# Assignment 2

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***Abstract*— This document calculate the $n^{th}$ power of given matrix A, using Cayley Hamilton theorem.**

Download all python codes from https://github.com/cs19resch11004/hari
Download all Latex-tikz codes from https://github.com/cs19resch11004/hari

## I. Problem

If $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$ then prove that $A^n = \begin{pmatrix} 1 + 2n & -4n \\ n & 1 - 2n \end{pmatrix}$, where n is any positive integer.

## II. Solution

Characteristic equation of A is $\lambda^2 - 2\lambda + 1 = 0$. By using Cayley Hamolton theorem, given matrix A is a solution of its own characteristic equation. So $A^2 - 2A + I = 0$.

$$A^2 = 2A - I \tag{1}$$
$$A^3 = 3A - 2I \tag{2}$$
$$A^4 = 4A - 3I \tag{3}$$
$$A^5 = 5A - 4I \tag{4}$$

By the substitution,

$$A^n = nA - (n-1)I \tag{5}$$

Substituting matrix A = $\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$ in above equation,

$$A^n = n\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} - (n-1)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$
$$= \begin{pmatrix} 3n & -4n \\ n & -n \end{pmatrix} - \begin{pmatrix} (n-1) & 0 \\ 0 & (n-1) \end{pmatrix} \tag{7}$$
$$= \begin{pmatrix} 2n+1 & -4n \\ n & 1-2n \end{pmatrix} \tag{8}$$

So, $A^n = \begin{pmatrix} 2n+1 & -4n \\ n & 1-2n \end{pmatrix}$