

Hypotheses and Hypothesis Testing Intro

February 20, 2020

Data Science CSCI 1951A

Brown University

Instructor: Ellie Pavlick

HTAs: Josh Levin, Diane Mutako, Sol Zitter

Announcements

- Final Project Pitches and Feedback
- Data Deliverable—***not*** a ceremonial checkin, please start soon!
- Grades, regrades—read policy on Piazza
- Map Reduce out later today—follow announcements about the cluster
- Map Reduce lab released today, no sections until next week. One optional lab.

Today

- What is a hypothesis?
- Some definitions/notation
- Intuition behind modeling/hypothesis testing

Today

Not to be dramatic, but...

THIS IS PROBABLY THE MOST IMPORTANT
LECTURE IN THE WHOLE COURSE!!!!

- What is a hypothesis?
- Some definitions/notation
- Intuition behind modeling/hypothesis testing

What is a hypothesis?

- #1 most important thing: *falsifiable*

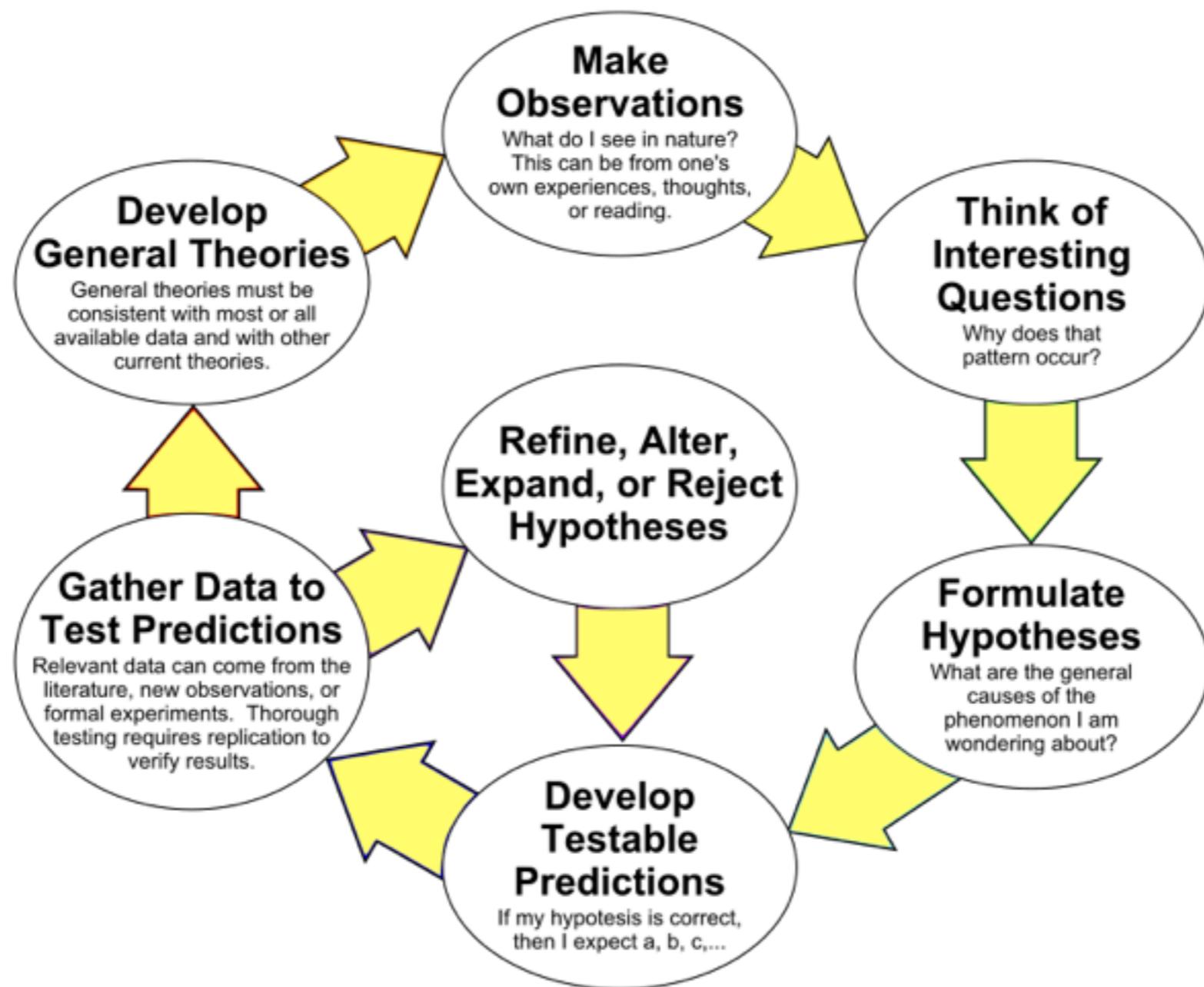
What is a hypothesis?

- #1 most important thing: *falsifiable*
- But also, should be:
 - Disputed (at least a little bit). I.e. it should be tied to a question people are actually asking

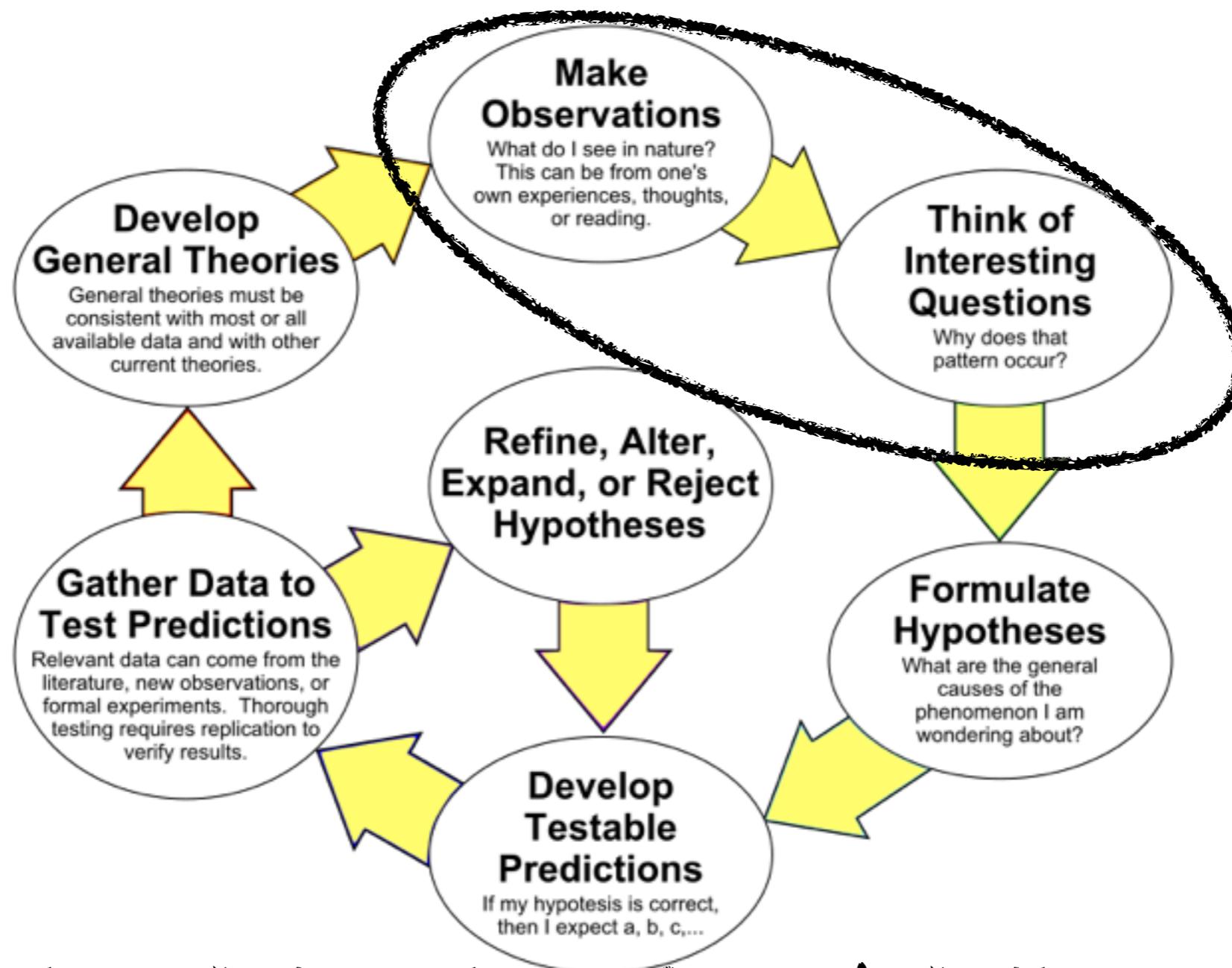
What is a hypothesis?

- #1 most important thing: *falsifiable*
- But also, should be:
 - Disputed (at least a little bit). I.e. it should be tied to a question people are actually asking
 - Specific. Avoid subjective terms like “better than”

What is a hypothesis?

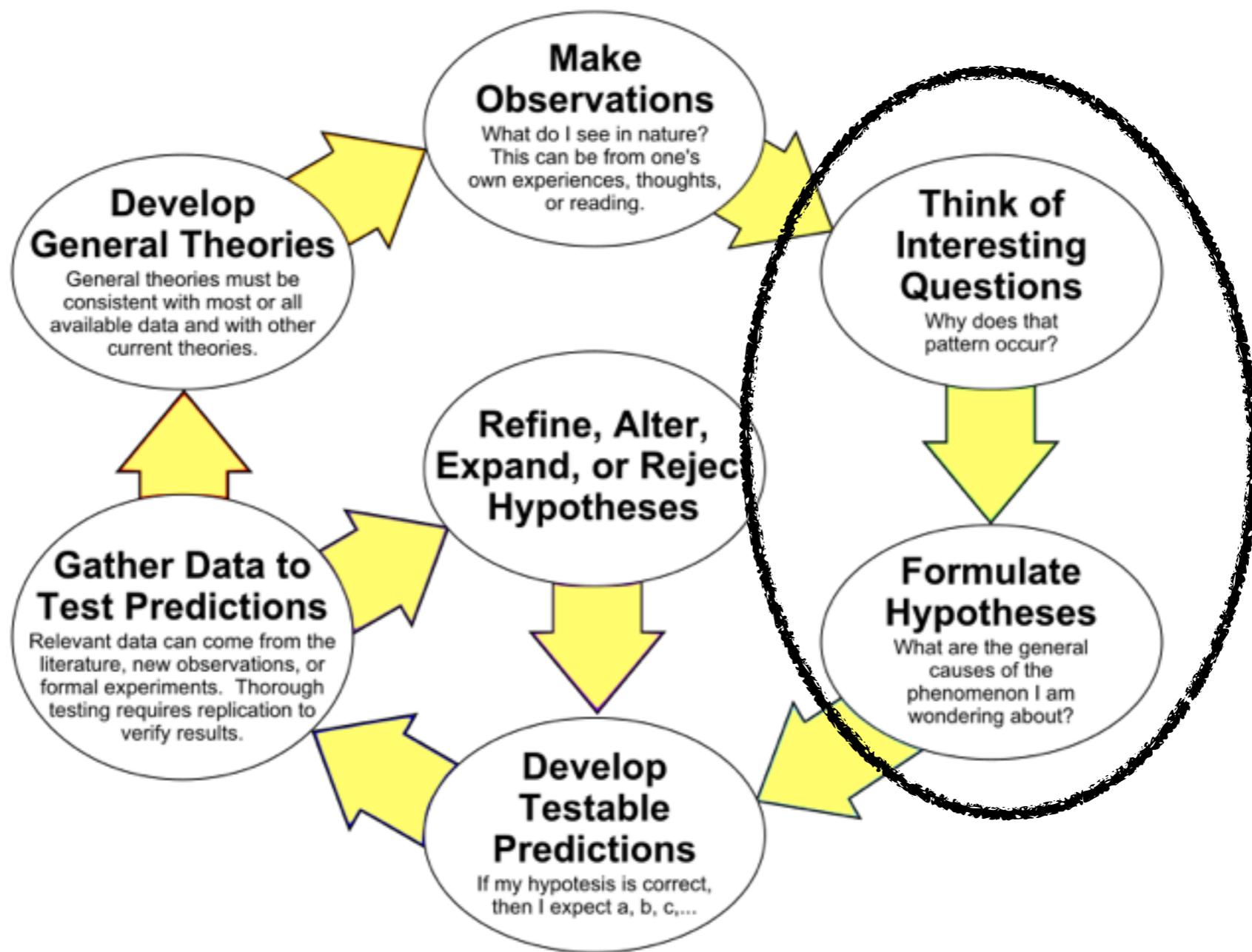


What is a hypothesis?



“explore”, “analyze trends”, “look for patterns”, “visualize”

What is a hypothesis?



Literally the hardest part!

Clicker Question!

Clicker Question!

“Look for differences in political affiliations between universities”

Is this a valid hypothesis?

- a) Yes
- b) No

Clicker Question!

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Is this a valid hypothesis?

- a) Yes
- b) No

Clicker Question!

~~"Look for differences in political affiliations between universities"~~

"There are differences in political affiliations between universities"

Is this a valid hypothesis?

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Clicker Question!

~~"Look for differences"~~

"There are differences"

yes, testable.

no, probably not really
in question.
(though could be)

Is this a valid hypothesis?

- a) Yes
- b) No

Clicker Question!

~~"Look for differences in political affiliations between universities"~~

~~"There are differences in political affiliations between universities"~~

"Coastal universities are more liberal than universities in the heartland"

Is this a valid hypothesis?

- a) Yes
- b) No

Clicker Question!

~~"Look for differences in political affiliations between universities"~~

~~"There are differences in political affiliations between universities"~~

"Coastal universities are more liberal than universities in the heartland"

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Clicker Question!

~~"Look for differences in political affiliations between universities"~~

~~"There are differences in political affiliations between universities"~~

"Coastal universities are more liberal than universities in the **heartland**"

~~need to define these~~

Clicker Question!

~~"Look for differences in political affiliations between universities"~~

~~"There are differences in political affiliations between universities"~~

"Coastal universities are more **liberal** than universities in the heartland"

and this: party affiliation? voting record? general opinions?

Clicker Question!

~~"Look for differences in political affiliations between universities"~~

- scary truth #1: no single right way to do this

"Coastal universities are more **liberal** than universities in the heartland"

and this: party affiliation? voting record? general opinions?

Clicker Question!

~~"Look for differences in political affiliations between~~

scary truth #2: you are biased, you have to work hard to not let yourself just "find what you want to find"

"Coastal universities are more **liberal** than universities in the heartland"

and this: party affiliation? voting record? general opinions?

Clicker Question!

"Look for differences in political affiliations between

scary truth #2: you are biased, you have to work hard to not let yourself just "find what you want to find"

"Coastal universities are more liberal than universities in the
and this: part (in a few lectures) g
record? general opinions!"

to be continued...

g

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- Some definitions/notation
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Statistics vs. Prob. Theory

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- Probability theory: mathematical theory that describes uncertainty.

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 - Distributions/parameters are known

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 - Distributions/parameters are known
- Statistics: techniques for extracting useful information from data.

Statistics vs. Prob. Theory

- Probability theory: mathematical theory that describes uncertainty.
 - Distributions/parameters are known
- Statistics: techniques for extracting useful information from data.
 - Distributions/parameters are generally unknown and need to be estimated from the data

The bigger picture

- Start with real world phenomenon/observations
- Make assumptions about the underlying model
- Fit the parameters of the model based on data

The bigger picture

Whether a coin is heads or tails

What a person will say next

Whether someone will click on an ad

- Start with real world phenomenon/observations
- Make assumptions about the underlying model
- Fit the parameters of the model based on data

The bigger picture

You ***always*** make assumptions about
the structure of the process that is
generating the data

- Start with real world phenomenon/observations
- Make assumptions about the underlying model
- Fit the parameters of the model based on data

"All models are bad, but some are useful"

The bigger picture

Goal is always to "explain the data"

- Start with real world phenomenon/observations
- Make assumptions about the underlying model
- Fit the parameters of the model based on data

Two typical (different) use cases:

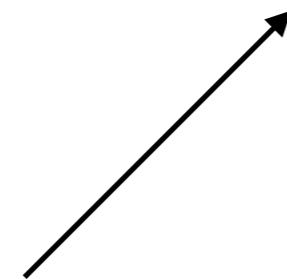
- 1) understand the underlying model better
- 2) make predictions

Probability Space

$$\langle \Omega, \mathcal{F}, P \rangle$$

Probability Space

$$\langle \Omega, F, P \rangle$$



the set of all
possible
outcomes of
the random
process
modeled

Probability Space

$$\langle \Omega, F, P \rangle$$



A family of sets F representing the allowable events, where each set in F is a subset of the sample space Ω

$$F = \{E_i \subseteq \Omega\}_i$$

Probability Space

$$\langle \Omega, F, P \rangle$$



A family of sets F representing the allowable events, where each set in F is a subset of the sample space Ω

$$F = \{E_i \subseteq \Omega\}_i$$

$$F = 2^\Omega$$

Probability Space

$$\langle \Omega, F, P \rangle$$

Probability function which
assigns a real number to each
event in F

$$P : F \rightarrow \mathbb{R}$$

Probability Space

Valid Probability Function:

$$0 \leq P(E) \leq 1 \quad \forall E \in F$$

$$P(\Omega) = 1$$

$$P\left(\bigcup_i E_i\right) = \sum_i P(E_i)$$

$$\langle F, P \rangle$$

Probability function which
gives a real number to each
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$$P : F \rightarrow \mathbb{R}$$

