Assignment 3

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***Abstract*— This document calculate the circle equation such that circle is passing through given two points and the centre of the circle is placed on given straight line.**

Download all python codes from https://github.com/cs19resch11004/hari
Download all Latex-tikz codes from https://github.com/cs19resch11004/hari

I. Problem

Find the equation to the circle which passes through the points $\begin{pmatrix}1\\-2\end{pmatrix}$ and $\begin{pmatrix}4\\-3\end{pmatrix}$ and which has its centre on the straight line $\begin{pmatrix}3 & 4\end{pmatrix}x = 7$.

II. solution

The equation of circle can be expressed as

$$\vec{x}^T\vec{x} - 2\vec{C}^T\vec{x} + f = 0 \tag{1}$$

$\vec{C}$ is the centre and substituting the points in the equation of circle we get

$$2\begin{pmatrix}1 & -2\end{pmatrix}\vec{C} - f = 5 \tag{2}$$
$$2\begin{pmatrix}4 & -3\end{pmatrix}\vec{C} - f = 25 \tag{3}$$
$$\begin{pmatrix}3 & 4\end{pmatrix}\vec{C} = 7 \tag{4}$$

can be expressed in matrix form

$$\begin{pmatrix}3 & 4 & 0\\2 & -4 & -1\\8 & -6 & -1\end{pmatrix}\begin{pmatrix}\vec{C}\\f\end{pmatrix} = \begin{pmatrix}7\\5\\25\end{pmatrix} \tag{5}$$

Row reducing the augmented matrix

$$\begin{pmatrix}3 & 4 & 0 & 7\\2 & -4 & -1 & 5\\8 & -6 & -1 & 25\end{pmatrix}\xleftrightarrow{R_1\leftarrow R_1/3}\begin{pmatrix}1 & \frac{4}{3} & 0 & \frac{7}{3}\\2 & -4 & -1 & 5\\8 & -6 & -1 & 25\end{pmatrix} \tag{6}$$

$$\xleftrightarrow[R_3\leftarrow R_3-8R_1]{R_2\leftarrow R_2-2R_1}\begin{pmatrix}1 & \frac{4}{3} & 0 & \frac{7}{3}\\0 & \frac{-20}{3} & -1 & \frac{1}{3}\\0 & \frac{-50}{3} & -1 & \frac{19}{3}\end{pmatrix} \tag{7}$$

$$\xleftrightarrow{R_2\leftarrow\frac{-3}{20}R_2}\begin{pmatrix}1 & \frac{4}{3} & 0 & \frac{7}{3}\\0 & 1 & \frac{3}{20} & \frac{-1}{20}\\0 & \frac{-50}{3} & -1 & \frac{19}{3}\end{pmatrix} \tag{8}$$

$$\xleftrightarrow{R_3\leftarrow R_3+\frac{50}{3}R_2}\begin{pmatrix}1 & \frac{4}{3} & 0 & \frac{7}{3}\\0 & 1 & \frac{3}{20} & \frac{-1}{20}\\0 & 0 & \frac{3}{2} & \frac{11}{2}\end{pmatrix} \tag{9}$$

$$\vec{C} = \begin{pmatrix}\frac{47}{15}\\\frac{-3}{5}\end{pmatrix} f = \frac{11}{3} \tag{10}$$

$$f = \frac{11}{3} \tag{11}$$

The required circle equation,

$$\vec{x}^T\vec{x} - 2\begin{pmatrix}\frac{47}{15} & \frac{-3}{5}\end{pmatrix}\vec{x} + \frac{11}{3} = 0 \tag{12}$$

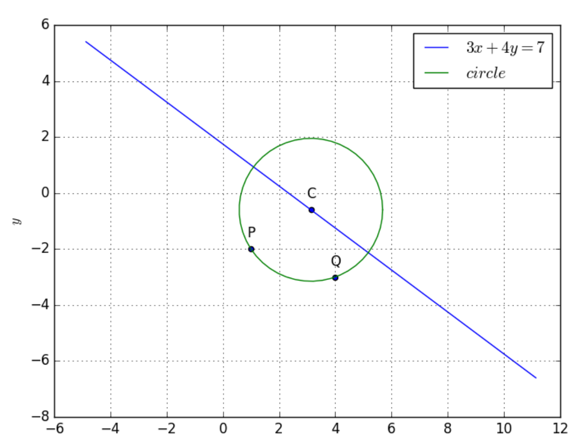

Fig. 1. Circle passing through point P and Q also centre lie on the line 3x+4y=7