Online Assignment 2

Due: 2:00am on Saturday, September 13, 2014

To understand how points are awarded, read the Grading Policy for this assignment.

Electric Potential and Potential Energy

A particle with charge 6.40×10^{-19}C is placed on the *x* axis in a region where the electric potential due to other charges increases in the +*x* direction but does not change in the *y* or *z* direction.

Part A

The particle, initially at rest, is acted upon only by the electric force and moves from point *a* to point *b* along the *x* axis, increasing its kinetic energy by 1.60×10^{-19}J . In what direction and through what potential difference $V_b - V_a$ does the particle move?

Hint 1. How to approach the problem

Because no forces other than the electric force act on the particle, the positively charged particle must move in the direction parallel to the electric field, and the field must do positive work on the particle. Recall that when the electric field does positive work on a charged particle, the potential energy of the particle decreases. Thus, the particle must move in the direction in which its potential energy decreases (which is consistent with the fact that the particle's kinetic energy increases as it moves from *a* to *b*). Moreover, from the definition of potential and the energy conservation equation, you can directly calculate the potential difference $V_a - V_b$.

Hint 2. Electric potential

The electric potential V at any point in an electric field is the electric potential energy U per unit charge associated with a test charge q' at that point:

$$V = \frac{U}{q'}.$$

Hint 3. Find the change in potential energy of the particle

What is the change in potential energy of the particle, $U_b - U_a$, as it moves from *a* to *b*?

Express your answer in joules.

Hint 1. Energy conservation

Recall that the total mechanical energy (kinetic plus potential) is conserved. That is,

$$K_a + U_a = K_b + U_b,$$

where the subscripts refer to points *a* and *b*, and K and U are the corresponding kinetic and potential energies.

Hint 2. Find the change in kinetic energy of the particle

What is the change in kinetic energy of the particle, $K_b - K_a$, as it moves from *a* to *b*? Recall that particle is initially at rest, and its kinetic energy at *b* is 1.60×10^{-19}J .

Express your answer in joules.

ANSWER:

$K_b - K_a$ = 1.60×10^{-19} J

ANSWER:

$U_b - U_a$ = -1.60×10^{-19} J

ANSWER:

- ○ The particle moves to the left through a potential difference of $V_b - V_a = 0.250\text{V}$.
- ◉ The particle moves to the left through a potential difference of $V_b - V_a = -0.250\text{V}$
- ○ The particle moves to the right through a potential difference of $V_b - V_a = 0.250\text{V}$.
- ○ The particle moves to the right through a potential difference of $V_b - V_a = -0.250\text{V}$.
- ○ The particle moves to the left through a potential difference of $V_b - V_a = 2.50\text{V}$.
- ○ The particle moves to the right through a potential difference of $V_b - V_a = -2.50\text{V}$.

Correct

In general, if no forces other than the electric force act on a positively charged particle, the particle always moves toward a point at lower potential.

Part B

If the particle moves from point *b* to point *c* in the *y* direction, what is the change in its potential energy, $U_c - U_b$?

Hint 1. How to approach the problem

Recall that the electric potential increases in the +*x* direction but does not change in the *y* or *z* direction.

ANSWER:

- ○ $+ 1.60 \times 10^{-19}\text{J}$
- ○ $- 1.60 \times 10^{-19}\text{J}$
- ◉ 0

Correct

Every time a charged particle moves along a line of constant potential, its potential energy remains constant and the electric field does no work on the particle.

Moving a Charge

Part A

A point charge with charge q_1 = 2.50$\mu\mathrm{C}$ is held stationary at the origin. A second point charge with charge q_2 = -5.00$\mu\mathrm{C}$ moves from the point (0.140m , 0) to the point (0.245m , 0.265m). How much work W is done by the electric force on the moving point charge?

Express your answer in joules. Use k = 8.99×10^9$\mathrm{N\cdot m^2/C^2}$ for Coulomb's constant: $k = \frac{1}{4\pi\epsilon_0}$.