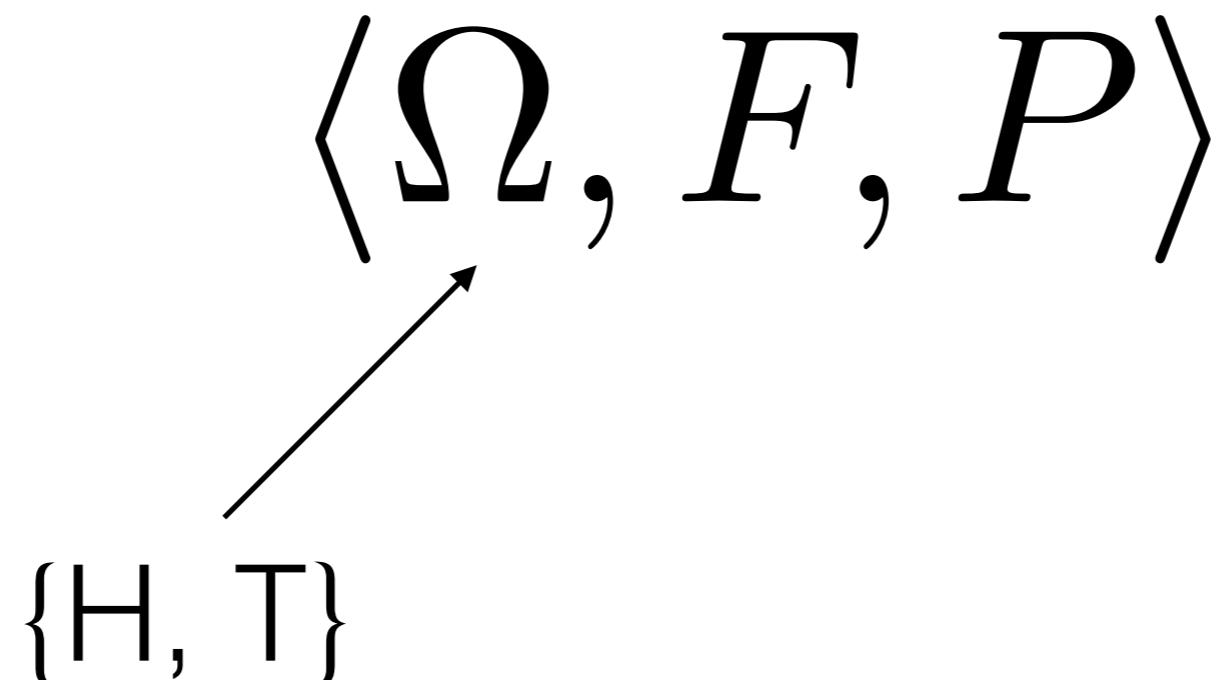
Over-used Example

Tossing a fair coin once

$$\langle \Omega, F, P \rangle$$

Over-used Example

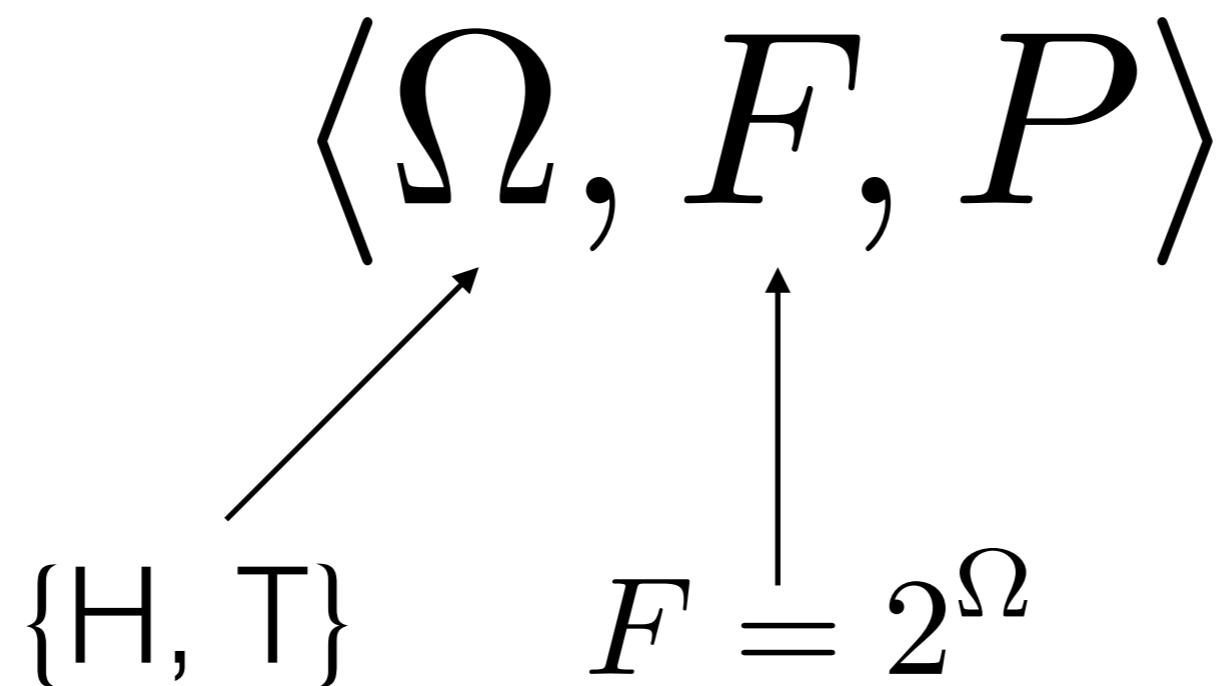
Tossing a fair coin once

$$\langle \Omega, F, P \rangle$$


The diagram illustrates a probability space. At the bottom, there is a set of outcomes represented by the symbol $\{\text{H}, \text{T}\}$. An arrow points from this set upwards towards the probability triple $\langle \Omega, F, P \rangle$, which is positioned at the top.

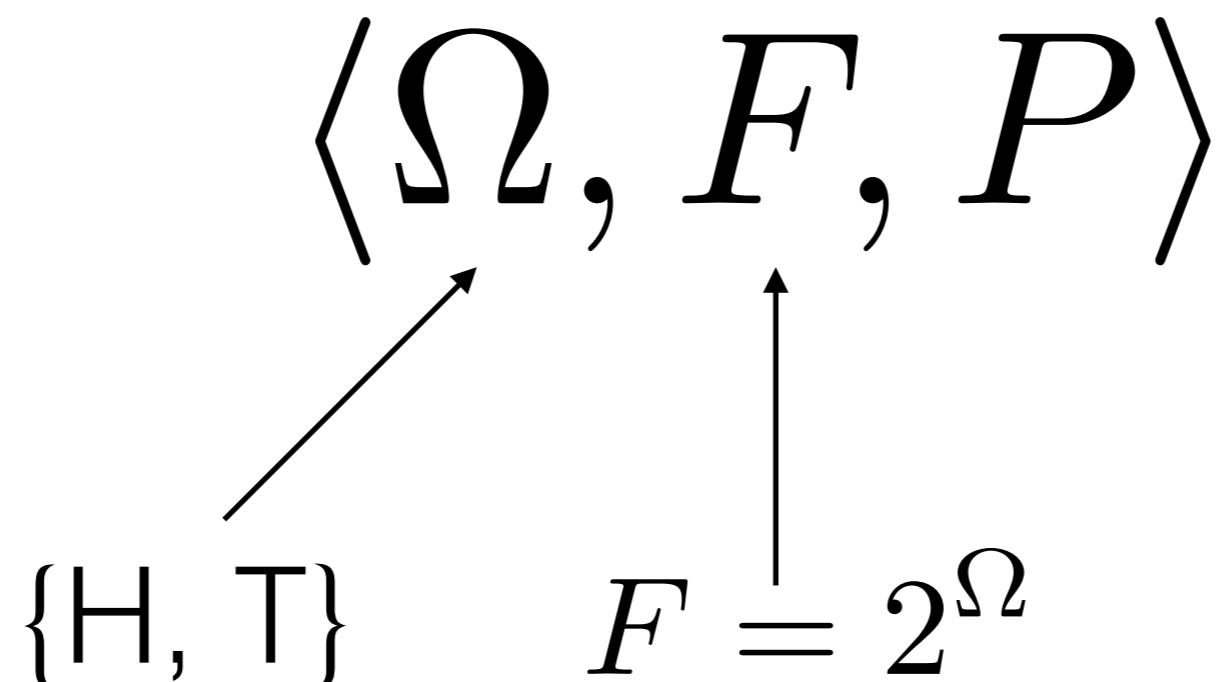
Over-used Example

Tossing a fair coin once



Over-used Example

Tossing a fair coin once



$$\{\emptyset, \{H\}, \{T\}, \{H, T\}$$

$\{\} \rightarrow 0$
 $\{H\} \rightarrow 0.5$
 $\{T\} \rightarrow 0.5$
 $\{H, T\} \rightarrow ???$

sed Example

ng a fair coin once

$$\langle \Omega, F, P \rangle$$

\uparrow

\uparrow

$\{H, T\}$ $F = 2^\Omega$

$\{\}, \{H\}, \{T\}, \{H, T\}$

$\{\} \rightarrow 0$
 $\{H\} \rightarrow 0.5$
 $\{T\} \rightarrow 0.5$
 $\{H, T\} \rightarrow ???$

sed Example

ng a fair coin once

$$\langle \Omega, F, P \rangle$$

$\Omega = \{H, T\}$

$F =$

$\{\}, \{H\}, \{T\}$

Valid Probability Function:

$$0 \leq P(E) \leq 1 \quad \forall E \in F$$

$$P(\Omega) = 1$$

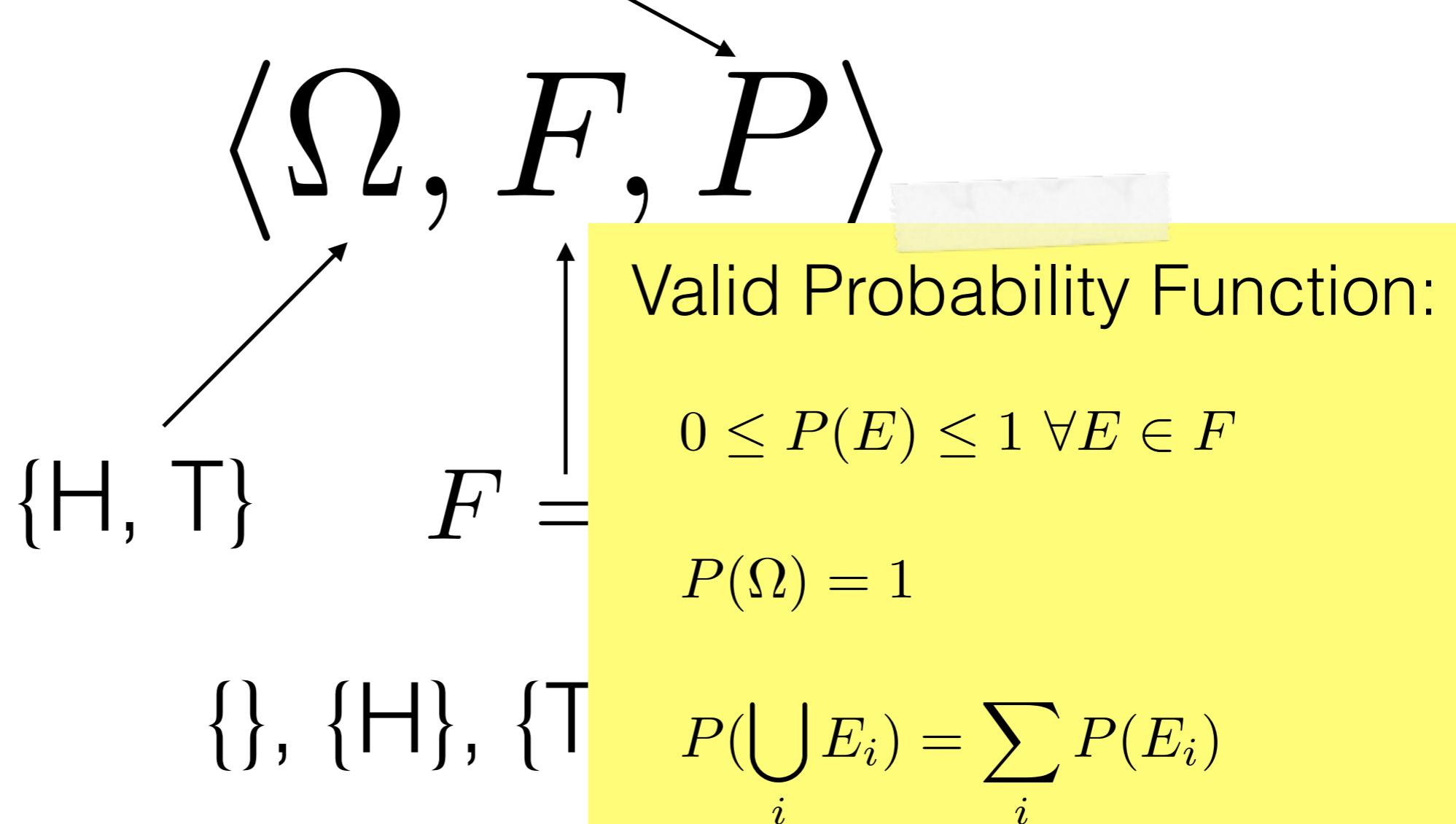
$$P\left(\bigcup_i E_i\right) = \sum_i P(E_i)$$

$$\{\} \rightarrow 0$$

$$\{H\} \rightarrow 0.5$$

$$\{T\} \rightarrow 0.5$$

$$\{H, T\} \rightarrow P(\{H\} \cup \{T\}) = P(\{H\}) + P(\{T\}) = 1$$



Random Variables

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- Random variable X assigns a number to each outcome: $X : \Omega \rightarrow \mathbb{R}$

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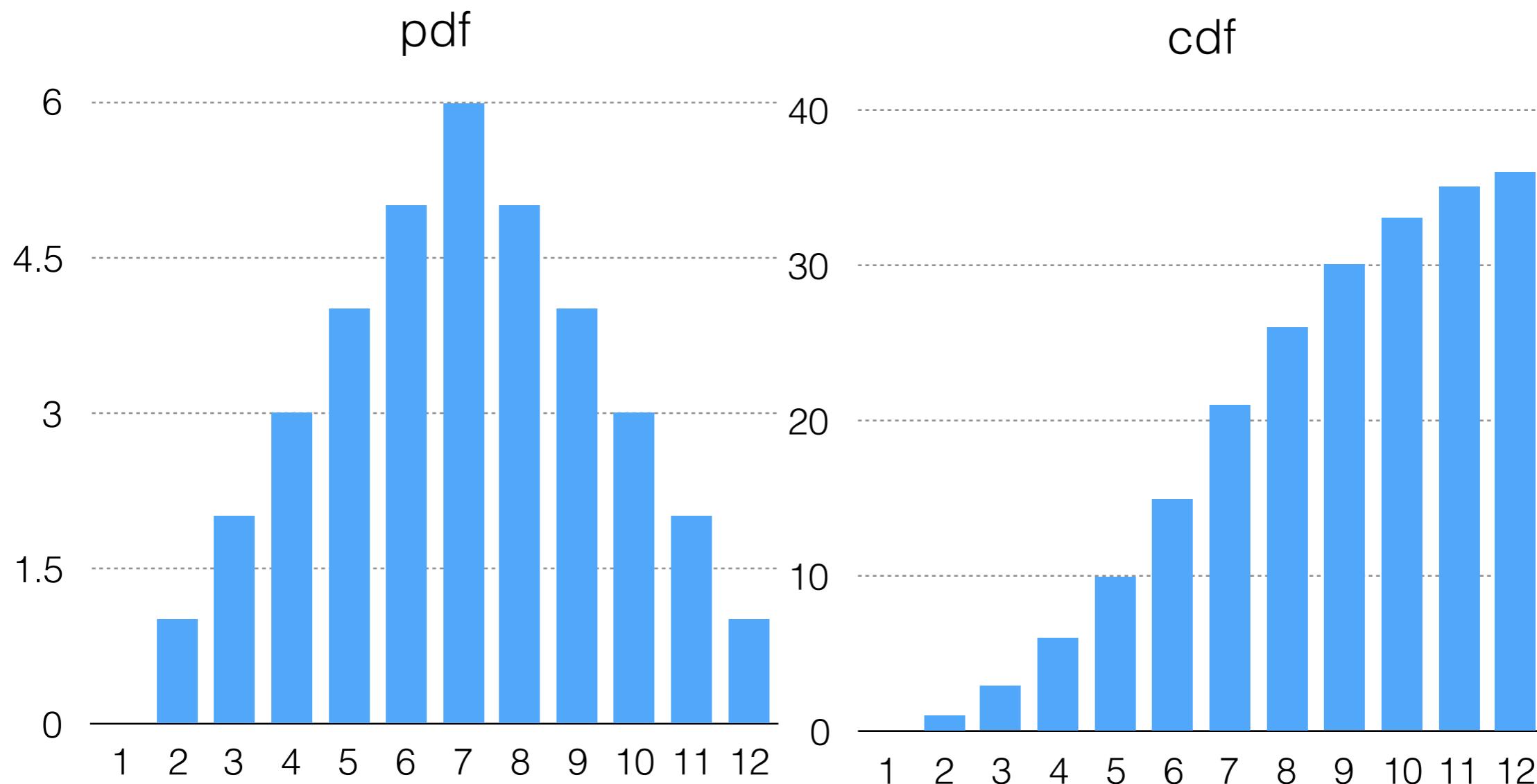
- Random variable X assigns a number to each outcome: $X : \Omega \rightarrow \mathbb{R}$
- Use $X = a$ to mean the event $\{\omega | X(\omega) = a\}$
- Probability mass function (pmf) gives probability that X takes the value a : $p(a) = Pr(X = a)$

Random Variables

- Random variable X assigns a number to each outcome: $X : \Omega \rightarrow \mathbb{R}$
- Use $X = a$ to mean the event $\{\omega | X(\omega) = a\}$
- Probability mass function (pmf) gives probability that X takes the value a : $p(a) = Pr(X = a)$
- Cumulative distribution function (cdf) gives probability that X takes any value up to a : $F(a) = Pr(X \leq a)$

Random Variables

$X = \text{sum of two dice}$



Clicker Question!

Clicker Question!

X is a random variable with the below cdf.

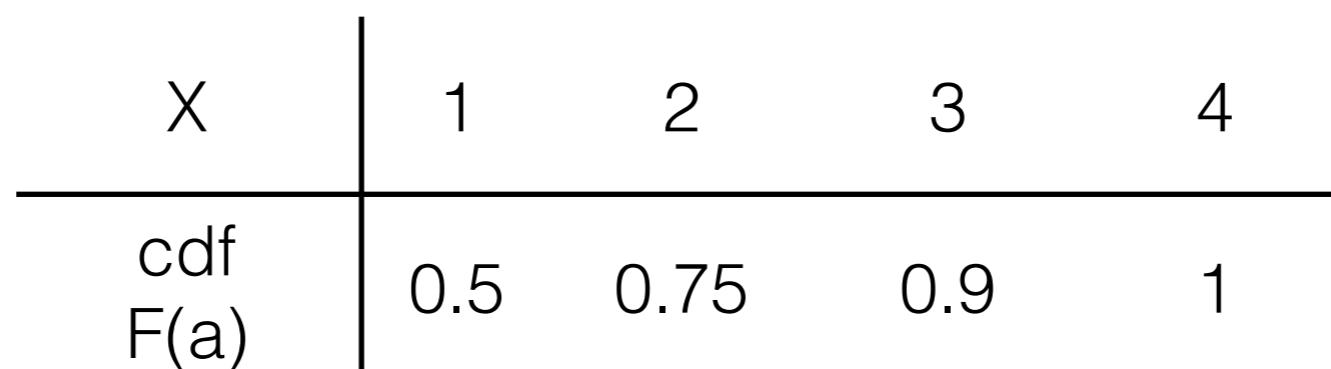


What is $P(X \leq 3)$?

- (a) 0
- (b) 0.15
- (c) 0.9
- (d) 1

Clicker Question!

X is a random variable with the below cdf.

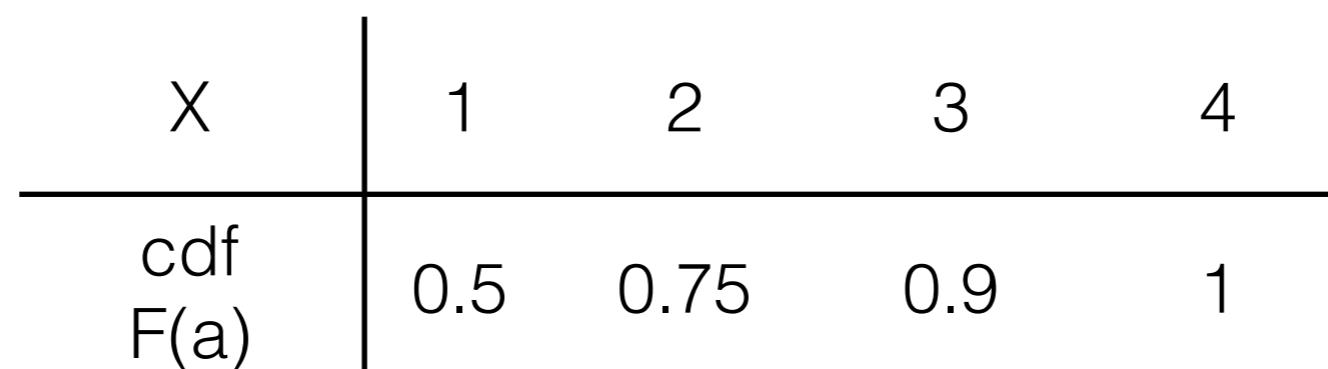


What is $P(X \leq 3)$?

