Assignment 3

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***Abstract*— This document calculate the circle equation such that circle is passing through given two points and the centre of the circle is placed on given straight line.**

Download all python codes from
https://github.com/cs19resch11004/hari
Download all Latex-tikz codes from
https://github.com/cs19resch11004/hari

I. Problem

Find the equation to the circle which passes through the points $\begin{pmatrix}1\\-2\end{pmatrix}$ and $\begin{pmatrix}4\\-3\end{pmatrix}$ and which has its centre on the straight line $\begin{pmatrix}3 & 4\end{pmatrix} X = 7$.

II. Solution

Given points $P = \begin{pmatrix}1\\-2\end{pmatrix}$ and $Q = \begin{pmatrix}4\\-3\end{pmatrix}$ and the straight line, which has centre of the circle is,

$$\begin{pmatrix}3 & 4\end{pmatrix} X = 7 \tag{1}$$

Let r be the radius of the circle. Let $C = \begin{pmatrix}h\\k\end{pmatrix}$ is the centre of the circle, then the circle equation is,

$$(x-h)^2 + (y-k)^2 = r^2 \tag{2}$$

Substituting the point P in equation 2,

$$(1-h)^2 + (-2-k)^2 = r^2 \tag{3}$$
$$h^2 + k^2 - 2h + 4k + 5 = r^2 \tag{4}$$

Substituting the point Q in equation 2,

$$(4-h)^2 + (-3-k)^2 = r^2 \tag{5}$$
$$h^2 + k^2 - 8h + 6k + 25 = r^2 \tag{6}$$

Eqution 4 - equation 6 gives,

$$3h - k = 10 \tag{7}$$

Substitutimg Centre C in equation 1,

$$3h + 4k = 7 \tag{8}$$

By solving equation 7 and equation 8, we will get the centre of the circle. as

$$C = \begin{pmatrix}\frac{47}{15}\\ \\ \frac{-3}{5}\end{pmatrix} \tag{9}$$

Radius r of the circle is the distance between points C and P.

$$r = \sqrt{(\frac{47}{15} - 1)^2 + (\frac{-3}{5} + 2)^2} \tag{10}$$
$$r = \frac{\sqrt{1465}}{15} \tag{11}$$

Required Resultant circle equation is,

$$X^T X - 2\begin{pmatrix}\frac{47}{15}\\ \\ \frac{-3}{5}\end{pmatrix} X + \frac{825}{225} = 0$$