AI1103 : Assignment 5

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Download latex-tikz codes from

https://github.com/cs20btech11007/Assigment−5/blob/main/Assignment%205.tex

Problem-(CSIR UGC NET EXAM) (Dec-2014),Q-116

Let Y follows multivariate normal distribution $N_n(0, I)$ and let A and B be a $n \times n$ symmetric,idempotent matrices. Then which of the following statements are true?

1.if $AB = 0$,then Y'AY and Y'BY are independently distributed.

2.if Y'(A+B)Y has chi square distribution then Y'AY, Y'BY are independently distributed.

3.Y'(A-B)Y has chi square distribution.

4.Y'AY, Y'BY has chi square distribution.

Solution

1.) $Y = (Y_1, Y_2, Y_3....., Y_n)$ has multivariate normal distribution with mean $\mu = 0$ and variance-covariance matrix $\sum = 0$.
let us assume Y'A_1Y is distributed as χ^2 with n_1 degrees of freedom.
let us assume Y'A_2Y is distributed as χ^2 with n_2 degrees of freedom.
Then Y'A_1Y and Y'A_2Y are independently distributed if $A_1 \sum A_2$=0.

2.) if Y'(A+B)Y has chi square distribution then $(\sum(A + B)$ must be a idempotent matrix.

$$(\sum(A + B))^2 = \sum(A + B) \tag{1}$$

We know that $\sum = I$ (identity matrix)

$$A^2 + B^2 + AB + BA = A + B \tag{2}$$

we know that A and B are idempotent matrices.

$$A^2 = A, B^2 = B \tag{3}$$

$$\cancel{A} + \cancel{B} + AB + BA = \cancel{A} + \cancel{B} \tag{4}$$

$$AB + BA = 0 \tag{5}$$

Then AB=BA=0,
Y'AY and Y'BY are independently distributed if $A \sum B$=0.
We know that $\sum = I$ (identity matrix)
so,AB=0

3.) we cannot say that Y'(A-B)Y has chi square distribution because we don't know about the nature of the $\sum(A - B)$ matrix .

4.$Y = (Y_1, Y_2, Y_3....., Y_n)$ has multivariate normal distribution with mean $\mu = 0$ and variance-covariance matrix $\sum = I$.
Then Y'AY and Y'BY has chi square distribution(χ^2) with k degree of freedom if and only if $\sum A$ and $\sum B$ are idempotent matrix.

we know that A and B are idempotent matrices.

$$A^2 = A, B^2 = B \tag{6}$$

We know that $\sum = I$ (identity matrix)

$$(\sum A)^2 = A, (\sum B)^2 = B \tag{7}$$

Hence ,$(\sum A), (\sum B)$ are idempotent so,they have chi square distribution.