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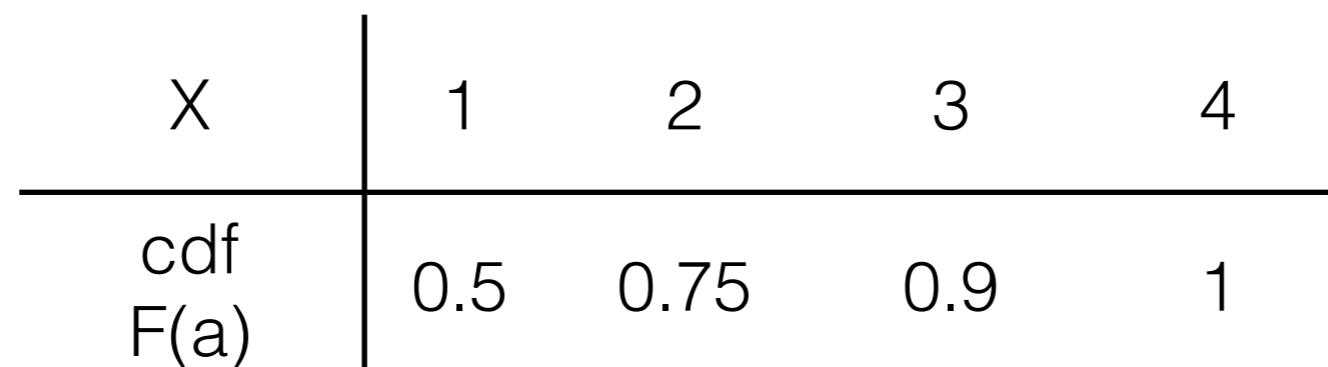


What is $P(X=3)$?

- (a) 0
- (b) 0.15
- (c) 0.9
- (d) 1

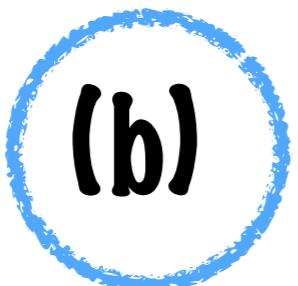
Clicker Question!

X is a random variable with the below cdf.



What is $P(X=3)$?

- (a) 0
- (b) 0.15
- (c) 0.9
- (d) 1



Expected Value

$$E(X) = \sum_i x_i Pr(x_i)$$

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$$E(X) = \int_i x_i Pr(x_i)$$

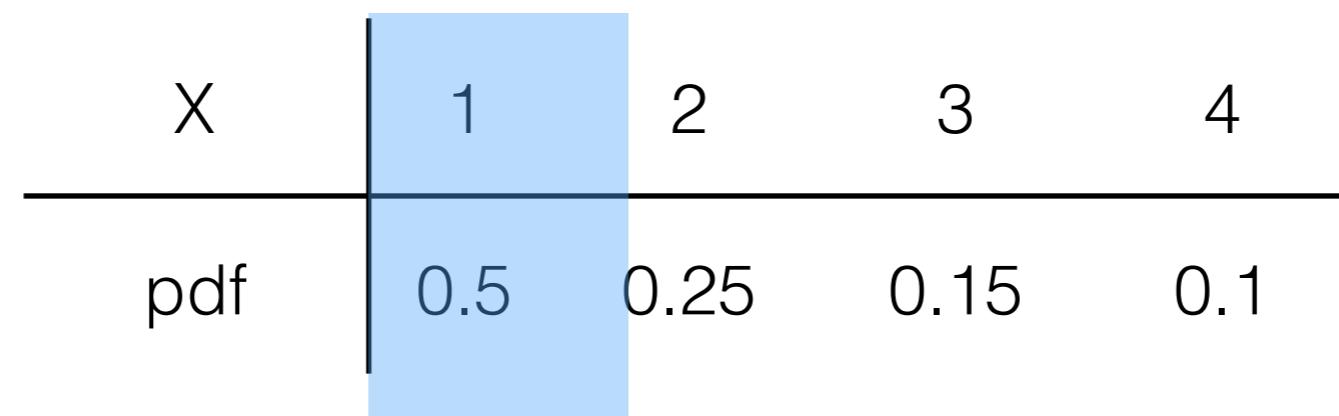
Expected Value

$$E(X) = \sum_i x_i Pr(x_i)$$



Expected Value

$$E(X) = \sum_i x_i Pr(x_i)$$



0.5

Expected Value

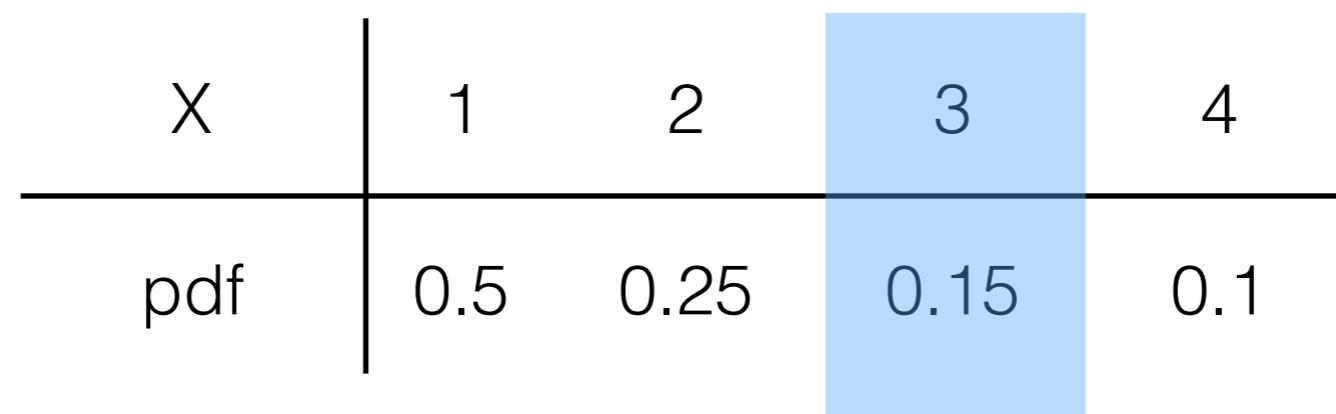
$$E(X) = \sum_i x_i Pr(x_i)$$



$$0.5 + 0.5$$

Expected Value

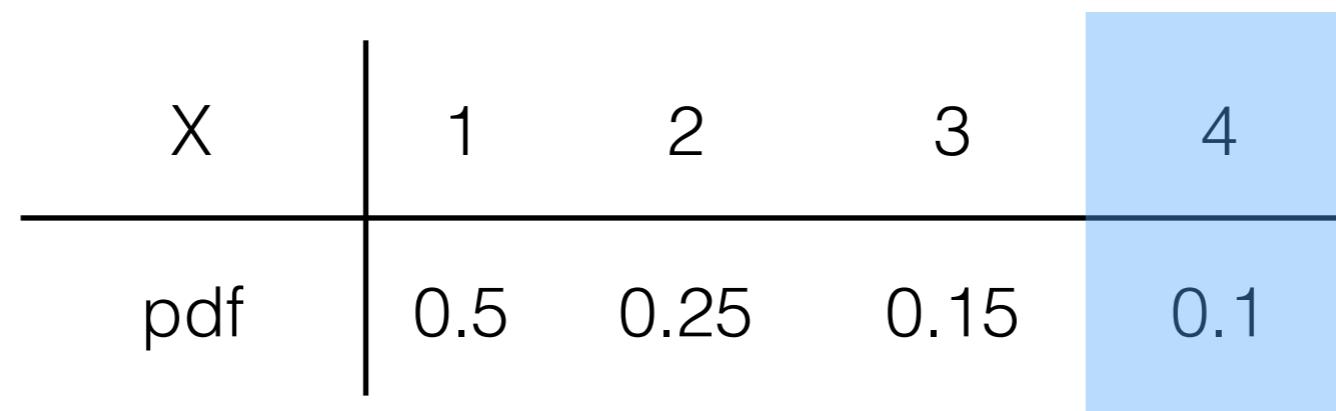
$$E(X) = \sum_i x_i Pr(x_i)$$



$$0.5 + 0.5 + 0.45$$

Expected Value

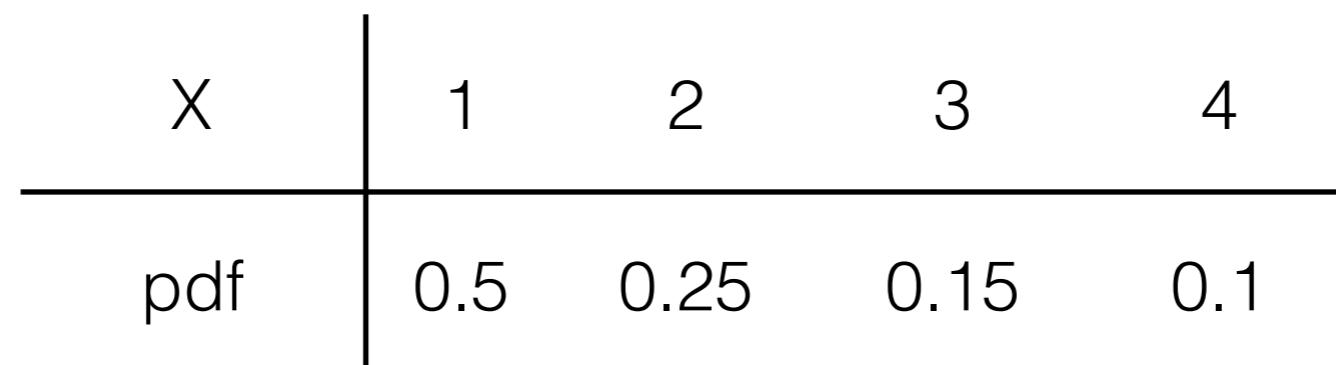
$$E(X) = \sum_i x_i Pr(x_i)$$



$$0.5 + 0.5 + 0.45 + 0.4$$

Expected Value

$$E(X) = \sum_i x_i Pr(x_i)$$



$$0.5 + 0.5 + 0.45 + 0.4 = 1.85$$

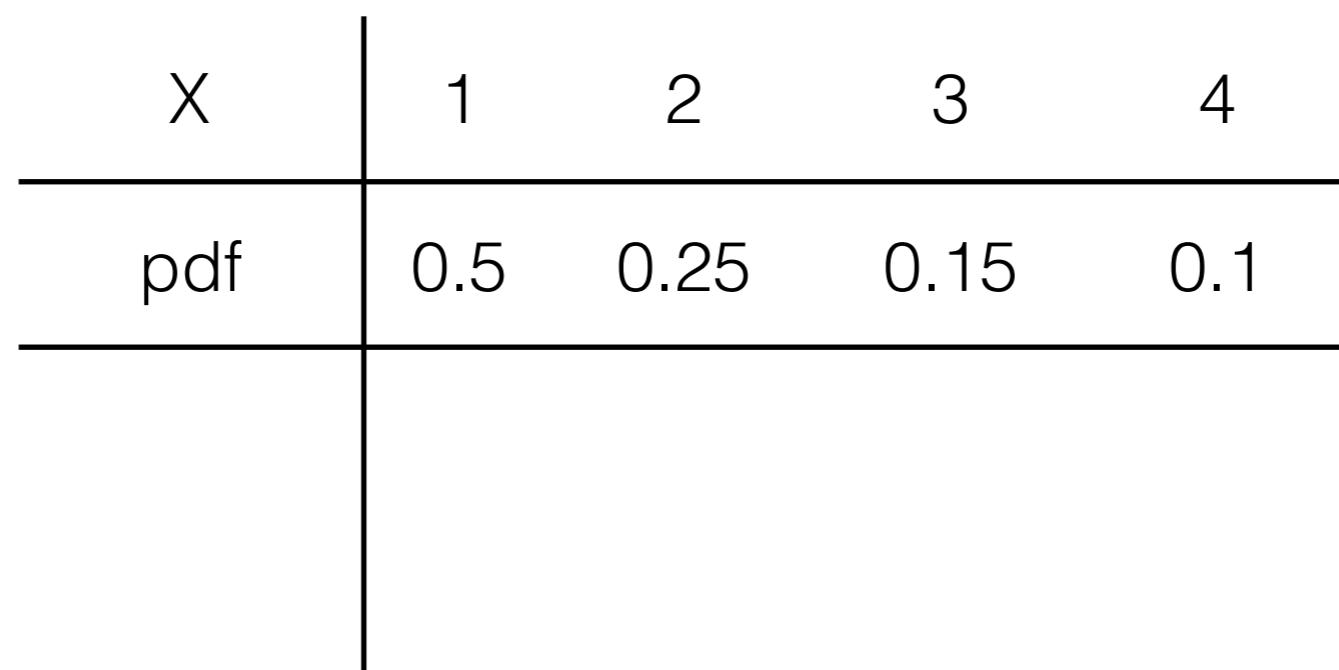
Variance

$$Var(X) = E((X - E(X))^2)$$

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Variance

$$Var(X) = E((X - E(X))^2)$$

$$E(X) = 1.85$$

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1
X - E(X)	-0.85	0.15	1.15	2.15
$(X - E(X))^2$	0.722	0.023	1.32	4.62

Variance

$$Var(X) = E((X - E(X))^2)$$

$$E(X) = 1.85$$

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pdf	0.5	0.25	0.15	0.1
X - E(X)	-0.85	0.15	1.15	2.15
$(X - E(X))^2$	0.722	0.023	1.32	4.62

0.361

Variance

$$Var(X) = E((X - E(X))^2)$$

$$E(X) = 1.85$$

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pdf	0.5	0.25	0.15	0.1
X - E(X)	-0.85	0.15	1.15	2.15
$(X - E(X))^2$	0.722	0.023	1.32	4.62

$$0.361 + .006$$

Variance

$$Var(X) = E((X - E(X))^2)$$

$$E(X) = 1.85$$

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1
X - E(X)	-0.85	0.15	1.15	2.15
$(X - E(X))^2$	0.722	0.023	1.32	4.62

$$0.361 + .006 + 0.198$$

Variance

$$Var(X) = E((X - E(X))^2)$$

$$E(X) = 1.85$$

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1
X - E(X)	-0.85	0.15	1.15	2.15
$(X - E(X))^2$	0.722	0.023	1.32	4.62

$$0.361 + .006 + 0.198 + 0.462$$

Variance

$$Var(X) = E((X - E(X))^2)$$

$$E(X) = 1.85$$

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1
X - E(X)	-0.85	0.15	1.15	2.15
$(X - E(X))^2$	0.722	0.023	1.32	4.62

$$0.361 + .006 + 0.198 + 0.462 = 1.027$$

Interpreting Expectation

Would you accept a gamble that offers a 10% chance to win \$95 and a 90% chance of losing \$5?

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$$E(\text{Payoff}) = (95 \times 0.10) - (5 \times 0.9)$$

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$$E(\text{Payoff}) = (9.5) - (4.5)$$

Interpreting Expectation

Would you accept a gamble that offers a 10% chance to win \$95 and a 90% chance of losing \$5?

$$E(\text{Payoff}) = (95 \times 0.10) - (5 \times 0.9)$$

$$E(\text{Payoff}) = (9.5) - (4.5)$$

$$E(\text{Payoff}) = 5$$

Clicker Question!

Clicker Question!

How much would you pay for a lottery ticket that offers a 10% percent chance of winning \$100 and a 90% chance of winning nothing?

- (a) \$0
- (b) no more than \$2
- (c) no more than \$5
- (d) no more than \$10

Clicker Question!

How much would you pay for a lottery ticket that offers a 10% percent chance of winning \$100 and a 90% chance of winning nothing?

- (a) \$0
- (b) no more than \$2
- (c) no more than \$5
- (d) no more than \$10

Clicker Question!

How much would you pay for a lottery ticket that offers a 10% percent chance of winning \$100 and a 90% chance of winning nothing?

- (a) \$0
- (b) no more than \$2
- (c) no more than \$5
- (d) no more than \$10

$$0 = 0.1(100 - \text{cost}) - 0.9(\text{cost})$$

$$0 = 10 - \text{cost}$$

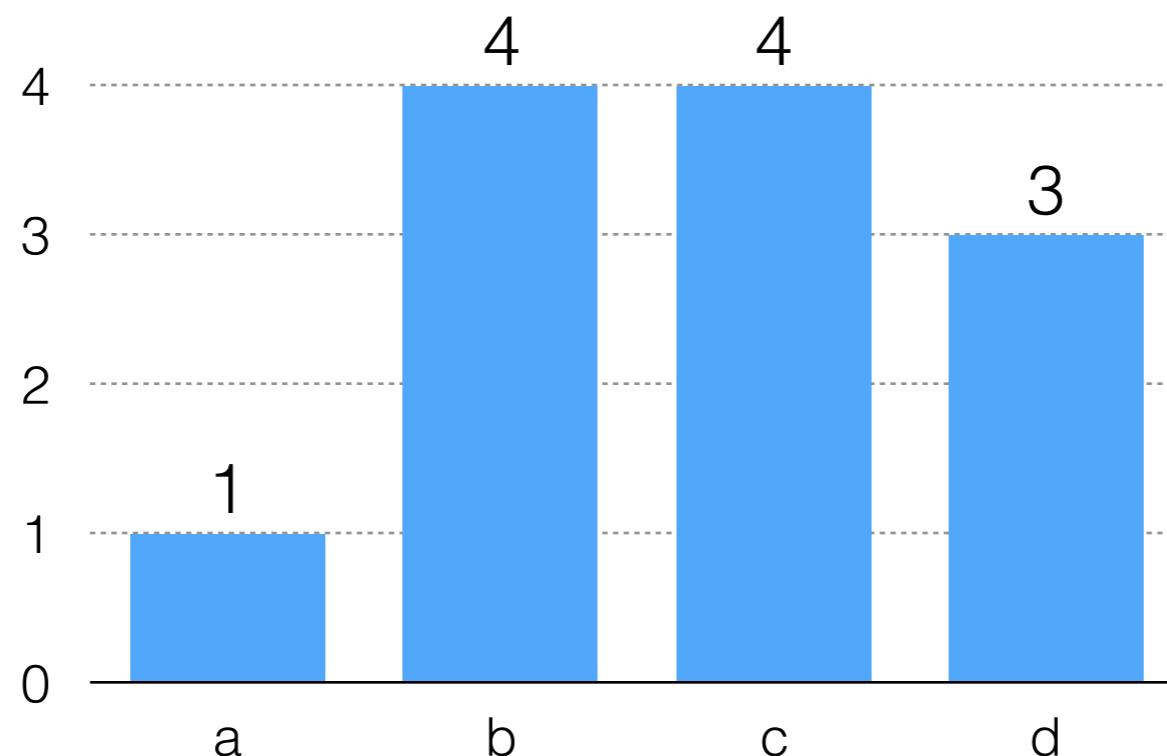
$$\text{cost} = 10$$

Today

- ~~What is a hypothesis?~~
- ~~Some definitions/notation~~
- Intuition behind modeling/hypothesis testing

Gaming Clicker Questions!

Are the answers to my clicker questions random?



Are the answers to my
clicker questions random?

"I swear literally like 80% of the answers are just (b)"

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c d b d
b a d c
b c b d

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c d b d
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b c b d

Probability of this?

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"

c	d	b	d
b	a	d	c
b	c	b	d

Probability of this?

The bigger picture

(Yes, I will belabor this point. Why do you ask?)

- Start with real world phenomenon/observations
- Make assumptions about the underlying model
- Fit the parameters of the model based on data

The bigger picture

- Start with real world phenomenon/observations
- Make assumptions about the underlying model
- Fit the parameters of the model based on data
 - Choose parameters of the model based on theories, do analysis to see if it's a good fit (hypothesis testing!!) AND/OR
 - Set parameters of the model based on data, try to make forecast for unseen/future data (prediction!!)

