AI1103 : Assignment 5

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Download latex-tikz codes from

https://github.com/cs20btech11007/Assignment−5/blob/main/Assignment%205.tex

Problem-(CSIR UGC NET EXAM) (Dec-2014),Q-116

Let Y follows multivariate normal distribution $N_n(0, I)$ and let A and B be a $n \times n$ symmetric,idempotent matrices. Then which of the following statements are true?

1.if $AB = 0$,then Y'AY and Y'BY are independently distributed.

2.if Y'(A+B)Y has chi square distribution then Y'AY, Y'BY are independently distributed.

3.Y'(A-B)Y has chi square distribution.

4.Y'AY, Y'BY has chi square distribution.

Solution

1.) $Y = (Y_1, Y_2, Y_3....., Y_n)$ has multivariate normal distribution with mean $\mu = 0$ and variance-covariance matrix $\sum = I$.
The distributions Y'AY and Y'BY are independently distributed if and only if $A \sum B$=0.

We know that $\sum = I$ (identity matrix)

$$A \sum B = AB = 0 \tag{1}$$

clearly we can say that the distributions Y'A_1Y and Y'A_2Y are independently distributed .

2.) if Y'(A+B)Y has chi square distribution,then $(\sum(A + B)$ must be an idempotent matrix.

$$(\sum(A + B))^2 = \sum(A + B) \tag{2}$$

We know that $\sum = I$ (identity matrix)

$$A^2 + B^2 + AB + BA = A + B \tag{3}$$

Given that A and B are idempotent matrices.

$$A^2 = A, B^2 = B \tag{4}$$

$$\cancel{A} + \cancel{B} + AB + BA = \cancel{A} + \cancel{B} \tag{5}$$

$$AB + BA = 0 \tag{5}$$

Then AB,BA must be equal to 0,
The distributions Y'A_1Y and Y'BY are independently distributed if $A \sum B$=0.

We know that $\sum = I$ (identity matrix)

$$A \sum B = AB = 0 \tag{6}$$

clearly we can say that the distributions Y'AY and Y'BY are independently distributed .

3.) For having chi square distribution for a normally distributed Y'(A-B)Y distribution .
Then the matrix $(\sum(A - B)$ must be an idempotent matrix.

$$(\sum(A - B))^2 = \sum(A - B) \tag{7}$$

We know that $\sum = I$ (identity matrix)

$$A^2 + B^2 - AB - BA = A - B \tag{8}$$

we know that A and B are idempotent matrices.

$$A^2 = A, B^2 = B \tag{9}$$

$$\cancel{A} + B - AB - BA = \cancel{A} + -B \tag{10}$$

$$AB + BA = 2B \tag{11}$$

for having chi square distribution the above equation(11) need to be satisfied.
but it is not mentioned in the option 3.
so,we cannot say that the distribution Y'(A-B)Y has chi square distribution.

4.) $Y = (Y_1, Y_2, Y_3....., Y_n)$ has multivariate normal distribution with mean $\mu = 0$ and variance-covariance matrix $\sum = I$.
Then Y'AY and Y'BY has chi square distribution(χ^2) if and only if $\sum A$ and $\sum B$ are idempotent matrix.

we know that A and B are idempotent matrices.

$$A^2 = A, B^2 = B \tag{12}$$

We know that $\sum = I$ (identity matrix)

$$(\sum A)^2 = A, (\sum B)^2 = B \tag{13}$$

Then Y'AY and Y'BY has chi square distribution(χ^2).

Statements 1,2,4 are true.