

AI1103-Assignment 4

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Download latex-tikz codes from

<https://github.com/cs20btech11007/Challenging-Problem-3/blob/main/Challenging%20Problem%203.tex>

and python codes from

<https://github.com/cs20btech11007/Challenging-Problem-3/blob/main/Challenging%20Problem%203.py>

\Rightarrow the probability of getting X_1 maximum among X_1, X_2, X_3, X_4, X_5 is

$$\Pr(X_1) = \frac{1}{5}. \quad (0.0.2)$$

\Rightarrow the probability of getting X_2 2nd maximum among X_2, X_3, X_4, X_5 is

$$\Pr(X_2 > X_3 > X_4 > X_5) = \frac{1}{4}. \quad (0.0.3)$$

UGC/MATH 2019, Q.50

Let X_1, X_2, X_3, X_4, X_5 be *i.i.d.* random variables having a continuous distribution function. Then

$$\Pr(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = \max(X_1, X_2, X_3, X_4, X_5))$$

equals ____.

- 1) $\frac{1}{4}$
- 2) $\frac{1}{5}$
- 3) $\frac{1}{4!}$
- 4) $\frac{1}{5!}$

\Rightarrow the probability of getting X_3 3rd maximum among X_3, X_4, X_5 is

$$\Pr(X_3 > X_4 > X_5) = \frac{1}{3}. \quad (0.0.4)$$

\Rightarrow the probability of getting X_4 4rd maximum among X_4, X_5 is

$$\Pr(X_4 > X_5) = \frac{1}{2}. \quad (0.0.5)$$

\Rightarrow the probability of getting X_5 least $\Pr(X_5) = 1$.

we know that X_1, X_2, X_3, X_4, X_5 are independently distributed.

$$\Pr(X_1 > X_2 > X_3 > X_4 > X_5 \cap X_1)).$$

$$\Rightarrow \Pr(X_1) \times \Pr(X_2 > X_3 > X_4 > X_5) \Pr(X_4 > X_5) \times \Pr(X_5).$$

$$\Pr(X_1 > X_2 > X_3 > X_4 > X_5 \cap X_1)) = \frac{1}{5!}$$

\therefore Required probability is,

$$= \frac{\frac{1}{5!}}{\frac{1}{5}} \quad (0.0.6)$$

$$= \frac{1}{4!} \quad (0.0.7)$$

Required probability,

$$\Pr(X_1 > X_2 > X_3 > X_4 > X_5 | x_1 = \max(X_1, X_2, X_3, X_4, X_5)) \quad (1)$$

by applying conditional probability,

$$\Pr(X_1 > X_2 > X_3 > X_4 > X_5 | X_1) = \left(\frac{\Pr(X_1 > X_2 > X_3 > X_4 > X_5 \cap X_1))}{\Pr(X_1)} \right) \quad (0.0.1)$$

SOLUTION

X_1, X_2, X_3, X_4 and X_5 are identical and independently distributed random variables, they can be represented by a single random variable X .

Let

$$\{X_1, X_2, X_3, X_4, X_5\} \in X$$