PCAP Framework for Managing Complexity

Python has features that facilitate modular programming.

- **def** combines operations into a procedure and binds a name to it
- **lists** provide flexible and hierarchical structures for data
- **variables** associate names with data
- **classes** associate data (attributes) and procedures (methods)

	procedures	data
Primitives	`+, *, ==, !=`	numbers, booleans, strings
Combination	`if, while, f(g(x))`	lists, dictionaries, objects
Abstraction	`def`	classes
Patterns	higher-order procedures	super-classes, sub-classes

PCAP Framework for Managing Complexity

We will build on these ideas to manage complexity at higher levels.

- **Programming Styles** for dealing with complexity
- PCAP in Higher-Level Abstractions: **State Machines**

Reading: Course notes, chapters 3–4

Programming Styles for Managing Complexity

Structure of program has significant effect on its modularity.

Imperative (procedural) programming

- focus on step-by-step instructions to accomplish task
- organize program using structured conditionals and loops

Functional programming

- focus on procedures that mimic mathematical functions, producing outputs from inputs without side effects
- functions are **first-class objects** used in data structures, arguments to procedures, and can be returned by procedures

Object-oriented programming

- focus on collections of related procedures and data
- organize programs as hierarchies of related classes and instances

Example Program

Task: Find a sequence of operations (either `increment` or `square`) that transforms the integer i (initial) to the integer g (goal).

Example: applying the sequence

`increment increment increment square`

to **1** yields **16**

apply `increment` to **1** $\rightarrow$ **2**

apply `increment` to **2** $\rightarrow$ **3**

apply `increment` to **3** $\rightarrow$ **4**

apply `square` to **4** $\rightarrow$ **16**

Check Yourself

What is the minimum length sequence of **increment** and **square** operations needed to transform **1** to **100**?

1: **<4** 2: **4** 3: **5** 4: **6** 5: **>6**

Check Yourself

What is the minimum length sequence of `increment` and `square` operations needed to transform `1` to `100`?

Try to use as many squares (especially big ones) as possible.

apply `increment` to **1** $\rightarrow$ **2**
apply `increment` to **2** $\rightarrow$ **3**
apply `square` to **3** $\rightarrow$ **9**
apply `increment` to **9** $\rightarrow$ **10**
apply `square` to **10** $\rightarrow$ **100**

Five operations.

Check Yourself

What is the minimum length sequence of **`increment`** and **`square`** operations needed to transform **`1`** to **`100`**? **`3: 5`**

1: **`<4`** 2: **`4`** 3: **`5`** 4: **`6`** 5: **`>6`**

Imperative (Procedural) Programming

Solve the previous problem by writing an imperative program to step through all possible sequences of length 1, 2, 3, ...

```
def increment(n):
    return n+1
def square(n):
    return n**2
def findSequence(initial,goal):
    # construct list of "candidates" of form ('1 increment increment',3)
    candidates = [(str(initial),initial)]
    # loop over sequences of length "i" = 1, 2, 3, ...
    for i in range(1,goal-initial+1):
        newCandidates = []
        # construct each new candidate by adding one operation to prev candidate
        for (action,result) in candidates:
            for (a,r) in [(' increment',increment),(' square',square)]:
                newCandidates.append((action+a,r(result)))
                print i,': ',newCandidates[-1]
                if newCandidates[-1][1] == goal:
                    return newCandidates[-1]
        candidates = newCandidates

answer = findSequence(1,100)
print 'answer =',answer
```

Imperative (Procedural) Programming

```
1 :  ('1 increment', 2)
1 :  ('1 square', 1)
2 :  ('1 increment increment', 3)
2 :  ('1 increment square', 4)
2 :  ('1 square increment', 2)
2 :  ('1 square square', 1)
3 :  ('1 increment increment increment', 4)
3 :  ('1 increment increment square', 9)
3 :  ('1 increment square increment', 5)
3 :  ('1 increment square square', 16)
3 :  ('1 square increment increment', 3)
3 :  ('1 square increment square', 4)
3 :  ('1 square square increment', 2)
3 :  ('1 square square square', 1)
4 :  ('1 increment increment increment increment', 5)
4 :  ('1 increment increment increment square', 16)
4 :  ('1 increment increment square increment', 10)
4 :  ('1 increment increment square square', 81)
4 :  ('1 increment square increment increment', 6)
4 :  ('1 increment square increment square', 25)
4 :  ('1 increment square square increment', 17)
4 :  ('1 increment square square square', 256)
4 :  ('1 square increment increment increment', 4)
4 :  ('1 square increment increment square', 9)
```

```
4 :  ('1 square increment square increment', 5)
4 :  ('1 square increment square square', 16)
4 :  ('1 square square increment increment', 3)
4 :  ('1 square square increment square', 4)
4 :  ('1 square square square increment', 2)
4 :  ('1 square square square square', 1)
5 :  ('1 increment increment increment increment increment', 6)
5 :  ('1 increment increment increment increment square', 25)
5 :  ('1 increment increment increment square increment', 17)
5 :  ('1 increment increment increment square square', 256)
5 :  ('1 increment increment square increment increment', 11)
```

