CS 130

Long Exam 3

October 2, 2013

General Instructions

- Answer the items completely. Show your solutions.
- Express non-whole numbers (i.e. those with decimal parts) in your answers as fractions. Failure to do so will merit a deduction.
- Write as legibly as possible. Illegible or unreadable answers and solutions may not merit any points.
- Refrain from making unnecessary motions and sounds during the exam. Any suspicious behavior will be dealt with accordingly.
- Direct all questions to the proctor.
- If you need to go to the CR, hand your questionnaire, answer sheet, and scratch paper to the proctor before heading out. Only one person at any given time is allowed to go out.
- Once you're done with the exam (one way or the other), place your scratch papers and the questionnaire inside your blue book.

Questions

1. Given the following system of ODEs:

$$\frac{dy}{dt} = -5y + z + 6e^{2t}$$

$$\frac{dz}{dt} = 4y - 2z - e^{2t}$$

   (a) Solve for the general solution using the matrix method

   (b) Verify your answer to (a) by solving for the general solution again, this time using the substitution method
   ANSWER (for both (a) and (b)):

$$\begin{pmatrix} y \\ z \end{pmatrix} = A \begin{pmatrix} 1 \\ 4 \end{pmatrix} e^{-t} + B \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-6t} + \begin{pmatrix} \frac{23}{24} \\ \frac{17}{24} \end{pmatrix} e^{2t}$$

   (c) Solve for the particular solution using Laplace transforms where $y(0) = 0$ and $z(0) = 0$
   ANSWER:

$$\begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} \\ -\frac{4}{3} \end{pmatrix} e^{-t} + \begin{pmatrix} -\frac{5}{8} \\ \frac{5}{8} \end{pmatrix} e^{-6t} + \begin{pmatrix} \frac{23}{24} \\ \frac{17}{24} \end{pmatrix} e^{2t}$$

2. Given $f(x) = e^{-x}$ on the interval $[-\pi, \pi]$

   (a) What is the coefficient of the Fourier sine series of f(x)?
   ANSWER:
   $$B_n = \frac{n(-1)^n(e^{\pi} - e^{-\pi})}{\pi(n^2 + 1)}$$

   (b) What are the coefficients of the Fourier cosine series of f(x)? *(HINT: You may use values you have already solved for during this exam.)*
   ANSWER:
   $$A_0 = \frac{e^{\pi} - e^{-\pi}}{2\pi}$$
   $$A_n = \frac{(-1)^n(e^{\pi} - e^{-\pi})}{\pi(n^2 + 1)}$$