The bigger picture

- Start with real world phenomenon/observations
 - Make assumptions about the underlying model
 - Fit the parameters of the model based on data
- Now** 
- Choose parameters of the model based on theories, do analysis to see if its a good fit (hypothesis testing!!) AND/OR
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The bigger picture

- Start with real world phenomenon/observations
 - Make assumptions about the underlying model
 - Fit the parameters of the model based on data
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- Choose parameters of the model based on theories, do analysis to see if its a good fit (hypothesis testing!!) AND/OR
 - Set parameters of the model based on data, try to make forecast for unseen/future data (prediction!!)
-  **Later**

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c	d	b	d
b	a	d	c
b	c	b	d

Probability of this?

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"I swear literally like 80% of the answers are just (b)"

c	d	b	d
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b	c	b	d

$$\langle \Omega, \mathcal{F}, P \rangle$$

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"

c	d	b	d
b	a	d	c
b	c	b	d

$$\langle \Omega, F, P \rangle$$

↑
 $\{b, \text{not } b\}$

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"

c	d	b	d
b	a	d	c
b	c	b	d

$$\langle \Omega, F, P \rangle$$

\uparrow \uparrow

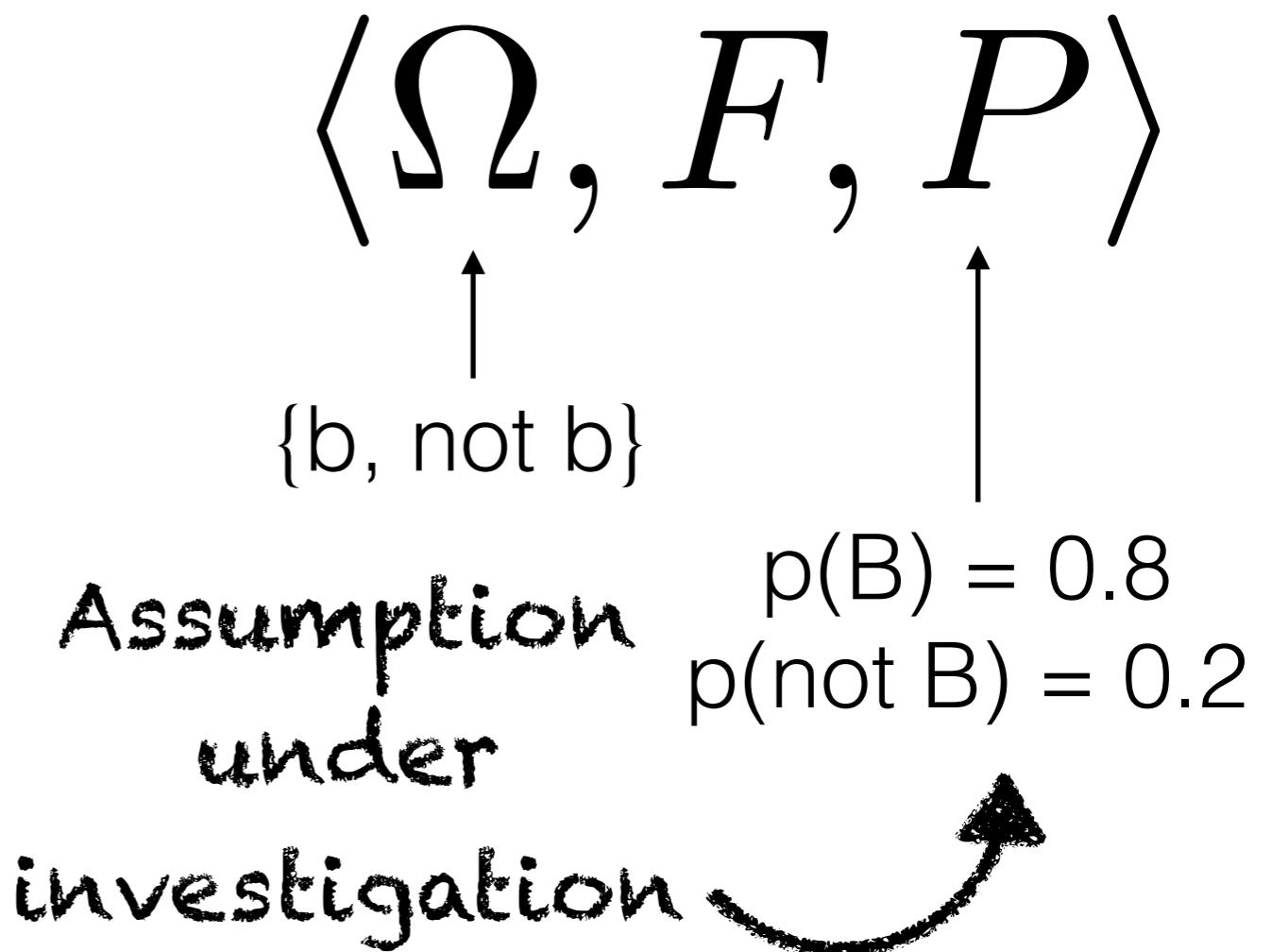
$\{b, \text{not } b\}$ $p(B) = 0.8$

$p(\text{not } B) = 0.2$

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"

c	d	b	d
b	a	d	c
b	c	b	d



Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"

c d **b** d
b a d c
b c **b** d

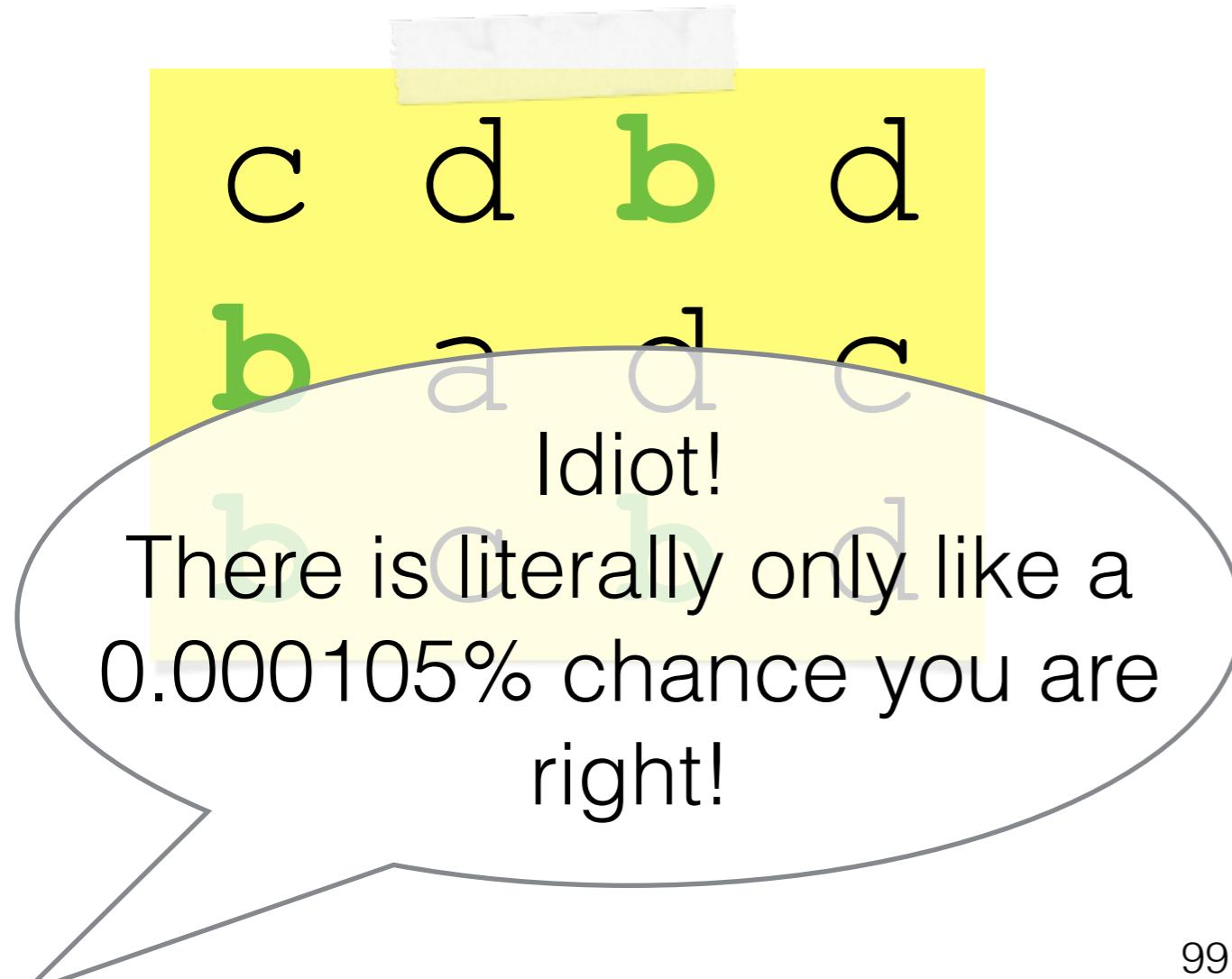
$$\langle \Omega, F, P \rangle$$

$$\uparrow \quad \uparrow$$
$$\{b, \text{not } b\} \quad p(B) = 0.8$$

$$0.2 \times 0.2 \times 0.8 \times 0.2 \times \\ 0.8 \times 0.2 \times 0.2 \times 0.2 \times \\ 0.8 \times 0.2 \times 0.8 \times 0.2 = \\ 0.00000105$$

Are the answers to my clicker questions random?

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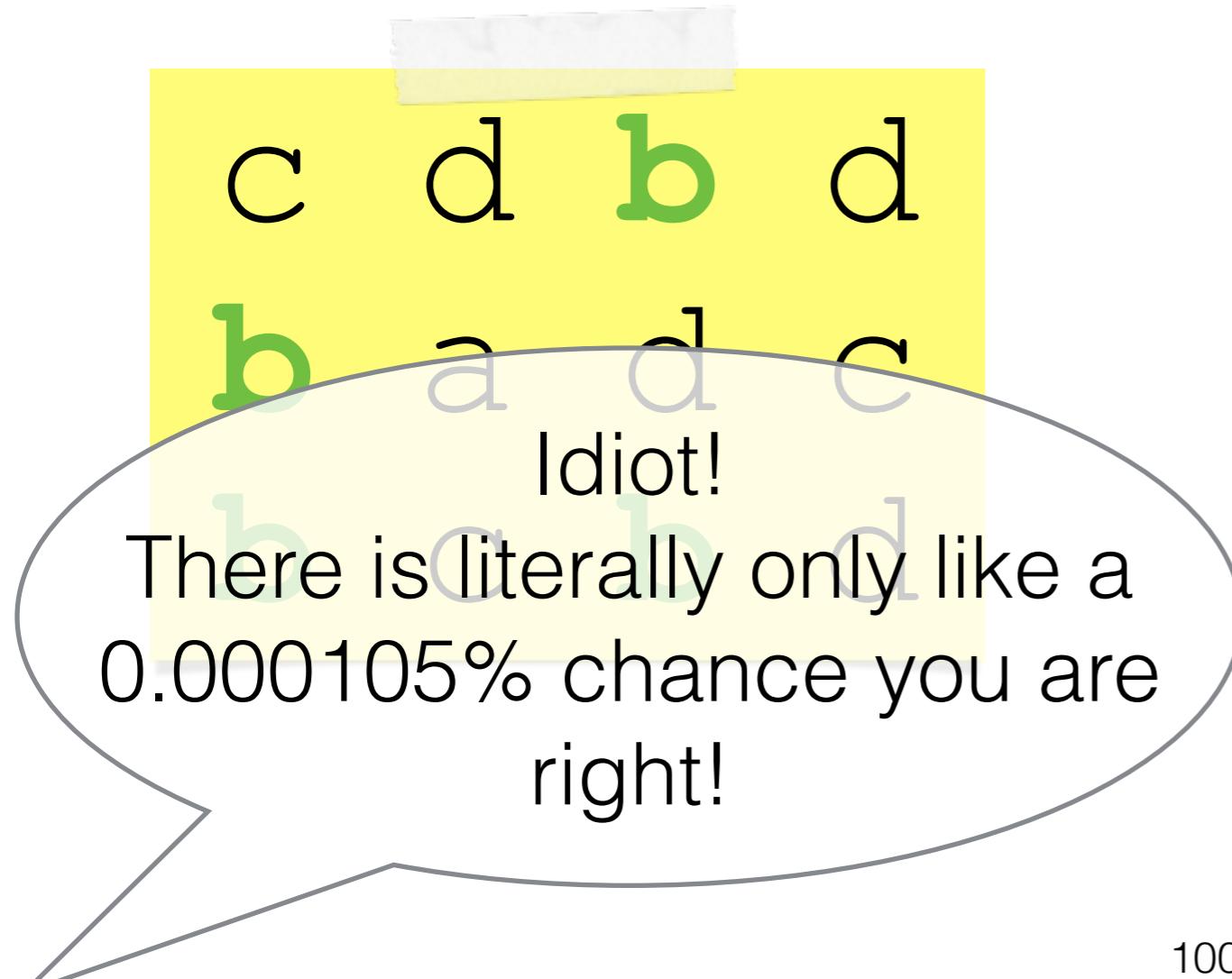
$$\langle \Omega, F, P \rangle$$

$$\begin{matrix} \uparrow & \uparrow \\ \{b, \text{not } b\} & p(B) = 0.8 \end{matrix}$$

$$\begin{aligned} 0.2 \times 0.2 \times 0.8 \times 0.2 \times \\ 0.8 \times 0.2 \times 0.2 \times 0.2 \times \\ 0.8 \times 0.2 \times 0.8 \times 0.2 = \\ 0.00000105 \end{aligned}$$

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"



$$\langle \Omega, F, P \rangle$$

\uparrow \uparrow

$$\{b, \text{not } b\} \quad p(B) = 0.8$$

$$0.2 \times 0.2 \times 0.8 \times 0.2 \times \\ 0.8 \times 0.2 \times 0.2 \times 0.2 \times \\ 0.8 \times 0.2 \times 0.8 \times 0.2 = \\ 0.00000105$$

Clicker Question!

Clicker Question!

"I swear literally like 80% of the answers are just (b)"

What is the probability of this event?

b b b d
b a b b
b b b b

- a) 1.0
- b) 0.4
- c) 0.04
- d) 0.004

Clicker Question!

"I swear literally like 80% of the answers are just (b)"

What is the probability of this event?

b b b d
b a b b
b b b b

- a) 1.0
- b) 0.4
- c) 0.04
- d) 0.004

Clicker Question!

"I swear literally like 80% of the answers are just (b)"

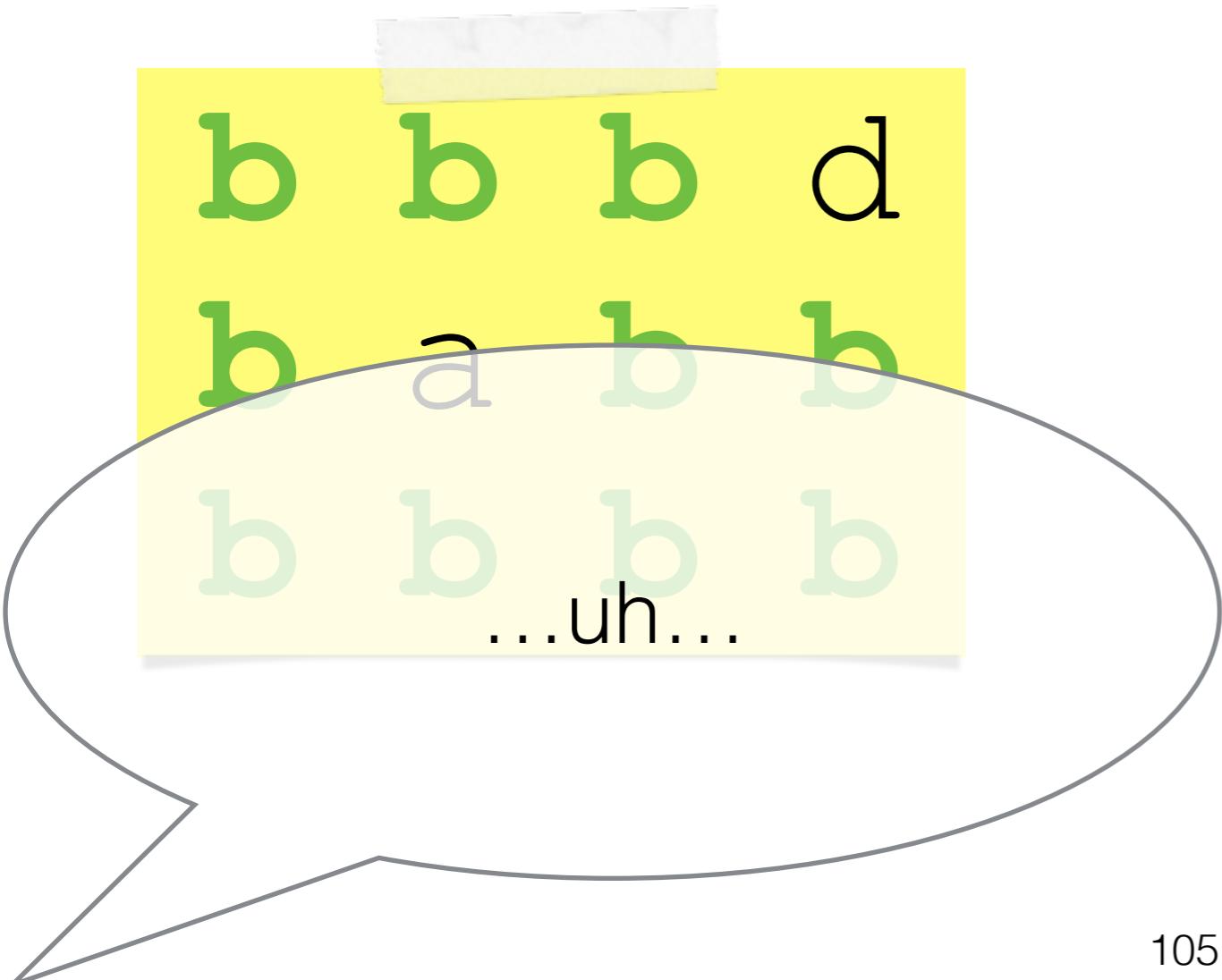
What is the probability of this event?

b b b d
b a b b
b b b b

$$\begin{aligned} & 0.8 \times 0.8 \times 0.8 \times 0.2 \times \\ & 0.8 \times 0.2 \times 0.8 \times 0.8 \times \\ & 0.8 \times 0.8 \times 0.8 \times 0.8 = \\ & 0.004 \end{aligned}$$

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"



$$\langle \Omega, F, P \rangle$$

$$\{b, \text{not } b\} \quad p(B) = 0.8$$

$$\begin{aligned} & 0.8 \times 0.8 \times 0.8 \times 0.2 \times \\ & 0.8 \times 0.2 \times 0.8 \times 0.8 \times \\ & 0.8 \times 0.8 \times 0.8 \times 0.8 = \\ & 0.004 \end{aligned}$$

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"

c	d	b	d
b	a	d	c
b	c	b	d

Probability of this?

