Functional Programming in F*

KC Sivaramakrishnan

Spring 2025

IIT MADRAS

Program verification
Shall the twain ever meet?

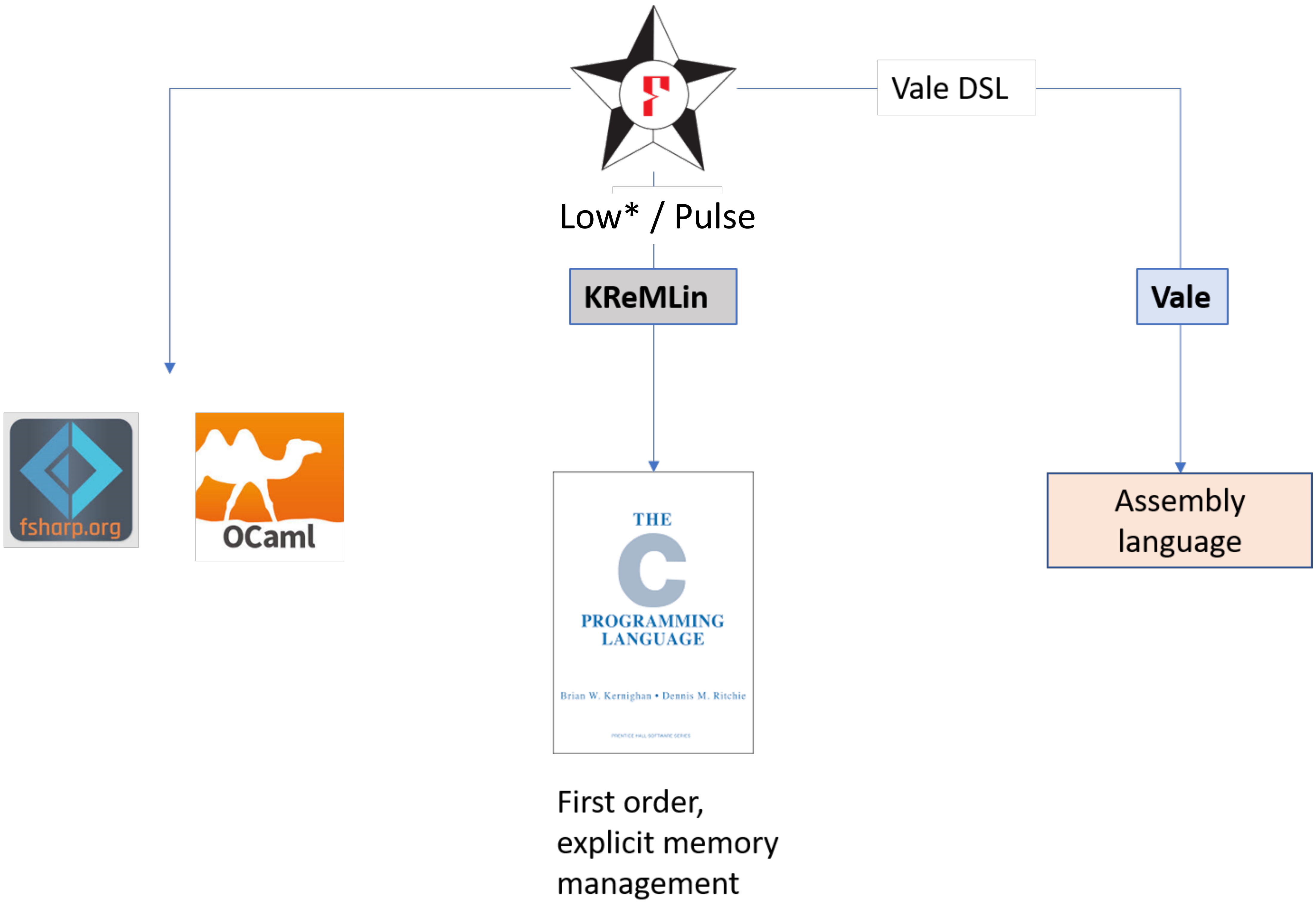
Interactive proof assistants		Semi-automated verifiers of imperative programs
Coq,	*air*	Dafny,
Agda,		FramaC,
Lean,	*gap*	Why3,
Isabelle		Liquid Types

- **Left corner:** very expressive higher-order logics, interactive proving, tactics, but mostly only purely functional programming
- **Right corner:** effectful programming, SMT-based automation, but only very weak logics

Bridging the Gap: F*

- Functional programming language with effects
  - ✦ Like OCaml, Haskell, F#,…
- Semi-automated verification system using SMT
  - ✦ Like Dafny, Frama-C, Why3, Liquid Types,…
- Expressive core language based on dependent type theory
  - ✦ Like Coq, NuPRL, Agda, Lean,…
- A metaprogramming and tactic framework for interactive proof and user-defined automation
  - ✦ Like Coq, NuPRL, Isabelle, HOL, Lean, …

Running F* programs



Uses of F* in Project Everest

- **Project Everest:** verify and deploy new, efficient HTTPS stack
  - **miTLS:** Verified reference implementation of TLS (1.2 and 1.3)
  - **EverParse:** Verified parsers and formatter generators
  - **EverCrypt:** Agile Cryptographic Provider
  - **HACL*:** High-Assurance Cryptographic Library
  - **Vale:** Verified Assembly Language for Everest
- Verified Everest code deployed in
  - Firefox (Mozilla NSS crypto, HACL*)
  - Windows (WinQUIC, EverParse)
  - Azure Confidential Consortium (Verified Merkle tree for the blockchain), WireGuard VPN, Zinc crypto library for Linux, Tezos and Concordium blockchains