5 : ('1 increment increment square increment square', 100)

```
answer = ('1 increment increment square increment square', 100)
```

Imperative (Procedural) Programming

This imperative version of the program has three levels of looping.

```
def findSequence(initial,goal):
    # construct list of "candidates" of form ('1 increment increment',3)
    candidates = [(str(initial),initial)]
    # loop over sequences of length "i" = 1, 2, 3, ...
    for i in range(1,goal-initial+1):
        newCandidates = []
        # construct each new candidate by adding one operation to prev candidate
        for (action,result) in candidates:
            for (a,r) in [(' increment',increment),(' square',square)]:
                newCandidates.append((action+a,r(result)))
                print i,': ',newCandidates[-1]
                if newCandidates[-1][1] == goal:
                    return newCandidates[-1]
        candidates = newCandidates
```

This approach is straightforward, but nested loops can be confusing.

Challenge is to get the indices right.

Functional Programming

This version focuses on functions as primitives.

```
def apply(opList,arg):
    if len(opList)==0:
        return arg
    else:
        return apply(opList[1:],opList[0](arg))

def addLevel(opList,fctList):
    return [x+[y] for y in fctList for x in opList]

def findSequence(initial,goal):
    opList = [[]]
    for i in range(1,goal-initial+1):
        opList = addLevel(opList,[increment,square])
        for seq in opList:
            if apply(seq,initial)==goal:
                return seq

answer = findSequence(1,100)
print 'answer =',answer
```

Functional Programming

The procedure **apply** is a "pure function."

```
def apply(opList,arg):
    if len(opList)==0:
        return arg
    else:
        return apply(opList[1:],opList[0](arg))
```

Its first argument is a list of functions. The procedure applies these functions to the second argument **arg** and returns the result.

```
>>> apply([],7)
7
>>> apply([increment],7)
8
>>> apply([square],7)
49
>>> apply([increment,square],7)
64
```

This list of procedures uses functions as first-class objects.

Functional Programming

The procedure **addLevel** is also a pure function.

```
def addLevel(opList,fctList):
    return [x+[y] for y in fctList for x in opList]
```

The first input is a list of sequences-of-operations, each of which is a list of functions.

The second input is a list of possible next-functions.

It returns a new list of sequences.

```
>>> addLevel([[increment]],[increment,square])
[[<function increment at 0xb7480aac>, <function increment at 0xb7480aac>],
[<function increment at 0xb7480aac>, <function square at 0xb747b25c>]]
```

Functional Programming

The answer is now a list of functions.

```
def apply(opList,arg):
    if len(opList)==0:
        return arg
    else:
        return apply(opList[1:],opList[0](arg))

def addLevel(opList,fctList):
    return [x+[y] for y in fctList for x in opList]

def findSequence(initial,goal):
    opList = [[]]
    for i in range(1,goal-initial+1):
        opList = addLevel(opList,[increment,square])
        for seq in opList:
            if apply(seq,initial)==goal:
                return seq

answer = findSequence(1,100)
print 'answer =',answer
```

answer = [<function increment at 0xb777ea74>, <function increment at 0xb777ea74>, <function square at 0xb7779224>, <function increment at 0xb777ea74>, <function square at 0xb7779224>]

Functional Programming

The functions **apply** and **addLevel** are easy to check.

```
def apply(opList,arg):
    if len(opList)==0:
        return arg
    else:
        return apply(opList[1:],opList[0](arg))

def addLevel(opList,fctList):
    return [x+[y] for y in fctList for x in opList]

>>> apply([],7)
7
>>> apply([increment],7)
8
>>> apply([square],7)
49
>>> apply([increment,square],7)
64
>>> addLevel([[increment]],[increment,square])
[[<function increment at 0xb7480aac>, <function increment at 0xb7480aac>],
[<function increment at 0xb7480aac>, <function square at 0xb747b25c>]]
```

Greater modularity reduces complexity and simplifies debugging.