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"

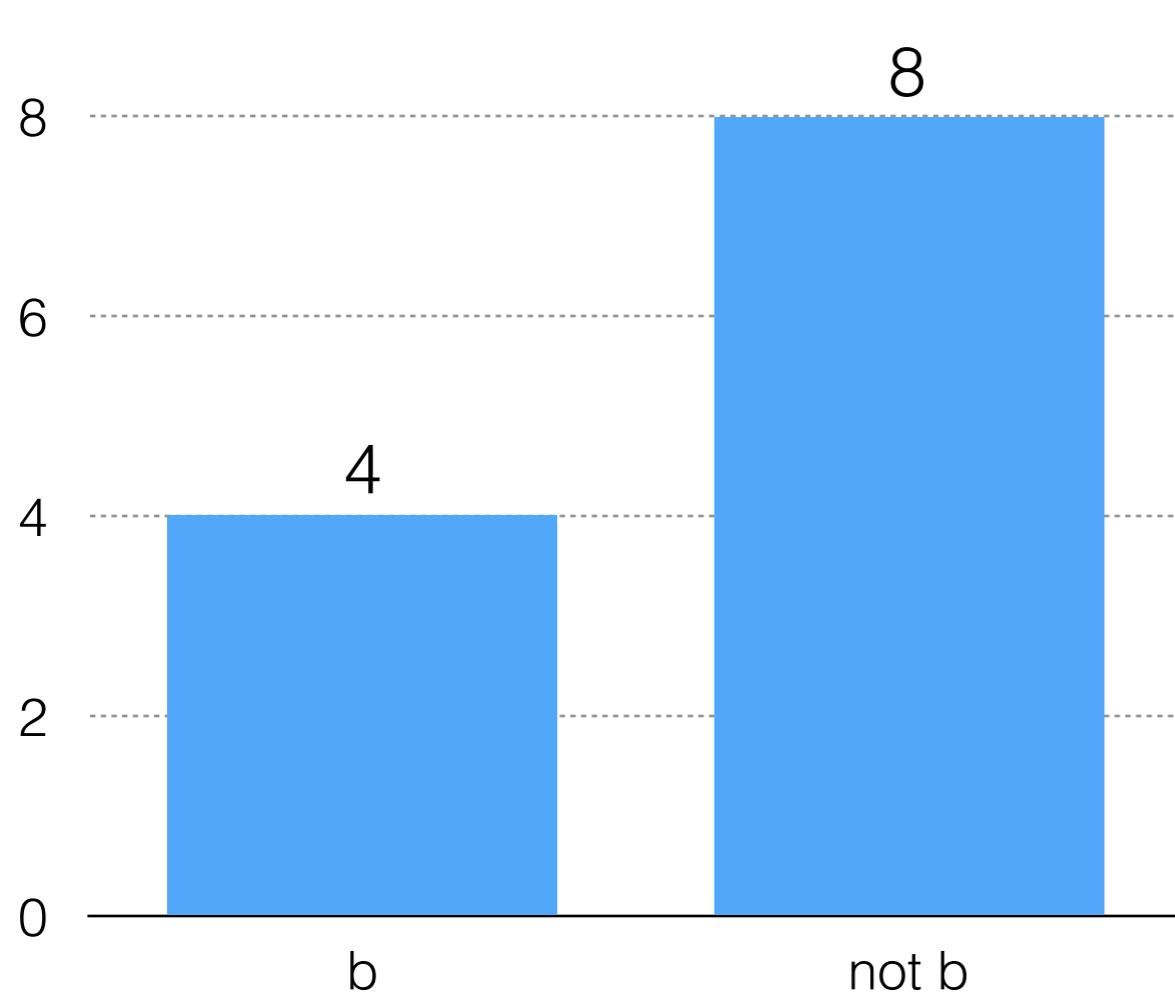
c	d	b	d
b	a	d	c
b	c	b	d

~~Probability of this?~~

Probability of anything as surprising as this

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"



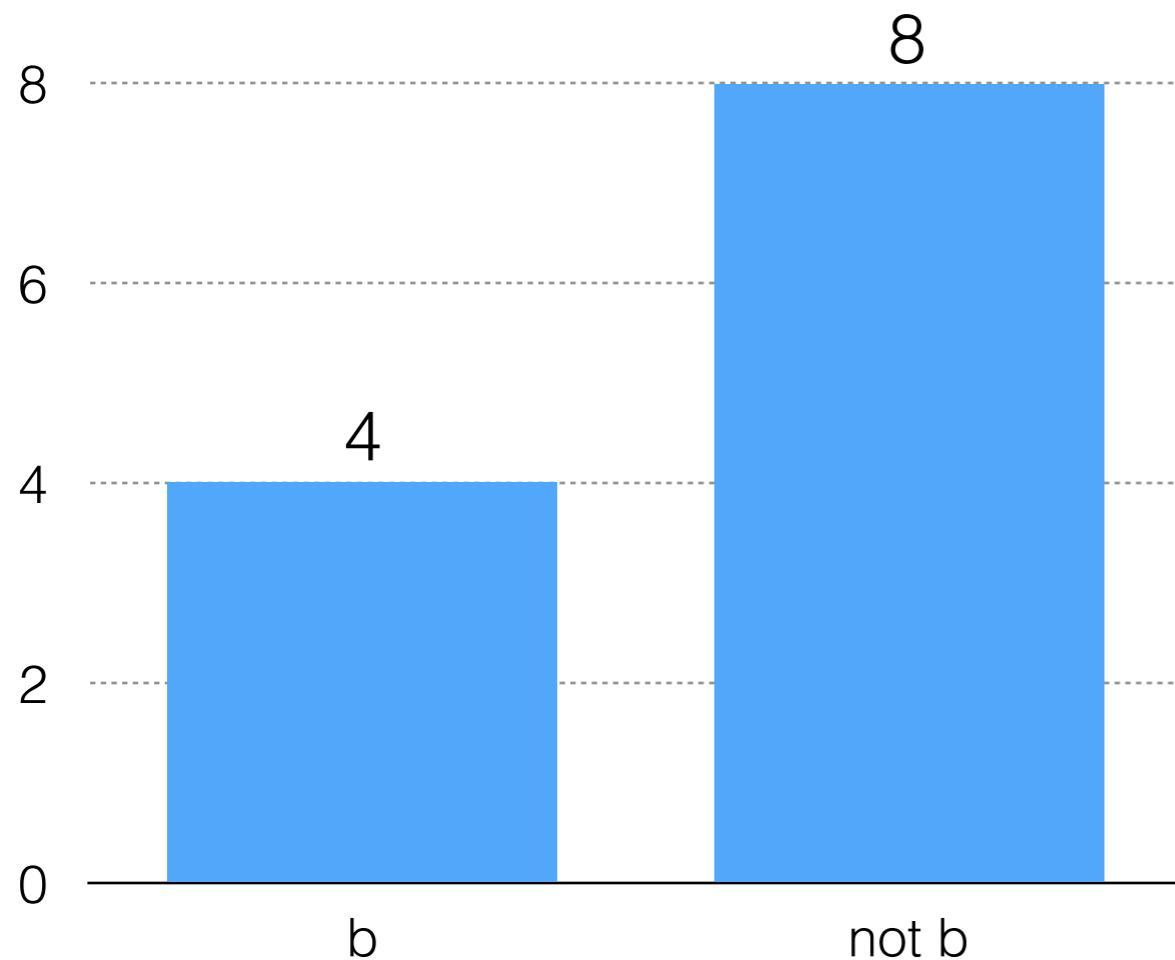
$$\langle \Omega, F, P \rangle$$

$$\{b, \text{not } b\} \quad p(B) = 0.8$$

$X = \text{number of (b)s}$

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"

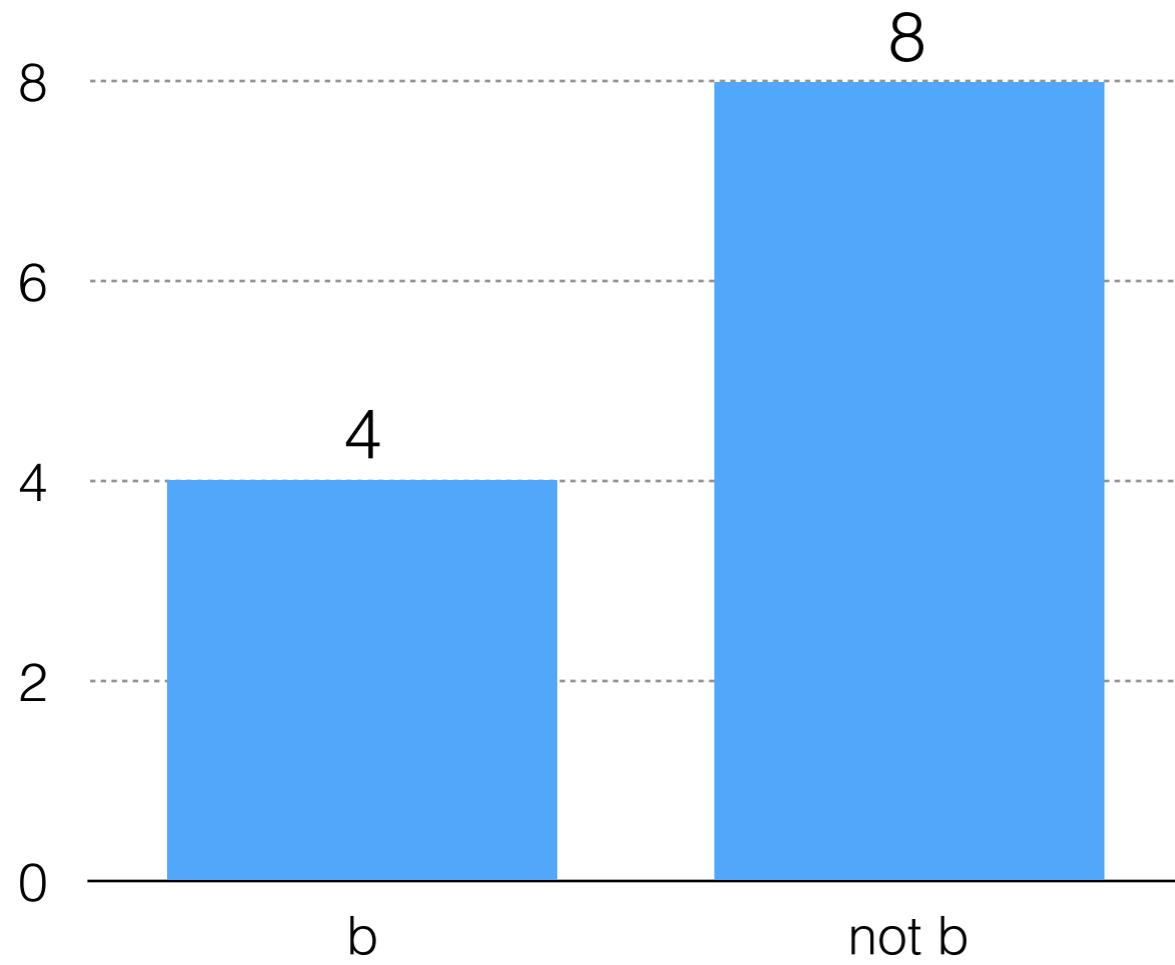


$X = \text{number of (b)s}$

How can we define
the pdf for this?

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"



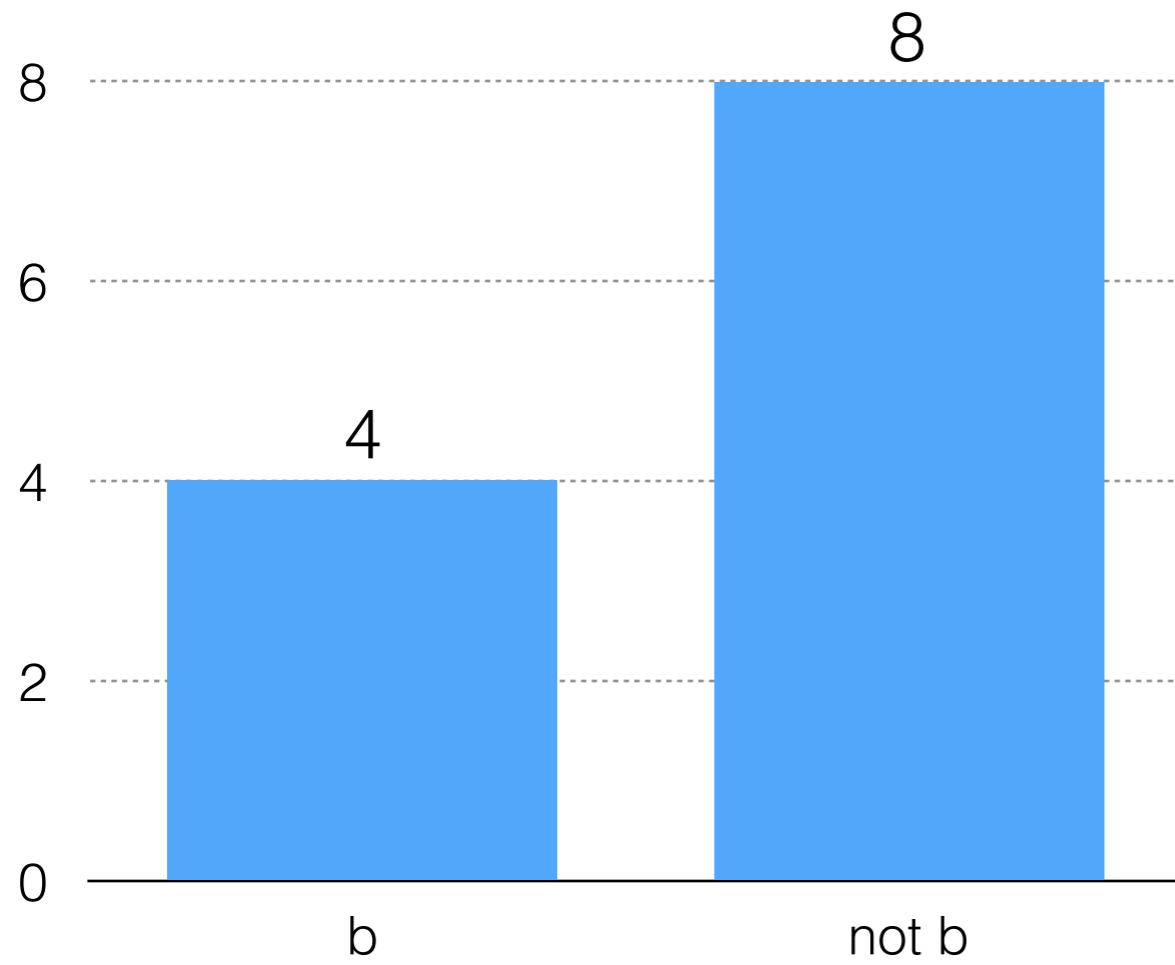
$X = \text{number of (b)s}$

How can we define
the pdf for this?

i.e. how do we model
 $P(\# \text{ bs} = k)$ for any
value of k ?

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"



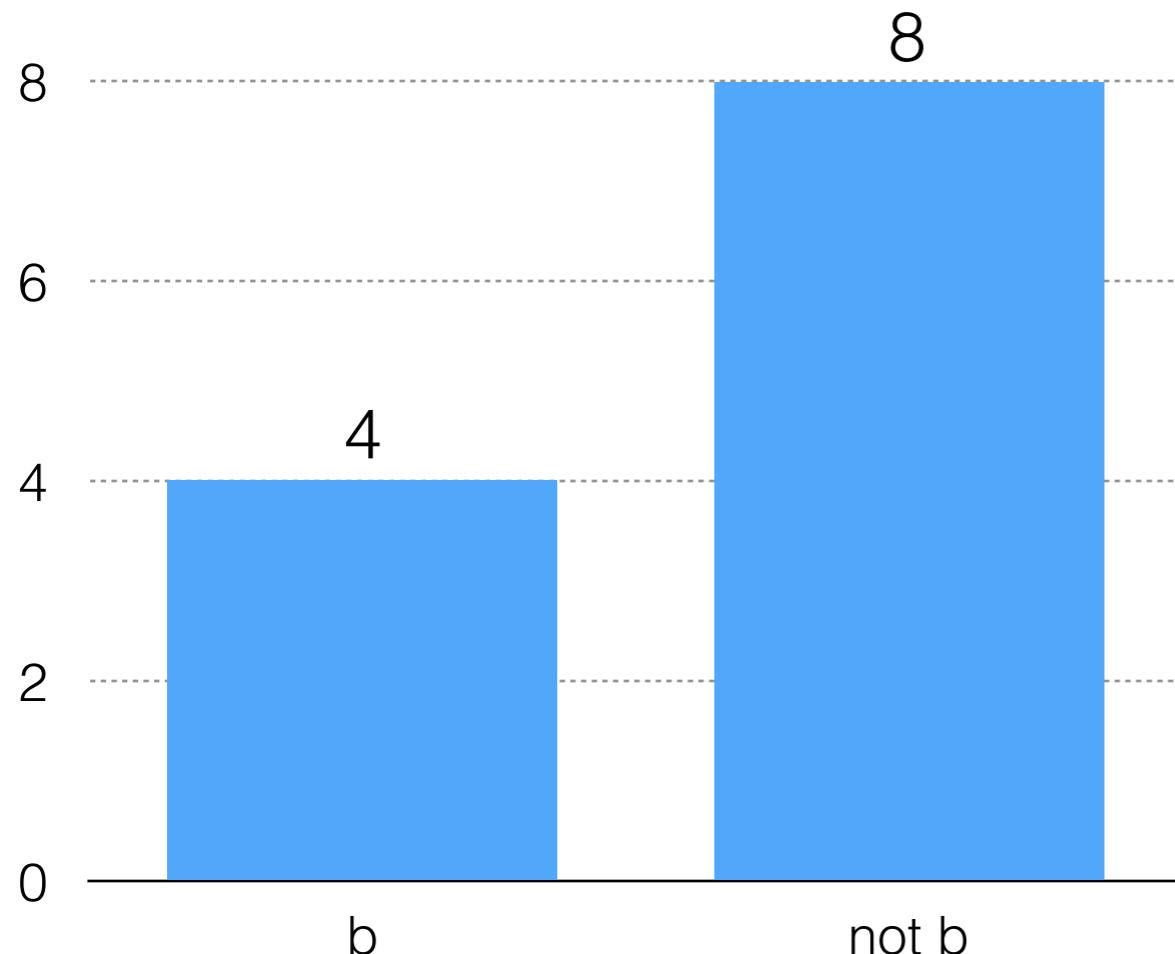
$X = \text{number of (b)s}$

$$f(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

binomial distribution!
("biased coin" distribution)

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"