Hint 1. How to approach the problem

Use the equation for the electric potential energy between two point charges to calculate the work done by the electric force. Recall that the work done by a *conservative* force is $W = U_\mathrm{i} - U_\mathrm{f} = -\Delta U$, the difference between the initial and final potential energies. A conservative force is one for which the work done on a particle by the force is independent of the path taken and depends only on the initial and final points. The electric force is a conservative force. Gravity is another example of a conservative force.

Hint 2. Calculate the initial electric potential energy

Calculate the initial electric potential energy U_i when the moving point charge is at the point (0.140m , 0).

Express your answer in joules. Use k = 8.99×10^9$\mathrm{N\cdot m^2/C^2}$ for Coulomb's constant: $k = \frac{1}{4\pi\epsilon_0}$.

Hint 1. Derivation of electric potential energy

The force between two point charges q and Q is given by Coulomb's law as $F_r = kqQ/r^2$, where r is the separation between the charges and $k = 1/4\pi\epsilon_0$. The work done by the electric force between the charges as one charge moves from point a to point b and the other is held fixed is calculated using $W_{ab} = \int_a^b \vec{F}\cdot d\vec{l}$. Since the force depends only on the distance between the charges, it follows that

$$W_{ab} = \int_{r_a}^{r_b} \frac{kqQ}{r^2}\,dr = kqQ\left(\frac{1}{r_a} - \frac{1}{r_b}\right),$$

where r_a and r_b are the distances between the fixed charge and points a and b, respectively. Since the work done is equal to the change in potential energy, this equation is consistent with defining the electric potential energy between two point charges a distance r apart by $U = \frac{kqQ}{r}$.

ANSWER:

U_i = -0.803 J

Hint 3. Calculate the final electric potential energy

Calculate the final electric potential energy U_f when the moving charge is at the point (0.245m , 0.265m).

Express your answer in joules. Use k = 8.99×10^9$\mathrm{N\cdot m^2/C^2}$ for Coulomb's constant: $k = \frac{1}{4\pi\epsilon_0}$.

Hint 1. Derivation of electric potential energy

The force between two point charges q and Q is given by Coulomb's law as $F_r = kqQ/r^2$, where r is the separation between the charges and $k = 1/4\pi\epsilon_0$. The work done by the electric force between the charges as one charge moves from point a to point b and the other is held fixed is calculated using

$W_{ab} = \int_a^b \vec{F} \cdot d\vec{l}$. Since the force depends only on the distance between the charges, it follows that

$$W_{ab} = \int_{r_a}^{r_b} \frac{kqQ}{r^2}\, dr = kqQ\left(\frac{1}{r_a} - \frac{1}{r_b}\right),$$

where r_a and r_b are the distances between the fixed charge and points a and b, respectively. Since the work done is equal to the change in potential energy, this equation is consistent with defining the electric potential energy between two point charges a distance r apart by $U = \frac{kqQ}{r}$.

ANSWER:

U_f = -0.311 J

ANSWER:

W = -0.491 J

Correct

Finding the Capacitance

A parallel-plate capacitor is filled with a dielectric whose dielectric constant is K, increasing its capacitance from C_1 to KC_1. A second capacitor with capacitance C_2 is then connected in series with the first, reducing the net capacitance back to C_1.

Part A

What is the capacitance C_2 of the second capacitor?

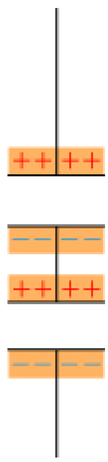
Express your answer in terms of K, C_1, and constants.

Hint 1. What do capacitors in series have in common?

Which of the following quantities is necessarily the same for capacitors connected in series?

Hint 1. Conservation laws

Consider the following system: the negative plate of the first capacitor, the positive plate of the second capacitor, and the wire that connects them. Initially, before the capacitors are charged, the system has no net charge. This system is isolated, so charge must be conserved within it. What, then, must be the relationship between the charge on the negative plate of the first capacitor and the charge on the positive plate of the second capacitor?



