & 2 & 0 \\ 2 & -3 & 2 & -3 & 5 \end{pmatrix}$$

   (a) What is the determinant of A?

   (b) If A is invertible, compute A^{-1}. If not, compute $A^{+} = (A^T A)^{-1} A^T$.

2. Consider the set $V_{10} = \left\{ \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} | v_i \in \{0,1,2,3,4,5,6,7,8,9\} \right\}$. Next, we define an analog vector addition $+_{10}$ where, given two vectors from V_{10}, say $s = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{pmatrix}$ and and $t = \begin{pmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{pmatrix}$, $t +_{10} u = \begin{pmatrix} (s_1 + t_1) \mod 10 \\ (s_2 + t_2) \mod 10 \\ \vdots \\ (s_n + t_n) \mod 10 \end{pmatrix}$, where mod is the modulo operator. Next, we define an analog scalar multiplication $\cdot_{10}$, where given a scalar $c \in \{0,1,2,3,4,5,6,7,8,9\}$ and $v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \in V_{10}, c \cdot_{10} v = \begin{pmatrix} (cv_1) \mod 10 \\ (cv_2) \mod 10 \\ \vdots \\ (cv_n) \mod 10 \end{pmatrix}$. Note that given two scalars, say $c, d \in \{0,1,2,3,4,5,6,7,8,9\}, c \cdot_{10} d = (cd) \mod 10$. Is V_{10} a vector space? If your answer is yes, show that it preserves all properties of a vector space. If no, show only the property/ies violated by V_{10} and the analog vector operations.

3. Given the following matrix below:

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

   (a) Compute for the eigenvalues and the corresponding eigenspaces of B.

   (b) Is B diagonalizable? Justify your answer.

4. Let $L : \mathbb{R}^3 \to \mathbb{R}^2, L\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} xy \\ z \end{pmatrix}$. Is L a linear transformation?

For items 5 and 6, consider the following matrix:

$$C = \begin{pmatrix} 8 & 2 & 9 \\ 4 & 9 & 4 \\ 6 & 7 & 9 \end{pmatrix}$$

5. Compute the LU factorization of C by hand using the method shown in class. Next, use the *lu* function in Scilab to solve for the LU factorization of the matrix above. Is the output of Scilab's *lu* function the same as the manually calculated L and U matrices?

6. Compute the QR factorization of C by hand using the method shown in class. Next, use the *qr* function in Scilab to solve for the QR factorization of the matrix above. Is the output of the *qr* function the same as the manually calculated Q and R matrices?

7. We will now do some basic image compression using Scilab. Firstly, download the signal toolbox and the general toolbox, downloadable from our class web page. Next, unzip the contents and make sure to take note of the paths of the folders (absolute or relative to your Scilab workspace). We then load the toolboxes like so:

```
--> getd("toolbox_general/");
--> getd("toolbox_signal/");
```

   The semi-colon at the end of each statements makes the execution of the function non-verbose, i.e. it will not display the details of the execution, or in the case of variable assignments, the contents of the variable/s. Next we load the sample image in the signal toolbox with filename *lena.bmp*. To do this, we use the *load_image* function:

```
--> M = load_image("lena", 256);
```

The function reads the image file, then saves it as a matrix (represented by the variable M). The second input parameter (256) specifies the size of the image in pixels, and thus assumes a square image. To view the image, we use the following statements:

```
--> clf;
--> imageplot(M);
```

We then get the singular value decomposition of the matrix representing the image:

```
--> [U S V] = svd(M);
```

To perform image compression, the idea is to delete the last k singular values by setting them to zero. The compressed image is then formed by reconstituting the image matrix, i.e. $M' = U\Sigma' V^T$ where M' is the compressed image matrix and Σ' is the matrix with the last k singular values set to zero.

(a) Write a Scilab script that will perform the compression on M, with specifiable input parameter k that would represent the last k singular values to be set to zero, and returns the compressed matrix M'. Name the R script *myImgCmprsn.R*

(b) What is the maximum value of k such that the girl in the image is still recognizable? Display the compressed image as proof.