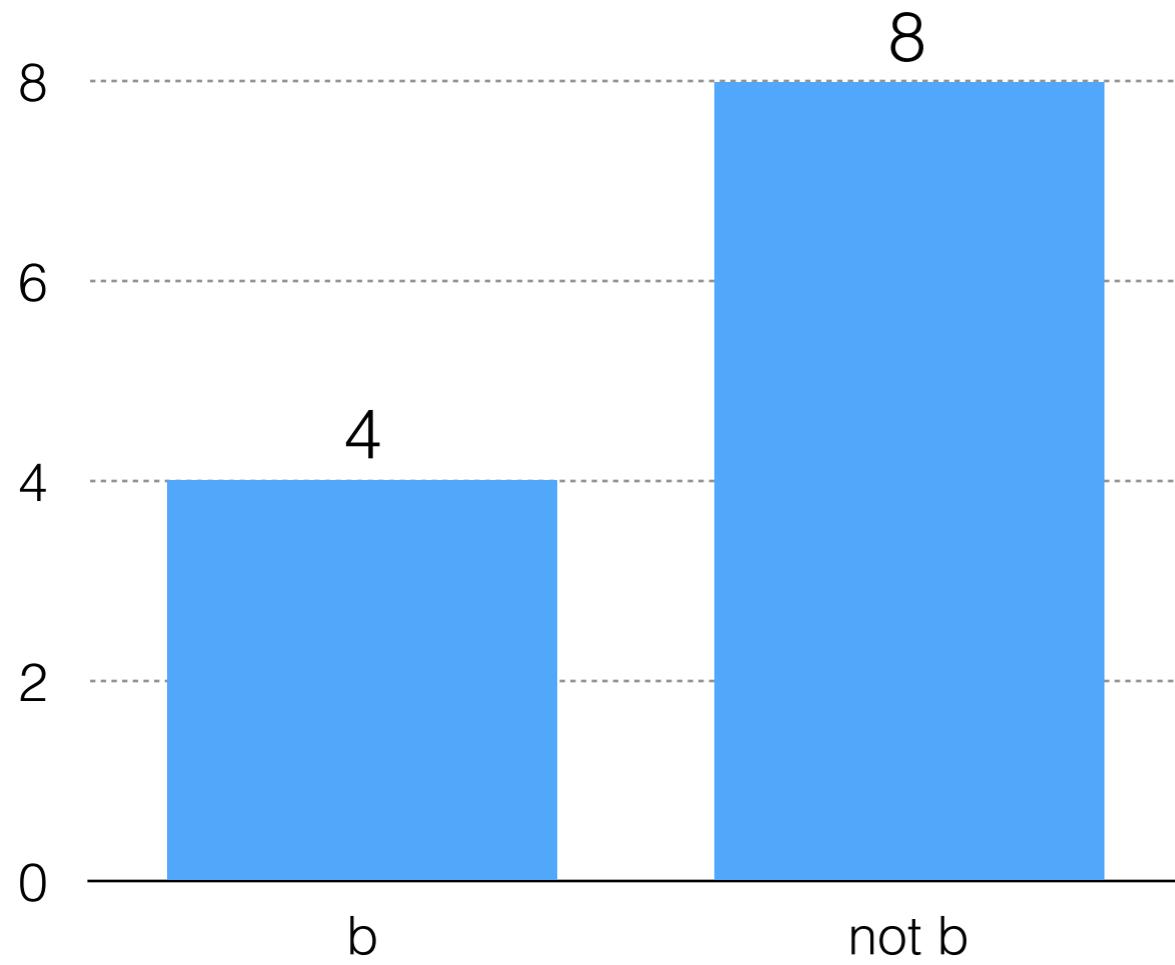
$X = \text{number of (b)s}$

$$f(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$P(\#\text{(b)s} = k)$$

Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"



$X = \text{number of (b)s}$

$$f(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

account for all
the positions the k
bs could occur in

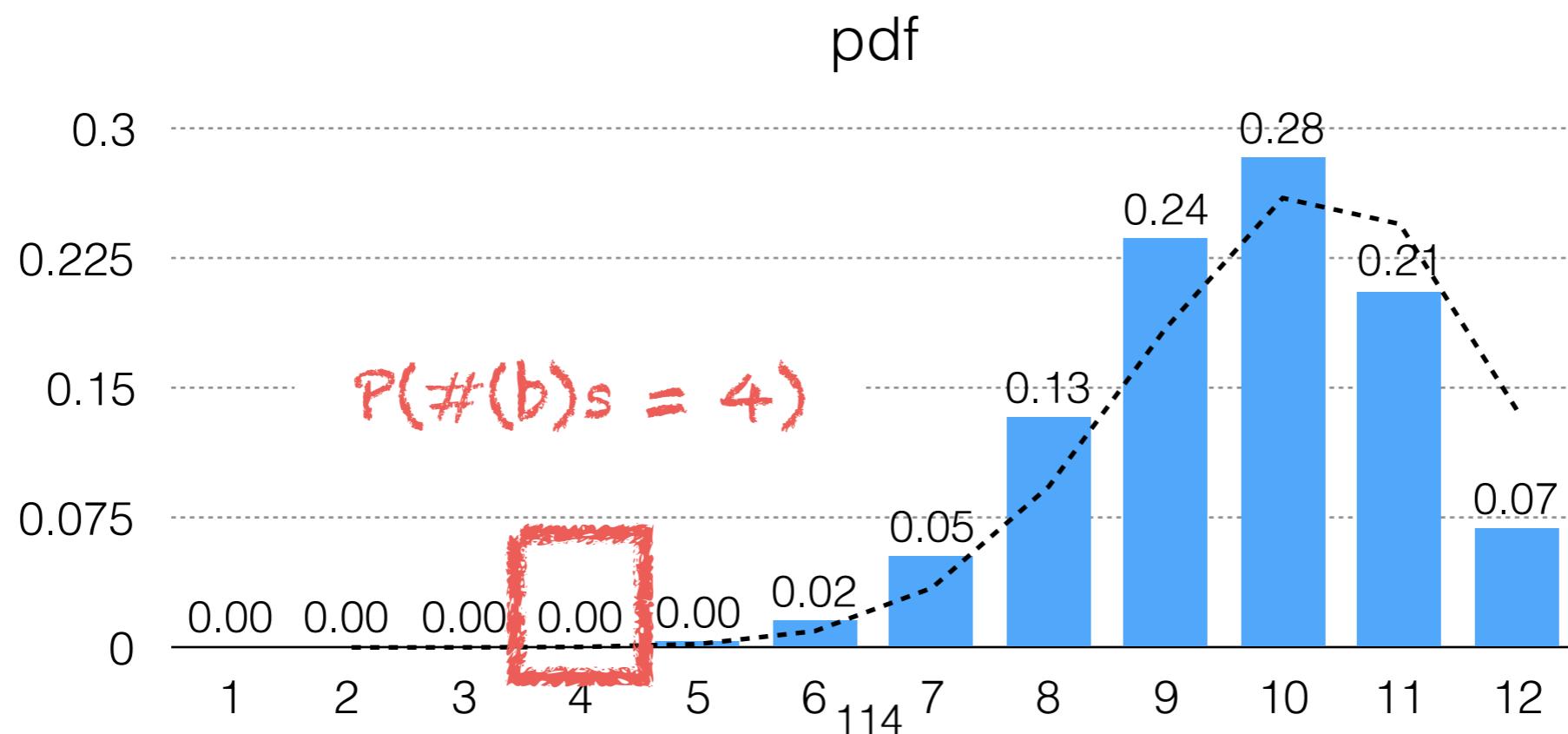
Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"

$$f(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\langle \Omega, F, P \rangle$$

↑
 $\{\text{b, not b}\} \quad p(\text{B}) = 0.8$

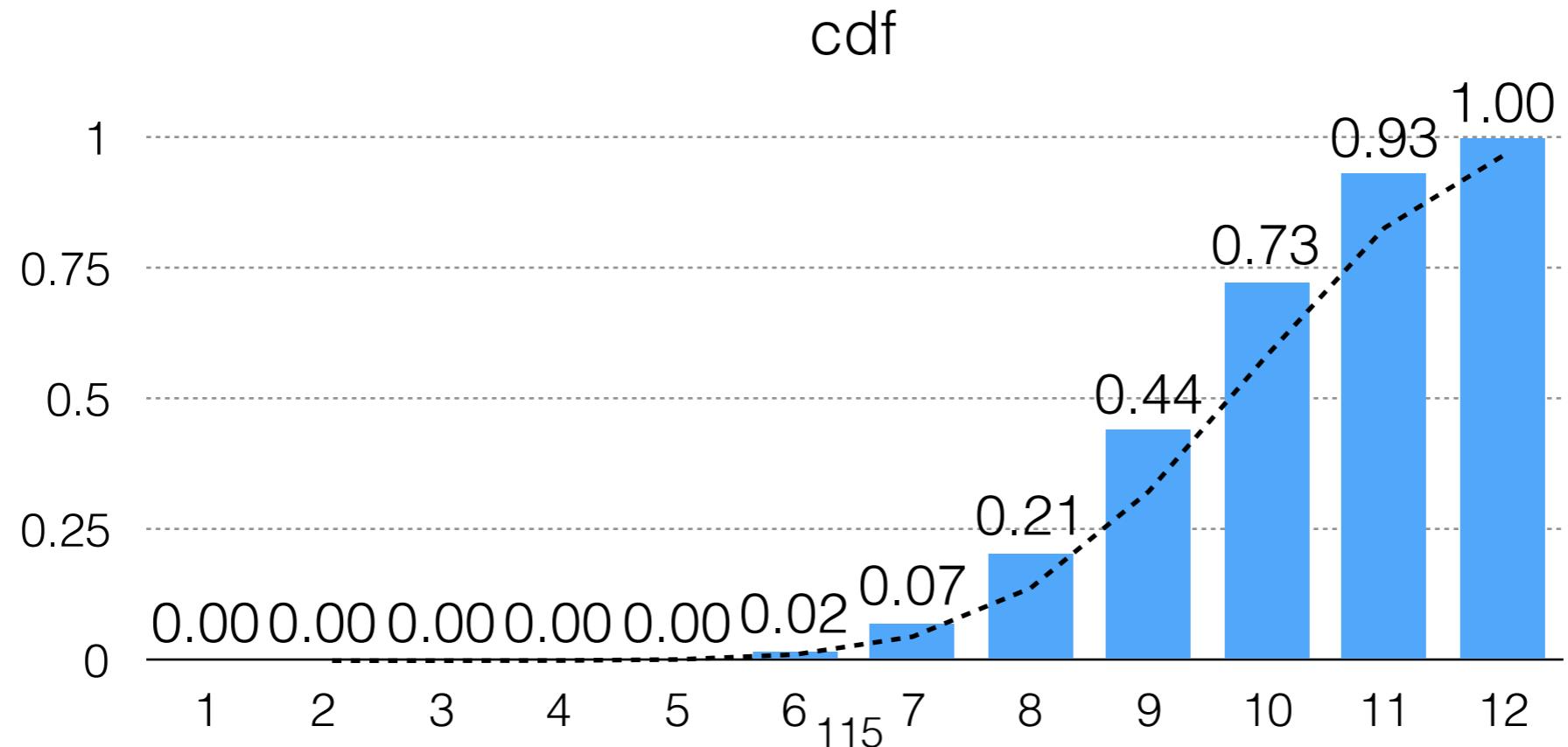


Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"

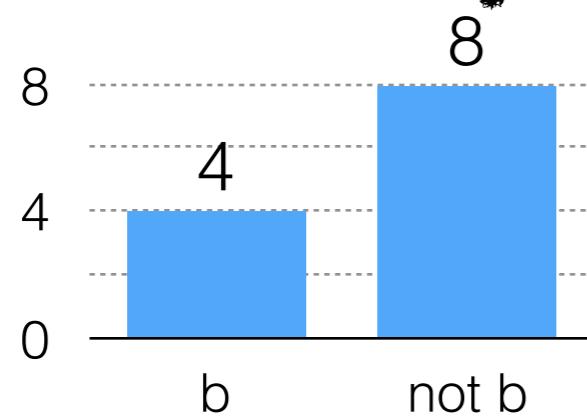
$$\langle \Omega, F, P \rangle$$

$$\{b, \text{ not } b\} \quad p(B) = 0.8$$

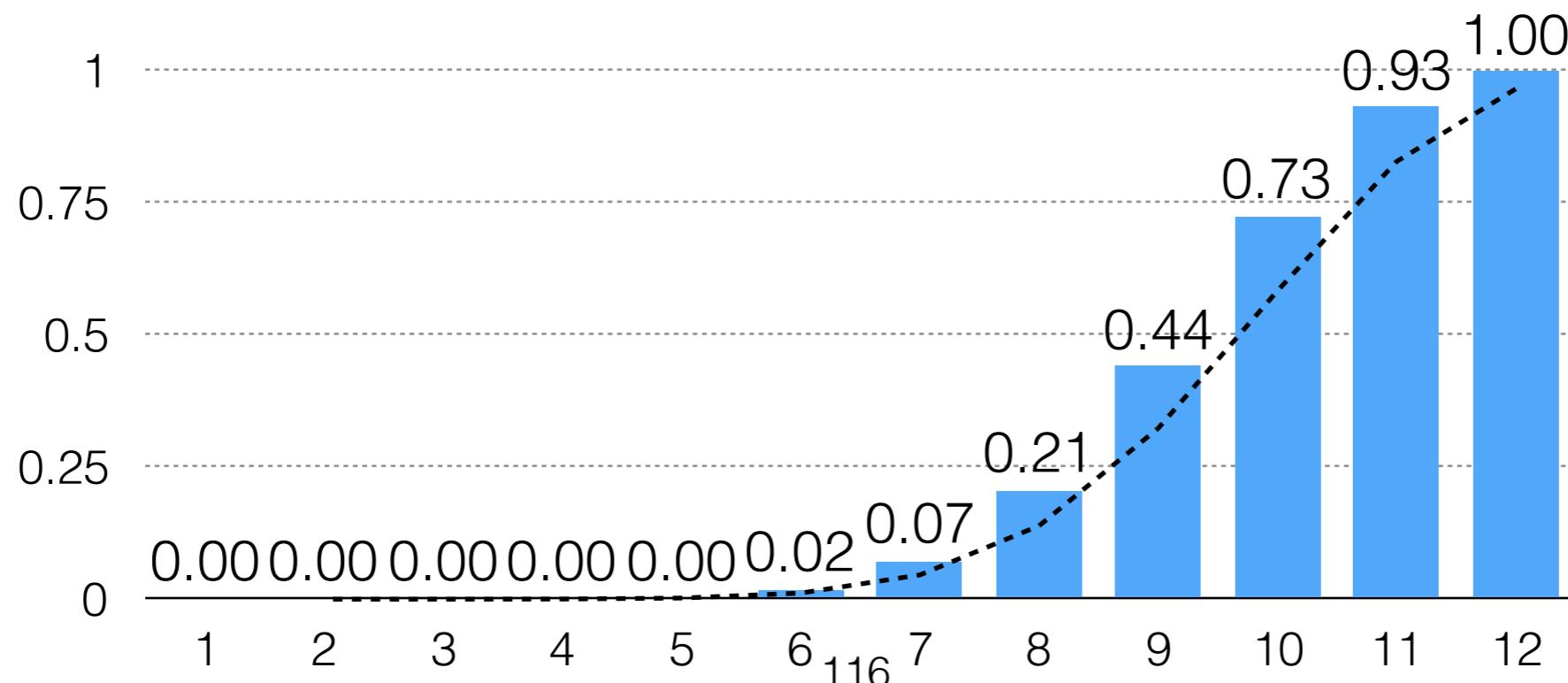


Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"

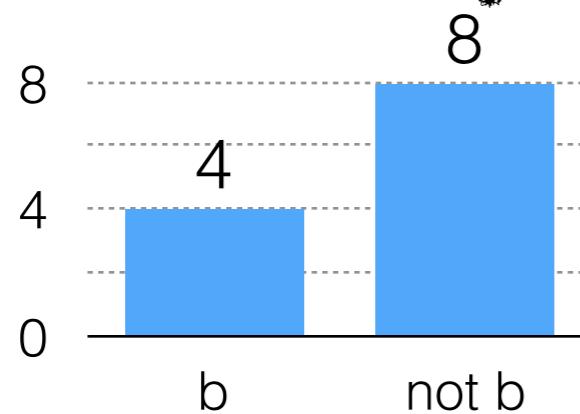


Is the 4 (b)s we observed significantly lower than what we would expect by chance, assuming that in fact 80% of answers are (b)?



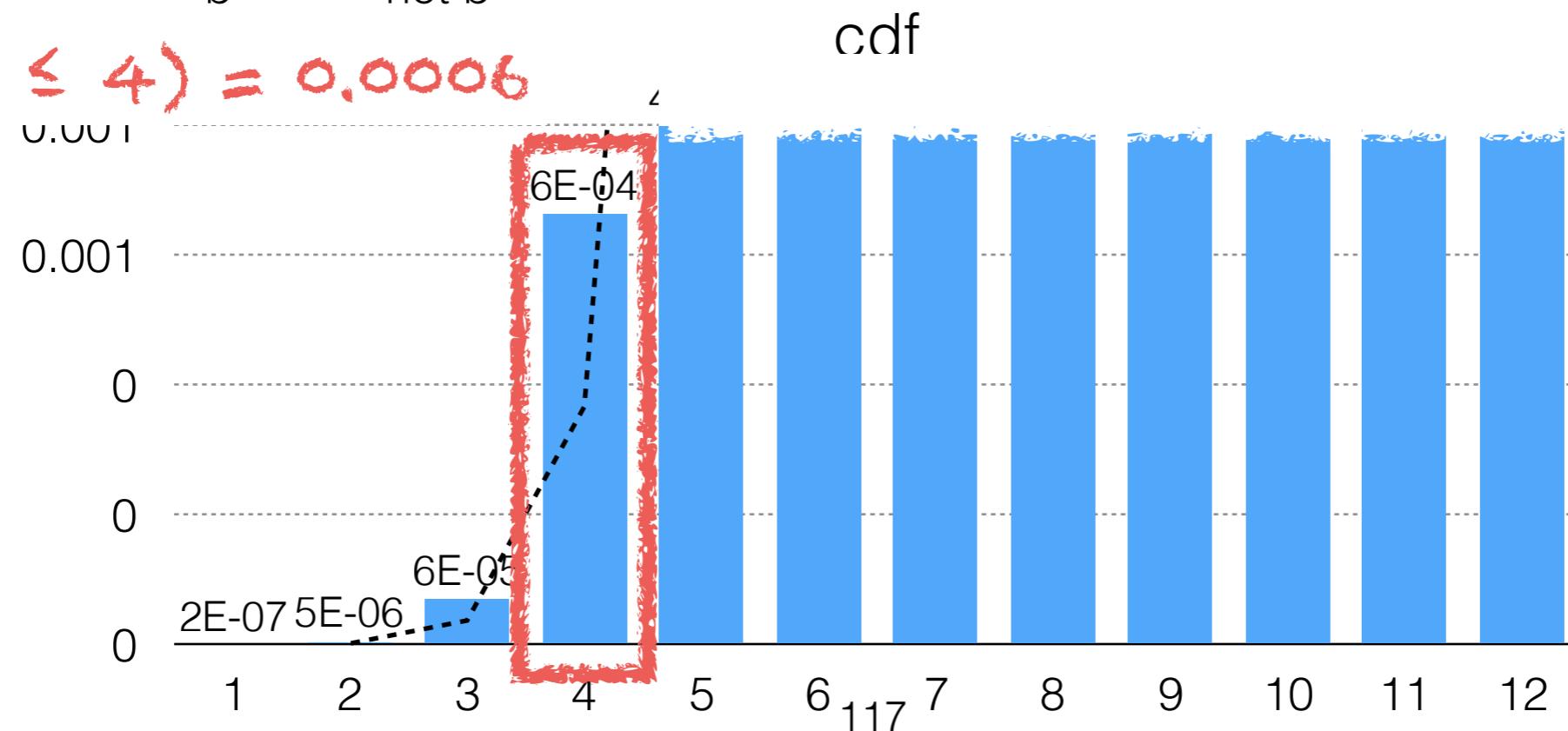
Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"



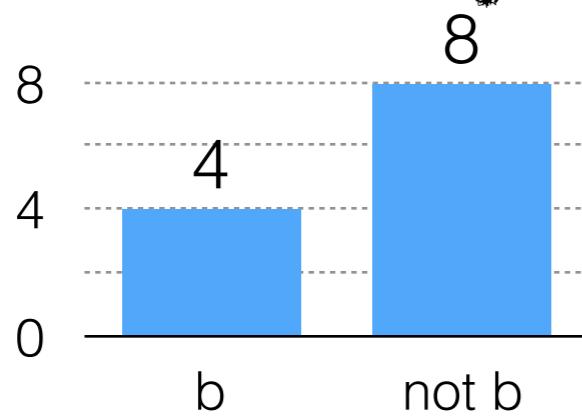
Is the 4 (b)s we observed significantly lower than what we would expect by chance, assuming that in fact 80% of answers are (b)?