ANSWER:

- Potential
- Charge (selected)
- Capacitance
- Area

Hint 2. Find the net capacitance

The potential drop V_{net} across the two capicators is the sum of the potential drops V_1 and V_2 across each capacitor separately. Using the relation $V = Q/C$ from the definition of capacitance, find an expression for the net capacitance C_{net} of the series.

Express your answer in terms of K, C_1, and the unknown capacitance C_2.

Hint 1. How to approach the problem

If the charge on each capacitor is Q, then the net capacitance C_{net} relates to the total voltage as

$$C_{\mathrm{net}} = \frac{Q}{V},$$

where, as mentioned in the question, $V = V_1 + V_2$. In turn,

$$KC_1 = \frac{Q}{V_1} \text{ and } C_2 = \frac{Q}{V_2}.$$

Use these equations to find a relation for C_{net} in terms of K, C_1, and C_2.

ANSWER:

$C_{\mathrm{net}} = C_1 = \frac{KC_1C_2}{KC_1+C_2}$

ANSWER:

$C_2 = \frac{KC_1}{K-1}$

Correct

As a check that this result makes sense, note that if $K \gg 1$, which corresponds to the first capacitor being replaced by a conductor, then $C_2 \approx C_1$. In the limit as K approaches 1 (but is still greater than 1), C_2 becomes very large, effectively making the second capacitor a conductor. This is also expected, since if $K = 1$, then no change was made to the first capacitor.

± Capacitance and Electric Field of a Spherical Capacitor

A spherical capacitor is formed from two concentric spherical conducting shells separated by vacuum. The inner sphere has a radius of $r_{\rm a}$ = 12.2$\rm cm$, and the outer sphere has a radius of $r_{\rm b}$ = 15.0$\rm cm$. A potential difference of 120$\rm V$ is applied to the capacitor.

Part A

What is the capacitance of the capacitor?

Use ϵ_0 = 8.85×10^{-12}$\rm F/m$ for the permittivity of free space.

Hint 1. How to approach the problem

Since a spherical capacitor is used, the equations for a parallel-plate capacitor are not valid, and the correct equations need to be determined using Gauss's law. Find an expression for the electric field between the charged spheres; then use this to calculate the potential; and finally, use the potential to calculate the capacitance.

Hint 2. Which Gaussian surface to use

Since the capacitor has spherical symmetry, use a sphere of radius r, where $r_{\rm a} < r < r_{\rm b}$, as the Gaussian surface. That way, by symmetry, the electric field at any point on the surface will be constant in magnitude and parallel to the normal surface vector at that point. Also, note that for $r < r_{\rm a}$, there is no enclosed charge, since the charge rests only on the surface of the inner sphere, and for $r > r_{\rm b}$, the total enclosed charge is $+Q - Q = 0$, since the inner and outer spheres have equal but opposite charges. This means that there will be no electric field outside the capacitor, so only the inner region will contribute to the capacitance.

Hint 3. Evaluate the integral in Gauss's law

For our system, Gauss's law states that $\oint_S \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$, where Q is the charge on the inner sphere, enclosed by our Gaussian surface S. Which of the following expressions for the integral on the left side of the equation is correct?

Hint 1. Calculating the dot product

Since the Gaussian surface is a sphere, by symmetry the electric field will be normal (and pointing outward) to the surface at all points on that surface, so $\vec{E} \cdot d\vec{A} = E\,dA$, where E is the magnitude of the electric field in the radial direction on the surface. Since we know that the Gaussian sphere chosen has a constant radius, E will be a constant over the surface of integration.

ANSWER:

- ○ $\pi r^2 E$
- ◉ $4\pi r^2 E$
- ○ πE
- ○ $\frac{E}{4\pi}$
- ○ $\frac{E}{4\pi r}$
- ○ $\frac{E}{\pi r^2}$

Hint 4. Find an expression for the potential difference

Which of the following is a correct expression for the potential difference $V_{\rm ab}$ between the inner and outer spheres?