Functional Programming

Also notice that the definition of **apply** is **recursive**:
the definition of **apply** calls **apply**.

```
>>> def apply(oplist,arg):
...     if len(opList)==0:
...         return arg
...     else:
...         return apply(opList[1:],opList[0](arg))
```

Recursion is

- an alternative way to implement iteration (looping)
- a natural generalization of functional programming
- powerful way to think about PCAP

Recursion

Express solution to problem in terms of simpler version of problem.

Example: raising a number to a non-negative integer power

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ b \cdot b^{n-1} & \text{if } n > 0 \end{cases}$$

functional notation:

$$f(n) = \begin{cases} 1 & \text{if } n = 0 \\ b\, f(n-1) & \text{if } n > 0 \end{cases}$$

Python implementation:

```
def exponent(b,n):
    if n==0:
        return 1
    else:
        return b*exponent(b,n-1)
```

Recursive Exponentiation

Invoking **exponent(2,6)** generates 6 more invocations of **exponent**.

```
def exponent(b,n):
    if n==0:
        return 1
    else:
        return b*exponent(b,n-1)

exponent(2,6)
    calls exponent(2,5)
        calls exponent(2,4)
            calls exponent(2,3)
                calls exponent(2,2)
                    calls exponent(2,1)
                        calls exponent(2,0)
                        returns 1
                    returns 2
                returns 4
            returns 8
        returns 16
    returns 32
returns 64
64
```

Number of invocations increases in proportion to **n** (i.e., linearly).

Fast Exponentiation

There is a straightforward way to speed this process:

If **`n`** is even, then square the result of raising **`b`** to the **`n/2`** power.

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ b \cdot b^{n-1} & \text{if } n \text{ odd} \\ \left(b^{n/2}\right)^2 & \text{otherwise} \end{cases}$$

functional notation:

$$f(n) = \begin{cases} 1 & \text{if } n = 0 \\ bf(n-1) & \text{if } n \text{ odd} \\ (f(n/2))^2 & \text{otherwise} \end{cases}$$

Fast Exponentiation

Implement in Python.

```
def fastExponent(b,n):
    if n==0:
        return 1
    elif n%2==1:
        return b*fastExponent(b,n-1)
    else:
        return fastExponent(b,n/2)**2
```

Check Yourself

```
def fastExponent(b,n):
    if n==0:
        return 1
    elif n%2==1:
        return b*fastExponent(b,n-1)
    else:
        return fastExponent(b,n/2)**2
```

How many invocations of **`fastExponent`** is generated by **`fastExponent(2,10)`**?

1. 10
2. 8
3. 7
4. 6
5. 5

Recursive Exponentiation

Implement recursion in Python.

```
def fastExponent(b,n):
    if n==0:
        return 1
    elif n%2==1:
        return b*fastExponent(b,n-1)
    else:
        return fastExponent(b,n/2)**2

fastExponent(2,10)
    calls fastExponent(2,5)
        calls fastExponent(2,4)
            calls fastExponent(2,2)
                calls fastExponent(2,1)
                    calls fastExponent(2,0)
                    returns 1
                returns 2
            returns 4
        returns 16
    returns 32
returns 1024
1024
```

The number of calls increases in proportion to **`log n`** (for large **`n`**).

Check Yourself

```
def fastExponent(b,n):
    if n==0:
        return 1
    elif n%2==1:
        return b*fastExponent(b,n-1)
    else:
        return fastExponent(b,n/2)**2
```

How many invocations of **`fastExponent`** is generated by **`fastExponent(2,10)`**? 5

1. 10
2. 8
3. 7
4. 6
5. 5

Recursive Exponentiation

Functional approach makes this simplification easy to spot.

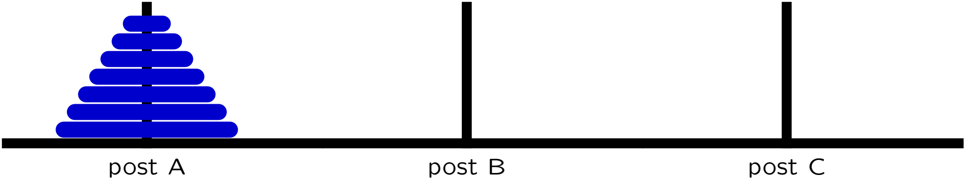
```
def fastExponent(b,n):
    if n==0:
        return 1
    elif n%2==1:
        return b*fastExponent(b,n-1)
    else:
        return fastExponent(b,n/2)**2

fastExponent(2,10)
    calls fastExponent(2,5)
        calls fastExponent(2,4)
            calls fastExponent(2,2)
                calls fastExponent(2,1)
                    calls fastExponent(2,0)
                    returns 1
                returns 2
            returns 4
        returns 16
    returns 32
returns 1024
1024
```

Functional approach is "expressive."