$$P(\#\text{(b)s} \leq 4) = 0.0006$$



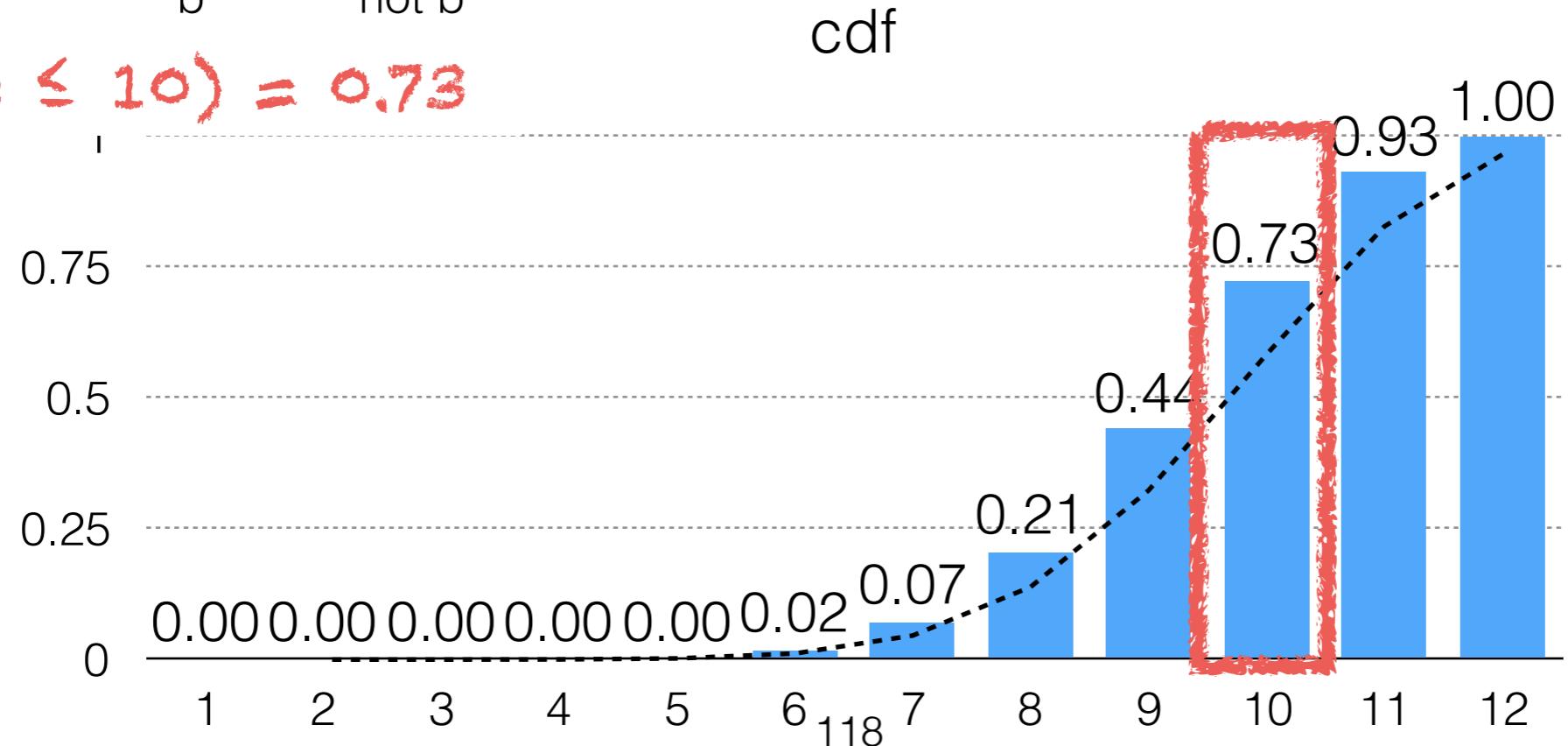
Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"



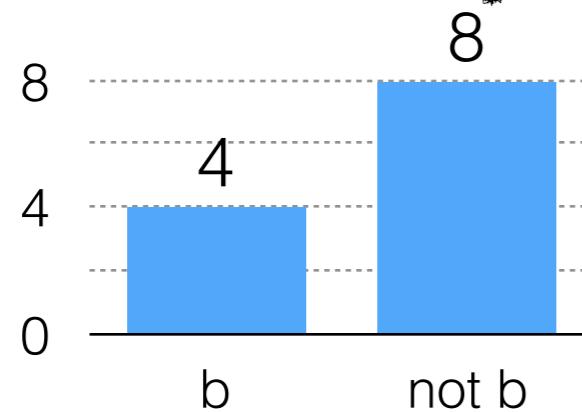
$$P(\#\text{(b)s} \leq 10) = 0.73$$

Is the 4 (b)s we observed significantly lower than what we would expect by chance, assuming that in fact 80% of answers are (b)?



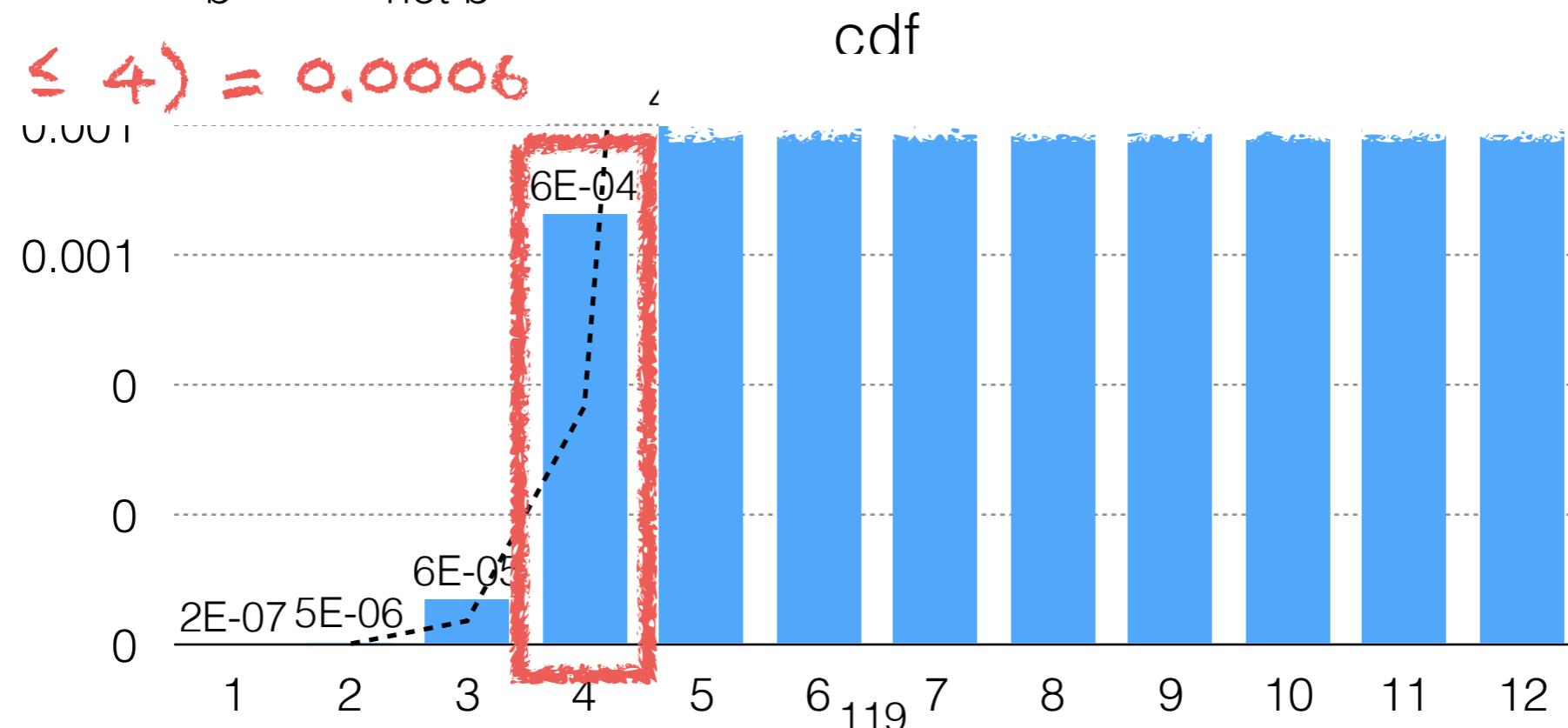
Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"



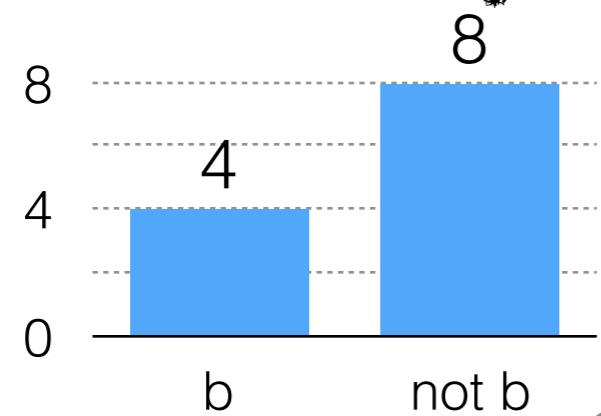
Is the 4 (b)s we observed significantly lower than what we would expect by chance, assuming that in fact 80% of answers are (b)?

$$P(\#\text{(b)s} \leq 4) = 0.0006$$



Are the answers to my clicker questions random?

"I swear literally like 80% of the answers are just (b)"

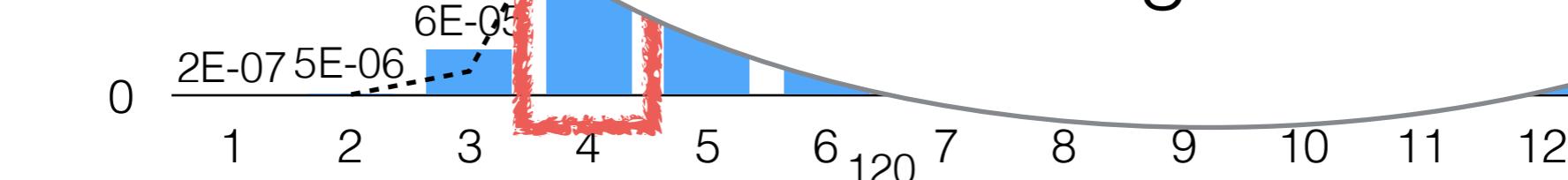


$$P(\#\text{(b)s} \leq 4) = 0.0001$$

Is the 4 (b)s we observed significantly lower than what we would expect, hence, assuming they are random?

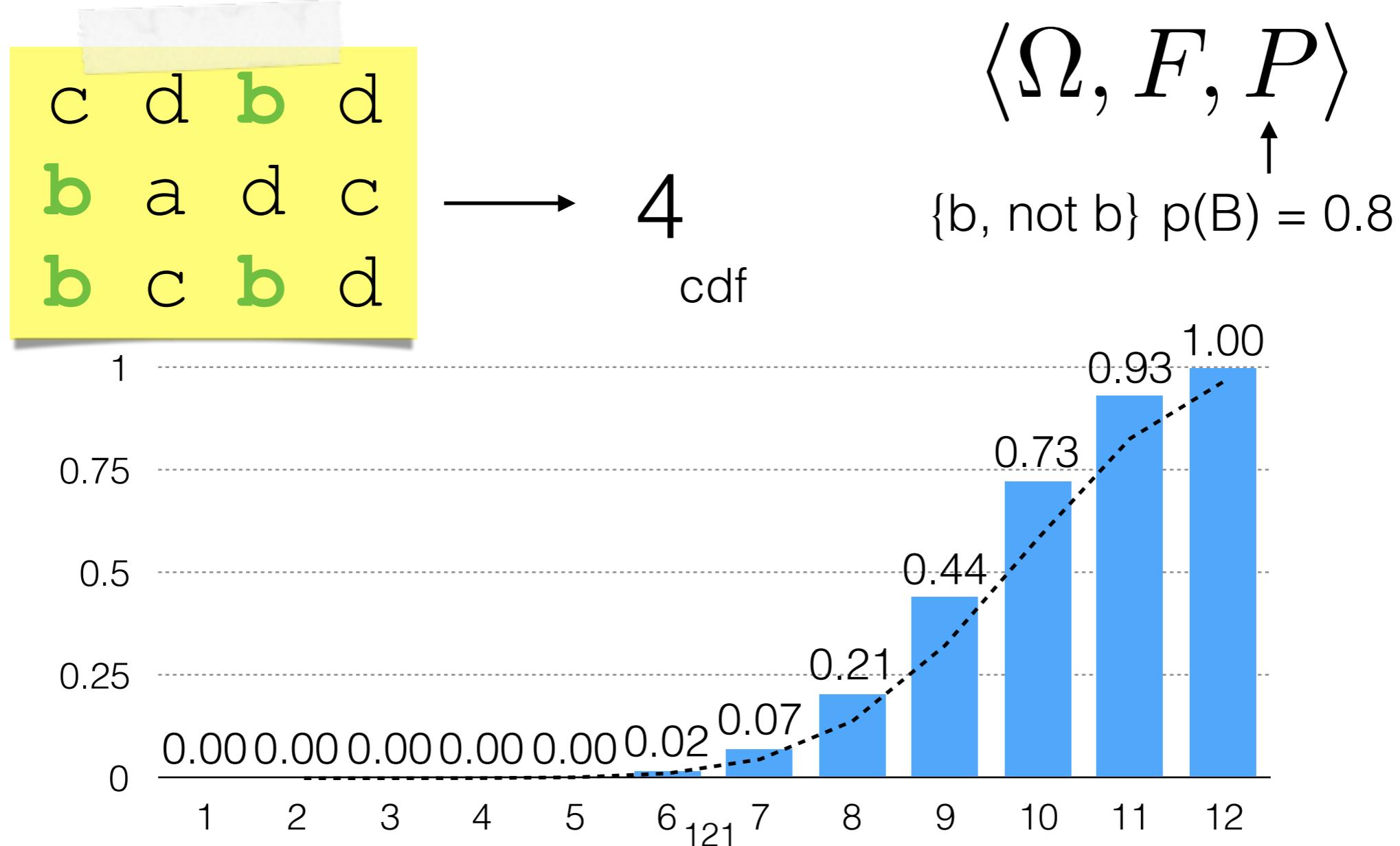
Dear friend, assuming this claim of yours is true, there is a very small chance of observing an event like the one we have just observed. Thus, I am inclined to reject your hypothesis.

Regards.



Hypothesis Testing

"I swear literally like 80% of the answers are just (b)"



Hypothesis Testing

Hypothesis

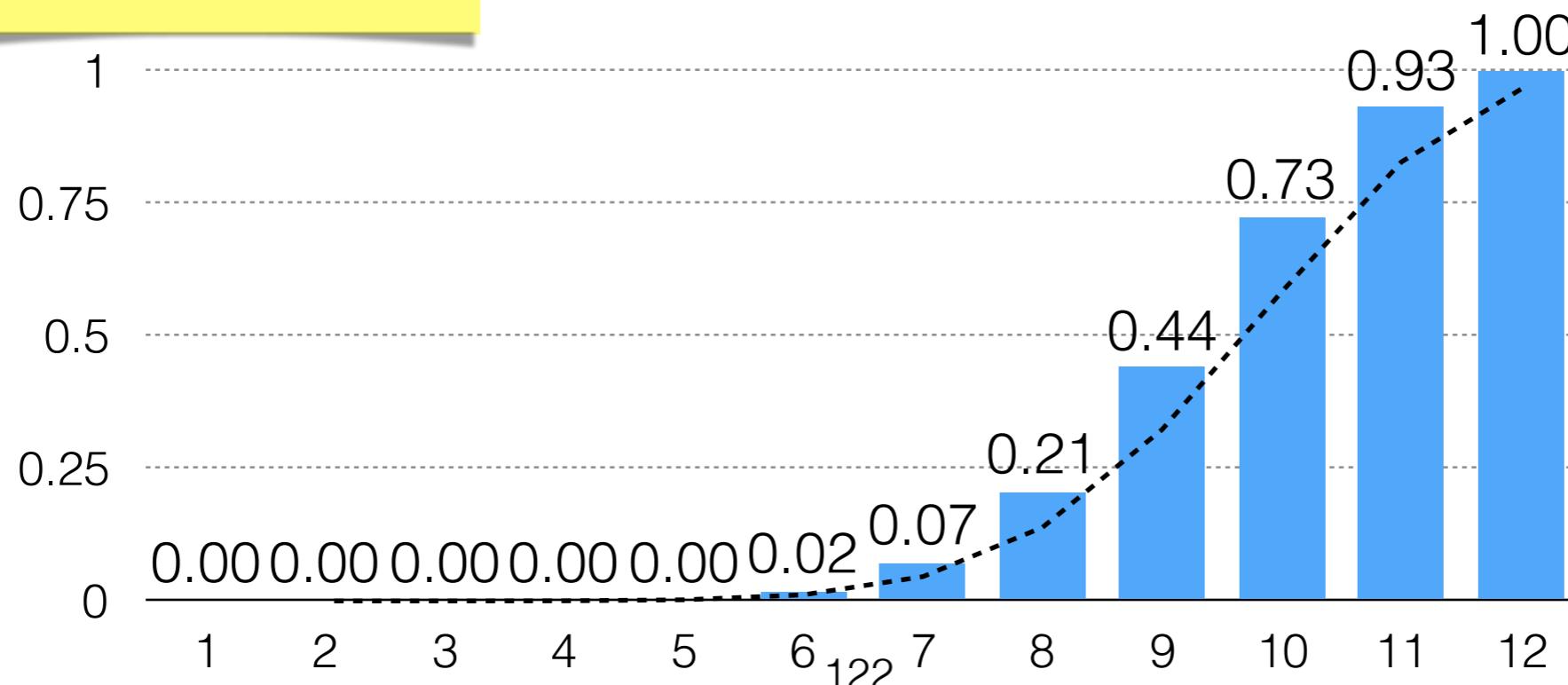
"I swear literally like 80% of the answers are just (b)"

c	d	b	d
b	a	d	c
b	c	b	d

→ 4
cdf

$$\langle \Omega, F, P \rangle$$

$$\{b, \text{not } b\} \quad p(B) = 0.8$$



Hypothesis Testing

Observation/Sample

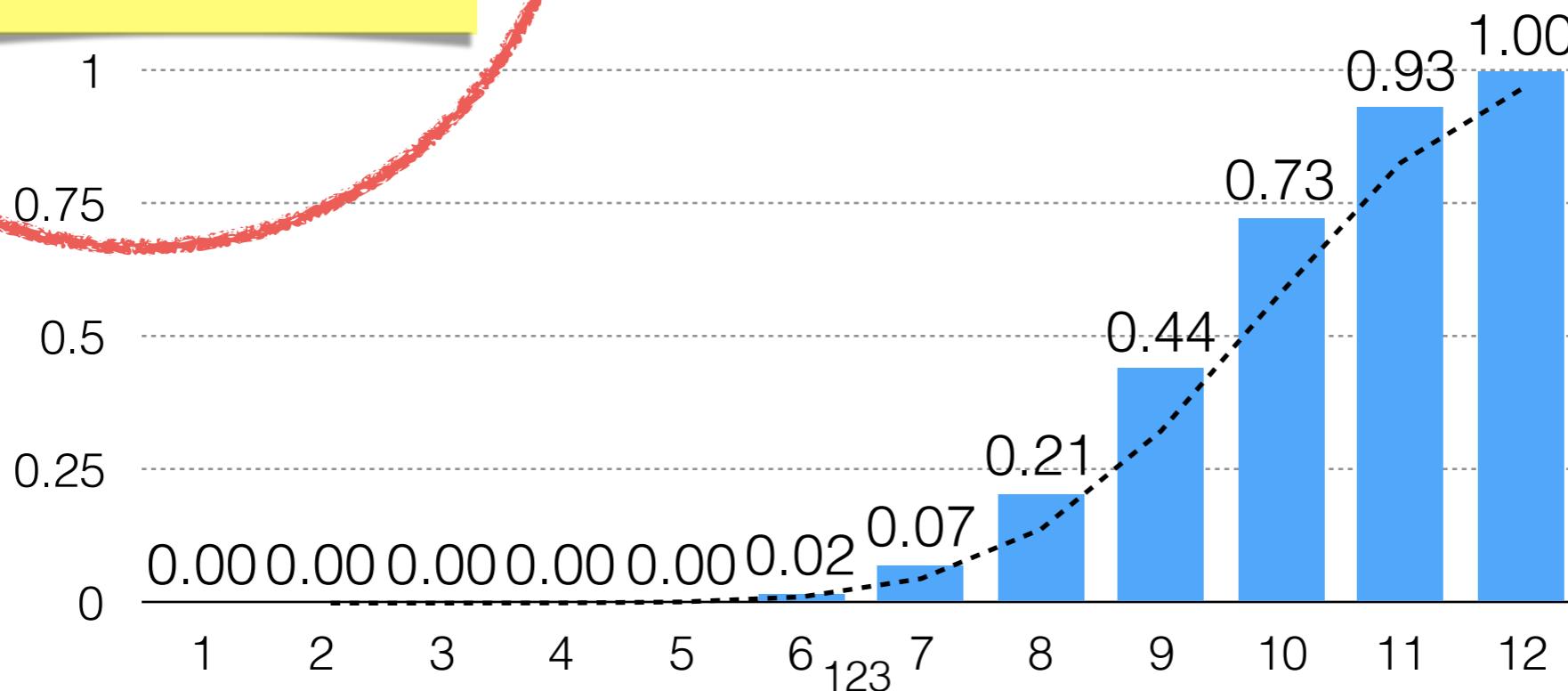
"I swear literally like 80% of the answers are just (b)"

