CS 130

Long Exam 2

August 30, 2013

General Instructions

- Answer the items completely. Show your solutions. **Items with no solutions will not merit any points regardless of the correctness of the final answer**.
- Express non-whole numbers (i.e. those with decimal parts) in your answers as fractions. Failure to do so will merit a deduction.
- Write as legibly as possible. Illegible or unreadable answers and solutions may not merit any points.
- Refrain from making unnecessary motions and sounds during the exam. Any suspicious behavior will be dealt with accordingly.
- Direct all questions to the proctor.
- If you need to go to the CR, hand your questionnaire, answer sheet, and scratch paper to the proctor before heading out. Only one person at any given time is allowed to go out.
- Once you're done with the exam (one way or the other), place your scratch papers and the questionnaire inside your blue book.

Questions

1. What is the general solution for the ODE $(2y - x)\frac{dy}{dx} = 2x + y$?
   ANSWER: $-\frac{1}{2}\ln\left(\left(\frac{y}{x}\right)^2 - \frac{y}{x} - 1\right) = \ln x + C$

2. Show that the particular solution of $\cos t\frac{dx}{dt} + x\sin t = 2\cos^3 t\sin t - 1$ subject to the condition $x = 3\sqrt{2}$ when $t = \frac{\pi}{4}$ is $x\sec t = -\frac{1}{2}\cos 2t - \tan t + 7$. *(HINT: $\sin 2t = 2\sin t\cos t$)*
   ANSWER:
   Divide both sides by $\cos t:$ $\frac{dx}{dt} + x\tan t = 2\cos^2 t\sin t - \sec t$, a linear DE.
   Integrating factor is $e^{\int \tan t dt} = e^{\ln(\sec t)} = \sec t$.
   Multiplying the integrating factor to both sides:
   $\sec t\frac{dx}{dt} + x\sec t\tan t = 2\cos t\sin t - \sec^2 t \rightarrow \frac{d(x\sec t)}{dt} = 2\cos t\sin t - \sec^2 t \rightarrow \frac{d(x\sec t)}{dt} = \sin 2t - \sec^2 t$
   Integrating both sides: $x\sec t = -\frac{1}{2}\cos 2t - \tan t + C$
   Letting $x = 3\sqrt{2}$ and $t = \frac{\pi}{4}:$ $3\sqrt{2}(\sec\frac{\pi}{4}) = -\frac{1}{2}\cos\frac{\pi}{2} - \tan\frac{\pi}{4} + C \rightarrow 6 = 0 - 1 + C \rightarrow 7 = C$
   Therefore, the particular solution for the ODE is $x\sec t = -\frac{1}{2}\cos 2t - \tan t + 7$.

3. Solve for the general solution of $y'y^2 = 5y^3 + e^{-2x}$
   ANSWER: $y^3e^{-15x} = -\frac{3}{17}e^{-17x} + C$

4. Solve for the general solution of $y'' - 4y' - 12y = e^{6t}$
   ANSWER: $y = \frac{1}{8}te^{6t} + Ae^{-2t} + Be^{6t}$

5. Show that the general solution of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = \cosh 3x - \sinh 3x$ is $y = \frac{\cosh 3x - \sinh 3x}{10} + Ae^{2x} + Be^{-5x}$ *(HINT:* $\frac{d(\cosh x)}{dx} = \sinh x,\ \frac{d(\sinh x)}{dx} = \cosh x$*)*

   ANSWER:

   Complementary Function v(x):

   Auxiliary function for corresponding homogenous equation is $m^2+3m-10 = 0 \rightarrow (m-2)(m+5) = 0.$

   Hence, $v(x) = Ae^{2x} + Be^{-5x}$

   Particular Integral u(x):

   Let $u(x) = \alpha \sinh 3x + \beta \cosh 3x,\ u'(x) = 3\alpha \cosh 3x + 3\beta \sinh 3x,\ u''(x) = 9\alpha \sinh 3x + 9\beta \cosh 3x$

   $u''(x)+3u'(x)-10u(x) = \cosh 3x - \sinh 3x \rightarrow (9\alpha-\beta)\cosh 3x + (-\alpha+9\beta)\sinh 3x = \cosh 3x - \sinh 3x$

   Using the coefficient of $\cosh 3x$ we have: $9\alpha - \beta = 1 \rightarrow \beta = 9\alpha - 1$

   Substituting to the coefficient of $\sinh 3x:\ -\alpha + 9\beta = -1 \rightarrow 80\alpha - 9 = -1 \rightarrow \alpha = \frac{1}{10}$

   Solving for $\beta:\ \beta = 9\alpha - 1 = \frac{9}{10} - 1 = -\frac{1}{10}$

   Therefore, $u(x) = \frac{cosh3x - \sinh 3x}{10}$. And thus, $y = u(x) + v(x) = \frac{cosh3x - \sinh 3x}{10} + Ae^{2x} + Be^{-5x}$