Assignment 2

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1 Problem

$X(t)$ is a random process with a constant mean value of 2 and the autocorrelation function:

$$R_x(\tau) = 4\left[e^{-0.2|\tau|} + 1\right] \tag{1.0.1}$$

Let X be the Gaussian Random Variable obtained by sampling the process at $t = t_i$, and let

$$Q(\alpha) = \int_{\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}}\, dy \tag{1.0.2}$$

Find $\Pr(X \le 1)$

(A) $1 - Q(0.5)$ (B) $Q(0.5)$

(C) $Q(\frac{1}{2\sqrt{2}})$ (D) $1 - Q(\frac{1}{2\sqrt{2}})$

2 Solution

X is a normal random variable defined by

$$X \sim N\left(2, \sigma_x^2\right) \tag{2.0.1}$$

Thus, from (1):

$$Var(X) = \sigma_x^2 = R_x(0) \tag{2.0.2}$$
$$\sigma_x^2 = 8 \tag{2.0.3}$$
$$\sigma_x = 2\sqrt{2} \tag{2.0.4}$$

Converting X to a standard normal random variable using :

$$Z = \frac{X - \mu_x}{\sigma_x} \tag{2.0.5}$$
$$\Pr(X \le 1) \tag{2.0.6}$$
$$= \Pr\left(\frac{X-2}{2\sqrt{2}} \le \frac{1-2}{2\sqrt{2}}\right) \tag{2.0.7}$$
$$= \Pr\left(Z \le \frac{-1}{2\sqrt{2}}\right) \tag{2.0.8}$$

where Z is a standard normal random variable defined by $Z \sim N(0,1)$

Due to symmetry of the bell curve graph:

$$\Pr\left(Z \le \frac{-1}{2\sqrt{2}}\right) = \Pr\left(Z \ge \frac{1}{2\sqrt{2}}\right) \tag{2.0.9}$$

From (1.0.2),

$$Q(\alpha) = \int_{\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}\, dz = \Pr(Z \ge \alpha) \tag{2.0.10}$$

Thus,

$$\Pr\left(Z \ge \frac{1}{2\sqrt{2}}\right) = Q\left(\frac{1}{2\sqrt{2}}\right) \tag{2.0.11}$$