Towers of Hanoi

Transfer a stack of disks from post A to post B by moving the disks one-at-a-time, without placing any disk on a smaller disk.




```
def Hanoi(n,A,B,C):
  if n==1:
      print 'move from ' + A + ' to ' + B
  else:
      Hanoi(n-1,A,C,B)
      Hanoi(1,A,B,C)
      Hanoi(n-1,C,B,A)
```

Towers of Hanoi

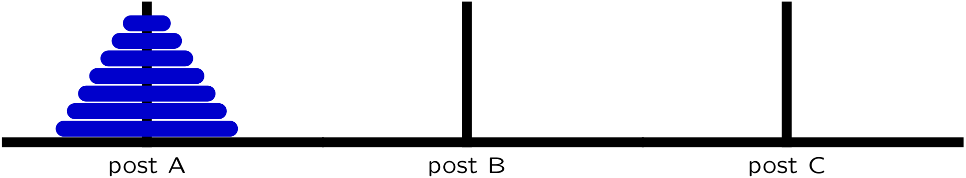
Towers of height 3 and 4.

```
> > > Hanoi(3,'a','b','c')
move from a to b
move from a to c
move from b to c
move from a to b
move from c to a
move from c to b
move from a to b

> > > Hanoi(4,'a','b','c')
move from a to c
move from a to b
move from c to b
move from a to c
move from b to a
move from b to c
move from a to c
move from a to b
move from c to b
move from c to a
move from b to a
move from c to b
move from a to c
move from a to b
move from c to b
```

Towers of Hanoi

Transfer a stack of disks from post A to post B by moving the disks one-at-a-time, without placing any disk on a smaller disk.




```
def Hanoi(n,A,B,C):
  if n==1:
       print 'move from ' + A + ' to ' + B
  else:
       Hanoi(n-1,A,C,B)
       Hanoi(1,A,B,C)
       Hanoi(n-1,C,B,A)
```

Recursive solution is "expressive" (also simple and elegant).

Back to the Earlier Example

Task: Find a sequence of operations (either **`increment`** or **`square`**) that transforms the integer i (initial) to the integer g (goal).

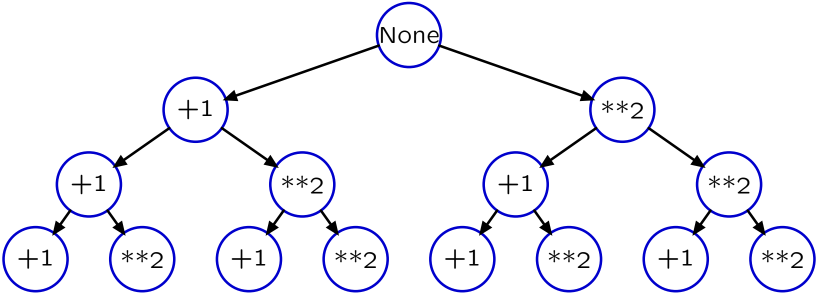
Imperative (procedural) approach $\surd$

Functional approach $\surd$

Object-oriented approach

OOP

Represent all possible sequences in a tree.



Define an object to repesent each of these "nodes":

```
class Node:
    def __init__(self,parent,action,answer):
        self.parent = parent
        self.action = action
        self.answer = answer
    def path(self):
        if self.parent == None:
            return [(self.action, self.answer)]
        else:
            return self.parent.path() + [(self.action,self.answer)]
```

OOP

Systematically create and search through all possible **Nodes**

```
def findSequence(initial,goal):
    q = [Node(None,None,1)]
    while q:
        parent = q.pop(0)
        for (a,r) in [('increment',increment),('square',square)]:
            newNode = Node(parent,a,r(parent.answer))
            if newNode.answer==goal:
                return newNode.path()
            else:
                q.append(newNode)
    return None

answer = findSequence(1,100)
print 'answer =',answer
```

answer = [(None, 1), ('increment', 2), ('increment', 3), ('square', 9), ('increment', 10), ('square', 100)]

Focus on constructing objects that represent pieces of the solution.

More later, when we focus on effective **search** strategies.

Programming Styles for Managing Complexity

Task: Find a sequence of operations (either **`increment`** or **`square`**) that transforms the integer i (initial) to the integer g (goal).

Imperative (procedural) approach

- structure of search was embedded in loops

Functional approach

- structure of search was constructed in lists of functions

Object-oriented approach

- structure of search was constructed from objects

Structure of program has significant effect on its modularity.

Now consider abstractions at even higher levels.