

AI1103 : Assignment 5

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Download latex-tikz codes from

https://github.com/cs20btech11007/Assignment-5/blob/main/Assignment_5.tex

PROBLEM-(CSIR UGC NET EXAM) (DEC-2014),Q-116

Let Y follows multivariate normal distribution $N_n(0, I)$ and let A and B be a $n \times n$ symmetric, idempotent matrices. Then which of the following statements are true?

1. if $AB = 0$, then $Y'AY$ and $Y'BY$ are independently distributed.
2. if $Y'(A+B)Y$ has chi square distribution then $Y'AY$, $Y'BY$ are independently distributed.
3. $Y'(A-B)Y$ has chi square distribution.
4. $Y'AY$, $Y'BY$ has chi square distribution.

SOLUTION

1. let us consider $X_1 = Y'AY$ and $X_2 = Y'BY$.
 X_1, X_2 are said to be independently distributed if and only if $A \sum B = 0$.
 given $\mu = 0$, $\sum = I$ (identity matrix).
 so, $A \sum B = AB = 0$.
2. if $Y'(A+B)Y$ has chi square distribution then $Y'AY$, $Y'BY$ distribution must be independent.

$$Y'(A + B)Y = Y'AY + Y'BY \quad (1)$$

3. if $Y'(A-B)Y$ has chi square distribution then $Y'AY$, $Y'BY$ distribution must be independent.

$$Y'(A - B)Y = Y'AY - Y'BY \quad (2)$$

4. the necessary condition for the $Y'AY$, $Y'BY$ chi square distributed is $A^2 = A$, $B^2 = B$ and we know that A , B are idempotent matrices.

so, 1, 2, 4 are the true statements.