

Assignment 3

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Abstract— This document calculate the circle equation such that circle is passing through given two points and the centre of the circle is placed on given straight line.

Download all python codes from
<https://github.com/cs19resch11004/hari>
 Download all Latex-tikz codes from
<https://github.com/cs19resch11004/hari>

I. PROBLEM

Find the equation to the circle which passes through the points $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and which has its centre on the straight line $(3 \ 4) x = 7$.

II. SOLUTION

The equation of circle can be expressed as

$$\vec{x}^T \vec{x} - 2\vec{C}^T \vec{x} + f = 0 \quad (1)$$

\vec{C} is the centre and substituting the points in the equation of circle we get

$$2 \begin{pmatrix} 1 & -2 \end{pmatrix} \vec{C} - f = 5 \quad (2)$$

$$2 \begin{pmatrix} 4 & -3 \end{pmatrix} \vec{C} - f = 25 \quad (3)$$

$$\begin{pmatrix} 3 & 4 \end{pmatrix} \vec{C} = 7 \quad (4)$$

can be expressed in matrix form

$$\begin{pmatrix} 3 & 4 & 0 \\ 2 & -4 & -1 \\ 8 & -6 & -1 \end{pmatrix} \begin{pmatrix} \vec{C} \\ f \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \\ 25 \end{pmatrix} \quad (5)$$

Row reducing the augmented matrix

$$\begin{pmatrix} 3 & 4 & 0 & 7 \\ 2 & -4 & -1 & 5 \\ 8 & -6 & -1 & 25 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1/3} \begin{pmatrix} 1 & \frac{4}{3} & 0 & \frac{7}{3} \\ 2 & -4 & -1 & 5 \\ 8 & -6 & -1 & 25 \end{pmatrix} \quad (6)$$

$$\xrightarrow{\begin{matrix} R_2 \leftarrow R_2 - 2R_1 \\ R_3 \leftarrow R_3 - 8R_1 \end{matrix}} \begin{pmatrix} 1 & \frac{4}{3} & 0 & \frac{7}{3} \\ 0 & -\frac{20}{3} & -1 & \frac{1}{3} \\ 0 & -\frac{50}{3} & -1 & \frac{19}{3} \end{pmatrix} \quad (7)$$

$$\xrightarrow{R_2 \leftarrow -\frac{3}{20} R_2} \begin{pmatrix} 1 & \frac{4}{3} & 0 & \frac{7}{3} \\ 0 & 1 & \frac{3}{20} & -\frac{1}{20} \\ 0 & -\frac{50}{3} & -1 & \frac{19}{3} \end{pmatrix} \quad (8)$$

$$\xrightarrow{R_3 \leftarrow R_3 + \frac{50}{3} R_2} \begin{pmatrix} 1 & \frac{4}{3} & 0 & \frac{7}{3} \\ 0 & 1 & \frac{3}{20} & -\frac{1}{20} \\ 0 & 0 & \frac{3}{2} & \frac{11}{2} \end{pmatrix} \quad (9)$$

$$\vec{C} = \begin{pmatrix} \frac{47}{15} \\ -\frac{3}{5} \end{pmatrix} f = \frac{11}{3} \quad (10)$$

$$f = \frac{11}{3} \quad (11)$$

The required circle equation,

$$\vec{x}^T \vec{x} - 2 \begin{pmatrix} \frac{47}{15} & -\frac{3}{5} \end{pmatrix} \vec{x} + \frac{11}{3} = 0 \quad (12)$$

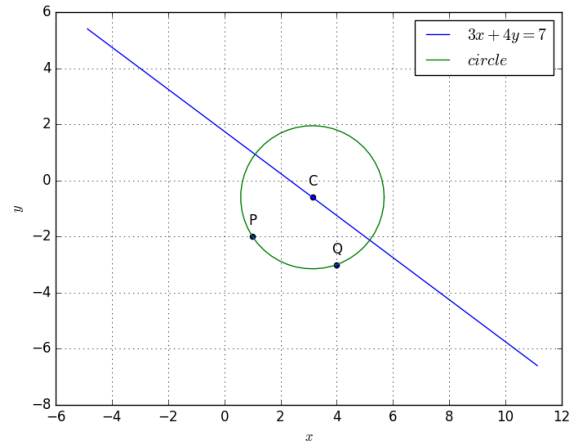


Fig. 1. Circle passing through point P and Q also centre lie on the line $3x+4y=7$