c	d	b	d
b	a	d	c
b	c	b	d

→ 4
cdf

$$\langle \Omega, F, P \rangle$$

$$\{b, \text{not } b\} \quad p(B) = 0.8$$

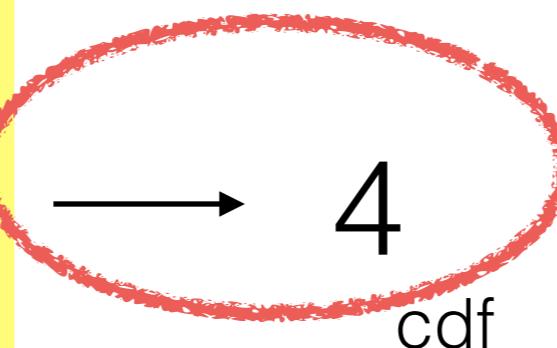


Hypothesis Testing

Test Statistic

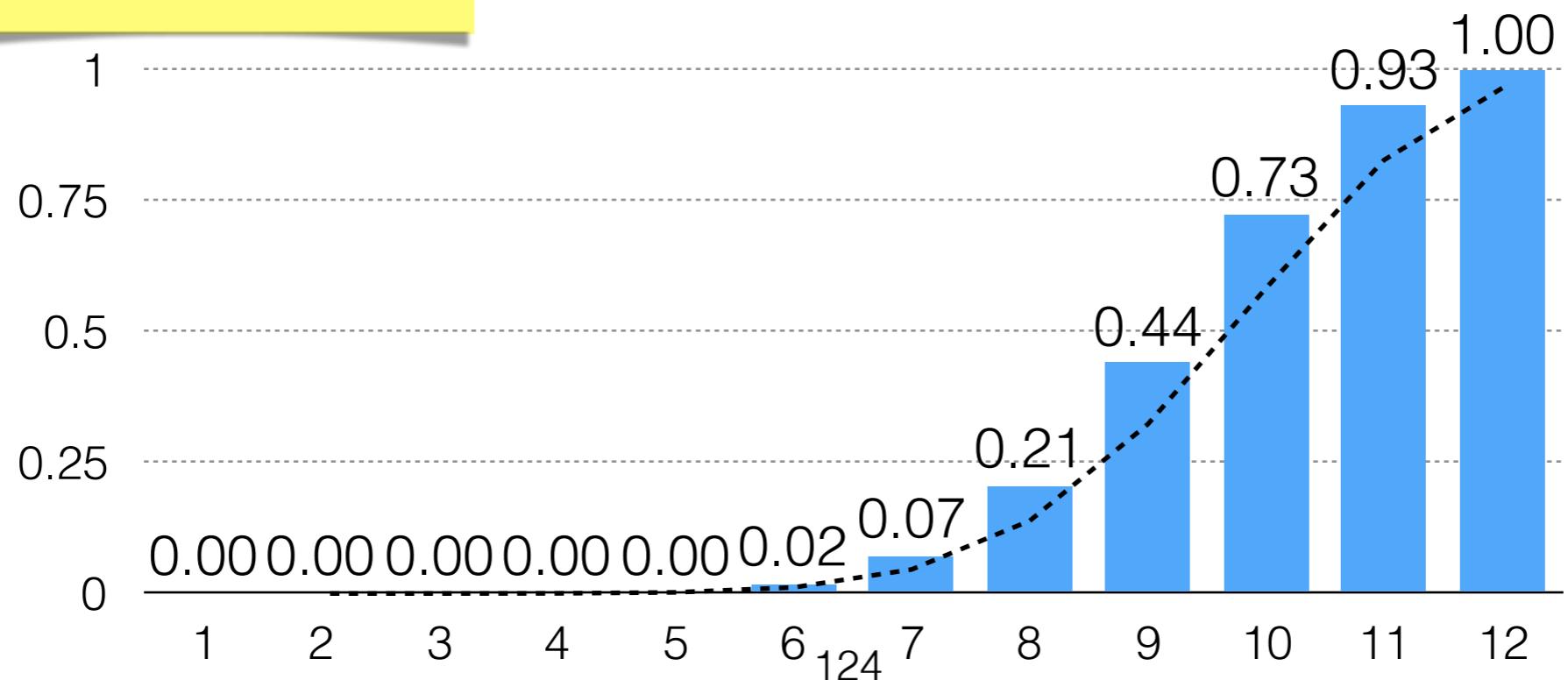
"I swear literally like 80% of the answers are just (b)"

c	d	b	d
b	a	d	c
b	c	b	d



$$\langle \Omega, F, P \rangle$$

$$\{b, \text{not } b\} \quad p(B) = 0.8$$



Hypothesis Testing

Theoretical Distribution

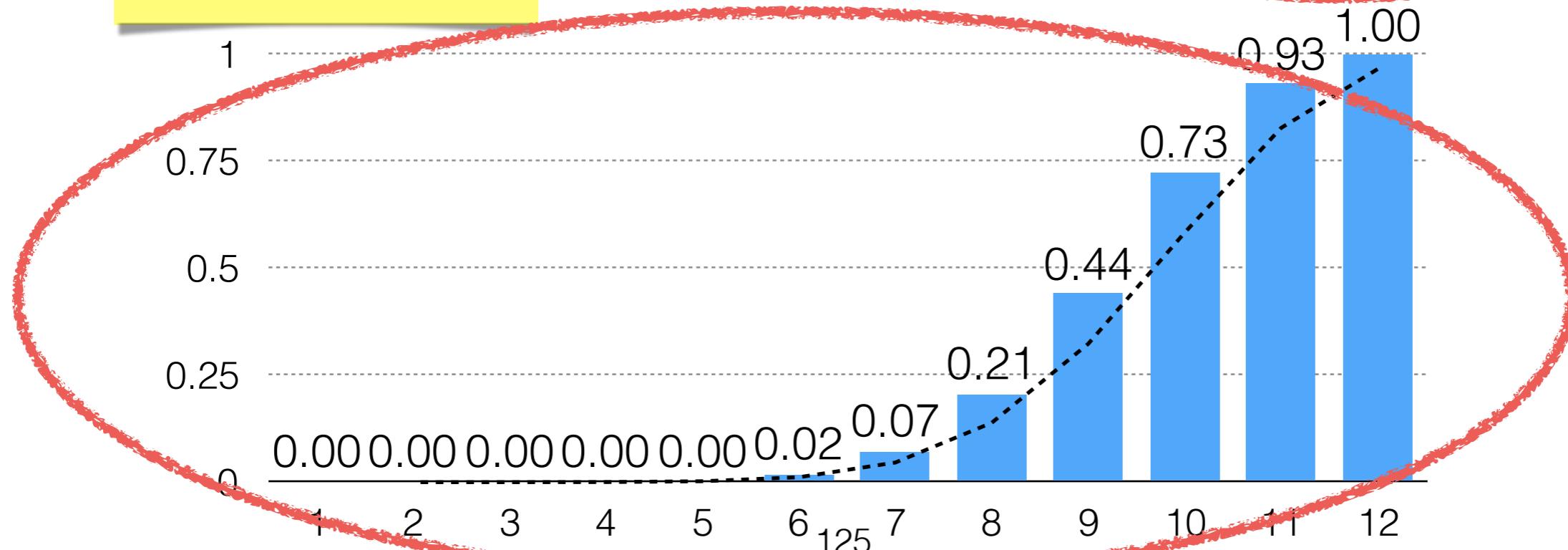
"I swear literally like 80% of the answers are just (b)"

c	d	b	d
b	a	d	c
b	c	b	d

→ 4
cdf

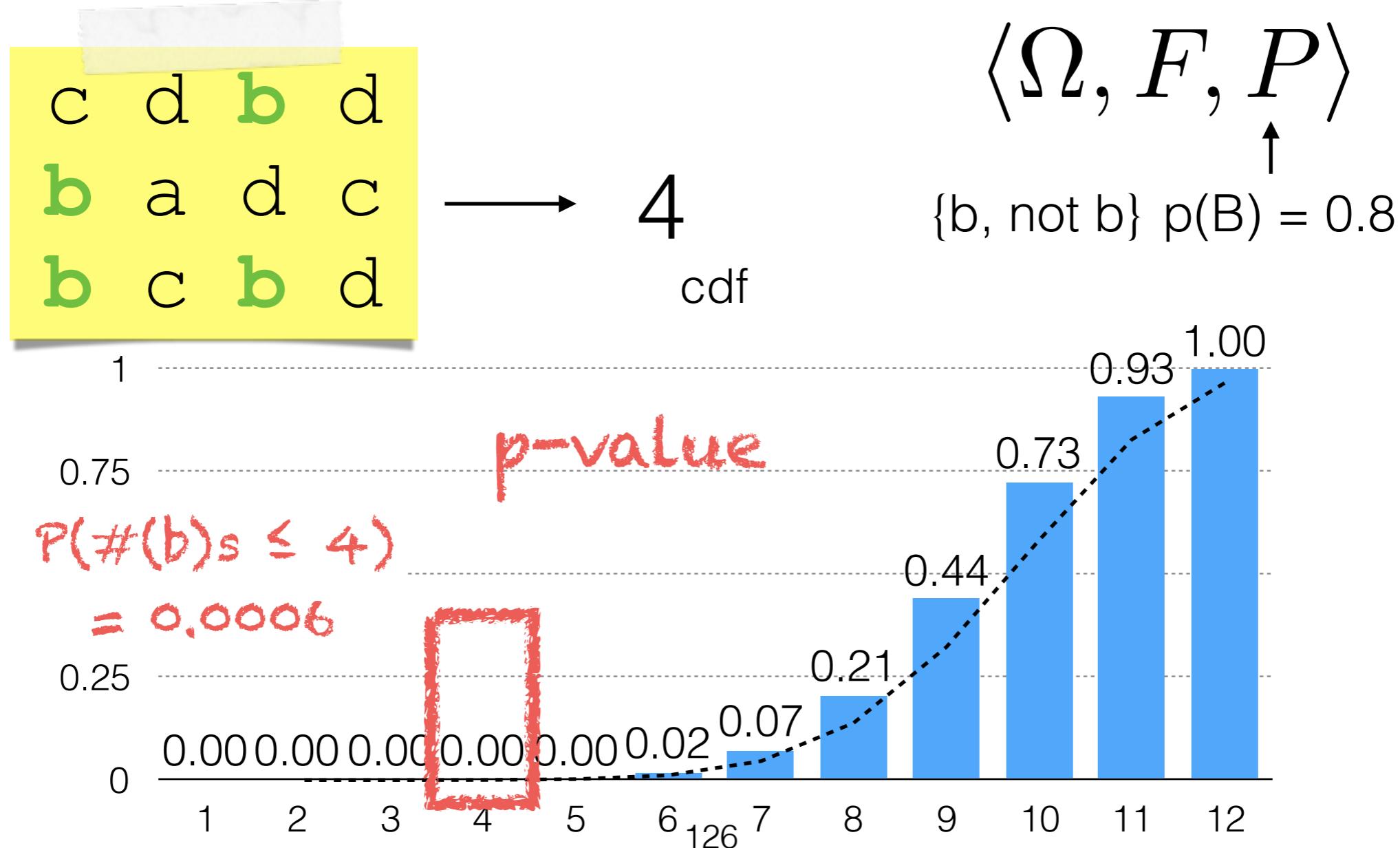
$$\langle \Omega, F, P \rangle$$

$$\{b, \text{not } b\} \quad p(B) = 0.8$$



Hypothesis Testing

"I swear literally like 80% of the answers are just (b)"



Clicker Question!

Clicker Question!

Given all of this, is your friend wrong?

- a) Yes!
- b) No...

"I swear literally like 80% of the answers are just (b)"

c	d	b	d
b	a	d	c
b	c	b	d



4
cdf

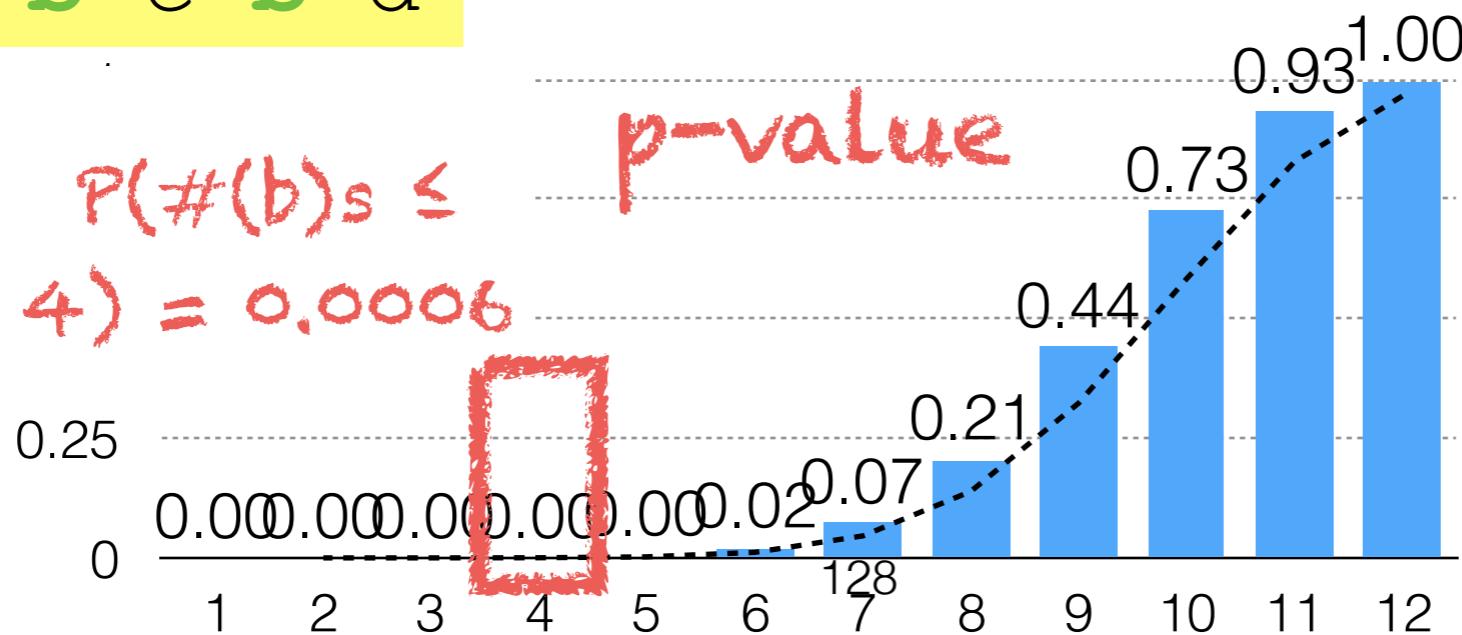
$\langle \Omega, F, P \rangle$

{b, not b}

$p(B) = 0.8$

$P(\#\{\text{b}\} \leq 4) = 0.0006$

p-value



Discussion Question!

Given all of this, is your friend wrong?

- a) Yes!
b) No...

"I swear literally like 80% of the answers are just (b)"

c	d	b	d
b	a	d	c
b	c	b	d

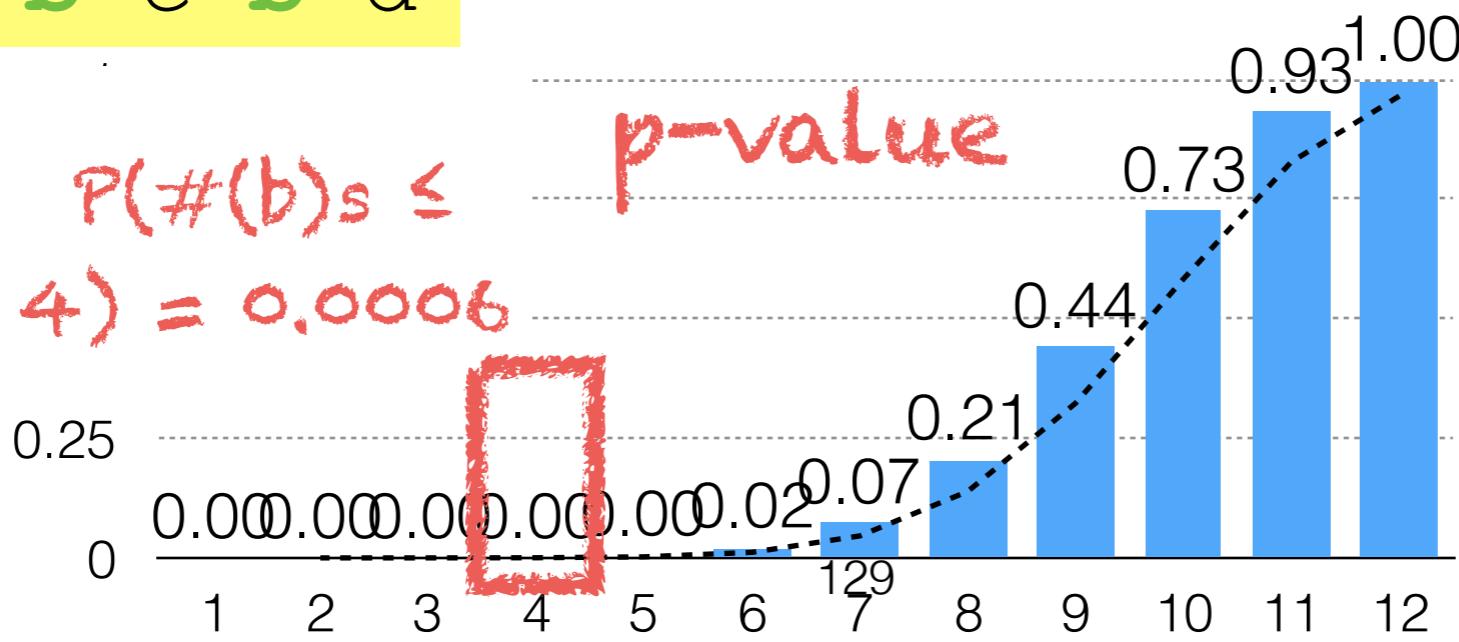


4
cdf

$\langle \Omega, F, P \rangle$
↑
 $\{b, \text{not } b\}$ $p(B) = 0.8$

$P(\#\{\text{b}\} \leq 4) = 0.0006$

p-value



okie done now