

CS 130 Exercises

Simultaneous ODEs

February 16, 2013

1.

$$\begin{aligned}\frac{dy}{dx} &= y + 2z + 2x \\ \frac{dz}{dx} &= 3y + 2z - 4x\end{aligned}$$

ANSWER:

$$\begin{pmatrix} y \\ z \end{pmatrix} = A \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + B \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix} e^{4t} + \begin{pmatrix} -\frac{11}{5} \\ \frac{23}{8} \end{pmatrix} + \begin{pmatrix} 3 \\ -\frac{5}{2} \end{pmatrix} x$$

2.

$$\begin{aligned}\frac{dx_2}{dt} + 2x_1 &= e^x \\ \frac{dx_1}{dt} - 2x_2 &= 1 + t\end{aligned}$$

ANSWER:

$$\begin{aligned}x_1 &= A \cos 2t + B \sin 2t + \frac{2}{5}e^t + \frac{1}{4} \\ x_2 &= B \cos 2t - A \sin 2t + \frac{1}{5}e^t - \frac{1}{2} - \frac{1}{2}t\end{aligned}$$

3.

$$\begin{aligned}\frac{dy}{dx} &= -5y + z + 6e^{2x} \\ \frac{dz}{dx} &= 4y + -2z - e^{2x}\end{aligned}$$

ANSWER:

$$\begin{pmatrix} y \\ z \end{pmatrix} = A \begin{pmatrix} 1 \\ 4 \end{pmatrix} e^{-x} + B \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-6x} + \begin{pmatrix} \frac{23}{24} \\ \frac{17}{24} \end{pmatrix} e^{2x}$$

4.

$$\begin{aligned}\frac{dx_1}{dt} - x_2 &= \sin t \\ \frac{dx_2}{dt} + x_1 &= \cos t\end{aligned}$$

ANSWER:

$$\begin{aligned}x_1 &= A \cos t + (B + t) \sin t \\ x_2 &= (B + t) \cos t - A \sin t\end{aligned}$$

5.

$$\frac{dy}{dx} = 2z$$

$$\frac{dz}{dx} = 8y$$

ANSWER:

$$y = Ae^{-4x} + Be^{4x}$$

$$z = -2Ae^{-4x} + 2Be^{4x}$$

6.

$$\frac{dx_1}{dt} = 8x_1 + x_2 + \sin t$$

$$\frac{dx_2}{dt} = -5x_1 + 6x_2 + \cos t$$

ANSWER:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{7t} \left\{ \cos 2t \begin{pmatrix} A \\ -A + 2B \end{pmatrix} + \sin 2t \begin{pmatrix} B \\ -B - 2A \end{pmatrix} \right\} + \begin{pmatrix} -\frac{17}{145} \\ -\frac{10}{145} \end{pmatrix} \sin t + \begin{pmatrix} \frac{1}{145} \\ -\frac{25}{145} \end{pmatrix} \cos t$$