Hint 1. Integrating to find the potential difference

To find the potential difference $V_{\rm ab} = V_{\rm a} - V_{\rm b}$ as a function of the electric field $\vec{E}$, calculate the integral $\int_a^b \vec{E} \cdot d\vec{L}$, where $d\vec{L}$ represents the path from a point a on the inner sphere to a point b on the outer sphere. For convenience, take the path to be purely radial, since the electric field is already pointing radially outward. Since the field points in the same direction as the chosen path, $\int_a^b \vec{E} \cdot d\vec{l} = \int_{r_{\rm a}}^{r_{\rm b}} E\,dL$.

ANSWER:

- (selected) $\frac{Q}{4\pi\epsilon_0}\frac{r_{\rm b}-r_{\rm a}}{r_{\rm a}r_{\rm b}}$
- $\frac{Q}{4\pi\epsilon_0}\frac{r_{\rm a}-r_{\rm b}}{r_{\rm a}r_{\rm b}}$
- $\frac{Q}{2\pi\epsilon_0}\frac{r_{\rm b}-r_{\rm a}}{r_{\rm a}r_{\rm b}}$
- $\frac{Q}{2\pi\epsilon_0}\frac{r_{\rm a}-r_{\rm b}}{r_{\rm a}r_{\rm b}}$

V

Hint 5. Definiton of capacitance

Recall that the amount of charge Q that a capacitor with voltage V across it can hold on each plate is given by its capacitance C, defined as

$$C = \frac{Q}{V}.$$

ANSWER:

C = 7.27×10^{-11} F

Correct

Part B

What is the magnitude E_1 of the electric field $\vec{E}$ at radius $r = 12.8 \rm cm$, just outside the inner sphere?

Hint 1. How to approach the problem

Using the expression for the potential difference from Part A.4, find the charge on the inner sphere; then use this value to calculate the field at the given radial distance.

Hint 2. Calculate the charge on the inner sphere

Calculate the charge Q on the inner sphere from the equation found for the potential difference between the inner and outer surfaces of the spherical capacitor.

Express your answer in coulombs

ANSWER:

Q = 8.72×10^{-9} C

ANSWER:

E_1 = 4790 $\mathrm{V/m}$

Correct

Part C

What is the magnitude of $\vec{E}$ at $r = 14.7\mathrm{cm}$, just inside the outer sphere?

Hint 1. How to approach the problem

Using the expression for the potential difference from Part A.4, find the charge on the inner sphere; then use this value to calculate the field at the given radial distance.

ANSWER:

E_2 = 3630 $\mathrm{V/m}$

Correct

In a parallel-plate capacitor, the field between the plates is uniform. In a spherical capacitor, however, the field is a function of the radial distance from the center, so the field is not uniform between the spheres.

Exercise 24.28 - 1

A capacitor of capacitance C is charged to a potential difference V_0. The terminals of the charged capacitor are then connected to those of an uncharged capacitor of capacitance $C/2$.

Part A

Compute the original charge of the system.

Express your answer in terms of the variables C and V_0.

ANSWER:

Q = CV_0

Correct

Part B

Compute the final potential difference across C.

Express your answer in terms of the variables C and V_0.

ANSWER:

V = $\frac{2}{3}V_0$

Correct

Part C

Compute the final potential difference across $C/2$.

Express your answer in terms of the variables C and V_0.

ANSWER:

V = $\frac{2}{3}V_0$

Correct

Part D

Compute the final energy of the system.

Express your answer in terms of the variables C and V_0.

ANSWER:

U = $\frac{1}{3}C{V_0}^2$

Correct

Part E

Compute the decrease in energy when the capacitors are connected.

Express your answer in terms of the variables C and V_0.

ANSWER:

U = $\frac{1}{6}C{V_0}^2$

Correct

Score Summary:

Your score on this assignment is 104%.
You received 52.17 out of a possible total of 50 points.