AI1103-Assignment 4

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Let X_1, X_2, X_3, X_4, X_5 be *i.i.d.* random variables having a continuous distribution function. Then

$$\Pr(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = max(X_1, X_2, X_3, X_4, X_5)$$

equals ____.

1) $\frac{1}{4}$
2) $\frac{1}{5}$
3) $\frac{1}{4!}$
4) $\frac{1}{5!}$

Solution

X_1, X_2, X_3, X_4 and X_5 are identical and independently distributed random variables, they can be represented by a single random variable X.
Let

$$\{X_1, X_2, X_3, X_4, X_5\} \in X$$

Required probability,

$$\Pr(X_1 > X_2 > X_3 > X_4 > X_5 | x_1 = max(X_1, X_2, X_3, X_4, X_5) \tag{1}$$

by applying conditional probability,

$$\Pr(X_1 > X_2 > X_3 > X_4 > X_5 | X_1) = \left(\frac{\Pr(X_1 > X_2 > X_3 > X_4 > X_5) \cap X_1))}{\Pr(X_1)} \right) \tag{0.0.1}$$

⇒the probability of getting X_1 maximum among X_1, X_2, X_3, X_4, X_5 is

$$\Pr(X_1) = \frac{1}{5}. \tag{0.0.2}$$

⇒the probability of getting X_2 2nd maximum among X_2, X_3, X_4, X_5 is

$$\Pr(X_2 > X_3 > X_4 > X_5) = \frac{1}{4}. \tag{0.0.3}$$

⇒the probability of getting X_3 3rd maximum among X_3, X_4, X_5 is

$$\Pr(X_3 > X_4 > X_5) = \frac{1}{3}. \tag{0.0.4}$$

⇒the probability of getting X_4 4rd maximum among X_4, X_5 is

$$\Pr(X_4 > X_5) = \frac{1}{2}. \tag{0.0.5}$$

⇒the probability of getting X_5 least $\Pr(X_5) = 1$.

we know that X_1, X_2, X_3, X_4, X_5 are independently distributed.
$\Pr(X_1 > X_2 > X_3 > X_4 > X_5) \cap X_1))$.

⇒ $\Pr(X_1) \times \Pr(X_2 > X_3 > X_4 > X_5) \Pr(X_4 > X_5) \times \Pr(X_5)$.

$\Pr(X_1 > X_2 > X_3 > X_4 > X_5) \cap X_1)) = \frac{1}{5!}$

∴ Required probability is,

$$= \frac{\frac{1}{5!}}{\frac{1}{5}} \tag{0.0.6}$$

$$= \frac{1}{4!} \tag{0.0.7}$$