Assignment 2

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***Abstract*—This document calculate the n^{th} power of given matrix A, using Cayley Hamilton theorem.**

Download all python codes from $\boxed{\text{https://github.com/cs19resch11004/hari}}$
Download all Latex-tikz codes from $\boxed{\text{https://github.com/cs19resch11004/hari}}$

I. Problem

If $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$ then prove that $A^n = \begin{pmatrix} \text{1 + 2n} & \text{-4n} \\ \text{n} & \text{1 - 2n} \end{pmatrix}$, where n is any positive integer.

II. Solution

Characteristic equation of A is $\lambda^2 - 2\lambda + 1 = 0$. By using Cayley Hamolton theorem, given matrix A is a solution of its own characteristic equation. So $A^2 - 2A + I = 0$.

$$A^2 = 2A - I \tag{1}$$
$$A^3 = 3A - 2I \tag{2}$$
$$A^4 = 4A - 3I \tag{3}$$
$$A^5 = 5A - 4I \tag{4}$$

By the substitution,

$$A^n = nA - (n-1)I \tag{5}$$

1) Proving above equation is true for n=2,

$$A^2 = 2A - I \tag{6}$$

2) Assume it is true for n = k

$$A^k = kA - (k-1)I \tag{7}$$

3) Proving it is true for n = k+1

$$A^{k+1} = A^k A \tag{8}$$
$$= (kA - (k-1)I)A \tag{9}$$
$$= kA^2 - (k-1)A \tag{10}$$
$$= k(2A - I) - (k-1)A \tag{11}$$
$$= (2k - k + 1)A - kI \tag{12}$$
$$= (k+1)A - kI \tag{13}$$

So, $A^n = nA - (n-1)I$

Substituting matrix A = $\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$ in the equation5,

$$A^n = n\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} - (n-1)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{14}$$
$$= \begin{pmatrix} 3n & -4n \\ n & -n \end{pmatrix} - \begin{pmatrix} (n-1) & 0 \\ 0 & (n-1) \end{pmatrix} \tag{15}$$
$$= \begin{pmatrix} 2n+1 & -4n \\ n & 1-2n \end{pmatrix} \tag{16}$$

So, $A^n = \begin{pmatrix} 2n+1 & -4n \\ n & 1-2n \end{pmatrix}$