Switch to Code

Refinement Types

```
type nat = x:int{x >= 0}
```

- Refinements introduced by type annotations (code unchanged)

```
val factorial : nat -> nat
let rec factorial n = if n = 0 then 1 else n * (factorial (n - 1))
```

- Logical obligations discharged by SMT (for else branch, simplified)

```
n >= 0, n <> 0 |= n - 1 >= 0
n >= 0, n <> 0, (factorial (n - 1)) >= 0 |= n * (factorial (n - 1)) >= 0
```

Refinement Types

- Refinements eliminated by **subtyping**: `nat <: int`

```
let i : int = factorial 42

let f : x:nat{x > 0} -> int = factorial
```

$$\frac{\texttt{A' <: A} \qquad \texttt{B <: B'}}{\texttt{A -> B <: A' -> B'}}$$

$$\frac{\texttt{x:nat\{x > 0\} <: nat} \qquad \texttt{nat <: int}}{\texttt{nat -> nat <: x:nat\{x > 0\} -> int}}$$

Switch to Code

Total Functions

- The F* functions we saw so far were all **total**
- `Tot` effect (default) = no side-effects, terminates on all inputs

```
val factorial : nat -> Tot nat
let rec factorial n =
  if n = 0 then 1 else n * (factorial (n - 1))
```

- **Quiz:** How about giving this weaker type to factorial?

```
val factorial : int -> Tot int
```

```
let rec factorial n = if n = 0 then 1 else n * (factorial (n - 1))
                                                             ^^^^^
Subtyping check failed; expected type (x:int{(x << n)}); got type int
```

`factorial (-1)` loops! (`int` type in F* is unbounded)

Switch to Code

Semantic Termination Checking

- based on **well-founded ordering on expressions** (<<)
  - naturals related by < (negative integers unrelated)
  - inductives related by subterm ordering
  - lex tuples %[a;b;c] with lexicographic ordering
- order constraints discharged by the SMT solver
- arbitrary total expression as **decreases metric**

```
val ackermann: m:nat -> n:nat -> Tot nat (decreases %[m;n])
let rec ackermann m n =
  if m = 0 then n + 1
  else if n = 0 then ackermann (m - 1) 1
  else ackermann (m - 1) (ackermann m (n - 1))
```

- default metric is lex ordering of all (non-function) args

```
val ackermann: m:nat -> n:nat -> Tot nat
```

Switch to Code

Divergence Effect (Dv)

- We might not want to prove that our code always terminates

```
val factorial : int -> Dv int
```

- Some useful code might not always terminate
  - ✦ An evaluator for lambda expressions

```
val eval : exp -> Dv exp
let rec eval e =
  match e with
  | App (Lam x e1) e2 -> eval (subst x e2 e1)
  | App e1 e2         -> eval (App (eval e1) e2)
  | Lam x e1          -> Lam x (eval e1)
  | _                 -> e

let main () = eval (App (Lam 0 (App (Var 0) (Var 0)))
                        (Lam 0 (App (Var 0) (Var 0))))
```

  - ✦ A webserver

- In Coq, we had to write such code as an inductive type and not a Fixpoint

Switch to Code

Effect System

- Pure code cannot call potentially divergent code
- Only pure code can appear in specifications

```
val factorial : int -> Dv int

type tau = x:int{x = factorial (-1)}
```

```
type tau = x:int{x = factorial (-1)}
                 ^^^^^^^^^^^^^^^^^^
Expected a pure expression; got an expression ... with effect "DIV"
```

- **Sub-effecting:** `Tot t <: Dv t`
- So, divergent code can include pure code

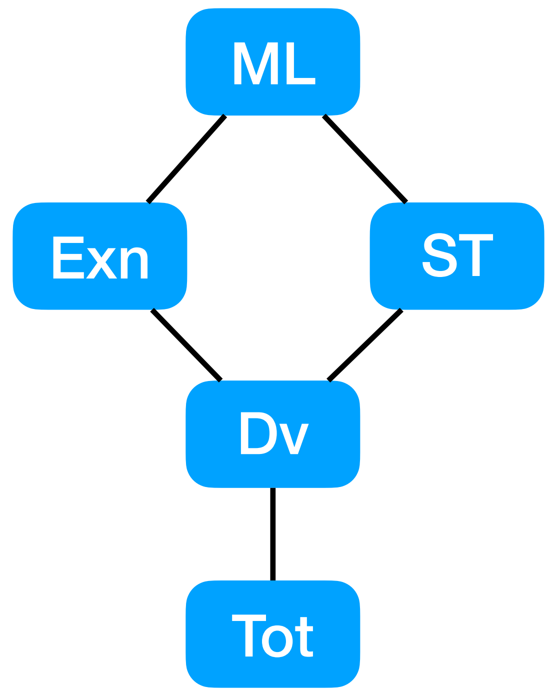
```
incr 2 + factorial (-1) : Dv int
```

Switch to Code

Effect System : Other effects

- Tot and Dv are just two of the possible effects. Some others include:
  - ST — the effect of a computation that may diverge, read, write or allocate new references in the heap
  - Exn — the effect of a computation that may diverge or raise an exception
  - ML — the effect of a computation that may diverge, read, write or allocate, or raise an exception

Effect System: Lattice



- **Sub-effecting:** Computations at a particular effect can be considered to be computations at an effect higher up the lattice

```
let baz () : Dv int = incr 2 + factorial3 (-1)
```

Switch to Code Example

Fin!